Axial acoustic field barrier for fluidic particle manipulation

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An acoustic field barrier integrated within a flow tubing system to achieve high-throughput separation of particles in fluid is reported in this work. We investigate the axial acoustic field of a piezo-tube with an inside diameter 34mm, length 25mm and operating frequency 1.15MHz. Energy concentrates within the tube and leakage at the ends provides a sharp monotonic acoustic pressure field within a fluidic circuit. This process is not the conventional standing wave mechanism; instead the geometry produces a spatially stable filtering action without fouling. This powerful filtering action is confirmed theoretically via a COMSOL simulation and demonstrated experimentally by concentrating suspensions of 5μm proteoglycan tracer particles at a flow rate of 20mL/min: The corresponding acoustic contrast factor is 0.243 and trapping force is 11pN. This tube geometry tackles the limitations of microfluidic standing wave based acoustic concentrators, namely complex extraction, low-throughput and distributed focus, by harnessing a stable monotonic field profile.

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Acoustic pressure fields offer non-contact manipulation opportunities, with levitation in air receiving attention recently with stable single beam levitation, mid-air trapping and control, single axis levitation and non-spherical particles in artificial field profiles. Levitation of microparticles in liquids has also emerged within advanced healthcare, biotechnology research, and industrial applications such as cell culturing, early diagnosis of diseases, biomass harvesting and food quality control. These standing wave pressure fields have high particle concentration efficiency, rapid processing and maintenance of cell viability. The primary advantage is access to physical forces that localise particles at an equilibrium point in a similar fashion as acoustic tweezers. The most successful application is miniaturised lab-on-a-chip geometries with standing wave patterns in submillimetre sized channels. Particle separation is well controlled for half-wavelength standing wave gaps with two monotonic pressure field profiles straddling a focussing point. Widening standing wave gaps to increase throughput produces polytonic regions (FIG.1.(a)) with distributed focus. This loss of focus and potential de-tune indicates standing waves in larger structures have an Achilles’ heel, i.e., particles don’t move to a single equilibrium point. Whereas living organisms sticking together via acoustic pressure waves improves harvesting and throughput. Nevertheless filtration, centrifugation, flocculation, sedimentation remain attractive options.

We present a piezo-tube method to create monotonic pressure profiles with single focussing action (FIG.1.(b)). It patches the Achilles’ heel of the standing wave microfluidic filtration devices. Here we use the piezo-tube end to pass or capture particle collectives reproducing porous filter or chromatographic medium character. COMSOL simulations and tests of proteoglycan accumulation within a monotonic force field are described below.

The piezo-tube is excited by a radio frequency signal and generates a monotonic pressure gradient at its end. The oscillations of the electric field induce the mechanical vibrations in the wall. This energy transfers axially within the adjoining walls between the piezo-tube and adjacent elastomer coupled glass tubes and radially as an internal pressure wave. Energy along the tube walls is governed by the equations of mechanical displacement field $U_S$ and the electric displacement field $D_{EI}$:

$$\nabla \cdot T_S = -\rho_S \omega^2 U_S$$

$$\nabla \cdot D_{EI} = 0$$

where $\rho_S$ is the the mass density of the solid, $\omega$ is the angular frequency of the applied voltage and $T_S$ is the mechanical stress tensor induced within the solid.

In a piezoelectric solid, $T_S$ and $D_{EI}$ are coupled by the linear piezoelectric constitutive matrix. The stress-charge form of these relations can be expressed as:

$$\begin{bmatrix}
T_S \\
D_{EI}
\end{bmatrix} =
\begin{bmatrix}
c_E & -e^T \\
e & \varepsilon_S
\end{bmatrix}
\begin{bmatrix}
\nabla U_S \\
-\nabla V_S
\end{bmatrix}$$

where $V_S$ is the electric potential field within the solid, $c_E$ is the elasticity matrix of the material, $e$ is the electromechanical coupling matrix of the material, and $\varepsilon_S$ is the electric permittivity matrix of the material. While in a non-piezoelectric material of the flow tubing, the electromechanical coupling matrix $e$ is absent, so the two energy fields can be expressed as:

$$\begin{bmatrix}
T_S \\
D_{EI}
\end{bmatrix} =
\begin{bmatrix}
c_E & 0 \\
0 & \varepsilon_S
\end{bmatrix}
\begin{bmatrix}
\nabla U_S \\
-\nabla V_S
\end{bmatrix}$$

The frequency domain of the acoustic pressure field $p_L$ in the liquid can be expressed by the Helmholtz equation:

$$(\nabla^2 + k^2)p_L = 0$$

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where \( k = \omega / c_L \) is the angular wave number, and \( c_L \) is the velocity of sound in the liquid.

The levitating pressure field in the fluid affects a particle’s path: Generally the glass tubes vibrate with non-uniform energy that interferes with the standing wave fields of the piezo-tube. The overall pressure field of the piezo-tube at frequencies where radial vibrations dominate over axial vibrations, leads to negligible end radiation and a sharp pressure gradient of significant force.

\[
\mathbf{F}_{Ac} = -\frac{\pi}{6} d_P^3 (0.5 f_1 \beta_L \nabla < p_L^2 > - 0.75 f_2 \rho_L \nabla < v_L^2 >)
\]  

where \( \beta_L \) and \( \rho_L \) are respectively the isothermal compressibility and the mass density of the liquid, \( v_L \) is the medium molecular velocity. The coefficients \( f_1 \) and \( f_2 \) can be defined as functions of the liquid and particle compressibilities and densities respectively:

\[
f_1 = 1 - (\beta_P / \beta_L)
\]
\[
f_2 = \frac{2[(\rho_P / \rho_L) - 1]}{2(\rho_P / \rho_L) + 1}
\]

FIG. 1. The pressure gradient profiles in (a) a typical lab-on-a-chip acoustic separation setup with multiple inlets and outlets, where \( w_1 \) is usually hundreds of microns and (b) the reported high-throughput acoustic filter, where \( w_2 \) and \( w_3 \) are 34mm and 25mm respectively. A COMSOL simulation of the axial acoustic pressure distribution is demonstrated on (c). The acoustic pressure field is symmetric about the mid-plane perpendicular to \( z \)-axis. It is at its strongest at \( z=0 \) and decreases along the \( z \)-axis to both the positive and negative directions until the pressure approaches a minimum value in liquid within the glass tubes. The tubing geometry is consisted in the sequence of glass tube, O-ring, piezo-tube, O-ring and glass tube.

This \( z \)-axis pressure profile is confirmed by a COMSOL simulation: In FIG.1.(c) an axisymmetric model is developed. Equations (1), (2), (3) and (4) are solved simultaneously. The pressure field in the liquid is driven by continuity conditions of stress and pressure at the tube inner wall. Plane wave radiation conditions are imposed at the upper and lower walls of the geometry to model energy leakage outside the piezo-tube into the flow tubing system.

The particle forces from the non-periodic monotonic gradient of its ends are determined as follows. If the diameter of the particle, \( d_P \) is smaller than the acoustic wavelength, the particle is subject to a time averaged acoustic radiation force \( \mathbf{F}_{Ac} \), according to Gorkov’s equation as:

\[
\Phi_{Ac} = f_1 + 1.5 f_2
\]

Here, the energy density multiplied by a sinusoidal profile gives a force periodicity less than 1mm. Whereas the piezo-tube, with its non-periodic, monotonic acoustic pressure field as predicted by the COMSOL simulation, generates monotonic gradients separated by the piezo-tube length matching the form of Equation (5). This gradient behaviour is consistent with a fraction of acoustic energy leaking out of the ends of the piezo-tube. This profile overlays the underlying radial component of the Bessel field within the fluid volume. This hybrid acoustic force structure supports a particle concentration mode via the tube end geometry as the force determining variable.

The uniqueness of the present work lies in the physics of the tube-end pressure gradient for trapping particles and against a co-linear fluid flow. Whereas the remaining terms involving fluid pressure acting on the particle is based on known physics, i.e. the diameter of the particle \( d_P \) to the 3rd power, the energy density of the acoustic field \( E_{Ac} \) and finally the acoustic contrast factor \( \Phi_{Ac} \).
Overall transiting particles experience a significant net force field \( \mathbf{F}_{\text{Total}} \), which is a vector sum of the acoustic radiation force \( \mathbf{F}_{\text{Ac}} \) upwards and the Stokes’ drag force \( \mathbf{F}_{\text{Drag}} \) downwards due to the flow rate \( \mathbf{u}_L \), i.e.:

\[
\mathbf{F}_{\text{Total}} = \mathbf{F}_{\text{Ac}} + \mathbf{F}_{\text{Drag}}
\]

\[
\mathbf{F}_{\text{Drag}} = 3\pi\eta_L d_p \mathbf{u}_L
\]  

(9)

where \( \eta_L \) is the dynamic viscosity of the liquid.

Thus particle paths resulting in concentration or leakage processes, is significantly influenced by the field \( \mathbf{F}_{\text{Total}} \) which needs to exceed zero to concentrate particles. For flow rates sufficiently strong or for weak acoustic powers, \( \mathbf{F}_{\text{Drag}} \) dominates over \( \mathbf{F}_{\text{Ac}} \) and the particles flow along without being manipulated. As one keeps increasing the particle diameter or power, the acoustic trapping forces increase more strongly (cubic scaling) than the fluid flow-induced drag forces (linear scaling). Hence for a given acoustic wave power and a fluid flow rate, one would have a critical particle diameter below which the acoustic forces are too weak to trap particles against the drag forces.

\[
\frac{\rho_p}{\rho} - \frac{\eta}{\eta_L} \left( \frac{d_p^3}{2} \right) \left( \frac{\omega}{c} \right)^2
\]

| particle          | \( \rho_p \) \( \text{kg/m}^3 \) | \( \beta_p \) \( \text{Pa}^{-1} \) | \( \Phi_{\text{Ac}} \) | \( | \mathbf{F}_{\text{Ac}} |_{\text{max}} \) \( \text{pN} \) |
|-------------------|-------------------------------|----------------|----------------|---------------------------------|
| Microphyte\(^2\) | 1100                          | 3.83 \times 10^{-10} | 0.243          | 0.088                           |
| WBC\(^2\) \(^b\) | 1090                          | 3.59 \times 10^{-10} | 0.287          | 0.119                           |
| Fused silica\(^2\) | 2210                          | 1.00 \times 10^{-11} | 1.648          | 0.577                           |
| Polystyrene\(^2\) | 1050                          | 2.49 \times 10^{-10} | 0.495          | 0.263                           |
| Gold\(^2\)       | 19300                         | 5.56 \times 10^{-12} | 2.374          | 0.584                           |

\(^a\) COMSOL predicted numbers

\(^b\) White blood cell

are loci where the axial trapping force can vanish. These permeable regions are expected to occur along inner cylindrical planes representing the pressure nodes of the field.

Concentration or leakage is also dependent on particle contrast factors when suspended in water as shown in TABLE I. This presents the estimated traction forces in the centre of the tubular geometry according to particle type. This demonstrates dense rigid particles such as gold and fused silica are entrained by significantly higher forces, whilst biological cells have the least effect. Distinguishing from standing waves, it is important to note the axial forces are not periodic along the fluid flow direction. This results in an advantageous single concentration phase.

Particle flow rate, according to Poiseuille/laminar flow (Reynolds number for the flow rate and tube ID used in this work is 12.482), is expected to vary from the centre to the edge. At the centre it is likely to be flowing faster but also working against a stronger field (Bessel function distribution), hence there is a degree of compatibility between the flow rate and force field distribution. Overall choice of different particles of \( \beta_p \), \( \rho_p \) result in different \( \Phi_{\text{Ac}} \), together with \( d_p \) lead to a variation in the \( \mathbf{F}_{\text{Ac}} \). The larger the \( | \mathbf{F}_{\text{Ac}} | \), the easier it is to manipulate the particle. TABLE I indicates that biological cells suspended in water are the most difficult particle to capture with acoustic waves, hence for our initial experiments we have selected a biological particle; microphyte as the most challenging test for the system.

An experimental setup for demonstrating the anticipated field-particle coupling of a suspension comprising proteoglycan tracer particles (microphytes \( d_p \) of 5\( \mu \)m) is shown in FIG.2.(a). The flow tubing geometry consists of a piezo-tube with an OD 38mm, ID 34mm and height 25mm (APC International Ltd., Type I) co-linear with two 40mm glass tubes of the same OD and ID positioned above and below. To minimise acoustic transmis-
An O-ring is inserted between adjoining active and passive tubes. The outer ends of the glass tubes are sand-blasted and wick between two customised acrylic sheets with liquid. Inlet (top) and outlet (bottom) respectively. A sinusoidal signal from the signal generator (Agilent 33120A Function Generator) drives the piezo tube at its radial mode resonant frequency and amplified by a power amplifier (EIN 310L) to approximately 30Vpp. The flow of the particle suspension is driven by a peristaltic pump (Masterflex). The glass tubes allow visualisation of the particle concentration effect.

These demonstration experiments are setup to cold-ict and optical data from the tubes via a high-dropphone and camera respectively: For measuring the pressure field a PVDF hydrophone (Dr Mueller Instruments, Müller-Platte Needle Probe) was placed at the tube axial centre (FIG.2.(b).I) and the sampled signal amplified by a voltage amplifier (Dr Mueller Instruments, MVA 10). The result is displayed on an oscilloscope covering the corresponding plot shown in FIG.2.(b).II. and confirming the acoustic presence sure falls significantly outside the piezo-tube.

Under the same conditions we pass the tracer particle suspension through the energised piezo-tube to get the concentration effect. Of note is the pumps flow rate, which is significantly greater than typical lab-on-chip devices by process by 1000 to 10000 times at 20mL/min. As the tube shape and depth prevent straightforward concentration measurements, the signature chromatic RGB technique was chosen based on its sensitive with assessing green foliage levels from satellite images and the potential to profile a smaller photo-synthetic system. Here the $S_{GC}$ carries the RGB colour information of an image correlated with the concentration process. A 75s video is converted into an image sequence; and a consistent region of interest (ROI) is analysed ysed frame by frame. The RGB information is recorded and the $S_{GC}$ is calculated, ratioing the signature value of digit number over the sum of the red, green and blue digit numbers. The green colour is the most representative parameter, as long as the lighting levels remain fixed.

At this stage we observe a concentration effect which agrees with the form of the theoretical prediction, thus making it a suitable candidate geometry for high throughput solid-liquid separation. A detailed COMSOL simulation also confirms the trajectory of microparticles in the flow tubing, which shows the same accumulation of behaviour of the particles at the entrance of the piezo-tube as expected (FIG.2.(b).I).

Particle concentration and other motional effects were also observed: Particles influenced by the piezo-tube in both either on-state or off-state is presented in FIG.3.(a). This shows the on-state creates a particle concentration effect as indicated by the relative concentrations via ROI data collected from a vertically oriented tube with downward flow. Here frames are taken of four quadrant-rafts: above and below an active tube, above and below an inactive tube (referred as on above, on below, off above and off below). Interpreting FIG.3.(b), there is a slight difference between baselines relating to small background differences in the first few frames. For the off-state as time progresses, the green and brown dashed lines indicate matching contrast above and below the tube. Whereas the on-state results in an increasing difference: The solid green line confirms that the particles accumulate above the piezo-tube, i.e. unable to enter the piezo-tube. Around frame 40 a minor leakage is visible, which indicates partial percolation of tracer particles through the field.

Leakage and streaming were minor relative to the trapping effect observed. For the former permeability of the field is associated with particle trajectories along the radial nodal planes, where the axial forces are weak. We hypothesise that the leakage is encouraged when the axil radiation pressure depletes along the nodal planes. As a result, we expect the leakage to be positively correlated to the number of the radial modes, although no tests were done to confirm this. However it was observed that leakage operates uniformly over the width, but is difficult to quantify due to agglomeration effects. Outside of the tube, streaming had minimal contribution to the motion of either entrant or leaked particles. From the gradient of the ROI $S_{GC}$, the accumulation rate in the active tube beyond frame 70 (where the count is most accurate) is doubled, further confirming the trapping effect.

We believe this is the first demonstration of a particle trajectory arrested by an acoustic barrier derived from a large scale monotonic field. The sample flowing within this tubing system is subject to a dominant localising effect, where it concentrates at a significant volume and flow rate of 20mL/min alongside the supernatant phase. These experiments demonstrate that low contrast bioparticles can be filtered, the most challenging case, which bodes well for entities of higher contrast. A clear focal point is produced at the tube end. We no longer need to depend on the characteristics of the standing wave alone, which can become problematic at small scales and high frequencies as the particle distorts the wavefront and leads to tuning variabilities.

The membrane like filter action is the most prominent characteristic of the profile leading to an accessible accumulation which is relatively straightforward to tap. Batch type fluidic operations can immediately be applied and is amenable for series or parallel operation, with the opportunity to boost very low concentrations.

Applications include plant cell collection, animal cell purification, bioreactor systems, geological separations, agricultural processing and sensing enhancement due to the concentration factor. The approach is especially useful for particles with high acoustic contrast. High high-throughput retrofitted systems benefit from less tuning than standing wave approaches. Thus it can work within micro- or macro-liquid processing systems and support low maintenance, non-clogging and low power applications. These are powerful foundations for processing bi-
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