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New tests for experiments producing pentaquarks

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Abstract

The distribution of the squared momentum difference $|P_A|^2 - |P_B|^2$ between the momenta in the laboratory system of two experimentally observed particles A and B provides a test for whether an observed mass peak indicates a real resonance rather than nonresonant background or kinematic reflection. The angular distribution of the relative momenta in the center-of-mass system exhibits a forward-backward symmetry in the production and decay of any resonance with a definite parity. This symmetry is not expected in other nonresonant processes and can be expressed without needing angular distributions in terms of the easily measured momenta in the laboratory frame that are already measured and used to calculate the invariant mass of the system. Our test is especially useful for low statistics experiments where the full angular distribution cannot be determined. It can be applied to both fixed-target and collider searches for the Θ^+ and Θ_c pentaquarks.

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The recent experimental discovery [1] of an exotic 5-quark KN resonance Θ^+ with $S = +1$, a mass of ∼1540 MeV, a very small width $\lesssim$ 20 MeV and a presumed quark configuration $uudd\bar{s}$ has given rise to a number of experiments with contrary results [2].

At this point it seems crucial to analyze and extend both the positive and negative experiments to either establish the Θ^+ as a real particle and understand this contradiction or to find good credible reasons against its existence.

Many detailed theoretical pentaquark models have been proposed, but few address the problem of why certain experiments see it and others do not. We therefore do not consider them here. The ball is definitely in the experimental court. Our purpose is to establish communication between theorists who know which measurements are of theoretical interest and experimenters who know which measurements are possible with available facilities.

In this context we note a simple experimental test for the production of any two-body resonance having a definite parity. The angular distribution of the relative momentum in the rest frame of the resonance exhibits a forward-backward symmetry for the production and decay of a resonance and this symmetry generally is not present in nonresonant background. We present here a method to test experimental data for this symmetry while avoiding the difficulties of measuring angular distributions with poor statistics.

For a simple example of the basic physics, consider a peak arising in the invariant mass of the two-particle system of particles denoted by A and B with masses M_A, and M_B in a multiparticle final state. If this is a real resonance with a definite parity, the angular distribution of the relative momentum in the AB center-of-mass frame must exhibit a forward-backward symmetry with respect to any external direction; e.g. the direction of the total momentum of the AB system in the laboratory. We now show how this forward-backward symmetry can be checked easily by measurements of the magnitudes of the momenta of the two particles in the laboratory system.

In the AB center-of-mass system which is moving with a velocity denoted by $\vec{v}$ with respect to the laboratory system, the total momentum of the AB system is zero. Consider the case where the momenta of particles A and B are perpendicular to the direction of the incident momentum in the laboratory system. In a nonrelativistic approximation the longitudinal components of the A and B momenta which are zero in the AB center-of-mass system are just the products of mass and velocity in the laboratory system,

$$\frac{\vec{v}}{v} \cdot \vec{P}_A^{NR} = M_A v; \qquad \frac{\vec{v}}{v} \cdot \vec{P}_B^{NR} = M_B v \tag{1}$$

where $\vec{P}_A^{NR}$ and $\vec{P}_B^{NR}$ denote the momenta respectively of particles A and B. Then the difference of the momenta squared satisfies

$$\frac{|P_A^{NR}|^2 - |P_B^{NR}|^2}{|\vec{P}_A^{NR} + \vec{P}_B^{NR}|^2} = \frac{M_A^2 - M_B^2}{(M_A + M_B)^2} \tag{2}$$

where we note that the contributions of the transverse components of A and B momenta which are equal and opposite cancel out in both the numerator and denominator of the lhs of eq. (2).

Eq. (2) gives the value of the squared laboratory momentum difference $|P_A^{NR}|^2 - |P_B^{NR}|^2$ which corresponds to a resonance decay in which particles A and B are both moving in a direction in the AB center-of mass system perpendicular to the direction of the total laboratory momentum. Events having a larger value of $|P_A^{NR}|^2 - |P_B^{NR}|^2$ correspond to forward emission of particle A; a smaller value corresponds to backward emission. Thus measurements of the laboratory momenta of particles A and B give information about the angular distribution in the center-of-mass system without any angular measurements.

A full relativistic treatment of the angular distribution is given below. However, we note that for the particular case of transverse momenta in the AB center-of mass system the relativistic corrections to the ratio (2) are simply expressed by replacing the masses M_A and M_B by the center-of-mass energies $E_A(cm)$ and $E_B(cm)$ and noting that in the center-of-mass system the total energy is just the invariant mass M while the momenta are equal and opposite and cancel out in the squared difference. Thus the relativistic generalization of eq. (2) is

$$\frac{|P_A|^2 - |P_B|^2}{|\vec{P}_A + \vec{P}_B|^2} = \frac{E_A(cm)^2 - E_B(cm)^2}{[E_A(cm) + E_B(cm)]^2} = \frac{M_A^2 - M_B^2}{M^2} \tag{3}$$

We now derive the full relativistic generalization of this simple approach and apply it to the particular case of production via a K or K^* exchange on a nucleon at rest and in the reaction $K^+p \to \pi^+\Theta^+ \to \pi^+K^+n$ [3].

In these exchange diagrams for Θ^+ production the reactions at the baryon vertex are

$$K + N \to \Theta^+ \to K + N; \qquad K^* + N \to \Theta^+ \to K + N \tag{4}$$

When a Θ^+ spin-1/2 baryon resonance is produced by the reactions (4) the angular distribution of the kaon momentum in the center-of-mass frame of the final KN system is isotropic. This isotropy produces a forward-backward symmetry relative to any axis and in particular the axis defined by the center-of-mass momentum in the laboratory frame.

This forward-backward symmetry holds for the production and decay of any resonance with a definite parity. The amplitudes for two final states differing only by a reversal of the relative momentum in the center-of-mass system, denoted by $\vec{p}_{cm}$ can differ only by a real phase which depends upon the parity of the resonance.

$$A(\vec{P}, \vec{p}_{cm}) = \pm A(\vec{P}, -\vec{p}_{cm}) \tag{5}$$

where $\vec{P}$ denotes the total momentum of the system in the laboratory frame. On the other hand, if the final KN state is produced by a nonresonant peripheral reaction like meson exchange, the kaon angular momentum is strongly peaked forward in the center-of mass system. The difference between a symmetric and a forward-peaked distribution can be checked without angular measurements by expressing the condition (5) in terms of the magnitudes of the nucleon and kaon momenta in the laboratory frame.

Let $\vec{P}_K$, E_K, $\vec{P}_N$ and E_N denote the momenta and energies of the kaon and nucleon in the laboratory frames. The total momentum and the momentum difference are four-vectors defined as

$$\begin{aligned} \vec{P} &= \vec{P}_K + \vec{P}_N; \qquad & E &= E_K + E_N \\ \vec{p} &= \vec{P}_K - \vec{P}_N; \qquad & \epsilon &= E_K - E_N \end{aligned} \tag{6}$$

The values of the sums and differences in the center-of-mass system are given by the Lorentz transformation with a velocity $\vec{v}$

$$\vec{P}_{cm} = \gamma[\vec{P} - \vec{v}E] = 0; \qquad E_{cm} = \gamma[E - \vec{v}\cdot\vec{P}] = M \tag{7}$$

$$\begin{aligned} \vec{p}_{cm} &= \gamma[\vec{p} - \vec{v}\epsilon] = \gamma\left[\vec{p} - \vec{P}\cdot\frac{|P_K|^2 - |P_N|^2 + M_K^2 - M_N^2}{E^2}\right]; \\ \epsilon_{cm} &= \gamma[\epsilon - \vec{v}\cdot\vec{p}] = \gamma\left[\epsilon - \frac{\vec{P}}{E}\cdot\vec{p}\right] \end{aligned} \tag{8}$$

where M is the invariant mass of the KN system,

$$\gamma = \frac{1}{\sqrt{1-v^2}} \tag{9}$$

and we have used eq. (7) to obtain and substitute in eq. (8)

$$\vec{v}\epsilon = \frac{\vec{P}}{E}\epsilon = \frac{\vec{P}}{E^2}\cdot\epsilon E = \vec{P}\cdot\frac{E_K^2 - E_N^2}{E^2} = \vec{P}\cdot\frac{|P_K|^2 - |P_N|^2 + M_K^2 - M_N^2}{E^2} \tag{10}$$

To express the condition (5) in terms of the laboratory momenta $\vec{P}_K$, $\vec{P}_N$ and kaon, nucleon and resonance masses denoted respectively by M_K, M_N and M, we note that

$$\begin{aligned} \vec{p}_{cm}\cdot\vec{P} &= \gamma\left[\vec{p}\cdot\vec{P} - |P|^2\cdot\frac{|P_K|^2 - |P_N|^2 + M_K^2 - M_N^2}{E^2}\right] = \\ &= \gamma\left[M^2\cdot\frac{|P_K|^2 - |P_N|^2}{E^2} - |P|^2\cdot\frac{M_K^2 - M_N^2}{E^2}\right] \end{aligned} \tag{11}$$

where we used $\vec{p}\cdot\vec{P} = |P_K|^2 - |P_N|^2$. The squared momentum difference $|P_K|^2 - |P_N|^2 \equiv \Delta P_{KN}^2$ is thus given by

$$\Delta P_{KN}^2 = \frac{M_K^2 - M_N^2}{M^2}\cdot|P|^2 + \frac{E^2}{\gamma M^2}\,\vec{p}_{cm}\cdot\vec{P} \equiv \overline{\Delta P_{KN}^2} + \frac{E^2}{\gamma M^2}\,\vec{p}_{cm}\cdot\vec{P} \tag{12}$$

where $\overline{\Delta P_{KN}^2} = (M_K^2 - M_N^2)\cdot|P|^2/M^2$.

The condition (5) implies that for any given total momentum value $\vec{P} = \vec{P}_1$ the counting rate observed at a value of the squared momentum difference $\Delta P_{KN}^2 = \overline{\Delta P_{KN}^2} + \delta P_{KN}^2$ will be equal to the counting rate observed at $\Delta P_{KN}^2 = \overline{\Delta P_{KN}^2} - \delta P_{KN}^2$

$$N(\Delta P_{KN}^2 = \overline{\Delta P_{KN}^2} + \delta P_{KN}^2) = N(\Delta P_{KN}^2 = \overline{\Delta P_{KN}^2} - \delta P_{KN}^2) \tag{13}$$

The mean value of the squared momentum difference $|P_K|^2 - |P_N|^2$ is given by

$$\langle|P_K|^2 - |P_N|^2\rangle = \frac{M_K^2 - M_N^2}{M^2}\cdot|P|^2 = \overline{\Delta P_{KN}^2} \tag{14}$$

and the distribution of the squared momentum difference $|P_K|^2 - |P_N|^2$ is symmetric around the mean value (14).

This mean is negative and proportional to the kaon-nucleon mass difference, because a boost with the same velocity for the kaon and nucleon increases the nucleon momentum more than the kaon momentum. A higher value of $P_K^2 - P_N^2$ corresponds to forward scattering, a lower to backward scattering. Thus a forward-peaked background will show up with higher values of the difference between the kaon and nucleon momenta $P_K^2 - P_N^2$ in the laboratory system.

For the case of an isotropic angular distribution in the center of mass system and a very narrow resonance, $\vec{p}_{cm}$ is constant in magnitude and

$$\vec{p}_{cm} \cdot \vec{P} = |\vec{p}_{cm}| \cdot |\vec{P}| \cos\theta \tag{15}$$

where θ is the angle between $\vec{p}_{cm}$ and $\vec{P}$. The distribution of the squared momentum difference $|P_K|^2 - |P_N|^2$ is flat between the limits corresponding to $\cos\theta = \pm 1$.

$$\overline{\Delta P_{KN}^2} - \frac{E^2}{\gamma M^2}\, |\vec{p}_{cm}| \cdot |\vec{P}| \;\leq\; \Delta P_{KN}^2 \;\leq\; \overline{\Delta P_{KN}^2} + \frac{E^2}{\gamma M^2}\, |\vec{p}_{cm}| \cdot |\vec{P}| \tag{16}$$

The question now arises whether eqs. (13) and (14) can provide useful information in real data with all the acceptance restrictions and give a simple test to see whether it works at all.

We first note that alternative mechanisms that have been suggested using kinematic reflections to explain the Θ^+ mass peak [4] will generally not have the forward-backward asymmetry nor satisfy eqs. (13) and (14). Comparing their predicted squared momentum difference $|P_K|^2 - |P_N|^2$ distributions with measured data can provide additional checks on these alternatives.

Further investigation of possible uses with real data raise two questions:

1. Is there a significant difference between the angular distributions of signal and background events?

2. If the answer to (1) is yes, is the difference still significant when only events that meet the detector acceptance are included?

If the answers to (1) and (2) are yes, there may be ways to improve the signal/background ratio by cutting out events which are mainly background.

For a simple test to see whether this makes sense at all in a real experiment with real detector acceptance limitations, consider the following.

Separate the events into two bins, with half of the events in each. Put the events with the highest values of kaon momentum in one bin, those with the lowest values in the other,

Now plot the mass distributions separately for each of the two bins. The signals from a resonance are expected to be equal in the two by eq. (13). But if the background is mainly peaked forward in the center of mass system, there should be more background events in the bin with the higher kaon momentum.

If the two mass distributions turn out to be the same, there is no point in following this further. But if the two mass distributions are different, it will be worth while trying to develop this approach further.

The basic idea suggesting that there might be a difference is the assumption that the kaons going forward in the center-of-mass frame are mainly background. These will be the kaons having the highest momentum in the laboratory system. If this is true, the signal to background ratio can be improved by cutting out the events with the highest kaon momentum.

If there is forward peaking, indicating a strong nonresonant background, cuts removing the events having the most positive values of ΔP^2_{KN} can eliminate the background coming from strongly forward events, without excessively harming the resonant signal.

In many experiments where the Θ^+ resonance is produced, other well-known resonances are also produced. In particular the $\Lambda(1520)$ resonance has often been used for comparison with the Θ^+ [2]. The momentum distributions of such final states of other resonances like the $\Lambda(1520)$ must certainly satisfy the conditions (13) and (14). Any deviations from these conditions must be due to variations in the detector acceptance as a function of the momenta. These can provide useful information on the detector acceptances for the Θ^+ data. In the particular case of the $\Lambda(1520)$, the CLAS data [2] show a very strong peak with comparatively low background. Measuring the squared momentum difference $|P_K|^2 - |P_N|^2$ distributions both in the resonance peaks and in the backgrounds on both sides of the resonance can provide interesting insight into the applicability of this method.

The conditions (13) and (14) hold for any experiment in which a resonance is produced, not only in production from a target at rest. They may not be useful if the background also satisfies these conditions and does not have a strong forward or backward peaking. However they may still supply useful checks and information in all cases.

We believe our approach is especially useful for low statistics experiments where the full angular distribution cannot be determined. It can be applied to both fixed-target experiments, such as photoproduction, pp, pA, KN, KA and νA collisions, as well as eN and e^+e^- collider searches for the Θ^+ [2] and Θ_c [5]-[7] pentaquarks.

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