

High sensitivity of indirect noise predictions in non-isentropic nozzles

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ABSTRACT

In aircraft engine combustors, incomplete mixing and air cooling give rise to flow inhomogeneities. When accelerated through the nozzle guide vane downstream of the combustor, these inhomogeneities give rise to acoustic waves. This is commonly known as indirect combustion noise. When these sound waves are reflected off the outlet and travel upstream of the combustor, they can lead to thermoacoustic oscillations. To predict indirect noise, models proposed in the literature assume the wavelength of impinging disturbances to be large with respect to the nozzle spatial extent (compact nozzle assumptions). However, in reality, the assumption might not hold; hence, a mismatch between the model and experimental results can be observed. The overarching objective of this study is to show that the acoustic transfer functions are sensitive to the compact-nozzle assumption. We show this on a realistic model of nozzle flow, which contains dissipation (non-isentropic) effects. First, we show that indirect-noise predictions are sensitive to a small change in the Helmholtz number for a nearly compact nozzle, particularly in the subsonic flow regime. Second, we show that modelling dissipation is key to mitigating the unphysical sensitivity of the compact-nozzle assumption. This study highlights the importance of the non-compact and non-isentropic assumptions for the accurate prediction of indirect noise transfer functions and thermoacoustic instability in aeronautical gas turbines.

1. INTRODUCTION

Combustion noise is one of the major sources of aero-engine noise. On the one hand, the fluctuations in the unsteady heat release of the flame in the combustor give rise to the direct combustion noise [e.g., 1, 2]. On the other hand, the acceleration of flow inhomogeneities downstream of the combustor generates indirect combustion noise [e.g., 3, 4, 5, 6, 7, 8, 9]. Indirect

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combustion noise generated by an advected temperature inhomogeneity is referred to as entropy noise [3, 10, 11, 12], whereas, noise generated by an advected compositional inhomogeneity is referred to as compositional noise [9, 13]. These sound waves are perceived as noise when they are emitted downstream of the engine components. Importantly, when these sound waves reflect back to the combustor, they can contribute to the thermoacoustic feedback [e.g., 7, 8, 14, 15]. Hence, research efforts aim at estimating the noise transfer functions accurately with low order models.

Marble & Candel (1977) [3] proposed a low-order model to predict entropy noise in a compact nozzle flow, in which the length of the nozzle is negligible as compared to the wavelength of the acoustic waves. Assuming an isentropic flow, the jump conditions were formulated to calculate the transfer functions [3, 16]. The analysis was extended to non-compact nozzles by Leyko et al. (2009) and Duran & Moreau (2013) [12, 17]. The non-compactness of a nozzle is measured using the Helmholtz number, He , which is the between the wavelength of the advected perturbations and that of the acoustic waves ($He = \omega L/c$, where L is the characteristic nozzle length, ω is the perturbation angular frequency and c is the reference speed of sound). The compact nozzle assumption ($He = 0$) assumes the nozzle as a discontinuity in the flow, hence algebraic jump conditions are imposed across the nozzle to conserve mass, momentum and energy. For non-compact nozzles ($He > 0$), the equations become differential, which needs to be solved numerically or semi-analytically.

All the above-mentioned studies assume the flow to be isentropic. However, factors such as flow recirculation and wall friction make the flow non-isentropic. The effect of non-isentropicity on indirect noise was shown experimentally by De Domenico et al. (2019) [18], who observed a mismatch between experimental observations and the isentropic model for a subsonic flow. Using conservation laws, Jain & Magri (2022) [19] developed a physical model to predict indirect noise in nozzles with dissipation, which was validated with experiments of the literature [18].

In this work, we (i) present the governing equations for entropy noise generation in non-isentropic flows; (ii) analyse the sensitivity of the entropic-acoustic transfer functions with respect to the Helmholtz number for isentropic and non-isentropic flows. The paper is structured as follows. Section 2 describes the mathematical model to predict entropy noise in nozzles with dissipation. Section 3 discusses the sensitivity of the nozzle transfer functions. Conclusions end the paper.

2. MATHEMATICAL MODEL

We consider a single component, calorically perfect, ideal and adiabatic flow. The effects of dissipation (non-isentropicity) averaged over the nozzle cross-section are considered using the friction factor [19]. The cut-off frequency is considered sufficiently large to model the acoustics as quasi-one-dimensional. The equations of conservation of mass, momentum and energy are [19], respectively

$$\frac{D\rho}{Dt} + \rho \frac{\partial u}{\partial x} + \frac{\rho u}{A} \frac{dA}{dx} = 0, \quad (1)$$

$$\frac{Du}{Dt} + \frac{1}{\rho} \frac{\partial p}{\partial x} = -\frac{4f}{D} \frac{u^2}{2}, \quad (2)$$

$$\frac{Ds}{Dt} = R \frac{f}{f\gamma M^2 - 2 \tan \alpha} \left(\frac{\gamma(1 - M^2)}{2 \left(1 + \frac{\gamma-1}{2} M^2\right)} \right) \frac{DM^2}{Dt}, \quad (3)$$

where, u is the velocity, M is the Mach number, x is the nozzle axial coordinate ($x = 0$ at the inlet), p is the pressure, ρ is the density, s is the entropy, T is the temperature, R is the gas constant, γ is the heat capacity ratio, A is the cross-sectional area, D is the nozzle diameter, $\tan \alpha$ is the spatial derivative of the nozzle profile ($\tan \alpha = (1/2)(dD/dx)$), and f is the Fanning friction factor. These equations are closed by Gibbs equation,

$$ds = c_p \frac{dT}{T} - R \frac{dp}{p}, \quad (4)$$

where, c_p is the specific heat at constant pressure.

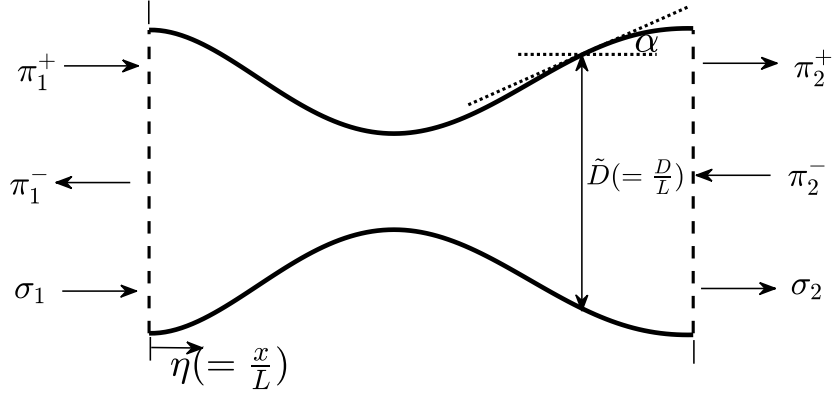


Figure 1: Nozzle schematic with nomenclature.

2.1. Linearisation

The acoustics are modeled as linear perturbations that develop over a steady mean flow. A generic variable is decomposed as $(.) = (\bar{.}) + (.)'$, where $(\bar{.})$ is the steady mean flow term and $(.)'$ is the first-order perturbation. On linearising Equation 1-Equation 3, we obtain [19],

$$\frac{D}{D\tau} \left(\frac{p'}{\gamma \bar{p}} \right) + \tilde{u} \frac{\partial}{\partial \eta} \left(\frac{u'}{\bar{u}} \right) - \frac{D}{D\tau} \left(\frac{s'}{\bar{c}_p} \right) = 0, \quad (5)$$

$$\frac{D}{D\tau} \left(\frac{u'}{\bar{u}} \right) + \frac{\tilde{u}}{\bar{M}^2} \frac{\partial}{\partial \eta} \frac{p'}{\gamma \bar{p}} + \left(2 \frac{u'}{\bar{u}} + (1 - \gamma) \frac{p'}{\gamma \bar{p}} - \frac{s'}{\bar{c}_p} \right) \left(\frac{\partial \tilde{u}}{\partial \eta} + \frac{4f}{\tilde{D}} \frac{\tilde{u}}{2} \right) = 0, \quad (6)$$

$$\frac{D}{D\tau} \left(\frac{s'}{\bar{c}_p} \right) = C_1 \left[2 \left(\bar{M}^2 \frac{D}{D\tau} \frac{M'}{\bar{M}} + 2\bar{M} \frac{M'}{\bar{M}} \frac{D\bar{M}}{D\tau} \right) + 2\bar{M} \frac{D\bar{M}}{D\tau} \left[-\frac{M'}{\bar{M}} \left(\frac{2\bar{M}^2}{1 - \bar{M}^2} + \frac{\bar{C}}{\bar{A}} \right) \right] \right] \quad (7)$$

where,

$$C_1 = \frac{(\gamma - 1)(1 - \bar{M}^2)f}{\left(1 + \frac{\gamma - 1}{2} \bar{M}^2 \right) (\gamma \bar{M}^2 f - 2 \tan \alpha)}, \quad (8)$$

$$\bar{A} = 2 \left(1 + \frac{\gamma - 1}{2} \bar{M}^2 \right) (\gamma \bar{M}^2 f - 2 \tan \alpha), \quad (9)$$

$$\bar{C} = 2\gamma \bar{M}^2 \left(-2f \left(1 + \frac{\gamma - 1}{2} \bar{M}^2 \right) + \frac{\gamma - 1}{\gamma} (2 \tan \alpha - \gamma \bar{M}^2 f) \right). \quad (10)$$

and the linearized Gibbs' equation is [3]

$$\frac{\rho'}{\bar{\rho}} = \frac{p'}{\gamma \bar{p}} - \frac{s'}{\bar{c}_p}. \quad (11)$$

The equations are non-dimensionalized with $\tau = t f_a$, $\eta = x/L$, $\tilde{D} = D/L$ and $\tilde{u} = \bar{u}/c_{ref}$, where f_a is the frequency of the advected perturbations, L is the nozzle length, and c_{ref} is the reference speed of sound. The non-dimensional material derivative is $\tilde{D}/D\tau = H e \partial/\partial t + \tilde{u} \partial/\partial \eta$. For zero friction

($f \rightarrow 0$) the equations tend to that of the isentropic flow [12]. The equations can be solved with a travelling wave approach,

$$\pi^\pm = \frac{1}{2} \left(\frac{p'}{\gamma \bar{p}} \pm \bar{M} \frac{u'}{\bar{u}} \right) \quad \sigma = \frac{s'}{\bar{c}_p} \quad (12)$$

Finally, from here, the variables in the differential equations refer are Fourier transformed as $(\cdot)(x, t) \rightarrow (\cdot)(x) \exp(2\pi i \tau)$. The equations can be solved to calculate the indirect noise transfer functions.

3. SENSITIVITY TO THE HELMHOLTZ NUMBER

We study the effect of the Helmholtz number on the entropic-acoustic transfer functions,

$$S_R = \pi_1^- / \sigma, \quad S_T = \pi_2^+ / \sigma \quad (13)$$

where S_R and S_T are the entropic-acoustic reflection and transmission coefficients, respectively. The transfer functions are complex, thus they have both phases and magnitudes. The magnitude and the phase of the reflected wave are important parameters to analyse the effect of indirect noise on thermoacoustic stability. Whereas, the magnitude and phase of the transmitted wave contribute to the noise. We use the Taylor expansion in this section to analyse the sensitivity of the nozzle transfer function with the Helmholtz number,

$$S_R(He) = S_R(He_0) + \frac{dS_R}{dHe}(He - He_0) + \frac{1}{2} \frac{d^2 S_R}{dHe^2} (He - He_0)^2 + O(He^3), \quad (14)$$

The transfer functions are calculated with an expansion at $He = He_0$. The Taylor expansion of the solution corresponds to the terms of the asymptotic expansion [13].

3.1. Subsonic flow

We analyse a convergent-divergent nozzle with the inlet area that is twice as large as the exit area.

Figure 2 shows the entropic-acoustic transfer functions. The magnitude of the transfer functions in subsonic isentropic nozzle flows in nearly compact nozzles ($He < 0.05$, in the analysed configuration) is significantly different for a small change in the Helmholtz number. For instance, the reflection coefficient is ≈ 35 times larger at $He = 0.1$ as compared to that at $He = 0$. Similar observations can be made for the transmission coefficient. Figure 2 shows the expansion of first, second and third order in a subsonic isentropic ($f = 0$) nozzle flow. The predictions do not match with an expansion at Helmholtz number, $He = 0$ (Figure 2 (a - d)). However, the expansion predictions tend to match the model predictions with an expansion at $He \approx 0.1$, especially for higher orders (Figure 2 (e - f)). On the contrary, in a non-isentropic flow ($f = 0.08$), the third-order expansion matches the model predictions for both expansions at $He = 0$ (Figure 3 (a - d)) and $He = 0.1$ (Figure 3 (e - h)).

This is because the acoustic waves are generated by the strong acceleration of the mean flow in the nozzle ($\partial \bar{u} / \partial \eta$ in Equation 6). In a compact nozzle ($He = 0$), the flow goes through positive acceleration in the convergent part to a negative acceleration in the divergent part. Hence, the indirect noise generated in the convergent section is cancelled out by the noise generated in the divergent section. However, as the Helmholtz number increases, there is a phase difference associated with the waves in convergent and divergent sections. This does not allow cancellation and a drastic change in the magnitudes of the transfer functions are observed [12]. Hence the compact nozzle analysis is limited to very low frequencies in an isentropic subsonic flow. It should be treated with care.

In a flow with dissipation, the indirect noise transfer functions are not as highly sensitive to the change in Helmholtz number. For instance, the reflection coefficient is ≈ 1.2 times larger at $He = 0.1$ as compared to the compact nozzle (Figure 3). Similar observations can be made for the transmission

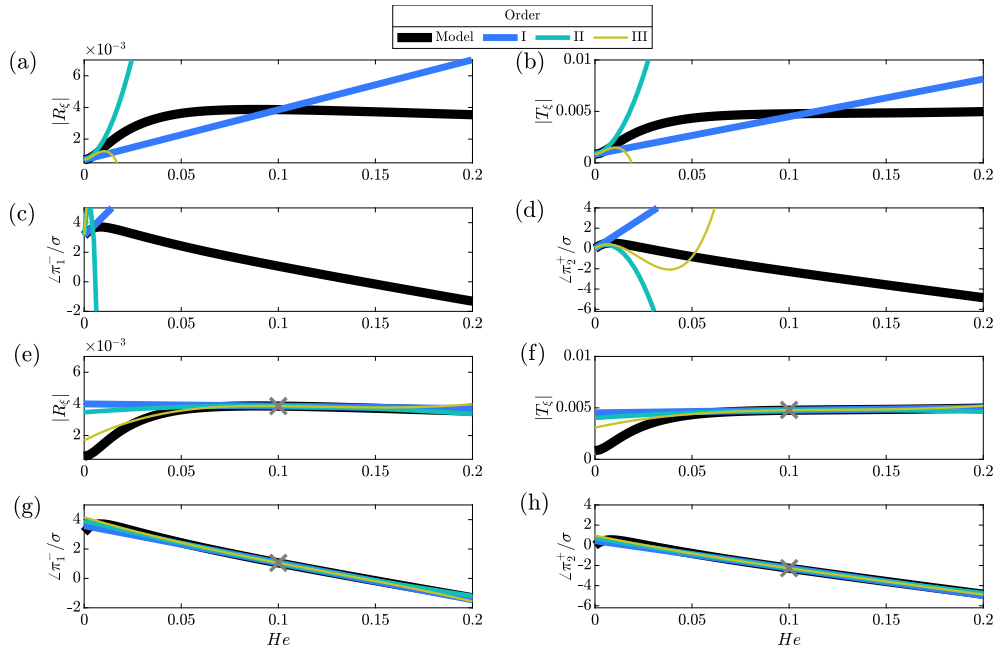


Figure 2: Entropic-acoustic reflection (a, c, e, g) and transmission (b,d, f, h) magnitudes and phase in an isentropic ($f = 0$) subsonic flow ($M_t = 0.6$). The black solid lines are the model predictions of Jain & Magri (2022) [19]. The solid line show the predictions using the Taylor expansion. The cross indicates the expansion point.

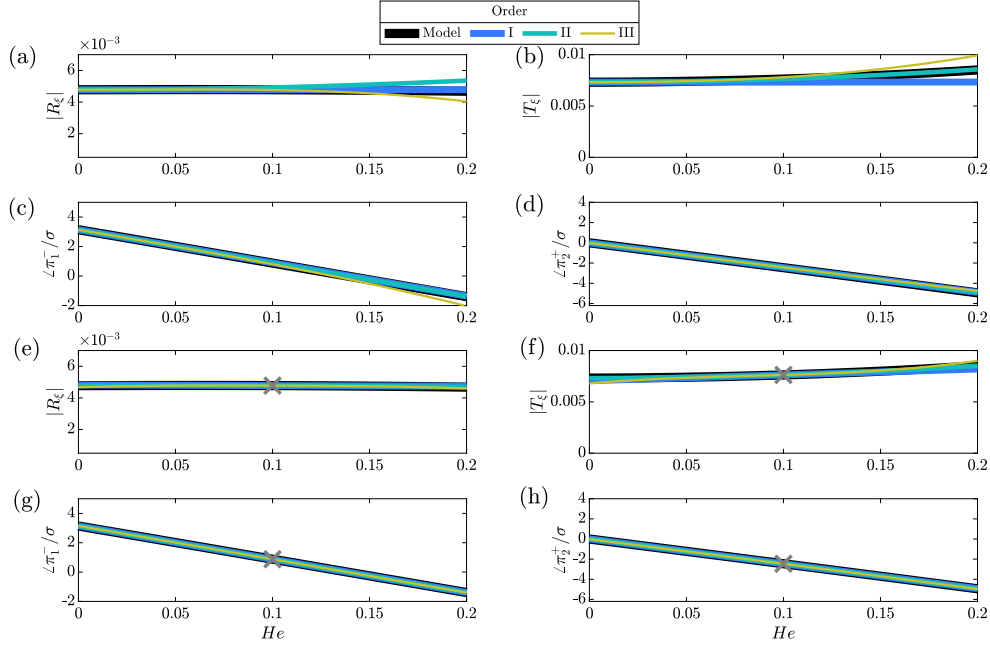


Figure 3: Entropic-acoustic reflection (a, c, e, g) and transmission (b,d, f, h) magnitudes and phase in a non-isentropic ($f = 0.08$) subsonic flow ($M_t = 0.6$). The black solid lines are the model predictions of Jain & Magri (2022) [19]. The solid line show the predictions using the Taylor expansion. The cross indicates the expansion point.

coefficient. Moreover, the predictions of the Taylor expansion match the model predictions for higher orders for expansion at $He = 0$ (Figure 3 (a - d)) and for all orders for an expansion at $He = 0.1$ (Figure 3 (e - h)). This is because, there is an additional term in the momentum equation ($\partial \tilde{u} / \partial \eta + 4f\tilde{u} / 2\tilde{D}$ in Equation 6). Physically, the dissipation has the effect of decelerating the mean flow and leads to a change in magnitude and phase of the indirect noise transfer functions. Hence, modelling dissipation can help mitigate of the non-physical sensitivity observed in the case with the isentropic flow assumption.

In conclusion, the transfer functions are highly sensitive to the Helmholtz number for small Helmholtz numbers ($He < 0.05$, in the analysed case) in an isentropic subsonic flow. The sensitivity reduces in a flow with dissipation. This highlights the importance of correct estimation of the Helmholtz number and modelling dissipation, even in the nearly compact nozzle regime.

4. CONCLUSIONS

Various indirect noise models in the literature consider the nozzle to be compact. In this work, we highlight the importance of non-compact assumptions. We also analyse the effect of dissipation on this assumption. To do so, first, we show the governing equations for a single component flow with dissipation derived using conservation laws. Second, we analyse the sensitivity of the entropic-acoustic transfer functions to changes in the Helmholtz number. The analysis is performed for both isentropic and non-isentropic flows. We observe that the transfer functions are highly sensitive around the Helmholtz number, $He = 0$ in an isentropic subsonic flow. The sensitivity reduces as the Helmholtz number increases. In a subsonic flow with dissipation, the transfer functions are less sensitive to changes in the Helmholtz number. Therefore, dissipation needs to be modelled for the predictions not to be overly sensitive to the compact-nozzle assumption.

In conclusion, the compact nozzle assumptions may result in inaccurate indirect noise predictions, especially in isentropic subsonic flows. This highlights the importance of modelling Helmholtz number effects with dissipation.

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