Origin of DC voltage in type II superconducting flux pumps: field, field rate of change, and current density dependence of resistivity

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Superconducting flux pumps are the kind of devices which can generate direct current into superconducting circuit using external magnetic field. The key point is how to induce a DC voltage across the superconducting load by AC fields. Giaever [1] pointed out flux motion in superconductors is origin of the DC voltage, and demonstrated a rectifier model. Klundert et al. [2, 3] reviewed various flux pumps which rely on inducing the normal state in at least part of the superconductor. In this letter, following their work, we reveal that the variation in the resistivity of type II superconductors can contribute to the origin of a DC voltage in flux pumps. The variation in resistivity is due to the fact that flux flow is influenced by current density, field intensity, and field rate of change. We proposed a general circuit analogy for travelling wave flux pumps, and provided a mathematical analysis of the DC voltage. Several existing superconducting flux pumps which rely on the use of a travelling magnetic wave can be explained using the analysis enclosed. This work can also throw light on the design and optimization of flux pumps.

1 Introduction

Under development for some years Coated Conductors (CC) have struggled to find application in high field magnet systems such as Magnetic Resonance Imaging (MRI) [3] and Nuclear Magnetic Resonance (NMR) [4]. This has chiefly stemmed from the fact that coils constructed from CC are difficult to operate in persistent mode due to the relatively low n-value [5] and the relatively high resistance in joints [6]. If a flux pump [7-17] is used then current leads and persistent current switches [18] are not required. The magnet's field can be maintained using the pump and the coil can be operated in persistent mode. The idea of using a travelling magnetic wave to gradually magnetize a type-II superconductor was firstly proposed by Coombs [7, 8]. After that, several High-T_c Superconducting (HTS) flux pumps based on travelling wave were developed for CC coils [9-15]. These flux pumps use a piece of CC (CCs) connecting to a superconducting load. When magnetic field travels across the CC, flux gradually accumulates in the load. The key point of these flux pumps is how a DC voltage is induced by external fields, which has also been confusing for years. In this work, we will reveal that nonlinearity in the resistivity of type II superconductors is the origin of the DC voltage and therefore flux pumping. The proposed principle can well explain existing travelling wave flux pumps.

2 Basic principle

Travelling wave flux pumps comprise a superconducting loop connecting to a superconducting load which we want to magnetize. The superconducting loop is subjected to magnetic fields which vary in time and space, these induce a voltage across the branch which is connected to the load, as shown in Fig. 1(a). If the open circuit voltage $v(t)$ has a DC component V_{DC}, it will generate a increasing DC current in the inductive load. Therefore, the DC component is substantial in flux pumping. If the perimeter of the loop is considerably larger than the width of branches, and the resistance of this loop along its length is considerably larger than its inductance, Fig. 1(a) can be described by Fig. 1(b) as a circuit model, where the right branch of the circuit represent the branch *ab* in Fig. 1(a), the left branch in the circuit represents branch *adc* in Fig. 1(a), $v_1(t)$ and $v_2(t)$ represent the induced EMF forces in each branches, and $R_1(t)$ and $R_2(t)$ represent resistance of the branches accordingly.

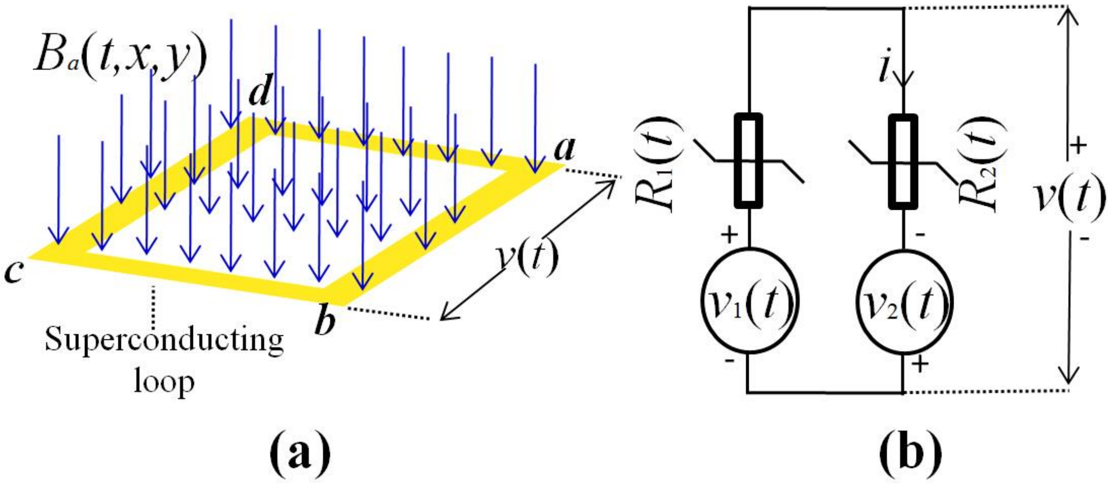


FIG. 1. Schematic drawing of open circuit voltage of travelling wave flux pump. (a) Magnetic field varying in time and space is applied to a superconducting loop, part of which will be connected to a superconducting load. (b) Circuit analogy of travelling wave flux pump, where $v_1(t)$ and $v_2(t)$ represent the induced EMF forces in each branches, and $R_1(t)$ and $R_2(t)$ represent resistance of the branches.

According to Faraday's Law:

$$v_1(t)+v_2(t)=\int_{adcb}\vec{E}\cdot d\vec{l}+\int_{ba}\vec{E}\cdot d\vec{l}=\oint_l \vec{E}\cdot d\vec{l}=-\int_S \frac{dB}{dt}ds=-d\Phi/dt \qquad (1)$$

Where l is the perimeter of the loop, S is the area of the loop, B is the applied field, and Φ is the total flux applied to the loop.

The open circuit voltage $v(t)$ across the branch is :

$$v(t)=iR_2(t)-v_2(t)=\frac{v_1(t)+v_2(t)}{R_1(t)+R_2(t)}R_2(t)-v_2(t) \qquad (2)$$

The DC component in $v(t)$ is:

$$V_{DC}=\frac{1}{T}\int_0^T v(t)dt=\frac{1}{T}\int_0^T \frac{-d\Phi/dt}{R_1(t)+R_2(t)}R_2(t)dt \qquad (3)$$

Where T is the period of the applied field in the area of the loop. Here we should consider $\int_0^T v_2(t)dt = 0$ (Otherwise an AC magnetic field would induce a DC electric field, which is against Faraday's Law). It should be noticed that the same conclusion can be drawn by analyzing the left branch in Fig. 1(b).

In Eq. (3), if R_2(t)/(R_1(t)+R_2(t)) is constant, then we can get:

$$V_{DC} = \frac{1}{T}\frac{R_2(t)}{R_1(t)+R_2(t)}\int_0^T \text{-}d\Phi / dtdt = 0 \quad (4)$$

The problem of flux pumping is now simplified as how to make Eq. (3) non-zero. To make it clear, we re-write Eq. (3) by separating Φ increasing and Φ decreasing processes:

$$V_{DC} = \text{-}\frac{1}{T}\left(\int_0^{t_1}\frac{R_2(t)}{R_1(t)+R_2(t)}d\Phi / dtdt + \int_{t_1}^T \frac{R_2(t)}{R_1(t)+R_2(t)}d\Phi / dtdt\right) = \frac{1}{T}(p_{dec} - p_{inc})\Delta\Phi$$

(5)

Where t_1 is the time Φ is minimum, and time zero is defined as when Φ is maximum. $\Delta\Phi$ is the peak-peak value of Φ applied to the loop. According to Eq. (5), if the ratio of $R_1(t)$ and $R_2(t)$ changes in the flux increasing process and flux decreasing process, a DC component in the open circuit voltage may occur.

For practical use, we want V_{DC} as large as possible. Various ways can increase the value. By changing the value of $R_1(t)$ and $R_2(t)$, p_{dec}-p_{inc} varies in the region of (-1,1). For example, if $R_2(t)$ is much larger than $R_1(t)$ when Φ is decreasing and $R_2(t)$ is much smaller than $R_1(t)$ when Φ is increasing, then p_{dec}-p_{inc}≈-1. By increasing field frequency, 1/T can be increased. By increasing field magnitude or area, $\Delta\Phi$ can be increased.

For a superconducting loop which has a large inductance, the situation is slightly different. But the inductance only influences the relative phase between the current and the applied flux in the loop. It will not change the substance that the variation of resistivity in branches generates a DC component in the open circuit voltage.

For a type-II superconductor, its resistivity is variable against current density, field intensity and field rate of change. Firstly, the branch resistance depends on current density and applied field, as described in *E-J* power law [19] under Kim's Model [20]:

$$R = \int_l \rho S dl = \int_l E_0 J^{n-1} / \left(\frac{J_{c0}}{1+B/B_0}\right)^n S dl \qquad (6)$$

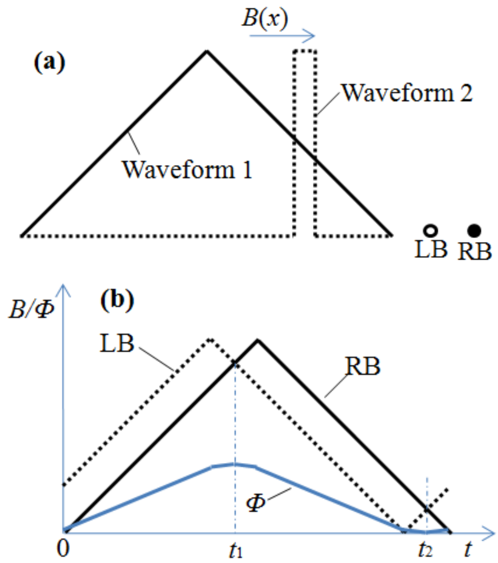
Where ρ is the resistivity, S is the cross section of the branch, and l is the length of the branch. Secondly, loss can be generated by AC current or applied field, which generates an equivalent AC loss resistance [21] or dynamic resistance [22-27]:

$$R_{dyn} = \frac{2afl}{I_c}(B_a\text{-}B_{a,th}) \qquad (7)$$

Last but not least, crossed-magnetic-field effect [28] and flux cutting effect [29] can also contribute to the variation of resistance, and thus contributes to pumping. For type II superconductor, if the geometry of the branches is the same and they are in homogeneous AC magnetic field, $R_2(t)$/($R_1(t)$+$R_2(t)$) is also constant, so no DC voltage can be generated either.

3 Explanation of travelling wave flux pumps

Several existing traveling wave flux pumps can be considered as the realization of the proposed principle. For travelling wave based flux pumps, inhomogeneous AC magnetic field travels across the superconducting branches. In the following we explain it in two ways: travelling wave influences the resistance of branches because of field dependency of critical current density, and travelling wave influence dynamic resistance of the branches. As shown in Fig. 2(a), two different types of magnetic wave travels across a superconducting loop, which is formed by two branches infinite long into the paper. As shown in Fig. 2(b), the two branches experience symmetrical triangular wave field. During time zero to t_1, total flux in the loop increases, and at the same time, the flux density experienced by the left branch is greater than that experienced by the right branch; during time t_1 and t_2, total flux in the loop decreases, and at the same time, the flux density experienced by the left branch is smaller than that experienced by the right branch. Therefore, if we consider the field dependency of critical current density, according to Eq. (5) and Eq. (6), the ratio of branch resistances changes with flux variation. This process will generate a DC voltage across the branch. For a narrow rectangular wave, in which the distance between the rise edge and the falling edge is short than the distance between the branches, the process is shown in Fig. 2(c). During time t_1 and t_2 the total flux in the loop increases, and the flux density in the left branch goes up and down, which generate a hysteresis loss (which can be considered as a dynamic resistance since the field in left branch changes much faster than the current in the loop, otherwise it can be considered as an AC loss resistance). During time t_3 and t_4 the total flux in the loop drops, and the flux density in the right branch goes up and down, which generate a hysteresis loss. So the ratio of resistances in the branches changes during flux rising and falling, thus resulting in a DC component in the open circuit voltage.



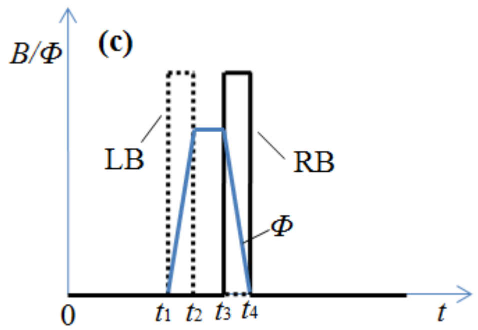


FIG. 2. Schematic drawing of Superconducting loop experiencing travelling magnetic wave. (a) two travelling magnetic fields with different waveforms are proceeding towards a superconducting loop, which is formed by two branches infinitely long into the paper. LB denotes left branch, and RB denotes right branch. (b) and (c), flux density experienced by the two branches against time, and total flux applied to the loop under symmetrical triangular wave and narrow rectangular wave respectively.

We developed a linear travelling wave based flux pump, as shown in Fig. 3(a).The flux pump consists of four pairs of copper poles which can generate travelling magnetic wave when the poles are powered alternatively, as shown in Fig. 3(b). Four parallel placed CC tapes experiencing travelling magnetic wave orientated perpendicular to their faces are connected to a CC load. The set up can be considered as two separate loops working in parallel. We tried a triangular magnetic wave with different dutyratios (i.e. we varied the ratio of the rise time to the period), the results are shown in Fig. 3(c). The load current can be well fitted by first order equations, which indicates the existence of a DC voltage in the flux pump. Results show that different waveforms end up with different load current. The difference between field rise time and fall time influences not only the final load current magnitude but also the polarity. For symmetrical waveform, which is similar with the description in Fig 2(b), a very small amount of current is pumped into the load. The result indicates that field dependency of critical current density is not the key influential factor in the experiment. Instead, it will generate more loss when the fast changing field edge is applied to a particular branch, thus changing that branches resistance more effectively. This is similar to the process described in Fig. 2(b), which becomes the dominant factor in the experiment.

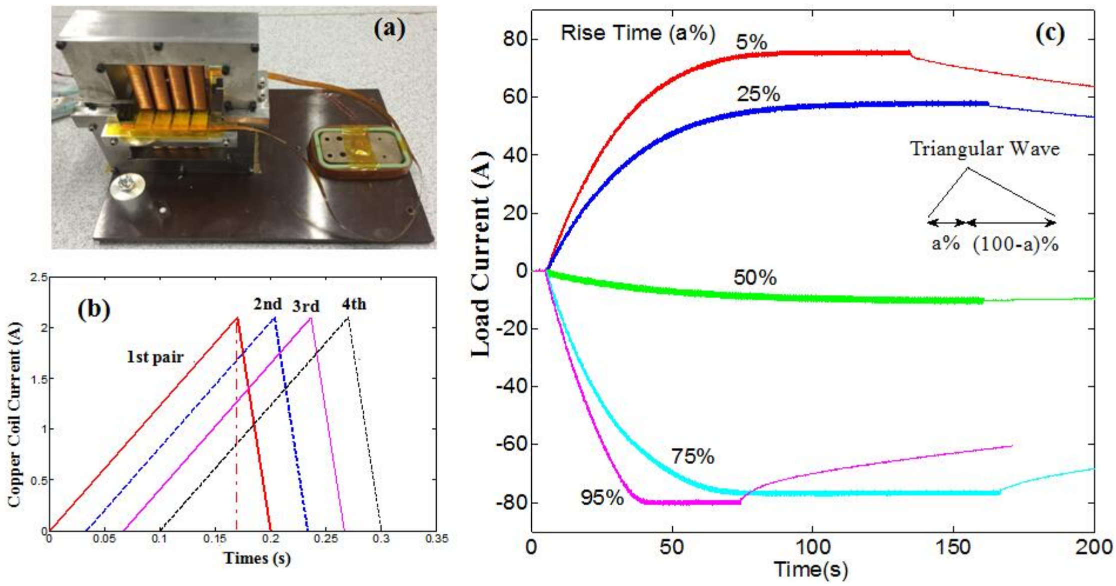


FIG. 3. Linear HTS flux pump device and flux pumping result. (a) The picture of the flux pump, which has 4 pole pairs that can generate a travelling magnetic wave. (b) The waveform of current in each pole pair. (c) The load current under different waveforms.

For moving magnets based flux pumps [10-14], if more than one piece of tape forms the superconducting loop, it is very similar to the description in Fig. 2(c). The published results [10, 12] show that the pumping speed is nearly proportional to rotating frequency, and the load current polarity is related to rotating direction, which proves our assumption. If only a single piece of tape experiences the moving magnetic field, it may also be considered as a loop as the induced currents will circulate within the tape [1]. In this case there is an additional factor as the size of the two branches is not fixed.

Our recent publication [17] is a very specific application of the proposed principle, although it is not based on travelling magnetic wave. As shown in Fig. 4, we used a transformer to input flux into the charging loop. A current with low frequency is induced. A high frequency perpendicular AC magnetic field is intermittently applied to a bridge (which will correspond to one branch from the above explanation), and this generates a dynamic resistance, as shown in Fig. 4. The very slight difference between this and the model described in Fig.1 is that the charging loop in Fig. 4 has a large inductance, but the substance is the same.

Fig. 4 can also be considered an analogue of travelling wave based flux pumps, but it has significant advantages. In a travelling wave based flux pump, a single travelling wave controls the flux variation in the charging loop or loops and the development of the resistivity in the branches. Since wavelength, branch geometry, travelling direction, and wave shape all influence the result, it is desirable to separate the charging mechanism from the one which induces the resistivity in the branches. This is achieved by an arrangement such as that shown In Fig. 4. In this way, we can achieve flexible control of the pumping speed and final load current.

4 Conclusion

In conclusion, we have revealed that nonlinear resistivity in type II superconductors is the origin of the DC component of the open circuit voltage of travelling wave flux pumps. Because the resistivity of type II superconductors is influenced by the magnitude and frequency of the applied field and current, when magnetic fields with different magnitudes and frequencies in space are applied to a superconducting loop, there is different and differing resistivity around the loop. This results in a DC open circuit voltage and is the origin of flux pumping in these devices. In a type I superconducting flux pump, a normal spot is used to transport flux into a superconducting loop, whereas in a type II or travelling wave type flux pump the normal spot is not necessary. The nonlinear resistivity property is particularly evident in High Temperature Superconductors making them ideal candidates for this type of pump.

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