Multi-Purpose Consumption and Functional Differentiation : Why has the Vibrant Galleria replaced the Good Old Fashioned Department Store?*

Susanna E. Sällström Matthews†

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Abstract

A very striking change in product selection over the last century has been the increased degree of specialisation of durable goods. To analyse these changes this paper introduces a new form of product differentiation called *functional*. It is shown that when a homogeneous population demands multiple locations (rather than consumers being heterogeneous) several standard results are reversed. A monopoly has an incentive to offer excessively specialised goods and delay innovation. It is in a duopoly that product characteristics will be efficient. Entry of a third firm will be more profitable in the fringes. Furthermore entry results in too much variety. Finally, the paper presents a novel argument in favour of bundling.

Key words: entry, innovation, optimum diversity, functional differentiation, bundling

JEL Classification: D42, D43, O31, L11

Consumers use durable goods for various purposes under different conditions. For example a pair of shoes can be used to walk in the street, run in the forest or hike in the mountain under different weather conditions. A century ago most people would have used the same pair of shoes for all activities, whereas an increased supply of specialised varieties indicate that consumers to a larger extent use different shoes when walking, running, cycling or hiking nowadays. Since it is annoying using a good that is not fit for purpose

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†Faculty of Economics, University of Cambridge, and St John's College, Cambridge CB2 1TP, UK. Phone: +44 1223 337721, Fax: +44 1223 335475, Email: SE.Sallstrom@econ.cam.ac.uk.

but costly to get several specialised varieties to match different conditions, the hypothesis would be that the higher the real income of the consumer, the more specialised will his consumption be, e.g. a special outfit for each activity he does. Thus an increase in the real income should result in an increase in demand for functional differentiation even if consumers are identical. The question is how the supply side will respond to this. Is it the department store or the galleria that will take the lead? Will there be any distortions in product selection? Why are mobile phones[1] bundles of an increasing number of functionally differentiated varieties, whereas shoes are not? What should the policy towards bundling of functionally differentiated goods be?

Hitherto the literature on product differentiation has confined attention to single purpose consumption, i.e. situations when the utility from a good for one particular consumer is independent on circumstances. Functional differentiation, in contrast, is a response to the multi-purpose nature of consumption of many durable goods. Existing models of product differentiation are therefore not directly applicable. However, I show that one can construct a general framework for the analysis of functional differentiation by introducing an annoyance function which is sub additive in the number of varieties. This function is constructed using building blocks from models of horizontal (Hotelling (1929)) and vertical differentiation (Mussa and Rosen (1978)) which makes the analysis directly comparable with previous work.

Functional differentiation differs from all other models of product differentiation in one important respect: it cannot be generated with a characteristics model and heterogeneous consumers.[2] On the contrary, this paper shows that it does matter whether the demand for differentiation is due to individuals having different needs to be met, or whether it is due to consumers being different. The reason for this difference is that all other models are based on the assumption of single-purpose consumption, including the love of variety approach.[3] With single-purpose consumption there will be no functional overlap and therefore it will not matter whether different consumers or the same consumer consumes the different varieties.

[1]Phone, text message, photograph and email are all examples of different varieties of a good that allows information to be communicated. Which one is preferred depends on circumstances.

[2]In an important paper Anderson et.al. (1989) showed that all existing models used in empirical and analytical work could be generated with a characteristics model and heterogeneous consumers, implying that the source to the demand for heterogeneity would not matter.

[3]Spence (1976) and Dixit-Stiglitz (1977) modeled preferences for love of variety. For these preferences all varieties are equally substitutable, whereas functionally differentiated goods are more or less substitutable depending on their functional overlap.

In particular, I show that two standard results are reversed. First, the incentive to differentiate too much horizontally, arises in a monopoly rather than in a duopoly.[4] Second, socially efficient locations are chosen in a duopoly rather than in a monopoly.[5]

Dynamically this has interesting implications. Whereas a monopoly will offer too few varieties unless it can bundle, a market with free entry will be characterised by too much variety.[6] Furthermore, the model predicts that entry will be more profitable in the fringes and more profitable than bundling. An incumbent firm can therefore not credibly deter entry by bundling. However, in the presence of barriers to entry the ability to bundle has positive effects on welfare. In a monopoly it takes away all the distortions in product space, leaving consumers' surplus unchanged. In the case of a duopoly there is a positive effect on welfare as well as consumers' surplus. This is because bundling has implications for the firm's choice of product characteristics, and which combinations can be supported as a Nash equilibrium as well as the number of varieties on offer.

This is a novel argument in favour of bundling. When there are high barriers to entry due to for example research and development, there will be less distortions in product selection over time if firms can bundle. Hence, by bundling their software Microsoft will have less distorted incentives to optimse the characteristics of each individual software, and to add new software at an efficient rate. This is an argument which complements the literature on bundling following Adams and Yellen's (1976). This literature has looked at various ways in which bundling enables a firm to extract more surplus for given product characteristics through leverage or price discrimination, or through strategic effects on price or quantity.[7] For functionanally differentiated goods bundling implies that the number of varieties will have optimal characteristics and be closer to the optimal number of varieties.

However, when there are no barriers to entry, bundling cannot be used as a credible

[4]The result with maximum differentiation occurs when the cost of transport is quadratic as was shown by d'Aspremont, Gabszewicz and Thisse (1979). Whereas the result in this paper is more general and thus applies both to the linear and the quadratic cases.

[5]For a heterogeneous population a monopoly would offer optimal locations. See Tirole (1989).

[6]This result is interesting since it differs from the conclusions in endogenous growth models based on the love of variety approach. Innovation of horizontally differentiated goods (Judd (1985), Romer (1990), Grossman and Helpman (1989,1991), Aghion and Howitt (1990)) results in too few varieties over time. However, a shared feature of my model, Reinganum (1982) one-shot patent race model, and Grossman and Helpman (1991)) is that the incentive to innovate is stronger for an entrant than the incumbent.

[7]Carbajo, de Meza and Seidmann (1990) look at the strategic effects on price and quantity, and find that the welfare effects will depend on the nature of product market competition. Whinston (1990) explored a justification for the leverage theory through its strategic implications.

commitment to deter entry, since it will be profitable to enter before it will be profitable to bundle. This is because there is a replacement effect for incumbent firms, even if they can bundle. This result can be compared with Klemperer and Padilla (1997), who showed that when consumers prefer to concentrate their purchases at a single supplier their new products may be introduced to foreclose competing firms from the market. The reason for this difference is that increasing the product line in their model does not reduce the price that can be charged for existing varieties, which it will in the case of functionally differentiated goods. Choi (1996) and Choi and Stefandis (2001) showed how the leverage theory could be understood through its impact on innovation in the case of complementary goods. In this case there will be a negative effect on consumers' surplus as well as welfare. Again an example of the opposite result, which is because functionally differentiated goods are both substitutes and complements, and bundling being used to enabling efficient characteristics rather than to deter entry.

The results in my paper are more general than standard models of horizontal differentiation based on Hotelling's approach since they only require the utility function to be sub-additive in the number of varieties. Hence, it does not matter whether the annoyance is linear or convex in the distance. The most important implication from sub-additivity is that it is not possible to extract all consumers' surplus from a set of varieties, since the consumer can choose to buy any subset. Hence, an equilibrium condition for the prices is that they have to be compatible with the consumer buying all varieties. This gives rise to a binding incentive compatibility constraint, not because of heterogeneous consumers, but because of the option of buying a subset.[8] The price that can be charged for a given variety will therefore be more constrained the larger the functional overlap with other available varieties.

The monopoly internalises the externality and therefore chooses maximum differentiation if it differentiates. The firms competing in a duopoly, on the other hand, do not internalise this externality on competitors price and as a result end up with optimal characteristics. However, the binding incentive compatibility constraint gives rise to another distortion in this case, which is that the duopoly will specialise too early. This is because the price they can charge depends on their competitor's equilibrium choice of product char-

[8]In the literature on quantity discrimination with high and low types, there will be different sized bundles and therefore an opportunity to buy a subset of the high quantity bundle. In particular, when the possibility to buy several small sized packages is allowed as in Alger (1999), who showed that this was necessary to get an unambiguous result regarding quantity discounts.

acteristics rather than the increase in surplus from moving from general to specific goods, which is what determines what is socially optimal.

The outline of the paper is as follows. I present an analytical framework for multi-purpose consumption and functional differentiation in Section 1. This is followed by a derivation of the social optimum in Section 2. Section 3 presents a strategic game between an incumbent monopoly and an entrant. This game is solved backwards in three stages: the price in sub section 3.1, characteristics in sub section 3.2, and finally number of varieties in sub section 3.3 Section 4 analyses bundling of functionally differentiated goods. Section 5 Addresses welfare issues. Section 6 discusses the results. There is an appendix with a summary of payoffs and how to calculate total annoyance.

1 Preferences for Multi-purpose Consumption

Consider needs such as protecting and supporting ones feet in various activities like dancing, cycling, hiking and walking under different conditions such as temperature, humidity and surface. Let these needs be summarized as exogenously given states $s \in [0,1]$ with distribution function $F(s)$ and continuous density $f(s)$. [9] Durable goods have characteristics which make them more suitable in some states and less suitable in other states. For example suede soles are perfect for ballroom dancing, but are a disaster outdoors on a wet surface, whereas rubber soles with good grip are ideal for hiking but would spoil the dancing experience.[10] Let these characteristics be summarized by $\theta_i \in [0,1]$ for variety i.

To use a good in a state that it is not perfectly suited for is annoying. The annoyance experienced by a consumer is a function of the state and the characteristics $a(z)$, where $z_i = \mid s - \theta_i \mid$. The consumer is happy when the characteristic exactly matches the state, $a(0) = 0$ and becomes increasingly annoyed the poorer the match, $a'(z) > 0$ at a constant or increasing rate $a''(z) \geq 0$.[11] If the consumer has n varieties, the consumer will use variety θ_i in all states in which it causes the smallest annoyance among available choices, i.e. $\theta_i = \arg\min\{a(\theta_j)\}_{j \in n}$. Let $\Theta_n = (\theta_1, \theta_2, \ldots \theta_n)$ denote a set of n different varieties,

[9] Thus the analysis does not include situations where the distribution of states is endogenously determined.

[10] Hence, the ranking of two varieties depends on the state, and no variety is preferred to all other varieties regardless of state.

[11] If the marginal annoyance increases with distance, e.g. the sunnier it is the more annoying it is on the margin to wear a raincoat, the function is convex $a''(z) > 0$. Note that the annoyance function is a more general representation of the cost of transport in models of spatial differentiation, and therefore encapsulates both the standard linear and quadratic cases.

where $\theta_i < \theta_{i+1}$. The average characteristics of two varieties is denoted $\bar{\theta}_{i,j} = \frac{\theta_i + \theta_j}{2}$. When needed θ_{nj} will denote the j'th variety when there are n varieties in total.

A consumer who is equipped with n varieties with characteristics Θ_n, will experience a total annoyance given by

$$A(\Theta_n) = \left[\int_0^{\bar{\theta}_{1,2}} a(z_1) f(s) ds + \int_{\bar{\theta}_{1,2}}^{\bar{\theta}_{2,3}} a(z_2) f(s) ds + \cdots + \int_{\bar{\theta}_{n-1,n}}^{1} a(z_n) f(s) ds \right].$$

This is a set function which is sub-additive in each variety.

Lemma 1 *$A(\Theta_n)$ is sub-additive, i.e. $A(\theta_1) + A(\Theta_n \setminus \theta_1) > A(\Theta_n)$.*

Proof:

$$A(\theta_1) + A(\Theta_n \setminus \theta_1) - A(\Theta_n) = \int_0^{\bar{\theta}_{1,2}} a(z_2) f(s) ds + \int_{\bar{\theta}_{1,2}}^{1} a(z_1) f(s) ds > 0. \tag{1}$$

Q.E.D.

This is an important property which follows from the multi-purpose nature of consumption. Since each variety could potentially be used in all states there will be a functional overlap between each variety. When a consumer owns several varieties, each variety will therefore be used less frequently but in states where it causes less annoyance. Hence, the consumer's utility from a set of goods depends on how well various states can be matched given the set of available varieties and their characteristics, rather than the goods *per se*.[12]

Functionally differentiated goods are therefore both complements and substitutes. The smaller the functional overlap the more complementary the varieties become, e.g. if a consumer owns a pair of cycling shoes that cannot be used to walk in, the more the consumer will value having a second pair of shoes for walking. Whereas when the functional over-lap is large the varieties are close substitutes.

The substitutability furthermore implies that the marginal value of an additional variety will be diminishing in the total number of varieties.

Lemma 2 *The larger the number of varieties the smaller the marginal reduction in annoyance from adding variety j, i.e. $A(\Theta_n \setminus \theta_j) - A(\Theta_n) \leq A(\Theta_{n-1} \setminus \theta_j) - A(\Theta_{n-1}) \forall n$*

Proof: Consider without loss of generality the marginal reduction in annoyance from adding variety 1, when all available varieties are being used except 1, is given by

$$A(\Theta_n \setminus \theta_1) - A(\Theta_n) = \int_0^{\bar{\theta}_{1,2}} [a(z_2) - a(z_1)] f(s) ds \tag{2}$$

[12]This is conceptually in line with Lancaster's (1966) characteristics approach to consumer demand.

If θ_1 is added to a subset $\Theta_{n-1} \subset \Theta_n$ instead, the reduction will be the same or larger depending on whether the closest substitute $\theta_2 \in \Theta_{n-1}$ or not.

$$A(\Theta_{n-1} \setminus \theta_1) - A(\Theta_{n-1}) = \begin{cases} \int_0^{\bar{\theta}_{1,2}} [a(z_2) - a(z_1)] f(s) ds & \text{if } \theta_2 \in \Theta_{n-1} \\ \int_0^{\bar{\theta}_{1,3}} [a(z_3) - a(z_1)] f(s) ds & \text{otherwise.} \end{cases} \tag{3}$$

Q.E.D.

The marginal value of an additional pair of shoes depends on how close the nearest varieties are. If a smaller subset contains the closest varieties the marginal reduction is the same, whereas if it does not, the marginal reduction in annoyance will be higher.

If an individual were to get an infinite number of different varieties to match all possible states, the individual would never be annoyed and gain maximum utility mV. However, for a finite number of varieties durable goods will be multi-purpose in use. Assume that $a(1) \leq V$, i.e. the individual gets non-negative utility even for the worst possible match, e.g. the individual is always better off using shoes whatever their characteristics to being barefoot in all states. The utility can then be written

$$U(\Theta_n) = m[V - A(\Theta_n)], \tag{4}$$

with the marginal change in utility from adding one variety equal to

$$U(\Theta_{n+1}) - U(\Theta_n) = m\left[A(\Theta_n) - A(\Theta_{n+1})\right]. \tag{5}$$

The parameter m captures how much the individual values the marginal reduction in annoyance.[13] Hence, $U(\cdot)$ is a set function with the same properties as the annoyance function.

1.1 Example

The general properties of these preferences can be illustrated for a uniform distribution of states, and linear annoyance. Consider $\Theta_3 = \{\theta_1, \theta_2, \theta_3\}$. The total annoyance from all possible combinations of these three goods is:

$$A(\theta_i) = a\left[\frac{1}{2} - \theta_i(1 - \theta_i)\right], \tag{6}$$

$$A(\theta_i, \theta_j) = a\left[\frac{1}{2} + \theta_i^2 - \theta_j(1 - \theta_j) - \left(\frac{\theta_i + \theta_j}{2}\right)^2\right], \tag{7}$$

$$A(\Theta_3) = a\left[\frac{1}{2} - \theta_3 + \sum_{j=1}^{3} \theta_j^2 - \frac{1}{4}\left[(\theta_1 + \theta_2)^2 + (\theta_2 + \theta_3)^2\right]\right]. \tag{8}$$

[13] This parameter is equivalent to the taste parameter in models of vertical differentiation. One interpretation (see e.g. Tirole (1989)) is that it is the inverse of the marginal utility of income, such that it will be higher the wealthier the consumer.

These can be used to illustrate sub additivity,

$$U(\theta_1) + U(\theta_2) - U(\theta_1, \theta_2) = m\left[V + \theta_1 - \frac{1}{2} - \left(\frac{\theta_1 + \theta_2}{2}\right)^2\right] \tag{9}$$

and

$$U(\theta_1, \theta_3) + U(\theta_2) - U(\Theta_3) = m\left[V - \frac{1}{2} + \theta_2 - \frac{\theta_2(\theta_2 + \theta_3) - \theta_1(\theta_3 - \theta_2)}{2}\right], \tag{10}$$

which are both positive since $V > a$. These expressions can furthermore be used to illustrate the marginal value of adding variety two to a bundle of one and two goods respectively.

$$U(\Theta_3) - U(\theta_1, \theta_3) < U(\Theta_2) - U(\theta_1) \tag{11}$$

subtracting $U(\theta_2)$ from both sides this can be written

$$U(\theta_1, \theta_3) + U(\theta_2) - U(\Theta_3) > U(\theta_1) + U(\theta_2) - U(\theta_1, \theta_2)$$

Hence,

$$m\left[V - \frac{1}{2} + \theta_2 - \frac{\theta_2(\theta_2 + \theta_3) - \theta_1(\theta_3 - \theta_2)}{2}\right] > m\left[V + \theta_1 - \frac{1}{2} - \left(\frac{\theta_1 + \theta_2}{2}\right)^2\right] \tag{12}$$

collecting terms

$$m\left[(\theta_2 - \theta_1 - \frac{1}{4}\left(\theta_2^2 - \theta_1^2 + 2\theta_3(\theta_2 - \theta_1)\right)\right] \tag{13}$$

which can be simplified further to

$$m(\theta_2 - \theta_1)\left[1 - \frac{1}{4}\left(\theta_2 + \theta_1 + 2\theta_3\right)\right] \geq 0 \tag{14}$$

which is strictly positive for $\theta_1 < 1$.

The expression is clearly decreasing in θ_1 and θ_3. For θ_2 there are two effects. Taking the derivative with respect to θ_2 gives,

$$m\left[1 - \frac{\theta_2 + \theta_3}{2}\right] \geq 0. \tag{15}$$

Thus it is increasing in θ_2 for $\theta_2 < 1$.

When two is added to a bundle with only variety one, it will be used in states for which it is best suited as well as in states where three otherwise would have been used.

To conclude. Preferences for multi-purpose consumption brings together three strands of the literature on product differentiation. The utility being a set function which is sub-additive, captures the notion introduced by Lancaster (1966) that consumers value a set of goods for the characteristics they jointly possess, rather than the goods *per se*. Furthermore the notion that consumers use durable goods under different conditions implies

that demand for different locations, i.e. horizontal differentiation, can arise even in a homogeneous population.[14] Finally, a bundle of specialised goods versus a general purpose good, is equivalent to choosing between a high quality good and a low quality good, where a consumer with a higher income would be willing to pay more for the the higher quality product as well as the improvement in quality as in Mussa and Rosen's (1979) model with vertically differentiated goods.

2 Social Optimum

Deriving the social optimum entails two steps: first a derivation of the optimal characteristics for a given set of n varieties; second, a derivation of the number of varieties which generates the highest surplus net of costs of producing n varieties.

Suppose that there is a constant marginal cost c_t for producing any variety. Hence, all locations cost the same and there are no economies of scale or scope, but we allow them to vary over time. Furthermore, let there be a time index on m_t to capture the notion that peoples ability to pay for greater comfort (i.e. less annoyance) can vary over time.

Optimal characteristics solve

$$\Theta_n^* = \arg\min A(\Theta_n) \tag{16}$$

since the utility is maximised for

$$\max_{\theta_i \in \Theta_n} U(\Theta_n) = m_t[V - A(\Theta_n)] \tag{17}$$

with first order conditions

$$-m_t A_{\theta_i}(\Theta_n) = 0. \tag{18}$$

When varieties are chosen optimally, the reduction in annoyance from adding an additional variety will be decreasing in the number of varieties.

Lemma 3 *The marginal reduction in annoyance from increasing the number of varieties is strictly decreasing in number of varieties when characteristics are chosen optimally,* $A(\Theta_{n-1}^*) - A(\Theta_n^*) > A(\Theta_n^*) - A(\Theta_{n+1}^*)$

Proof: Adding variety i will reduce annoyance in states where it is used instead of variety $i-1$ and variety $i+1$. Hence, the benefit is

$$\int_{\bar{\theta}_{i-1.i}^*}^{\theta_i^*} [a(z_{i-1}) - a(z_i)] + \int_{\theta_i^*}^{\bar{\theta^*}_{i,i+1}} [a(z_{i+1}) - a(z_i)] f(s) ds \tag{19}$$

[14] Note that the annoyance function is a more general formulation which includes both the linear and the quadratic cost of transportation case in Hotelling's (1929) Linear city model.

When n increases

$$\theta^*_{i+1} - \theta^*_{i-1}$$

decreases, if $a(\cdot)$ and $f(\cdot)$ are continuous. This has two implications. First annoyance will be reduced for a smaller number of states. Second, the varieties the new variety replaces will be closer substitutes, e.g. $\mid z_k - z_i \mid$ for $k = i-1, i+1$. Hence, the marginal benefit in each state will also be smaller. Q.E.D.

To illustrate let us consider the case where the marginal annoyance is constant and calculate the optimal characteristics for one, two and n characteristics.

In the case of one characteristic total annoyance can be written:

$$A(\theta) = a\left[\int_0^\theta (\theta - s)f(s)ds + \int_\theta^1 (s-\theta)f(s)ds\right]. \tag{20}$$

Integration by parts yields,

$$A(\theta) = a\left[\int_0^\theta F(s)ds + (1-\theta) - \int_\theta^1 F(s)ds\right]. \tag{21}$$

The first order condition for a minimum is

$$a\left[2F(\theta) - 1\right] = 0. \tag{22}$$

The second order condition for a minimum is

$$a2f(\theta) > 0. \tag{23}$$

Hence optimal characteristics for one variety are such that

$$F(\theta^*) = \frac{1}{2}. \tag{24}$$

Total annoyance for two varieties can be written

$$A(\theta_1, \theta_2) = a\left[\int_0^{\theta_1} (\theta_1 - s)f(s)ds + \int_{\theta_1}^{\bar{\theta}_{1,2}} (s-\theta_1)f(s)ds + \int_{\bar{\theta}_{1,2}}^{\theta_2} (\theta_2 - s)f(s)ds + \int_{\theta_2}^1 (s-\theta_2)f(s)ds\right]. \tag{25}$$

Integration by parts yields

$$A(\theta_1, \theta_2) = a\left[\int_0^{\theta_1} F(s)ds - \int_{\theta_1}^{\bar{\theta}_{1,2}} F(s)ds + (1-\theta_2) + \int_{\bar{\theta}_{1,2}}^{\theta_2} F(s)ds - \int_{\theta_2}^1 F(s)ds\right]. \tag{26}$$

The first order conditions for a minimum are

$$\begin{aligned} a\left[2F(\theta_1) - F(\bar{\theta}_{1,2})\right] &= 0, & (27)\\ a\left[2F(\theta_2) - F(\bar{\theta}_{1,2}) - 1\right] &= 0. & (28) \end{aligned}$$

The second order conditions for a minimum are

$$a\left[2f(\theta_1) - \frac{1}{2}f(\bar{\theta}_{1,2})\right] > 0. \tag{29}$$

and the determinant of the Hessian of second derivatives $detH > 0$. This condition is for example satisfied for the uniform distribution.

Combining first order conditions

$$F(\theta_2) - F(\theta_1) = \frac{1}{2}. \tag{30}$$

Hence, the frequency of different states determine the optimal characteristics.

Let $\Theta_n = \{\theta_1, \theta_2, \ldots, \theta_n\}$ denote a set with n varieties. Total annoyance can then be written

$$\begin{aligned} A(\Theta_n) &= a\left[\int_0^{\theta_1}(\theta_1 - s)f(s)ds + \int_{\theta_1}^{\bar{\theta}_{1,2}}(s - \theta_1)f(s)ds + \int_{\bar{\theta}_{1,2}}^{\theta_2}(\theta_2 - s)f(s)ds + \right. \\ &\left. \int_{\theta_2}^{\bar{\theta}_{2,3}}(s - \theta_2)f(s)ds + \cdots + \int_{\bar{\theta}_{n-1,n}}^{\theta_n}(\theta_n - s)f(s)ds + \int_{\theta_n}^{1}(s - \theta_n)f(s)ds\right]. \end{aligned}$$

Integration by parts yields

$$\begin{aligned} A(\Theta_n) &= a\left[\int_0^{\theta_1}F(s)ds - \int_{\theta_1}^{\bar{\theta}_{1,2}}F(s)ds + \int_{\bar{\theta}_{1,2}}^{\theta_2}F(s)ds - \int_{\theta_2}^{\bar{\theta}_{2,3}}F(s)ds + \cdots \right. \\ &\left. + (1 - \theta_n) + \int_{\bar{\theta}_{n-1,n}}^{\theta_n}F(s)ds - \int_{\theta_n}^{1}F(s)ds\right]. \end{aligned}$$

The first order conditions for a minimum are

$$a\left[2F(\theta_1) - F(\bar{\theta}_{1,2})\right] = 0, \tag{31}$$

$$a\left[2F(\theta_2) - F(\bar{\theta}_{1,2}) - F(\bar{\theta}_{2,3})\right] = 0, \tag{32}$$

$\vdots$

$$a\left[2F(\theta_n) - F(\bar{\theta}_{n-1,n}) - 1\right] = 0. \tag{33}$$

Combining first order conditions one gets

$$a\left[2\sum_{i=1}^{n}(-1)^{i+1}F(\theta_i) + (-1)^n\right] = 0 \tag{34}$$

The solution to this system of equations is denoted Θ_n^*, and is unique if the second order conditions for a minimum are satisfied. These are that the Hessian of second derivatives is positive definite, i.e. that all sub matrices along the diagonal are positive.

$$a\left[2f(\theta_1) - \frac{1}{2}f(\bar{\theta}_{1,2})\right] > 0. \tag{35}$$

When some states are more frequent than other states, the optimal set of varieties will be such that all varieties are used with the same frequency. For example a consumer who lives in a hot climate and thus mainly experience hot weather will optimally be equipped with varieties to cope with small variations in hot weather and use the warmest pair on the rare occasions of cold weather.

For a uniform distribution optimal characteristics are such that $z_i \leq 1/2n$. Hence $A(\Theta_n^*) = a\frac{1}{2}(\frac{1}{2n})^2 2n = a\frac{1}{4n}$, since total annoyance is then made up from $2n$ triangles of width and height $1/2n$. The marginal value of an additional variety is thus

$$A(\Theta_n^*) - A(\Theta_{n+1}^*) = a\left[\frac{1}{4n} - \frac{1}{4(n+1)}\right] = a\frac{1}{4n(n+1)} \tag{36}$$

which is clearly decreasing in n. Hence the marginal benefit from adding an additional variety is

$$A(\Theta_1^*) - A(\Theta_2^*) = a\frac{1}{4} - a\frac{1}{8} = a\frac{1}{8} \tag{37}$$

$$A(\Theta_2^*) - A(\Theta_3^*) = a\frac{1}{8} - a\frac{1}{12} = a\frac{1}{24} \tag{38}$$

$$A(\Theta_3^*) - A(\Theta_4^*) = a\frac{1}{12} - A\frac{1}{16} = a\frac{1}{48} \tag{39}$$

etc.

2.1 Optimal number of varieties

The optimal number of varieties solves

$$n^* = \arg\max\{U(\Theta_n^*) - nc\}. \tag{40}$$

where the solution depends on m_t/c_t, i.e. the ratio between how much the individual values a reduction in annoyance, to what it would cost to reduce it by producing an additional variety. To see this note that one variety is better than no variety if

$$U(\Theta_1^*) - c_t > 0. \tag{41}$$

Two varieties is better than one if

$$U(\Theta_2^*) - 2c_t \geq U(\Theta_1^*) - c_t. \tag{42}$$

Or more generally,

$$U(\Theta_n^*) - nc_t \geq U(\Theta_{n-1}^*) - (n-1)c_t. \tag{43}$$

This implies that there exists critical values of m/c, which is when m/c equals the inverse of the marginal change in annoyance from an additional variety. Let $r = m/c$, then

$$r_1^* = \frac{1}{V - A(\Theta_1^*)} \tag{44}$$

$$r_2^* = \frac{1}{A(\Theta_1^*) - A(\Theta_2^*)} \tag{45}$$

$$r_n^* = \frac{1}{A(\Theta_{n-1}^*) - A(\Theta_n^*)}. \tag{46}$$

Since $r_{n+1}^* > r_n^*$ follows from the property that the marginal utility of an increase in the degree of optimised specialisation decreases with the number of varieties, there will exist an optimum. [15] The optimal degree of specialisation at time t is to produce n varieties for $r_t \in \{r_n^*, r_{n+1}^*\}$.

For the uniform distribution one gets

$$r_n^* = \frac{4n(n-1)}{a}. \tag{47}$$

Hence, as real income r_t goes up, consumers should optimally consume a larger number of specialised varieties. However, is it the department store or the galleria that will lead and move from general purpose varieties to specialised varieties?

3 Strategic product selection

What are the incentives to innovate in imperfectly competitive markets when durable goods are multi-purpose? Does an incumbent monopolist have an incentive to innovate and offer more specialised varieties at an optimal rate, and with optimal characteristics? Furthermore does the possibility of selling a specialised variety to match states in which the currently available varieties do poorly, open up opportunities for profitable entry? If, yes, what is the optimal response of an incumbent monopolist to such entry?

All these issues can be covered in one model which is a game between an incumbent monopolist and a potential entrant.

The players are:

- I - Incumbent

[15] The intuition behind this property is that the scope for improving the match in specific states will be smaller the larger the number of varieties designed to cover that subset of states. Hence, the marginal utility from serious hiking boots will be smaller the larger the number of varieties already selected to cater for rough surfaces outdoors.

- E - Entrant

The timing of decisions is:

1. E decides whether or not to enter
2. I observes E decision, and I and E choose number of varieties $X_j \in \{0, 1, 2\}$, $j = I, E$.[16]
3. I and E observe X_j, $j = E, I$ and choose characteristics $\theta \in [0, 1]$ of each variety in X_j, $j = I, E$.
4. I and E choose the price p_j of each variety.

The objective of each firm is to maximise its profit

$$\Pi_j(X_j, X_i) = \sum_{k=1}^{X_j} (p_k - c)d_k, \tag{48}$$

given the strategy of the other player. The demand for each variety is $d_k = \{0, 1\}$. Hence, the size of the population is normalised to one since consumers are identical and the marginal cost is assumed to be constant and independent of number of varieties produced by the same firm.[17]

This game is solved using backward induction. Thus we start by determining the profit maximising price.

3.1 Price

In this section it is shown how the equilibrium prices depend on the number of varieties and their individual characteristics.

If there is only one variety available the consumer will buy it as long as

$$U(\theta) - p \geq 0. \tag{49}$$

Hence, a monopoly can extract all surplus.

Now suppose that there are two varieties available. A consumer can choose to buy any combination it chooses, i.e. neither, both, or just one of them. The equilibrium constraints on prices are

$$U(\theta_1, \theta_2) - p_1 - p_2 \geq 0, \tag{50}$$

$$U(\theta_1, \theta_2) - p_1 - p_2 \geq U(\theta_1) - p_1, \tag{51}$$

$$U(\theta_1, \theta_2) - p_1 - p_2 \geq U(\theta_2) - p_2. \tag{52}$$

[16] $X_E = 0$ is equivalent to E not entering.

[17] I.e. there are no economies of scale or scope.

If all these constraints are satisfied the consumer will buy both varieties on offer. In this case it will not be possible to extract all consumers' surplus, since a consumer could always choose to buy only one variety. Adding (51) and (52) gives $2U(\theta_1, \theta_2) - p_1 - p_2 \geq U(\theta_1) + U(\theta_2)$. If (50) binds this gives $U(\theta_1, \theta_2) \geq U(\theta_1) + U(\theta_2)$. This is a contradiction since $U(\cdot)$ is sub additive. Hence, the equilibrium prices will be given by

$$p_j = U(\theta_1, \theta_2) - U(\theta_i), j \neq i. \tag{53}$$

Similarly for three available varieties, consumers could choose to buy one, two or three, which implies that there will be seven equilibrium constraints,

$$\begin{aligned}
U(\theta_1, \theta_2, \theta_3) - p_1 - p_2 - p_3 &\geq 0, && (54)\\
U(\theta_1, \theta_2, \theta_3) - p_1 - p_2 - p_3 &\geq U(\theta_1) - p_1, && (55)\\
U(\theta_1, \theta_2, \theta_3) - p_1 - p_2 - p_3 &\geq U(\theta_2) - p_2, && (56)\\
U(\theta_1, \theta_2, \theta_3) - p_1 - p_2 - p_3 &\geq U(\theta_3) - p_3, && (57)\\
U(\theta_1, \theta_2, \theta_3) - p_1 - p_2 - p_3 &\geq U(\theta_1, \theta_2) - p_1 - p_2, && (58)\\
U(\theta_1, \theta_2, \theta_3) - p_1 - p_2 - p_3 &\geq U(\theta_2, \theta_3) - p_2 - p_3, && (59)\\
U(\theta_1, \theta_2, \theta_3) - p_1 - p_2 - p_3 &\geq U(\theta_1, \theta_3) - p_1 - p_3. && (60)
\end{aligned}$$

The only constraints which can be binding without violating the others, are the last three ones. To see this note that adding (55),(56) and (57) gives

$$3U(\Theta_3) - 2\sum_{j=1}^{3} p_j \geq \sum_{j=1}^{3} U(\theta_j). \tag{61}$$

Adding (58),(59) and (60) gives

$$3U(\Theta_3) - \sum_{j=1}^{3} p_j \geq U(\theta_1, \theta_2) + U(\theta_1, \theta_3) + U(\theta_2, \theta_3). \tag{62}$$

If (54) is binding, (61) becomes $U(\Theta_3) \geq \sum_{i=1}^{3} U(\theta_i)$ which from Lemma 1 can be seen to be a contradiction. Similarly if (61) is binding, (62) becomes $3U(\Theta_3) + \sum_{j=1}^{3} U(\theta_j) \geq 2\left(U(\theta_1, \theta_2) + U(\theta_1, \theta_3) + U(\theta_2, \theta_3)\right)$. This can be rewritten as follows

$$3U(\Theta_3) - U(\theta_1, \theta_2) + U(\theta_1, \theta_3) + U(\theta_2, \theta_3) \geq U(\theta_1, \theta_2) + U(\theta_1, \theta_3) + U(\theta_2, \theta_3) - \sum_{j=1}^{3} U(\theta_j) \tag{63}$$

On the left hand we have the sum of the marginal increment in utility from adding each variety to a bundle of two, whereas on the right hand side we have the sum when each

variety is added to a bundle of one. From Lemma 2 follows that the right hand side must be strictly larger. Hence, we have arrived at a contradiction.

The results can be generalised to n varieties since they follow from general properties, i.e. sub-additivity, of the annoyance function. Thus equilibrium prices for n varieties must satisfy

$$p_j = U(\Theta_n) - U(\Theta_n \setminus \theta_j) = m\left[A(\Theta_n \setminus \theta_j) - A(\Theta_n)\right] \tag{64}$$

Hence, sub-additivity and free consumer choice imply that the larger the choice set available to the consumer the less surplus will be extracted in equilibrium.

The equilibrium price is equivalent to the reduction in annoyance in the states where the additional variety will be used in place of the existing ones that would otherwise have been used. This can be illustrated in the case of linear annoyance

$$\begin{aligned} p_j &= ma\left[\int_{\theta_{j-1}}^{\bar{\theta}_{j-1,j+1}}(s-\theta_{j-1})f(s)ds + \int_{\bar{\theta}_{j-1,j+1}}^{\theta_{j+1}}(\theta_{j+1}-s)f(s)ds - \int_{\theta_{j-1}}^{\bar{\theta}_{j-1,j}}(s-\theta_{j-1})f(s)ds\right. \\ &\quad \left. - \int_{\bar{\theta}_{j-1,j}}^{\theta_j}(\theta_j-s)f(s)ds - \int_{\theta_j}^{\bar{\theta}_{j,j+1}}(s-\theta_j)f(s)ds - \int_{\bar{\theta}_{j,j+1}}^{\theta_{j+1}}(\theta_{j+1}-s)f(s)ds\right] \\ &= ma\left[\int_{\bar{\theta}_{j-1,j}}^{\bar{\theta}_{j-1,j+1}}(s-\theta_{j-1})f(s)ds + \int_{\bar{\theta}_{j-1,j+1}}^{\bar{\theta}_{j,j+1}}(\theta_{j+1}-s)f(s)ds\right. \\ &\quad \left. - \int_{\bar{\theta}_{j-1,j}}^{\theta_j}(\theta_j-s)f(s)ds - \int_{\theta_j}^{\bar{\theta}_{j,j+1}}(s-\theta_j)f(s)ds\right] \end{aligned}$$

Integration by parts yields

$$\begin{aligned} p_j &= ma\left[\int_{\theta_j}^{\bar{\theta}_{j,j+1}}F(s)ds - \int_{\bar{\theta}_{j-1,j}}^{\theta_j}F(s)ds - \int_{\bar{\theta}_{j-1,j}}^{\bar{\theta}_{j-1,j+1}}F(s)ds + \int_{\bar{\theta}_{j-1,j+1}}^{\bar{\theta}_{j,j+1}}F(s)ds\right] \\ &= ma2\left[\int_{\theta_j}^{\bar{\theta}_{j,j+1}}F(s)ds - \int_{\bar{\theta}_{j-1,j}}^{\bar{\theta}_{j-1,j+1}}F(s)ds\right] \end{aligned}$$

For variety 1

$$p_1 = ma2\int_{\theta_1}^{\bar{\theta}_{1,2}}F(s)ds \tag{65}$$

whereas for variety n

$$p_n = ma\left[\theta_n - \theta_{n-1} - 2\int_{\bar{\theta}_{n-1,n}}^{\theta_n}F(s)ds\right] \tag{66}$$

The equilibrium prices are a function of characteristics as well as number of available varieties. Firms will therefore take this into account when they decide on number of varieties and their characteristics.

3.2 Characteristics

After having observed how many varieties the competitor has chosen to produce, firms simultaneously decide on characteristics of these varieties. There are two classes of sub games which will be considered in this section. First, the case with no entry, in which case the incumbent is a monopoly. Second the case with entry in which case there is a duopoly.

3.2.1 Monopoly

First, consider the sub games with $X_E = 0$. There are two possibilities here. The monopoly will either have chosen to produce one or two varieties.

Proposition 1 *If the monopoly offers only one variety it will have optimal characteristics, whereas if it offers two, they will be too specialised.*

Proof: For one variety the profit maximisation problem of the monopoly is

$$\max_{\theta} \Pi(1,0) = m[V - A(\theta)] - c. \tag{67}$$

F.o.c.

$$-mA_{\theta}(\theta_1) = 0 \tag{68}$$

which is satisfied for the socially efficient choice θ^*.

For two varieties it becomes,

$$\max_{\theta_1,\theta_2} \Pi(2,0) = p_1 + p_2 - 2c = m[A(\theta_1) + A(\theta_2) - 2A(\theta_1,\theta_2)] - 2c. \tag{69}$$

F.o.c.

$$\begin{aligned} m[A_{\theta_1}(\theta_1) - 2A_{\theta_1}(\theta_1,\theta_2)] &= 0 && (70)\\ m[A_{\theta_2}(\theta_1) - 2A_{\theta_2}(\theta_1,\theta_2)] &= 0 && (71)\\ & && (72) \end{aligned}$$

The second term is zero at θ_1^*, θ_2^*, whereas the first term reaches its minimum in between these two values. Hence, the first order condition reveals that the profit is decreasing in θ_1 whereas it is increasing in θ_2 at θ_1^*, θ_2^*. Thus, $\theta_1^M < \theta_1$, and $\theta_2^M > \theta_2^*$. Q.E.D. This result can be illustrated for linear annoyance,

$$\begin{aligned} \Pi(2,0) &= ma\left[\sum_{i=1}^{2}\left(\int_0^{\theta_i}(\theta_i - s)f(s)ds + \int_{\theta_i}^{1}(s-\theta_i)f(s)ds\right) - \right. && (73)\\ & \left. 2a\left(\int_0^{\theta_1}(\theta_1 - s)f(s)ds + \int_{\theta_1}^{\bar{\theta}_{12}}(s-\theta_1) + \int_{\bar{\theta}_{12}}^{\theta_2}(\theta_2 - s)f(s)ds + \int_{\theta_2}^{1}(s-\theta)f(s)ds\right)\right] - 2c && (74) \end{aligned}$$

which can be simplified to

$$\Pi(2,0) = ma\left[\int_0^{\theta_1}(\theta_2-\theta_1)f(s)ds+\int_{\theta_1}^{\bar{\theta}_{12}}(\theta_1+\theta_2-2s)f(s)ds+\int_{\bar{\theta}_{12}}^{\theta_2}(2s-\theta_1-\theta_2)f(s)ds+\int_{\theta_2}^{1}(\theta_2-\theta_1)f(s)ds\right]-2c. \quad (75)(76)$$

Integration by parts gives

$$\Pi(2,0) = ma\left[\theta_2-\theta_1+2\left(\int_{\theta_1}^{\bar{\theta}_{12}}F(s)ds-\int_{\bar{\theta}_{12}}^{\theta_2}F(s)ds\right)\right]-2c \quad (77)$$

Product characteristics maximise the profit if first order conditions are satisfied. These can be written

$$F(\bar{\theta}_{12})-F(\theta_1) = \frac{1}{2}, \quad (78)$$

$$F(\theta_2)-F(\bar{\theta}_{12}) = \frac{1}{2}. \quad (79)$$

Combining these gives

$$F(\theta_2)-F(\theta_1)=1. \quad (80)$$

Hence, the profit maxmising combination of characteristics is $\theta_1=0$ and $\theta_2=1$. Maximum differentiation allows the monopoly to charge the highest price for each variety since it minimises functional overlap. However, the monopoly may be better off not differentiating at all if it is not possible to bundle.

Compare the price the monopoly can charge if it offers one variety with the prices it can charge if it offers two. Can it charge more in total when it offers two varieties?

$$p_1+p_2-p^* = m[A(\theta_1^M)+A(\theta_2^M)-2A(\theta_1^M,\theta_2^M)-V+A(\theta^*)] \quad (81)$$

This is positive if

$$A(\theta^*)-A(\theta_1^M,\theta_2^M) > V+A(\theta_1^M,\theta_2^M)-A(\theta_1^M)-A(\theta_2^M). \quad (82)$$

Hence, a necessary condition for differentiation in a monopoly is that the increase in social surplus i.e. reduction in annoyance, the left hand side, is greater than the rent to the consumer.

If this condition is satisfied there exists an r^M,

$$r^M = \frac{1}{A(\theta_1^M)+A(\theta_2^M)-2A(\theta_1^M,\theta_2^M)-V+A(\theta^*)} \quad (83)$$

such that the monopoly differentiates for $r \geq r^M$. However, since $r^M > r_2^*$ the monopoly will innovate with a delay.

Note that $r^M > r_2$ if

$$A(\theta^*) - A(0,1) - [V + A(0,1) - A(0) - A(1)] < A(\theta^*) - A(\theta_1^*, \theta_2^*) \tag{84}$$

which is equivalent to stating that the maximum increase in social surplus, the right hand side, has to be greater than the increase in surplus for the monopoly minus the rent to the consumer, which by definition is true.

However, there are instances when the necessary condition will not be satisfied.

Proposition 2 (A conservative monopoly) *If all states are equally likely, the monopoly has no incentive to offer more specialised durable goods.*

Proof: With a uniform distribution of states there is no increase in social surplus when the monopoly differentiates since the annoyance is unchanged $A(0,1) = A(1/2)$. Hence, the monopoly will always be strictly worse off differentiating in this case. QED.

However, with a slight modification to the uniform distribution, the monopoly will innovate but with a delay.

Suppose that the various states are distributed according to the following distribution function $s \in [0,1]$

$$F(s) = \begin{cases} s\dfrac{1-h(1-2z)}{2z} & \text{if } s \in [0,z) \\ \dfrac{1-h(1-2s)}{2} & \text{if } s \in [z,1-z] \\ 1-(1-s)\dfrac{1-h(1-2z)}{2z} & \text{if } s \in (1-z,1] \end{cases} \tag{85}$$

with density

$$f(x) = \begin{cases} \dfrac{1-h(1-2z)}{2z} & \text{if } x \in [0,z) \\ h & \text{if } x \in [z,1-z] \\ \dfrac{1-h(1-2z)}{2z} & \text{if } x \in (1-z,1], \end{cases} \tag{86}$$

where $h > 0$ and $z \in (\frac{h-1}{2h}, \frac{1}{2})$. This is a symmetric distribution which becomes the uniform distribution for $h = 1$. For $h < 1$, it has two flat tails in the regions $s < z$ and $s > 1 - z$ with higher density than the flat middle section. Hence, it represents a situation where the consumer tends to be more frequently using the good in states with special needs rather than states with average needs. For example if the individual lives in a country where its

either seriously cold, or very hot, rather than constant drizzling rain. Whereas if $h > 1$ the opposite applies.

For this class of distribution functions total annoyance becomes

$$\begin{aligned} A(\theta_1) &= \int_0^z (\theta_1 - s)\frac{1-h(1-2z)}{2z}ds + \int_z^{\theta_1}(\theta_1 - s)hds + \quad (87)\\ &\quad \int_{\theta_1}^{1-z}(s-\theta_1)hds + \int_{1-z}^{1}(s-\theta_1)\frac{1-h(1-2z)}{2z}ds \quad (88)\\ &= \frac{1-h}{2}\left[1-z\right] + h\left[\theta_1^2 - \theta_1 + \frac{1}{2}\right] \quad (89)\end{aligned}$$

in the case of one variety and

$$\begin{aligned} A(\theta_1,\theta_2) &= \int_0^{\theta_1}(\theta_1 - s)\frac{1-h(1-2z)}{2z}ds + \int_{\theta_1}^{z}(s-\theta_1)\frac{1-h(1-2z)}{2z}ds + \int_z^{\bar{\theta}_{1,2}}(s-\theta_1)hds + \\ &\quad \int_{\bar{\theta}_{1,2}}^{1-z}(\theta_2 - s)hds + \int_{1-z}^{\theta_2}(\theta_2 - s)\frac{1-h(1-2z)}{2z}ds + \int_{\theta_2}^{1}(s-\theta_2)\frac{1-h(1-2z)}{2z}ds \end{aligned}$$

in the case of two.

If the tails of the distribution are thick and short enough, the monopolist will be able to extract enough surplus but it will happen with a delay.

Proposition 3 *Let $z < \frac{2}{7}$ and $h < \frac{4-14z}{5-14z}$ then there exists an*

$$r^M = \frac{1}{a\left[(1-h)(1-\frac{7}{2}z) - \frac{h}{4}\right]} \quad (90)$$

such that a monopoly offers two specialised varieties for $r > r^M$.

Proof: The monopoly will offer two specialised varieties if $\Pi(1,0) > \Pi(2,0)$. For the stepped distribution we have

$$\begin{aligned} A(\frac{1}{2}) &= a\left[\frac{1}{2} - \frac{h}{4} - \frac{1-h}{2}z\right] \quad (91)\\ A(0) &= a\left[\frac{1}{2} - \frac{1-h}{2}z\right] \quad (92)\\ A(1) &= a\left[\frac{1}{2} - \frac{1-h}{2}z\right] \quad (93)\end{aligned}$$

$$\begin{aligned} A(0,1) &= a\left[\frac{h}{4} + \frac{1-h}{2}z\right] \quad (94)\\ A(0,1/2) &= a\left[\frac{4-h}{16}\right] \quad (95)\\ & \quad (96)\end{aligned}$$

Monopoly profit with one good is

$$\Pi(1,0) = \left(\frac{1}{2}\right) = m\left(V - a\left[\frac{1}{2} - \frac{h}{4} - \frac{1-h}{2}z\right]\right) - c \tag{97}$$

Monopoly profit with two varieties sold separately at prices $p_1 = m\left[A(1) - A(0,1)\right]$, $p_2 = m\left[A(0) - A(0,1)\right]$ is

$$\Pi^M(0,1) = 2ma\left[\frac{1}{2} - \frac{h}{4} - 2\frac{1-h}{2}z\right] - 2c. \tag{98}$$

$$\Pi(2,0) - \Pi(1,0) = m\left[a\left(\frac{3}{2} - \frac{3}{4}h - \frac{5}{2}(1-h)z\right) - V\right] - c \geq 0 \tag{99}$$

Two specialised varieties generates higher profit than one general purpose if

$$r > \frac{1}{a\left[\frac{3}{2} - \frac{3}{4}h - \frac{5}{2}(1-h)z\right] - V} \tag{100}$$

the denominator is positive if

$$h \leq \frac{2\left(3 - 2\frac{V}{a}\right) - 10z}{3 - 10z} \tag{101}$$

$$z \leq \frac{3a - 2V}{5a} \tag{102}$$

Q.E.D.

Hence the distribution of states is crucial for whether the monopoly differentiates or not.

3.2.2 Duopoly

Now suppose that entry occurred, i.e. $X_E \geq 1$, so that there are two competing firms in the market.

If each firm will produce one variety, characteristics are chosen simultaneously by each firm to maximise

$$\max_{\theta_i} \pi_i = m[A(\theta_j) - A(\theta_i, \theta_j)] - c \tag{103}$$

which gives first order conditions

$$-mA_{\theta_i}(\theta_i, \theta_j) = 0, \tag{104}$$

i.e. the socially efficient ones.

Thus there is no distortion in characteristics. Each firm tries to minimise annoyance since this will maximise the consumers willingness to pay. The maximised payoff to each firm j is therefore $\Pi_i(1,1) = m[A(\theta_i^*) - A(\theta_1^*, \theta_2^*)] - c$ A firm makes non-negative profit if

$$r \geq r_2^E = \frac{1}{a\left[A(\theta_j^*) - A(\Theta_2^*)\right]}. \tag{105}$$

Proposition 4 *With free entry there will be too much variety in equilibrium.*

Proof: There will be too much variety if $r_2^E < r_2^*$, i.e. if

$$\frac{1}{a\left[A(\theta_{2j}^*) - A(\Theta_2^*)\right]} < \frac{1}{a\left[A(\theta_{11}^*) - A(\Theta_2^*)\right]} \tag{106}$$

which simplifies to $A(\theta_{2j}^*) > A(\theta_{11}^*)$. This is true by definition since θ_{11}^* minimises annoyance for one variety. Q.E.D. However, it is not going to be optimal to enter with a third variety before it is socially optimal to offer 2, i.e. $r_2^* < r_3^E$ which is equivalent to

$$A(\Theta_2^*) < \frac{A(\theta_{21}^*) + A(\Theta_3^*)}{2}. \tag{107}$$

This again is satisfied since annoyance is reduced at a decreasing rate when more varieties are added.

Next consider the case where one of the firms offers two varieties.

Proposition 5 *If one firm offers two varieties and the other firm only one, there is a unique Nash equilibrium in characteristics in which one firm produces variety θ_1^* and θ_3^*, and the other firm θ_2^*, making profits*

$$\Pi^*(2,1) = m\left[A(\theta_1^*, \theta_2^*) + A(\theta_2^*, \theta_3^*) - 2A(\theta_1^*, \theta_2^*, \theta_3^*)\right] - 2c, \tag{108}$$

$$\Pi^*(1,2) = m\left[A(\theta_1^*, \theta_3^*) - A(\theta_1^*, \theta_2^*, \theta_3^*)\right] - c. \tag{109}$$

Proof: If one firm has chosen to produce two varieties and the other one only one, there are two possibilities: the one who only sells one offers a 'general purpose' i.e. variety θ_2, or does a very specialised one i.e. θ_1. The proof involves showing that, the first is a Nash equilibrium whereas the latter is not.

In the first case, for the firm offering one general purpose variety with characteristics θ_2^*, there is no profitable deviation for $\theta \in [\theta_1^*, \theta_3^*]$ since the profit is by definition maximised for θ_2^*. Could the firm increase profit by offering something more specialised such as $\theta' < \theta_1^*$? Such a deviation would result in a profit

$$\Pi^{dev}(1,2) = m\left[A(\theta_1^*, \theta_3^*) - A(\theta', \theta_1^*, \theta_3^*)\right] - c. \tag{110}$$

Since $A(\Theta_3)$ is minimised for Θ_3^*, this profit is strictly less than the equilibrium profit.

Similarly for the firm offering two varieties. If it offers a one and a three, θ_1^* and θ_3^* maximise the profit by definition. The question is whether positioning to the left of the

general purpose variety would result in a higher profit, e.g. $\theta' < \theta_1^* < \theta'' < \theta_2^*$. This would result in strictly lower prices for θ^*,

$$p(\theta') = A(\theta'', \theta_2^*) - A(\theta', \theta'', \theta_2^*) < A(\theta_2^*, \theta_3^*) - A(\Theta_3^*) \quad (111)$$

$$p(\theta'') = A(\theta', \theta_2^*) - A(\theta', \theta'', \theta_2^*) < A(\theta_1^*, \theta_2^*) - A(\Theta_3^*). \quad (112)$$

Hence, the socially optimal characteristics do form a Nash equilibrium.

Can a situation where one firm offers one specialised variety and the other two specialised varieties at the other end be a Nash equilibrium?

Optimal characteristics if the firms anticipate that the single variety firm will produce variety one and the multi-product firm variety two and three would solve the following problem.

$$\max_{\theta_2,\theta_3} p_2 + p_3 - 2c = ma\left[\theta_3 - \theta_2 + 2\left(\int_{\theta_2}^{\bar{\theta}_{2,3}} F(s)ds - \int_{\bar{\theta}_{1,2}}^{\bar{\theta}_{1,3}} F(s)ds - \int_{\bar{\theta}_{2,3}}^{\theta_3} F(s)ds\right)\right] - 2c$$

$$\max_{\theta_1} p_1 - c = ma2\int_{\theta_1}^{\bar{\theta}_{1,2}} F(s)ds - c$$

First order conditions in this case are:

$$ma\left[F(\theta_{1,2}) - 2F(\theta_1)\right] = 0 \quad (113)$$

$$ma\left[-1 + 2F(\bar{\theta}_{2,3}) - 2F(\theta_2) + F(\bar{\theta}_{1,2})\right] = 0 \quad (114)$$

$$ma\left[1 + 2F(\bar{\theta}_{2,3}) - 2F(\theta_3) - F(\bar{\theta}_{1,3})\right] = 0 \quad (115)$$

Combining these conditions gives

$$1 - F(\theta_3) + F(\theta_2) - F(\theta_1) = \frac{F(\bar{\theta}_{1,3})}{2}. \quad (116)$$

The outcome is neither the socially optimal one nor the monopoly outcome for three varieties. This is, however, not a Nash equilibrium. The single good firm will have an incentive to deviate and offer a general purpose good instead.

This can be illustrated using the uniform distribution. In this case the solution is

$$\theta_1 = \frac{1}{12}, \quad (117)$$

$$\theta_2 = \frac{1}{4}, \quad (118)$$

$$\theta_3 = \frac{11}{12}. \quad (119)$$

However, the single good firm would do better for these choices of characteristics to produce a general purpose variety with characteristics $\theta = \frac{7}{12}$ instead of $\theta_1 = \frac{1}{12}$, since

$$\Pi(\frac{7}{12}, \frac{1}{4}, \frac{11}{12}) = ma2\left[\int_{\frac{7}{12}}^{\frac{3}{4}} \theta d\theta - \int_{\frac{5}{12}}^{\frac{7}{12}} \theta d\theta\right] - c = \frac{ma}{18} - c \quad (120)$$

which is clearly higher than

$$\Pi(\frac{1}{12},\frac{1}{4},\frac{11}{12}) = ma2\int_{\frac{1}{12}}^{\frac{1}{6}} \theta d\theta - c = \frac{ma}{48}. \tag{121}$$

Q.E.D.

The reason why one firm offering two adjacent specialised varieties is not an equilibrium is because this firm internalises the negative externality on the price and therefore has an incentive to differentiate these varieties as much as possible. However, the more differentiated they are, the more profitable it is for the competitor to offer a general purpose variety rather than a specialised variety, which is why this will not be a Nash equilibrium.

A similar argument can be used for the case where both firms produce two varieties. Again if they compete in product space by offering 1 and 3, and 2 and 4 respectively, there will be no profitable deviation, and the socially efficient characteristics will form a Nash equilibrium.

The results in this section are summarised in the matrix below.

X_I, X_E	Incumbent	Entrant
1,0	θ^*	-
2,0	θ_1^M, θ^M	-
1,1	θ_1^*	θ_2^*
1,2	θ_2^*	θ_1^*, θ_3^*
2,1	θ_1^*, θ_3^*	θ_2^*
2,2	θ_1^*, θ_3^*	θ_2^*, θ_4^*

The intuition for these results is that when firms are not choosing adjacent characteristics they do not internalise the negative externality from their choices on adjacent varieties. The price that they can charge will therefore be maximised for the socially optimal characteristics, which minimises the annoyance $A(\Theta_n)$. The only instance where a firm chooses adjacent varieties is when it is a monopoly producing two varieties. In this case product characteristics will be distorted and result in too much differentiation.

3.3 Number of varieties

The final sub games to consider are those in which the firms decide how many varieties to offer. In the sub game with no entry it was shown that the monopoly would delay or not differentiate at all depending on the distribution of states. In the sub game with entry the question is whether entry will be accommodated or not, i.e will the incumbent choose one or two varieties?

If an incumbent could credibly commit to offering two varieties this would deter entry. However, due to the replacement effect, the incentive to differentiate will be stronger for an entrant than an incumbent.

Lemma 4 *An incumbent can deter entry by offering two varieties if*

$$r < r^D = \frac{1}{A(\theta_1^*, \theta_3^*) - A(\Theta_3^*)} \tag{122}$$

Proof: entry is deterred if, the entrant would get a negative payoff

$$\Pi(1,2) = m\left[A(\theta_1^*, \theta_3^*) - A(\Theta_3^*)\right] - c < 0. \tag{123}$$

Q.E.D.

Lemma 5 *Deterring entry is a credible strategy if*

$$r > \bar{r} = \frac{1}{A(\theta_{31}^*, \theta_{32}^*) + A(\theta_{32}^*, \theta_{33}^*) - 2A(\Theta_3^*) - A(\theta_{22}^*) + A(\Theta_2^*)} \tag{124}$$

Proof: Threatening to offer two varieties rather than one if entry occurs is only credible if $\Pi(2,1) > \Pi(1,1)$, i.e. if

$$m[A(\theta_{31}^*, \theta_{32}^*) + A(\theta_{32}^*, \theta_{33}^*) - 2A(\Theta_3^*) - 2c > m[A(\theta_{22}^*) - A(\Theta_2^*)] - c. \tag{125}$$

Q.E.D.

Proposition 6 *If the states are uniformly distributed and the inconvenience is linear it is never credible to deter entry.*

*Proof:*This is true since one variety is strictly better than two for all parameter values,

$$\Pi(1,1) - \Pi(2,1) = ma\left[\frac{3}{16} - \frac{1}{6}\right] + c = \frac{ma}{48} + c > 0. \tag{126}$$

Q.E.D.

Hence, entry could potentially be deterred for $\bar{r} < r < r^D$. However, entry will already have happened at a point when it would be optimal to deter it.

Proposition 7 *At the point when entry becomes profitable r_2^E, it is not credible to deter it.*

Proof: This is true if $\bar{r} > r_2^E$, i.e. if

$$A(\theta_{21}^*) + A(\theta_{22}^*) - 2A(\Theta_2^*) > A(\theta_{31}^*, \theta_{32}^*) + A(\theta_{32}^*, \theta_{33}^*) - 2A(\Theta_3^*) \tag{127}$$

which follows from Lemma 3. Q.E.D.

The model also gives a clear prediction as to where entry would happen if a third firm were to enter

Lemma 6 *An entrant who positions himself in a fringe will get a higher profit than an entrant who positions himself in between the existing firms in product space.*

Proof: An entrant who positions himself as 3 in product space gets a higher profit since

$$m[A(\theta_1^*, \theta_2^*) - A(\Theta_3^*)] - c > m[A(\theta_1^*, \theta_3^*) - A(\Theta_3^*)] - c \tag{128}$$

$$A(\theta_1^*, \theta_2^*) - A(\theta_1^*, \theta_3^*) = a\left[\theta_3 - \theta_2 + 2\left(\int_{\bar{\theta}_{1,2}}^{\theta_2} F(s)ds - \int_{\theta_2}^{\theta^3} F(s)ds\right)\right] \tag{129}$$

Q.E.D. Due to there being a replacement effect, the incentive to innovate will always be stronger for an entrant than the incumbent.

Proposition 8 *Let $r_2^E < r \leq \bar{r}$, then there is a unique sub game perfect equilibrium in which E enters, and I and E chooses one variety each with socially efficient characteristics.*

Proof: If $r < \bar{r}$ entry will be accommodated. If $r > r_2^E$ it will be profitable to enter. Thus the Nash equilibrium conditions

$$\Pi^*(1,1) \geq \max\{\Pi_x(0,1), \Pi_x(2,1)\}, \tag{130}$$

are satisfied.

Q.E.D.

The reason why a monopoly could not credibly commit to offering two varieties to deter entry is because of the effect on equilibrium prices from such a strategy. For functionally differentiated goods there is a disincentive to differentiate because of the negative externality new varieties have on the general price level. With linear annoyance and a uniform distribution of states, the effect on price is so dramatic that even though the monopoly would be selling two varieties, the sum of the prices he could get for those, would be less than the price he could charge for one variety. This is regardless of whether there would be a competitor selling one variety or not.

4 Bundling

The results in the previous section were derived on the assumption that bundling was not feasible. This section investigates what will happen when bundling is feasible, in the case of monopoly and duopoly respectively. It turns out that the most important effect from bundling is on product characteristics and the incentive to offer more specialised goods, rather than its effect on price.

Lemma 7 *If a firm can bundle it will choose socially efficient characteristics regardless of whether it is a monopoly or a duopoly.*

Proof: The monopoly can charge $p_B(\theta_1, \theta_2) = U(\theta_1, \theta_2)$ for a bundle. First order conditions to profit maximisation are therefore $-mA_{\theta_i}(\Theta_2) = 0$, i.e. socially efficient.

The firm offering a bundle in a duopoly can charge $p_B(\theta_j, \theta_k) = m[A(\theta_i) - A(\theta_j, \theta_k, \theta_i)]$. First order conditions are again the socially optimal ones $-mA_{\theta_j}(\Theta_3) = 0$, since this is what minimises the annoyance. Q.E.D.

Bundling internalises the negative externality on price, and therefore takes away the incentive to distort product characteristics. This has two important implications. The first is that the number of varieties will be closer to optimal.

Corollary 1 *A firm who can bundle has a less distorted incentive to innovate and offer more specialised varieties.*

The second is that it makes it possible for a firm to offer two specialised varieties on one side of the market, which is something that could not be supported as a Nash equilibrium if sold separately due to the incentive to distort product characteristics as was shown in the previous section.

Lemma 8 *When the firm offering two varieties in an equilibrium with three varieties can bundle, any combination of varieties is a Nash equilibrium in characteristics space.*

Proof: From Proposition 5 follows that θ_2^* is a best response to θ_1^* and θ_3^*. This is also true if they are sold as a bundle, since the price that can be charged for variety 2 in equilibrium solely depends on the characteristics of the other available varieties and not their price.

If firm one offers a bundle with two specialised varieties at the same end of the market θ_1^*, θ_2^*, the question is whether the firm two who offers θ_3^* could do better by offering a variety $\theta' \in (\theta_1, \theta_2)$ instead. This would result in a price

$$p(\theta') = m[A(\theta_1^*, \theta_2^*) - A(\theta_1^*, \theta', \theta_2^*)] \tag{131}$$

which is strictly less than $p(\theta_3^*)$ since $A(\Theta_3^*)$ is minimised for the optimal characteristics. Q.E.D.

This result is vital, since it is only in the 'new' equilibria that bundling results in a higher price, for the firm who bundles as well as the firm who does not. Thus it is not bundling *per se* that allows firms to extract more surplus in this case, but the fact that bundling makes efficient bundles strategically feasible.

Proposition 9 *The firm can only extract more surplus if the bundle contains adjacent varieties.*

Proof: The prices that can be charged by the firm who offers a bundle are

$$p_B(\theta_1^*, \theta_2^*) = m\left[A(\theta_3^*) - A(\Theta_3^*)\right] \tag{132}$$

$$p_B(\theta_1^*, \theta_3^*) = m\left[A(\theta_2^*) - A(\Theta_3^*)\right] \tag{133}$$

Comparing these with the individual prices $p(\theta_i) = m[A(\theta_j^*, \theta_k^*) - A(\Theta_3^*)]$ it can be verified that $p_B(\theta_1^*, \theta_3^*) = p(\theta_1^*) + p(\theta_3^*)$ since

$$A(\theta_2^*) - A(\theta_1^*, \theta_2^*) = A(\theta_2^*, \theta_3^*) - A(\Theta_3^*) \tag{134}$$

and that $p_B(\theta_1^*, \theta_2^*) > p(\theta_1^*) + p(\theta_2^*)$ since

$$A(\theta_3^*) - A(\theta_2^*, \theta_3^*) > A(\theta_1^*, \theta_3^*) - A(\Theta_3^*) \tag{135}$$

which follows from Lemma 2. Q.E.D.

The difference in price between a bundle and the price that can be charged for each good when sold separately is the difference between adding that variety to a bundle of one and two goods respectively. If the closest variety in the smaller and the larger bundle is the same the marginal value of adding variety one is the same, and there are no gains from bundling. This will be the case if the varieties that are bundled are not adjacent. If they are adjacent, the price for the bundle will be higher since the marginal value of adding variety two to a small bundle will be higher than the value of adding it to a larger bundle. This is because there will be closed substitutes in the larger bundle.

When firms can bundle there is thus a stronger incentive to offer more specialised varieties in a duopoly. In particular bundling changes the prediction for the uniform case.

Lemma 9 *If the firms can bundle in a duopoly, offering one variety ceases to strictly dominate offering two when the states are uniformly distributed and annoyance is linear.*

Proof: For a uniform distribution the firm can charge

$$p_B(\theta_1, \theta_2) = a\left[\frac{13}{36} - \frac{1}{12}\right] = ma\frac{5}{18} \tag{136}$$

if it offers a bundle. It is better to bundle if $\Pi_B(2,1) - \Pi^*(1,1) > 0$ i.e.

$$ma\frac{5}{18} - 2c - \left[ma\frac{3}{16} - c\right] = ma\frac{13}{(12)^2} - c \geq 0. \tag{137}$$

Q.E.D.

More generally the incumbent prefers to offer a bundle with θ_1^*, θ_2^* if $r > r_B$ where

$$r_B = \frac{1}{A(\theta_{33}^*) - A(\Theta_3^*) - A(\theta_{22}^*) + A(\Theta_2^*)} \tag{138}$$

Whilst bundling implies a monopoly will specialise at an optimal rate, there will still be a delay in the duopoly.

Proposition 10 *If a monopoly can bundle it will innovate at an optimal rate. If a firm in a duopoly can bundle it will innovate earlier but still with a delay.*

Proof: Since the monopoly can extract all consumers' surplus when bundling it will specialise when $U(\theta_1^*, \theta_2^*) \geq U(\theta^*)$, i.e. when it is socially optimal.

It will be optimal to offer three varieties in a duopoly when one firm can bundle at r_B. it will be delayed if $r_B > r_3^*$, i.e. if

$$\frac{1}{A(\theta_{33}^*) - A(\Theta_3^*) - A(\theta_{22}^* + A(\Theta_2^*)} > \frac{1}{A(\Theta_3^*) - A(\Theta_2^*)} \tag{139}$$

This simplifies to

$$A(\theta_{33}^* > A(\theta_{22}^*) \tag{140}$$

which is true by definition, since annoyance for one variety is higher the more specialised the variety, and $\theta_{33}^* > \theta_{22}^*$ e.g. more specialised. Q.E.D.

Hence, bundling reduces the distortions in product selection in the monopoly and the duopoly. Does it also help the incumbent to credibly deter entry?

Proposition 11 *An incumbent monopolist will not deter entry even if it can bundle.*

Proof: It is only credible to deter entry if $r > r_B$. Since $r_2^E < r_2^*$, it follows from the proof of Proposition 10 that $r_B > r_2^E$. Hence, entry cannot be credibly deterred, since by the time it can credibly be deterred it will already have happened. Q.E.D.

Even though bundling makes entry deterrence more profitable, it is still not sufficient to make up for the overall reduction in prices needed to support three varieties in equilibrium.

To conclude. If there are no barriers to entry, and there are no economies of scope, functionally differentiated goods will not be bundled, since there is an incentive to enter before it becomes profitable to differentiate through bundling. However, if there are barriers to entry, the analysis shows that there is a strong incentive to bundle functionally differentiated goods. This incentive would be further strengthened by economies of scope. The added functions on mobile phones is an excellent example of a good satisfying both of these criteria. There are clearly economies of scope, and due to high sunk costs of research and development it is also an industry with high barriers to entry.

5 Welfare

In the previous sections it was shown that:

- A monopolist who can not bundle will offer inefficient characteristics with a delay.
- A monopolist who can bundle will offer efficient characteristics with no delay.
- If there is a second firm, bundling enables one firm to offer efficient characteristics with a delay.
- Specialisation due to entry will result in efficient characteristics with specialisation happening too early, e.g. too much variety.

The question is what are the effects on consumers' surplus. Consumers' surplus is given by

$$S = U(\Theta_n) - \sum_i^n p_i \tag{141}$$

Even though a monopoly who can bundle will maximise welfare since there will be no distortions in product selection, the consumer is left with zero surplus. When there are more than one firm three scenarios are of particular interest. First the one where there has been entry and each firm produces one variety,

$$S(1,1) = U(\theta_1^*) + U(\theta_2^*) - U(\Theta_2^*) = m\left[V + A(\Theta_2^*) - A(\theta_1^*) - A(\theta_2^*)\right]. \tag{142}$$

Second a situation where one firm offers a bundle, and the other firm one specialised variety

$$S_B(2,1) = U(\theta_3^*) + U(\theta_1^*, \theta_2^*) - U(\Theta_3^*) = m\left[V + A(\Theta_3^*) - A(\theta_3^*) - A(\theta_1^*, \theta_2^*)\right]. \tag{143}$$

Third a situation where three varieties are supplied due to entry of a third firm

$$\begin{aligned} S(1,1,1) &= U(\theta_1^*, \theta_3^*) + U(\theta_1^*, \theta_2^*) + U(\theta_2^*, \theta_3^*) - 2U(\Theta_3^*) & (144)\\ &= m\left[V + 2A(\Theta_3^*) - A(\theta_1^*, \theta_3^*) - A(\theta_2^*, \theta_3^*) - A(\theta_1^*, \theta_2^*)\right]. & (145) \end{aligned}$$

Whereas social welfare as well as the firm's incentive to specialise will depend on r, the consumers' surplus is independent on this factor. This is because more varieties will have a negative impact on prices which benefit the consumers.

These can be calculated for our running example which gives,

$$\begin{aligned} S(1,1) &= m\left[V - a\frac{1}{2}\right] & (146)\\ S(2,1) &= m\left[V - a\frac{4}{9}\right] & (147)\\ S(1,1,1) &= m\left[V - a\frac{11}{36}\right] & (148) \end{aligned}$$

Thus $S(1,1,1) > S(2,1) > S(1,1)$.

Consumers prefer free entry. If there are barriers to entry, such that there is only two firms in the market, bundling will have a positive effect on both welfare and consumers' surplus. This is because this will give firms an incentive to offer a larger selection of varieties at an earlier stage closer to optimum.

6 Discussion

This paper has shown that functional differentiation is distinct from other forms of product differentiation. Already in the simplest possible case with homogeneous consumers and constant returns to scale technology, several interesting results were derived. Hence, it is a model which opens up a new field of exploration into the world of products that are functionally differentiated. Apart from the fact that this is an important form of product differentiation empirically, it has also been proved in this paper to be of theoretical interest.

The paper also makes a conceptual contribution to the literature on product innovation, by adding a third class of successful innovations which encourages consumers to buy more goods rather than switching suppliers.[18] Functional differentiation is the introduction of specialised goods that are less suitable for general purpose, but more suitable for specific purposes, such as cycling shoes.[19] For the devoted cyclist this is a quality improvement and could therefore be treated as a combined horizontal and vertical improvement. However, the element not captured in a standard model of product innovation is that the consumer is likely to increase the overall consumption of shoes, rather than switching, since buying a pair of shoes that can only be used under one specific set of conditions for which shoes are required increases the total number of shoes needed to perform various functions. Thus there is an element of complementarity between goods specialised to match specific conditions, which arises as a result of the multi-purpose nature of consumption of several durable goods, such as computers, cycles, clothes and shoes. In this case successful product innovations entail identifying the various conditions under which the good will be used, and make varieties that match those conditions. Such innovations have two effects. First an increase in total

[18]The nature of successful product innovations is an issue that brings industrial economists and business strategists together (see Caves (1984)). Porter (1980) points out two ways for an innovation to be successful, either by improving quality (see e.g. Fudenberg et.al.(1983)) or to better match the taste of a market segment. In both cases some consumers will switch from one supplier to the new one.

[19]There are plenty of other examples e.g. a palm. Consumers own more and more computers of different sizes that match specific needs.

demand for the good, and second a shift from general purpose to specific purpose goods. If a consumer can afford two pairs of shoes, it is better to buy two specific purpose that complement one another, than one general purpose and one specific purpose that partly overlap in function.

This is a crucial difference from the love of variety approach that was initiated by Spence (1976) and Dixit and Stiglitz (1977) to study optimal product diversity under monopolistic competition. These preferences can be represented by the CES sub utility function[20], in which variety is valued per se and thus can be used to explain the increase in differentiation of goods that perform the same function, e.g. light summer clothes. There is in this case no reason to switch from one variety to two different new varieties, since all are equally substitutable. Thus there is a demand for variety as a result of taste for variety rather than a demand for variety to reduce the annoyance from using a variety under conditions for which it is less well suited, e.g. sandals in rain.

Multi-purpose consumption explains the success of companies who are able to identify the various ways in which a durable good could potentially be used, and manage to invent a variety which makes it perfect under one of those specific conditions. The introduction of such varieties lowers the profitability of existing goods, but increases the number of durable goods an individual decides to own.

Since the demand for specialised goods depends on the income of the individual, the model explains why the degree of specialisation and the over all consumption of durable goods has been increasing in parallel with the increase in real income over the last century.

Furthermore, it highlights why the good old-fashioned department store may be too conservative offering only multi-purpose varieties, whilst entry of independent suppliers with new specialised varieties in a galleria will induce existing suppliers to change their characteristics as well as result in an increase in the overall degree of specialisation. The prices in the galleria will also be more competitive in equilibrium. These are factors which can explain the success of gallerias at the expense of the department store.

The model also opens up for other interesting applications, such as the role of specialised goods when consumers are heterogeneous. For example, what is the optimal design, size range and pricing of baby clothes when consumers differ in terms of how much they are willing to pay for a perfect fit during their baby's first year? These are questions which are of theoretical as well as practical importance.

[20]See e.g. Helpman and Krugman (1985) chapter 6.

A Appendix

This appendix contains a summary of payoffs that are being used in the paper.

The payoff matrix for the three stage game is

	0	1	2
0	0,0	0, $\Pi^*(1,0)$	0, $\Pi^*(2,0)$
1	$\Pi^*(1,0)$, 0	$\Pi^*(1,1)$,$\Pi^*(1,1)$	$\Pi^*(1,2), \Pi^*(2,1)$
2	$\Pi^*(2,0)$,0	$\Pi^*(2,1)$, $\Pi^*(1,2)$	$\Pi^*(2,2), \Pi^*(2,2)$

where

$$\begin{aligned}
\Pi^*(0,X) &= 0 && (149)\\
\Pi^*(1,0) &= U(\theta^*) - c && (150)\\
\Pi^*(2,0) &= 2U(\theta_1^M, \theta_2^M) - U(\theta_1^M) - U(\theta_2^M) - 2c && (151)\\
\Pi^*(1,1) &= U(\theta_{21}^*, \theta_{22}^*) - U(\theta_{22}^*) - c && (152)\\
\Pi^*(1,2) &= U(\Theta_3^*) - U(\theta_{31}^*, \theta_{33}^*) - c && (153)\\
\Pi^*(2,1) &= 2U(\Theta_3^*) - U(\theta_{31}^*, \theta_{32}^*) - U(\theta_{32}^*, \theta_{33}^*) - 2c && (154)\\
\Pi^*(2,2) &= 2U(\Theta_4^*) - U(\theta_{42}^*, \theta_{43}^*, \theta_{44}^*) - U(\theta_{41}^*, \theta_{42}^*, \theta_{44}^*) - 2c && (155)\\
& && (156)
\end{aligned}$$

To calculate total annoyance for linear annoyance and a uniform distribution the following formulas can be used:

$$\begin{aligned}
A(\theta) &= \frac{1}{2} - \theta(1-\theta) && (157)\\
A(\theta_1, \theta_2) &= \frac{1}{2} + \theta_1^2 - \theta_2(1-\theta_2) - \left(\frac{\theta_1+\theta_2}{2}\right)^2 && (158)\\
A(\theta_1, \theta_2, \theta_3) &= \frac{1}{2} - \theta_3 + \sum_{j=1}^{3} \theta_j^2 - \frac{1}{4}\left[(\theta_1+\theta_2)^2 + (\theta_2+\theta_3)^2\right] && (159)
\end{aligned}$$

For the running example this gives,

$$\begin{aligned}
A(\theta_{11}^*) &= A(\frac{1}{2}) = a\frac{1}{4} && (160)\\
A(\theta_{21}^*) &= A(\frac{1}{4}) = a\frac{5}{16} && (161)\\
A(\theta_{31}^*) &= A(\frac{1}{6}) = a\frac{13}{36} && (162)\\
A(\theta_{21}^*, \theta_{22}^*) &= A(\frac{1}{4}, \frac{3}{4}) = a\frac{1}{8} && (163)\\
A(\theta_{31}^*, \theta_{32}^*) &= A(\frac{1}{6}, \frac{1}{2}) = a\frac{1}{6} && (164)
\end{aligned}$$

$$A(\theta_{31}^*, \theta_{33}^*) = A(\frac{1}{6}, \frac{5}{6}) = a\frac{5}{36} \quad (165)$$

$$A(\Theta_3^*) = A(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}) = a\frac{1}{12} \quad (166)$$

These can be used to calculate prices.

$$p_B(\theta_1, \theta_3) = ma\left[\frac{1}{4} - \frac{1}{12}\right] = a\frac{1}{6} \quad (167)$$

$$p_B(\theta_1, \theta_2) = ma\left[\frac{13}{36} - \frac{1}{12}\right] = a\frac{5}{18} \quad (168)$$

$$p(\frac{1}{4}) = ma\left[\frac{5}{16} - \frac{1}{8}\right] = a\frac{3}{16} \quad (169)$$

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