Price Informativeness and Corporate Investment: A Model of Costly Manipulation and Share Repurchases

Abstract

We characterize how managers can curb manipulation by signaling information via stock repurchases in an environment with costly short selling and feedback effects from trading to firms' access to capital. Without repurchases, manipulation coexists with informed trading at low shorting costs, reducing price informativeness and firm investment. Manipulation becomes less profitable as shorting costs increase, making prices more informative and boosting investment if speculators are less informed. Incentives to manipulate cease to exist at moderate short selling costs, but making shorting more costly reduces price informativeness and firm investment. Our model shows how contracts that pre-commit funding and condition future investment on stock prices can induce managers to eliminate manipulation through stock repurchases, facilitating optimal investment policies. An important policy implication is that when stock repurchases are allowed and shorting costs are low, short selling bans will typically reduce investment efficiency.

Keywords: short selling costs, stock manipulation, informed trading, corporate investment, share repurchases.

JEL classification: D82, D84, G14, G32.

1 Introduction

Downward price manipulation by unconstrained short sellers can distort the allocation of corporate resources across several dimensions (see Goldstein and Guembel (2008) and Goldstein et al. (2013)). Unsurprisingly, firm managers often seek to hinder the shorting of their companies' stocks (Lamont (2012)). While managers cannot control stock prices, they can influence trading. Unlike other agents, managers can affect the supply of company stocks going into the market: they can issue new shares and repurchase existing ones under company-sponsored programs. The latter allows managers to signal information to financial markets and counter the impact of actions such as the initiation of short positions by activist investors.[1]

This paper is the first to show how manipulative trading affects prices, contracts, and investment in a setting where firms can repurchase shares and speculators face shorting costs. It does so by integrating several real-world features of the stock market. Following a short sale, a trader must borrow the stock from a lender (e.g., a pension or index fund) to fulfill settlement obligations. Borrowing fees are a function of search and matching costs in the equity lending market (see Duffie et al. (2002) and Kolasinski et al. (2013)). They can be measurably high and constrain arbitrage activity (Porras Prado et al. (2016)). As we demonstrate, managers can be induced to repurchase shares and signal firm value to capital providers by contracts that pre-commit funds and condition future investment on stock prices, such as revocable credit lines (von Thadden (1995)), IPOs combining equity with warrants (Chemmanur and Fulghieri (1997)), and convertible debt (Chakraborty and Yilmaz (2011)). This prevents manipulation by speculators, leading to the implementation of optimal investment policies. The analysis we advance stands in contrast to the existing theoretical literature, which has looked at interactions between speculators in the absence of shorting costs and ignored firms' ability to repurchase their shares. Ignoring these considerations is problematic. We argue that it may lead to empirical modeling that misinforms regulators when adopting policies meant to improve market efficiency.

[1] A well-known case is the large short position taken on Herbalife by William Ackman's Pershing Square. The market capitalization of Herbalife dropped 38% after the announcement of Pershing's position. The company's managers responded by implementing an aggressive repurchase program to support equity prices.

Our theory builds on the framework of Goldstein and Guembel (2008) in which manipulation arises from profit that relies on the feedback effect of costless trading on managers' investment decisions and on multiple rounds of trade.[2] In our richer setting, managers can curb manipulation by signaling information via stock repurchases in an environment with costly short selling and feedback effect from trading to firms' access to capital (Baker et al. (2003)).[3] Stock repurchases by the firm may also be interpreted similarly to corporate insiders buying shares to signal corporate value and prevent attempts by short sellers to manipulate prices. This feedback effect arises in our model because managers derive private benefits from making investments and thus have incentives to undertake negative-NPV projects. This makes access to capital dependent on the information learned by capital providers from stock prices. Our optimal contracting model adopts the standard mechanism design approach in the presence of multiple parties, which requires the implementation of efficient investment by the existence of an equilibrium of the game induced by the contract that maximizes the firm value.[4]

We start our analysis by showing a non-monotonic relation between investment and short selling costs when firms cannot access pre-committed financing. Specifically, when shorting costs are low, an increase in costs curbs stock manipulation at the expense of hindering informed trading.[5] In this case, the effect of shorting cost on investment is negative if speculators are likely to possess private information about project fundamentals but positive if they are uninformed. Our model shows how the effect of short selling costs on investment varies across

[2]Our results are robust to adopting the novel one-round setting with non-uniform noise orders from Gao et al. (2020) that simplifies the manipulation analysis. We adopt the standard two-round framework of Goldstein and Guembel (2008) to clarify and contextualize the essential assumptions that drive our new results.

[3]In our setting, we can show that results on the ex-ante contracting (i.e., when buybacks are allowed) implementing efficient investment are robust to endogenizing short selling costs. Adding an equity lending market and jointly solving for the equilibrium stock price and the short selling fee bring significant modeling challenges beyond the scope of this paper (e.g., Blocher et al. (2013) and Atmaz et al. (2023)).

[4]We take the standard assumption of the literature on contract and mechanism design that the parties play the efficient equilibrium. The argument is that, since this is a focal equilibrium, it will likely be the played one (see Myerson (2009), Palfrey (1992), and Schelling (1980)). While we adopt the indirect mechanism, following the *evidence game* setup of Hart et al. (2017) allows us to show that our outcome is the same as the one obtained by the direct revelation mechanism.

[5]Shorting costs are affected by regulations designed to restrain shorting via price tests (Diether et al. (2009)) and outright bans (Beber and Pagano (2013)), as well as by policies that impact search and matching costs in the lending market via choices between over-the-counter *versus* centralized lending platforms (Chague et al. (2017) and Huszár and Prado (2019)) and caps on institutional ownership concentration (Porras Prado et al. (2016)).

dimensions relevant to the design of empirical research and regulation. It sheds new light on the received notion that constraining short sales prevents price manipulation while preserving informed trading when shorting costs are low — the case of the vast majority of stocks (see Porras Prado et al. (2016)).[6] Critically, this conjecture is incorrect when speculators are likely to possess relevant information for investment efficiency.

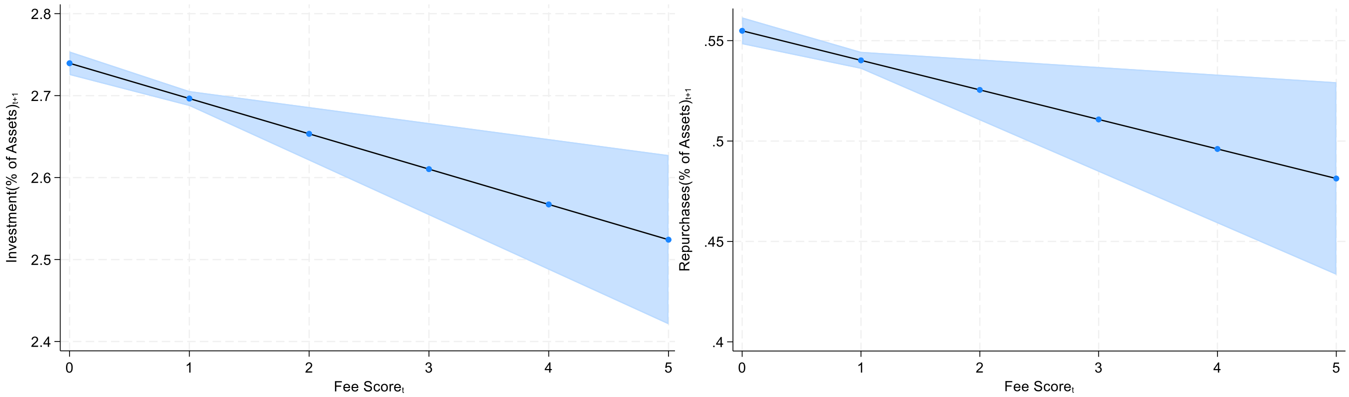
Next, our analysis shows how ex-ante contracting induces managers to curb manipulation via stock repurchases following decreases in shorting costs. This result reveals how financial policies commonly employed by managers impact market trading and ultimately allow for optimal investment funding. The relation between shorting costs and investment decreases monotonically when ex-ante financial contracting is feasible. These predictions rationalize two salient empirical correlation patterns shown in Figure 1. Namely, short selling costs are negatively associated with corporate investment (see panel A) and stock repurchases (panel B).[7] Our analysis further demonstrates how share repurchase programs are shaped by features such as stock liquidity and cash flow uncertainty. In light of drastic policy interventions on shorting activity adopted worldwide (see Beber and Pagano (2013)), our model's results show that short selling restrictions can be *detrimental* to market efficiency. They also point to the negative consequences of regulation restricting managers' ability to signal value through stock repurchases, such as "safe harbor" provisions in Rule 10b-18 of the Securities Exchange Act.

Our model encompasses two contracting settings. First, without ex-ante financial contracting, firms must raise reinvestment funds on the spot from investors who learn about project fundamentals through stock prices. This base setting is relevant since it closely captures the

[6]While securities lending transactions can be opaque and over-the-counter, several data providers collect data on shorting costs. This makes it easier for market participants (including managers and traders) to similarly assess those costs.

[7]Using data from 2005 to 2016, we estimate quarterly regressions, including firm and year-quarter fixed effects, of investment and stock repurchases (both scaled by assets) as a function of the Fee Score. This variable is computed from Markit data and measures short selling costs, where zero indicates the cheapest and five the most expensive stocks to borrow. Estimations include firm- and time-fixed effects in addition to a set of control variables: firm size (log of Compustat's ATQ), market-to-book ($(PRCC \times CSHO) \div CEQ$), cash flow ($IBQ + DPQ$) scaled by lagged assets, Amihud's (2002) stock liquidity measure, and the fraction of total institutional ownership. Marginal effects and associated confidence intervals are plotted in the figure. While these regressions only show correlations without providing evidence of causality and can potentially suffer from omitted variables bias, they motivate our theoretical work.

Figure 1. Estimated Effects of Short Selling Costs on Corporate Investment and Repurchases



(a) Investment

(b) Repurchases

This figure shows the estimated level of corporate investment (left panel) and stock repurchases (right panel) at different levels of Fee Score (a proxy for shorting costs) based on OLS-FE regressions. 95% confidence intervals are shown in grey.

circumstances of financially constrained firms — often young and opaque — that either cannot raise funds above investment outlays or do not have access to pre-committed financing.

In this base setting, uninformed speculators may profit by short selling the stock when shorting costs are low. The reason is that lower prices are interpreted as a negative signal about project fundamentals by market investors, who refuse to fund value-creating investment plans. This further reduces share prices and allows short sellers to cover their initial positions profitably. As shorting costs increase, the scope for profitable manipulation declines. Yet, an equilibrium with uninhibited informed trading is not obtained since, in such an equilibrium, prices are higher and give incentives for uninformed speculators to deviate and short the stock. As a result, manipulation incentives drive away informed trading. When shorting costs are moderate, manipulation incentives are no longer present. It follows that prices are more informative, and reinvestment plans are only canceled under selling pressure by negatively informed speculators. Finally, short sales may be unprofitable even for negatively informed speculators when shorting costs are high. The decrease in price informativeness raises the chance of value-destroying reinvestment, reducing profitability and constraining initial investment outlays.

Next, we extend the model with a richer framework featuring ex-ante contracting in which investors can pre-commit funds and make future payments contingent on stock prices. This induces managers to signal value to investors through stock repurchases for moderately low shorting costs, offsetting manipulation gains, and improving price informativeness and financing capacity. Notably, contracts allowing stock repurchases may not be optimal relative to spot refinancing for moderately high shorting costs. In this case, manipulation incentives are no longer present, but agency problems encourage managers to shore up stock prices against informed shorting. Inflated prices lead to financing value-destroying investment plans, reducing firm value and financing capacity. Capital providers anticipate this behavior and do not offer contracts that fund stock repurchases. Our model predicts that repurchases should occur less often as shorting costs become larger, consistent with recent evidence by Schultz (2023). We also expect that contracts allowing for stock repurchases are more likely to be used when doing so increases firm value.

As we discuss below, our model's results are consistent with empirical findings showing how companies implement anti-shorting strategies (Lamont (2012)) and modify managerial incentives (De Angelis et al. (2017)) to mitigate the effects of unrestrained short selling. Our model also reconciles evidence on the impact of shorting regulations on corporate investment (Grullon et al. (2015), Deng et al. (2023), Tsai et al. (2021), Chen et al. (2023), and Zhdanov et al. (2024)). It further predicts that firms will not obtain financing over investment needs if funds can be used to repurchase shares for managerial private gains. In many firms, managers can have enough capital to buy back shares but still need external financing. In these cases, our model implies that the board of directors has the incentive to restrict stock buybacks to prevent managers from using repurchases as a signaling device to fund inefficient investments and extract private benefits. These predictions find support in Bargeron and Bonaime (2020), which show that repurchases following increases in short selling are motivated by releases of positive private information rather than attempts at supporting over-valued prices (see also

Billet and Xue (2007) and Rubio (2019)). We conclude our analysis by discussing several novel testable hypotheses derived from our model.

We contribute to the literature on the feedback effects from prices to real decisions (see the literature review in Bond et al. (2012)). Our model brings stock repurchases to the forefront of the strand that studies the impact of short selling on price efficiency, investment, and firm value (e.g., Goldstein and Guembel (2008), Khanna and Mathews (2012), Goldstein et al. (2013), Cornelli and Yilmaz (2016), and Boulatov et al. (2020)). Critically, we depart from the analyses in these papers in important ways. While Khanna and Mathews (2012) focus on trading among uninformed speculators, we study strategic interactions between both informed and uninformed speculators. While Goldstein and Guembel (2008) focus on feedback effects generated by managers learning from stock prices, our richer framework allows for analyzing feedback effects engendered by management–creditor agency problems. In our setting, managers receive the same private signal as speculators and rely on financing from uninformed capital providers, who cannot perfectly monitor managerial actions and learn about the firm from stock prices. Accordingly, they design contracts that mitigate this agency problem, such as state-contingent financing. This integrates into the model a well-documented real-world feature: managers often attempt to signal firm value through stock repurchase programs (see Brav et al. (2005) and Bargeron and Bonaime (2020)). Compared to the existing papers, we push the literature forward by analyzing the simultaneous impact of short selling costs on multiple corporate policies.[8]

In many instances, regulators have tried to ban short selling to reduce market manipulation (Edwards et al. (2024)). While it helps informed investors to more precisely reveal their positive information by buying stocks, it prevents them from bringing their negative information to

[8]There is a large literature on the relation between differences of opinion and short selling. Disagreement can arise for several reasons. One comes from investors receiving different information signals that induce trading (e.g., pessimists trading with optimists as in Miller (1977) and Reed et al. (2021)) after they update their priors about fundamental value. A parallel between our work and this literature relates to the magnitude of trading demand by informed investors due to differences of opinion about the value of an investment between investors. While we do not endogeneize shorting costs, another parallel comes from how differences of opinion can affect the demand for shorting and the costs of short selling.

the market through short sales. The net effect of a shorting ban can be negative by reducing investment efficiency. We show that shorting bans will likely bring about investment inefficiency when shorting costs are low, as firms can curb manipulation through stock repurchases.

Investors' ability to engage in speculative shorting has long challenged our understanding of optimal capital market functioning. The effects of short selling are likely far-reaching, bearing real-side implications by shaping corporate financing and spending. Our analysis brings these connections to light. Among other new insights, it shows how the relation between shorting costs and investment hinges on stock price efficiency and firms' ability to participate in trading their own stocks via repurchase programs.

There are several costs associated with short selling, with lending fees paid by short sellers to borrow shares being one of them. These fees are an outcome of supply and demand in the securities lending market (Blocher et al. (2013) and Atmaz et al. (2023)). Fees can be endogenously affected due the firm's shareholders actions through trading shares or constraining supply, such as those by corporate insiders (Massa et al. (2015)), institutional investors (Attari et al. (2006)), and other informed traders (Subrahmanyam and Titman (2001)). Our model focuses on exogenous short selling costs that capture real-world exogenous-like situations such as taxes on short sales, differences arising from having centralized *vs.* over-the-counter lending markets, and disclosure requirements that impose costs on short sellers. In all, we contribute to the current regulatory debate on shorting activity stemming from exogenous shifts in shorting costs.

2 The Model

Our theory innovates on two dimensions. First, it explicitly shows how short selling costs affect price manipulation incentives and investment outcomes.[9] In particular, it reveals a non-monotonic relation between short selling costs and firm investment that has not been studied before. Second, while past models focus on financing and investment decisions after the firm's

[9]The literature on feedback models does not incorporate asymmetric costs between buying and shorting when studying the effect of information in stock prices on corporate policies. Goldstein and Guembel (2008), for example, examine a setting where short sales are costless.

shares are traded in the market, our model shows that ex-ante financial contracting can prevent manipulation and lead to optimal investment policies. Contracts that pre-commit funds and condition future investments on stock prices induce managers to repurchase stock and signal the firm's value to investors, offsetting potential manipulation by speculators. To the extent that ex-ante contracting is feasible, our theory yields the key prediction that stock investment and repurchases are decreasing in short selling costs.

2.1 Setup

The economy has four periods $t \in \{0, 1, 2, 3\}$ and a firm whose stock is traded in the financial market. Following Holmstrom and Tirole (1998), the firm has an investment opportunity that requires an investment of I in $t = 0$ and a reinvestment of K in $t = 3$. The value of the firm is given by $V(k, \omega)$, where $k \in \{0, K\}$ is the reinvestment policy and $\omega \in \{l, h\}$ is the state of the economy. If no reinvestment is made, the firm's value equals $V(0, \omega) = 0$. If the firm reinvests, it is worth $V(K, l) = V^- > 0$ when the state is "low" and $V(K, h) = V^+ > K > V^-$ when the state is "high." Both states are equally likely. The firm has no funds and must borrow from a risk-neutral capital provider that requires an expected rate of return of at least zero. Financial contracts are signed in $t = 0$, equity trading occurs in $t \in \{1, 2\}$, and spot financing takes place in $t = 3$.

In $t = 0$, the capital provider offers a contract $\mathcal{C}$ that maximizes firm value and yields a non-negative expected rate of return. Conditional on the verifiable information, the contract specifies: (i) if the investment is made; (ii) the amount of funds lent; (iii) if and when reinvestment occurs; and (iv) the share of the firm value that goes to the capital provider. Following Lin et al. (2019), the firm's manager derives a positive private benefit from investment which is a proportion $\delta \in (0, 1 - K^{-1}V^-)$ of the amount invested (empire-building motive). Thus, the capital provider faces an agency problem since, regardless of the state of the economy, the firm's manager always wants to inefficiently reinvest if enough funds can be raised. Contract terms are observable to all participants.

In $t = 1$, the state of the economy is realized, and the firm's stock begins to trade in the market. There are four agents in the equity market: (i) a risk-neutral speculator; (ii) a noise trader; (iii) the firm's manager; and (iv) a risk-neutral market maker. The manager and the speculator are privately informed about the state of the economy. To streamline the analysis, we assume they both observe the same signal $s \in \{l, h, \emptyset\}$. The results are qualitatively similar but much more intractable under less than perfectly correlated signals.[10] The signal s is perfectly informative ($s \in \{l, h\}$) with probability $\alpha \in (0, 1)$ and uninformative ($s = \emptyset$) with probability $1 - \alpha$. The size of order flows is fixed at $\pi > 0$. The speculator submits order flows $u_t \in \{-1, 0, 1\}$, which represent, respectively, the decision to: short, not trade, or buy π units of the firm's stock. We follow Glosten and Harris (1988) and take π as a proxy for the stock's liquidity, such that a lower π reflects more illiquid shares.

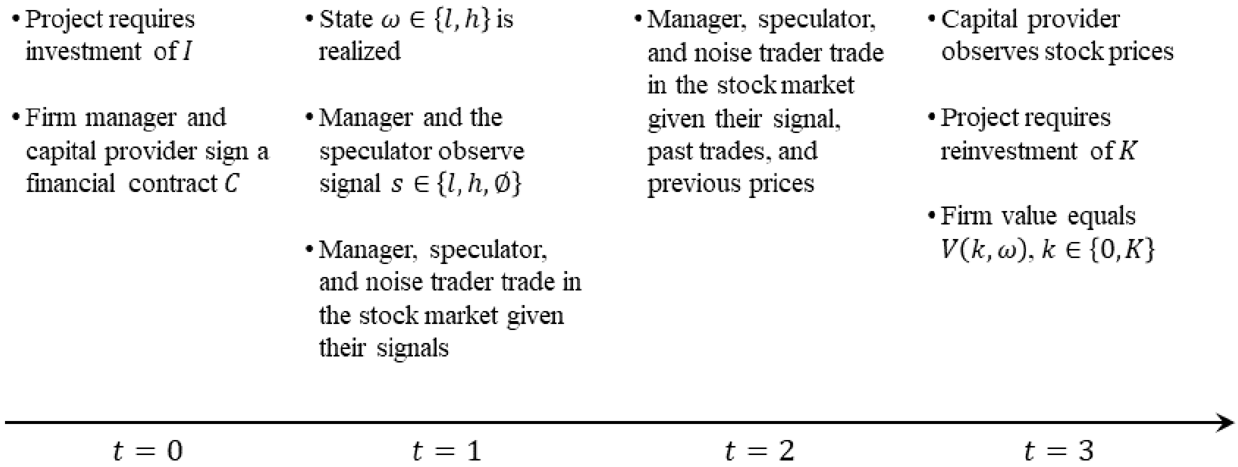
While buying by the speculator is unconstrained, shorting faces an observable fixed cost $c > 0$. This measure reflects searching and matching costs in the equity lending market (e.g., Duffie et al. (2002), Kolasinski et al. (2013), and Porras Prado et al. (2016)).[11] The noise trader does not act strategically and submits serially uncorrelated random orders $n_t \in \{-1, 1\}$ with equal probability. The manager is not allowed to short sell but may repurchase shares in the open market by submitting orders $r_t \in \{0, 1\}$.

As in Kyle (1985), orders are submitted simultaneously at each trading period to a market maker. The market maker observes only the aggregate order flow $Q_t = u_t + n_t + r_t$ and behaves competitively, setting the price p_t to earn zero expected profits conditional on all the public information available in t. It follows that $p_1 (Q_1, \mathcal{C}) = E [V (k, \omega) - k | Q_1, \mathcal{C}]$ and $p_2 (Q_1, Q_2, \mathcal{C}) = E [V (k, \omega) - k | Q_1, Q_2, \mathcal{C}]$. The speculator and the manager choose their trading strategies contingent on their own signals, past actions, and previously observed prices to maximize their payoff, given the price-setting rule. The capital provider only observes prices

[10] Due to positive private benefits derived from investment, the manager always wants to reinvest regardless of his information about the state of the economy. Relatedly, when the manager's signal is uninformative, he always has an incentive to try to reveal his type to avoid manipulation by potentially uninformed speculators. Therefore, our main results are very similar regardless of the signal structure.

[11] There is considerable dispersion in short selling costs across market participants (e.g., Kolasinski et al. (2013) and Chague et al. (2017)).

Figure 2. Game Timeline



and requires to at least break even in expectation to provide funds. A summary description of the game timeline is presented in Figure 2.

We restrict our attention to pure-strategy equilibria.[12] An equilibrium consists of the following: (i) a contract that maximizes firm value given the trading strategy of the speculator, the trading and reinvestment strategies of the manager and the price-setting rule of the market maker; (ii) the speculator's and manager's strategies are best responses to each other given the price-setting rule of the market maker; (iii) the price-setting rule of the market maker allows the market maker to break even given other players' strategies; and (iv) the beliefs of all players are consistent with all strategies and derived from Bayes' rule.

To make the model more interesting and to simplify its solution, we assume that reinvestment is sufficiently profitable in the absence of news but has negative NPV when the order flow reveals that the speculator is not informed about the high state:

Assumption 1 $\frac{1}{2} < \frac{V^{+}+V^{-}-2K}{V^{+}-K} < \alpha.$

[12]We note that the equilibrium impacts of shorting cost, the possibility of the speculator being informed, and stock repurchase on price informativeness and investment efficiency are qualitatively similar when mixed strategies are considered.

As in Goldstein and Guembel (2008), this assumption ensures that stock markets are sufficiently informative about investment fundamentals so that trades by an uninformed speculator play an important allocational role. It provides a sufficient condition for the existence of a feedback effect from the stock market to investment decisions. It also implies that firm value is lower without strategic trading than when the speculator trades only if informed about the high state. Since buy-side manipulation is not profitable and buy orders are not subject to costs, our analysis focuses on more efficient equilibria where the speculator informed about the high state buys in the second round of trading when the first round does not reveal her type.

2.2 Equilibrium *Without* Ex-ante Contracting

We begin with a benchmark case where the firm and its capital provider agree on their reinvestment arrangements in $t = 3$; after trading takes place (spot financing). Following the previous literature (e.g., Goldstein and Guembel (2008) and Goldstein et al. (2013)), we first establish the relation between investment and short selling *without* the possibility of stock repurchases. We subsequently investigate the role of financial contracting by allowing reinvestment arrangements to be made in $t = 0$; before markets are open. This allows managers to use funds to repurchase shares in $t = 1, 2$. Given our focus on how financial contracting can help firms to implement efficient investment policies (e.g., by setting up repurchase programs), we adopt the efficiency refinement for equilibrium characterization.

2.2.1 Equilibrium Characterization

Since the firm has no endowed funds in $t = 0$, the manager cannot repurchase stock when funding for reinvestment is only available after markets close. We use efficiency refinement for selection among multiple equilibria. Proposition 1 characterizes the most efficient trade and investment equilibrium strategies within the subset of equilibria where the speculator who is informed about the high state buys in $t = 2$ if her type is not revealed in $t = 1$ ($p_1 < V^+ - K$).[13]

[13] All model proofs are collected in the Appendix.

Proposition 1 *If $c < \frac{V^+ - V^-}{4}$, an equilibrium where the speculator who is informed about the high state buys in $t = 2$ when $p_1 < V^+ - K$ always exists. The most efficient trade and investment equilibrium strategies within this class of equilibria are characterized below.*

(i) *For $c < \underline{c} \equiv \frac{\alpha(V^+ - K)}{12}$, there is an equilibrium where the speculator informed about the high state buys in $t = 1$. In any equilibrium where the speculator informed about the high state buys in $t = 1$: the uninformed speculator and the speculator informed about the low state sell in $t = 1$ and sell again in $t = 2$ when $p_1 > 0$. Investment occurs if $I \leq \underline{I} \equiv \frac{3\alpha}{8}(V^+ - K) + \frac{1}{4}\left(\frac{V^+ + V^-}{2} - K\right)$.*

(ii) *For $\underline{c} < c < \widehat{c} \equiv \frac{V^+ - K}{12}$, in any equilibrium: no speculator trades in $t = 1$, the uninformed speculator does not trade in $t = 2$, and the speculator informed about the low state sells in $t = 2$. Investment occurs if $I \leq I' \equiv \frac{\alpha}{4}(V^+ - K) + \frac{2-\alpha}{2}\left(\frac{V^+ + V^-}{2} - K\right)$.*

(iii) *For $\widehat{c} < c < c' \equiv \frac{V^+ - K}{12} + \frac{V^+ - V^-}{6}$, there is an equilibrium where the speculator informed about the high state buys in $t = 1$. In any equilibrium where the speculator informed about the high state buys in $t = 1$: the uninformed speculator does not trade in $t = 1$ and the speculator informed about the low state sells in $t = 1$ and sells again in $t = 2$ when $p_1 > 0$. Investment occurs if $I \leq I^* \equiv \frac{3\alpha}{8}(V^+ - K) + \frac{4-3\alpha}{4}\left(\frac{V^+ + V^-}{2} - K\right)$.*

(iv) *For $c > c'$, in any equilibrium: no speculator trades in $t = 1$, the uninformed speculator does not trade in $t = 2$, and the speculator informed about the low state sells in $t = 2$. Investment occurs if $I \leq I'$.*

For low short selling costs ($c < \underline{c}$), both informed and manipulative short sales may occur. The latter happens when an uninformed speculator establishes a short position in $t = 1$ and then sells again in $t = 2$, when order flows in $t = 1$ do not reveal that she is not informed about the high state. Selling pressure in $t = 2$ may reduce the firm's access to financing, leading to the cancellation of reinvestment and driving firm value to zero in $t = 3$. The reason is that when prices reveal that the speculator is not informed about the high state, investors cannot distinguish between a speculator informed about the low state and an uninformed speculator, in which case the expected income is insufficient for financing to be arranged, $\frac{V^{-}+(1-\alpha)V^{+}}{2-\alpha} < K$. Manipulation results in a loss of c to the speculator in each trading period. However, the period-1 stock price is positive, as the market expects that the period-2 stock price may reveal that the speculator is informed about the high state, allowing the firm to raise funds for reinvestment. It follows that manipulation is profitable if short selling costs are small.

For moderately low short selling costs $c \in (\underline{c}, \widehat{c})$, short sales by uninformed speculators are no longer profitable in equilibrium. Yet, when only informed speculators trade, the expected firm value is higher when the order flow in $t = 1$ does not reveal the speculator's type. As a result, the period-1 price is large enough to compensate for the costs of establishing a short position in $t = 1$, creating manipulation incentives. Therefore, an equilibrium exists only in the absence of speculative trading in $t = 1$, in which case price informativeness declines. This result suggests that adopting even modest short selling costs may be enough to drive out informed trading and investment. It also highlights the relevance of a full analysis of the equilibrium consequences of marginal short selling costs on the behavior of both informed and uninformed speculators. Such analysis is missing from the existing literature, which either looks at the interaction between speculators in the absence of shorting costs (Goldstein and Guembel (2008)), or considers the effect of limits on trade sizes on the behavior of uninformed speculators alone (Khanna and Mathews (2012)).

Moderately high short selling costs ($c \in (\widehat{c}, c'))$ are enough to eliminate manipulation incentives by uninformed speculators but insufficient to reduce informed speculation. When costs on short sales become high $c > c'$, short selling in $t = 1$ is not profitable in equilibrium even for speculators informed about the low state. However, if only speculators informed about the high state trade in $t = 1$, the period-1 price becomes so high that it makes informed short sales attractive. It follows that an equilibrium exists only if no speculator trades in $t = 1$.

2.2.2 The Relation between Short Selling Constraints and Investment

We next assess the relation between short selling costs and *investment capacity*: the cutoff level of I below which investment occurs. This relation depends not only on the level of shorting costs (as described by Proposition 1), but also on the informativeness of the speculator's signal about fundamentals (α) as characterized below:

Proposition 2 *Consider the equilibria in Proposition 1. The relation between shorting costs and investment capacity depends on the informativeness of the speculator's signal as follows:*

(i) For $\alpha < \alpha' \equiv \frac{3\left(V^{+}+V^{-}-2K\right)}{2(V^{+}+V^{-}-2K)+V^{+}-K}$, $I^{*} > I' > \underline{I}$.

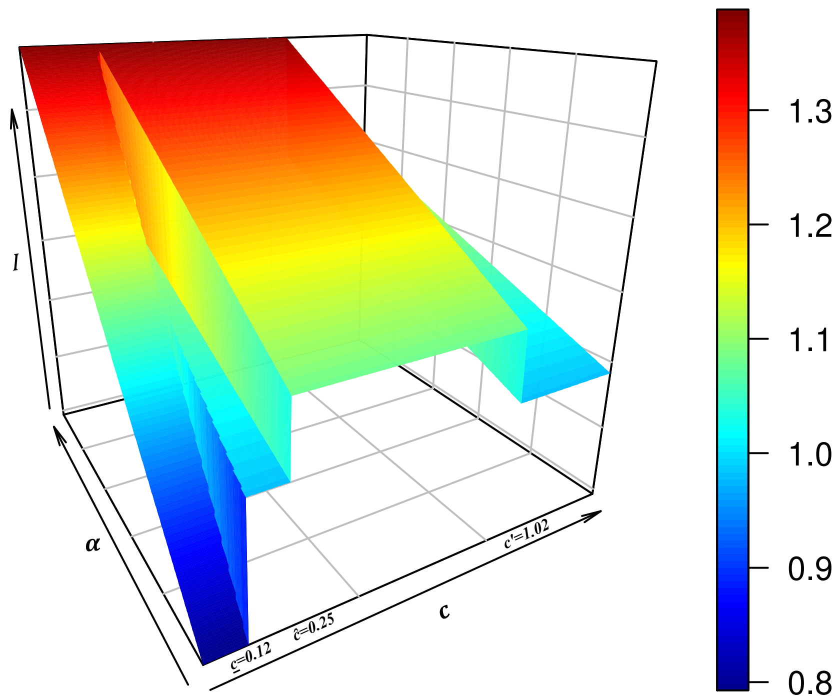
(ii) For $\alpha > \alpha'$, $I^{*} > \underline{I} > I'$.

(iii) As $\alpha \to 1$, $\underline{c} \to \widehat{c}$ *and* $\underline{I} \to I^{*}$.

Proposition 2 reveals that the effect of short selling costs (c) on investment capacity (I) is non-monotonic under spot financing. Critically, the sign of this effect is also a function of the probability α that speculators are informed about the true state of the world. In the benchmark case as $\alpha \to 1$, the region in which manipulation by an uninformed speculator occurs becomes maximal ($\underline{c} \to \widehat{c}$). Yet, the inefficiency resulting from manipulation vanishes since the speculator is almost always informed ($\underline{I} \to I^{*}$). As a result, increasing short selling costs always reduces investment capacity, since all it does is to drive away informed trading.

Figure 3 depicts the more general relation between short selling costs (c) and investment capacity (I) as a function of the informativeness of the speculator's signal (α). When the probability that speculators are informed is low ($\alpha < \alpha'$), the effect of short selling costs on investment is positive if costs are low to moderately high ($c < c'$), and negative if costs are moderately high to high ($c > \widehat{c}$). When the probability that speculators are informed is high ($\alpha > \alpha'$), the effect of short selling costs on investment is negative if costs are low to moderately low ($c < \widehat{c}$), positive if costs are moderate ($\underline{c} < c < c'$), and negative if costs are moderately high to high ($c > \widehat{c}$). We formalize these implications of Proposition 2 in a corollary, after which we discuss the intuition behind them.

Figure 3. Investment Without Ex-ante Contracting



This figure shows investment (I) as a function of short selling costs (c) and the probability that the speculator is informed (α) when funds for reinvestment are raised exclusively ex post (i.e., after prices incorporate speculators' information).

Corollary 1 *Under the equilibria in Proposition 1, the relation between investment capacity and short selling costs is non-monotonic. The sign of the effect of short selling costs on investment capacity depends on the informativeness of the speculator's signal as follows:*

(i) For low to moderately low cost levels ($c < \widehat{c}$), the sign is positive if the signal is less informative ($\alpha < \alpha'$), but negative if it is more informative ($\alpha > \alpha'$).

(ii) For moderate cost levels ($\underline{c} < c < c'$), the sign is positive regardless of the of the informativeness of the signal.

(iii) For moderately high to high cost levels ($c > \widehat{c}$), the sign is negative regardless of the informativeness of the signal.

(iv) In the limit as $\alpha \to 1$, the sign is negative. That is, investment is monotonically decreasing in short selling costs.

A common feature across all levels of accuracy of the speculator's signal is that investment capacity always increases when short selling costs become moderately high ($\widehat{c} < c < c'$) either from below or above. When short selling costs are moderately low ($\underline{c} < c < \widehat{c}$), manipulation is not profitable in equilibrium, but it drives away informed trading in $t = 1$. As a result, speculators informed about the high or low states are revealed less frequently by stock prices. As shorting costs become moderately high ($\widehat{c} < c < c'$), manipulation incentives are no longer present and informed trading takes place in both trading periods. Because of the higher trading frequency, stock prices more often identify speculators with perfectly informative signals, which improves investment efficiency. As short selling costs become high ($c > c'$), informed trading is unprofitable in $t = 1$ even in the absence of manipulation incentives. Thus, prices become less informative, and investment efficiency decreases.

Let us turn to the case in which shorting costs become low ($c < \underline{c}$) from above. In this case, manipulation becomes profitable and coexists with informed trading in both trading periods. Higher trading frequency allows speculators informed about the high state to be revealed more often, but manipulation makes it impossible to identify when speculators are uninformed. While the former effect boosts investment efficiency, the latter reduces positive net present value reinvestments. When the chance that the speculator is informed about the state is low ($\alpha < \alpha'$), the latter effect dominates and overall investment efficiency drops. On the flip side, when the probability that the speculator is informed about the state is high ($\alpha > \alpha'$), the former effect dominates and overall investment efficiency rises.

2.3 Equilibrium *With* Ex-ante Contracting

Our analysis thus far has shown that the manager's ability to finance investment after trading in financial markets depends on the capital provider's beliefs about the firm's value given observed stock prices. Prices are less informative and lead to underinvestment when short selling costs are either high or low to moderately low. Informativeness is low when short selling is constrained because prices do not often reflect the information of informed speculators. It is also low when short sales are relatively unconstrained due to manipulation by uninformed speculators. This raises the question of whether the impact of manipulation can be resolved by contracts that provide the firm with access to funding beyond investment needs, allowing the manager to repurchase stock in the market and signal firm value to investors.

2.3.1 Investment Policies of the Optimal Mechanism

We start deriving the outcome from a mechanism design that maximizes the investment capacity of the firm. The capital provider (principal) chooses a reinvestment policy $k(m)$ and assigns a reward $\rho(m) \equiv E[V(k(m), \omega) - k(m)|m]$ to the manager (agent) based on the received message m and the trade strategies in the financial market given the reward scheme. The information that can be revealed depends on the type of the manager with signal s, which is identified by the pieces of evidence that can be provided from the set E_s. Public firms disclosing false information is a criminal act and, while they may be capable to fully or partially disclose the available pieces of verifiable information, they might be allowed to withhold them. Thus, there are two possible scenarios depending on the available information and the pieces of verifiable evidence that can be disclosed. In the first one, the type of the informed manager $s \in \{l, h\}$ can fully reveal it, $E_s = \{s, \emptyset\}$, and in the second ones only partially so, $E_s = \{\{l, h\}, \emptyset\}$. In both cases, the type of the uninformed manager provides no verifiable evidence, $E_\emptyset = \{\emptyset\}$. It follows that the sets of possible messages given the type of the manager are the following: the informed manager $s \in \{l, h\}$ can either provide all the available evidence or no evidence ($m \in M_s = \{s, \emptyset\}$), whereas the uninformed speculator cannot provide any evidence ($m \in M_s = \{\emptyset\}$).

In our model, the manager always wants to obtain a reward as high as possible (including the investment private benefit) regardless of the obtained information. Furthermore, reinvestment is clearly optimal if the net present value when $k(m) = K$ is positive ($\rho(m) > 0$) and not optimal if negative ($\rho(m) < 0$). This is equivalent to the principal having a single-peaked utility $h(s) = -[\rho - E(V(K, \omega) - K|s)]^2$, so that the reward $\rho \geq 0$ maximizing the expected utility based on the probability $q_s(m)$ of the manager's type being s given m is the following:

$\rho = 0$ if $\sum_{s\in\{l,h,\emptyset\}} q_s(m)E\left(V(K,\omega) - K|s\right) < 0$, and $\rho = \sum_{s\in\{l,h,\emptyset\}} q_s(m)E\left(V(K,\omega) - K|s\right)$ if $\sum_{s\in\{l,h,\emptyset\}} q_s(m)E\left(V(K,\omega) - K|s\right) > 0$. This makes our setup equivalent to the *"evidence game"* of Hart et al. (2017), who show that the *"truth-leaning equilibrium"* yields the same unique outcome of the optimal direct mechanism.

Proposition 3 *The optimal mechanism outcome yields the following investment capacity:*

(i) When the perfectly informative signal $s \in \{l, h\}$ can by fully revealed by the manager disclosing all the available pieces of evidence ($E_s = \{s, \emptyset\}$), the investment capacity equals $\tilde{I} \equiv \frac{\alpha}{2}\left(V^+ - K\right)$ for all $c > 0$.

(ii) When the perfectly informative signal $s \in \{l, h\}$ can by only partially revealed by the manager disclosing all the available pieces of evidence ($E_s = \{\{l, h\}, \emptyset\}$), the investment capacity equals I^* for $c < c'$ and I' for $c > c'$.

From Proposition 3, it is clear that there is no equilibrium when $E_s = \{s, \emptyset\}$ in which information is fully revealed and investment is made if and only if the manager is informed about the high state or uninformed. Without further information from trade strategies, the manager informed about the low state has an incentive to pretend that it has no evidence as it would result in reinvestment. In the unique truth-leaning equilibrium, the manager informed about the high state provides full evidence, while the one informed about the low state provides no evidence and cannot be distinguished from the uninformed one. Based on the information disclosure depending on the manager's type, the trade strategies must reveal at least the same information so that the stock price reflects the reinvestment decision. Otherwise the market maker can obtain access to the disclosed pieces of verifiable evidence and identify when the speculator is informed about the high state. If the financial market can reveal more information, reinvestment is made when the manager is identified to be uninformed, and canceled when recognized to be informed about the low state. However, there is no trade equilibrium that reveals when the manager is uninformed. It follows that reinvestment is made if and only if the manager is informed about the high state, in which case the investment capacity equals $\tilde{I}$.

While informed managers reveal that they know the state in the unique truth-leaning equilibrium when $E_s = \{\{l, h\}, \emptyset\}$ and there is no further information from trade strategies, the fully disclosed verifiable evidence is not sufficient to identify the state. The trade strategies must at least reveal when the speculator is uninformed, since otherwise the market maker can identify when the speculator is uninformed by accessing the verifiable evidence disclosed by the manager. If the financial market can provide further information, reinvestment is made when the manager is revealed to be informed about the high state, and canceled when identified to be informed about the low state. It tuns out that the most efficiency trade equilibrium is the one in Proposition 1 (iii) for $c < c'$ and the one in Proposition 1 (iv) for $c > c'$. This yields the investment capacity equal to I^* for $c < c'$ and I' for $c > c'$. Moreover, the manager informed about the high state has no incentive to pretend to be uninformed (reinvestment would also always be made, but the reward would be lower). Furthermore, the manager informed about the low state does not have the incentive to pretend being uninformed: while reinvestment would always occur, the reward without deviation more than offsets the extra private benefit from deviation.

Given that the optimal investment capacity depends on the scenario of available pieces of evidence that can be disclosed, which scenario yields the highest investment capacity? Does it depend on the likelihood of the manager being informed about the state and the short selling cost? This comparison is formalized in the next corollary.

Corollary 2 *If the optimal mechanism investment capacity is higher when the perfectly informative signal $s \in \{l, h\}$ can by fully ($E_s = \{s, \emptyset\}$) or partially ($E_s = \{\{l, h\}, \emptyset\}$) revealed by the manager depends on the short selling cost and the chance of being perfectly informed:*

(i) If $\alpha < \underline{\alpha} \equiv \frac{2\left(V^+ + V^- - 2K\right)}{(V^+ + V^- - 2K) + V^+ - K}$, I^* for $c < c'$ and I' for $c > c'$.

(ii) If $\underline{\alpha} < \alpha < \overline{\alpha} \equiv \frac{4\left(V^+ + V^- - 2K\right)}{3(V^+ + V^- - 2K) + V^+ - K}$, I^* for $c < c'$ and $\tilde{I} \equiv \frac{\alpha}{2}\left(V^+ - K\right)$ for $c > c'$.

(ii) If $\alpha > \overline{\alpha}$, $\tilde{I}$ for all $c > 0$.

The main takeaway of Corollary 2 is that, while a scenario in which the manager can only partially reveal informative signals ($E_s = \{\{l, h\}, \emptyset\}$) has the benefit of identifying the uninformative ones, it has the cost of not fully identifying the positive ones. The benefit comes from not canceling investment when the manager is either informed about the high state or uninformed, while the cost arises from the positive chance of not canceling when the manager is informed about the low state. The cost is increasing in the short selling cost c. While a scenario allowing the manager to fully reveal informative signals ($E_s = \{s, \emptyset\}$) has the benefit of identifying the positive ones, it has the cost of not distinguishing the negative and uninformative ones. The cost is independent of the short selling cost c. The benefit comes from efficiently canceling investment when the manager is informed about the low state, and the cost is due to inefficiently canceling when the manager is uninformed. When the chance of the manager being informed increases (higher α), the benefit of scenario ($E_s = \{s, \emptyset\}$) increases and the cost decreases, while the benefit of scenario $E_s = \{\{l, h\}, \emptyset\}$ decreases and the cost increases. It follows that the investment capacity of scenario $E_s = \{\{l, h\}, \emptyset\}$ compared to the one of scenario $E_s = \{s, \emptyset\}$ is higher when chance of informative signal is low ($\alpha < \underline{\alpha}$), higher for low to moderately low shorting costs ($c < c'$) and lower for high shorting costs ($c > c'$) when the chance of informative signal is moderate ($\underline{\alpha} < \alpha < \overline{\alpha}$), and lower when chance of informative signal is high ($\alpha > \overline{\alpha}$).

The results of Proposition 3 and Corollary 2 post an important question: can the highest possible investment capacity be implemented by financial policies employed by managers in a given scenario, and intervention policies taken by regulators that can switch the scenarios?

2.3.2 Contractual Implementation of Efficient Investment Policies

Pre-committed funds are a necessary condition for contracts to implement the outcome of Proposition 1(iii) under moderately low to low short selling costs (i.e., $c < \widehat{c}$). However, they are not sufficient. Due to private benefits from investment, the manager always reinvests irrespective of the state of the economy. The resulting investment capacity equals $\widehat{I} \equiv \frac{V^{+}+V^{-}}{2} - K$, which is lower than that under manipulation since $\underline{I} - \widehat{I} =$

$\frac{3}{8}\left[\alpha\left(V^{+}-K\right)-\left(V^{+}+V^{-}-2K\right)\right]>0$. The reason is that prices still play an important allocational role in the presence of manipulation, as they reflect the information the speculator provided about the low state. Thus, the implementation of the outcome of Proposition 1(ii) requires contracts to condition reinvestment on stock prices, such that reinvestment is canceled whenever the expected income net of investment costs is negative. Under this contingency, reinvestment is less likely to occur after manipulative selling pressure by an uninformed speculator. Hence, an uninformed manager has the incentive to signal his information by supporting prices through stock repurchases since the manager's payoff is positive if and only if reinvestment takes place. This leads to the next proposition:

Proposition 4 *Consider the following contract:*

(i) The manager borrows an amount b_t (repays when $b_t<0$) in $t\in\{0,3\}$, where $b_0 = I+2\pi\frac{V^{+}+(1-\alpha)V^{-}-(2-\alpha)K}{2-\alpha}$ and $b_0+b_3=I+\sum_{t=1}^{2}r_t\pi p_t+k$.

(ii) Reinvestment occurs ($k=K$) if the expected income conditional on the order flows is greater than the reinvestment outlay, $E\left[V\left(K,\omega\right)|Q_1,Q_2,\mathcal{C}\right]\geq K$, and does not occur if otherwise ($k=0$).

(iii) Repayments are such that the capital provider at least breaks even in expectation.

If short selling costs are moderately low to low ($c<\widehat{c}$), there exists an equilibrium such that: the speculator informed about the high state buys in $t=1$ and buys again in $t=2$ when $p_1<V^{+}-K$; the uninformed speculator does not trade in $t=1$ and $t=2$ when $p_1>0$; the speculator informed about the low state sells in $t=1$ and sells again in $t=2$ when $p_1>0$; and the manager trades if and only if he is uninformed about the state, in which case the manager buys in $t=1$ and buys again in $t=2$ when $p_1>0$. This equilibrium implements the investment capacity specified in Proposition 1(iii).

According to Proposition 4, there is an equilibrium with financial contracting that induces the manager to signal firm value through buybacks if and only if he is uninformed. In this equilibrium, buying pressure is seen as the outcome of either repurchases by an uninformed manager or informed trading by a speculator informed about the high state. Moderate order flows result from short sales by a speculator informed about the low state. As a result, manipulative short sales by an uninformed speculator counters the buying pressure from buybacks and results in moderate order flows, driving prices to zero at $t = 1$. This offsets manipulation gains and makes it an unattractive strategy, efficiently boosting investment for moderately low to low short selling costs. Moreover, the manager does not have an incentive to repurchase shares when informed about the low state. If successful, such strategy would mitigate negative information coming through prices and enable the firm to raise funds and reinvest, allowing the manager to obtain private benefits. Yet, because of the selling pressure by the speculator informed about the low state, repurchases only lead to moderate order flows, driving the price down to zero and leading to the cancellation of reinvestment.

It is important to highlight that while stock repurchase by the manager makes the signal from positive order-flows noisier in the equilibrium described by Proposition 4, its ex-ante impact on price is positive as expected. Setting up repurchase programs ex ante signals that the firm is willing to support its stock against sell-side manipulation. Intuitively, buybacks eliminate manipulative short sales and manipulation incentives altogether for moderately low to low short selling costs ($c < \widehat{c}$), which makes negative order flows more informative and yields ex-ante borrowing capacity I^* over this range. In addition, Proposition 2 implies that I^* is the maximum investment capacity under spot financing from moderately high to low short selling costs ($c < c'$) for all α. Therefore, it is optimal for the capital provider to offer such a contract whenever $c < \widehat{c}$, increasing firm investment capacity to I^* over the entire range $c < c'$.

For high short selling costs ($c > c'$), ex-ante contracting cannot improve upon ex-post spot financing. In this case, short selling costs make shorting the stock in $t = 1$ unprofitable even for a speculator informed about the low state, completely driving out informed trading in $t = 1$. Under spot financing, informed trading in $t = 2$ secures enough funds for reinvestment when the speculator is either informed about the high state or uninformed, leading to the cancellation of reinvestment when the state is low with probability $\frac{\alpha}{4}$. Under ex-ante contracting, repurchases by an uninformed manager result in buying pressure that is interpreted as coming either from trading by a speculator informed about the high state or from uninformed buybacks, generating enough income for reinvestment. Repurchases by a manager informed about the low state are not advantageous since they offset the selling pressure from informed speculation and result in moderate order flows, driving the stock price to zero and leading to cancellation of reinvestment. It follows that reinvestment policies and investment capacity are the same in both settings.

The discussion above implies that ex-ante contracting that allows for stock repurchases is optimal for moderately low to low short selling costs ($c < \widehat{c}$). One potential situation where shorting costs are low arises when short sellers have short trading horizons. In this case, manipulative shorting is more likely without stock repurchases, as described in Proposition 1. This situation is where allowing firms to signal value through repurchases is the most helpful to enable efficient investment outcomes by curbing manipulation, as described in Proposition 4. In contrast, ex-post spot financing is optimal for moderately high to high short selling costs ($c > \widehat{c}$). We formalize this in turn.

Proposition 5 *Consider the equilibria described in Propositions 1 and 4. The relation between short selling costs and investment capacity with ex-ante contracting is characterized by:*

(i) For moderately low to low short selling costs ($c < \widehat{c}$), the contract described in Proposition 4 is optimal and investment capacity equals I^*.

(ii) For moderately high to high short selling costs ($c > \widehat{c}$), spot financing is optimal and investment capacity equals I^* for $\widehat{c} < c < c'$, and I' for $c > c'$.

Figure 4 depicts the relation between investment and short selling costs when ex-ante contracting is allowed. It is worth noting the stark contrast between Figure 4 and Figure 3 above. Under ex-ante contracting, investment capacity is insensitive to changes in short selling costs for moderately high to low costs ($c < c'$), and decreasing for moderately high to high costs

$(c > \widehat{c})$. As a result, the relation between investment and short selling costs is monotonically decreasing regardless of the informativeness of the speculator's signal. We formalize this result in the following corollary.

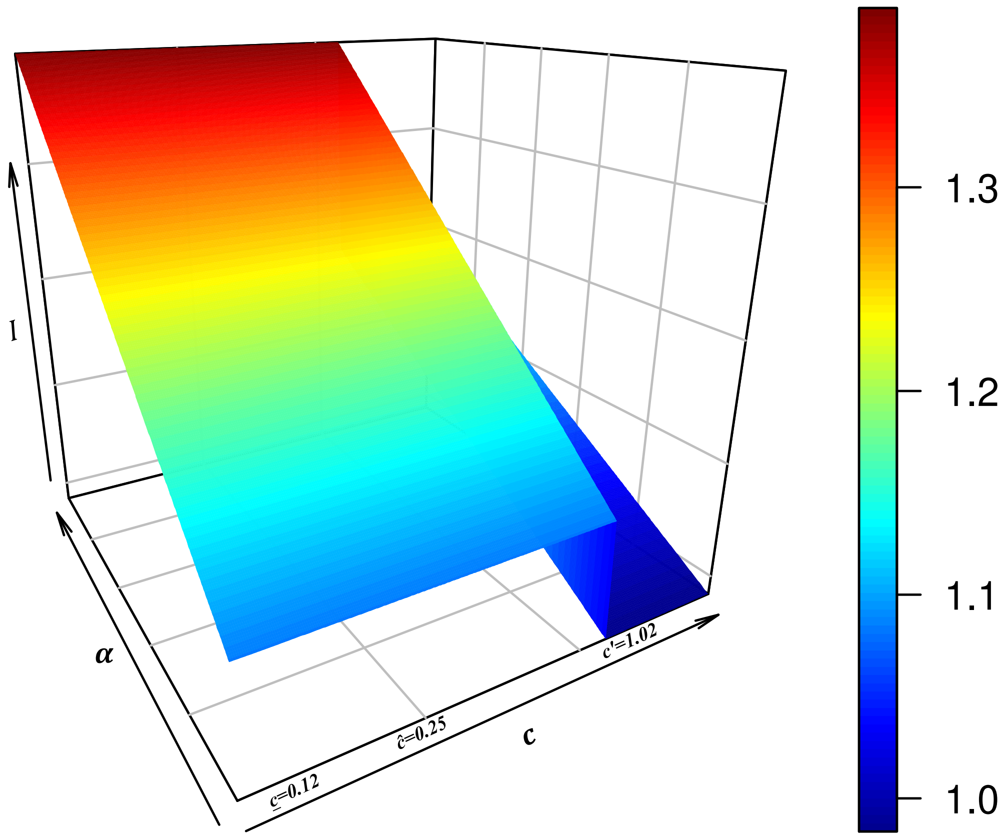
Corollary 3 *Under the equilibria described in Proposition 5, stock repurchases and investment capacity monotonically decrease in short selling costs.*

An important policy implication concerns short selling bans. The following proposition characterizes conditions in which short selling ban reduces or increases efficiency.

Corollary 4 *If $\alpha < \underline{\alpha}$, investment capacity with short selling ban is lower. If $\alpha \in (\underline{\alpha}, \overline{\alpha})$, investment capacity with short selling ban is lower for $c < c'$, and higher for $c > c'$. If $\alpha > \overline{\alpha}$, investment capacity with short selling ban is higher.*

The main takeaway of Corollary 4 is the contribution of repurchases relative to short selling ban to investment efficiency when the short selling cost is not high $(c < c')$. While short selling ban has the benefit of allowing perfect identification of positive private information, it has the cost of not allowing the distinction between uninformed and negatively informed trading. The benefit comes from efficiently canceling investment when the speculator is informed about the low state (not canceled in the absence of a ban when $Q_1 = 0$ and $Q_2 = 0$, which occurs with probability $\frac{1}{4}$) and equals $\frac{\alpha}{2}\frac{1}{4}(K - V^-)$. The cost is due to due to inefficiently canceling investment when the speculator is uninformed (never canceled in the absence of a ban) and equals $(1-\alpha)\left(\frac{V^+ + V^-}{2} - K\right)$. When the chance of the speculator being informed increases (higher α), the short selling ban cost reduces while the benefit increases. It follows that, when the chance of informed trading is not too high $(\alpha < \overline{\alpha})$, the cost outweighs the benefit since, in the absence of short selling ban, repurchases implement investment efficiency by curbing manipulation and revealing negative private information $(I^* > \tilde{I})$. Short selling ban is efficient when the chance of informed trading is too high $(\alpha > \overline{\alpha})$, as in this case the cost becomes very small $(I^* < \tilde{I})$.

Figure 4. Investment With Ex-ante Contracting



This figure shows investment (I) as a function of short selling costs (c) and the probability that the speculator is informed (α) for the two alternative settings of the model: (a) when funds for repurchases and reinvestment are raised ex ante (i.e., before prices incorporate speculators' information); and (b) when funds for reinvestment are raised exclusively ex post.

2.3.3 Contract Feasibility

We now turn our attention to the feasibility of ex-ante contracting. The contract described in Proposition 4 requires excess funds to allow an uninformed manager to repurchase the firm's stock in both trading periods. The highest overall repurchase cost occurs when the aggregate order flow in $t = 1$ reveals that the speculator is not informed about the low state ($Q_1 = 2$). In this case, the stock price in each period equals $p_1 = p_2 = \frac{V^+ + (1-\alpha)V^- - (2-\alpha)K}{2-\alpha}$, and the capital provider needs to provide excess funds amounting to $b_0 - I = 2\pi \frac{V^+ + (1-\alpha)V^- - (2-\alpha)K}{2-\alpha}$. Because excess funds are returned to the capital provider when the borrower does not conduct repurchases (i.e., when he is informed about the state), the resulting expected value of excess funds equals $(1-\alpha)(b_0 - I)$. Since the investment capacity is I^*, ex-ante contracting is feasible if and only if $I^* - I \geq (1-\alpha)(b_0 - I)$; that is, when the investment capacity in $t = 0$ is enough to cover the investment outlay and the expected amount lent for stock repurchases. This result is characterized in Proposition 6.

Proposition 6 *The contract described in Proposition 4 is feasible if and only if*

$$I^* - I \geq (1-\alpha)\, 2\pi \frac{V^+ + (1-\alpha)\, V^- - (2-\alpha)\, K}{2-\alpha}.$$

This condition is never satisfied for $I \geq I^*$. For $I < I^*$, there exists $\pi^*(\alpha, \Delta) > 0$ such that it is satisfied if and only if $\pi \leq \pi^*$, where π^* is increasing both in α and in $\Delta \equiv \frac{V^+ - V^-}{2}$.

Proposition 6 engenders several heterogeneous implications for the impact of short selling costs on financial contracting and stock repurchases. We formalize these direct implications in the corollary below.

Corollary 5 *The contract described in Proposition 4 is more likely to be feasible for firms with lower stock liquidity (lower π), higher chance of having informed traders (higher α), and higher cash flow uncertainty (higher Δ).*

According to Corollary 5, financial contracting is used more often when the speculator's signal becomes more informative (higher α). This results from a rise in the beneficial impact of offsetting manipulation and a decline in the expected cost of doing so through repurchases. Moreover, financial contracting is more feasible when stock liquidity is lower (lower π). This result is intuitive as the manager needs to repurchase fewer shares to offset manipulative selling orders of lower sizes, which in turn requires capital providers to commit less capital above the initial investment outlay. Finally, financial contracting is more likely when there is more uncertainty concerning investment prospects (higher Δ). This is because higher uncertainty increases the positive impact on firm value when prices reveal that the speculator is not informed about the low state, whereas the cost of providing such a signal is incurred only when the speculator is uninformed.

2.3.4 Efficiency of Stock Repurchases

One could raise the question of whether the contract described in Proposition 4 yields an alternative equilibrium in which the manager inefficiently repurchases shares to offset the selling pressure from negatively informed speculators. Since the manager derives private benefits from investment, he wants to maximize the probability of reinvestment when informed about the low state. As it turns out, an equilibrium exists in which the manager never trades when informed about the high state, but always repurchases shares when uninformed or negatively informed about the state of the economy. While these inefficient repurchases are likely to arise when firms have excessive cash reserves, our model suggests that repurchases are efficient when firms need external finance to invest and institute repurchase programs.

Intuitively, under this equilibrium, a speculator with private information about the high state always buys but never trades when uninformed or privy about the low state. Notably, prices are uninformative and the firm always invests regardless of private signals and the level of short selling costs, with investment capacity equal to $\widehat{I}$. However, it follows from Proposition 2 that investment capacity is at least $\min\{\underline{I}, I'\} > \widehat{I}$ under spot financing (i.e., without ex-ante contracting). The capital provider anticipates this ex-post misalignment of incentives and refrains from offering contracts allowing repurchasing shares. It follows that contracts that allow firms to set up repurchase programs are only implemented when doing so increases firm value relative to spot financing. We formalize this result in Proposition 7.

Proposition 7 *For moderately low short selling costs ($c < \widehat{c}$), there is an equilibrium under the contract described in Proposition 4 such that: the speculator informed about the high state buys in $t = 1$ and buys again in $t = 2$ when $p_1 < V^+ - K$; the uninformed speculator and the* *speculator informed about the low state do not trade in $t = 1$ and $t = 2$ when $p_1 > 0$; and the manager trades if and only if he is uninformed or informed about the low state, in which case the manager buys in $t = 1$ and buys again in $t = 2$ when $p_1 > 0$. In this equilibrium, investment capacity is $\widehat{I} < \min\{\underline{I}, I'\}$, which is less efficient than that under spot financing in Proposition 1.*

According to Proposition 7, it is sub-optimal for the capital provider to lend in excess of initial investment outlays if the manager is expected to use these additional funds to buy back shares to inefficiently support prices. As a result, a contract allowing for stock repurchases is signed in $t = 0$ only if the resulting firm value in equilibrium is at least as high as that achieved under spot financing. This reflects our assumption that the capital provider (principal) has all the bargaining power and makes a take-it-or-leave offer that achieves the highest possible firm value in $t = 0$. It follows that stock repurchases are more likely to be efficient when the firm's need for external finance is high, or the capital provider has strong bargaining power.

Corollary 6 *A contract allowing for stock repurchases is signed in $t = 0$ only if the resulting firm value is at least as high as that under spot financing. It follows that investment capacity is lower in the absence of financial markets.*

Corollary 6 says that financial markets never decrease investment capacity despite the negative impact of manipulation, which is in line with Goldstein and Guembel (2008). This is consistent with an environment in which shareholders or board of directors act as a principal that wants to maximize firm value. Thus, they choose to restrain the managers from conducting buybacks when anticipating inefficient investment consequences. In this case, the firm value in the absence of short selling ban is characterized in Proposition 1.

3 Empirical and Policy Implications

We must flesh out the empirical and policy implications of our model. To date, the analysis of how shorting costs affect markets has been limited to considerations about market liquidity and price discovery, ignoring consequences for managerial incentives and corporate decision-making. Our model takes a different path and yields several novel testable predictions.

Our analysis fully incorporates shorting costs into manipulative trading strategies. This allows us to evaluate the impact of short selling on investment efficiency differently from the existing literature. We show that the relation between investment and shorting costs depends not only on how informed stock traders are but also on firms' ability to engage in share repurchases. On that note, the first set of empirical implications derived from our model highlights the case when repurchasing stock is infeasible due to funding constraints as stated in Proposition 2.

Implication 1A: *When the probability of informed trading is low, the effect of short selling costs on investment is positive if shorting costs are low and negative if shorting costs are high.*

Implication 1B: *When the probability of informed trading is high, the effect of short selling costs on investment is V-shaped if shorting costs are low and negative if shorting costs are high.*

In short, our model points to a non-monotonic relation between investment and short selling costs in the absence of stock repurchases, a relation modulated by the informativeness of speculators' private signals (as depicted in Figure 3). Several studies estimate the probability of informative trading from stock returns and order flows (e.g., Easley et al. (1996), Odders-White and Ready (2008), and Back et al. (2018)). Our model's implications can be tested using existing proxies for the probability of informed trading, along with data on shorting costs. The latter has been shown to exhibit significant time-series and cross-sectional variation (see Saffi and Sigurdsson (2011)), a feature that facilitates direct testing of our model's predictions.

Next, we discuss how firms' ability to repurchase their own stock affects the relationship between investment and short selling costs. It is worth stressing that the existing theoretical literature has not considered the role of stock repurchase programs in modulating the impact of manipulative short sales on firm policies. We begin with an empirical prediction about how repurchases are related to the cost of shorting stocks.

Implication 2: *Stock repurchases are decreasing in short selling costs. This effect is more pronounced for firms with more illiquid stocks, a higher degree of private information, and higher cash flow uncertainty.*

This prediction follows from Corollary 5. The main driver for this result is the firm's ability to convey information through stock repurchases, signaling to the market its valuation view against short sellers. We note that while Khanna and Mathews (2012) highlight the role of a large blockholder in deterring manipulative shorts, our model shows that firms themselves can fulfill that deterrence role. This result is consistent with recent evidence by Schultz (2023) and finds support in corporate policy practice.[14]

While the proportion of firms employing buybacks has increased in the last 20 years, it still hovers below 50% (see Farre-Mensa et al. (2014)). Firms often lack the resources to engage in buyback programs, but our model highlights a novel reason for those that do. Those programs work as a credible threat against manipulative short selling. Importantly, for testing purposes, we show that the feasibility of stock repurchases is related to several measurable firm characteristics. For example, those lacking internal funds but with high borrowing capacity (e.g., firms with tangible assets and good credit ratings) can more easily threaten to buy back shares to deter short selling.

We stress that short selling is quite cheap in the U.S. For instance, the median (75^{th} percentile) lending fee of U.S. stocks in the 2002-2020 period is 0.12% p.y. (0.23% p.y.), with the median utilization (i.e., the number of shares on loan divided by lendable supply) is 8.3%. These low costs are precisely the environment in which shorting manipulators might be able to profit from uniformed trading, as described in Goldstein and Guembel (2008).

Another implication of our model is that the effect of short selling costs on investment is negative when firms can repurchase their shares. This obtains regardless of the informativeness of speculators' signals. Let us formally state this prediction, which stems from Corollary 3:

[14]For example, Brav et al. (2005) report that the most common rationale for stock repurchases is "to convey information about our company to investors".

Implication 3: *Firm investment is unaffected by shorting costs when those costs are low, but declines as shorting costs become large.*

Consistent with this implication, Grullon et al. (2015) and Boulatov et al. (2020) report that investment spending declined following the removal of shorting constraints under SEC's 2005 Regulation SHO. This reform boosted the demand for short selling and raised stock loan fees, as shown by Campello et al. (2019), making short selling more expensive afterward. In contrast, the investment of large firms was unaffected by that change. Lending market data show that the average shorting fee for small firms is much higher than that of large firms (e.g., Bekjarovski (2018)). In light of these salient data patterns, our model explains why large firms' investment plans are insensitive to changes in shorting constraints.

More generally, our model's implications challenge policymakers when devising short selling regulations. We show that the impact of shorting constraints on investment can be non-linear: it will depend on the ability of firms to repurchase stocks and how costly shorting the firm stock is. This calls into question "one-size-fits-all" policy interventions used in many countries.[15] For researchers, our results can guide the design of empirical strategies and the identification of firms more likely affected by changes in shorting costs or other general shorting constraints.

We now turn to the motivation behind managers' decisions to repurchase shares. Managerial compensation is commonly tied to stock price levels. As a result, stock repurchase programs may be used by managers to support stock prices inefficiently. Repurchases may also be used to avoid negative information about firm value being revealed rather than as a mechanism to convey positive information about the firm's prospects. Our model predicts that firms that require external financing will obtain funds in excess of investment needs only if used for repurchase programs that prevent manipulative shorting and improve price informativeness. As a result, these firms are more likely to conduct repurchases jointly with debt issuance when manipulation incentives are high due to low shorting costs. These predictions, summarized below, are directly implied by Proposition 7.

[15] Beber and Pagano (2013) describe several instances when regulators have imposed short selling bans in an attempt to support stock prices, often with detrimental consequences for stock liquidity and price efficiency.

Implication 4: *Stock repurchases are more likely to signal private information and investment efficiency for firms that need external finance and face capital providers with strong bargaining power.*

Implication 5: *Stock repurchases coupled with debt issuance are more likely when short selling costs are low for firms that need external finance and face capital providers with strong bargaining power.*

The empirical literature supports these implications. Bargeron and Bonaime (2020), for example, report that managers choose to initiate stock repurchase programs when short selling demand is high, and that these repurchases are followed by positive abnormal stock returns. This is consistent with our model, where managerial repurchasing decisions are motivated by signaling positive private information rather than by a myopic defense of inflated prices. Billet and Xue (2007) show evidence that the signaling motive is stronger and more effective for firms that need external funding and are constrained, which puts them in a weak bargaining position relative to capital providers. Rubio (2019) shows that repurchase programs initiated by firms that rely on external finance are followed by higher investment spending.

Finally, our model predicts that the risk of manipulative shorting provides another reason why companies raise capital coupled with stock repurchases. This ever-growing real-world phenomenon has received some theoretical attention (see Bond and Zhong (2016)) but remains understudied by empirical researchers. Our analysis suggests that firms that need external finance must engage in repurchases concomitantly with debt issuance to prevent manipulation and secure subsequent funding rounds.

Empirically testing these implications is challenging because it requires identifying market feedback effects unrelated to changes in expectations about firm fundamentals. Wardlaw (2020) highlights how the econometric approach of using mutual fund flows as a source of exogenous price pressure cannot clearly identify nonfundamental shocks (e.g., Edmans et al. (2012)). We leave the task of testing our empirical implications and addressing the endogeneity issues for future research.

4 Concluding Remarks

We show how manipulative feedback effects from stock prices shape financial contracts and corporate policies. Our model innovates on two main dimensions. First, it explicitly accounts for the cost of short selling and its effect on manipulation incentives and investment outcomes. In particular, it reveals a non-monotonic relation between short selling costs and investment when firms cannot repurchase stock due to funding constraints. Such a relation has not been identified in prior research. Second, while past models focus on financing and investment decisions that follow trading in financial markets, we allow for contracts that pre-commit funds and condition future investments on stock prices. This is shown to induce managers to repurchase stock and signal firm value to investors, offsetting potential manipulation by speculators and leading to the implementation of optimal investment policies. The ability to pre-commit funds yields a negative relation between investment and shorting costs. Importantly, contracts that allow firms to achieve efficient investment levels are more likely to be feasible when the income is higher and the costs to signal value through stock repurchases are lower. Our model predicts such contracts will be more often observed when firms have more illiquid stocks and higher cash flow uncertainty.

Understanding the impact of short selling on corporate policies is important for researchers, managers, and policymakers alike as capital markets evolve and present new challenges to all of its participants. The relationship between short selling costs and investment efficiency critically hinges on considerations such as the degree of private information traders have and on firms' ability to repurchase their own stocks. Ignoring such real-world considerations can lead to misspecified empirical models, which may ultimately result in inefficient policy interventions.

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Appendix

Proof of Proposition 1. We first analyze sequentially rational strategies in $t = 2$ for any reachable information set. Next we examine the optimal strategies in $t = 1$.

Trading in $t = 2$

The possible information sets after trading takes place in $t = 1$ are as follows: (i) the order flow Q_1 perfectly reveals the speculator's information; (ii) the order flow Q_1 reveals that she is not informed about the low state; (iii) the order flow Q_1 reveals that she is not informed about the high state; (iv) the order flow Q_1 reveals that the speculator is not uninformed; and (v) the order flow does not reveal any information. Conditional on the information set, the speculator chooses u_2 to maximize her payoff, $u_1 \pi E\left[V(k,\omega) - k - p_1(Q_1)\,|s, Q_1, u_2\right] - 1_{\{u_1=-1\}}\pi c + u_2 \pi E\left[V(k,\omega) - k - p_2(Q_1,Q_2)\,|s, Q_1, u_2\right] - 1_{\{u_2=-1\}}\pi c$. Note that the speculator's strategy in $t = 2$ is independent from: (a) the order size π, such that we take $\pi = 1$ without loss of generality; and (b) the action chosen in $t = 1$ unless u_2 affects the firm value $V(k,\omega) - k$ through the reinvestment policy k.

Case (i): The market price reflects the expected firm value given the speculator's information, $p_1(Q_1) = E\left[p_2(Q_1,Q_2)\,|s, Q_1, u_2\right] = E\left[V(k,\omega) - k|s, Q_1, u_2\right]$. Thus, the speculator is indifferent between buying and not trading in $t = 2$ as both yield a zero profit. If speculator sells, there is a loss of $-c$. Therefore, her only sequentially rational strategies given beliefs consistent on the path is to either buy or not trade in $t = 2$.

Case (ii): Note that in this case $E\left[V(K,\omega) - K|Q_1,Q_2\right] > 0$, which implies that the firm always invests ($k = K$) and the speculator's strategy in $t = 2$ is independent from what happened in $t = 1$. Thus, we examine only the period-2 trade profit. We proceed as follows: (a) we characterize the sequentially rational strategy profiles with beliefs consistent on the path in which the speculator informed about the high state buys in $t = 2$; (b) we show that there is no sequentially rational strategy profile with beliefs consistent on the path in which the speculator informed about the high state sells in $t = 2$; and (c) we characterize the sequentially rational strategy profiles with beliefs consistent on the path in which the speculator informed about the high state does not trade in $t = 2$.

Let us start with (a). Consider sequentially rational strategy profiles given beliefs consistent on the path in which the speculator informed about the high state buys in $t = 2$. Suppose by way of contradiction that the uninformed speculator buys in $t = 2$. In this case, the uninformed speculator loses money: the period-2 price is $p_2(\cdot,\cdot) = \frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)$ and her profit equals $\frac{V^+ + V^-}{2} - K - p_2(\cdot,\cdot) < 0$. Hence, the uninformed speculator has an incentive to deviate and not trade, which secures a payoff of zero. This leads to a contradiction.

Suppose the uninformed speculator does not trade, in which case her profit is zero. If she deviates and buys, $p_2(\cdot,\cdot) = V^+ - K$ and her profit equals $\frac{V^+ + V^-}{2} - K - p_2(\cdot,\cdot) < 0$, which implies that she does not have an incentive to deviate and buy. If she deviates and sells, with probability $\frac{1}{2}$ the order flow equals $Q_2 = 0$ and $p_2(\cdot, 0) = V^+ - K$; with probability $\frac{1}{2}$ the order flow equals $Q_2 = -2$ and $p_2(\cdot, -2)$ depends on the beliefs associated with $Q_2 = -2$. Her expected profit is at least $\frac{1}{2}\left[p_2(\cdot, 0) - \left(\frac{V^+ + V^-}{2} - K\right)\right] - c = \frac{V^+ - V^-}{4} - c$ (which occurs if the beliefs associated with $Q_2 = -2$ are such that $p_2(\cdot, -2) = \frac{V^+ + V^-}{2} - K$), and at most $\frac{V^+ - V^-}{2} - c$ (which occurs if the beliefs associated $Q_2 = -2$ are such that $p_2(\cdot, -2) = V^+ - K$). Therefore, the uninformed speculator has the incentive to deviate and sell for $\frac{V^+ - V^-}{4} > c$; for $\frac{V^+ - V^-}{4} \le c \le \frac{V^+ - V^-}{2}$, there exist beliefs such that she does have the incentive to deviate, and beliefs such that she does not; for $c > \frac{V^+ - V^-}{2}$, she does not have the incentive to deviate.

In turn, if the uninformed speculator does not trade, the expected profit of the speculator informed about the high state when she buys is zero. Therefore, the speculator informed about the high state does not have an incentive to deviate and not trade, as in this case her profit is also zero. If she deviates and sells, with probability $\frac{1}{2}$ the order flow equals $Q_2 = 0$ and $p_2(\cdot, 0) = V^+ - K$; with probability $\frac{1}{2}$ the order flow equals $Q_2 = -2$ and $p_2(\cdot, -2)$ depends on the beliefs associated with $Q_2 = -2$. Her expected profit equals $\frac{1}{2}\left[p_2(\cdot, 0) - (V^+ - K)\right] + \frac{1}{2}\left[p_2(\cdot, -2) - (V^+ - K)\right] - c \le c$. Therefore, the speculator informed about the high state does not have an incentive to deviate and sell.

Now, suppose that the uninformed speculator sells. Her profit is determined as follows: with probability $\frac{1}{2}$ the order flow is $Q_2 = 0$ and $p_2(\cdot, 0) = \frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)$, generating a profit of $p_2(\cdot, 0) - \left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{2} - c$; with probability $\frac{1}{2}$ the order flow is $Q_2 = -2$ and $p_2(\cdot, -2) = \frac{V^+ + V^-}{2} - K$, yielding a profit of $p_2(\cdot, -2) - \left(\frac{V^+ + V^-}{2} - K\right) - c = -c$; hence, her expected profit is $\frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c$. If she deviates and does not trade, her payoff is zero.

If she deviates and buys, then: with probability $\frac{1}{2}$ the order flow equals $Q_2 = 0$ and $p_2(\cdot, 0) = \frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)$; and with probability $\frac{1}{2}$ the order flow equals $Q_2 = 2$ and $p_2(\cdot, 2) = V^+ - K$; hence, her expected profit equals $\frac{V^+ + V^-}{2} - K - \left(\frac{p_2(\cdot,0)}{2} + \frac{p_2(\cdot,2)}{2}\right) < 0$. Therefore, the uninformed speculator does not have an incentive to deviate for $\frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} > c$; for $c > \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4}$, she has an incentive to deviate and not trade.

In turn, if the uninformed speculator sells, the profit of the speculator informed about he high state when she buys is determined as follows: with probability $\frac{1}{2}$ the order flow equals $Q_2 = 0$ and $p_2(\cdot, 0) = \frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)$; and with probability $\frac{1}{2}$ the order flow equals $Q_2 = 2$ and $p_2(\cdot, 2) = V^+ - K$; hence her expected profit is $V^+ - K - \left(\frac{p_2(\cdot,0)}{2} + \frac{p_2(\cdot,2)}{2}\right) > 0$. Since her expected profit is positive if she buys, she does not have an incentive to deviate and not trade, as this gives a payoff of zero. If she deviates and sells, then: with probability $\frac{1}{2}$ the order flow equals $Q_2 = 0$ and $p_2(\cdot, 0) = \frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)$; and with probability $\frac{1}{2}$ the order flow equals $Q_2 = -2$ and $p_2(\cdot, -2) \leq V^+ - K$; hence, her expected payoff is at most $\frac{p_2(\cdot,0)}{2} + \frac{1}{2}(V^+ - K) - (V^+ - K) - c < 0$, which implies that she does not have an incentive to deviate and sell.

The collection of the results above implies that within the class of sequentially rational strategy profiles with beliefs consistent on the path in which the speculator informed about the high state buys in $t = 2$, we have the following: for $\frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} > c$, the only sequentially rational strategy profile given beliefs consistent on the path consists of the speculator informed about the high state buying and the uninformed speculator selling; for $c > \frac{V^+ - V^-}{4}$, the only sequentially rational strategy profile given beliefs consistent on the path consists of the speculator informed about the high state buying and the uninformed speculator not trading.

We now turn to (b). Suppose by way of contradiction that the speculator informed about the high state sells in $t = 2$ under a sequentially rational strategy profile with beliefs consistent on the path. In this case, her expected payoff equals $\frac{1}{2}[p_2(\cdot, -2) - (V^+ - K)] + \frac{1}{2}[p_2(\cdot, 0) - (V^+ - K)] - c \leq -c$. Thus, she has an incentive to deviate and secure a zero payoff by not trading, which leads to a contradiction.

We now turn to (c). Suppose by way of contradiction that there exists a sequentially rational strategy profile with beliefs consistent on the path in which the speculator informed about the high state does not trade in $t = 2$ while the uninformed speculator buys, in which case both speculators have a payoff of zero. If the speculator informed about the high state deviates and buys, her expected payoff is $\frac{1}{2}\left[V^{+}-K-p_2\left(\cdot,2\right)\right]+\frac{1}{2}\left[V^{+}-K-p_2\left(\cdot,0\right)\right]=\frac{1}{2}\left[V^{+}-K-\left(\frac{V^{+}+V^{-}}{2}-K\right)\right]+\frac{1}{2}\left[V^{+}-K-\left(\frac{V^{+}+V^{-}}{2}-K\right)\right]=\frac{V^{+}-V^{-}}{2}>0$. Thus, the speculator informed about the high state has an incentive to deviate, leading to a contradiction.

Suppose by way of contradiction that there exists a sequentially rational strategy profile with beliefs consistent on the path, in which the speculator informed about the high state does not trade $t=2$ while the uninformed speculator sells. In this case, the payoff of the uninformed speculator equals $-c<0$. Therefore, the uninformed speculator has an incentive to deviate and secure a payoff of zero by not trading, which leads to a contradiction.

The results above imply that a sequentially rational strategy profile with beliefs consistent on the path in which the speculator informed about the high state does not trade exists only if the uninformed speculator does not trade in $t=2$. Suppose that the uninformed speculator and the speculator informed about the high state do not trade in $t=2$, in which case both payoffs are equal to zero. If the speculator informed about the high state deviates and sells, her payoff is $\frac{1}{2}\left[p_2\left(\cdot,-2\right)-\left(V^{+}-K\right)\right]+\frac{1}{2}\left[p_2\left(\cdot,0\right)-\left(V^{+}-K\right)\right]-c\leq -c$; if the uninformed speculator deviates and buys, her payoff is $\frac{1}{2}\left[\frac{V^{+}+V^{-}}{2}-K-p_2\left(\cdot,2\right)\right]+\frac{1}{2}\left[\frac{V^{+}+V^{-}}{2}-K-p_2\left(\cdot,0\right)\right]\leq 0$; therefore the speculator informed about the high state does not have an incentive to deviate and sell, while the uninformed speculator does not have the incentive to deviate and buy. If the speculator informed about the high state deviates and buys, her payoff is $\frac{1}{2}\left[V^{+}-K-p_2\left(\cdot,2\right)\right]+\frac{1}{2}\left[V^{+}-K-p_2\left(\cdot,0\right)\right]$; therefore she has the incentive to deviate unless $p_2\left(\cdot,2\right)=p_2\left(\cdot,0\right)=V^{+}-K$. For $p_2\left(\cdot,0\right)=V^{+}-K$, the payoff of the uninformed speculator if she deviates and sells is $\frac{1}{2}\left[p_2\left(\cdot,-2\right)-\left(\frac{V^{+}+V^{-}}{2}-K\right)\right]+\frac{1}{2}\left[p_2\left(\cdot,0\right)-\left(\frac{V^{+}+V^{-}}{2}-K\right)\right]-c=\frac{1}{2}\left[p_2\left(\cdot,-2\right)-\left(\frac{V^{+}+V^{-}}{2}-K\right)\right]+\frac{V^{+}-V^{-}}{4}-c$, which is lowest and equal to $\frac{V^{+}-V^{-}}{4}-c$ for $p_2\left(\cdot,-2\right)=\frac{V^{+}+V^{-}}{2}-K$. Therefore, a sequentially rational strategy profile with beliefs consistent on the path in which the speculator informed about the high state does not trade in $t=2$ exists if and only if $c\geq\frac{V^{+}-V^{-}}{4}$.

Case (iii): We first show that the speculator informed about the low state does not sell in $t = 2$ in any sequentially rational profile given beliefs consistent on the path. Suppose by way of contradiction that she does sell in $t = 2$. In this case, her profit is $-p_1(\cdot) - c$ if she buys in $t = 1$, $-c$ if she does not trade, and $p_1(\cdot) - 2c$ if she sells. If she deviates and does not trade in $t = 2$, her payoff is $0 > -c$ if she does not trade in $t = 1$ and at least (it is higher if the beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment occurs) $p_1(\cdot) - c > p_1(\cdot) - 2c$ if she sells. Therefore, she has an incentive to deviate and not trade in $t = 2$ if she either does not trade or sells in $t = 1$. If she deviates and buys in $t = 2$, her payoff if she buys in $t = 1$ is at least (it is higher if the beliefs associated with $Q_2 = 2$ are such that investment occurs) $-p_1(\cdot) > -p_1(\cdot) - c$. Therefore, she has an incentive to deviate and buy in $t = 2$ if she buys in $t = 1$. We conclude that selling in $t = 2$ is not sequentially rational for the speculator informed about low state given beliefs consistent on the path.

The previous result implies that reinvestment does not occur in any sequentially rational strategy profile given beliefs consistent on the path when the speculator is informed about the low state, in which case her payoff equals $-p_1(\cdot)$ if she buys in $t = 1$, zero if she does not trade, and $p_1(\cdot) - c$ if she sells. This follows because consistency of beliefs on the path results in an expected income of $V^- - K < 0$ when the order flow Q_2 reveals that the speculator is informed about the low state, and $\frac{(1-\alpha)V^+ + V^- - (2-\alpha)K}{2-\alpha} < 0$ when the order flow Q_2 does not reveal the speculator's type.

We now show that both the speculator informed about the low state and the uninformed speculator not trading in $t = 2$ constitute a sequentially rational strategy profile given beliefs consistent on the path. In this case, their payoffs equal $-p_1(\cdot)$ if they buy in $t = 1$, zero if they do not trade, and $p_1(\cdot) - c$ if they sell. If either speculator deviates and sells instead, her payoff assuming that the beliefs associated with $Q_2 \in \{-2, 0\}$ are such that investment does not occur equals $-p_1(\cdot) - c$ if she buys in $t = 1$, $-c$ if she does not trade, and $p_1(\cdot) - 2c$ if she sells. If either speculator deviates and buys instead, her payoff assuming that the beliefs associated with $Q_2 \in \{0, 2\}$ are such that investment does not occur equals $-p_1(\cdot)$ if she buys in $t = 1$, zero if she does not trade, and $p_1(\cdot) - c$ if she sells. Therefore, neither speculator has an incentive to deviate.

Case (iv): We derive the sequentially rational strategy profiles with beliefs consistent on the path in which the speculator informed about the high state buys in $t = 2$.

- Sequential rationality of the speculator informed about the low state with beliefs consistent on the path given that the speculator informed about the high state buys in $t = 2$

Suppose by contradiction that the speculator informed about the low state buys in $t = 2$. In this case, her profit is $V^- - K - p_1(\cdot) + V^- - K - \left(\frac{V^+ + V^-}{2} - K\right) = V^- - K - p_1(\cdot) - \frac{V^+ - V^-}{2}$ if she buys in $t = 1$, $V^- - K - \left(\frac{V^+ + V^-}{2} - K\right) = -\frac{V^+ - V^-}{2}$ if she does not trade, and $p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right)$ if she sells; if she deviates and does not trade, her profit is at least (it is higher if beliefs are such that investment does not occur when $Q_2 \in \{-1, 1\}$) is $V^- - K - p_1(\cdot) > V^- - K - p_1(\cdot) - \frac{V^+ - V^-}{2}$ if she buys in $t = 1$, $0 > -\frac{V^+ - V^-}{2}$ if she does not trade, and at least (it is higher if beliefs are such that investment occurs when $Q_2 \in \{-1, 1\}$) $p_1(\cdot) - c > p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right)$ if she sells. Therefore, the speculator informed about the low state does not buy in $t = 2$.

Suppose that she does not trade in $t = 2$. In this case, her profit is $-p_1(\cdot)$ if she buys in $t = 1$, zero if she does not trade, and $p_1(\cdot) - c$ if she sells. If she deviates and sells in $t = 2$, her profit is at least (it is higher if beliefs are such that investment occurs when $Q_2 = -2$) $\frac{V^- - K}{2} - p_1(\cdot) + \frac{1}{2}\left[p_2(\cdot, 0) - (V^- - K)\right] - c = -p_1(\cdot) + \frac{V^+ - K}{2} - c = -p_1(\cdot) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) + \frac{V^+ - V^-}{4} - c$ if she buys in $t = 1$, $\frac{1}{2}\left[p_2(\cdot, 0) - (V^- - K)\right] - c = \frac{V^+ - V^-}{2} - c$ if she does not trade, and $p_1(\cdot) - \frac{V^- - K}{2} - c + \frac{1}{2}\left[p_2(\cdot, 0) - (V^- - K)\right] - c = p_1(\cdot) - c - (V^- - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) + \frac{V^+ - V^-}{4} - c$ if she sells. If she deviates and buys in $t = 2$, her payoff is always lower than that under the proposed equilibrium in which she does not trade in $t = 2$: it equals $V^- - K - p_1(\cdot) - (V^+ - V^-) < p_1(\cdot)$ if she buys in $t = 1$, $-(V^+ - V^-) < 0$ if she does not trade, and $p_1(\cdot) - c - (V^+ - K) < p_1(\cdot) - c$ if she sells. Therefore, she does not have an incentive to deviate and buy in $t = 2$. The deviation when she sells in $t = 2$ yields a net payoff relative to not trading in $t = 2$ equal to $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) + \frac{V^+ - V^-}{4} - c$ if she buys in $t = 1$, $\frac{V^+ - V^-}{2} - c$ if she does not trade, and $-(V^- - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) + \frac{V^+ - V^-}{4} - c$ if she sells.

Now, suppose that she sells in $t = 2$. Her profit is $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - p_1(\cdot) - c$ is she buys in $t = 1$, $\frac{V^+ - V^-}{4} - c$ if she does not trade, and $p_1(\cdot) - \frac{V^- - K}{2} + \frac{V^+ - V^-}{4} - 2c$ if she sells. If she deviates and does not trade in $t = 2$, her least favorable payoff if she buys in $t = 1$ is $V^- - K - p_1(\cdot)$

(her payoff is higher and equal to $-p_1(\cdot)$ if the beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment does not occur), her payoff if she does not trade in $t = 1$ is zero, and her least favorable payoff if she sells in $t = 1$ is $p_1(\cdot) - c$ (her payoff is higher and equal to $p_1(\cdot) - (V^- - K) - c$ if the beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment occurs). If she deviates and buys in $t = 2$, her payoff is always lower than that when she deviates and does not trade: $V^- - K - p_1(\cdot) + \frac{1}{2}\left[V^- - K - (V^+ - K)\right] + \frac{1}{2}\left[V^- - K - \left(\frac{V^+ + V^-}{2} - K\right)\right] < V^- - K - p_1(\cdot)$ if she buys in $t = 1$, $\frac{1}{2}\left[V^- - K - (V^+ - K)\right] + \frac{1}{2}\left[V^- - K - \left(\frac{V^+ + V^-}{2} - K\right)\right] < 0$ if she does not trade, and $p_1(\cdot) - \frac{1}{2}\left[V^+ - K + \left(\frac{V^+ + V^-}{2} - K\right)\right] - c < p_1(\cdot) - c$ if she sells. Therefore, the best deviation is when she does not trade in $t = 2$, which yields a net payoff relative to selling in $t = 2$ of $\frac{V^- - K}{2} - \left(\frac{V^+ - V^-}{4} - c\right)$ if she buys in $t = 1$, $-\left(\frac{V^+ - V^-}{4} - c\right)$ if she does not trade, and $\frac{V^- - K}{2} - \left(\frac{V^+ - V^-}{4} - c\right)$ if she sells.

The collection of results above implies that the speculator informed about the low state has an incentive to deviate if she buys in $t = 2$; hence, buying in $t = 2$ is not sequentially rational given beliefs consistent on the path. For $\frac{V^+ - V^-}{4} > c$, she has an incentive to deviate if she does not trade in $t = 2$, but does not have an incentive to deviate if she sells in $t = 2$; thus, the only sequentially rational strategy given beliefs consistent on the path is for her to sell in $t = 2$.

- Sequential rationality with beliefs consistent on the path for the speculator informed about the high state to buy if the speculator informed about the low state sells in $t = 2$

The expected profit of the speculator informed about the high state when she buys in $t = 2$ is $\frac{1}{2}\left[V^+ - K - p_1(\cdot) + V^+ - K - p_2(\cdot, 2)\right] + \frac{1}{2}\left[V^+ - K - p_1(\cdot) + V^+ - K - p_2(\cdot, 0)\right] = V^+ - K - p_1(\cdot) + \frac{V^+ - V^-}{4}$ if she buys in $t = 1$, $\frac{V^+ - V^-}{4}$ if she does not trade, and $p_1(\cdot) - c - \frac{p_2(\cdot,2)}{2} - \frac{p_2(\cdot,0)}{2} = p_1(\cdot) - c - \frac{V^+ - K}{2} - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$ if she sells. If she deviates and sells, her payoff is always lower: it equals $\frac{p_2(\cdot,0)}{2} + \frac{p_2(\cdot,-2)}{2} - p_1(\cdot) - c = \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - p_1(\cdot) - c < V^+ - K - p_1(\cdot) + \frac{V^+ - V^-}{4}$ if she buys in $t = 2$, $\frac{1}{2}\left[p_2(\cdot, 0) - (V^+ - K)\right] - c = -\frac{V^+ - V^-}{4} - c < \frac{V^+ - V^-}{4}$ if she does not trade, and $\frac{1}{2}\left[p_1(\cdot) - (V^+ - K) + p_2(\cdot, 0) - (V^+ - K) - 2c\right] + \frac{1}{2}\left[p_1(\cdot) - 2c\right] = p_1(\cdot) - c - \frac{V^+ - K}{2} - \frac{V^+ - V^-}{4} - c < p_1(\cdot) - c - \frac{V^+ - K}{2} - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$ if she sells. Therefore, she does not have an incentive to deviate and sell. If she deviates and does not trade, her payoff if beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment occurs equals $V^+ - K - p_1(\cdot) < V^+ - K - p_1(\cdot) + \frac{V^+ - V^-}{4}$ if she buys in $t = 1$, $0 < \frac{V^+ - V^-}{4}$ if she does not trade, and $p_1(\cdot) - c - (V^+ - K) < p_1(\cdot) - c - \frac{V^+ - K}{2} - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$ if she sells. Therefore, she does not have an incentive to deviate and not trade.

- Sequentially rational strategy profiles with beliefs consistent on the path in which the speculator informed about the high state buys in $t = 2$

For $\frac{V^+ - V^-}{4} > c$, we have the following: (i) if the speculator informed about the high state buys in $t = 2$, then the only sequentially rational strategy given beliefs consistent on the path for the speculator informed about the low state is to sell in $t = 2$; and (ii) if the speculator informed about the low state sells in $t = 2$, then it is sequentially rational given beliefs consistent on the path for the speculator informed about the high state to buy in $t = 2$.

Case (v): Consider sequentially rational strategy profiles with beliefs consistent on the path in which the speculator informed about the high state buys in $t = 2$. First, we derive the sequentially rational strategies of the speculator informed about the low state with beliefs consistent on the path given the strategies of the uninformed speculator. Second, we derive the sequentially rational strategies of the uninformed speculator with beliefs consistent on the pat given the strategies of the speculator informed about the low state. Lastly, we derive the sequentially rational strategy profiles with beliefs consistent on the path.

- Sequentially rational strategies of the speculator informed about the low state with beliefs consistent on the path given that the uninformed speculator does not trade in $t = 2$

Suppose by way of contradiction that the speculator informed about the low state buys in $t = 2$. In such an equilibrium, her profit is $V^- - K - p_1(\cdot) + V^- - K - \left(\frac{V^+ + V^-}{2} - K\right) = V^- - K - p_1(\cdot) - \frac{V^+ - V^-}{2}$ if she buys in $t = 1$, $V^- - K - \left(\frac{V^+ + V^-}{2} - K\right) = -\frac{V^+ - V^-}{2}$ if she does not trade, and $p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right)$ if she sells. If she deviates and does not trade, her profit is $V^- - K - p_1(\cdot) > V^- - K - p_1(\cdot) - \frac{V^+ - V^-}{2}$ if she buys in $t = 1$, $0 > -\frac{V^+ - V^-}{2}$ if she does not trade, and $p_1(\cdot) - (V^- - K) - c > p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right)$ if she sells. Therefore, the speculator informed about the low state does not buy in $t = 2$.

Suppose the speculator informed about the low state does not trade in $t = 2$. In this case, the analysis is the same as that in Case (iv): she does not have an incentive to deviate and buy; the deviation when she sells in $t = 2$ yields a net payoff relative to not trading in $t = 2$

equal to $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) + \frac{V^+ - V^-}{4} - c$ if she buys in $t = 1$, $\frac{V^+ - V^-}{2} - c$ if she does not trade, and $-(V^- - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) + \frac{V^+ - V^-}{4} - c$ if she sells.

Now, suppose the speculator is informed about the low state sells in $t = 2$. In such an equilibrium, her profit is $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - p_1(\cdot) - c$ is she buys in $t = 1$, $\frac{V^+ - V^-}{4} - c$ if she does not trade, and $p_1(\cdot) - \frac{V^- - K}{2} + \frac{V^+ - V^-}{4} - 2c$ if she sells. If she deviates and does not trade in $t = 2$, her payoff if she buys in $t = 1$ is $V^- - K - p_1(\cdot)$, her payoff is she does not trade in $t = 1$ is zero, and her payoff if she sells in $t = 1$ is $p_1(\cdot) - (V^- - K) - c$. If she deviates and buys in $t = 2$, her payoff is always lower than that when she deviates and does not trade: $V^- - K - p_1(\cdot) + \frac{1}{2}\left[V^- - K - (V^+ - K)\right] + \frac{1}{2}\left[V^- - K - \left(\frac{V^+ + V^-}{2} - K\right)\right] < V^- - K - p_1(\cdot)$ if she buys in $t = 1$, $\frac{1}{2}\left[V^- - K - (V^+ - K)\right] + \frac{1}{2}\left[V^- - K - \left(\frac{V^+ + V^-}{2} - K\right)\right] < 0$ if she does not trade, and $p_1(\cdot) - \frac{1}{2}\left[V^+ - K + \left(\frac{V^+ + V^-}{2} - K\right)\right] - c < p_1(\cdot) - (V^- - K) - c$ if she sells. Therefore, the best deviation is when she does not trade in $t = 2$, which yields a net payoff relative to selling in $t = 2$ equal to $\frac{V^- - K}{2} - \left(\frac{V^+ - V^-}{4} - c\right)$ if she buys in $t = 1$, $-\left(\frac{V^+ - V^-}{4} - c\right)$ if she does not trade, and $-\left[\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c\right]$ if she sells.

The collection of results above implies that the speculator informed about the low state has an incentive to deviate if she buys in $t = 2$. If $\frac{V^+ - V^-}{4} > c$ and she either buys or does not trade in $t = 1$, she has an incentive to deviate if she does not trade in $t = 2$, but does not have an incentive to deviate if she sells in $t = 2$; therefore, she sells in $t = 2$. If $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) > c$, she has an incentive to deviate if she does not trade in $t = 2$, but does not have an incentive to deviate if she sells in $t = 2$; therefore, she sells in $t = 2$.

- Sequentially rational strategies of the speculator informed about the low state with beliefs consistent on the path given that the uninformed speculator sells in $t = 2$.

Suppose by way of contradiction that the speculator informed about the low state buys in $t = 2$. In such an equilibrium, her profit is $V^- - K - p_1(\cdot) + V^- - K - \left(\frac{V^+ + V^-}{2} - K\right) = V^- - K - p_1(\cdot) - \frac{V^+ - V^-}{2}$ if she buys in $t = 1$, $V^- - K - \left(\frac{V^+ + V^-}{2} - K\right) = -\frac{V^+ - V^-}{2}$ if she does not trade, and $p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right)$ if she sells. Suppose she deviates and does not trade instead. In that case, her profit is at least (it is higher if the beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment does not occur) $V^- - K - p_1(\cdot) > V^- - K - p_1(\cdot) - \frac{V^+ - V^-}{2}$ if she buys in $t = 1$, $0 > -\frac{V^+ - V^-}{2}$ if she does not trade, and at least (it is higher if beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment occurs) $p_1(\cdot) - c > p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right)$ if she sells. Therefore, the speculator informed about the low state does not buy in $t = 2$.

Suppose that the speculator informed about the low state does not trade. In such equilibrium, her profit is $-p_1(\cdot)$ if she buys in $t = 1$, zero if she does not trade, and $p_1(\cdot) - c$ if she sells. If she deviates and sells in $t = 2$ instead, her profit is $-p_1(\cdot) + \frac{1}{2}p_2(\cdot, 0) + \frac{1}{2}p_2(\cdot, -2) - c = -p_1(\cdot) + \frac{1}{2}\left[\frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)\right] + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c = -p_1(\cdot) + \left(\frac{V^+ + V^-}{2} - K\right) + \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c$ if she buys in $t = 1$, $\frac{1}{2}p_2(\cdot, 0) + \frac{1}{2}p_2(\cdot, -2) - (V^- - K) - c = \frac{V^+ - V^-}{2} + \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c$ if she does not trade, and $p_1(\cdot) - (V^- - K) - c + \frac{1}{2}p_2(\cdot, 0) + \frac{1}{2}p_2(\cdot, -2) - (V^- - K) - c = p_1(\cdot) - c - (V^- - K) + \frac{V^+ - V^-}{2} + \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c$ if she sells. If she deviates and buys in $t = 2$, her payoff is always lower than that in the proposed equilibrium in which she does not trade in $t = 2$: it equals $V^- - K - p_1(\cdot) + V^- - K - \frac{1}{2}p_2(\cdot, 2) - \frac{1}{2}p_2(\cdot, 0) = V^- - K - p_1(\cdot) - \frac{V^+ - V^-}{2} - \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} < -p_1(\cdot)$ if she buys in $t = 1$, $V^- - K - \frac{1}{2}p_2(\cdot, 2) - \frac{1}{2}p_2(\cdot, 0) = -\frac{V^+ - V^-}{2} - \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} < 0$ if she does not trade, and $p_1(\cdot) - c - \frac{1}{2}p_2(\cdot, 2) - \frac{1}{2}p_2(\cdot, 0) = p_1(\cdot) - c - \frac{V^+ - V^-}{2} - \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} < p_1(\cdot) - c$ if she sells. Therefore, she does not have an incentive to deviate and buy. The deviation when she sells in $t = 2$ yields a net payoff relative to not trading in $t = 2$ equal to $\left(\frac{V^+ + V^-}{2} - K\right) + \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c$ if she buys in $t = 1$, $\frac{V^+ - V^-}{2} + \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c$ if she does not trade, and $-(V^- - K) + \frac{V^+ - V^-}{2} + \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c$ if she sells.

Now suppose that the speculator informed about the low state sells in $t = 2$. In this case, the analysis is the same as that in Case (iv): the best deviation is when she does not trade in $t = 2$, which yields a net payoff relative to selling in $t = 2$ of $\frac{V^- - K}{2} - \left(\frac{V^+ - V^-}{4} - c\right)$ if she buys in $t = 1$, $-\left(\frac{V^+ - V^-}{4} - c\right)$ if she does not trade, and $\frac{V^- - K}{2} - \left(\frac{V^+ - V^-}{4} - c\right)$ if she sells.

The collection of results above implies that the speculator informed about the low state has an incentive to deviate if she buys in $t = 2$. If $\frac{V^+ - V^-}{4} > c$ and she does not trade or sell in $t = 1$, she has an incentive to deviate if she does not trade in $t = 2$, but does not have an incentive to deviate if she sells in $t = 2$; therefore, she sells in $t = 2$. If $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) > c$, she has an incentive to deviate if she does not trade in $t = 2$, but does not have an incentive to deviate if she sells in $t = 2$; therefore, she sells in $t = 2$.

- Sequentially rational strategies of the speculator informed about the low state with beliefs consistent on the path given that the uninformed speculator buys in $t = 2$

Suppose by contradiction that the speculator informed about the low state buys in $t=2$. In this case, the analysis is the same as that in Case (iv): she has an incentive to deviate and not trade, which implies that she does not buy in $t=2$.

Suppose that the speculator informed about the low state does not trade. In such equilibrium, her profit is $-p_1(\cdot)$ if she buys in $t=1$, zero if she does not trade, and $p_1(\cdot)-c$ if she sells. If she deviates and sells in $t=2$ instead, her profit is at least (it is higher if beliefs are such that investment occurs when $Q_2=-2$) $-p_1(\cdot)+\frac{1}{2}p_2(\cdot,0)-c=-p_1(\cdot)+\frac{1}{2}\left[\frac{\alpha}{2-\alpha}(V^+-K)+\frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^++V^-}{2}-K\right)\right]-c=-p_1(\cdot)+\frac{1}{2}\left(\frac{V^++V^-}{2}-K\right)+\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{4}-c$ if she buys in $t=1$, $\frac{1}{2}\left[p_2(\cdot,0)-(V^--K)\right]-c=\frac{V^+-V^-}{4}+\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{4}-c$ if she does not trade, and $p_1(\cdot)-\frac{V^--K}{2}-c+\frac{1}{2}\left[p_2(\cdot,0)-(V^--K)\right]-c=p_1(\cdot)-c-(V^--K)+\frac{1}{2}\left(\frac{V^++V^-}{2}-K\right)+\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{4}-c$ if she sells. If she deviates and buys, her payoff is always lower than that in the proposed equilibrium in which she does not trade in $t=2$: it equals $V^--K-p_1(\cdot)+V^--K-\frac{1}{2}p_2(\cdot,2)-\frac{1}{2}p_2(\cdot,0)=V^--K-p_1(\cdot)-\frac{V^+-V^-}{2}-\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{2}<-p_1(\cdot)$ if she buys in $t=1$, $-\frac{V^+-V^-}{2}-\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{2}<0$ if she does not trade, and $p_1(\cdot)-c-\frac{1}{2}p_2(\cdot,2)-\frac{1}{2}p_2(\cdot,0)=p_1(\cdot)-c-\left(\frac{V^++V^-}{2}-K\right)-\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{2}<p_1(\cdot)-c$. Therefore, she does not have an incentive to deviate and buy. The deviation when she sells in $t=2$ yields a net payoff relative to not trading equal to $\frac{1}{2}\left(\frac{V^++V^-}{2}-K\right)+\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{4}-c$ if she buys in $t=1$, $\frac{V^+-V^-}{4}+\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{4}-c$ if she does not trade, and $-(V^--K)+\frac{1}{2}\left(\frac{V^++V^-}{2}-K\right)+\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{4}-c$ if she sells.

Assume now that the speculator informed about the low state sells in $t=2$. In such an equilibrium, her profit is $\frac{1}{2}\left(\frac{V^++V^-}{2}-K\right)-p_1(\cdot)-c$ is she buys in $t=1$, $\frac{V^+-V^-}{4}-c$ if she does not trade, and $p_1(\cdot)-\frac{V^--K}{2}+\frac{V^+-V^-}{4}-2c$ if she sells. Suppose she deviates and does not trade in $t=2$. In that case, her payoff is at least (it is higher and equal to $-p_1(\cdot)$ if the beliefs associated with $Q_2\in\{-1,1\}$ are such that investment does not occur) $V^--K-p_1(\cdot)$ if she buys in $t=1$, zero if she does not trade, and at least (it is higher and equal to $p_1(\cdot)-(V^--K)-c$ if the beliefs associated with $Q_2\in\{-1,1\}$ are such that investment does occur) $p_1(\cdot)-c$ if she sells. If she deviates and buys in $t=2$, her payoff is always lower than that when she deviates and does not trade: it equals $V^--K-p_1(\cdot)+V^--K-\frac{1}{2}\left[\frac{\alpha}{2-\alpha}(V^+-K)+\frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^++V^-}{2}-K\right)\right]-\frac{1}{2}\left(\frac{V^++V^-}{2}-K\right)=V^--K-p_1(\cdot)-\frac{V^+-V^-}{2}-\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{4}<V^--K-p_1(\cdot)$ if she buys in $t=1$, $-\frac{V^+-V^-}{2}-\frac{\alpha}{2-\alpha}\frac{V^+-V^-}{4}<0$ if she does not trade, and $p_1(\cdot) - \frac{1}{2}\left[\frac{\alpha}{2-\alpha}\left(V^+ - K\right) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right) + \left(\frac{V^+ + V^-}{2} - K\right)\right] - c = p_1(\cdot) - c - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} < p_1(\cdot) - c$ if she sells in $t = 1$. Therefore, the best deviation is when she does not trade in $t = 2$, which yields a net payoff relative to selling in $t = 2$ equal to $\frac{V^- - K}{2} - \left(\frac{V^+ - V^-}{4} - c\right)$ if she buys in $t = 1$, $-\left(\frac{V^+ - V^-}{4} - c\right)$ if she does not trade, and $\frac{V^- - K}{2} - \left(\frac{V^+ - V^-}{4} - c\right)$ if she sells.

The collection of results above implies that the speculator informed about the low state has an incentive to deviate if she buys in $t = 2$. For $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) > c$, she has an incentive to deviate if she does not trade in $t = 2$, but does not have an incentive to deviate if she sells in $t = 2$; therefore, she sells in $t = 2$.

- Sequentially rational strategies of the uninformed speculator with beliefs consistent on the path given that the speculator informed about the low state sells in $t = 2$

Suppose by way of contradiction that if the uninformed speculator either does not trade or sells in $t = 1$, then she buys in $t = 2$. In such an equilibrium, her profit is $-\frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4}$ if she does not trade and $p_1(\cdot) - c - \frac{1}{2}p_2(\cdot, 2) - \frac{1}{2}p_2(\cdot, 0) = p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right) - \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4}$ if she sells. If she deviates and does not trade instead, her profit is $0 > -\frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4}$ if she does not trade in $t = 1$ and at least (it is higher if beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment does not occur) $p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right) > p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right) - \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4}$ if she sells. Therefore, if the uninformed speculator does not trade or sells in $t = 1$, she does not buy in $t = 2$.

Suppose that the uninformed speculator does not trade. In such equilibrium, her profit is $\left(\frac{V^+ + V^-}{2} - K\right) - p_1(\cdot)$ if she buys in $t = 1$, zero if she does not trade, and $p_1(\cdot) - \left(\frac{V^+ + V^-}{2} - K\right) - c$ if she sells. If she deviates and sells in $t = 2$, her profit is $\frac{1}{2}p_2(\cdot, 0) + \frac{1}{2}p_2(\cdot, -2) - p_1(\cdot) - c = \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - p_1(\cdot) - c$ if she buys in $t = 1$, $\frac{1}{2}\left[p_2(\cdot, 0) - \left(\frac{V^+ + V^-}{2} - K\right)\right] + \frac{1}{2}\left[p_2(\cdot, -2) - 0\right] - c = -c$ if she does not trade, and $\frac{1}{2}\left[p_1(\cdot) - \left(\frac{V^+ + V^-}{2} - K\right) - c + p_2(\cdot, 0) - \left(\frac{V^+ + V^-}{2} - K\right) - c\right] + \frac{1}{2}\left[p_1(\cdot) - 0 - c + p_2(\cdot, -2) - 0 - c\right] = p_1(\cdot) - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - 2c$ if she sells. If she deviates and buys in $t = 2$, her payoff is always lower than that in the proposed equilibrium in which she does not trade in $t = 2$: it equals $\frac{V^+ + V^-}{2} - K - p_1(\cdot) + \frac{V^+ + V^-}{2} - K - \frac{1}{2}p_2(\cdot, 2) - \frac{1}{2}p_2(\cdot, 0) = \frac{V^+ + V^-}{2} - p_1(\cdot) - \frac{V^+ - V^-}{4}$ if she buys in $t = 1$, $\frac{V^+ + V^-}{2} - K - \frac{1}{2}p_2(\cdot, 2) - \frac{1}{2}p_2(\cdot, 0) = -\frac{V^+ - V^-}{4} < 0$ if she does not trade, and $p_1(\cdot) - c - \frac{1}{2}p_2(\cdot, 2) - \frac{1}{2}p_2(\cdot, 0) = p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right) - \frac{V^+ - V^-}{4} <$

$p_1(\cdot) - \left(\frac{V^+ + V^-}{2} - K\right) - c$ if she sells. Therefore, she does not have an incentive to deviate and buy. The deviation when she sells in $t = 2$ yields a net payoff relative to not trading in $t = 2$ equal to $-\left(\frac{V^+ + V^-}{2} - K + c\right)$ if she buys in $t = 1$, $-c$ if she does not trade, and $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c$ if she sells.

Now suppose that the uninformed speculator sells in $t = 2$. In such equilibrium, her profit if she does not trade in $t = 1$ is $\frac{1}{2}\left[p_2(\cdot, 0) - \left(\frac{V^+ + V^-}{2} - K\right)\right] + \frac{1}{2}\left[p_2(\cdot, -2) - 0\right] - c = -c$ and her profit if she sells is $\frac{1}{2}\left[p_1(\cdot) - \left(\frac{V^+ + V^-}{2} - K\right) - c + p_2(\cdot, 0) - \left(\frac{V^+ + V^-}{2} - K\right) - c\right] + \frac{1}{2}\left[p_1(\cdot) - 0 - c + p_2(\cdot, -2) - 0 - c\right] = p_1(\cdot) - \left(\frac{V^+ + V^-}{2} - K\right) - 2c$. If she deviates and does not trade in $t = 2$ instead, her profit is zero if she does not trade in $t = 1$ and at least (it is higher if the beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment does not occur) $p_1(\cdot) - \left(\frac{V^+ + V^-}{2} - K\right) - c$ if she sells. If she deviates and buys in $t = 2$, her payoff is always lower than that when she deviates does not trade in $t = 2$: it equals $\frac{V^+ + V^-}{2} - K - \frac{1}{2}p_2(\cdot, 2) - \frac{1}{2}p_2(\cdot, 0) = -\frac{V^+ - V^-}{4} < 0$ if she does not trade in $t = 1$ and $p_1(\cdot) - c - \frac{1}{2}p_2(\cdot, 2) - \frac{1}{2}p_2(\cdot, 0) = p_1(\cdot) - c - \left(\frac{V^+ + V^-}{2} - K\right) - \frac{V^+ - V^-}{4} < p_1(\cdot) - \left(\frac{V^+ + V^-}{2} - K\right) - c$ if she sells. Therefore, the best deviation is when she does not trade in $t = 2$, which yields a net payoff relative to selling in $t = 2$ equal to c if she does not trade in $t = 1$ and $-\left[\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c\right]$ if she sells.

The collection of results above implies that if the uninformed speculator does not trade or sell in $t = 1$, she has an incentive to deviate if she buys in $t = 2$. If she buys in $t = 1$, she does not have an incentive to deviate if she does not trade in $t = 2$. If she does not trade in $t = 1$, she has an incentive to deviate if she sells in $t = 2$, but does not have an incentive to deviate if she does not trade in $t = 2$; therefore, she does not trade in $t = 2$. If she sells in $t = 1$ and $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) > c$, she has an incentive to deviate if she does not trade in $t = 2$, but does not have an incentive to deviate if she sells in $t = 2$; therefore, she sells in $t = 2$. If she sells in $t = 1$ and $c > \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$, she has an incentive to deviate if she sells in $t = 2$, but does not have an incentive to deviate if she does not trade in $t = 2$; therefore, she does not trade in $t = 2$.

- Sequential rationality with beliefs consistent on the path for the speculator informed about the high state to buy if either the uninformed speculator does not trade in $t = 2$ and speculator informed about the low state sells, or both the uninformed speculator and the speculator informed about the low state sell in $t = 2$

Suppose that the speculator informed about the high state buys and (a) either the uninformed speculator does not trade in $t = 2$ and speculator informed about the low state sells, or (b) both the uninformed speculator and the speculator informed about the low state sell in $t = 2$. Moreover, assume that in (b) beliefs are that the speculator is uninformed when $Q_2 \in \{-1, 1\}$. In this case, the analysis showing that it is sequentially rational given beliefs on the path for the speculator informed about the high state to buy in $t = 2$ is the same as that in Case (iv).

- Sequentially rational strategy profiles with beliefs consistent on the path

For $\frac{V^+ - V^-}{4} > c$, we have the following: (i) if the speculator informed about the high state buys in $t = 2$ and the speculator informed about the low state sells, then the only sequentially rational choice for the uninformed speculator with beliefs consistent on the equilibrium path is not to trade in $t = 2$ if she does not trade in $t = 1$; (ii) if the speculator informed about the high state buys in $t = 2$ and the speculator informed about the low state sells, then not trading in $t = 2$ is sequentially rational for the uninformed speculator given beliefs consistent on the equilibrium path if she buys in $t = 1$; (iii) if the speculator informed about the high state buys in $t = 2$ and the uninformed speculator does not trade, then the only sequentially rational strategy for the speculator informed about the low state with beliefs consistent on the equilibrium path is to sell in $t = 2$ if she either buys or does not trade in $t = 1$; (iv) if the uninformed speculator does not trade in $t = 2$ and the speculator informed about the low state sells, then buying in $t = 2$ is sequentially rational for the speculator informed about the high state given beliefs consistent on the path; and (v) for $c > \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$, if the speculator informed about the high state buys in $t = 2$ and the speculator informed about the low state sells, then the only sequentially rational choice for the uninformed speculator with beliefs on the path is to no trade in $t = 2$ if she sells in $t = 1$. For $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) > c$, a strategy profile in which the speculator informed about the high state buys in $t = 2$ is sequentially rational given beliefs consistent on the path only if the speculator informed about the low state sells in $t = 2$; moreover, we have: (i) if the speculator informed about the low state sells in $t = 2$, then the only sequentially rational strategy for the uninformed speculator with beliefs consistent on the equilibrium path is to sell in $t = 2$ if she sells in $t = 1$; (ii) if the uninformed speculator sells in $t = 2$, then the only sequentially rational strategy for the speculator informed about the low state with beliefs consistent on the equilibrium path is to sell in $t = 2$; and (iii) if both the uninformed speculator and the speculator informed about the low state sell in $t = 2$, then buying in $t = 2$ is sequentially rational for the speculator informed about the high state given beliefs consistent on the path.

Trading in $t = 1$

We characterize equilibria where the speculator informed about the high state buys in $t = 2$.

Claim 1 *If* $\frac{V^+ - V^-}{2} - c > 0$*, there is no equilibrium in which the speculator informed about the high state sells in* $t = 1$*.*

Suppose by way of contradiction that she sells in $t = 1$ in equilibrium. In such an equilibrium, the speculator informed about the low state sells in $t = 1$. To see this, suppose instead that the speculator informed about the low state either (a) buys or (b) does not trade. Let us first consider (a). If the uninformed speculator does not trade, then with probability $\frac{1}{2}$ the order flow equals $Q_1 = 2$ and reveals that the speculator is informed about the low state, in which case her profit is zero; with probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$, in which case sequential rationality implies that the speculator informed about the low state either does not trade or sells in $t = 2$; in the former case her profit is $-p_1(0) = -\frac{V^+ - K}{2} < 0$; in the latter case it equals $\frac{1}{2}[V^- - K - p_1(0) + p_2(0,0) - (V^- - K) - c] + \frac{1}{2}[-p_1(0) - c] = \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \left[\frac{1}{4}(V^+ - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)\right] - c = -\frac{V^+ - K}{4} - c < 0$; therefore, the expected profit is negative. If the uninformed speculator buys, then with probability $\frac{1}{2}$ the order flow equals $Q_1 = 2$, in which case sequential rationality results in a profit of $-p_1(2)$; with probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$, in which case sequential rationality implies that the speculator informed about the low state either does not trade or sells in $t = 2$; in the former case her profit is $-p_1(0) < 0$; in the latter case it equals $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - p_1(0) - c < 0$, where the inequality follows because $p_1(0) > \frac{V^+ + V^-}{2} - K$; therefore, the expected profit is negative. If the uninformed speculator sells, then with probability $\frac{1}{2}$ the order flow equals $Q_1 = 2$ and reveals the speculator is informed about the low state; with $\frac{1}{2}$ the order flow equals $Q_1 = 0$, in which case sequential rationality implies that the speculator informed about the low state either does not trade or sells in $t = 2$; in the former case her profit is $-p_1(0) < 0$; in the latter case it equals $\frac{1}{2}\left(\frac{V^{+}+V^{-}}{2}-K\right)-p_1(0)-c<0$, where the inequality follow because $p_1(0)>\frac{V^{+}+V^{-}}{2}-K$; therefore, the expected profit is negative. We conclude that in all possibilities, the expected profit of the speculator informed about the low state is negative. She has an incentive to deviate and not trade in both periods, securing a zero payoff. This contradicts (a).

Under (b), let us first assume that the uninformed speculator either buys or sells. In this case, the order flow $Q_1 \in \{-1,1\}$ reveals that the speculator is informed about the low state, yielding her profit of zero. If she deviates and sells in $t=1$ instead, with probability $\frac{1}{2}$ the order flow equals $Q_1=0$ and $p_1(0)=\frac{\alpha}{2-\alpha}\left(V^{+}-K\right)+\frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^{+}+V^{-}}{2}-K\right)$. In this case, she can profit by following the strategy of the uninformed speculator in $t=2$, which yields her a profit from the period-2 trade of at least zero (her profit is higher if $\frac{\alpha}{2-\alpha}\frac{V^{+}-V^{-}}{4}>c$, in which case the uninformed speculator sells while the speculator informed about the high state buys in $t=2$); thus, her overall profit is at least $p_1(0)-\left(V^{-}-K\right)-c=\frac{V^{+}-V^{-}}{2}+\frac{\alpha}{2-\alpha}\frac{V^{+}-V^{-}}{2}-c>0$. With probability $\frac{1}{2}$ the order flow equals $Q_1=-2$. If the uninformed speculator buys, then $p_1(-2)=V^{+}-K$; in this case, she can profit by not trading in $t=2$, receiving an overall payoff of $V^{+}-K-\left(V^{-}-K\right)-c=V^{+}-V^{-}-c>0$. If the uninformed speculator sells, then $p_1(-2)=\frac{\alpha}{2-\alpha}\left(V^{+}-K\right)+\frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^{+}+V^{-}}{2}-K\right)$; in this case, she can profit by following the strategy of the uninformed speculator in $t=2$, which yields her a profit from the period-2 trade of at least zero (her profit is higher if $\frac{\alpha}{2-\alpha}\frac{V^{+}-V^{-}}{4}>c$, in which case the uninformed speculator sells while the speculator informed about the high state buys in $t=2$); thus, her overall profit is at least $p_1(-2)-\left(V^{-}-K\right)-c=\frac{V^{+}-V^{-}}{2}+\frac{\alpha}{2-\alpha}\frac{V^{+}-V^{-}}{2}-c>0$.

Let us assume that the uninformed speculator does not trade under (b). In this case, sequential rationality implies that the overall profit of the speculator informed about the low state is zero. If she deviates and sells in $t=1$ instead, with probability $\frac{1}{2}$ the order flow equals $Q_1=0$ and $p_1(0)=V^{+}-K$, while with probability $\frac{1}{2}$ the order flow equals $Q_1=-2$ and $p_1(-2)=V^{+}-K$. In both cases, she can profit by not trading in $t=2$, receiving an overall payoff of $V^{+}-K-\left(V^{-}-K\right)-c=V^{+}-V^{-}-c>0$. It follows from the analysis above that the speculator informed about the low state trades, contradicting (b).

Therefore, in any equilibrium in which the speculator informed about the high state sells in $t=1$, the speculator informed about the low state also sells in $t=1$. However, in this case the speculator informed about the high state has an incentive to deviate. To see this, note that in such an equilibrium the order flow equals $Q_1 = -2$ and $Q_1 = 0$ with equal probability. First, let us consider the situation in which the uninformed speculator buys in $t = 1$. In this case the speculator informed about the high state can profit by buying in $t = 1$, following the uninformed speculator's equilibrium strategy in $t = 2$ if $Q_1 = 2$ (probability $\frac{1}{2}$), and conforming with her period-2 equilibrium strategy if $Q_1 = 0$ (probability $\frac{1}{2}$). Since this deviation and the equilibrium strategy yield the same payoff if $Q_1 = 0$, it suffices to show that the deviation payoff if $Q_2 = 2$ is higher than the equilibrium payoff when $Q_1 = -2$. If $Q_1 = -2$, sequential rationality implies that the speculator informed about the low state sells in $t = 2$, in which case the expected profit of the speculator informed about the high state is $p_1(-2) - c - \left(\frac{p_2(-2,2)}{2} + \frac{p_2(-2,0)}{2}\right) = \frac{1}{4}(V^+ - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c - \left[\frac{1}{2}(V^+ - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)\right] < 0$. If $Q_1 = 2$, her deviation strategy yields at least $V^+ - K - p_1(2) = V^+ - K - \left(\frac{V^+ + V^-}{2} - K\right) > 0$ (her payoff is higher if the uninformed speculator buys in $t = 2$). Therefore, the speculator informed about the high state has an incentive to deviate.

Next, let us consider the situations in which the uninformed speculator either does not trade or sells in $t = 1$, in which case the expected profit of the speculator informed about the high state equals $p_1(\cdot) - \left(\frac{p_2(\cdot,2)}{2} + \frac{p_2(\cdot,0)}{2}\right) - c$. If the uninformed speculator does not trade $t = 1$, sequential rationality implies that the speculator informed about the low state sells in $t = 2$ and the expected profit of the speculator informed about the high state equals $\frac{V^+ - K}{4} + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \frac{1}{2}\left[V^+ - K + \left(\frac{V^+ + V^-}{2} - K\right)\right] - c < 0$; thus, the speculator informed about the high state has an incentive to deviate and not trade in periods $t = 1, 2$ to secure a profit of zero. If the uninformed speculator sells in $t = 1$, sequential rationality implies that the speculator informed about the low state does not buy in $t = 2$. The speculator informed about the low state sells in $t = 2$ if the uninformed speculator either does not trade or sells in $t = 2$; in the former case, the profit of the speculator informed about the high state is $\frac{\alpha}{4}(V^+ - K) + \frac{2-\alpha}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \frac{1}{2}\left[V^+ - K + \left(\frac{V^+ + V^-}{2} - K\right)\right] - c = \frac{-\left[V^+ + (1-\alpha)V^- - (2-\alpha)K - 2(1-\alpha)\left(V^- - K\right)\right]}{4} - c < 0$; in the latter, it equals $\frac{\alpha}{4}(V^+ - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \frac{1}{2}\left[V^+ - K + \left(\frac{V^+ + V^-}{2} - K\right)\right] - c < 0$. If the uninformed speculator buys in $t = 2$, the profit of the speculator informed about the high state is $\frac{\alpha}{2}(V^+ - K) + \frac{2(1-\alpha)}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \left[\frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)\right] - c < 0$ if the speculator informed about the low state does not trade in $t = 2$, and equals

$\frac{\alpha\left(V^{+}-K\right)}{4}+\frac{2(2-\alpha)}{4}\left(\frac{V^{+}+V^{-}}{2}-K\right)-\frac{1}{2}\left[\frac{\alpha\left(V^{+}-K\right)}{2-\alpha}+\frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^{+}+V^{-}}{2}-K\right)+\left(\frac{V^{+}+V^{-}}{2}-K\right)\right]-c=\frac{-\alpha\left[V^{+}+(1-\alpha)V^{-}-(2-\alpha)K\right]}{4(2-\alpha)}-c<0$ if the speculator informed about the low state sells in $t=2$. Therefore, the speculator informed about the high state has an incentive to deviate and not trade in periods $t=1,2$ to secure a profit of zero.

Claim 2 *Suppose the speculator informed about the high state buys in $t=1$ in equilibrium. It is not sequentially rational with beliefs consistent on the path for the speculator informed about the low state to buy in $t=1$. If $\frac{V^{+}-V^{-}}{4}-c>0$, it is not sequentially rational given beliefs consistent on the path for the speculator informed about the low state to not trade in $t=1$.*

Suppose instead that speculator informed about the low state buys in $t=1$. If the uninformed speculator does not trade in $t=1$, then sequential rationality implies that the speculator in formed about the low state either does not trade or sells in $t=2$: in the former case, her profit equals $-p_1(\cdot)=-\frac{V^{+}-K}{2}<0$; in the latter case, it equals $\left(\frac{p_2(\cdot,0)}{2}+\frac{p_2(\cdot,-2)}{2}\right)-p_1(\cdot)-c=\frac{1}{2}\left(\frac{V^{+}+V^{-}}{2}-K\right)-\left[\frac{V^{+}-K}{4}+\frac{1}{2}\left(\frac{V^{+}+V^{-}}{2}-K\right)\right]-c<0$. If the uninformed speculator buys in $t=1$, sequential rationality implies that the speculator informed about the low state either does not trade or sells in $t=2$; in the former case her profit is $-p_1(\cdot)<0$; in the latter case it equals $\frac{1}{2}\left(\frac{V^{+}+V^{-}}{2}-K\right)-p_1(\cdot)-c<0$, where the inequality follow because $p_1(\cdot)>\frac{V^{+}+V^{-}}{2}-K$; therefore, the expected profit is negative. If the uninformed speculator sells in $t=1$, then with probability $\frac{1}{2}$ the order flow equals $Q_1=2$, in which case sequential rationality implies that that the speculator in formed about the low state either does not trade or sells in $t=2$: in the former case, her profit equals $-p_1(2)=-\frac{V^{+}-K}{2}<0$; in the latter case, it equals $\left(\frac{p_2(2,0)}{2}+\frac{p_2(2,-2)}{2}\right)-p_1(2)-c=\frac{1}{2}\left(\frac{V^{+}+V^{-}}{2}-K\right)-\left[\frac{V^{+}-K}{4}+\frac{1}{2}\left(\frac{V^{+}+V^{-}}{2}-K\right)\right]-c<0$. With probability $\frac{1}{2}$ the order flow equals $Q_1=0$, in which case sequential rationality implies that the speculator informed about the low state either does not trade or sells in $t=2$; in the former case her profit is $-p_1(0)<0$; in the latter case it equals $\frac{1}{2}\left(\frac{V^{+}+V^{-}}{2}-K\right)-p_1(0)-c<0$, where the inequality follow because $p_1(0)>\frac{V^{+}+V^{-}}{2}-K$. Therefore, the expected profit of the speculator informed about the low state is always negative, which implies that she can profit by not trading in periods $t=1,2$ to secure a profit of zero. This leads to a contradiction.

Next, suppose that $\frac{V^{+}-V^{-}}{4}-c>0$ and the speculator informed about the low state does not trade in $t=1$. If uninformed speculator does not trade, then sequential rationality implies that the profit of the speculator informed about the low state is zero. In this case, she can profit by selling in $t = 1$ and not trading in $t = 2$: for $Q_1 = 0$, we have $p_1(0) = V^+ - K$, in which case this deviation yields her a payoff of $p_1(0) - (V^- - K)) - c = V^+ - V^- - c$; for $Q_1 = -2$, this deviation generates a profit of at least $-c$ (her payoff is higher if beliefs conditional of $Q_1 = -2$ are such that investment occurs with positive probability when she sells in $t = 2$); therefore, the expected profit from the deviation is at least $\frac{V^+ - V^-}{2} - c > 0$. If the uninformed speculator either buys or sells in $t = 1$, then the order flow $Q_1 \in \{-1, 1\}$ reveals the type of the speculator informed about the low state, resulting in a profit of zero. In this case, she can profit by selling in $t = 1$, not trading in $t = 2$ if $Q_1 = -2$, and following the equilibrium strategy of the uninformed speculator if $Q_1 = 0$: for $Q_1 = 0$, we have $p_1(0) = \frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)$, in which case this deviation yields her a profit from the period-2 trade of at least zero (her profit is higher if $\frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} > c$, as in this case the uninformed speculator sells in $t = 2$), and an overall profit of at least $p_1(0) - (V^- - K) = \frac{V^+ - V^-}{2} + \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{2} - c$; for $Q_1 = -2$, this deviation generates a profit of at least $-c$ (her payoff is higher if beliefs conditional on $Q_2 \in \{-1, 1\}$ are such that investment occurs); thus, the expected profit from the deviation is at least $\frac{V^+ - V^-}{4} + \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c > 0$. This leads to a contradiction.

Claim 3 *Suppose the speculator informed about the high state buys in $t = 1$ in equilibrium. If the speculator informed about the low state does not trade in $t = 1$, it is not sequentially rational given beliefs consistent on the path for the uninformed speculator to buy in $t = 1$. If $\frac{V^+ - V^-}{2} - c > 0$ and the speculator informed about the lows state sells in $t = 1$, it is not sequentially rational given beliefs consistent on the path for the uninformed speculator to buy in $t = 1$.*

Suppose the speculator informed about the low state does not trade in $t = 1$ and the uninformed speculator buys in $t = 1$. In this case, sequential rationality implies that either that the uninformed speculator sells in $t = 2$ while the speculator informed about the high state buys, or that both the uninformed speculator and the speculator informed about the high state do not trade in $t = 2$; the former case yields an overall payoff of $\frac{1}{2}\left[p_2(\cdot, 0) + p_2(\cdot, -2)\right] - p_1(\cdot) - c = \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) + \frac{1}{2}\left[\frac{\alpha\left(V^+ - K\right)}{2-\alpha} + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)\right] - \left[\frac{\alpha\left(V^+ - K\right)}{2-\alpha} + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)\right] - c = -\frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c < 0$; the latter case yields an overall payoff of $\frac{V^+ + V^-}{2} - K - p_1(\cdot) =$

$-\frac{\alpha}{2-\alpha}\frac{V^{+}-V^{-}}{2} < 0$. Therefore, the uninformed speculator has an incentive to deviate and not trade in periods $t = 1, 2$ to secure a profit of zero, which leads to a contradiction.

Next, suppose that $\frac{V^{+}-V^{-}}{2} - c > 0$ and the speculator informed about the low state sells in $t = 1$, while the uninformed speculator buys in $t = 1$. With probability $\frac{1}{2}$ the order flow equals $Q_1 = 2$, in which case sequential rationality implies that either that the uninformed speculator sells in $t = 2$ while the speculator informed about the high state buys, or that both the uninformed speculator and the speculator informed about the high state do not trade in $t = 2$; the former case yields an overall payoff of $\frac{1}{2}\left[p_2(2,0) + p_2(2,-2)\right] - p_1(2) - c = \frac{1}{2}\left(\frac{V^{+}+V^{-}}{2} - K\right) + \frac{1}{2}\left[\frac{\alpha\left(V^{+}-K\right)}{2-\alpha} + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^{+}+V^{-}}{2} - K\right)\right] - \left[\frac{\alpha\left(V^{+}-K\right)}{2-\alpha} + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^{+}+V^{-}}{2} - K\right)\right] - c = -\frac{\alpha}{2-\alpha}\frac{V^{+}-V^{-}}{4} - c < 0$; the latter case yields an overall payoff of $\frac{V^{+}+V^{-}}{2} - K - p_1(2) = -\frac{\alpha}{2-\alpha}\frac{V^{+}-V^{-}}{2} < 0$. With probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$, in which case the speculator informed about the low state either does not trade or sells in $t = 2$; if the uninformed speculator buys in $t = 2$, her overall profit is $\frac{V^{+}+V^{-}}{2} - K - p_1(0) + \frac{V^{+}+V^{-}}{2} - K - \frac{p_2(0,2)}{2} - \frac{p_2(0,0)}{2} = \frac{V^{+}+V^{-}}{2} - K - p_1(0) - \frac{V^{+}-V^{-}}{4} < 0$ if the speculator informed about the low state sells and $\frac{V^{+}+V^{-}}{2} - K - p_1(0) - \frac{V^{+}-V^{-}}{2} < 0$ if she does not trade; if the uninformed speculator either does not trade or sells in $t = 2$, then sequential rationality implies that the speculator informed about the low state sells in $t = 2$ (as per the analysis of Case (v)), which implies that overall the payoff of the uninformed speculator equals $\frac{V^{+}+V^{-}}{2} - K - p_1(0) < 0$ if she does not trade in $t = 2$ and $\frac{p_2(0,0)}{2} + \frac{p_2(0,-2)}{2} - p_1(0) - c = \frac{1}{2}\left(\frac{V^{+}+V^{-}}{2} - K\right) - p_1(0) - c < 0$ if she sells; therefore the overall of the uninformed speculator is always negative. It follows that the expected profit of the uninformed speculator is negative when she buys in $t = 1$, which implies that she has an incentive to deviate and not trade in periods $t = 1, 2$ to secure a profit of zero, leading to a contradiction.

Claim 4 *Suppose the speculator informed about the high state buys in $t = 1$ in equilibrium. If $\frac{V^{+}-V^{-}}{4} - c < 0$ and the speculator informed about the low state does not trade in $t = 1$, then it is not sequentially rational given beliefs on the path for the uninformed speculator to sell in $t = 1$. If $\frac{V^{+}-K}{12} > c$ and the speculator informed about the low state sells in $t = 1$, then it is not sequentially rational given beliefs on the path for the uninformed speculator not to trade in $t = 1$.*

First, suppose that $\frac{V^{+}-V^{-}}{4} - c < 0$ and the speculator informed about the low state does not trade in $t = 1$, while the uninformed speculator sells in $t = 1$. With probability $\frac{1}{2}$ the order flow equals $Q_1 = -2$ and reveals that the speculator is uninformed, which implies that her profit equals $-c$; with probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$ and sequential rationality implies that the speculator informed about the high state buys in $t = 1$ while the uninformed speculator does not trade (as per the analysis of Case (ii)), which implies that the profit of the uninformed speculator equals $p_1(0) - \left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right) - \left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{2} - c$; therefore, the expected profit of the uninformed speculator equals $\frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} - c < 0$ so that she has an incentive to deviate and not trade in periods $t = 1, 2$ in order to secure a profit of zero, which leads to a contradiction.

Next, suppose $\frac{V^+ - K}{12} = \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \frac{1}{4}\left(\frac{2}{3}V^+ + V^- - \frac{5}{3}K\right) > c$ and the speculator informed about the low state sells in $t = 1$. Note that $\frac{2}{3}V^+ + V^- - \frac{5}{3}K = \frac{1}{2}V^+ + V^- - \frac{3}{2}K + \frac{V^+ - K}{6} > 0$, where the last inequality follows from assumption that $\frac{1}{2} < \frac{V^+ + V^- - 2K}{V^+ - K}$. Therefore, $\frac{V^+ - K}{12} < \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$. Suppose by way of contradiction that the uninformed speculator does not trade in $t = 1$, which implies that the order flow $Q_1 \in \{-1, 1\}$ reveals her type and results in a payoff of zero. Consider a deviation in which she sells in $t = 1$ and conforms with the strategy of the speculator informed about the low state in $t = 2$. The order flow equals $Q_1 = -2$ with probability $\frac{1}{2}$, in which case her profit from the period-2 trade is zero. Her profit from the period-1 trade is $p_1(-2) - 0 - c = -c$. Thus, her overall profit when $Q_1 = -2$ is $-c$. With probability $\frac{1}{2}$, the order flow equals $Q_1 = 0$, in which case her profit from the period-2 trade is $-c$; her profit from the period-1 trade is $p_1(0) - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{V^+ - K}{4} + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{V^+ - K}{4} - c$; thus, her overall profit when $Q_1 = 0$ is $\left(\frac{V^+ - K}{4} - c\right) - c$. It follows that the expected profit of the uninformed speculator from the deviation is $\frac{1}{2}\left(\frac{V^+ - K}{4} - c\right) - c$, which is positive if and only if $c < \frac{V^+ - K}{12}$. Therefore, she has an incentive to deviate if $c < \frac{V^+ - K}{12}$.

Claim 5 *If $\alpha\frac{V^+ - K}{12} > c$, then the speculator informed about the high state buying in $t = 1$ and in $t = 2$ and both the speculator informed about the low state and the uninformed speculator selling in $t = 1$ and selling again in $t = 2$ if $p_1(\cdot) > 0$ constitute a sequentially rational strategy profile with beliefs consistent on the path.*

Suppose the speculator informed about the high state buys in $t = 1$ and the speculator informed about the low state sells. If the uninformed speculator sells in $t = 1$, then $Q_1 = -2$

with probability $\frac{1}{2}$, in which case sequential rationality implies that profit of the uninformed speculator from the period-2 trade is at most zero (it is lower and equal to $-c$ if she sells in $t = 2$). Her profit from the period-1 trade is as follows: it equals $p_1(-2) - \left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right) - \left(\frac{V^+ + V^-}{2} - K\right) - c = -\frac{\alpha}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right) - c$ if the period-2 trades always reveal her type; $p_1(-2) - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{1-\alpha}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right) - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c = -\frac{\alpha}{2(2-\alpha)}\left(\frac{V^+ + V^-}{2} - K\right) - c$ if the period-2 trades sometimes reveal her type; and $p_1(-2) - 0 - c = -c$ if the period-2 trades never reveal her type (which is shown to be sequentially rational given beliefs consistent on the path in $t = 2$). Thus, her overall profit when $Q_1 = -2$ is at most $-c$. With probability $\frac{1}{2}$, the order flow equals $Q_1 = 0$, in which case sequential rationality implies that both the uninformed speculator and the speculator informed about the low state sell in $t = 2$. Her profit from the period-2 trade is $-c$. Her profit from the period-1 trade is $p_1(0) - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{\alpha}{4}(V^+ - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{\alpha}{4}(V^+ - K) - c$; therefore, her overall profit is when $Q_1 = 0$ is $\frac{\alpha}{4}(V^+ - K) - 2c$. It follows the highest expected profit of the uninformed speculator is $\frac{1}{2}\left[\frac{\alpha}{4}(V^+ - K) - c\right] - c$, which is positive if and only if $\alpha\frac{V^+ - K}{12} > c$.

If the uninformed speculator deviates and buys, with probability $\frac{1}{2}$ the order flow equals $Q_1 = 2$ and $p_1(2) = p_2(2, \cdot) = V^+ - K$. In this case, her period-1 trade profit is $\frac{V^+ + V^-}{2} - K - p_1(2) = -\frac{V^+ - V^-}{2}$. Her period-2 trade profit is $\frac{V^+ + V^-}{2} - K - p_2(2, \cdot) = -\frac{V^+ - V^-}{2}$ if she buys in $t = 2$, zero if she does not trade, and $p_2(2, \cdot) - \left(\frac{V^+ + V^-}{2} - K\right) - c = \frac{V^+ - V^-}{2} - c$ if she sells. Thus, her overall profit is at most $-c$. With probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$ and we have $p_1(0) = \frac{\alpha}{4}(V^+ - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$, in which case her overall profit is determined as follows: it equals $\frac{1}{2}\left[\frac{V^+ + V^-}{2} - K - p_1(0) + \frac{V^+ + V^-}{2} - K - p_2(0, 2)\right] + \frac{1}{2}\left[\frac{V^+ + V^-}{2} - K - p_1(0) + \frac{V^+ + V^-}{2} - K - p_2(0, 0)\right] = \frac{V^- - K}{2} - \frac{\alpha\left(V^+ - K\right)}{4} < 0$ if she buys in $t = 2$, at most $\frac{V^+ + V^-}{2} - K - p_1(0) = \frac{(1-\alpha)V^+ + V^- - (2-\alpha)K}{4} < 0$ if she does not trade (it is lower if the beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment does not occur), and $\frac{p_2(0,0)}{2} + \frac{p_2(0,-2)}{2} - p_1(0) - c = -\frac{\alpha\left(V^+ - K\right)}{4} - c < 0$ if she sells. Therefore, her expected profit is negative, so she does not have an incentive to deviate and buy. Moreover, the uninformed speculator does not have an incentive to deviate and not trade if the beliefs associated with $Q_1 \in \{-1, 1\}$ are that the speculator is informed about the low state, as in this case investment does not occur and her profit equals zero. Therefore, it is sequentially rational given beliefs consistent on the path for the uninformed speculator to sell in $t = 1$ and sell again $t = 2$ when $p_1(\cdot) > 0$.

Let us assume that the speculator informed about the high state buys in $t = 1$ and the uninformed speculator sells. If the speculator informed about the low state sells in $t = 1$, then $Q_1 = -2$ with probability $\frac{1}{2}$, in which case sequential rationality implies that the speculator informed about the low state does not sell in $t = 2$ and that overall profit is $p_1(-2) - c$: it equals $\frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right) - c$ if the period-2 trades always reveal her type; $\frac{1-\alpha}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right) - c$ if the period-2 trades sometimes reveal her type; and $-c$ if the period-2 trades never reveal her type (which is shown to be sequentially rational given beliefs consistent on the path in $t = 2$). Thus, her overall profit when $Q_1 = -2$ is at least $-c$. With probability $\frac{1}{2}$, the order flow equals $Q_1 = 0$, in which case sequential rationality implies that both the uninformed speculator and the speculator informed about the low state sell in $t = 2$, yielding her an overall profit of $\frac{1}{2}\left[p_1(0) - (V^- - K) + p_2(0,0) - (V^- - K) - 2c\right] + \frac{1}{2}\left[p_1(0) - 2c\right] = \frac{\alpha}{4}\left(V^+ - K\right) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - \frac{V^- - K}{2} + \frac{V^+ - V^-}{4} - 2c = \frac{\alpha}{4}\left(V^+ - K\right) + \frac{V^+ - V^-}{2} - 2c$. Therefore, the expected profit of the speculator informed about the low state is at least $\frac{1}{2}\left[\frac{\alpha}{4}\left(V^+ - K\right) + \frac{V^+ - V^-}{2} - c\right] - c$, which is positive if and only if $\frac{\alpha\left(V^+ - K\right)}{12} + \frac{V^+ - V^-}{6} > c$.

If the speculator informed about the low state deviates and buys, with probability $\frac{1}{2}$ the order flow equals $Q_1 = 2$ and $p_1(2) = p_2(2,\cdot) = V^+ - K$. In this case, her period-1 trade profit is $V^- - K - p_1(2) = -\left(V^+ - V^-\right)$. Her period-2 trade profit is $V^- - K - p_2(2,\cdot) = -\left(V^+ - V^-\right)$ if she buys in $t = 2$, zero if she does not trade, and $p_2(2,\cdot) - (V^- - K) - c = \left(V^+ - V^-\right) - c$ if she sells. Thus, her overall profit is at most $-c$. With probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$ and we have $p_1(0) = \frac{\alpha}{4}\left(V^+ - K\right) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$, in which case her overall profit is determined as follows: it equals $\frac{1}{2}\left[V^- - K - p_1(0) + V^- - K - p_2(0,2)\right] + \frac{1}{2}\left[V^- - K - p_1(0) + V^- - K - p_2(0,0)\right] = \frac{V^- - K}{2} - \left(V^+ - V^-\right) - \frac{\alpha\left(V^+ - K\right)}{4} < 0$ if she buys in $t = 2$, at most $-p_1(0) = -\left(\frac{V^+ + V^-}{2} - K\right) + \frac{(1-\alpha)V^+ + V^- - (2-\alpha)K}{4} < 0$ if she does not trade (it is lower the beliefs associated with $Q_2 \in \{-1, 1\}$ are such that investment occurs), and $\frac{p_2(0,0)}{2} + \frac{p_2(0,-2)}{2} - p_1(0) - c = -\frac{\alpha\left(V^+ - K\right)}{4} - c < 0$ if she sells. Therefore, her expected profit is negative, so she does not have an incentive to deviate and buy. Moreover, the speculator informed about the low state does not have the incentive to deviate and not trade if the beliefs associated with $Q_1 \in \{-1,1\}$ are that the speculator is informed about the low state, as in this case investment does not occur and her profit equals zero. Therefore, it is sequentially rational given beliefs consistent on the path for the speculator informed about the low state to sell in $t=1$ and sell again $t=2$ when $p_1(\cdot)>0$.

Now let us assume that the uninformed speculator and the speculator informed about the low state sell in $t=1$. If the speculator informed about the high state buys in $t=1$, then $Q_1=2$ with probability $\frac{1}{2}$, in which case the order flow reveals her type and results in a zero profit. With probability $\frac{1}{2}$, the order flow equals $Q_1=0$ and $p_1(0)=\frac{\alpha}{4}(V^+-K)+\frac{1}{2}\left(\frac{V^++V^-}{2}-K\right)$, in which case it is sequentially rational for both the uninformed speculator and the speculator informed about the low state to sell in $t=2$ and for the speculator informed about the high state to buy, yielding her an overall profit of $\frac{1}{2}\left[V^+-K-p_1(0)+V^+-K-p_2(0,2)\right]+\frac{1}{2}\left[V^+-K-p_1(0)+V^+-K-p_2(0,0)\right]=V^+-K-p_1(0)+\frac{V^+-V^-}{4}$. Therefore, the expected profit of the speculator informed about the high state is at least $\frac{1}{2}\left(V^+-K-p_1(0)+\frac{V^+-V^-}{4}\right)=\frac{1}{2}\left[\frac{(2-\alpha)\left(V^+-K\right)}{4}+\frac{V^+-V^-}{2}\right]>0$.

If the speculator informed about the high state deviates and sells, with probability $\frac{1}{2}$ the order flow equals $Q_1=-2$. In this case, her overall profit is at most (it is lower if the period-2 trades are such that investment occurs with positive probability) $p_1(-2)-c=-c$ if she buys in $t=2$, $p_1(-2)-c=-c$ if she does not trade, and $p_1(-2)-2c=-2c$ if she sells. With probability $\frac{1}{2}$ the order flow equals $Q_1=0$ and we have $p_1(0)=\frac{\alpha}{4}(V^+-K)+\frac{1}{2}\left(\frac{V^++V^-}{2}-K\right)$. In this case, sequential rationality with beliefs that the speculator is uninformed for $Q_2\in\{-1,1\}$ (as used in the analysis of Case (v)) results in a payoff of $p_1(0)-(V^+-K)-c<-c$ if she buys in $t=2$, $p_1(0)-(V^+-K)-c<-c$ if she does not trade, and $p_1(0)-\frac{V^+-K}{2}+\frac{1}{2}\left[\frac{V^++V^-}{2}-K-(V^+-K)\right]-2c=-\frac{(1-\alpha)\left(V^+-K\right)}{4}-\frac{V^+-V^-}{4}+\frac{V^--K}{4}-2c<0$ if she sells. Therefore, her expected profit is negative, so she does not have an incentive to deviate and buy. Moreover, the speculator informed about the high state does not have the incentive to deviate and not trade if the beliefs associated with $Q_1\in\{-1,1\}$ are that the speculator is informed about the low state, as in this case investment does not occur and her profit equals zero. Therefore, it is sequentially rational given beliefs consistent on the path for the speculator informed about the high state to buy in $t=1$ and $t=2$.

- Sequentially rational strategy profiles with beliefs consistent on the path

The collection of the previous results implies the following. For $\mathbf{c} < \frac{\alpha}{\mathbf{12}}(\mathbf{V}^+ - \mathbf{K})$, there exists an equilibrium in which the speculator informed about the high state buys in $t = 1$ if and only if the uninformed speculator and the speculator informed about the low state sell in $t = 1$, and sell again in $t = 2$ when their types are not revealed by the period-1 trade (Claim 5).

For $\frac{\alpha}{\mathbf{12}}(\mathbf{V}^+ - \mathbf{K}) < \mathbf{c} < \frac{\mathbf{V}^+ - \mathbf{K}}{\mathbf{12}}$, there is no equilibrium in which the speculator informed about the high state buys in $t = 1$: the combination of Claims 2, 3, and 4 implies that an equilibrium in which the speculator informed about the high state buys in $t = 1$ exists only if both the uninformed speculator and the speculator informed about the low state sell in $t = 1$; however, the analysis of Claim 5 shows that the uninformed speculator has an incentive to deviate and not trade when she sells $t = 1$. It follows from Claim 1 that an equilibrium exists only if the speculator informed about the high state does not trade in $t = 1$.

Moreover, an equilibrium in which the speculator informed about the high state does not trade in $t = 1$ exists only if neither the uninformed speculator nor the speculator informed about the low state sells in $t = 1$. To see this, first note that the analysis of Claim 5 shows that when the uninformed speculator sells in $t = 1$, and the period-1 trade only reveals that the speculator is not informed about the high state, the uninformed speculator's profit is at most $-c$; when the uninformed speculator sells in $t = 1$, and the period-1 trade reveals her type, then her profit is $-c$; therefore, the uninformed speculator always has the incentive to deviate and not trade in both periods to secure a profit of zero when she sells in $t = 2$. Next, note that when the speculator informed about the low state sells in $t = 1$ and the period-1 trade always reveals her type (i.e., the uninformed speculator does not trade), then her profit is $-c$; therefore, the speculator informed about the low state has an incentive to deviate and not trade in both periods to secure a payoff of zero. Lastly, when the speculator informed about the low state sells in $t = 1$ and the period-1 trade only sometimes reveals her type (i.e., the uninformed speculator buys), sequential rationality implies that her expected profit is at most (it is lower if the period-2 trade not always reveals her type) $\frac{1}{2}(p_1(\cdot) - 2c) + \frac{1}{2}(-c) = \frac{1}{2}\left[\frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right) - c\right] - c$; therefore, the speculator informed about the low state has an incentive to deviate and not trade in $t = 1$ and sell in $t = 2$, which yields her a profit of at least (it is higher if the beliefs associated with $Q_2 = -2$ are such that investment occurs) $\frac{1}{2}\left[p_2(\cdot,0) - (V^- - K)\right] - c = \frac{V^+ - V^-}{2} - c = \frac{V^+ + V^-}{2} - K - (V^- - K) - c > \frac{1}{2}\left[\frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right) - c\right] - c$

In addition, an equilibrium in which the speculator informed about the high state does not trade in $t = 1$ exists only if neither uninformed speculator nor the speculator informed about the low state buy in $t = 1$. To see this, first note that if the speculator informed about the low state buys in $t = 1$ and the period-1 trade always reveals her type (i.e., the uninformed speculator does not trade), then her profit is 0; in this case, she has an incentive to deviate and not trade in $t = 1$ and sell in $t = 2$, which gives her an expected profit of $\frac{1}{2}\left[p_2(\cdot,0) - (V^- - K)\right] + \frac{1}{2}\left[p_2(\cdot,-2) - (V^- - K)\right] - c = \frac{\alpha}{2-\alpha}\frac{V^+ - V^-}{4} + \frac{V^+ - V^-}{2} - c > 0$. If the speculator informed about the low state buys in $t = 1$ and the period-1 trade never reveals her type (i.e., the uninformed speculator buys), then sequential rationality implies that she does not sell in $t = 2$ and her profit equals $-p_1(\cdot) \leq 0$; in this case, she has an incentive to deviate and not trade in $t = 1$ and sell in $t = 2$, which gives her an expected payoff of at least $\frac{1}{2}\left[p_2(0) - (V^- - K)\right] - c = \frac{V^+ - V^-}{2} - c > 0$ (it is higher if the beliefs associated with $Q_2 = -2$ are such that investment occurs).

Next, consider an equilibrium in which both the speculators informed about the high and low states do not trade in $t = 1$ and the uninformed speculator buys. Sequential rationality implies that the profit of the speculator informed about the high state equals $\frac{1}{2}\left[V^+ - K - p_2(\cdot,2)\right] + \frac{1}{2}\left[V^+ - K - p_2(\cdot,0)\right] = \frac{V^+ - V^-}{4}$; in this case, she has an incentive to deviate and buy in $t = 1$ and follow the uninformed speculator's strategy in $t = 2$, which gives her a payoff of $V^+ - K - \left(\frac{V^+ + V^-}{2} - K\right) = \frac{V^+ - V^-}{2} > \frac{V^+ - V^-}{4}$.

Given the results above, an equilibrium when $\frac{\alpha}{12}(V^+ - K) < c < \frac{V^+ - K}{12}$ exists only if neither speculator trades in $t = 1$. Now we check the existence of an equilibrium in which neither speculator trades in $t = 1$ and the beliefs following $Q_1 \in \{-2, 0, 2\}$ are such that speculator is informed about the low state. Following the period-1 trade, a sequentially rational strategy profile in which the speculator informed about the high state buys in $t = 2$ exists if and only if the uninformed speculator does trade in $t = 2$ and the speculator informed about the low state sells in $t = 2$. In this case, the profit equals $\frac{1}{2}\left[V^+ - K - p_2(\cdot,2)\right] + \frac{1}{2}\left[V^+ - K - p_2(\cdot,0)\right] = \frac{V^+ - V^-}{4} > 0$ for speculator informed about the high state, zero for the uninformed speculator, and $\frac{1}{2}\left[p_2(\cdot,0) - (V^- - K)\right] - c = \frac{V^+ - V^-}{4} - c > 0$ for the speculator informed about the low state. It follows that no speculator has an incentive to deviate and trade in $t = 1$ as in this case investment does not occur and yields a profit of zero.

Therefore, given the results above, we conclude that for $\frac{\alpha}{12}(V^+ - K) < c < \frac{V^+ - K}{12}$ an equilibrium exists if and only if no speculator trades in $t = 1$, the uninformed speculator does not trade in $t = 2$, and the speculator informed about the low state sells in $t = 2$.

For $\mathbf{\frac{V^+ - K}{12} < c < \frac{V^+ - K}{12} + \frac{V^+ - V^-}{6}}$ an equilibrium in which the speculator informed about the high state buys in $t = 1$ exists only if the speculator informed about the low state sells in $t = 1$ and the uninformed speculator does not buy $t = 1$ (Claims 2 and 3). If the uninformed speculator sells in $t = 1$ and $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) > c$, the analysis of Claim 5 shows that the uninformed speculator has an incentive to deviate. Let $c > \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$ and suppose that the uninformed speculator sells in $t = 1$. With probability $\frac{1}{2}$, the order flow equals $Q_1 = 0$, in which case sequential rationality given beliefs consistent on the path implies the following (as per the analysis of Case (v)): the speculator informed about the state must sell in $t = 2$; in turn, the uninformed speculator must not trade in $t = 2$; however, in this case the speculator informed about the low state has an incentive to deviate and not trade in $t = 2$. Therefore, an equilibrium in this case exists only if the uninformed speculator does not trade in $t = 1$.

Consider an equilibrium in which the uninformed speculator does not trade in $t = 1$ while the speculator informed about the high and low states buy and sell in $t = 1$ respectively, where the beliefs off the path are as follows: the beliefs associated with $Q_2 \in \{-1, 1\}$ when $Q_1 \notin \{-1, 1\}$ are that the speculator is uninformed; and the beliefs associated with $Q_2 \in \{-2, 0, 2\}$ when $Q_1 \in \{-1, 1\}$ are that the speculator is informed about the low state. In this case, sequential rationality implies that the expected profit of the speculator informed about the low state is $\frac{1}{2}\left[p_1(0) - \frac{V^- - K}{2} + \frac{p_2(0,0) - \left(V^- - K\right)}{2} - 2c\right] + \frac{1}{2}\left[p_1(-2) - c\right] = \frac{1}{2}\left[\frac{1}{4}(V^+ - K) + \frac{V^+ - V^-}{2} - c\right] - c > 0 \iff c < \frac{V^+ - K}{12} + \frac{V^+ - V^-}{6}$. The analysis showing that the speculator informed about the low state does not have the incentive to deviate and buy is the same as that of Claim 5 substituting $p_1(0)$ for $\frac{1}{4}(V^+ - K) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - k\right)$, which yields a lower payoff when she deviates and buys in $t = 1$. Moreover, the speculator informed about the low state does not have an incentive to deviate and not trade in $t = 1$, as in this case investment does not occur and her profit is zero if she either buys or sells in $t = 2$, while her profit is also zero is she does not trade in $t = 2$.

The expected payoff of the speculator informed about the high state in the proposed equilibrium is $\frac{1}{2}\left[V^+ - K - p_1(0) + \frac{V^+-K-p_2(0,2)}{2} + \frac{V^+-K-p_2(0,0)}{2}\right] = \frac{1}{2}\left(\frac{V^+-K}{4} + \frac{V^+-V^-}{2}\right) > 0$. If she deviates and sells then with probability $\frac{1}{2}$ the order flow equals $Q_1 = -2$, in which case her overall profit is $-c$. With probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$ and we have $p_1(0) = \frac{1}{4}(V^+ - K) + \frac{1}{2}\left(\frac{V^++V^-}{2} - K\right)$, in which her overall payoff is $p_1(0) - \frac{1}{2}(p_2(0,2) + p_2(0,0)) - c < -c$ if she buys in $t = 2$, $p_1(0) - (V^+ - K) - c < -c$ if she does not trade, and $p_1(0) - \frac{V^+-K}{2} + \frac{1}{2}\left[\frac{V^++V^-}{2} - K - (V^+ - K)\right] - 2c = -\frac{V^+-V^-}{4} + \frac{V^--K}{4} - 2c < 0$ if she sells. Therefore, her expected profit is negative so that she does not have an incentive to deviate and buy. Moreover, the speculator informed about the high state does not have an incentive to deviate and not trade in $t = 1$, as in this case, investment does not occur and her profit is zero if she either buys or sells in $t = 2$, while her payoff is also zero is she does not trade in $t = 2$.

Lastly, the payoff of the uninformed speculator in the equilibrium proposed here is zero. The analysis showing that the uninformed speculator does not have the incentive to deviate and buy is the same as that of Claim 5 substituting $p_1(0)$ for $\frac{1}{4}(V^+ - K) + \frac{1}{2}\left(\frac{V^++V^-}{2} - k\right)$, which yields a lower payoff when she deviates and buys in $t = 1$. If she deviates and sells, then $Q_1 = -2$ with probability $\frac{1}{2}$, in which case beliefs that the speculator is informed about the low state for $Q_2 \in \{-2, -1, 0, 1, 2\}$ imply that investment does not occur and her profit from the period-2 trade is at most zero (it is lower and equal to $-c$ if she sells in $t = 2$); thus her overall profit is at most $-c$. With probability $\frac{1}{2}$, the order flow equals $Q_1 = 0$, in which case her profit from the period-2 trade is $\frac{1}{2}\left[\frac{V^++V^-}{2} - K - p_2(0,2)\right] + \frac{1}{2}\left[\frac{V^++V^-}{2} - K - p_2(0,0)\right] = -\frac{V^+-V^-}{4} < 0$ if she buys in $t = 2$, zero if she does not trade, and $\frac{1}{2}\left[p_2(0,0) - \left(\frac{V^++V^-}{2} - K\right)\right] - c = -c < 0$ if she sells; thus, her overall profit is at most $-c$. Therefore, her expected payoff if she deviates and sells is at most $-c$, implying she is not incentivized to deviate and sell.

Therefore, we conclude that for $\frac{V^+-K}{12} < c < \frac{V^+-K}{12} + \frac{V^+-V^-}{6}$ an equilibrium in which the speculator informed about the high state buys in $t = 1$ exists if and only if the speculator informed about the low state sells in $t = 1$ and sells again $t = 2$ if $p_1(\cdot) > 0$ and the uninformed speculator does not trade in $t = 1$.

For $\mathbf{\frac{V^+-K}{12} + \frac{V^+-V^-}{6} < c < \frac{V^+-V^-}{4}}$, Claims 2 and 3 imply that an equilibrium in which the speculator informed about the high state buys in $t = 1$ exists only if the speculator informed about the low state sells in $t = 1$ and the uninformed speculator does not buy $t = 1$.

However, the previous analysis of the range $\frac{V^+-K}{12} < c < \frac{V^+-K}{12} + \frac{V^+-V^-}{6}$ shows that: the uninformed speculator has the incentive to deviate when she sells in $t = 1$, which implies that the uninformed speculator must not trade in $t = 1$; in this case, the expected profit of speculator informed about the low state is negative, which implies that she has the incentive to deviate and not trade in periods $t = 1, 1$ to secure a profit of zero. Therefore, there is no equilibrium in which the speculator informed about the high state buys in $t = 1$. If follows from the analysis of the range $\frac{\alpha}{12}(V^+ - K) < c < \frac{V^+-K}{12}$ that an equilibrium exists if and only if no speculator trades in $t = 1$, the uninformed speculator does not trade in $t = 2$, and the speculator informed about the low state sells in $t = 2$. ■

Proof of Proposition 2. First, $I^* = I' - \frac{\alpha}{8}(V^- - K) > I'$, where the inequality follows since $V^- < K$. Second, $I^* = \underline{I} + \frac{3(1-\alpha)}{8}[V^+ + V^- - 2K] > \underline{I}$, where the inequality follows from Assumption 1. Lastly, $I' = \underline{I} + \frac{\left[3\left(V^+ + V^-\right) - 6K\right] - \alpha\left[2\left(V^+ + V^-\right) - 4K + V^+ - K\right]}{8} > \underline{I} \iff \alpha < \alpha' \equiv \frac{3\left(V^+ + V^-\right) - 6K}{2(V^+ + V^-) - 4K + V^+ - K}$. ■

Proof of Proposition 3. We start showing the unique truth-leaning equilibrium without further information from trade strategies in the financial market. First, let $E_s = \{s, \emptyset\}$. Suppose by way of contradiction that there is an equilibrium that fully separates the manager types $s \in \{l, h, \emptyset\}$. If the manager's type is $s = l$, $E(V(K, l) - K|l) = V^- - K < 0$, so that reinvestment is not made, the reward equals $\rho(l) = 0$, and the manager's payoff is zero. Thus, the manager $s = l$ has an incentive to deviate and send a message $m = \emptyset$, as in this case $E(V(K, \emptyset) - K|\emptyset) = \frac{V^+ + V^-}{2} - K > 0$, so that reinvestment is made, the reward equals $\rho(\emptyset) = \frac{V^+ + V^-}{2} - K > 0$, and the manager's payoff equals $\rho(\emptyset) + \delta K > 0$. Suppose by way of contradiction that there is a truth-leaning equilibrium in which all types of managers send the message $m = \emptyset$. This contradicts the truth-leaning equilibrium assumptions $A0$ and $P0$ of Hart et al. (2017), as it requires the manager (agent) always to reveal the type when it gives a payoff at least as high, and the capital provider (principal) to believe that a message m not sent in equilibrium comes from type $s = m$. Given these assumptions, the manager of type $s = h$ has incentive to deviate and send a message $m = h$ as the $E(V(K, h) - K|h) = V^+ - K > \frac{V^+ + V^-}{2} - K = E(V(K, \emptyset) - K|\emptyset)$, so that reinvestment is made, the reward equals $\rho(h) = V^+ - K$, and the manager's payoff equals $\rho(h) + \delta K > \rho(\emptyset) + \delta K > 0$. Last, the manager sending the message $m = s$ when the type is $s = h$ and the message $m = \emptyset$ when the types are $s = l$ and $s = \emptyset$ is the unique truth-leaning equilibrium. In this case, the types $s = l$ and $s = \emptyset$ cannot deviate as they do not have the evidence provided by the type $s = h$. In addition, the type $s = h$ does not have an incentive to deviate and send the message $m = \emptyset$, as $E\left(V(K,\emptyset) - K|\emptyset\right) = \frac{V^- + (1-\alpha)V^+}{2-\alpha} - K < 0$, the reinvestment is not made so that $\rho(\emptyset) = 0$, and the payoff equals zero. Thus, this is the unique truth-leaning equilibrium as there is no equilibrium left, and the resulting investment capacity equals $\frac{\alpha}{2}\left(V^+ - K\right)$.

Second, let $E_s = \{\{l,h\},\emptyset\}$. Given that the there are only two different messages that can be sent by the three types, there is no equilibrium that fully separates the manager types $s \in \{l,h,\emptyset\}$. Suppose by way of contradiction that there is a truth-leaning equilibrium in which either the manager of type $s = h$ or type $s = l$ (or both) sends the message $m = \emptyset$. In this case, $E\left(V(K,\emptyset) - K|\emptyset\right) = \frac{V^+ + V^-}{2} - K > 0$ so that reinvestment is made, the reward of equals $\rho(\emptyset) = \frac{V^+ + V^-}{2} - K > 0$, and the manager's payoff equals $\rho(\emptyset) + \delta K > 0$. This contradicts the truth-leaning equilibrium assumptions $A0$ and $P0$ of Hart et al. (2017), as it requires the manager (agent) always to reveal the type when it gives a payoff at least as high, and the capital provider (principal) to believe that a message m not sent in equilibrium comes from type $s = m$. Given these assumptions, the manager of type $s \in \{l,h\}$ has incentive to deviate and send a message $m = s$ as the $E\left(V(K,s) - K|s\right) = \frac{V^+ + V^-}{2} - K = E\left(V(K,\emptyset) - K|\emptyset\right)$, so that reinvestment is made, the reward equals $\rho(s) = \frac{V^+ + V^-}{2} - K$, and the manager's payoff equals $\rho(s) + \delta K = \rho(\emptyset) + \delta K > 0$. Thus, since the payoff is the same, the manager of type $s \in \{l,h\}$ has incentive to deviate given the strict preference to truthfully reveal the type. It follows that there is a unique truth-leaning equilibrium in which the manager sends the message $m = s$ when the type is $s \in \{l,h\}$, and $m = \emptyset$ when the type is $s = \emptyset$. The resulting investment capacity equals $\frac{V^+ + V^-}{2} - K$.

We now move to show whether the investment capacity from the unique truth-leaning equilibrium given E_s can be improved from trade strategies. First, when $E_s = \{s,\emptyset\}$, there is no improvement possibility. The trade strategies must at least reveal when the speculator is informed about the high state, since otherwise the market maker learn it by accessing the disclosed pieces of verifiable evidence by the manager. Suppose by way of contradiction that there is a trade equilibrium that can reveal whether the speculator is either informed about the low state or uninformed. Since the type $s = h$ must be revealed in $t = 1$, due to the short selling cost $c > 0$ either the speculator buys in $t = 1$ when $s = h$ and does not trade when $s \in \{l, \emptyset\}$, or does not trade when $s = h$ and buys when $s \in \{l, \emptyset\}$. In both cases, the speculator of type $s = l$ will never sell in $t = 2$, since regardless of whether the type is fully revealed or not distinguished from the type $s = \emptyset$, reinvestment will not be made and impose a loss of $c > 0$. The revelation requirement does imply that one of the types $s = l$ and $s = \emptyset$ must buy in $t = 2$ and the other not trade. Since reinvestment is made when $s = \emptyset$ is revealed and not made when $s = \ell$ is revealed, it follows that $p_1 = \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)$ when $s \in \{l, \emptyset\}$, and $p_1 = V^+ - K$ when $s = h$. If the speculator $s = h$ buys in $t = 1$, the speculator $s = l$ has an incentive to deviate and sell in $t = 1$ as with probability $\frac{1}{2}$ the flow equals $Q_1 = 0$, which yields an expected profit at least as high as $\frac{V^+ - V^-}{2} - c > 0$ (assuming $p_1 = 0$ when $Q_1 = -2$). If the speculator $s = h$ does not trade in $t = 1$, the speculator $s = l$ faces a negative profit $-p_1 = -\frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)$ and thus has an incentive to deviate and not trade in $t = 1$. It follows that there is no equilibrium that distinguishes whether the speculator is either informed about the low state or uninformed.

Second, when $E_s = \{s, \emptyset\}$, the trade strategies must at least reveal when the speculator is uninformed, since otherwise the market maker learn it by accessing the disclosed pieces of verifiable evidence by the manager. There exists a trade equilibrium that imposes a positive chance of distinguishing when the speculator is informed about the high state ($s = h$) or informed about the low state ($s = l$). The most efficient is clearly the one of Proposition 1 (iii) for $c < c'$, which yields the investment capacity of I^*, and the ones of Proposition 1 (iv) for $c > c'$, which yields the investment capacity of I'. Given these efficient trade strategies, the manager of type $s = h$ has no incentive to deviate and send a message $m = \emptyset$: while reinvestment would also always be made, the reward would be lower. The manager of type $s = l$ also does not have an incentive to deviate and send a message $m = \emptyset$. The reward and private benefits without deviation ($m = l$) net of those with deviation ($m = \emptyset$) based on Proposition 1 (iii) and Proposition 1 (iv) are respectively the following: $I^*(\alpha = 1) + \frac{5}{8}\delta K - \left(\frac{V^- + V^+}{2} - K + \delta K\right) = \frac{3\left(K - V^- - \delta K\right)}{8} > 0$ and $I'(\alpha = 1) + \frac{3}{4}\delta K - \left(\frac{V^- + V^+}{2} - K + \delta K\right) = \frac{K - V^- - \delta K}{4} > 0$. Both are positive as the net present value from reinvesting when the state is low is negative even after including the manager's private benefit, $\delta \in (0, 1 - K^{-1}V^-)$. This concludes the proof. ■

Proof of Proposition 4. Let $c < \frac{V^+ - K}{12}$. Consider the following strategies' profile: the speculator informed about the high state buys in $t = 1$ and buys again in $t = 2$ if $p_1 < V^+ - K$; the uninformed speculator does not trade in $t = 1$ and does not trade in $t = 2$ if $p_1 > 0$; the speculator informed about the low state sells in $t = 1$ and sells again in $t = 2$ if $p_1 > 0$; the manager buys in $t = 1$ if and only if he is uninformed and buys in $t = 2$ if $p_1 > 0$ if and only if he is uninformed; beliefs assign probability one to the speculator being informed about the low state for the trade histories $\{\{Q_1\}, \{Q_1, Q_2\}\}$ for $Q_1 \in \{-2, -1, 1, 3\}$ and $Q_2 \in \{-3, -2, -1, 0, 1, 2, 3\}$, and for $Q_1 \in \{0, 2\}$ and $Q_2 \in \{-2, -1, 1, 3\}$.

The payoff of the uninformed speculator under the proposed equilibrium is 0. If she deviates and sells in $t = 1$, the order flow equals $Q_1 \in \{-1, 1\}$: her profit from the period-1 trade is $-c$; her period-2 trade profit is at most 0 (it is $-c$ if she sells); hence, she does not have an incentive to deviate and sell. If she deviates and buys, the order flow equals $Q_1 \in \{1, 3\}$: her profit from the period-1 trade is 0; her profit from the period-2 trade is at most 0 (it is $-c$ if she sells); hence, she does not have an incentive to buy. Therefore, she does not have an incentive to deviate from her strategy in the proposed equilibrium.

If the speculator informed about the low state deviates and does not trade the order flow equals $Q_1 \in \{-1, 1\}$: her profit from the period-1 trade is 0; her period-2 trade profit is at most 0 (it is $-c$ if she sells); hence, her overall profit is at most 0. If she deviates and buys in $t = 1$, then with probability $\frac{1}{2}$ the order flow equals $Q_1 = 2$: in this case, $p_1(2) = p_2(2, Q_2 \in \{2, 0\}) = \frac{\alpha}{2-\alpha}(V^+ - K) + \frac{2(1-\alpha)}{2-\alpha}\left(\frac{V^+ + V^-}{2} - K\right)$; if she buys in $t = 2$, her profit equals $2(V^- - K) - p_1(2) - p_2(2, Q_2 \in \{2, 0\}) < 0$; if she does not trade in $t = 2$, then $Q_2 \in \{-1, 1\}$ and her profit equals $-p_1(2) < 0$; and if she sells in $t = 2$, her profit equals $\frac{p_2(2,0)}{2} - p_1(2) - c < 0$. With probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$: in this case, $p_2(0, 2) = p_2(2, Q_2 \in \{2, 0\})$, $p_2(0, 0) = \frac{V^+ + V^-}{2} - K$, and $p_1(0) = \frac{2-\alpha}{4} p_2(2, Q_2 \in \{2, 0\}) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right)$; if she buys in $t = 2$, her profit equals $2(V^- - K) - p_1(0) - \frac{p_2(0,2)}{2} - \frac{p_2(0,0)}{2} < 0$; if she does not trade in $t = 2$, then $Q_2 \in \{-1, 1\}$ and her profit equals $-p_1(0) < 0$; and if she deviates and sells her profit is $-\frac{p_2(0,0)}{2} - p_1(0) - c < 0$. It follows that her overall profit if she deviates from her strategy in the proposed equilibrium is at most 0, which implies that she does not have the incentive to deviate since in the proposed equilibrium she makes a profit of $\frac{1}{2}\left[p_1(0) + \frac{p_2(0,0)}{2} - (V^- - K) - c\right] - c = \frac{1}{2}\left(\frac{V^+ + (1-\alpha)V^- - (2-\alpha)K}{4} + \frac{V^+ - V^-}{2} - c\right) - c > 0$.

The payoff of the speculator informed about the high state in the proposed equilibrium is $\frac{1}{2}\left[2(V^+ - K) - p_1(2) - \left(\frac{p_2(2,2)}{2} + \frac{p_2(2,0)}{2}\right)\right] + \frac{1}{2}\left[2(V^+ - K) - p_1(0) - \left(\frac{p_2(0,2)}{2} + \frac{p_2(0,0)}{2}\right)\right]$. If she deviates and does not trade the order flow equals $Q_1 \in \{-1, 1\}$: her profit from the period-1 trade is 0; her period-2 trade profit is at most 0 (it is $-c$ if she sells); hence, she does not have an incentive to deviate and not trade. If she deviates and sells in $t = 1$ the order flow equals $Q_1 = -2$ with probability $\frac{1}{2}$: her profit from the period-1 trade is $-c$ and her period-2 trade profit is at most 0 (it is $-c$ if she sells). With probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$: if she buys in $t = 2$, her profit equals $p_1(0) - \left(\frac{p_2(0,2)}{2} + \frac{p_2(0,0)}{2}\right) - c < 0$; if she does not trade, her profit equals $p_1(0) - c$; and if she sells her profit equals $p_1(0) + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - (V^+ - K) - 2c$. It follows that her overall profit when she deviates and sells in $t = 1$ is at most $\frac{p_1(0)}{2} - c$. Subtracting the best deviation payoff from her payoff in the proposed equilibrium yields $\frac{1}{2}\left[2(V^+ - K) - p_1(2) - \left(\frac{p_2(2,2)}{2} + \frac{p_2(2,0)}{2}\right)\right] + \frac{1}{2}\left[(V^+ - K) - \frac{(2-\alpha)(V^- - K)}{2} - \left(\frac{p_2(0,2)}{2} + \frac{p_2(0,0)}{2}\right)\right] + c > 0$. It follows that she does not have an incentive to deviate and sell. Therefore, she does not have an incentive to deviate from her strategies in the proposed equilibrium.

Lastly, we consider the deviation incentives of the manager. Because the manager derives private benefits from investment, once the project is financed in $t = 0$, he has an incentive to change his repurchase strategy if and only if it leads to an increase in the probability of reinvestment in $t = 2$. If the manager is either informed about the high state or uninformed, reinvestment always occurs under the proposed equilibrium; hence he has no incentive to deviate. If the manager is informed about the low state, reinvestment does no occur when $Q_1 = -2$ and when $\{Q_1 = 0, Q_2 = -2\}$. If he deviates and buys, the order flow equals $Q_1 \in \{-1, 1\}$. In this case, beliefs assign probability one to the speculator being informed about the low state, which implies that investment does not occur. Therefore, he does not have an incentive to deviate and change his repurchase strategy. ■

Proof of Proposition 5. See the text. ■

Proof of Proposition 6. If $I \geq I^*$, it follows that $I^* - I \leq 0 < (1-\alpha)\, 2\pi \frac{V^+ + (1-\alpha)V^- - (2-\alpha)K}{2-\alpha}$, which violates the condition of the proposition. If $I < I^*$, we can use $V^+ - K = 2\widehat{V} - (V^- - K)$ to rewrite the condition of the proposition as

$$-\frac{3\alpha}{8}(V^- - K) + \widehat{I} - 2\pi \left[(1-\alpha) \frac{2\widehat{I} - \alpha (V^- - K)}{2-\alpha} \right] - I \geq 0. \tag{A.1}$$

The left-hand side of A.1 is clearly decreasing in π. The derivative of the term inside the brackets of A.1 with respect to α is $-\frac{2\widehat{I} - \left(V^- - K\right)\left(-\alpha^2 + 4\alpha - 2\right)}{(2-\alpha)^2}$. This derivative is negative since the numerator is positive; this follows from our parametric assumption $\alpha > \frac{V^+ + V^- - 2K}{V^+ - K} > \frac{1}{2}$, which implies that $(-\alpha^2 + 4\alpha - 2) > -1$ and the numerator is at least $2\widehat{I} + (V^- - K) > 0$. Therefore, the left-hand side of A.1 is increasing in α. Since the left-hand side of A.1 is positive for π close to 0 and negative for π large enough, there exists $\pi^* > 0$ such that the equality obtains, which implies that the condition is satisfied for $\pi \leq \pi^*$. Write this equality as

$$-\alpha \left(V^- - K\right) \left[\frac{3}{8} - \frac{2(1-\alpha)\pi^*}{2-\alpha} \right] + 2\widehat{I} \left[\frac{1}{2} - \frac{2(1-\alpha)\pi^*}{2-\alpha} \right] - I = 0.$$

Note that the second term on the left-hand side of the equality must be positive; if it were negative, then the first term would also be negative, violating the equality. Moreover, $\widehat{I} = V^- - K + \Delta$. Thus, it is easy to see (and simple comparative statics can be used for verification) that π^* is increasing both in α and in Δ. ■

Proof of Proposition 7. We first show that, under the contract described in Proposition 4, there exists an equilibrium in which the following holds: (i) the speculator always buys when informed about the high state, but never trades when informed about the low state or uninformed; and (ii) the manager never trades when informed about the high state, but always repurchases shares when informed about the low state or uninformed.

Let $c < \frac{V^+ - K}{12}$. Consider the following strategies' profile: the speculator informed about the high state buys in $t = 1$ and buys again in $t = 2$ if $p_1 < V^+ - K$; the uninformed speculator does not trade in $t = 1$ and does not trade in $t = 2$ if $p_1 > 0$; the speculator informed about the low state does not trade in $t = 1$ and does not trade in $t = 2$ if $p_1 > 0$; the manager buys in $t = 1$ if and only if he is informed about the low state or uninformed, and buys in

$t = 2$ if $p_1 > 0$ if and only if he is informed about the low state or uninformed; beliefs assign probability one to the speculator being informed about the low state for the trade histories $\{\{Q_1\}, \{Q_1, Q_2\}\}$ for $Q_1 \in \{-2, -1, 1, 3\}$ and $Q_2 \in \{-3, -2, -1, 0, 1, 2, 3\}$, and for $Q_1 \in \{0, 2\}$ and $Q_2 \in \{-2, -1, 1, 3\}$.

The payoff of the speculator informed about the low state or uninformed under the proposed equilibrium is 0. If she deviates and sells in $t = 1$, the order flow equals $Q_1 \in \{-1, 1\}$: her profit from the period-1 trade is $-c$; her period-2 trade profit is at most 0 (it is $-c$ if she sells); hence, she does not have an incentive to deviate and sell. If she deviates and buys, the order flow equals $Q_1 \in \{1, 3\}$: her profit from the period-1 trade is 0; her profit from the period-2 trade is at most 0 (it is $-c$ if she sells); hence, she does not have an incentive to buy. Therefore, she does not have an incentive to deviate from her strategy in the proposed equilibrium.

Given that in the proposed equilibrium $p_1(0) = p_2(0, Q_2 \in \{0, 2\}) = p_2(2, Q_2 \in \{0, 2\}) = \frac{V^+ + V^-}{2} - K$, the payoff of the speculator informed about the high state is $2(V^+ - K) - (V^+ + V^- - 2K)$. If she deviates and does not trade the order flow equals $Q_1 \in \{-1, 1\}$: her profit from the period-1 trade is 0; her period-2 trade profit is at most 0 (it is $-c$ if she sells); hence, she does not have an incentive to deviate and not trade. If she deviates and sells in $t = 1$ the order flow equals $Q_1 = -2$ with probability $\frac{1}{2}$: her profit from the period-1 trade is $-c$ and her period-2 trade profit is at most 0 (it is $-c$ if she sells). With probability $\frac{1}{2}$ the order flow equals $Q_1 = 0$: if she buys in $t = 2$, her profit equals $-c$; if she does not trade, her profit equals $\frac{V^+ + V^-}{2} - K - c$; and if she sells her profit equals $\frac{V^+ + V^-}{2} - K + \frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - (V^+ - K) - 2c$. It follows that her overall profit when she deviates and sells in $t = 1$ is at most $\frac{1}{2}\left(\frac{V^+ + V^-}{2} - K\right) - c$. Subtracting the best deviation payoff from her payoff in the proposed equilibrium yields $\frac{V^+ - V^- - 2\left(V^- - K\right)}{2} + c > 0$. It follows that she does not have an incentive to deviate and sell. Therefore, she does not have an incentive to deviate from her strategies in the proposed equilibrium.

Because the manager derives private benefits from investment, once the project is financed in $t = 0$, he has an incentive to change his repurchase strategy if and only if it increases the probability of reinvestment in $t = 2$. Since reinvestment always occurs regardless of his information, he has no incentive to deviate and change his repurchase strategy.

Lastly, because prices are uninformative and the firm always reinvests regardless of private signals and short selling costs, investment capacity equals $\widehat{I}$. However, it follows from Proposition 2 that investment capacity is at least $\min\{\underline{I}, I'\} > \widehat{I}$ under spot financing (i.e., without ex-ante contracting) characterized in Proposition 1. Therefore, investment capacity is less efficient compared to spot financing. ■