# Product Selling Versus Pay-Per-Use Service: A Strategic Analysis of Competing Business Models

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We present a model that suggests possible explanations for the observed proliferation of "pay-per-use" (PPU) business models over the last two decades. Delivering "fractions" of a product as a service offers a cost advantage to customers with lower usage but requires extra delivery costs. Previous research focused on information goods (with negligible production costs) and predicted that PPU, when arising as a differentiation to selling in equilibrium, fundamentally achieves lower profits than selling. We extend the theory by covering goods with any production cost, in duopolistic competition. We show that PPU business models can be more profitable than selling (especially at mid-range production costs), as long as their delivery costs are not too high, a requirement that is more easily fulfilled as new technologies reduce these costs. Moreover, if firms are imperfectly informed about their customers' usage profiles, PPU's effective pricing of customers' varying usage offers an additional advantage over selling. This requires companies to employ accounting methods that do not inappropriately allocate production costs over stochastic usage levels. If PPU service provision suffers from queueing inefficiencies, this does not fundamentally change the relative profitability of the PPU and selling models, provided that PPU providers can attract sufficiently high demand to benefit from pooling economies.

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## 1. Introduction

Companies compete not only with the quality and price of their products and services but also with their business models. A business model is defined by the content, structure, and governance of transactions (Amit and Zott 2001, Zott and Amit 2007). It describes how firms carry out their economic activity, "the way they do business." Most prominently, this is captured by the profit model, which describes the pricing mechanism in the economic exchange with customers (Cachon 2018, Christensen et al. 2016, Gassmann et al. 2014). In addition to innovations in products and services, business model innovations have attracted much managerial attention in the last decade (Girotra and Netessine 2011, Christensen et al. 2016). Pricing mechanisms offer one differentiation route to lower competition intensity; for example, profit models have exhibited a shift away from *product selling* toward the *selling of product usage*, referred to as "Pay-Per-Use (PPU)."[1] Charging a customer a price per "usage" has, of course, existed for a long time: for example, Xerox has rented copiers to corporate clients, who pay a price per copy made, since the 1970s (Chesbrough and Rosenbloom 2002), and Rolls Royce pioneered the aircraft engine as a service, paid per hour of aircraft flight (Kim et al. 2007). These examples are not outliers despite the fact that a PPU service may sometimes carry sizeable usage-related delivery costs such as monitoring, tracking, or billing.

The starting point of our study is that PPU models have become much more widespread over the last decade; see, for example, Signify's selling of lighting instead of light bulbs (Lorenzo and Sánchez 2007), or Hilti's construction solutions instead of power tools (Johnson 2010). Today, companies are offering usage-based pricing on everything from cars to car insurance, software, hardware infrastructure, music and films, as well as machinery and industrial products. The question that our study examines is *why* this is the case.

Previous academic literature has proposed that PPU business models may arise as differentiation mechanisms in competitive equilibria, in "information goods" with zero production costs. Although they may produce positive profits, PPU models are shown to be less profitable than selling (Bala and Carr 2010, Balasubramanian et al. 2015). These findings imply that PPU models should not be sought as a priority by companies. Therefore, the existing literature does not offer an explanation of the recent *proliferation* of PPU pricing mechanisms across many industries.

One possible explanation for this proliferation may relate to the costs of offering a PPU model. Companies nowadays *can* offer PPU mechanisms because technology lowers the delivery costs required. Technological advances, such as ubiquitous sensors, Internet speed, and cloud computing, facilitate the tracking, billing, and, in some cases, delivery of smaller and more dynamic increments of use across a variety of customer contexts (Kavadias et al. 2016). Moreover, it seems that some consumer segments (for example, "millennials") prefer paying small increments for a usage unit, rather than paying for the whole product (Kotler 2011). However, reduced PPU delivery costs alone cannot explain a proliferation of PPU business models, as the same technologies that reduce logistics costs may *also* reduce the distribution costs for the seller, and, again render selling more attractive than PPU, as the information goods literature predicts.

This leads to the research question of our study: Is there an explanation, beyond differentiation, for the wide adoption of PPU business models across industries (many with non-zero production costs)? In other words, under what cost conditions can a PPU business model be more profitable than a selling model in a competitive equilibrium?

[1] This shift has been given different names in different industries, such as *servicization* (Kim et al. 2007, Agrawal and Bellos 2016, Örsdemir et al. 2018) or *product-as-a-service*, to name a few.

In order to answer this question, we develop a normative model that compares the profitability of a PPU and a selling business model in a competitive duopoly. PPU models require, at a minimum, *some* consumers who benefit from the fractional use of an asset (whether or not they are heterogeneous in their asset use). We explicitly account for the manufacturing costs and delivery costs of monitoring and managing consumer usage faced by PPU business models. It turns out that manufactured goods markets behave very differently than information goods markets, a fact that previous studies have not considered. For manufactured goods, being able to use the product without owning it is valuable to customers and can make the PPU model *more* profitable than selling.

We examine the PPU model's profitability relative to the selling model across a variety of industrial settings, namely:

1. when consumers are heterogeneous *both* in their usage and in valuation,
2. when consumers are sensitive to pricing and endogenously decide on their usage quantity,
3. when consumers' usage is not uniformly distributed,
4. when there is uncertainty in the usage quantities,
5. when the "level of service" (represented by product availability) and queuing effects are accounted for.

### 1.1. Literature Review

Whereas the literature on the competitiveness of different business models has been primarily qualitative (Christensen et al. 2016, Girotra and Netessine 2011, Gassmann et al. 2014), economists have approached business models through the analysis of key variables that drive profitability, for example, quality, quantity, and price. A large and influential body of literature has addressed this topic through product quality and market differentiation (Maskin and Riley 1984, Moorthy 1988, Mussa and Rosen 1978, Wilson 1993). Only a small amount of sub-literature has addressed PPU pricing, and it has largely focused on information goods, where manufacturing costs are zero or negligibly small.

In this literature, three notable studies are closely related to our work: first, Varian (2000) asks whether usage-based pricing of an information good can actually make a monopolist better off. He answers positively under certain circumstances: when sharing the good is cheap; when the informational content is viewed only a limited number of times, so that buying is "inefficient"; and when sharing allows market segmentation. Second, Sundararajan (2004) shows that between two pricing schemes (fixed fee and usage-based pricing) a monopolist's optimal pricing model is mixed, including a fixed fee plus a usage-based fee. Finally, Balasubramanian et al. (2015) consider a competitive duopoly where one firm sells information goods (software) and the other offers a

PPU version of the software package. Users differ in their usage quantity, and they face a psychological disutility from PPU (the stressful "ticking-meter" effect). In this duopoly equilibrium, selling earns higher profits than PPU — PPU is a fundamentally less profitable business model in the presence of competition. Our model adds to this literature by introducing non-zero production costs. This was previously dismissed with the argument that production costs are fixed and sunk and thus not decision-relevant (Balasubramanian et al. 2015, p.222). However, production costs are almost always apportioned and thus "made variable" in PPU markets. We retain several appropriate characteristics from the information goods models: like Varian (2000), the product can be efficiently shared by low-usage customers; we model consumer usage as in Balasubramanian et al. (2015), and we consider duopolistic competition as they do. A special case of our model with zero production costs is consistent with their findings. However, the presence of even small production costs fundamentally changes the competitive dynamics.

Another related literature is concerned with two-part tariffs, the variable component of which represents usage-based pricing. This literature has also focused largely on information goods. Essegaier et al. (2002) examine information goods both in a monopoly and a duopoly, and they find that two-part pricing might not be optimal compared to a pure selling business model. Fishburn and Odlyzko (1999) examine two service providers that use different business models, namely, a fixed fee and PPU, in a multi-period repeated game. They also find that the selling firm (fixed subscription fee) performs better than the PPU firm. Closer to our work, a few studies have examined manufactured goods with a non-zero unit cost. In two seminal studies, Bulow (1982) and Stokey (1981) examine a durable goods monopolist's choice between the business models of product selling and product leasing (which is similar but not the same as usage-based pricing because payments are by period rather than by usage). In a two-period model, they show that the selling business model does not do as well as the leasing model. Hayes (1987) examines perfect competition, where producers price at marginal cost, and finds that a two-part tariff scheme might be preferred over fixed pricing if the consumers are uncertain about their consumption levels. Consumers choose PPU as a "hedge" against a scenario where they consume very little and would overpay if they bought the "whole" product. Finally, Desai and Purohit (1999) examine competition in durable goods markets, where firms compete in a two-period game by selling and leasing a percentage of their products. The more intense the competition, the lower the percentage of leasing. There is also a stream of literature on durable goods with $c > 0$ that examines whether servicization is greener than selling (Agrawal et al. 2012, Agrawal and Bellos 2016, Örsdemir et al. 2018). These models are based on multi-period games and examine profitability as well as the effects of selling and servicization on the environment, but they are all restricted to monopolies.

### 1.2. Main Results and Contribution

Our study extends this literature along three dimensions: first, we explicitly consider the usage quantity (the number of uses), as opposed to time-based leasing contracts. Second, we generalize previous studies by considering arbitrary production costs and their allocation across the different uses – information goods with zero production costs are a special case of our model. Third, we focus on competition between two firms (which may employ different business models). To our knowledge, previous work has not addressed this combination, although it represents a large fraction of industrial settings.

As in previous studies, the differentiating feature of PPU over product selling lies in its ability to serve consumers with low usage quantities (for whom buying a "whole" product would be inefficient). Our base model suggests that a monopolist prefers the PPU business model to product selling when the usage delivery cost is below a threshold fraction of the unit production cost (consistent with Varian (2000)). In a competitive duopoly equilibrium, the PPU business model is more profitable when the delivery cost is lower than a threshold that depends on the production cost, with the threshold at its highest at medium production costs. For very cheap products, PPU is not worthwhile, as the full product satisfies all customers effectively at very low prices; for very expensive products, the amortized part of the unit production cost becomes too high relative to the consumer willingness to pay for a single use, which together with the additional delivery cost squeezes the potential for a profit margin.

The finding that PPU is more profitable for a significant range of (production cost, delivery cost) instances is a fundamental deviation from previous literature. Our model predicts that if changes in both types of cost have moved a set of competitors within an industrial setting into the appropriate region of the cost space, PPU can become more profitable than selling. For example, the evolution of technology could reduce both delivery costs (e.g., tracking) and production costs (e.g., materials, automation). Overall, one can expect that under certain industry settings PPU would be adopted more widely by companies. This area of competition between PPU and selling, with the effects of the configuration of both production and delivery costs, is not intuitive and has not been shown. Thus, our model suggests a possible explanation of the observed increase in the use of PPU models.

Extensions of the model confirm the wide applicability of our results. We first consider customers who vary not only in their usage quantity but also in their valuation of a single usage. Our qualitative findings continue to hold; there are regions in the (production cost, delivery cost) space in which PPU is more profitable than selling and the threshold value of the delivery cost is non-monotone. The same continues to hold when users are heterogeneous, both in usage and in valuation, and are price-sensitive (they choose usage endogenously).

If customers are not only heterogeneous but also *uncertain* in their usage frequency, or if the maximum number of uses a product can support is uncertain, the PPU business model becomes even more attractive (it wins over selling at higher levels of delivery costs). This happens because a customer under PPU naturally purchases individual uses without explicit concerns about the maximum number of uses, so the PPU provider is isolated from the usage uncertainty and gains market share from the seller, who must hedge against the usage uncertainty. This depends, however, on the capability of the service provider to make pricing and capacity decisions not based on allocating the production cost over the (a priori stochastic) realized number of uses but instead over the expected number of uses (standard costing). Straight allocation based on the realized number of uses makes the per-use cost uncertain, which forces the PPU provider to hedge as well, neutralizing its natural flexibility-based competitive advantage over the seller. We show that allocation of the production cost over the expected number of uses (standard costing) allows the PPU provider to employ the number of assets required to match expected supply with demand.

Finally, we examine PPU inefficiencies arising from impatient customers having to queue for the service. PPU providers need to deploy excess capacity, which is linked to the service level they opt to offer. Although the PPU model becomes relatively less attractive, the delay disadvantage of the PPU model diminishes with economies of scale and, moreover, customer impatience may also be characterized by heterogeneity, which enables the PPU provider to discriminate on customer service delays and improve its market share and profitability.

### 1.3. Organization of the Paper

In Section 2, we present the key assumptions and the mathematical set-up of the base model, for which Section 3 describes the results. Then, we explore three extensions: Section 4 illustrates the robustness of the base case results in the presence of customers that are heterogeneous in both usage valuations (willingness to pay) and usage needs; it also explores a different model, in which heterogeneous consumers in valuation choose their usage quantities endogenously. Section 5 examines the effects of uncertainty in the usage level distributions, whereas Section 6 addresses PPU inefficiencies arising from queuing effects. Section 7 draws conclusions and outlines future work.

## 2. Model Set-up

Our base model consists of the following elements: the firms' production structure, the market structure with the associated consumer utilities, and the two business models. In the selling business model, upfront payment gives the consumer ownership and unrestricted use of a product. In the PPU business model, the customer pays (usually *ex post*) for use of the product. The PPU model carries an extra delivery/logistics[2] cost per use. Firms can choose either the PPU or the selling business models. We compare the optimum profitability of the two business models in monopoly, and we analyze the duopoly equilibrium business model choices for all admissible values of the production and delivery costs.

### 2.1. Production Structure

Firms incur a constant unit production cost, $c > 0$ for producing or sourcing the product. The *product* is a physical or digital entity that enables multiple *uses* (e.g., multiple car trips undertaken by the owner). Consumers may also be given the option of buying a single use (e.g., a single car trip) without becoming product owners. Each product, as an individual asset, can deliver a maximum of $q_m$ uses before becoming obsolete.[3]

PPU providers also incur a delivery or logistics cost per use, $\delta$. It includes all expenses that a service provider incurs to offer the PPU service, for example, monitoring and billing of usage (as in the case of telecommunications providers who offer pay-as-you-go plans), or maintenance, or insurance costs (as in the case of car rental services and power-by-the-hour services for jet engines), or even coordination costs (as in the case of shared car service schemes where firms need to schedule, plan, and manage a distributed fleet of shared cars). The logistics cost can also result from service design decisions to "give away" some value to build market share. For example, Zipcar started by offering a Honda Civic for \$8.50 per hour including 25 free miles (Frei 2005). Estimating the car's gasoline mileage at 35 mpg and the price of a gallon of fuel at \$2.80, the free mileage decision amounted to adding \$2 (=2.80*25/35) on top of other logistics costs for offering the service.

### 2.2. Market Structure

All customers have (for now) the same valuation, or *willingness to pay* (WTP), $a$, for a single use of the product, but they differ in their desired *usage quantity*, $q$, the number of uses over the product's useful life, which is the time horizon of our model. We assume that $q \in [0, q_m]$, where $q_m$ is the maximum meaningful number of uses that the product can deliver. We assume that customers' desired usage quantity, $q$, is uniformly distributed over the range $[0, q_m]$, with density function $f(q) = {}^1/_{q_m}$.

[2] In the remainder of the paper, we will interchangeably use the terms logistics costs and delivery costs.

[3] The quantity $q_m$ does not necessarily reflect the technological limit of the particular product but instead the maximum meaningful use over the product's lifetime. For operational or marketing reasons, the marketable lifetime of a product is usually less than its technological lifetime. For example, car rental companies use rental cars for a set number of years (typically 1–2 years) although a car can last much longer.We have explicitly assumed that uses and time are related in our model. It is indeed true that in reality units of the same product would deliver different maximum numbers of uses within a period of time. However, as a first order analysis we consider that every product unit delivers the same amount $q_m$ within a time period. We relax the fixed nature of obsolescence $q_m$ in Section 5, where we treat the parameter $q_m$ as stochastic.

Customer utilities associated with the different business models are as follows: $U_s(q;p_s) = qa - p_s$ is the utility of a customer with usage quantity $q$ who purchases the product at a price, $p_s$, and $U_u(q;p_u) = q(a-p_u)$ is the utility of a customer with usage quantity $q$ who uses the product under a PPU scheme with price $p_u$ per use. Note that both business models deliver the same gross utility, $qa$, but differ in the net utility provided, once prices are applied.

### 2.3. Business Models

We examine first the profit structure of the selling and the pay-per-use business models. The selling business model with price $p_s$ produces profits

$$\Pi_s(p_s) = \int_D (p_s - c) f(q)\, dq$$

where the domain of integration $D$ is $D = \{q \mid U_s(q) \geqslant 0\}$ in monopoly and $D = \{q \mid U_s(q) \geqslant 0 \text{ and } U_s(q) \geqslant U_u(q)\}$ in duopoly. A PPU service provider (either in monopoly or duopoly) buys a number of assets (products), say $N$, from manufacturers at cost $c$, pools the total customers' usage, and offers a service charging $p_u$ per use, incurring a logistic cost, $\delta$, per use . The expected profits $\Pi_u$ for the PPU firm are

$$\Pi_u(p_u) = -cN + \int_D (p_u - \delta) q f(q)\, dq$$

where the domain of integration $D = \{q \mid U_u(q) \geqslant 0\}$ in monopoly and $D = \{q \mid U_u(q) \geqslant 0 \text{ and } U_u(q) \geqslant U_s(q)\}$ in duopoly.

In allocating the production cost to the uses, we are assuming that the PPU firm operates in a *strong pooling* regime, as described in Agrawal and Bellos (2016). Strong pooling has two elements. First, an asset, even if purchased or produced at the outset, does not have its production cost sunk; rather, production costs can be allocated to uses. This assumption is appropriate for a strategic analysis: for example, car rental companies lease the cars, as do airline companies with their airplanes. Moreover, part of these costs can often be recovered if the assets are not used to their end of life. For example, many consumer goods are re-manufactured, or traded in secondary markets. Therefore, asset costs are almost universally allocated to uses.

Second, strong pooling makes an operational assumption, namely that the PPU provider is able to use exactly the number of assets required to fulfill demand at 100% utilization: If one asset can deliver $q_m$ uses and the demand is not overlapping, then the number of products the PPU provider has to buy is $N = {}^{\int_D q f(q)\, dq}/_{q_m}$. For non-overlapping demand, even when the PPU provider cannot exactly match supply with demand and ends up with some unused capacity the strong pooling assumption offers a good approximation of the per use cost when the demand is high.[4] In Section 6 we relax this strong pooling assumption by accounting for the queueing inefficiencies arising from overlapping customer demand, which require the PPU provider to hold excess capacity to achieve targeted service levels.

For now, under the strong pooling assumption, we can write the profits of the PPU firm as follows

$$\Pi_u(p_u) = -cN + \int_D (p_u - \delta) q f(q) = \int_D (p_u - c_u) q f(q)\, dq,$$

where, $c_u$, is the cost per-use, $c_u = c/q_m + \delta$, and includes the logistics cost, $\delta$, of delivering a single use plus the amortized unit production cost spread over the maximum number of uses. Profits are positive for both models if $c \leqslant aq_m$ and $c_u \leqslant a$: the production cost must be below the maximum value that customers obtain by purchasing the product, and the fully loaded cost per use must be smaller than the customer's benefit from that use.

A monopolist's maximization problem can be written as follows:

$$\max\{\max_{p_s} \Pi_s(p_s), \max_{\substack{p_u \\ p_u \leqslant a}} \Pi_u(p_u)\}.$$

In the duopoly setting, each firm $i$ maximizes its profit subject to the competitive choice of the other firm ($-i$ may use either $s$ or $u$ as a business model). Therefore, the respective profits can be written as follows:

$$\Pi_s(p_s; p_{-i}) = \int_{\substack{U_i \geqslant 0 \\ U_s \geqslant U_{-i}}} (p_s - c) f(q) dq \quad \text{and} \quad \Pi_u(p_u; p_{-i}) = \int_{\substack{U_u \geqslant 0 \\ U_u \geqslant U_{-i}}} (p_u - c_u) q f(q) dq.$$

In line with previous literature, we assume that firms first simultaneously choose their business model, $s$ or $u$, and then, knowing the business model choices, simultaneously choose prices. Our sequential timeline reflects the mere reality that business model choices imply irreversible investments (e.g., setting up a usage monitoring and/or a billing system) and therefore precede the price decisions.

### 2.4. Base Model Assumptions

Our base model is parsimonious, and it rests upon a few key assumptions. With respect to the demand, we assume that the willingness to pay for a single use is constant and common across both business models. With respect to the supply, we assume that the PPU provider can apply

[4] To see this, consider an asset that can deliver $q_m = 10$ uses and costs \$1. The per-use cost under strong pooling is $c/q_m = 0.10$. When a PPU firm faces a low demand, say 12 units, the firm has to use 2 assets to meet this demand. Then, the per-use cost is $2/12 =$ \$0.1667, which is 67% higher than the strong pooling equivalent cost. This is because the fraction of unused product capacity is significant. However, when the firm faces a high demand, say 402 units, then it has to use 41 assets to meet that demand. In this case, the per-use cost is $41/402 =$ \$0.1019, or 2% higher than the strong pooling equivalent.

strong pooling (exact match of demand with supply and full asset utilization). Finally, we assume that the providers know upfront the *deterministic* maximum useful lifetime of an asset.

In the next section we analyze the duopoly equilibrium choices for our base model, and we further discuss the relative profitability of the two business models in the case of asymmetric duopoly equilibria. Then, in the subsequent sections we relax these key assumptions in order to see their effect on the relative profitability of the selling and the PPU models. We enrich the demand structure in two distinct ways. First, in Section 4.1 we allow for the WTP apart from the usage needs to vary independently. Thus, we cater for customers with continuously varying (lower or higher) valuation and large or small usage needs. Second, in Section 4.2 we allow for an endogenous emergence of the customer usage needs based on their variable willingness to pay (WTP) and the price charged. We also relax the assumption that the consumer needs are uniformly distributed in Section 4.3. Furthermore, in Section 6 we allow for the WTP to differ across the two business models: we account for impatient customers who experience reduced value because they have to wait for the service. The PPU provider needs to trade off more assets (overcapacity) for better customer service delivery. Finally, in Section 5 we explore the effect of variability in the asset's useful lifetime on the relative profitability of the business models.

## 3. PPU Versus Selling: Base Model Analysis

We examine first the monopoly problem and identify when a monopolist should choose the service model (PPU) or the selling model. Next, we examine the duopoly case, where we establish the competitive game equilibrium and compare the profits of PPU and selling model for all admissible values of production cost $c$ and logistics cost $\delta$.

### 3.1. The Monopolist's Business Model Choice

Apart from serving as a useful benchmark, the monopolist's optimal choice also allows us to understand the key profit enablers across the two business models. The monopolist's optimal prices, $p_i^*$, and profits, $\Pi_i^*(p_i)$, for $i = s, u$ are derived in Appendix A.1.1.

The PPU service model covers the entire market and extracts all consumer surplus at the expense of the logistics costs. The selling model, in contrast, faces a lower limit in market coverage, which has been observed previously in the vertical differentiation literature (Lancaster 1998). Customers with usage needs above the limit are considering buying. Proposition (1) characterizes the optimal business model choice.

Proposition 1. *There exists a threshold, $\bar{\bar{\delta}}(c) \in [0, a]$, in logistics cost such that for all $\delta \leqslant \bar{\bar{\delta}}(c)$, $\Pi_u^*(p_u^*) \geqslant \Pi_s^*(p_s^*)$, otherwise $\Pi_u^*(p_u^*) < \Pi_s^*(p_s^*)$. The threshold value is $\bar{\bar{\delta}}(c) = (a^2 q_m^2 - c^2)/2aq_m^2$, and it decreases concavely in $c$.*

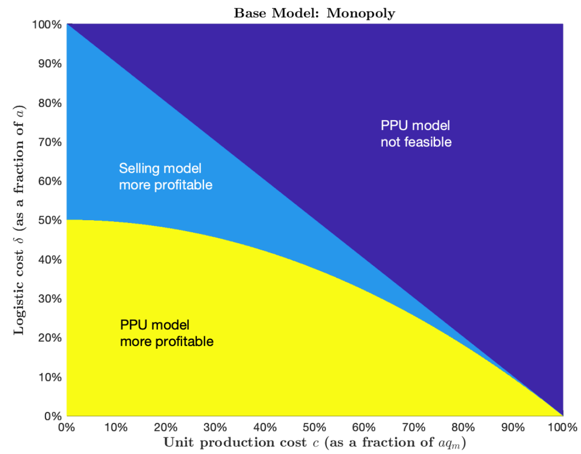


**Figure 1** Base model monopoly: areas in the $(c, \delta)$ space where one model is more profitable than the other. Yellow area: PPU model more profitable, Light blue area: Selling model more profitable, Dark Blue area: PPU model not feasible.

The proof is shown in Appendix A.1.2. The iso-profit line in the $(c, \delta)$ space can be expressed both by the relationship $\bar{c}(\delta) = q_m\sqrt{a(a-2\delta)}$ and by its inverse relationship, $\bar{\delta}(c) = {(a^2q_m^2 - c^2)}/{2aq_m^2}$, which gives a $\bar{\delta}$ threshold as a function of the unit production cost $c$. Figure 1 shows the areas of superior relative profitability for the two business models in the $(c, \delta)$ space. Note that when $c_u = \frac{c}{q_m} + \delta > a$, the PPU business model is infeasible, as the service provider faces a higher cost per use than the willingness to pay.

It is not surprising that the PPU model is a better choice for relatively lower service delivery costs, whereas delivery costs above a certain threshold make product selling more profitable. What is less intuitive is that the $\bar{\delta}(c)$ threshold shrinks in $c$, which means that the service model becomes less attractive for higher levels of production costs. To explain this, recall that the PPU monopolist covers the entire market. As the production cost increases, the PPU profit decreases linearly in $c$, whereas the seller's profit decreases quadratically, resulting in the observed threshold.

### 3.2. Equilibria of the Business Model Choice Game

In this subsection we analyze the duopoly game, in which each firm chooses a business model, and then based on their choice they set their competing prices. We establish the game's sub-game perfect equilibria, as well as the overall equilibrium. Each firm can use either or both business models. We assume (for now) that the firms offer identical goods and there is no market friction. This allows us to focus on the competition between the business models. The pure-strategy and mixed-strategy equilibria for the overall game are summarized in the following proposition.

Proposition 2. *Base model equilibrium analysis*

*(i) There are two asymmetric pure-strategy Nash equilibria in the game, where the two firms offer different business models.*

*(ii) There are two symmetric sub-game perfect equilibria, in which both firms sell or both offer a PPU service. In these equilibria, price competition reduces the firms' profits to zero. These sub-game perfect equilibria are not an overall game equilibrium.*

*(iii) There is also a unique symmetric mixed-strategy Nash equilibrium, where each firm chooses the selling business model with probability $\lambda_s = \Pi_s/(\Pi_s + \Pi_u)$ and the PPU business model with probability $\lambda_u = \Pi_u/(\Pi_s + \Pi_u)$; where $\Pi_s$ is the resulting profit of a firm using the selling mechanism under a pure-strategy asymmetric Nash equilibrium, and $\Pi_u$, is the respective profit of offering a PPU service.*

The proof of this proposition is in Appendix A.2.2. The symmetric equilibria leave the firms without profits, and the mixed-strategy equilibria do not capture the fact that firms make choices with significant profit implications. Therefore, the two game equilibria that are relevant for a managerial discussion are the two asymmetric pure-strategy equilibria, where one firm sells and the other offers a PPU scheme. Which exact equilibrium arises depends on many factors outside the scope of this work. In reality, even in the absence of an explicit coordination mechanism that guides the firms' business model choices, firms tend to choose different business models to differentiate their offerings. Thus, coordination may be achieved in the long run as firms learn from their coordination failures in the short run. One mechanism that may enable such coordination is "cheap talk" (Farrell 1987, 1995). Cooper et al. (1989) report experimental results in the battle-of-the-sexes game via cheap talk, and Crawford (1998) offers a survey of relevant experiments.

One might be tempted to suspect that the asymmetric equilibrium arises only when the goods offered are identical (perfect substitution) and the market has no friction. Indeed, many consumer markets exhibit some degree of price dispersion because market imperfections influence competition. Market imperfections may have many sources: different inventory costs between firms, cyclical fluctuations in costs or demand, or customers' search costs in seeking price information; other sources are summarized in Stiglitz (1979). We show in Appendix A.2.3 that even in imperfect markets, the asymmetric equilibria still arise for reasonable levels of market imperfection.[5] This suggests that the asymmetric competitive business model choice that we observe is not an artifact of the perfect substitution assumption but rather reflective of the structure of competitive duopolies.

[5] We follow a model of price dispersion proposed by Varian (1980), in which market imperfection stems from the coexistence of informed and uninformed customers in the market. Similar models have been developed by Salop and Stiglitz (1977) and Shilony (1977).

### 3.3. Comparison of Asymmetric Equilibrium Profits

Having established that asymmetric equilibria with differentiated business models are the key constellations of the duopoly game, we now compare the equilibrium profits. The optimal prices, $p_i^*$, and profits, $\Pi_i^*(p_i)$, for $i = s, u$ depend on the production cost, $c$, and the delivery cost, $\delta$, and are given in the following table. The proof is in Appendix A.2.1.

| | $0 \leqslant 2(\frac{c}{q_m} + \delta) \leqslant a$ | $a < 2(\frac{c}{q_m} + \delta)$ |
|---|---|---|
| $p_s^*$ | $\frac{1}{2}(3c + 2\delta q_m)$ | $\frac{1}{2}(c + aq_m)$ |
| $p_u^*$ | $2(\frac{c}{q_m} + \delta)$ | $a$ |
| $\Pi_s^*$ | $\frac{(c+2\delta q_m)^2}{8(c+\delta q_m)}$ | $\frac{(aq_m-c)^2}{4q_m a}$ |
| $\Pi_u^*$ | $\frac{(3c+2\delta q_m)^2}{32(c+\delta q_m)}$ | $\frac{(q_m(a-\delta)-c)(aq_m+c)^2}{8q_m^2 a^2}$ |

Competition between business models reveals three interesting structural properties of the strategic interaction between the two firms. First, the two business models segment the market in the following way: customers with high usage needs prefer the selling model, and customers with low usage needs opt to be served by the PPU model.

Second, two pricing regimes emerge with different competitive pricing choices, depending on the production cost. At low production costs, the PPU provider prices at twice the per-use cost, choosing to keep a constant profit margin $(p_u - c_u)/p_u$ at 50%. Increases in production cost push the service price, $p_u$, closer to the customer's willingness to pay for a single use, and a threshold is reached, forcing the PPU provider to price at $p_u^* = a$; recall that this is the monopoly price of the PPU provider. In the second regime the PPU's profit margin is decreasing in production cost $c$. In both regimes the PPU provider increases their market share in number of uses, namely, $(\bar{q}/q_m)^2$. The seller, on the other hand, has a decreasing profit margin $(p_s - c)/p_s$ and market share $1 - (\bar{q}/q_m)^2$ as the production cost increases in both regimes. Profit margins and market shares are displayed in Figure 2.

Third, the market split depends on the production cost. When the production cost is low, the seller can acquire a large proportion of the uses in the market; in fact, for $c = 0$ the seller has three times the share of uses compared to the PPU provider. Thus, the seller benefits from a low production cost. The low share of uses remaining to be served by the PPU provider does not allow them to exploit the fact that they can amortize the production cost over many uses. In other words, the PPU provider's pooling cost benefit (i.e., cost amortization over uses) is weak, limiting their profitability. However, at higher production cost, the indifferent customer's usage,

$\bar{q}$, is increasing, as shown in Figure2. As a result, the PPU provider benefits more from stronger pooling as the production cost increases, because he/she captures more market share (while the seller is losing market share) and can serve these uses with a cost advantage. This enables the PPU provider to increase profits faster than the seller. Note that the PPU's profits increase quadratically on the indifferent customer usage, $\bar{q}$, while the seller's profits depend linearly on $\bar{q}$. However, as the production cost increases further, the number of uses served by the PPU provider requires a significantly higher overall logistics cost. It is that increase in the logistics costs that lowers the PPU's profitability for very high production costs, as it starts eating into his/her margin — which is already capped by the maximum willingness to pay.

The relative profits together with market shares, profit margins, and indifferent customer usage as functions of the production cost are illustrated in Figure 2 for a particular value of the logistics cost, $\delta$. Proposition 3 further compares the optimal profits of the duopoly competitors across all admissible values of production cost $c$ and delivery cost $\delta$.

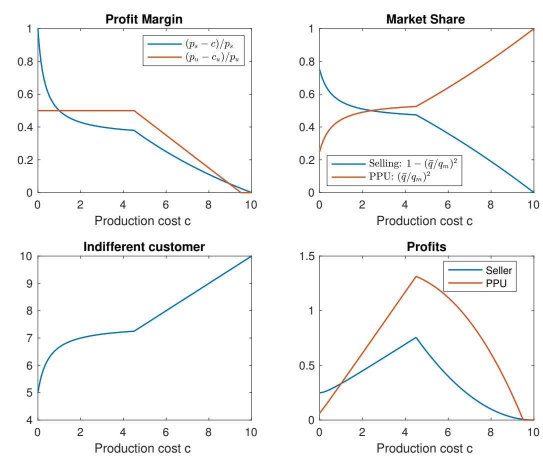


**Figure 2** Base model duopoly: example of seller's and PPU provider's profit margins, market share, indifferent customer, and profits as a function of the product cost $0 \leqslant c \leqslant aq_m$ for a particular per-use delivery cost, $\delta = 0.05a$. In this example we have taken $q_m = 10$ and $a = 1$.

PROPOSITION 3. *There exists a threshold level of delivery cost $\delta$, depending on production cost $c$, $\bar{\delta}(c)$, such that for $\delta \leqslant \bar{\delta}(c)$, the profits of the service provider are larger than the profits of the seller (i.e., $\Pi_u^*(c,\delta) \geqslant \Pi_s^*(c,\delta)$). For $\delta > \bar{\delta}(c)$, the profits of the service provider are smaller than the profits of the seller. The threshold value, $\bar{\delta}(c)$, is given by*

$$\bar{\delta}(c) = \begin{cases} \frac{c}{2q_m} & \textit{for} \quad 0 \leqslant c \leqslant \frac{1}{3}aq_m \\ a - \frac{c}{q_m} - 2a\left(\frac{aq_m - c}{aq_m + c}\right)^2 & \textit{for} \quad \frac{1}{3}aq_m < c \leqslant aq_m. \end{cases}$$

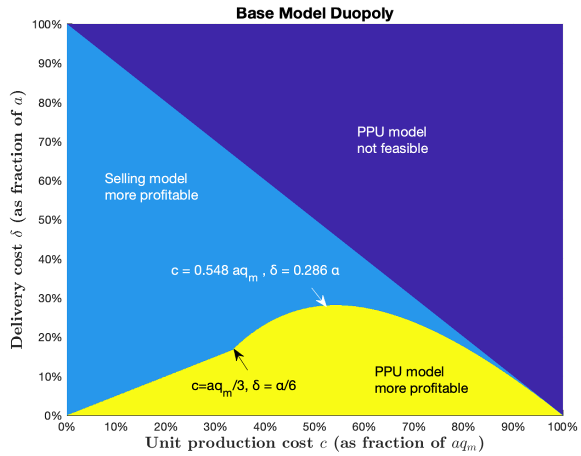


**Figure 3** Relative profitability of selling and PPU business models in $(c,\delta)$ space. Yellow area: PPU model more profitable. Light blue area: selling model more profitable. Dark blue area: PPU model not feasible.

The proof of the above proposition is shown in Appendix A.2.4. The threshold is zero for information goods ($c = 0$), which echoes the results of Balasubramanian et al. (2015), who find that PPU is less profitable than selling in information goods sectors.

Figure 3 shows the iso-profit line and the areas of relative profitability of the two business models in $(c,\delta)$ space. Interestingly, the PPU business model is the most profitable at intermediate production cost levels: the PPU provider has a cost advantage over the seller because they sell uses at fractions of the asset cost, an advantage that grows as the production cost increases. However, the PPU provider also has a disadvantage with respect to the seller: with an increasing production cost the PPU provider is servicing more uses at the expense of a higher overall logistics cost. The lower the per-use logistics cost, $\delta$ is, the wider the range of intermediate production cost values, $c$, for which PPU is more profitable than selling, as depicted in Figure 3. It must also be noted that the PPU provider manages to gain better profits in both pricing regimes, and, therefore, the fact of higher profitability cannot be attributed to the limit created by the constant willingness to pay.

Our analysis is economically meaningful, as a rough estimation with realistic numbers suggests. Suppose a car dealer can buy a mid-range car for £27,000 and then choose whether to rent it out per day (PPU) for a period of three years. In three years, the maximum usage quantity is approximately 1,000 days (with a rental almost every day), and suppose the willingness of customers to pay for the mid-range car is £50 per day. The highest point of the $\bar{\delta}$ line in Figure 3 is $(c,\delta) = (27000, 14)$, while the maximum value that a customer would be willing to pay for using it every day ($q_m a$) would be £50,000. Then in the model, the service-providing dealer (assuming that it could manage a delivery cost of £14 per day) would charge £50 per day, get a market share of 77%, and make a profit of £2,668, while a car-selling competitor would charge £38,500 for the car, get a market share of 23%, and make a profit of £2,645. Only customers with high usage would be willing to buy the car. If technological changes allow the PPU dealer to have a service delivery cost of less than £14 per day, it will achieve even higher profits and strongly outperform the seller. If the PPU dealer cannot deliver the car rental service at a cost at or below £14, they will make a lower profit than the selling dealer.

Our base model offers a novel justification for why PPU models arise across different industries. This happens not as a mere business model differentiation choice in which the service provider subsists but earns less than the seller. Instead, it happens because the PPU provider can be the *most* profitable competitor in the market if they manage to contain the delivery cost. Moreover, our results suggest possible reasons for why this PPU model advantage (and thus adoption) has grown over time: automation and digitization have reduced *both* the delivery and production costs such that many more PPU business models are now falling into the higher-profitability yellow zone, in the medium production-cost range, of Figure 3. This is a non-trivial observation — if production costs had become low across the board, and the threshold delivery cost, $\bar{\delta}$, had decreased this would have resulted in *less* adoption of the PPU business model. In addition, although our model analysis is static, it suggests possible industry shifts that may happen rapidly rather than gradually: as the PPU business model becomes economically more attractive, sellers will shift toward it. Once they do, they will quite quickly open up the large and under-serviced market of low-usage customers (as illustrated in the example in the previous paragraph). This may result in a disruptive competitive effect, as we have seen happen in many industries, from bicycles to tools (Hilti), cars (all car manufacturers), and aircraft engines (Rolls-Royce) (see Kim et al. 2007, Johnson 2010).

## 4. Extensions to Different Consumer Behaviors

In the base model, we have assumed that customers, although heterogeneous in their usage quantity, $q$, all exhibit the same willingness to pay $a$ for a single use of the product and that their usage quantity $q$ is exogenously determined. In this section, we explore the robustness of our insights for the duopoly case by relaxing those assumptions in three ways. First, we consider customers who are heterogeneous, both in their usage quantities and in their willingness to pay for a use of the product. Second, we consider customers that adjust their consumption, depending on the price charged, to maximize their utility.[6] Finally, we relax the assumption that customer usage is uniformly distributed, and we allow for alternative distributions.

[6] Following Wilson (1993), we assume in this extension that the utility is not linear in quantity, $q$, as in the base model, but has a quadratic term.

### 4.1. Consumer Heterogeneity in Usage *and* Valuation

As in the base model, we assume that $q \in [0, q_m]$ is uniformly distributed across the same usage quantity range, with $f(q) = 1/q_m$. We now suppose that the customer's willingness to pay for a single use is also uniformly distributed in $a \in [0,1]$, with density $g(a) = 1$. Our assumption captures an unbiased consumer market, where any customer type $(q,a) \in [0,q_m] \times [0,1]$ is present, including customers who have high or low valuation but low usage needs, and customers who have high or low valuation and high usage needs. The utility that a customer of type $(q,a)$ gains from buying the product at $p_s$ or using it for a per-use price, $p_u$, is unchanged from the base model. The cost structure also remains unchanged. The PPU model is able to attract *any* customer as long as $c_u = c/q_m + \delta \leqslant 1$. A similar setting has been introduced and analyzed in Bala and Carr (2010), for information goods with $c = 0$, where they find that in competition the seller outperforms the service provider. Our work extends the above-mentioned work to account for all costs ,$c \neq 0$, and shows that in a duopoly the service provider can outperform the seller in some areas of the $(c,\delta)$ space. In this subsection we present the duopoly discussion between the selling and the service business models. In Appendix A.3 we show the full analysis, both for monopoly and for duopoly.

We analyze the same subgame as in the base model, in which one firm sells the product at price, $p_s$, and the other uses a PPU business model pricing at $p_u$. A straightforward calculation shows the set of indifferent consumers to be $\{(q,a) \,|\, q = p_s/p_u \;\; and \;\; a \geqslant p_u\}$. The market segmentation is summarized in Figure 4. The service firm captures the market to the left of $q = p_s/p_u$ line (smaller usage quantities) and with a willingness to pay above $p_u$. Customers who buy the product are characterized by $qa - p_s \geqslant 0$. Note that in this model the market is not fully covered by the two providers. For a market to make economic sense, we must have $c < q_m$ and $\delta < 1$ ("1" being the maximum value of the willingness to pay $a$).

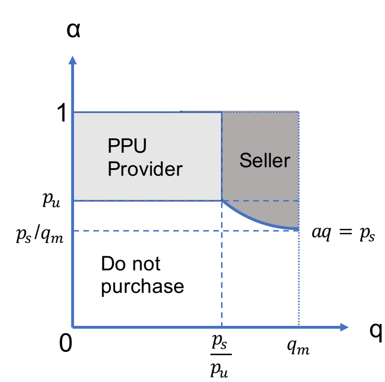


**Figure 4** Duopoly market segmentation in the $(q,a)$ space. Light grey area: customers prefer to pay-per-use. Dark grey area: customers prefer to buy. White area: customers abstain from the market.

In Appendix A.3.2 we show that the optimal prices and profits for the product provider and the service provider (PPU) are:

$$p_u^* = \frac{2c_u}{1+c_u} \qquad p_s^* = \frac{2c_u q_m}{1+c_u} \exp\left[\frac{c_u}{2} + \mathcal{W}\left(\frac{c(1+c_u)e^{c_u/2}}{4q_m c_u}; 0, \frac{2q_m - c}{c}\right)\right]$$

$$\Pi_u^* = \frac{(1-c_u)^2 p_s^{*2}}{8q_m c_u} \qquad \Pi_s^* = \frac{(p_s^* - c)^2 (q_m - p_s^*)}{q_m(2p_s^* - c)}$$

where $c_u = c/q_m + \delta$ and $\mathcal{W}$ is the generalized Lambert W function (Corless et al. 1996, Mező 2017).

Figure 5 shows areas in the $(c, \delta)$ space where the service model (PPU) is more profitable than the selling model. This result confirms that even when customers are heterogeneous in valuation, the relative profitability is similar to the base model. There exists a service delivery cost threshold, $\bar{\delta}(c)$, below which the service provider attains more profit than the seller. This $\bar{\delta}(c)$ increases in $c$ up to a maximum point and then decreases, as in the base model. Thus, our base model results are robust and apply even when the willingness to pay is variable. Notably, in the special case of information goods ($c = 0$) the selling model "always wins," as shown by Bala and Carr (2010). However, their insight cannot be extrapolated to non-zero production costs.

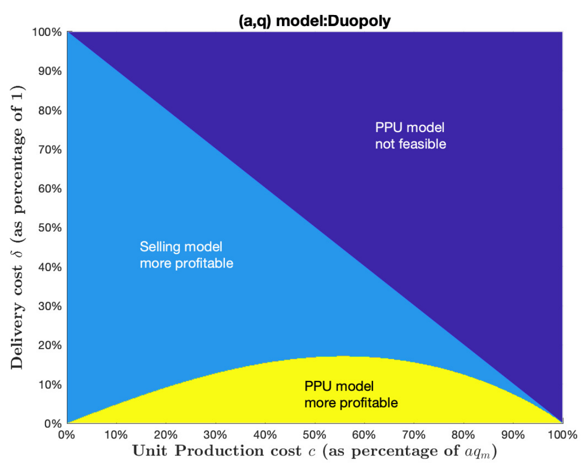


**Figure 5** Duopoly relative profits as a function of the marginal cost of $c$ and delivery cost $\delta$. Light blue area: product provider appropriates higher profits than pay-per-use one. Yellow area: higher profits for the PPU provider. Dark blue area: PPU model not feasible.

The major difference with the base model is the following:[7] the market is not covered in the heterogeneous valuation duopoly, while in the base model the market is fully covered. This is a direct consequence of heterogeneity in customer willingness to pay. As the valuation for a single use ranges from $[0, 1]$, for any usage price, $p_u \leqslant 1$, customers with valuation $a \leqslant p_u$ will abstain from the market. Hence, the service provider sets a price, $p_u$, to balance participation and abstinence.

[7] We offer a more elaborate discussion in the Appendix A.3.3.

This market share loss is reflected in the PPU provider profits and, in turn, reduces the region where the PPU business model is more attractive. Still, though, the tension between the strong pooling cost benefit versus the logistics cost extra burden drives a similar shape as to the one seen in Figure 3.

### 4.2. Consumer Usage Sensitivity to Pricing

So far, we have assumed that customer usage, $q$, is independent of price. In this subsection we extend the base model to accommodate consumers who are still heterogeneous in usage but decide on their usage depending on price (Wilson 1993, Sundararajan 2004, Agrawal and Bellos 2016). Customers are heterogeneous in two dimensions: with respect to their type, $a$, which is a proxy for their willingness to pay for a unit of usage, and indirectly with respect to the actual usage, $q$. We assume that $a$ is uniformly distributed in $[0,1]$. Customers with large $a$ derive more utility from the same usage than customers with small $a$. Moreover, customers observe the price posted by the seller and the PPU provider and then determine their usage by maximizing their net utility. The net utility that a customer of type $a$ derives from $q$ units of usage under the selling model is $U_s(a,q) = qa - \frac{1}{2}q^2 - p_s$ and $U_u(a,q) = qa - \frac{1}{2}q^2 - qp_u$ under the PPU model. Note that $U(a,q)$ is increasing and concave in $q$.

In Appendix A.4 we derive the optimum prices and the resulting profits for the seller and the PPU provider in duopoly. Figure 6 shows areas in the $(c,\delta)$ space where the service model (PPU) is more profitable than the selling model. This result shows that even when customers determine endogenously their usage quantities, the relative profitability of the PPU and the seller is similar to the base model. Once more, there exists a service delivery cost threshold, $\bar{\delta}(c)$, below which the service provider attains more profit than the seller. Moreover, the same tension as in the base model induces $\bar{\delta}(c)$ to increase in $c$ up to a maximum point and then to decrease. This indicates that our base model results are also robust for more general utility functions.

### 4.3. Alternative Distributions of Customer Usage Profiles

The value of a uniform distribution does not come solely from the tractability that it provides. A uniform distribution provides an additional conceptual benefit: its "equal weight to all possible occurrences" prevents the results from stemming from contextual properties of the market captured by specific distributional assumptions. For example, a distribution skewed toward high-usage customers could favor selling because high usage customers prefer to buy the product. Similarly, a distribution skewed toward low-usage customers will help the PPU model because it engages low usage customers. It is for that reason that a sizeable stream of literature in marketing has persistently used a uniform distribution to analyze models of product differentiation, since the uniform distribution introduces no prior bias in the customer demand. However, we do examine the effects

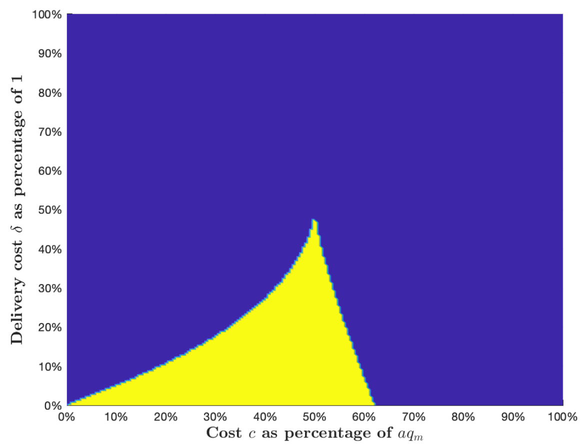


**Figure 6** Endogenous consumption model. Regions in $(c, \delta)$ space where the PPU model outperforms (underperforms) the selling model. Dark blue region: seller's profits are bigger than service provider profits. Yellow region: PPU provider's profits are bigger than seller's profits

of alternative distributions on the relative profitability. We numerically test a scenario where the customer usage follows a parametrized beta distribution where we vary the skew.

Suppose that customer usage is distributed according to $f(q)$ with support in $q \in [0,1]$[8] and that the willingness to pay is constant, $a$. If the firms choose prices $p_s$ and $p_u$ then the profits for the seller and the PPU provider are:

$$\Pi_s = (p_s - c)F(p_s/p_u) \quad \text{where} \quad F(x) = \int_x^1 f(q)\, dq$$

$$\Pi_u = (p_u - c_u)G(p_s/p_u) \quad \text{where} \quad G(x) = \int_0^x q f(q)\, dq$$

The best responses for the seller and the PPU provider can be found by solving the equations $\partial \Pi_s / \partial p_s = 0$ and $\partial \Pi_u / \partial p_u = 0$, which give the following system of nonlinear equations:

$$\begin{aligned} &(p_s - c)F'(p_s/p_u) + p_u F(p_s/p_u) = 0 \\ p_u^2 G(p_s/p_u) - &\frac{p_s}{p_u}(p_u - c_u)G'(p_s/p_u) = 0 \quad \text{subject to} \quad p_u \leqslant a \end{aligned}$$

Consider that $f(q)$ follows the beta distribution with two shape parameters, $\alpha$ and $\beta$, that is,

$$f(q; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} q^{a-1}(1-q)^{\beta - 1},$$

where $\Gamma(z)$ is the gamma function. The beta distribution has finite support, $q \in [0,1]$, and by choosing the shape parameters we can shift the consumer mass toward lower or higher usage. We numerically solve the nonlinear system of equations in two distinctive cases to compute the optimum

[8] Taking $q_m = 1$ without loss of generality.

prices, $p_s^*$ and $p_u^*$: (a) with distribution $f(q;1,2)$ (low-usage larger consumer mass); and (b) with distribution $f(q;2,1)$ (high-usage larger consumer mass). The resulting profits are displayed in the Figures 7a and 7b (which can be readily compared with the bottom-right panel of Figure 2. As expected, when customer distribution is shifted to low usage, the PPU provider is more profitable than the seller for a larger range of $c$ values (as the PPU firm is strongest for low-usage customers who do not benefit from owning a whole product). When the distribution is shifted to high usage, the range of $c$ for which PPU is more profitable is reduced; but notably it does not disappear.


(a) $f(q;1,2)$ (b) $f(q;2,1)$

**Figure 7** Profits of the seller and PPU provider as a function of the cost parameter $c$ for $\delta = 0.15a$

## 5. Uncertainty in the Usage Profiles: a Threat or an Opportunity?

The previous section has explored heterogeneity among consumers along their usage and their usage valuation, where the consumer's usage $q$ was uniformly distributed $\mathbb{U}(0, q_m)$ and the maximum usage $q_m$ was known to the firms. In this section, we allow for the consumers' usage distribution being unknown to the firms in advance.[9] In order to capture such uncertainty around the consumer's usage, as perceived by the firm, we build a model similar to Balasubramanian et al. (2015), where the usage quantities are distributed as $\mathbb{U}(0, q_m - v)$ with probability $1/2$ and $\mathbb{U}(0, q_m + v)$ with probability $1/2$, where $v \geqslant 0$ is a measure of the uncertainty or the knowledge gap of the firm about the market. Proposition 4 summarizes the impact of usage uncertainty on firms' profitability in both monopoly and duopoly.

PROPOSITION 4. *If there exists uncertainty, $v$, in each consumer's usage level:*

1. *In a monopoly, the seller behaves as if the maximum usage level was $q_m - v^2/q_m$; its optimum price and profit are described by the same formulas as in Section 3.1, adjusted by the transformation $q_m \to q_m - v^2/q_m$. The service provider is not affected by the uncertainty; its optimum price and profit are the same as in the base model.*

[9] Recall that $q_m$ does not represent the product's technical breakdown limit; most often, consumer products such as phones or cars are not kept until they break, but they are replaced according to (possibly implicit) write-down periods or because of style obsolescence. Consumers may know the total intended usage but the firm may not.

2. *In a duopoly, the seller behaves as if the maximum usage level were $q_m - v^2/q_m$; its optimum price and profit are described by the same formulas as in Section 3.3, adjusted by the transformation $q_m \to q_m - v^2/q_m$. The service provider is also affected by the uncertainty and behaves as if the maximum usage level were $q_m - v^2/q_m$ and as if the cost of the product were $c(q_m^2 + v^2)/(q_m(q_m^2 - v^2))$. The optimum price and profit of the service provider are described by the same formulas as in Section 3.3, adjusted by the transformations $q_m \to q_m - v^2/q_m$ and $c \to c(1 + v^2/q_m^2)$.*

The proof of the above proposition is in Appendix A.5. Proposition 4 offers two insights. First, the monopoly analysis reveals that uncertainty in customers' usage profiles, $q$, makes the PPU provider flexible: if a customer uses the product a bit less or a bit more, the service provider does not need to hedge against it because the revenue automatically adjusts. The service provider can use the same price as in the base model. The seller, in contrast, *must hedge* against the usage uncertainty by lowering its price and thus earning a lower profit. Thus, the service business model (PPU) becomes more attractive for the monopolist.

The second insight is that, in the duopoly, this attractiveness reverses. The service provider suffers *more* from the usage uncertainty than the seller. The reason is that, under competition, the service provider must also hedge, not only against a change in volume but also against a change in cost: the reader may recall that the cost per service unit is ${}^{c}\!/_{q_m} + \delta$. If $q_m$ is uncertain, the per-use cost becomes also uncertain, which forces the service provider to hedge. In contrast, the seller does not need to hedge against the cost because the unit production cost, $c$, is unaffected by the usage uncertainty. In hedging, the service provider behaves as if it faced a higher allocated production cost per use, which can be summarized as

$$\frac{c}{q_m} \to \frac{c}{q_m}\frac{q_m^2 + v^2}{q_m^2 - v^2} \geqslant \frac{c}{q_m}$$

As a result of this higher "equivalent" cost, the service provider increases its price when compared to the base model. This leads to a decreased market share and lower profits. In response, the seller then lowers its price and recoups market share and profit.

Figure 8, analogous to Figure 3, shows areas in the $(c, \delta)$ space where the service model achieves higher profits than the selling model in competition. For the reasons just explained, the area in which the service model is preferable is smaller in Figure 8 than in Figure 3.

It is of course intriguing why the service provider is indifferent to usage uncertainty in the monopoly setting but is affected from it in duopoly. As we see in the base model (see Appendix A.8 and A.2.1) in monopoly the profits of the PPU provider depend linearly on $q_m$ and hence variability cancels out. In duopoly, however, the profits of the PPU provider depend non-linearly

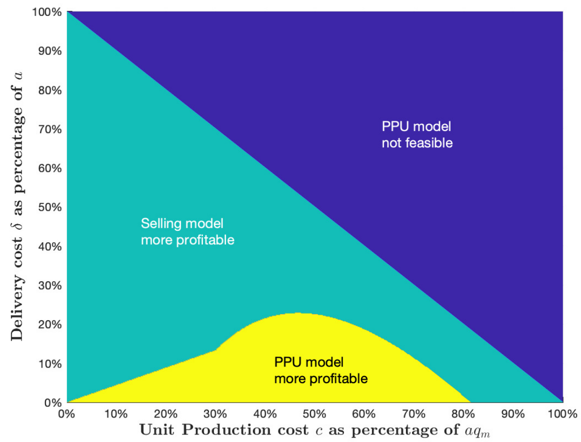


**Figure 8** Duopoly with uncertainty in $q_m$: areas in the $(c,\delta)$ space where the PPU model is better than the selling model. This example assumes a 10% uncertainty in $q_m$. Yellow area: PPU more profitable. Light green area: selling more profitable. Dark blue area: PPU model not feasible

in $q_m$. Uncertainty not only affects usage, but also affects the per-use production cost $c//q_m$. For example, if there is a 10% uncertainty in the maximum use frequency (i.e, $v^2 = 0.1q_m$), the service provider will behave as if the maximum use frequency is $0.9\,q_m$ and the production per-use cost is $1.1\,c/q_m$. A "naive" allocation of production cost, $c$, over a stochastic number of uses changes the service provider's pricing decisions and the associated competitive dynamics.

This corresponds to an important phenomenon in reality: while the allocation of production costs over usage occasions makes sense (the strategic element of strong pooling), allocating a production cost to a usage among a *stochastic* total usage amount might violate the principle of decision-relevant costing (Horngren et al. 2006, p. 383).

One method that simplifies accounting procedures and avoids allocating cost over stochastic uses is called "standard costing" (Horngren et al. 2006, p. 229). It uses "normal" unit cost levels (fixed or variable) rather than varying cost levels that are updated every period. If the service provider in our model uses standard costing, rather than a naive allocation of production cost $c$, they leave the service cost, $c_u$, unaffected by the usage uncertainty. This means that the pricing decision will be made based on a constant number for the per use cost, and, thus, the pricing decision of the service provider changes back to the same one as in the base case. In our previous example, the PPU provider will use a per-use production cost $c/q_m$ under standard costing, instead of $1.1\,c/q_m$. In other words, the service provider no longer hedges against per-use cost variations, while the seller (regardless of which costing method they use) *does* need to hedge against usage uncertainty. As a result, the service provider is unaffected by the usage uncertainty compared to the seller. Thus, if the service provider uses an appropriate costing method (which does not allocate the production cost to a stochastic number of uses), the service business model gains in relative profitability over the selling model (Figure 9).

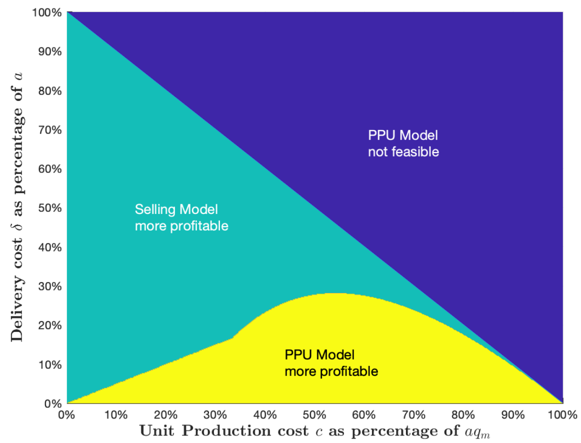


**Figure 9** Duopoly with uncertainty in $q_m$ under "standard costing": areas in the $(c, \delta)$ space where the service model is better than the selling model. This example assumes a 10% uncertainty in $q_m$. Yellow area: PPU more profitable. Light green area: selling more profitable. Dark Blue area: PPU model not feasible.

This insight points to an additional source of advantage of the PPU service business model over selling: the service business model provides a *source of flexibility against usage uncertainty* (this advantage may be widely relevant, as few products are bought in predetermined quantities). This, however, depends on the *decision-making approach* of the service provider, which is represented in our model either by the naive allocation of the production cost to service uses, or by standard costing.

## 6. Delay Effects in PPU and the Need for Overcapacity

### 6.1. Homogeneous Customer Base

The base model assumed strong pooling of customer demand, which enables perfect asset utilization. We now relax this assumption and examine the need for overcapacity to limit queueing effects.

PPU services sometimes suffer from inefficiencies such as travelling to a PPU provider's location, or waiting one's turn (queuing effects). In such cases, impatient customers suffer a utility degradation from delays and are less willing to pay. In response, a PPU provider may adjust its service level by deploying extra assets (i.e., excess capacity over strong pooling): an extra cost (from overcapacity) is traded for a reduced willingness to pay. The seller may, of course, also face inefficiencies in servicing its buyers, for example keeping a stock of assets to fulfill simultaneous demand. However, in this section we consider the one-sided ("conservative") situation where only the PPU provider faces delay inefficiencies.

Let $0 \leqslant \rho \leqslant 1$ denote the PPU's asset utilization. $A(\rho) \leqslant a$ denotes impatience, or the willingness to pay for a single use under the PPU model, where $a$ is the willingness to pay for a single use under the selling model or in the absence of any service inefficiencies. Naturally, suppose that when $\rho \to 0$, $A(\rho) \to a$ and when $\rho \to 1$, $A(\rho) \to 0$. In other words, infinite capacity would eliminate service delays, whereas full capacity utilization ($\rho \to 1$) wipes out any value for the customer. This is fulfilled by a simple service model such as the $M/M/1$ queue.

With $c$ the production cost and $\delta$ the delivery cost per use, as before, the cost per use for the PPU provider at asset utilization $\rho$ becomes $c_u(\rho) = {}^{c}/_{\rho q_m} + \delta$. Clearly, $c_u(\rho) \geqslant c_u$, and $c_u(\rho) = c_u$ when $\rho = 1$. The utilities for a customer under the selling ($U_s$) and the PPU model ($U_u$) are

$$U_s = qa - p_s \quad \text{and} \quad U_u = q(A(\rho) - p_u),$$

where $p_s$ and $p_u$ are the prices of the seller and the PPU provider, respectively. We treat the asset utilization parameter, $\rho$, as a decision variable; the PPU provider chooses first the utilization $\rho$ (which determines the service level), and subsequently both providers decide simultaneously on prices $p_u$ and $p_s$.

To analyze this further, we choose a reasonable function $A(\rho) = ae^{-k\rho/(1-\rho)}$. The customer's utility falls exponentially with a service-level surrogate measure captured by the queueing factor (the ratio of expected waiting divided by usage duration from the M/M/1 queue). The queueing factor grows superlinearly with the asset utilization, for example, a utilization $\rho = 90\%$ results in a queueing factor of $\rho/(1-\rho) = 9$ (an expected non-value added time that is nine times as high as the value added service time).[10] The utility reduction is moderated by the parameter, $k$, which represents the customer impatience. If we calibrate $k = 0.01$ as an example, then the 9-fold delay (and the need to call ahead) drops the customer's utility by 9% to $0.91a$, while at the same time increasing the cost per use by 11% when compared to full asset utilization. For this case, the duopoly problem is solved in Appendix A.6. The optimum utilization parameter, $\rho$, is given by $\rho = z/(z-k)$ where $z = 2\mathbb{W}_0\left(-\frac{1}{2}\sqrt{kc/(aq_m)}\right)$ with $\mathbb{W}_0$ be the Lambert function. The optimum price is $p_u = a - A(\rho) + 2c_u(\rho)$ for $c_u(\rho) \leqslant A(\rho) - a/2$ and $p_u = A(\rho)$ for $c_u(\rho) > A(\rho) - a/2$ where $A(\rho) = ae^{k\rho/(1-\rho)}$ and $c_u(\rho) = c/(\rho q_m) + \delta$. Finally, the seller's optimum price is $p_s = \frac{1}{2}(c + q_m(a - A(\rho) + p_u))$.

Figure 10 shows the areas in the $(c, \delta)$ space where the PPU model is more profitable than selling for the parameter values just discussed. It is apparent that the area in which the PPU model is more

[10] In real life, such as a car rental, this means that the customer should not just show up but call ahead a day or two in advance.

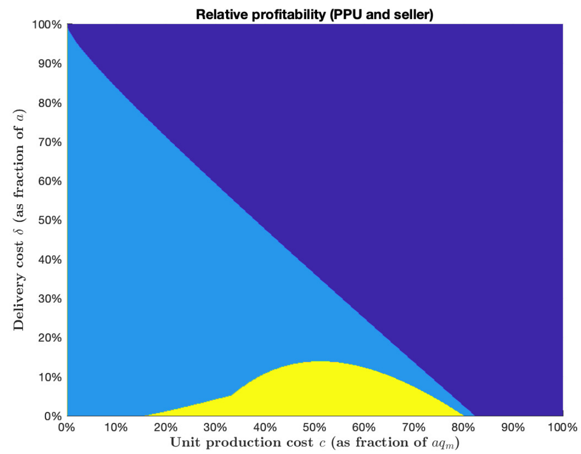


**Figure 10** Service-level model in a duopoly: Areas in $(c,\delta)$ space where one model is more profitable than the other. Dark blue area: the PPU model is not feasible. Light blue area: the selling model is more profitable. Yellow area: the PPU model is more profitable.

profitable is smaller than for the base model. Obviously, if the customers are more impatient (higher $k$) the area shrinks further. However, it is also apparent that the PPU-advantageous area remains structurally similar. The PPU provider remains more profitable in the mid-range of production costs. Hence, our main finding that there is a region in $(c,\delta)$ space where PPU is more profitable than selling holds even when PPU suffers from delay inefficiencies.

Moreover, the PPU delay inefficiency diminishes with the number of assets — a large PPU provider benefits from economies of scale, as already pointed out in Section 2. To see this, we model the PPU provider as a $M/M/n$ queuing system with inter-arrival times of mean $1/\lambda$, service times with mean $1/\mu$, and $n$ parallel identical servers, corresponding to the assets deployed (think of the PPU provider's car fleet). With $\rho=\lambda/(n\mu)$ the PPU provider's asset utilization ($\rho\leqslant 1$), the probability of waiting $P_w$ is given by Kleinrock (1975), where $\alpha=n\rho$

$$P_w = P_w(n,\alpha) = \frac{\frac{\alpha^n}{n!}\frac{n}{n-\alpha}}{\frac{\alpha^n}{n!}\frac{n}{n-\alpha}+\sum_{k=0}^{n-1}\frac{\alpha^k}{k!}}$$

The probability of waiting longer than $t$ is $P(W>t)=P_w e^{-n\mu(1-\lambda/(n\mu))t}=P_w e^{-(n\mu-\lambda)t}$. Then the service level, $S(t)$, the probability that a customer has to wait less than $t$, becomes:

$$S(t)=1-P(W>t)=1-P_w e^{-(n\mu-\lambda)t}.$$

Now consider two PPU providers of the same service (take car rentals as an example) with different demand levels. Provider (1) operates in a large town and faces $\lambda_1=50$ customers per day, and provider (2) operates in a small town and faces $\lambda_2=5$ customers per day. Both PPU providers have the same average usage time, $1/\mu = 2$ days (the time that customers keep the cars). Provider (1)'s minimum capacity (at 100% utilization, as is feasible under strong pooling) is $n_{01} = 100$ cars, while provider (2)'s minimum capacity is $n_{02} = 10$ cars. Suppose that each provider chooses a capacity (a fleet size) $n_1 > n_{01}$ and $n_2 > n_{02}$ such that customers, on average, wait 1/10 of the mean service time (of 2 days). The service levels $S_1$ and $S_2$ for the providers as a function of the servers $n_1$ and $n_2$ are

$$S_1 = 1 - P_w(n_1, a_1)e^{-0.1(n_1 - a_1)}$$
$$S_2 = 1 - P_w(n_2, a_2)e^{-0.1(n_2 - a_2)}$$

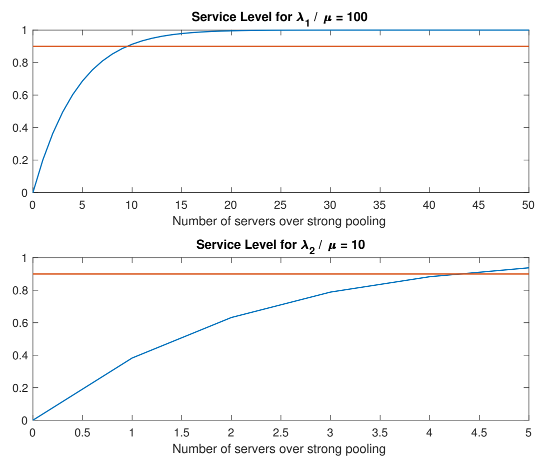


**Figure 11** Service level for two PPU providers as a function of servers over strong pooling

Figure 11 plots the service level as a function of excess servers (cars above the "strong pooling" capacity). We see that to achieve a service level of 90%, provider (1) needs to deploy 9 additional cars (an overcapacity of 9%), while provider (2) needs to deploy 5 additional cars (an overcapacity of 50%). For the large provider, a 9% overcapacity does not substantially affect its profitability. However, for the small PPU provider, the overcapacity represents an important hurdle. This illustrates the economies of scale at work, which reduce the delay inefficiencies for large PPU providers. This further supports our conclusion that a deviation from strong pooling may hurt the PPU provider but does not affect the main findings of our study.

### 6.2. Service and Price Discrimination

While delay inefficiencies may hurt the PPU provider, they also represent an opportunity if customers exhibit heterogeneity in their patience ($k$). This offers the PPU provider an opportunity to differentiate on the quality of service, following from the model setup presented in the previous subsection. Suppose that $w$% of customers are patient (with $k_1 = 0$) and $(1 - w)$% are impatient (with $k_2 > 0$, $A(\rho; k_2) < a$ ). While impatient customers need overcapacity and priority, patient customers are willing to book in advance according to a fixed schedule, which enables the PPU provider to keep them from overlapping and therefore to apply strong pooling. If the PPU provider can identify the customer's patience (for example, by asking them), it can discriminate and service one type only (i.e., the patient or the impatient), leaving the other type to the seller. Alternatively, the PPU provider can choose to service both types but price the service depending on the customer's type. Note that the seller has no such flexibility and addresses the whole market with a single price.

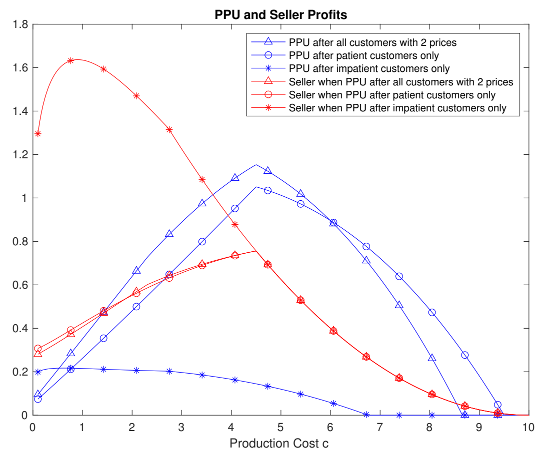


**Figure 12** PPU and seller's profits as a function of production cost $c$ for $\delta = 0.05a$ when PPU services (a) the patient customer only, (b) the impatient customers only, (c) both customer segments with different prices. The customer mix is 80% patient and 20% impatient.

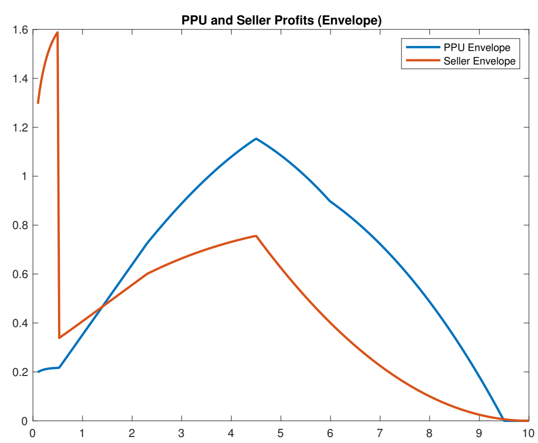


**Figure 13** PPU and seller's maximum profits as a function of production cost $c$ for $\delta = 0.05a$.The customer mix is 80% patient and 20% impatient.

Figure 12 shows the PPU provider's and seller's profits as a function of the production cost, $c$, when the PPU provider can select to service (a) the patient segment only, (b) the impatient segment only, or (c) both segments with two different prices. The derivations can be found in Appendix A.7. In Figure 13 we show the envelope profits (i.e., the best strategy given the production cost) for the seller and the PPU provider. Note that the PPU envelope profit is continuous, while the seller's respective curve is discontinuous. This happens because the seller, as seen even in the base model, responds to PPU pricing. Thus, as the PPU provider switches strategies, this response becomes discontinuous. The figure suggests two key insights. First, the PPU provider's chosen service strategy changes with the level of production cost, $c$. For very low $c$, the PPU provider optimally serves only the impatient customer segment. Overcapacity is cheap, while exploiting the strong pooling offers no advantage, as seen in the base model. In contrast, for high $c$ the PPU provider is better off servicing the patient customers only, as the cost of building overcapacity becomes prohibitive, and the strong pooling effect is very beneficial. Finally, in the medium production cost range, the PPU provider is better off price discriminating across both segments.

Second, the PPU business model is again superior to selling in the medium-production cost range, as in the base model. Notably, servicing the patient customers only allows the PPU provider to sustain profit dominance beyond what it would achieve by committing to servicing both types. By exploring the heterogeneity in customer patience, the PPU business model can achieve relative dominance, even for expensive assets.[11] Thus, service quality discrimination suggests an additional possible explanation for the recent proliferation of PPU models in industries with significant manufacturing costs (e.g., Kim et al. (2007)).

## 7. Discussion and Conclusion

The model proposed in this paper compares the profitability of two business models: traditional product selling versus the provision of the same product "as a service" with a pay-per-use (PPU) price. Customers derive utility from each use of the product, and they are heterogeneous with respect to how many uses they require. Firms (either as monopolists or in a competitive duopoly) buy or manufacture the product at a given unit cost, and then they sell the product or offer it as a PPU service. PPU service providers incur an additional service delivery or logistics cost for each use they provide.

Our work explores the circumstances under which the PPU business model is more profitable than product selling in competition. Previous work on the selling versus PPU business models has either considered only monopolists or focused solely on information goods (goods with negligible

[11] If the relative size of the two segments is 50-50 or 20-80, rather than 80-20 (80% of customers being patient and 20% impatient), the insights remain qualitatively unchanged.

manufacturing costs) and has concluded that product selling is more profitable than PPU in competitive equilibria (Balasubramanian et al. 2015, Bala and Carr 2010). This implies that the PPU may arise as a competitive differentiation strategy, but it represents the less attractive business model. Our analysis establishes that non-zero production costs fundamentally change the relative profitability of the two business models in equilibrium, with a significant area in the (production cost, delivery cost) space where the PPU model is more profitable. The area in the space of (production cost, logistics cost) where PPU business models are *more* profitable than selling comprises mid-range production costs combined with low delivery costs, which represent a "sweet spot" for PPU business models.

Indeed, PPU business models have become widely adopted across many industries in the last two decades (Kavadias et al. 2016, Crozet and Milet 2017). Our model suggests a possible explanation for why PPU models have spread across industries; PPU models are not mere competitive differentiators, but they can become more profitable ones provided a company operates in an appropriate cost regime.

The qualitative result of our base model remains valid when we extend it to account for consumers that exhibit additional heterogeneity in their willingness to pay for each use, or consumers that endogenously choose usage quantities with heterogeneous service valuations; and the model also remains valid when market friction weakens competition. The findings of the base model remain structurally similar even when we introduce delay inefficiencies into the PPU model. Although the PPU model becomes relatively less attractive, the delay disadvantage of the PPU model diminishes with economies of scale (if the PPU provider becomes larger); and, moreover, customer delays and impatience may also be characterized by patience heterogeneity, which enables the PPU provider to discriminate on customer service delays and improve his/her market share and profitability.

Furthermore, we show that PPU models further benefit from the PPU firm's flexibility toward unpredictable customers' usage quantities: the seller needs to hedge in its pricing against this uncertainty in usage. In contrast, the service provider offers a PPU contract, which linearly scales up and down if the customer uses more or fewer uses of the product. Thus, the structure of the PPU model does not require hedging in its pricing against demand uncertainty and allows for lowering prices, gaining market share, and driving profitability. However, this PPU advantage depends on an operational capability, namely, the use of standard costing that enables the firm to avoid naive allocation of a fixed production cost over a stochastic number of uses. Without this ability, the PPU business model loses its competitive advantage.

Our findings have clear managerial relevance. For information goods, PPU was shown to be a competitive response but inferior (less profitable than selling) in equilibrium. For manufactured goods, managers should be looking at PPU as a competitive weapon with the potential to gain dominance in equilibrium (over selling). However, this potential does not apply across the full product range of. First, PPU services offer little advantage for either very cheap products (where customer may as well buy outright), or very expensive products (where the production cost share per use is enough to deter cost sensitive users). Second, a critical *ratio* of costs can serve as a guide for identifying products that may benefit from PPU: the relative magnitude of logistic to production cost. This ratio provides a bound for the "allowable" logistics cost (as a fraction of production cost) so that a PPU service can be more profitable than selling. Third, a key source of value for a PPU business model lies in flexibility: flexibility with respect to varying (unpredictable) usage levels (against which a seller needs to hedge), and flexibility with respect to customer heterogeneity in impatience, which enable availability, service level and price discrimination. In summary, our model gives the manager a conceptual guide for a strategic analysis of business model choice over the product range.

At a broader level, our paper also contributes to an ongoing discussion about the effects of digitization on business models (Kavadias et al. 2016). Over the last twenty years, digital technologies have been credited with transforming traditional business models away from selling and towards PPU services, and a commonly cited reason is that "tracking has become cheaper". Our analysis qualifies this claim: digital technologies have shifted *both* production and logistic costs. The evolution is by no means one-directional – if production costs diminish, the position of the PPU model in the (production cost, logistics cost) space may actually be pushed *outside* its sweet zone and reduce its attractiveness. Examples are video streaming services being offered for a fixed subscription fee, or cellphone plans offered with a fixed fee for service, moving away from PPU. Overall, our insights suggest empirically testable hypotheses for the proliferation (or not) of PPU business models across different industrial settings.

As in any model of this type, we make several limiting assumptions. For example, we do not consider products that are rented until they "wear out," that is, where the maximum usage is determined technically rather than by customer conventions or procedures. This seems to be the case in a minority of industries, so we have left it out of our analysis, but we consider it an interesting topic for future research. Also, the two business models may trigger customer behavioral biases in different ways. An extensive literature shows that attributes of a pricing plan may affect behavior beyond direct cost implications (Ascarza et al. 2012, Lambrecht and Skiera 2006). The impact of biases on usage is a topic for future research. Another area of future work may be uncertainty in the technical maximum usage, which would have different effects than the imperfect firm's knowledge about the customer usage profiles that we currently analyze. In addition, our model is static. Important elements of competition change over time when there is technological progress that affects costs or functionality (and thus willingness to pay). These could be examined in a two-period model and would call for a separate study. Finally, our model does not fully address tactical issues such as the coordination and set-up costs across product switching, and the stochastic nature of the service delivery; customers might choose not to wait when queue waiting time is considerably long but to revisit the service provider. Future research should examine the implementation and limitations of strong pooling, as well as process effects that may influence strategic business model viability. We hope that this contribution opens up further relevant analytical or quantitative (rather than qualitative) work on service provision business models.

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# Appendix A: Appendices

## A.1. Base model:Monopoly

**A.1.1. Monopolist's prices and profits** When the monopolist sets a price $p_s$ for the product, a customer of consumption profile $q$ enjoys utility $U(q;p_s)=aq-p_s$. The customer of profile $q$ buys the product if $U\geqslant 0$. Hence, customers with frequency $q\geqslant p_s/a$ will purchase, and customers with $q<p_s/a$ do not buy. The maximization of the monopolist's profit $\Pi_s$ is can be written as follows: $\max_{p_s>0}\Pi_s(p_s)$ where $\Pi_s=\int_{\frac{p_s}{a}}^{q_m}(p_s-c)f(q)dq=\frac{1}{q_m}(p_s-c)(q_m-\frac{p_s}{a})$. The value $p_s$ that maximizes the profit $\Pi_s(p_s)$ satisfies the first order condition $\partial\Pi_s/\partial p_s=0$. It is straightforward to show that the price and the maximum profits of the product provider will respectively be $p_s^*=(aq_m+c)/2$ and $\Pi_s^*(p_s)=\frac{1}{4aq_m}(aq_m-c)^2$. The market share for the product provider is $M_s=1-p_s^*/(aq_m)$. Note that when $c=0$, customers with use profile $q\geqslant q_m/2$ buy the product and the market share is $q_m/2$. As the cost increases ($c\to aq_m$), the marginal customer shifts to higher use moving towards $q_m$ and the monopolist's market share diminishes gradually to zero.

Instead of selling, a monopolist can choose to deliver a service where she charges $p_u$ for a single use of the product. Then a customer with profile $q$ will enjoy utility $U(q;p_u)=q(a-p_u)$. Under such a selling mechanism, any customers enjoys a positive utility for every $p_u\leqslant a$. If $p_u>a$ no consumer purchases the service and the monopolist gets zero profit. The profit maximization problem can be written as follows: $\max_{p_u\leqslant a}\Pi_u(p_u)$ where $\Pi_u(p_u)=\int_0^{q_m}(p_u-c_u)qf(q)dq=\frac{1}{2}q_m(p_u-c_u)$ It is straightforward to establish that $\partial\Pi_u/\partial p_u=q_m/2>0$, in other words, $\Pi_u(p_u)$ is an increasing function of $p_u$ and hence the maximum profit is attained at $p_u=a$. Then the optimal price and the maximum profit for the pay-per-use scheme are respectively: $p_u^*=a$ and $\Pi_u^*=\frac{1}{2}\big(q_m(a-\delta)-c\big)$ and the market is covered (i.e, $M_u=1$). Note that because the service provider bears a cost per use $\delta$ is limited to offer the pay-per-use solution up to costs $c\leqslant q_m(a-\delta)$. The following table summarizes the optimum prices, market share, and profits for the seller and the PPU provider

| | Selling | Pay-per-use |
|---|---|---|
| Price | $p_s^*=\frac{1}{2}(aq_m+c)$ | $p_u^*=a$ |
| Market share | $M_s=\frac{1}{2}-\frac{c}{2aq_m}$ | $M_u=1$ |
| Profit | $\Pi_s^*=\frac{(aq_m-c)^2}{4aq_m}$ | $\Pi_u^*=\frac{1}{2}\big(q_m(a-\delta)-c\big)$ |

**A.1.2. Proof of Proposition 1** There exists a value of $c$ that makes the maximum profits of the pay per use and the selling mechanism equal, i.e., $\Pi_s^*=\Pi_u^*$. Using the expressions for the profits presented in section 3.1 it is straightforward to show that the value of $c$ that makes the profits equal is given by $c=q_m\sqrt{a(a-2\delta)}$ and that $\Pi_u>\Pi_s$ (i.e., the pay-per-use mechanism) outperforms the selling mechanism) for $c\leqslant q_m\sqrt{a(a-2\delta)}$.

### A.2. Base model: Duopoly

**A.2.1. Duopoly prices and profits** In a duopoly we assume that one firm acts as a product provider who uses the selling mechanism setting a price at $p_s$ offering unrestricted usage of the product, and the other firm acts as a service provider and employs the pay-per-use mechanism asking $p_u$ per one use of the product. The indifferent customer gets the same utility both from buying the product and from buying uses of the product. In other words, the indifferent customer is at frequency $\bar{q}$ for which $U_s(\bar{q};p_s)=U_u(\bar{q};p_u)$. Solving this equation for $\bar{q}$ it can be easily seen that $\bar{q}=p_s/p_u$. Customers with frequency $q\in[\bar{q},q_m]$ prefer to buy the product and customers with frequency $q\in[0,\bar{q}]$ prefer to use the service and to pay according to their use. The profits each firm expects $\Pi_s$ and $\Pi_u$ are given by:

$$\Pi_s(p_s,p_u)=\int_{\bar{q}}^{q_m}(p_s-c)f(q)dq=\frac{1}{q_m}(p_s-c)(q_m-\frac{p_s}{p_u}) \tag{1}$$

$$\Pi_u(p_s,p_u)=\int_0^{\bar{q}}(p_u-c_u)qf(q)dq=\frac{1}{2q_m}\frac{p_s^2}{p_u^2}(p_u-c_u) \tag{2}$$

The product provider will set the price $p_s$ as to maximize her profit and the condition $\partial\Pi_s/\partial p_s=0$ gives $p_s=\frac{1}{2}(c+q_m p_u)$ as the seller's best response. The service provider will choose its price $p_u$ as to maximize her profits $\Pi_u$ subject to the constraint $p_u\le a$. The Lagrangian for the service provider's constrained optimization problem is $\Lambda(p_u,\mu)=\Pi_u(p_u)+\mu(a-p_u)$. The solution for $p_u$ and $\mu$ must satisfy the KKT conditions

$$\frac{\partial\Pi_u}{\partial p_u}-\mu=0 \quad , \quad \mu(a-p_u)=0 \quad , \quad p_u\leqslant a \quad , \quad \mu\geqslant 0$$

When $\mu=0$ we have from the KKT conditions that $\partial\Pi_u/\partial p_u=0$. The derivative of $\Pi_u$ with respect to $p_u$ is

$$\frac{\partial\Pi_u}{\partial p_u}=\frac{p_s^2(2c_u-p_u)}{2q_m p_u^3}$$

Setting the above expression to zero gives the solution $p_{u,1}=2c_u$ which satisfies all the KKT conditions when $c_u<a/2$. When $\mu\neq 0$ we have $p_{u,2}=a$ which gives $\mu=p_s^2(2c_u-a)/(2q_m a^3)$ which is positive provided $c_u>a/2$. Therefore, when the cost $c\leqslant\frac{1}{2}q_m(a-2\delta)$, the service provider will ask a price per use at $p_u=2(\frac{c}{q_m}+\delta)$ (a price double than the cost of producing one use of the product or service), he will capture market form zero frequency up to $\frac{1}{2}q_m+\frac{1}{4}q_m c/(c+q_m\delta)$, and will end up with profits

$$\Pi_{u,d}=\frac{(3c+2q_m\delta)^2}{32(c+q_m\delta)}$$

In response, the product provider will set a price at $p_s=\frac{1}{2}(3c+2\delta q_m)$ and will capture profits

$$\Pi_{s,d}=\frac{(c+2q_m\delta)^2}{8(c+q_m\delta)}$$

When $c>\frac{1}{2}q_m(a-2\delta)$ the service provider cannot continue this policy (pricing at double the cost) as this price will exceed the customer's willingness to pay $a$. He will then price at $p_u=a$ and the product provider in response will price at $\frac{1}{2}(c+aq_m)$. The profits for the product provider (seller) and the service provider they will be

$$\Pi_s=\frac{1}{4q_m a}(aq_m-c)^2 \quad \text{and} \quad \Pi_u=\frac{1}{8q_m^2a^2}(q_m(a-\delta)-c)(aq_m+c)^2$$

The market shares for the product provider $M_s$ and the service provider $M_u$ are respectively, $M_s=1-\bar{q}/q_m$ and $M_u=\bar{q}/q_m$. The overall solution for the duopoly problem for both firms is given in the following table.

| | $c \leqslant q_m(\frac{a}{2}-\delta)$ | $c > q_m(\frac{a}{2}-\delta)$ |
|---|---|---|
| $p_s$ | $\frac{1}{2}(3c+2\delta q_m)$ | $\frac{1}{2}(c+aq_m)$ |
| $p_u$ | $2(\frac{c}{q_m}+\delta)$ | $a$ |
| $M_s^*$ | $\frac{1}{2}-\frac{c}{4(c+\delta q_m)}$ | $\frac{1}{2}-\frac{c}{2aq_m}$ |
| $M_u^*$ | $\frac{1}{2}+\frac{c}{4(c+\delta q_m)}$ | $\frac{1}{2}+\frac{c}{2aq_m}$ |
| $\Pi_s$ | $\frac{(c+2\delta q_m)^2}{8(c+\delta q_m)}$ | $\frac{(aq_m-c)^2}{4q_m a}$ |
| $\Pi_u$ | $\frac{(3c+2\delta q_m)^2}{32(c+\delta q_m)}$ | $\frac{(q_m(a-\delta)-c)(aq_m+c)^2}{8q_m^2 a^2}$ |

It can be easily verified the solutions for the profits of the service provider and for the product provider are continuous in $c$.

**A.2.2. Equilibrium analysis of the base model (Proof of Proposition 2)** If both firms use the same business model they compete themselves down to zero profits under Bertrand competition. Each firm can lower its selling price $p_s$ or pay-per-use fee $p_u$ to undercut the other. Therefore, the $(s,s)$ and $(u,u)$ sub-games will have zero payoffs for both firms. On the other hand, if each firm is committing to a different business model, they will assume non zero profits $\Pi_s > 0$ for the selling firm and $\Pi_u > 0$ for the firm using PPU. The expressions for profits $\Pi_s$ and $\Pi_u$ are derived in appendix (A.2.1).

We consider the "hybrid" case where one firm is offering both business models (sell and PPU) at prices $(p_{s_1}, p_{u_1})$ and the other is offering just PPU at $p_{u_2}$. The firm offering only pay-per-use will compete down to $c_u$ with the firm that is offering both models. and hence the pay-per-use fee will be $p_{u_1} = p_{u_2} = c_u$. The firm that is offering only PPU will realize zero profits and the firm that is offering both models will realize profits only form selling. Given that the PPU price is at $p_u = c_u$, the indifferent customer is at $\bar{q} = p_s/c_u$ and the profits for the firm that is offering both models $\Pi'_s$ will $\Pi'_s = \frac{1}{q_m}(p_s - c)(q_m - p_s/c_u)$. The firm will set $p_s$ as to maximize the profits. The optimum price will be $p_s = \frac{1}{2}(c + q_m c_u)$ and the maximum profits will be $\Pi'_s = \frac{1}{4q_m c_u}(q_m c_u - c)^2$. It is straight forward to show that $\Pi'_s < \Pi_s$.

The symmetric problem is when one firm offers both models at prices $(p_{s1}, p_{u1})$ and the other is offering just selling at $p_{s2}$. The firm offering only the selling model will compete down to $p_{s_1} = p_{s_2} = c$ with the firm that is offering both models. So the firm offering only selling will realize zero profits and the firm that offers both models will realize profits $\Pi'_u$ only from customers attracted by the pay-per-use model. Given that the selling price is at $p_s = c$, the indifferent customer is at $\bar{q} = c/p_u$ and the profits will be $\Pi'_u = \frac{c^2}{2q_m}\frac{p_u - c_u}{p_u^2}$. The firm will try to set $p_u$ as to maximize the profits. The optimum price will be $p_u = 2c_u$ for $c_u \leqslant a/2$ and $p_u = a$ for $c_u > a/2$ and the maximum profits will be $\Pi'_u = \frac{c^2}{8q_m c_u}$ for $c_u \leqslant a/2$ and $\Pi'_u = \frac{(a-c_u)c^2}{2q_m a^2}$ for $c_u > a/2$. It is straightforward to show that $\Pi'_u < \Pi_u$.

In case where both firms offer both business models (selling and PPU) they will compete down to cost for both the selling part and the PPU part. The prices will be $p_u = c_u$ and $p_s = c$ and both firms will end up with zero profits.

The normal form representation of the game is:

| | Sell | PPU | Both |
|---|---|---|---|
| Sell | 0,0 | $\Pi_s$,$\Pi_u$ | 0, $\Pi'_u$ |
| PPU | $\Pi_u$,$\Pi_s$ | 0,0 | 0,$\Pi'_s$ |
| Both | $\Pi'_u$,0 | $\Pi'_s$,0 | 0,0 |

There are two asymmetric pure-strategy Nash equilibria in the game where the two firms offer identical products, with one firm selling and the other using PPU business model.

For the mixed-strategy equilibrium for the normal game in which one firm is using selling and the competitor uses PPU we have the following. Each firm can adopt a mixed-strategy in which the probability $\lambda_s$ of selling makes the other firm indifferent between selling and PPU. This probability then satisfies $(1-\lambda_s)\Pi_s = \lambda_s\Pi_u$ which gives $\lambda_s = \Pi_s/(\Pi_s + \Pi_u)$.

**A.2.3. Imperfect Market** As in the base model, suppose customers want $q$ uses of the product, with $q$ uniformly distributed between zero and $q_m$. The customers have a reservation price $a$ per use. To model market imperfection, we assume that customers come in two types, informed and uninformed. Uninformed customers select a firm at random; a customer of type $q$ buys the product from a seller that asks a price $p_s$ if $aq \leqslant p_s$ or buys $q$ uses of the product from a PPU provider if the price $p_u \leqslant a$. Informed customers, in contrast, know the business models and prices and select that firm that maximizes their utility. Let $\lambda$ be the percentage of uninformed customers where $0 \leqslant \lambda \leqslant 1$. Firms attempt to price discriminate between informed and uninformed customers. In what follows we use the notation $\Pi_{im}$, $\Pi_{id}$ for the monopoly and duopoly profits for $i = \{s, u\}$ and $p_{im}$, $p_{id}$ for the corresponding optimal prices as presented in the base model. We repeat here the functional forms for convenience:

$$
\begin{aligned}
\Pi_{sm}(p_s) &= \tfrac{1}{q_m}(p_s - c) * (q_m - p_s/a) \\
\Pi_{sd}(p_s, p_u) &= \tfrac{1}{q_m}(p_s - c) * (q_m - p_s/p_u) \\
\Pi_{um}(p_u) &= \tfrac{1}{2}q_m(p_u - c_u) \\
\Pi_{ud}(p_s, p_u) &= \tfrac{1}{2q_m}(p_u - c_u)p_s^2/p_u^2
\end{aligned}
$$

**The** $(s,s)$ **and** $(u,u)$ **subgame**: Assume that two firms sell identical goods at prices $p_{s1}$ and $p_{s2}$. If firm 1 post a price $p_{s1} < p_{s2}$, she will get $\frac{\lambda}{2}$ customers that have a usage profile $q \leqslant p_{s1}/a$ and all the $1-\lambda$ customers with usage profile $q \leqslant p_{s1}/a$. If firm1 posts a price $p_{s1} > p_{s2}$ she will get only $\frac{\lambda}{2}$ of the uninformed customers whose $q$ profile is $q \leqslant p_{s1}/a$. If firms post the same price they split the market. In other words, firm 1 will get profits

$$
\Pi_{s1}(p_{s1}) = \begin{cases} \frac{\lambda}{2}\Pi_{sm}(p_{s1}) & p_{s1} > p_{s2} \\ \frac{1}{2}\Pi_{sm}(p_{s1}) & p_{s1} = p_{s2} \\ (1-\frac{\lambda}{2})\Pi_{sm}(p_{s1}) & p_{s1} < p_{s2} \end{cases}
$$

where $\Pi_{sm}(p_{s1})$ are the monopoly profits for price $p_{s1}$. In this duopoly game firms enter the market if they get at least $\frac{\lambda}{2}\Pi_{sm}(p_{sm})$ profits. Varian (1980, p. 654) shows this game has no symmetric price equilibrium in pure strategies. He assumes that firms choose prices randomly out of a distribution density $f(p_s)$ which has a cumulative distribution function $F(p_s)$. Varian (1980) shows that there exists a cumulative distribution function $F(p_s)$ that maximizes the expected profits for each firm. Moreover, the expected profits for each firms are the optimal monopolist profits for the uninformed customer segment. Therefore, the expected profits of both firms will be

$$\mathbb{E}(\Pi_{si}) = \tfrac{\lambda}{2}\Pi_{sm}(p_{sm}) = \frac{\lambda}{8aq_m}(aq_m - c)^2$$

for $i = 1, 2$ where $p_{sm}$ is the monopoly price $p_{sm} = \frac{1}{2}(c + aq_m)$. When there is no market friction ($\lambda = 0$), the expected profits are zero inline with Bertrand's competition for identical products result.

The $(u, u)$ sub-game can be treated in exactly the same way. The expected profits of both firms will be the optimal monopolist profits for the uninformed customer segment

$$\mathbb{E}(\Pi_{ui}) = \tfrac{\lambda}{2}\Pi_{um}(p_{um}) = \frac{\lambda}{4}q_m(a - c_u)$$

for $i = 1, 2$, where $p_{um}$ is the monopolist price $p_{um} = a$. When there is no market friction, ($\lambda = 0$) the expected profits drop to zero inline with Bertrand's competition result for identical products.

**The (s,u) subgame**: We assume that one firm sells the product at price $p_s$ and the other firms offers uses of the product at price $p_u$ per use. Each provider gets monopoly profits from $\frac{\lambda}{2}$ part of the uninformed customers and engages in competition for the $1 - \lambda$ uninformed customers from which she gets duopoly profits according to the posted prices $p_s$ and $p_u$. The profits for the seller $\Pi_{s\lambda}$ and the PPU provider $\Pi_{u\lambda}$ are therefore,

$$\Pi_{s\lambda}(p_s, p_u) = \tfrac{\lambda}{2}\Pi_{sm}(p_s) + (1-\lambda)\Pi_{sd}(p_s, p_u)$$
$$\Pi_{u\lambda}(p_s, p_u) = \tfrac{\lambda}{2}\Pi_{um}(p_s) + (1-\lambda)\Pi_{ud}(p_s, p_u)$$

The partial derivatives of seller's and PPU's profits with respect to the seller's and PPU's price are:

$$\frac{\partial \Pi_{s\lambda}}{\partial p_{s\lambda}} = \left(1 - \tfrac{\lambda}{2}\right)q_m - \left(\tfrac{\lambda}{2a} + \tfrac{1-\lambda}{p_u}\right)(2p_s - c)$$
$$\frac{\partial \Pi_{u\lambda}}{\partial p_{u\lambda}} = \tfrac{\lambda}{2}q_m^2 p_u^3 - (1-\lambda)p_s^2(p_u - 2c_u) = 0$$

The equilibrium prices $p_{s\lambda}$ and $p_{u\lambda}$ for $\lambda$ market imperfection have to satisfy the following equations

$$\left(1 - \tfrac{\lambda}{2}\right)q_m - \left(\tfrac{\lambda}{2a} + \tfrac{1-\lambda}{p_{u\lambda}}\right)(2p_{s\lambda} - c) = 0 \tag{3}$$
$$\tfrac{\lambda}{2}q_m^2 p_{u\lambda}^3 - (1-\lambda)p_{s\lambda}^2(p_{u\lambda} - 2c_u) = 0 \tag{4}$$
$$p_{u\lambda} \leqslant a \tag{5}$$

The equilibrium profits are $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda})$ and $\Pi_{u\lambda}(p_{s\lambda}, p_{u\lambda})$. Note that equations (3) and (4) form a system of nonlinear equations of fifth degree in $p_u$ and one cannot expect to track it analytically.

Equation (4) implies that $p_u > 2c_u$, otherwise the second term is negative and subtracted from the first term which is positive gives always a positive result. But if $c_u \geqslant a/2$ then $p_u \geqslant a$ and therefore the solution for $p_u$ is limited to $p_{u\lambda} = p_{sm} = a$. Note that when $p_{u\lambda} p_{um} = a$ then equation (3) gives $p_{s\lambda} = p_{sm} = \frac{1}{2}(c + aq_m)$. Therefore, when $c_u \geqslant a/2$ the equilibrium prices for the $\lambda$ problem are the monopoly prices allowing equilibrium profits to be expressed in closed form as shown below

$$\Pi_{s\lambda} = (1 - \tfrac{\lambda}{2})\Pi_{sm}(p_{sm}) = (1 - \tfrac{\lambda}{2})\frac{(aq_m - c)^2}{4aq_m}$$

$$\Pi_{u\lambda} = \tfrac{\lambda}{2}\Pi_{um}(a) + (1-\lambda)\Pi_{ud}(p_{sm}, a) = \frac{a - cu}{2q_m}\left(\tfrac{\lambda}{2}q_m^2 + (1-\lambda)\frac{(aq_m + c)^2}{4a^2}\right)$$

As is evident from the above equations, equilibrium profits are functions of $c$ and $c_u = c/q_m + \delta$.

For $c_u < a/2$ there are no closed form solutions. But we can compare the equilibrium profits. Let $p_{s\lambda}$ and $p_{u\lambda}$ denote the equilibrium prices. Because $p_{u\lambda}$ is the Nash price, any other price different from this will give lower profits. Therefore, $\Pi_{u\lambda}(p_{s\lambda}, p_{u\lambda}) > \Pi_{u\lambda}(p_{s\lambda}, a)$. But $\Pi_{u\lambda}(p_{s\lambda}, a) = \frac{\lambda}{2}\Pi_{um}(a) + (1-\lambda)\Pi_{ud}(p_{s\lambda}, a) > \frac{\lambda}{2}\Pi_{um}(a) > \frac{\lambda}{2}\Pi_{sm}(p_{sm})$. The last inequality comes from the fact that when $c_u < a/2$, the monopoly profits of PPU are greater than the monopoly profits of the selling scheme as we have shown in the base section. Therefore, for all $\lambda$ and for $c_u < a/2$, we have that $\Pi_{u\lambda}(p_{s\lambda}, p_{u\lambda}) > \frac{\lambda}{2}\Pi_{sm}(p_{sm})$. This implies that for $c_u < a/2$ the equilibrium can be either $(u, s)$, or $(s, u)$, or $(u, u)$. Said differently, the equilibrium cannot be $(s, s)$ for $c_u < a/2$.

For $c_u < a/2$, the equilibrium prices $p_{s\lambda}$ and $p_{u\lambda}$ are smaller than the monopoly prices so we have that $p_{s\lambda} < p_{sm}$ and $p_{u\lambda} < a$. We can show that $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda}) < \Pi_{s\lambda}(p_{sm}, p_{um})$. Indeed, $\Pi_{s\lambda}(p_{sm}, p_{um}) = \frac{\lambda}{2}\Pi_{sm}(p_{sm}) + (1-\lambda)\Pi_{sd}(p_{sm}, p_{um}) = \frac{\lambda}{2}\Pi_{sm}(p_{sm}) + (1-\lambda)\Pi_{sm}(p_{sm})$. On the other hand, $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda}) = \frac{\lambda}{2}\Pi_{sm}(p_{s\lambda}) + (1-\lambda)\Pi_{sd}(p_{s\lambda}, p_{u\lambda})$. But $\Pi_{sm}(p_{s\lambda}) < \Pi_{sm}(p_{sm})$ because $p_{sm}$ is the optimum monopoly price. Also, $\Pi_{sd}(p_{s\lambda}, p_{u\lambda}) < \Pi_{sd}(p_{s\lambda}, p_{um})$ because $\Pi_{sd}(p_s, p_u)$ is an increasing function in $p_u$[12]. Therefore, we have that $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda}) < \Pi_{s\lambda}(p_{sm}, p_{um})$. But $\Pi_{s\lambda}(p_{sm}, p_{um}) = (1 - \frac{\lambda}{2})\Pi_{sm}(p_{sm})$ and hence we have that $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda}) < (1 - \frac{\lambda}{2})\Pi_{sm}(p_{sm})$. Because $\Pi_{sm}(p_{sm}) < \Pi_{um}(p_{um})$ for $c_u < a/2$, it is straightforward to show[13] that there is $\lambda_0 < 1$ such that for $\lambda > \lambda_0$, $(1 - \frac{\lambda}{2})\Pi_{sm}(p_{sm}) < \frac{\lambda}{2}\Pi_{um}(p_{um})$. We have therefore shown that for $c_u < a/2$ and for $\lambda > \lambda_0$, $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda}) < \frac{\lambda}{2}\Pi_{um}(p_{um})$ which implies that the equilibrium is $(u, u)$.

We can also show that there exists $\lambda_1 < 1$ such that for $\lambda < \lambda_1$ $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda}) > \frac{\lambda}{2}\Pi_{um}(p_{um})$. Indeed, because $p_{s\lambda}$ is the equilibrium price, $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda}) > \Pi_{s\lambda}(p_{sd}, p_{u\lambda}) > (1-\lambda)\Pi_{s\lambda}(p_{sd}, p_{u\lambda}) > (1-\lambda)\Pi_{s\lambda}(p_{sd}, p_{ud})$. The last equality comes from the fact that $\Pi_{sd}$ is an increasing function in $p_u$ and that $p_{u\lambda} > p_{ud} = 2c_u$. It is straightforward to show[14] that there exists $\lambda_1 < 1$ such that for $\lambda < \lambda_1$, $(1-\lambda)\Pi_{sd}(p_{sd}, p_{ud}) > \frac{\lambda}{2}\Pi_{um}(p_{um})$. Therefore, there exists a $\lambda_1 < 1$ such that for $\lambda < \lambda_1$ we have asymmetric equilibrium is $(u, s)$ or $(s, u)$.

**The overall game** Each firm can choose one of the two strategies: either to play $(s)$ or to play $(u)$. The normal form representation of the game is given in the following table

| | Selling | PPU |
|---|---|---|
| Selling | $\frac{\lambda}{2}\Pi_{sm}(p_{sm})$ , $\frac{\lambda}{2}\Pi_{sm}(p_{sm})$ | $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda})$ , $\Pi_{u\lambda}(p_{s\lambda}, p_{u\lambda})$ |
| PPU | $\Pi_{u\lambda}(p_{s\lambda}, p_{u\lambda})$ , $\Pi_{s\lambda}(p_{s\lambda}, p_{u\lambda})$ | $\frac{\lambda}{2}\Pi_{um}(p_{um})$ , $\frac{\lambda}{2}\Pi_{um}(p_{um})$ |

[12] It is straightforward to show that $\partial\Pi_{sd}/\partial p_u > 0$.

[13] Indeed, $\lambda_0 = 2\Pi_{sm}(p_{sm})/(\Pi_{sm}(p_{sm}) + \Pi_{um}(p_{um})$. Because $\Pi_{sm}(p_{sm}) < \Pi_{um}(p_{um})$, then $\lambda_0 < 1$

[14] Indeed, $\lambda_1 = 2\Pi_{sd}(p_{sd}, p_{ud})/(2\Pi_{sd}(p_{sd}, p_{ud}) + \Pi_{um}(p_{um}))$ which is always positive and smaller than one.

The following figures depict the outcomes of a family of games for all feasible values of $c$ and $\delta$ for specific values of the market friction parameter $\lambda$. We present two scenarios: a small market friction $\lambda = 10\%$, where just 10% of the customers are uninformed, and significant market friction, where 50% of the customers are uninformed.

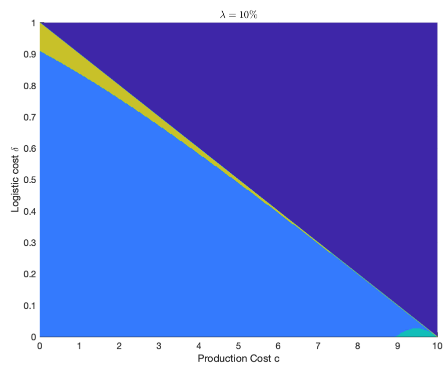


**Figure 14** Equilibria for small friction $\lambda = 10\%$ case in the $(c, \delta)$ space. Light Blue area: Equilibrium at $(u, s)$ or $(s, u)$, Yellow area: Equilibrium at $(s, s)$, Green area: Equilibrium at $(u, u)$, Dark Blue area: PPU not feasible.

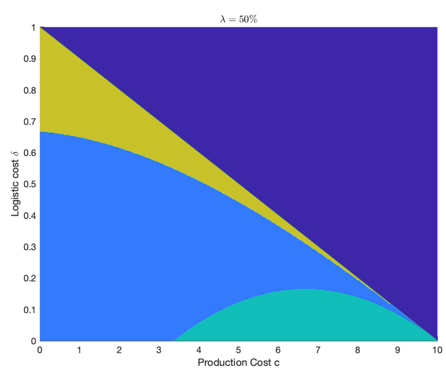


**Figure 15** Equilibria for significant friction $\lambda = 50\%$ case. Light Blue area: Equilibrium at $(u, s)$ or $(s, u)$, Yellow area: Equilibrium at $(s, s)$, Green area: Equilibrium at $(u, u)$, Dark Blue area: PPU not feasible

Figures 14 and 15 suggest that for small values of $c$ and large values of $\delta$, the equilibrium is $(s, s)$ (selling dominates even under head-to-head competition). For large values of $c$ and small values of $\delta$ the imperfect market game equilibrium becomes $(u, u)$ (PPU is more attractive even under head-to-head competition). For all other regions in the $(c, \delta)$ space, we get an asymmetric equilibrium $(u, s)$ or $(s, u)$ as in the base model. It is striking that even in the scenario of significant market friction (50% of customers are uninformed), the asymmetric equilibria arise across the largest portion of the feasible parameter space.

**A.2.4. Equilibrium profits comparison (Proof of Proposition 3)** The iso-profit line consists of points in $(c,\delta)$ space where the profits for the two mechanisms are equal. For $c \leqslant q_m(\frac{a}{2}-\delta)$ the condition $\Pi_s=\Pi_u$, after substituting the relevant expressions for the profits from Appendix (A.2.1) leads to the equation $(5c+6\delta q_m)(c-2\delta q_m)=0$, which gives $c=2\delta q_m$ or $\delta=c/(2q_m)$. The second solution of the quadratic equation gives a negative $\delta$. Because this is valid for $c \leqslant q_m(\frac{a}{2}-\delta)$ this leads to an upper bound for $\delta$ i.e., $2\delta q_m \leqslant q_m(\frac{a}{2}-\delta)$, which implies that $\delta \leqslant a/6$. The parameter $\delta$ achieves the value $a/6$ at $c=\frac{1}{3}aq_m$.

For $c > q_m(\frac{a}{2}-\delta)$ the condition $\Pi_s=\Pi_u$, after substituting the relevant expressions from Appendix (A.2.1), leads to the equation $2aq_m(aq_m-c)^2=(q_m(a-\delta)-c)(aq_m+c)^2$. Solving for $\delta$ we find

$$\delta = a - \frac{c}{q_m} - 2a\frac{(aq_m-c)^2}{(aq_m+c)^2}$$

The derivative with respect to $c$ is

$$\partial\delta/\partial c = \frac{c^3+3ac^2+11a^2q_m^2c-7a^3q_m^3}{q_m(aq_m+c)^3}$$

The condition $\partial\delta/\partial c=0$ gives the value of $c$ that maximizes $\delta$ which is $c\approx 0.542aq_m$ and the maximum value of $\delta$ is $\delta_{max}\approx 0.286a$.

### A.3. Heterogeneity in valuation

**A.3.1. Monopoly** The product provider will set a price $p_s$. Customers with $qa \geqslant p_s$ will buy the product and customers with $qa < p_s$ will abstain from the market. In this case the iso-utility curves are hyperbolas in the $(q,a)$ plane. The expected profit for the seller is

$$\Pi_s = \int_{p_s}^{q_m} f(q)dq \int_{p_s/q}^{1} (p_s-c)da = (p_s-c)\big(1-\frac{p_s}{q_m}+\frac{p_s}{q_m}\ln(p_s/q_m)\big) \tag{6}$$

The price $p_s$ that maximizes seller's profits must satisfy condition $\partial\Pi_s/\partial p_s=0$, which gives,

$$\frac{1}{q_m}(2p_s-c)\ln(p_s/q_m)+1-\frac{p_s}{q_m}=0 \tag{7}$$

Equation 7 is a transcendental equation that can be solved with the help of the following lemma.

Lemma 1. *Equations of the form $(Ax+C)\ln x - Bx = a$ for $A,B,a$ real numbers and $A\neq 0$ and $B\neq 0$ have solution given by the following expression*

$$x=\exp\left[\frac{B}{A}+\mathcal{W}\left(-\frac{C}{A}e^{-B/A};0,\frac{aA-BC}{AC}\right)\right]$$

*where $\mathcal{W}$ is the generalized Lambert function*

We note that equation 7 is in the form of the equation in Lemma 1 with $A=2$, $B=1$, $C=-c/q_m$, and $a=-1$. Applying the results of Lemma 1 we arrive at

$$p_s = q_m\exp\left[\frac{1}{2}+\mathcal{W}\left(\frac{c}{2q_m}e^{-1/2};0,\frac{2q_m-c}{2c}\right)\right] \tag{8}$$

The corresponding profits can be expressed in terms of the price $p_s$. We can solve equation 7 for $\ln(p_s/q_m)$ and substitute into equation 6 to express the seller's profits in terms of the price $p_s$ as

$$\Pi_s = \frac{(p_s-c)^2(q_m-p_s)}{q_m(2p_s-c)} \tag{9}$$

The service provider, in a monopoly setting, will set a price $p_u$ as to maximize his profits. Customers with $a \geqslant p_u$ i.e, customers that are willing to pay more than $p_u$ will participate in the market irrespective of the frequency of use. The service provider's profits are given by

$$\Pi_u = \int_0^{q_m} qf(q)dq \int_{p_u}^1 (p_u - c_u)g(a)da = \frac{q_m}{2}(p_u - c_u)(1 - p_u)$$

The condition $\partial \Pi_u / \partial p_u = 0$ gives as $p_u = \frac{1}{2}(1 + c_u)$. Substituting back into the equations for the profits, we find that the profits the service provider expects as a monopolist are $\Pi_u = \frac{q_m}{8}(1 - c_u)^2$.

**A.3.2. Duopoly** In duopoly one firm decides to offer a fixed-based pricing scheme at $p_s$ and the other to offer a pay-per use scheme at $p_u$. The two firms compete by truncating the others segment along the line $q = p_s / p_u$ which is the locus of the indifference between fixed and usage based pricing. The profits each firm expect are then given by

$$\Pi_u = \int_0^{\frac{p_s}{p_u}} \int_{p_u}^1 (p_u - c_u) qf(q) da dq \quad \text{and} \quad \Pi_s = \int_{\frac{p_s}{p_u}}^{q_m} \int_{\frac{p_s}{q}}^1 (p_s - c) f(q) da dq$$

Carrying out the integration for the profits of the service provider, it can be readily found that $\Pi_u = \frac{1}{2q_m}(p_u - c_u)(1 - p_u)(p_s/p_u)^2$ and that $\partial \Pi_u / \partial p_u = -p_s^2(c_u(p_u - 2) + p_u)/2q_m p_u^3$. The condition $\partial \Pi_u / \partial p_u = 0$ leads to $p_u = 2c_u/(1 + c_u)$. It can be readily seen that $p_u \leqslant 1$ as long as $c_u \leqslant 1$ and there is no need for a break point in prices as we has in the base model. Substituting the solution for the service provider's price to the equation for the service provider's profits we arrive at $\Pi_u = \frac{(1-c_u)^2 p_s^2}{8 q_m c_u}$. Carrying out the integration for the product provider's profits we can find that

$$\Pi_s = (p_s - c)\left(1 - \frac{p_s}{q_m p_u} + \frac{p_s}{q_m} \ln \frac{p_s}{q_m p_u}\right) \tag{10}$$

The condition $\partial \Pi_s / \partial p_s = 0$ gives

$$q_m - \frac{2p_s - c}{p_u} + (2p_s - c)\ln(p_s / q_m p_u) + (p_s - c) = 0 \tag{11}$$

Rearranging terms we can write the above equation as

$$\frac{2p_s - c}{q_m p_u} \ln \frac{p_s}{q_m pu} - (\frac{2}{pu} - 1)\frac{p_s}{q_m pu} = \frac{p_u(c - q_m) - c}{p_u^2 q_m}$$

We can use Lemma 1 with $A = 2$, $B = (2 - p_u)/pu$, $C = -c/(q_m p_u)$, and $a = (p_u(c - q_m) - c)/(p_u^2 q_m)$. The solution for the price of the seller then becomes

$$p_s = p_u q_m \exp\left[\frac{2 - p_u}{2p_u} + \mathcal{W}\left(\frac{c}{2q_m p_u} e^{(p_u - 2)/2p_u}; 0, \frac{2q_m - c}{c}\right)\right] \tag{12}$$

Substituting the expression for $p_u$ in the above formula we arrive at

$$p_s = \frac{2c_u q_m}{1 + c_u} \exp\left[\frac{c_u}{2} + \mathcal{W}\left(\frac{c(1 + c_u)}{4q_m c_u} e^{c_u/2}; 0, \frac{2q_m - c}{c}\right)\right] \tag{13}$$

Solving equation 11 for $\ln \frac{p_s}{q_m p_u}$ and substituting into equation 10, we can express the profits of the seller in terms of $p_s$ as

$$\Pi_s = \frac{(p_s - c)^2 (q_m - p_s)}{q_m (2p_s - c)}$$

**A.3.3. Comparison Between the Base Model and the Extension with Heterogeneous Valuation Customers** The base model and the heterogeneous valuation model cannot be directly compared as they have a different total willingness to pay and a different cost support. The base model can support costs up to $aq_m$, while the heterogeneous valuation model (with willingness to pay up to $a_m$) can support costs up to $a_m q_m$. For both models having the same cost support it is required that $a = a_m$. But then the models have a different total willingness to pay. In the base model, where the customers place a constant value of $a$ for the willingness to pay for a single use, the total willingness of the market to pay is

$$\int_0^{q_m} aqf(q)dq = \frac{1}{2}aq_m.$$

In the heterogeneous valuation model, in which customers are willing to pay up to $a_m$ per use, the total willingness of the market to pay is

$$\int_0^{a_m}\int_0^{q_m} aqf(q)g(a)dadq = \frac{1}{4}a_m q_m.$$

This means that the two models (uniform $a$ versus heterogeneous willingness to pay) can have the same total willingness to pay but they support different cost ranges, or they can have the same cost support but with different total willingness to pay.

### A.4. Endogenous $q$ model

In this section we examine a model the usage $q$ is endogenous. Customers observe the price under the given business model and decide on usage by maximizing their utility. Customers are heterogeneous in two parameters: the type $a$ which is roughly equivalent to the willingness to pay for a unit of usage and in usage $q$. Customers with large $a$ derive more utility from the same usage than customers with small $a$. We assume that both $a$ and $q$ are uniformly distributed in $[0,1]$. The net utility a customer of type $a$ derives from $q$ units of usage under the selling model is $U_s(a,q) = qa - \frac{1}{2}q^2 - p_s$ and $U_u(a,q) = qa - \frac{1}{2}q^2 - qp_u$ under the PPU model. Note that $U(a,q)$ is increasing and concave in $q$.

**A.4.1. Monopoly for the selling or the PPU model** Given a price $p_s$, a customer of type $a$ will choose $q_{s,m}$ that will maximize his utility. The first order condition $\partial U_s/\partial q = 0$ gives $q_{s,m} = a$ and the utility at the maximum usage is $U_{s,m}(a) = \frac{1}{2}a^2 - p_s$. Customers with type $a \geqslant \sqrt{2p_s}$ have a positive utility and participate in the market and customers with $a < \sqrt{2p_s}$ will abstain. The number of customers that will buy is $1-\sqrt{2p_s}$ and the profit the seller will realize will be $\Pi_s(p_s) = (p-c)(1-\sqrt{2p_s})$. The first order condition $\partial \Pi_s/\partial p_s = 0$ gives $p_s^* = (1+3c+\sqrt{6c+1})/9$ and maximum profit $\Pi_s^* = (1-18c+\sqrt{(6c+1)^3})/27$.

Given a price per use $p_u$, a customer of type $q$ will choose $q_{u,m}$ that will maximize his utility. The first order condition $\partial U_u/\partial q = 0$ gives $q_{u,m} = a - p_u$ and the utility at the maximum usage is $U_{u,m}(a) = \frac{1}{2}(a-p_u)^2$. As the usage has to be positive, customers with type $a \geqslant p_u$ participate in the market, while customers with $a < p_u$ abstain. The number of uses $M_u$ required by the customers is $M_u = \int_{p_u}^1 (a-p_u)\,da = \frac{1}{2}(1-p_u)^2$. The number of customers that participate in the market are $N_u = 1-p_u$. The profits for the monopolist are $\Pi_u(p_u) = M_u(p_u-\delta) - \lambda N_u c = \frac{1}{2}(p_u-\delta)(1-p_u)^2 - \lambda c(1-p_u)$ where $\lambda < 1$ is the pooling factor that determines the level of service of the PPU provider. The first order condition $\partial \Pi_u/\partial p_u = 0$ gives $p_u^* = (2+\delta+\sqrt{1+\delta^2-3\lambda c-2\delta})/3$. For a level of service equivalent to the one in the base model (strong pooling), we assume that the maximum uses the product can deliver is one and the cost per use is $c_u = c + \delta$. Therefore the profit for the PPU provider becomes $\Pi_u(p_u) = \frac{1}{2}(p_u - c_u)(1 - p_u)^2$. The first order condition $\partial \Pi_u / \partial p_u = 0$ gives the optimal price at $p_u^* = \frac{1}{3}(1 + 2c_u)$ and substituting into the profit equation, the maximum profit is at $\Pi_u^* = \frac{2}{27}(1 - c_u)^3$.

**A.4.2. Competition between selling and PPU model** Given prices $p_s$ and $p_u$ customers find the quantities $q_{s,m}$ and $q_{u,m}$ that maximize their utility. These optimum quantities are as in monopoly $q_{s,m} = a$ and $q_{u,m} = a - p_u$. Customers' utility at the optimum quantities are $U_{s,m} = \frac{a^2}{2} - p_s$ and $U_{u,m} = \frac{1}{2}(a - p_u)^2$. Customers choose among three options: not participating (remain inactive), buying uses from the PPU provider or buying the product from the seller. The indifferent customer $a_1$ between remain inactive and buying is at $U_{u,m}(a_1) = 0$ which gives $a_1 = p_u$. The indifferent customer $a_2$ between buying uses and buying product is at $U_{u,m}(a_2) = U_{s,m}(a_2)$, which gives $a_2 = \frac{1}{2}p_u + \frac{p_s}{p_u}$. So the number of customers in the inactive, PPU, selling segments are respectively $p_u, \frac{p_s}{p_u} - \frac{1}{2}p_u, 1 - \frac{p_s}{p_u} - \frac{1}{2}p_u$. The number of uses required by customers that adopt PPU model are $M_u = \int_{a_1}^{a_2}(a - p_u)\,da = \frac{1}{2}(\frac{p_s}{p_u} - \frac{p_u}{2})^2$. The profits for the seller and the PPU provider (under pooling factor $\lambda$) are

$$\Pi_s(p_s, p_u) = (p_s - c)\left(1 - \frac{p_s}{p_u} - \frac{p_u}{2}\right) \quad \text{and} \quad \Pi_u(p_s, p_u) = \tfrac{1}{2}(p_u - \delta)\left(\frac{p_s}{p_u} - \frac{p_u}{2}\right)^2 - \lambda c\left(\frac{p_s}{p_u} - \frac{p_u}{2}\right)$$

The first order conditions $\partial \Pi_s / \partial p_s = 0$ and $\partial \Pi_u / \partial p_u = 0$ give

$$p_u - \tfrac{1}{2}p_u^2 + c - 2p_s = 0$$

$$\frac{1}{8p_u^3}\left(8\lambda c p_s p_u + 4\lambda c p_u^3 + 8\delta p_s^2 - 2\delta p_u^4 - 4p_s^2 p_u - 4p_s p_u^3 + 3p_u^5\right) = 0$$

The last two equations are a system of nonlinear equations for the prices of $p_s$ and $p_u$ that has to be treated numerically for the optimum prices $p_s^*$ and $p_u^*$. Under strong pooling the profits of the PPU provider are

$$\Pi_u(p_s, p_u) = \tfrac{1}{2}(p_u - c_u)\left(\frac{p_s}{p_u} - \frac{p_u}{2}\right)^2$$

where $c_u = c + \delta$. The first order conditions $\partial \Pi_s / \partial p_s = 0$ and $\partial \Pi_u / \partial p_u = 0$ gives the following system of equations for the optimum prices

$$p_u - \tfrac{1}{2}p_u^2 + c - 2p_s = 0$$

$$3p_u^3 + 2p_s p_u - 2c_u(2p_s + p_u^2) = 0$$

Solving the first equation for $p_s$ and substituting into the second equation we arrive at a third degree equation for the price of the PPU provider $\frac{5}{2}p_u^3 + (1 - c_u)p_u^2 - (2c_u - c)p_u - 2cc_u = 0$. The cubic discriminant is $\Delta = \frac{q^2}{4} + \frac{p^3}{27}$, where $p = \beta - \frac{1}{3}\alpha^2$ and $q = \frac{2\alpha^3}{27} - \frac{\alpha\beta}{3} + \gamma$ and where $\alpha = \frac{2}{5}(1 - c_u)$, $\beta = -\frac{2}{5}(2c_u - c)$, and $\gamma = -\frac{4}{5}cc_u$. The cubic discriminant $\Delta$ depends on $c$ and $c_u$. It is straightforward to show numerically that there are regions in $(c, c_u)$ space in which $\Delta > 0$ and therefore there is only one real solution ($0 < p_u < 1$), and regions where $\Delta \leqslant 0$ and the cubic equation has three real solutions (one positive $0 < p_u < 1$ and two negatives). Therefore, for all values of $c$ and $c_u$ there is one solution $0 < p_u < 1$, which implies also that $0 < p_s < 1$ because $p_s$ has to satisfy $p_u - \frac{1}{2}p_u^2 + c - 2p_s = 0$.

### A.5. Uncertainty in the usage profile (Proof of proposition 4)

We assume that the maximum use frequency $q_m$ is a random variable $\tilde{q}_m$ that takes values $q_m - v$ with probability $\frac{1}{2}$ and $q_m + v$ with probability $\frac{1}{2}$ where $v \geqslant 0$ is a measure of the uncertainty in the value of the maximum use frequency.

**A.5.1. Monopoly** In the monopoly setting the seller will set a price $p_s$ and the profits she will expect will become

$$\begin{aligned}\Pi_s &= \frac{(p_s - c)(q_m + v - p_s/a)}{2(q_m + v)} + \frac{(p_s - c)(q_m - v - p_s/a)}{2(q_m - v)} \\ &= (p_s - c)\left(1 - \frac{q_m p_s}{a(q_m^2 - v^2)}\right)\end{aligned}$$

The price $p_s$ that maximizes the seller's profits satisfies the equation $\partial\Pi_s/\partial p_s = 0$. Comparing the seller's profits with the ones of the base model (A.1.1), we find the the seller behaves as if he is faced with a smaller maximum frequency $q'_m = (q_m^2 - v^2)/q_m \leqslant q_m$. Therefore, the optimum price and the expected profits under uncertainty will be smaller than the ones of the base model for the monopolist seller. It can readily shown that the optimum price $p_s^*$ and the maximum profits $\Pi_s^*$ for the seller in a monopoly situation are

$$p_s^* = \frac{1}{2}(c + aq'_m) \quad , \quad \Pi_s^* = \frac{1}{4aq'_m}(aq'_m - c)^2 \quad \text{where} \quad q'_m = (q_m^2 - v^2)/q_m$$

If the monopolist chooses to sell uses of the product and offer a pay-per-use service at a price $p_u$, he will face per use cost $c/(q_m + v) + \delta$ with probability $\frac{1}{2}$ and $c/(q_m - v) + \delta$ again with probability $\frac{1}{2}$. She will set the price $p_u$ that will maximize the expected profits

$$\begin{aligned}\Pi_u &= \frac{1}{4}(p_u - \delta - c/(q_m + v))(q_m + v) + \frac{1}{4}(p_u - \delta - c/(q_m - v))(q_m - v) \\ &= \frac{1}{2}(q_m(p_u - \delta) - c)\end{aligned}$$

Note that for the service provider, the effect of the uncertainty cancels out and the expression for the profits is the same as in the base model in A.1.1. Therefore, the optimum price and the profits for the service provider are exactly the same as in the base case. The seller on the other hand, faced with uncertainty, she will choose a lower price than the price chosen in the base model. She will price as if the maximum use frequency is $q'_m = (q_m^2 - v^2)/q_m$ and of course, $q'_m \leqslant q_m$. This choice brings more market share than the base model but less profits. Therefore, in monopoly the effect of uncertainty in the maximum use frequency $q_m$ make the service provider more attractive than the seller.

**A.5.2. Duopoly** In duopoly, the indifferent customer is not affected by uncertainty in the maximum use frequency $q_m$ and is, just like the base model, at $\bar{q} = p_s/p_u$. The expected profits as functions of the prices $p_s$ and $p_u$ for the seller and the service provider are respectively

$$\begin{aligned}\Pi_s(p_s, p_u) &= \tfrac{1}{2}(p_s - c)\left[\frac{1}{q_m - v}\left(q_m - v - \frac{p_s}{p_u}\right) + \frac{1}{q_m + v}\left(q_m + v - \frac{p_s}{p_u}\right)\right] \\ \Pi_s(p_s, p_u) &= (p_s - c)\left[1 - \frac{q_m p_s}{p_u(q_m^2 - v^2)}\right] \qquad (14)\end{aligned}$$

$$\Pi_u(p_s,p_u) = \tfrac{1}{2}\frac{p_s^2}{p_u^2}\left[\frac{1}{2(q_m-v)}\left(p_u-\delta-\frac{c}{q_m-v}\right)+\frac{1}{2(q_m+v)}\left(p_u-\delta-\frac{c}{q_m+v}\right)\right]$$
$$\Pi_u(p_s,p_u) = \tfrac{1}{2}\frac{p_s^2}{p_u^2}\left[(p_u-\delta)\frac{q_m}{q_m^2-v^2}-\frac{c(q_m^2+v^2)}{(q_m+v)^2(q_m-v)^2}\right] \tag{15}$$

Equation 14 can be written in the form $(p_s-c)(1-p_s/(q_m' p_u))$ with $q_m' = q_m - v^2/q_m$. In other words, the seller behaves as if the maximum use frequency is $q_m - v^2/q_m < q_m$. For example if $v^2 = 0.1q_m$ then the seller behaves as if the maximum frequency is $0.9q_m$. Similarly, equation 15 can be put in the form $\frac{1}{2q_m'}(p_u-\delta)p_s^2/p_u^2 - \frac{c'}{2q_m'^2}p_s^2/p_u^2$, where $q_m' = q_m - v^2/q_m$ and $c' = c(1+v^2/q_m^2)$. In other words, the service provider behaves as if she is facing a smaller maximum use frequency and a larger cost. Following the previous example, if there is a 10% uncertainty in the maximum use frequency (i.e, $v^2 = 0.1q_m$), the service provider will behave as if the maximum use frequency is $0.9q_m$ and the cost is $1.1c$.

The equilibrium $(p_s^*, p_u^*)$ satisfies the equations $\partial\Pi_s/\partial p_s = 0$ and $\partial\Pi_u/\partial p_u = 0$. The equation $\partial\Pi_s/\partial p_s = 0$ gives the seller's best response

$$p_s = \frac{1}{2}\left(c + \frac{p_u}{q_m}(q_m^2 - v^2)\right)$$

The service provider is constrained, as in the base case, by $p_u \leqslant a$. The solution to the maximization problem for the service provider gives

$$p_u^* = \begin{cases} 2\delta + \frac{2c(q_m^2+v^2)}{q_m(q_m^2-v^2)} & \text{for} \quad c \leqslant \frac{1}{2}q_m(a-2\delta)(q_m^2-v^2)/(q_m^2+v^2) \\ a & \text{for} \quad c > \frac{1}{2}q_m(a-2\delta)(q_m^2-v^2)/(q_m^2+v^2) \end{cases} \tag{16}$$

Finally, substituting the optimum price of the service provider to the seller's best response we find the optimum price for the seller

$$p_s^* = \begin{cases} \frac{1}{2q_m^2}(3cq_m^2 + 2cv^2 + 2\delta q_m(q_m^2-v^2)) & \text{for} \quad c \leqslant \frac{1}{2}q_m(a-2\delta)(q_m^2-v^2)/(q_m^2+v^2) \\ \frac{1}{2q_m}(cq_m + a(q_m^2-v^2)) & \text{for} \quad c > \frac{1}{2}q_m(a-2\delta)(q_m^2-v^2)/(q_m^2+v^2) \end{cases} \tag{17}$$

The optimum profits can be computed by substituting the optimum prices from equations 16 and 17 to the expressions for the profits 14 and 15.

### A.6. Overcapacity and queuing effects

We assume that customer's utility under the selling model is $U_s = qa - p_s$ where $a$ is the willingness to pay for a single use and $q$ is the desired usage. We also assume that customer's utility under the PPU model is $U_u = q(A(\rho) - p_u)$ where $A(\rho) = ae^{-k\rho/(1-\rho)}$ for $\rho \in [0,1]$. The parameter $\rho$ is a proxy for the service level and is a decision variable for the PPU model. $A(\rho)$ is the willingness to pay for a single use when the service level is $\rho$. When $\rho \to 0$, $A(\rho) \to a$ and when $\rho \to 1$, $A(\rho) \to 0$. Clearly $A(\rho) \leqslant a$. On the cost side we assume also that the cost per use depends on the service level and is given by $c_u(\rho) = c/(\rho q_m) + \delta$. Note that because $0 \leqslant \rho \leqslant 1$ we have that $c_u(\rho) \geqslant c/q_m + \delta$ i.e., the cost per use is greater than the strong pooling cost. The PPU provider chooses first the service level $\rho$ and subsequently the price $p_u$.

The indifferent customer $Q$ is at

$$Q = Q(p_s, p_u; \rho) = \frac{p_s}{a - A(\rho) + p_u}$$

and the profits for the seller $\Pi_s$ and the PPU provider $\Pi_u$ are respectively

$$\Pi_s = \Pi_s(p_s, p_u; \rho) = \frac{1}{q_m}(p_c - c)(q_m - Q)$$
$$\Pi_u = \Pi_u(p_s, p_u; \rho) = \frac{1}{2q_m}Q^2(p_u - c_u(\rho))$$

The seller's and the PPU provider's best response functions satisfy $\partial\Pi_s/\partial p_s = 0$ and $\partial\Pi_u/\partial p_u = 0$ which gives respectively

$$p_s = \tfrac{1}{2}\left(c + q_m(a - A(\rho) + p_u)\right)$$
$$p_u = a - A(\rho) + 2c_u(\rho)$$

The PPU provider is constrained to have $p_u \leqslant A(\rho)$ because if he chooses a price more than customer's willingness to pay he will get no customers and no profits. Therefore there are two cases to consider: case A where $p_u = A(\rho)$ which applies when $c_u(\rho) > A(\rho) - a/2$ and case B where $p_u = a - A(\rho) + 2c_u(\rho)$ which applies when $c_u(\rho) \leqslant A(\rho) - a/2$.

**Case A:** When $p_u = A(\rho)$ then $p_s = \frac{1}{2}(c + aq_m)$ and the indifferent customer is at $Q = \frac{1}{2a}(c + aq_m)$. Note that seller's price and the indifferent customer do not depend on $p_u$ or $\rho$; the seller behaves as in the monopoly case. The profits of the PPU provider are then given by

$$\Pi_u = \frac{1}{8q_m a^2}(c + aq_m)^2\,(A(\rho) - c_u(\rho))$$

The condition $\partial\Pi_u/\partial\rho = 0$ implies that $\partial A/\partial\rho - \partial c_u/\partial\rho = 0$, which in turns gives

$$\frac{k^2\rho^2}{(1-\rho)^2}e^{-k\rho/(1-\rho)} = \frac{kc}{aq_m}$$

Set $z = -k\rho/(1-\rho)$. Then the above equation becomes $z^2e^z = c/(aq_m)$ which has three possible solutions:

$$z_1 = 2\mathbb{W}_0\left(\tfrac{1}{2}\sqrt{kc/(aq_m)}\right) \text{ or } z_2 = 2\mathbb{W}_0\left(-\tfrac{1}{2}\sqrt{kc/(aq_m)}\right) \text{ or } z_3 = 2\mathbb{W}_{-1}\left(-\tfrac{1}{2}\sqrt{kc/(aq_m)}\right)$$

where $\mathbb{W}_j$ is jth branch of the Lambert function. Note that for the solutions to be a real number we must have $-1/e \leqslant -\frac{1}{2}\sqrt{kc/(aq_m)} \leqslant 0$ which in turn limits the parameter $kc$ to $0 \leqslant kc \leqslant 4aq_m/e^2 \approx 0.541aq_m$.

**Case B:** When $p_u = a - A(\rho) + 2c_u(\rho)$ then set $B(\rho) = a - A(\rho) + c_u(\rho)$. The seller's price is $p_s = \frac{1}{2}(c + 2q_mB(\rho))$ and the indifferent customer is at $Q = \frac{1}{4}(c + 2q_mB(\rho))/B(\rho)$. The profits of the PPU provider are then

$$\Pi_u = \frac{1}{32q_m}\frac{(c + 2q_mB(\rho))^2}{B(\rho)}$$

The derivative of PPU profits with respect to $\rho$ is then

$$\frac{\partial\Pi_u}{\partial\rho} = \frac{1}{32q_m}\frac{B'(\rho)(2q_mB(\rho) + c)(2q_mB(\rho) - c)}{B^2(\rho)}$$

Where $B'(\rho) = \partial B/\partial\rho$. Note that $B(\rho) > 0$ because $a > A(\rho)$ and $c_u(\rho) > 0$. Therefore, $2q_mB(\rho) + c > 0$. Similarly, $2q_mB(\rho) - c = 2q_m(a - A(\rho)) + \frac{2c}{\rho} - c > 0$ because $\rho \in [0,1]$. Hence for $\partial\Pi_u/\partial\rho = 0$, we must have $B'(\rho) = 0$ which gives us the same equation for $\rho$ as in case A above.

### A.7. Service and price discrimination

**A.7.1. Service Discrimination** The PPU provider can decide to service the patient customers only, leaving the impatient customers to the seller (Case A) or service the impatient customers only leaving the patient segment to the seller.

**Case A** When customers are patient the PPU provider can schedule service in a way that demand in non overlapping. In this case the cost per use is given by the strong pooling approximation as $c_u = c/q_m + \delta$. The PPU provider and the seller have to find prices $p_u$ and $p_s$ to maximize their profits. The profits of the seller and the PPU provider are given by

$$\Pi_u = \tfrac{1}{2q_m} w(p_u - c_u)(p_s/p_u)^2$$
$$\Pi_s = \tfrac{1}{q_m}(w(p_s - c)(q_m - p_s/p_u) + (1-w)(p_s - c)(q_m - p_s/a))$$

The condition $\partial \Pi_u / \partial p_u = 0$ gives $p_u = 2c_u$ for $c_u \leqslant a/2$ and $p_u = a$ for $c_u > a$. The condition $\partial \Pi_s / \partial p_s = 0$ gives

$$p_s = \frac{1}{2}\left(c + \frac{a q_m p_u}{wa + (1-w)p_u}\right)$$

**Case B** The PPU provider has to find a price $p_u$ and a service level $\rho$ and the seller has to find a price $p_s$ that will maximize their profits

$$\Pi_u = \tfrac{1}{2q_m}(1-w)(p_u - c_u(\rho))\frac{p_s^2}{(a - A(\rho) + p_u)^2}$$
$$\Pi_s = \tfrac{1}{q_m}\left(w(p_s - c)(q_m - p_s/a) + (1-w)(p_s - c)\left(q_m - \frac{p_s}{a - A(\rho) + p_u}\right)\right)$$

The problem for the PPU provider solves as in appendix (A.6). The optimum service level is $\rho = z/(z-k)$ where $z = 2\mathbb{W}_0\left(-\frac{1}{2}\sqrt{kc/(aq_m)}\right)$. The optimum price is $p_u = a - A(\rho) + 2c_u(\rho)$ for $c_u(\rho) \leqslant A(\rho) - a/2$ and $p_u = A(\rho)$ for $c_u(\rho) > A(\rho) - a/2$ where $A(\rho) = ae^{k\rho/(1-\rho)}$ and $c_u(\rho) = c/(\rho q_m) + \delta$. The condition $\partial \Pi_s / \partial p_s = 0$ gives

$$p_s = \frac{1}{2}\left(c + \frac{a q_m (a - A(\rho) + p_u)}{a - wA(\rho) + wp_u}\right)$$

**A.7.2. Price Discrimination** Assume that the PPU provider knows in which segment each customer belongs and decides to offer two prices $p_{u1}$ for the patient customers and $p_{u2}$ for the impatient ones. Note that for patient customers the PPU provider can schedule their demand and hence the cost per use is given by the strong pooling approximation $c_u = c/q_m + \delta$. For the impatient segment, the PPU provider has to build overcapacity $c_u(\rho)$ and decide on the service level $\rho$. The profits for the PPU provider and the seller are

$$\Pi_u = \tfrac{1}{2q_m}\left(w(p_{u1} - c_u)\frac{p_s^2}{p_{u1}^2} + (1-w)(p_{u2} - c_u(\rho))\frac{p_s^2}{(a - A(\rho) + p_{u2})^2}\right)$$
$$\Pi_s = \tfrac{1}{q_m}\left(w(p_s - c)(q_m - p_s/p_{u1}) + (1-w)(p_s - c)\left(q_m - \frac{p_s}{a - A(\rho) + p_{u2}}\right)\right)$$

It is straight forward to show that $p_{u1} = 2c_u$ for $c_u \leqslant a/2$ and $p_{u1} = a$ for $c_u > a/2$. The optimum $\rho$ is $\rho = z/(z-k)$ where $z = 2\mathbb{W}_0\left(-\frac{1}{2}\sqrt{kc/(aq_m)}\right)$. The optimum price for the impatient segment is $p_{u2} = a - A(\rho) + 2c_u(\rho)$ for $c_u(\rho) \leqslant A(\rho) - a/2$ and $p_{u2} = A(\rho)$ for $c_u(\rho) > A(\rho) - a/2$, where $A(\rho) = ae^{k\rho/(1-\rho)}$. Finally, the condition $\partial \Pi_s / \partial p_s = 0$ gives seller's price

$$p_s = \frac{1}{2}\left(c + \frac{q_m}{\frac{w}{p_{u1}} + \frac{1-w}{a - A(\rho) + p_{u2}}}\right)$$

### A.8. Proof of Lemma 1: The Generalized Lambert Function

Equations in the form $xe^x = \alpha$ are solved in terms of the Lambert $W$ function, and the solution is given as $x = W(\alpha)$. For more details on the properties of the Lambert W function see Corless et al. (1996). Equations of the form

$$(x-t)e^x = \beta(x-s) \tag{18}$$

are solved through the *generalized Lambert W function*. The solution of $(x-t)e^x = \beta(x-s)$ is $x = \mathcal{W}(\beta; t, s)$. For details and properties of the generalized Lambert W function see Siewert and Burniston (1974), Mező (2017), and Wright (1961). For solving the equation $(Ax+C)\ln x - Bx = a$ we let $y = \ln x$, or equivalently $x = e^y$. In terms of $y$ the previous equation becomes $(Ay-B)e^y + Cy = a$. We let $z = Ay - B$, or equivalently, $y = (z+B)/A$. In terms of $z$ the previous equation then becomes $ze^{z/A}e^{B/A} + \frac{C}{A}(z+B) = a$. Rearranging terms and multiplying both sides by $1/A$ we arrive at

$$\frac{z}{A}e^{z/A} + \frac{Ce^{-B/A}}{A}\frac{z}{A} = \frac{aA-BC}{A^2}e^{-B/A}$$

It can be readily shown that the above equation for $z/A$ is of the form of equation 18 with $t = 0$, $\beta = -Ce^{-B/A}/A$, and $s = (aA-BC)/(AC)$. The solution therefore for $z$ is

$$z = A\mathcal{W}\left(-\frac{C}{A}e^{-B/A}; 0; \frac{aA-BC}{AC}\right)$$

Carrying out the inverse transformations, the solution for $x$ is then

$$x = \exp\left[\frac{B}{A} + \mathcal{W}\left(-\frac{C}{A}e^{-B/A}; 0, \frac{aA-BC}{AC}\right)\right] \tag{19}$$