Resolving Tensions between Heterogeneous Investors in a Startup*

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Abstract

Legal scholars highlight the tensions that exist between different classes of shareholders in startups. We model a startup owned by undiversified investors with heterogeneous capital contributions and risk preferences. A social planner runs the firm on behalf of all investors. We compare investors' expected utility with a hypothetical first-best decentralized benchmark. The startup's optimal investment policy is pro-cyclical and a time-varying weighted average of shareholders' optimal investment policies. The optimal contracts issued to investors are tailor-made, interdependent, and include equity claims resembling preferred stock with heterogeneous payout caps, leading to a complex capitalization table as more investors join the startup.

Keywords: investment, payout, startup, risk preferences, group policy

JEL: G32, G34, G35

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1 Introduction

Theories of the firm have studied governance primarily in vertical, hierarchical terms focussing on conflicts between different classes of stakeholders (e.g. equityholders versus managers). Members within a stakeholder class are assumed to be homogeneous and having the same interests. Some legal scholars (e.g. Bartlett (2006), Pollman (2019)) are challenging this view and studying conflicts within a stakeholder group, with stockholders in startups being a focal point of attention.[1] Pollman (2019) claims that the horizontal conflicts in startups are between the holders of preferred and common stock, between preferred shareholders, and between common shareholders. She raises concerns that "recent case law requires startup directors to maximize value for common shareholders, without recognizing that in startups these shareholders do not have a monolithic set of interests and do not represent the firm value".[2] Corporate law scholars point out that the court's interpretation of fiduciary duty can give rise to inefficient outcomes. This raises the question how shareholders' divergent interests can be reconciled or aggregated in a way that makes the concept of fiduciary duty meaningful.

Shareholder tensions are significant within startups for several reasons. First, equity stakes in startups are illiquid. Prior to exit (e.g. through an IPO or takeover), walking away is not an option for unhappy shareholders. Second, investors in startups are usually undiversified, which amplifies the illiquidity issue. Third, startup valuations are

[1]Startups are an important source of economic growth and innovation. Apple, Alphabet, Microsoft, and Amazon all began as startups. In 2019, the aggregate amount of private equity capital invested in venture capital backed start-ups raising a Series C or higher round was $80 billion (see Ewens and Farre-Mensa (2022)).

[2]See e.g. the Trados Inc. shareholder litigation case following an acquisition in 2005 in which Trados' preferred stockholders received approximately $52 million, and its management received $8 million under an incentive plan that provided executives with a bonus to find and carry out the deal. Common stockholders received nothing. The question was whether, in negotiating and approving a sale that gave the common shareholders nothing, the directors breached their fiduciary duty. The court ruled that the directors owed a fiduciary duty to the common shareholders as the residual claimants.

subjective and dependent on shareholders' personal preferences.[3] In the absence of a stock market price that company directors can maximize, Fisher's separation theorem no longer applies. Consequently, it is no longer clear what the startup's objective function should be, leaving scope for legal disputes between shareholders. Similar tensions exist within family businesses which are often illiquid and owned by a few undiversified and heterogeneous individuals, whose livelihood may depend on the firm's income.[4]

This paper explores the following research questions. How can policy tensions in startups be resolved efficiently if its investors have different utility functions? What should be the objective function of the director(s) running the startup if investors rely on private valuations -and not a market price- that depend on personal preferences? How do tensions between different stockholders affect a startup's investment and payout decisions? Why do startups issue such a puzzling diversity of equity claims?

We consider a group of investors with heterogeneous risk preferences and capital endowments who invest in a startup.[5] We assume that investors are undiversified and wealth constrained.[6] We also assume that investors are unable privately to hedge the startup's risk by trading in securities or by creating homemade risk exposure. Although investors might try to hedge by trading in securities that are correlated with the price of the firm's output, the output price is only one determinant of the startup's future return. The commercial success of the startup and its innovation is subject to technological risk,

[3]TV programs such as Dragons Den (UK) or Shark Tank (US) illustrate that startup valuations differ significantly between entrepreneurs and venture capitalists (VCs), and even between VCs.

[4]Astrachan and Shanker (2003) estimate that about 60% of all partnerships and private corporations in the US can be deemed family businesses.

[5]The model applies to other forms of heterogeneity, such investors with different beliefs about the project's expected return, who agree to disagree regarding their beliefs (i.e. there is no learning).

[6]Many startups originate in the proverbial garage or dorm rooms, and initially operate on a shoestring, with founders putting most, if not all, of their money into the venture. Co-founders Larry Page and Sergey Brin started in their Stanford dorm rooms by building an internet search engine that they brought to market as Google. Jan Koum, co-founder of WhatsApp famously signed the agreement to sell WhatsApp to Facebook in front of the social services office where he once stood in line to collect food stamps. Venture capitalists are wealthier, but tend to be specialized in one industry to leverage their expertise.

regulatory risk, and new types of business risk that cannot be hedged (see our example of the independent energy supplier First Utility in Section 2). Therefore, each investor has a different implied private valuation for the startup that depends on her personal utility function. Consequently, it is not clear what the firm's overall objective function and its optimal policies should be. Heterogeneity also creates tensions as to how much the firm should invest in the risky project, how much to keep in cash, and how the returns should be shared among investors.

We present two solution approaches. We first derive each investor's first-best policy and expected utility by solving a decentralized problem. Next we present a central planner solution. We identify a scenario for which both solutions coincide. In the first part (the decentralized problem), we pretend "as if" each investor has access to the firm's technology and can use her endowment to run her own "division" within the firm according to her preferred investment and payout policies ("divisions" might represent different geographical regions, for example). Under suitable assumptions -investors having the same time-invariant, constant returns to scale technology- the startup's first-best investment (payout) level is then the sum of the investments (payouts) of all divisions. Using investors' first-best policies, we calculate their first-best expected utilities.

In the second part, we solve a social planner problem who acts on behalf of all investors and makes policy decisions at the firm level by maximizing a weighted average of investors' expected utility from the startup. Investors access the technology as part of the group, not individually. We initially take investors' utility weights as exogenously given and derive the firm's optimal policies as a function of those weights. Subsequently, we determine investors' utility weights as the solution to investors' participation constraints at inception. These constraints dictate that each investor's expected utility from joining the startup must equal at least her reservation utility from not joining, as determined by her bargaining power and outside investment opportunity.

We consider two polar cases for investors' outside oppportunity. In the first case, the least risk averse investor (the "entrepreneur") has exclusive access to the firm's technology (e.g. a patent or other type of intellectual property right). Co-investors' outside investment opportunity (e.g. the market portfolio) has a lower Sharpe ratio. Synergies can therefore be generated if investors join the startup. We show that the entrepreneur's bargaining power allows her to extract more synergy surplus, to increase her utility weight and equity stake in the startup, and to pursue a more aggressive investment policy.

Next, we consider another polar case where all investors are treated equally in terms of their access to the firm's technology, endowing each investor with a first-best reservation utility equal to the one derived under the decentralized problem. We show that there exists a unique set of utility weights for which the social planner solution provides each investor with her first-best expected utility. This set of utility weights applies to a special case for which there is no wealth redistribution (i.e. no book equity transfers) across investors at startup. The planner's optimal policies depend on the preferences of all investors. Therefore, Fisher's separation theorem no longer holds.

We derive the startup's optimal investment and payout policies within a continuous-time open-horizon model where investors have heterogeneous risk preferences. The continuous-time setting allows us to express the dynamics of the investment policy in a tractable, closed-form fashion. We show that the optimal dynamic policies for the firm are a time-varying weighted average of each individual investor's optimal policy. The time-varying weights (not to be confounded with the fixed utility weights) correspond to investors' book equity shares. As total book equity increases (decreases) in good (bad) times through retained earnings, the book equity share of the least (most) risk averse investor rises, causing the group's coefficient of risk aversion – defined in terms of the group's indirect utility function – to converge to the lowest (highest) coefficient of risk aversion within the group. Consequently, investment is procyclical. As the startup and the dollar value of its book equity grows, it depletes its cashholding and ultimately starts borrowing using a line of credit, building up a net debt position.

The social planner solution contractually fixes the utility weights at inception and specifies the optimal investment and payout policies as a function of the firm's total book equity. The firm's total book equity can be tracked through the company accounts as it is the result of investors' initial capital contributions and subsequent cumulative retained earnings. Unlike market values which are forward looking, book values are based on historical data that are observable and verifiable by investors. Therefore, book equity is a variable that the firm and investors can contract on, which enables investors in principle to commit to the optimal policies and to challenge contractual deviations in court.[7] Each investor receives a tailor-made contract that specifies her payout as a function of the firm's total book equity. The firm's policies are Pareto efficient in the sense that no improvement can be achieved at any point in time without hurting one of the investors.

Our model can accommodate both the case where payouts are spread over time, and the case where payouts are made all at once upon exit (e.g. at IPO). Investors that are relatively less risk averse receive contracts with higher upside potential but more downside risk. We find that the payout of the least (most) risk averse investor is a convex (concave) function of the firm's total book equity. For investors with intermediate risk aversion, payout is S-shaped in book equity (i.e. convex for low levels of book equity, and concave for high levels). Payouts converge to fixed caps as book equity increases, except for the least risk averse investor who features as a residual claimant. The claims of investors with higher levels of risk aversion resemble different classes of preferred equity with more risk averse investors having higher seniority but lower maximum payout. The diversity in equity contracts causes investors' book equity shares to be time-varying, and

[7] We say "in principle" as verifying and ensuring that the company director sticks to the optimal investment policy may be non-trivial in practice as it relies on our strong assumption that all investors have complete information.

the startup's governance and capital structure (or "cap table") to become increasingly more complicated as new investors join the firm. Congruent with our model, Pollman (2019) argues that going public is a way for mature startups to simplify a governance and cap table that have become unwieldy over time.[8]

Our model reconciles two competing hypotheses for group decisions. The group shift hypothesis (originally termed "risky shift", see Moscovici and Zavalloni (1969)) posits that groups exaggerate the initial positions of individual group members toward a more extreme position, leading to polarization. In contrast, an alternative hypothesis predicts that extreme preferences in a group are averaged out and teams eventually make less extreme decisions than individuals do. Our model reconciles both hypotheses as it shows that group choices are a time-varying weighted average of individuals' preferred decisions. Polarization is a limiting case in which (almost) all decision weight is put on one individual. Polarization occurs when the firm is performing very well or very poorly.

Our paper includes a number of extensions and robustness checks. First, we consider the scenario where investors do not receive intermediate payouts but cash out all at once through an IPO or takeover. Second, we consider the role of bankruptcy risk. Default does not arise in the baseline model because the startup disinvests and builds up cash reserves in response to adverse shocks. We extend the model by exposing the firm's risky asset base to an exogenous "crash" that destroys a fraction of the firm's risky assets and causes the startup to become insolvent. Third, we extend the model to allow for equity issues (i.e. new investors joining the startup). Finally, we discuss the scenario where the investor base is made up of subgroups (inside versus outside shareholders). Empirical implications, model limitations and challenges for future research are presented in the conclusions. The Internet Appendix includes further proofs and extensions, such as the

[8]Startups typically have different classes of preferred stock (Series A, Series B, ...) with different terms or valuation caps, in addition to common stock and other securities.

case where investors have CARA preferences (Internet Appendix D), and a discussion of the selection and matching of heterogeneous investors (Internet Appendix E).

Dynamic models of group decisions in corporate finance are rare.[9] A notable exception is Garlappi et al. (2017) who study a dynamic corporate investment problem by a group of agents who hold heterogeneous beliefs and adopt a utilitarian aggregation mechanism. Group decisions are dynamically inconsistent due to learning and this may lead to under-investment. Our policies are time-consistent because no learning takes place along the way. Garlappi et al. (2022) study a real option model where the decisions to invest in and abandon a project are made sequentially by a group of agents with heterogeneous beliefs who make decisions based on majority voting. Ebert et al. (2020) model a canonical real option investment problem under weighted discounting. Weighted discount functions may describe the discounting behavior of groups. Habib and Mella-Barral (2007) model the formation and duration of joint ventures when partners can acquire part of each other's knowhow. Antill and Grenadier (2019) develop a continuous-time dynamic bargaining model of corporate reorganization between equityholders and creditors. Some papers examine group decisions within a principal-agent setting (e.g. Grenadier et al. (2016) and Rivera (2020)). These papers do not consider payout. Furthermore, all agents have identical (risk neutral) risk preferences.[10] Investors do not vote nor trade in our model. Instead investors receive tailor-made contracts at startup, and a social planner runs the firm on behalf of all by adopting the optimal dynamic investment policy.

[9]Most models are static and focus on risk sharing between partners (see e.g. Wilson (1968), Eliashberg and Winkler (1981), Pratt and Zeckhauser (1989), Mazzocco (2004)). Hara et al. (2007) show within a static setting that heterogeneity in consumers' risk attitudes generates optimal sharing rules that are concave, convex, or initially convex and eventually concave, depending on investors' risk preferences.

[10]Some asset pricing papers study the effect of heterogeneous risk preferences on prices in a market equilibrium framework. Dumas (1989), Wang (1996), Cvitanić et al. (2012), Bhamra and Uppal (2014), and Chabakauri (2015) consider a pure exchange economy where agents trade their share in the exogenous, aggregate endowment of the consumption good and have access to a risk-free security. Liu et al. (2021) examine heterogeneity in preferences in a general-equilibrium production economy. Biais et al. (2021) study risk sharing in a dynamic exchange economy in the presence of incentive problems.

Our paper also relates to continuous-time papers in corporate finance that jointly model a firm's financial policies, such as Gryglewicz (2011), Bolton et al. (2011), Décamps et al. (2017), Hugonnier and Morellec (2017), Lambrecht and Myers (2017), Hartman-Glaser et al. (2020), Bolton et al. (2021), DeMarzo and He (2021), Benzoni et al. (2022), Geelen et al. (2022), among others (see Strebulaev and Whited (2012) for a review of the earlier literature). Some papers focus on entrepreneurial (rather than corporate) finance. Chen et al. (2010) study the effects of nondiversifiable risk on entrepreneurial finance by building on Leland (1994). Wang et al. (2012) develop an incomplete markets q-theoretic model to study entrepreneurship dynamics. Sorensen et al. (2014) analyze the portfolio-choice problem of a risk-averse Limited Partner (LP). Whereas these papers focus on the decisions of a single agent, and take contracts as exogenously given, our paper considers a group of investors, and endogenizes their optimal claims.

Finally, our model provides a new theory for the diversity of securities in firms (and startups in particular) that is not based on information asymmetry or "vertical" agency problems. Startups and VC contracts have proven to be rich hunting ground for contract theorists (see Metrick and Yasuda (2011), Da Rin et al. (2013) for literature reviews). Relatively few papers have gone beyond the vertical relationship between the entrepreneur and VC. Hellman and Wasserman (2017) examine the trade-off between efficiency and equality within the context of the division of founder equity in new ventures. The supporting proprietary survey data shows that entrepreneurial teams use a variety of equity agreements. Ewens et al. (2022) study how contracting terms influence a startup's probability of success and sharing of value between entrepreneurs and investors. Finally, Kagan et al. (2020) is an experimental study on equity contracts and incentive design in startup teams. They highlight within-team interaction. Their results suggest that personal characteristics are the primitive, and the contract form is the derived consequence, which is consistent with the approach adopted in our paper.

2 The Model Setup and Assumptions

Our model is applicable to a variety of businesses, but to make matters concrete we use the example of an independent energy supplier (see e.g. the case of First Utility).[11] The firm's business model is to buy energy (gas and electricity) in the wholesale market to sell it to retail customers at a profit. Delivery of the energy to customers happens through an existing energy grid. The firm derives its competitive edge by introducing innovative technology and better customer service (see First Utility case in footnote 11). The firm's size can be scaled up by expanding geographically.

Our model considers a startup that has n investors with an initial capital contribution W_{i0} (for $i = 1, ..., n$) at $t = 0$. All n investors have a power utility function. Investor i's utility function is given by $u_i(c_{it}) = \frac{c_{it}^{1-\gamma_i}}{1-\gamma_i}$ where $\gamma_i > 0$ for $i = 1, ..., n$.[12] All investors have a subjective discount rate ρ. Investors may have a different coefficient of risk aversion. Without loss of generality, we assume $0 < \gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$. Investors are undiversified and invest their entire endowment in the startup.

The startup invests in a constant-return-to-scale production technology whose return per unit of capital (before any payouts) has a constant gaussian distribution with fixed parameters μ and σ. The return shocks include technological or project risks that cannot be hedged. The startup can save (borrow) through a cash account (line of credit) at the risk free rate r.[13] L_t denotes the startup's net cashholding (a negative value for L_t

[11] The UK independent energy company First Utility was set up by two entrepreneurs in 2008 who announced a partnership with Google to bring the company's energy software tool, PowerMeter, to the UK. Google PowerMeter allows users to see a near real-time display of their energy usage through an iGoogle account on their computer or mobile phone. At the time, First Utility was the only energy supplier in the UK to provide free smart meters to its customers, to install smart thermostats and to lower the switch time to 2.5 weeks. By October 2009 First Utility had 30,000 customers, and by 2018 this had grown to 700,000 customers. In 2018 Shell acquired First Utility for around 240 million, with the two co-founders receiving about 48 million each, and the finance chief around 17 million, with the remainder going to friends, family and other investors.(Source: This is Money (24 December 2017))

[12] In Internet Appendix D we present the results for investors with exponential utility $u_i(c_{it}) = -exp(-\eta_i c_{it})/\eta_i$.

[13] We show that the firm never defaults under the firm's optimal policies. We consider bankruptcy and

indicates a net debt position), and A_t denotes the capital invested in the risky project (which includes, for example, the amount of energy purchased in the wholesale market). Investors' initial capital contribution constitutes the firm's initial total equity, some of which is allocated to the risky project (A_0) and the remainder to cash (L_0), i.e. $W_0 = \sum_{i=1}^{n} W_{i0} = A_0 + L_0$.

The firm achieves a Sharpe ratio $\xi = \frac{\mu - r}{\sigma}$, which reflects its risk adjusted performance from buying and selling energy, net of all trading and customer servicing costs. The firm achieves the profit margin and excess return through its superior customer service. Although the production inputs and startup's assets (e.g. gas and electricity) are traded, the startup's superior technology (a key source of its excess returns and risk) can only be accessed by joining the firm.

At each instant t after inception, investors receive payouts c_{it} with $c_t \equiv \sum_{i=1}^{n} c_{it}$, and decide on the amount A_t to invest in risky capital given the firm's book equity W_t. The firm's book equity can be tracked through the company accounts as it is the result of investors' initial capital contributions (W_0) and subsequent retained earnings. The book equity process (net of payouts) is given by:

$$dW_t = dA_t + dL_t - c_t dt = [(\mu - r) A_t + rW_t - c_t] dt + \sigma A_t dB_t \tag{1}$$

where B_t is a Brownian motion that reflects the earlier mentioned project risks. Define $\omega_t \equiv \frac{A_t}{W_t}$ as the fraction of the firm's book equity invested in risky assets. The process for W_t is now[14]

$$dW_t = [(\omega_t(\mu - r) + r) W_t - c_t] dt + \sigma \omega_t W_t dB_t \tag{2}$$

risky debt in Section 6.2 by introducing negative Poisson jumps to the firm's return on assets.

[14]A standard way to incorporate corporate taxes (τ_c) into the model is to assume that $\mu = \mu'(1 - \tau_c)$ and $\sigma = \sigma'(1 - \tau_c)$, where μ' and σ' are the pre-tax drift and volatility. This approach assumes that firms fully enjoy tax-loss offsets. In reality there may be restrictions on tax loss carry-forwards.

All investors have complete information (all model parameters, including investors' risk preferences, are common knowledge). The firm's total book equity W_t is observable and verifiable through the company accounts, allowing the implementation of contracts written on the state variable W_t.

At each instant t the startup sets its investment (ω_t) and payout (c_t) policies, and determines how this payout is distributed across investors (i.e. c_{it} for $i = 1, ..., n$). The problem is non-trivial because heterogeneous investors have different preferences regarding these policies.

In what follows we first solve a decentralized problem where the firm is hypothetically split into n "divisions" (e.g. linked to different geographical regions) that are each run by a single investor. Each division is funded with the investor's initial capital endowment W_{i0} and has access to the firm's technology with Sharpe ratio ξ. Thanks to our simplifying assumptions (such as constant returns to scale) the firm is the sum of its divisions, and the firm's aggregate investment (ω_t) and payout (c_t) policies are then determined by the quantities $A_t = \sum_{i=1}^n A_{it}$, $W_t = \sum_{i=1}^n W_{it}$, and $c_t = \sum_{i=1}^n c_{it}$. We call this the first-best benchmark as each investor can, under this ideal scenario, implement her preferred investment and payout policies.

Subsequently, we consider the more realistic, centralized problem, where a single agent (e.g. a company director) acts as a social planner and solves directly for the firm's overall investment policy ω_t and the payouts c_{it} to investors. The centralized policies are expressed as functions of the firm's observable total book equity W_t (not W_{it}), allowing them to be written in a contract. We explore whether a central planner can replicate the first-best expected utility that investors achieve under the decentralized scenario. We also examine the effect of investors' outside reservation utility and bargaining power on the optimal policies.

3 First-best Benchmark: A Decentralized Problem

In an ideal world each investor i ($i = 1, ..., n$) would have access to the firm's technology and be able to run a division with her initial endowment W_{i0}. Each investor would individually decide how much to invest in risky assets (A_{i0}) and cash (L_{i0}), with $W_{i0} = A_{i0} + L_{i0}$. Define $\omega^d_{it} \equiv \frac{A_{it}}{W_{it}}$ and c^d_{it} as investor i's preferred investment and payout policies, respectively. The process for investor i's equity, W_{it}, is now:

$$dW_{it} \;=\; \left[\left(\omega^d_{it}(\mu \,-\, r) \,+\, r\right) W_{it} \,-\, c^d_{it}\right] dt \,+\, \sigma\, \omega^d_{it}\, W_{it}\, dB_t \tag{3}$$

Each investor maximizes the expected life-time utility from her payouts with respect to the control variables ω^d_{it} and c^d_{it}. The maximum life-time utility $I_i(W_{it}|\xi)$ for investor i from the division is given by the familiar Merton (1969) solution:

$$\begin{aligned}
I_i(W_{it}|\xi) &= \max_{\{c^d_{it};\omega^d_{it}\}} E_t\left[\int_t^\infty e^{-\rho(s-t)} u_i(c^d_{is}) ds\right] = a_i(\xi)^{\gamma_i} \frac{W_{it}^{1-\gamma_i}}{1-\gamma_i} && (4)\\
\omega^d_{it}(W_{it}|\xi) &= \frac{\xi}{\sigma\gamma_i} \quad \text{and} \quad c^d_{it}(W_{it}|\xi) = \frac{W_{it}}{a_i(\xi)} \quad \text{where} && (5)\\
a_i(\xi_i) &= \frac{\gamma_i^2}{r\gamma_i^2 \,+\, (\rho + \frac{\xi^2}{2} \,-\, r)\gamma_i \,-\, \frac{\xi^2}{2}} \qquad \text{for } i = 1, ..., n && (6)
\end{aligned}$$

Aggregating investors' investment and payout across divisions gives the firm's first-best policies as stated in the below corollary.[15]

Corollary 1 *The group's first-best payout and investment policies for the decentralized problem are given by:*

$$c^d_t(W_t|\xi) \;=\; \sum_{i=1}^n c^d_{it}(W_{it}|\xi) \;=\; \sum_{i=1}^n \frac{W_{it}}{a_i(\xi)} \tag{7}$$

[15] The proofs of all the corollaries are in Internet Appendix A.

$$\omega_t^d(W_t|\xi) \;=\; \frac{\sum_{i=1}^n A_{it}}{W_t} \;=\; \sum_{i=1}^n \frac{W_{it}}{W_t}\omega_{it}^d(\xi) \;=\; \sum_{i=1}^n \frac{W_{it}}{W_t}\frac{\xi}{\sigma\gamma_i} \tag{8}$$

where $W_t = \sum_{i=1}^n W_{it}$. *The group's first-best aggregate life-time utility is given by* $I^d(W_t) \equiv \sum_{i=1}^n I_i(W_{it}|\xi)$. *Let* W_{it}^d *denote investor i's book equity under her first-best investment and payout policies. The evolution of* W_{it}^d *is given by:*

$$\frac{dW_{it}^d}{W_{it}^d} \;=\; \left[\frac{\xi^2}{\gamma_i} + r - \frac{1}{a_i(\xi)}\right] dt \;+ \frac{\xi}{\gamma_i} dB_t \tag{9}$$

Equations (7) and (8) show that investor i's optimal investment and payout policies both depend on her coefficient of risk aversion γ_i.[16] This makes it non-trivial for heterogeneous investors to agree on a common policy. The evolution of an investor's book equity depends on her coefficient of risk aversion, which makes investors' relative size within the firm time-varying. Since each investor's book equity stake follows a geometric Brownian motion (given by Equation (9)) under her optimal policies, it is easy to show that less risk averse investors on average grow faster than their more risk averse counterparts.

The group's joint investment and payout decisions are obtained as the sum of individual investors' preferred investment and payout decisions (i.e. $A_t = \sum_{i=1}^n A_{it}$ and $c_t = \sum_{i=1}^n c_{it}^d$).[17] This allows us to identify the group's first best policies, and investors' first-best life-time utility under these policies.

Although our decentralized scenario is useful from a conceptual viewpoint, it may be

[16] Investor i's preferred policies and process for her personal book equity stake W_{it}^d do not depend on co-investors' actions. Matters become more complicated if that is not the case. If, for example, the returns on the risky asset are subject to decreasing returns to scale and given by $(\mu - \phi\,(A_t/W_t)^{\nu-1})dt + \sigma dB_t$ then ω_{it}^d and c_{it}^d become a function of $\omega_t \equiv A_t/W_t = (\sum_{i=1}^n A_{it})/(\sum_{i=1}^n W_{it})$. This means that investor i's optimal investment policy depends on the investment policies of the other agents. In that case, we need to solve for a Nash equilibrium in which each investor's policies are optimal given the investment policies of the other $n-1$ agents.

[17] In our model, the group determines the amounts of investment and payout. This differentiates our model from those where the group votes whether or not an investment should be made (see e.g. Garlappi et al. (2022) for a model of this type).

impractical. Leaving aside the issue whether investors can run their own division within the firm, the first-best policies (7) and (8) require that one can observe and track each investor's book equity stake W_{it}. This may be impossible because many transactions take place centrally within the firm and allocating or transferring cashflows across divisions is not a clear-cut matter. Furthermore, company accounts are produced at a firm (not division) level. Given that investors' personal book equity stake W_{it} is not observable in practice, it is not possible to link contracts, and investors' ownership claims in particular, to W_{it}. In reality, contracts are written between investors and the firm (not the divisions), and linked to total book equity (W_t). In what follows we consider the more realistic scenario where investment and payout policies are set centrally. Investors' first-best life-time utility derived under the decentralized problem serves as a benchmark.

4 Group Policies: A Social Planner Solution

We now consider a central planner (e.g. a company director) who maximizes a social welfare function that encompasses the utility of all investors. We subsequently show a link between the social planner problem and the decentralized problem.

We adopt a standard approach in which the central planner maximizes a weighted average of investors' life-time utilities, and derives the group's optimal policies ($c^*_{1t}, ..., c^*_{nt}$ and ω^*_t) as the solution to the following optimization problem.

$$J(W_t) = \max_{\{c_{1t},...,c_{nt};\ \omega_t\}} E_t \left[\sum_{i=1}^{n} \lambda_i \int_t^{\infty} e^{-\rho(s-t)} u_i(c_{is})\, ds \right] = \sum_{i=1}^{n} \lambda_i J_i(W_t) \qquad (10)$$

subject to the transversality condition $\lim_{t\to\infty} [e^{-\rho t} J(W_t)] = 0$ and the budget constraint (2), for given λ_i. $J_i(W_t)$ denotes investor i's life-time utility from being part of a firm with n investors and total book equity W_t.

Objective function (10) is known as the weighted-sum-of-utilities welfare function, a generalization of the classical utilitarian welfare function. Investors derive utility from payouts c_{it} only (i.e. they do not sell their claim to generate income). In Section 6.1 we consider the other polar case where investors do not receive any intermediate payouts and derive utility exclusively from selling their claim upon exit (e.g. an IPO or takeover). The results remain in essence the same.

It is well known (see Duffie (2001), Chapter 10) that a necessary and sufficient condition for $(c_{1t}, ..., c_{nt})$ to be a Pareto efficient allocation is that there exist non-negative weights $(\lambda_1, \lambda_2, ..., \lambda_n)$ that solve the above optimization problem. We verify in Section 5.1 that such positive weights indeed exist and can be found as the solution to investors' participation constraints. Proposition 1 gives the solution to optimization problem (10).

Proposition 1 *The group's optimal investment (ω_t^*) and payout (c_{it}^*) policies for any positive utility weights λ_i are:*

$$\omega_t^* = \left(\frac{A_t}{W_t}\right)^* = \frac{\xi}{\sigma}\frac{W'(z_t)}{W(z_t)} = \sum_{i=1}^{n} \phi_i(z_t)\frac{\xi}{\sigma\gamma_i} \tag{11}$$

$$c_{it}^* = \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} = \phi_i(z_t)\frac{W_t}{a_i(\xi)} \qquad \text{for} \quad i = 1, ..., n \tag{12}$$

where the auxiliary state variable $z_t \equiv -ln(J'(W_t))$ is the solution to

$$W(z_t) = \sum_{i=1}^{n} a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} \equiv \sum_{i=1}^{n} \tilde{W}_{it}, \text{ and where}: \tag{13}$$

$$\phi_i(z_t) = \frac{a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}}}{\sum_{j=1}^{n} a_j(\xi)\lambda_j^{\frac{1}{\gamma_j}} e^{\frac{z_t}{\gamma_j}}} = \frac{\tilde{W}_{it}}{W_t} \quad \text{and} \quad \sum_{i=1}^{n} \phi_i(z_t) = 1 \tag{14}$$

$\tilde{W}_{it}$ and $\phi_i(z_t)$ denote investor i's book equity stake and book equity share, respectively.

$a_i(\xi)$ is defined in Equation (6)*. The group's joint weighted life-time utility, $J(W_t)$, is:*

$$J(W(z_t)) = \sum_{i=1}^{n} \lambda_i \, J_i(z_t) \; = \; \sum_{i=1}^{n} \lambda_i \left[\frac{a_i(\xi)}{1 \, - \, \gamma_i} (\lambda_i e^{z_t})^{\frac{1}{\gamma_i} - 1} \right] \tag{15}$$

The group policies are expressed as a function of an auxiliary variable z_t that can be interpreted as some kind of "normalized" book equity value and that allows solutions to be expressed in closed form. Total book equity is a continuous, monotonically increasing function of z as expressed by Equation (13). As z_t ranges from $-\infty$ to $+\infty$, W_t varies from 0 to $+\infty$, i.e. $\lim_{z_t \to -\infty} W(z_t) = 0$ and $\lim_{z_t \to +\infty} W(z_t) = +\infty$. The firm never defaults because zero equity is never reached.

4.1 Investment policy

The optimal investment policy ω_t^* of a group of investors is a weighted average of the optimal investment policies $\omega_i^d(\xi)$ of the individual investors. The weights are given by investors' book equity shares $\phi_i(z_t)$. In contrast to the utility weights λ_i, which are fixed at startup, the book equity shares ϕ_{it} are time-varying as they depend on the firm's total book equity W_t through the auxiliary variable z_t.

Corollary 2 *The startup's risky asset to book equity ratio, ω_t^*, increases in its book equity level, i.e. $\frac{\partial \omega_t^*}{\partial W_t} \geq 0$*

The corollary implies that the firm invests more aggressively in risky assets in good times (when its book equity has grown), and reduces its weight in risky assets during bad times. Consider the n-investor case where investor 1 (n) is least (most) risk averse (i.e. $\gamma_1 \leq \gamma_2 \leq ... \leq \gamma_n$). In that case,

$$\lim_{z_t \to +\infty} \omega^*(z_t) \; = \; \frac{\xi}{\sigma \gamma_1} \quad \text{and} \quad \lim_{z_t \to -\infty} \omega^*(z_t) \; = \; \frac{\xi}{\sigma \gamma_n} \tag{16}$$

As the firm accumulates (loses) more book equity, its investment policy converges towards the one of the least (most) risk averse investor. $\omega^*(z_t)$ is an S-shaped function situated between the asymptotes $\frac{\xi}{\sigma\gamma_1}$ and $\frac{\xi}{\sigma\gamma_n}$. $\omega^*(z_t)$ is constant for the knife-edge case of homogenous risk preferences ($\gamma_i = \gamma$ for all i), and given by the standard Merton (1969) ratio $\omega = \xi/(\sigma\gamma)$.

The startup's optimal cash-holding is given by $L_t = (1 - \omega_t^*)W_t$. If the cashholding is negative ($L_t < 0$) then the firm has a net debt position and uses its line of credit. The firm's net cash to book equity ratio ($L_t/W_t = 1 - \omega_t^*$) is countercyclical and decreases in W_t. For low levels of book equity (e.g. at startup) and assuming some investors are sufficiently risk averse, the firm initially keeps a net cash balance and only invests a fraction of its book equity in risky assets (i.e. $\omega_t^* < 1$). As the firm accumulates (depletes) retained earnings and increases (decreases) its book equity, the optimal cash to book equity ratio converges towards the one of the least (most) risk averse investor, which involves net borrowing (cash saving) if some investors have a sufficiently low (high) coefficient of risk aversion. Dynamic rebalancing in response to shocks keeps the firm solvent through economic downturns. In Section 6.2 we introduce negative jumps to the firm's asset returns that are sufficiently large to make the firm insolvent and bankrupt.

4.2 Payout policy

The payout (in dollars) to each investor is increasing but non-linear in the firm's book equity if investors have different coefficients of risk aversion. Since $c_{it}^* = \phi_{it} W_t / a_i(\xi)$ (see Equation (12)), the non-linearity is driven by the time-varying book equity share ϕ_{it}.

Proposition 2 *Payout is a convex (concave) increasing function of the firm's total book equity W_t for the least (most) risk averse investor. For investors with an intermediate level of risk aversion, there exists a critical book equity level W_i^* ($i = 2, 3, ..., n - 1$) such* *that payout is convex for low levels of book equity (i.e.* $W_t \leq W_i^*$*) and concave for high levels of book equity (i.e.* $W_t \geq W_i^*$*). Furthermore,* $\lim_{z_t \to -\infty} c_{it}^* = 0$ *for* $i = 1, ..., n$*, and*

$$\lim_{z_t \to +\infty} \frac{\partial c_{1t}^*}{\partial W_t} = \frac{1}{a_1(\xi)} \quad \textit{and} \quad \lim_{z_t \to +\infty} \frac{\partial c_{jt}^*}{\partial W_t} = 0 \quad \textit{for} \quad j = 2, 3, ..., n$$

It is Pareto optimal for the least (most) risk averse agent to have a convex (concave) payout function. Since all payouts are zero for $W_t = 0$, it follows that, among all investors, the most risk averse investor is paid the most for very low levels of book equity. The least risk averse investor is the residual claimant who gets the upside potential in good times, but is heavily exposed in bad times. All other co-investors are less exposed in downturns, but their payouts are capped as the firm grows and accumulates book equity. The diversity in payouts reconciles group members' heterogeneous risk preferences, and allows for efficient risk sharing.

It follows from Proposition 1 that the firm's payout yield, c_t^*/W_t, is time-varying, except if all investors have the same coefficient of risk aversion γ:

$$\frac{c_t^*}{W_t} = \sum_{i=1}^{n} \frac{c_{it}^*}{W_t} = \sum_{i=1}^{n} \frac{\phi_{it}}{a_i(\xi)} \tag{17}$$

4.3 Group risk aversion

Corollary 3 *The group's level of relative risk aversion (RRA_G) is endogenous and can be defined in terms of the value function as:*

$$RRA_G \equiv -W_t \frac{J_{WW}}{J_W} = \frac{W(z_t)}{W'(z_t)} = \frac{\xi}{\sigma\, \omega_t^*} \tag{18}$$

Using our comparative statics for ω_t^* (see Section 4.1), it follows that:

$$\frac{\partial RRA_G}{\partial W_t} \leq 0 \quad \text{and} \quad \lim_{z_t \to +\infty} RRA_G = \gamma_1 \quad \text{and} \quad \lim_{z_t \to -\infty} RRA_G = \gamma_n \tag{19}$$

Although each individual investor has a constant level of relative risk aversion, the group's level of relative risk aversion decreases in the startup's book equity, and converges to the level of the least (most) risk averse investor as book equity goes to infinity (zero). This explains the earlier described behavior of the firm's investment policy. More generally, it may also help explain why groups are very bullish during booms and extremely risk averse during recessions (see Shupp and Williams (2008)). Finally, $RRA_G = \gamma$ if all investors have the same coefficient of risk aversion γ.

5 Utility Weights, Synergies and Equity Stakes

5.1 Utility weights

Investors' utility weights λ_i are fixed at startup ($t = 0$) and depend on investors' outside investment opportunity and their bargaining power. The Sharpe ratio of investors' outside opportunity is denoted by $\beta(\leq \xi)$. We assume that the outside option and the startup are mutually exclusive. The welfare gains created by investors joining the startup are $J(W_0|\xi) - \sum_{i=1}^{n} I_i(W_{i0}|\beta)$. These gains or "synergies" are distributed at $t = 0$ by setting investors' initial book equity stakes, $\tilde{W}_{i0}$, which in turn fix investors' utility weights λ_i. Investor i's reservation utility is given by $I_i(W_{i0}|\beta)$, whereas the maximum life-time utility she could achieve with endowment W_{i0} and technology ξ equals $I_i(W_{i0}|\xi)$. Since

$I_i(W_{i0}|\beta) \leq I_i(W_{i0}|\xi)$ for $\beta \leq \xi$, there exists a reservation equity stake $\underline{W}_{i0}$ such that:

$$I_i(W_{i0}|\beta) \;=\; I_i(\underline{W}_{i0}|\xi) \;\; with \;\; \underline{W}_{i0} \;=\; W_{i0} \left(\frac{a_i(\beta)}{a_i(\xi)}\right)^{\frac{\gamma_i}{1-\gamma_i}} \leq W_{i0} \;\; for \; \beta \leq \xi \tag{20}$$

$W_{i0} - \underline{W}_{i0}$ represents the maximum amount of book equity that investor i would be willing to sacrifice in order to join the startup. The total "surplus" to be shared across all investors can then be written as:

$$\Delta \equiv \sum_{i=1}^{n} (W_{i0} - \underline{W}_{i0}) = \sum_{i=1}^{n} W_{i0} \left(1 - \left(\frac{a_i(\beta)}{a_i(\xi)}\right)^{\frac{\gamma_i}{1-\gamma_i}}\right) \tag{21}$$

The social planner can replicate all possible Pareto efficient allocations. Each allocation corresponds to a different sharing of the surplus Δ, which in turn maps into a sharing of the synergies (welfare gains). Let s_i denote agent i's share of the surplus (with $\sum_{i=1}^{n} s_i = 1$). s_i is exogenously given and reflects investor i's bargaining power. Investor i's initial equity stake in the startup $\tilde{W}_{i0}$ can then be written as:

$$\tilde{W}_{i0} \;=\; \underline{W}_{i0} + s_i \Delta \;\; with \; 0 \leq s_i \leq 1 \;\; and \;\; \sum_{i=1}^{n} s_i = 1 \; for \; i = 1, ..., n \tag{22}$$

Since $\sum_{i=1}^{n} \tilde{W}_{i0} = \sum_{i=1}^{n} W_{i0}$, the sharing rule s_i at startup does not change the firm's total book equity, but it redistributes equity across investors (unless $\tilde{W}_{i0} = W_{i0}$ for all i).

We now solve the social planner problem for a couple of interesting surplus sharing rules. Consider first the polar case in which an entrepreneur (labelled as lead investor 1) controls exclusive access to the firm's technology. The entrepreneur is able to run the technology on her own (using her endowment W_{10}), and can make take-it-or-leave-it offers to co-investors 2 to n who provide additional capital on a competitive basis. Therefore, the entrepreneur extracts the total synergy surplus, i.e. $s_1 = 1$ and $s_i = 0$ for $i = 2, ..., n$. In the absence of bargaining power, co-investors 2 to n are just left with their reservation utility $I_i(W_{i0}|\beta)$. The corresponding utility weights are the solution to the social planner problem (10) subject to the participation constraints $J_i(W_0) = I_i(\tilde{W}_{i0}|\xi) = I_i(\underline{W}_{i0}|\xi) = I_i(W_{i0}|\beta)$ for $i = 2, ..., n$ and the total equity constraint $\sum_{i=1}^{n} W_{i0} = W_0$.

Proposition 3 *If an entrepreneur (lead investor 1) controls exclusive access to the startup with Sharpe ratio ξ and can make take-it-or-leave-it offers to co-investors with reservation utility $I_i(W_{i0}|\beta)$ (for $i = 2, ..., n$) then investors' utility weights are given by:*

$$\lambda_1 = \left[\frac{\tilde{W}_{10}}{a_1(\xi)}\right]^{\gamma_1} e^{-z_0} = \left[\frac{W_{10} + \Delta}{a_1(\xi)}\right]^{\gamma_1} e^{-z_0} \tag{23}$$

$$\lambda_i = \left[\frac{\tilde{W}_{i0}}{a_i(\xi)}\right]^{\gamma_i} e^{-z_0} = \left[\frac{\underline{W}_{i0}}{a_i(\xi)}\right]^{\gamma_i} e^{-z_0} = W_{i0}^{\gamma_i}\left[\frac{a_i(\beta)^{\gamma_i}}{a_i(\xi)}\right]^{\frac{\gamma_i}{1-\gamma_i}} e^{-z_0} \quad \textit{for } i = 2, ..., n \tag{24}$$

The factor $e^{-z_0}(= J'(W_0))$ implies that the utility weights are uniquely defined in relative terms, not in absolute terms. Investors' utility weights are non-linear in the amounts of capital W_{i0} they invest at startup. Whereas co-investors who contribute no capital get zero utility weight, the entrepreneur receives a positive utility weight λ_1 even if she contributes no capital (i.e. $W_{10} = 0$). The entrepreneur's "excess" weight is a compensation for her superior Sharpe ratio (which results in the surplus Δ) in combination with her strong bargaining power ($s_1 = 1$).

Consider next the case where upon joining the startup all investors receive a book equity stake equal to their initial capital contribution ($\tilde{W}_{i0} = W_{i0}$ for $i = 1, ..., n$), i.e. there is no redistribution. From Equations (20) and (22) it follows that this scenario only happens if $\beta = \xi$, i.e. the outside opportunity and the startup have the same Sharpe ratio and therefore $\Delta = 0$. Solving social planner (10) subject to the participation constraints $J_i(W_0) = I_i(\tilde{W}_{i0}|\xi) = I_i(W_{i0}|\xi)$ leads to the following proposition.

Proposition 4 *If all investors have equal access to the startup's investment opportunity* *(i.e. $\beta = \xi$ for all investors) then there is no redistribution of book equity at startup (i.e. $\tilde{W}_{i0} = W_{i0}$) and investors' utility weights are given by:*

$$\lambda_i = \frac{J'(W(z_0))}{u'(c_{i0}^*)} = \left(\frac{W_{i0}}{a_i(\xi)}\right)^{\gamma_i} e^{-z_0} \quad \textit{such that} \quad \frac{u_i'(c_{i0}^*)}{\frac{dI(W_{i0}|\xi)}{dW_{i0}}} = 1 \quad \textit{for } i = 1, ..., n \tag{25}$$

Investors' life-time utility, book equity stake, and payout from the group are identical to the first-best outcome presented in Corollary 1 for the decentralized problem, i.e. $J_i(W_0) = I_i(W_{i0}|\xi)$, $\tilde{W}_{it} = W_{it}^d$, and $c_{it}^*(W_t) = c_{it}^d(W_{it}^d|\xi)$ for all t.

From Equation (25) it follows that the weighting normalizes the utility function by the investor's shadow price of wealth so that each investor's marginal utility of consumption for the rescaled utility equals 1. As such, our endogenous utility weights are similar to the Negishi (1960) welfare weights, which have the unique feature of preserving the initial wealth distribution.[18]

Proposition 4 shows that there exists positive utility weights for which the central planner solution coincides with the first-best outcome for the decentralized problem. These utility weights apply to a special case for which there is no redistribution across investors at startup. Typically first-best is not achieved (see Table 1 in Section 5.3 for illustrative numerical examples).

[18] Of the social welfare functions consistent with Pareto optimality only weighted combinations of individual utilities with Negishi weights replicate market outcomes for given initial resource allocations. The Negishi (1960) weights are equal to the inverse of the marginal utility of wealth for each individual evaluated at the maximizing (equilibrium) resource allocation.

5.2 Investors' book equity stakes

Let us consider next $\tilde{W}_{it}$, investor i's book equity stake in the startup, as defined by Equation (13) in Proposition 1. Recall that the utility weights λ_i are fixed at $t = 0$.

$$\tilde{W}_{it} \;=\; a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} \;=\; a_i(\xi)c_{it}^* \;=\; \phi_{it}\, W_t \;\text{ with }\; \sum_{i=1}^{n} \tilde{W}_{it} = W_t \tag{26}$$

The next corollary describes an individual investor's book equity stake. Since its value is a constant multiple of payout (i.e. $\tilde{W}_{it} = a_i(\xi)c_{it}^*$) the corollary mirrors the results we obtained in Proposition 2 for investors' payout policies c_{it}^*.

Corollary 4 *The book equity stake of the most (least) risk averse investor is a concave (convex) function of the firm's total book equity W_t. The book equity stake of investors with intermediate levels of risk aversion is S-shaped in the firm's book equity. Moreover,* $\lim_{z_t \to -\infty} \tilde{W}_{it} = 0$ *for $i = 1, ..., n$, and*

$$\lim_{z_t \to +\infty} \frac{\partial \tilde{W}_{1t}}{\partial W_t} = 1 \quad and \quad \lim_{z_t \to +\infty} \frac{\partial \tilde{W}_{it}}{\partial W_t} = 0 \quad for \quad i = 2, ..., n \tag{27}$$

$$\lim_{z_t \to -\infty} \frac{\partial \tilde{W}_{nt}}{\partial W_t} = 1 \quad and \quad \lim_{z_t \to -\infty} \frac{\partial \tilde{W}_{it}}{\partial W_t} = 0 \quad for \quad i = 1, ..., n-1 \tag{28}$$

Finally, $\exists\, W_i^* \in \mathbb{R}^+$ *such that* $\frac{\partial^2 \tilde{W}_{it}}{\partial W_t^2} \geq (\leq)\, 0 \;\Leftrightarrow\; W_t \leq (\geq)\, W_i^*$ *for $i = 2, ..., n-1$.*

As the startup grows, an additional dollar of book equity is almost entirely to the benefit of the least risk averse investor (i.e $\lim_{z_t \to +\infty} \frac{\partial \tilde{W}_{1t}}{\partial W_t} = 1$), whereas the other investors' book equity stakes converge to a fixed value. Conversely, for a startup with little or no book equity, the marginal dollar accrues almost entirely to the most risk averse investor (i.e $\lim_{z_t \to -\infty} \frac{\partial \tilde{W}_{nt}}{\partial W_t} = 1$).

The book equity stake of the least risk averse investor ($i = 1$) is strictly convex in total book equity and unbounded. It resembles the textbook call option shape of common equity.[19] The stakes of investors 2 to $n-1$ are convex (concave) for low (high) values of book equity, with each book equity stake converging towards a maximum value and resembling a preferred equity contract.[20] To put it simply, preferred equity resembles debt when the total amount of equity capital is high, and it resembles an equity contract when the amount of equity is low. This explains why preferred stock values can be convex and concave for, respectively, low and high values of total equity W_t.

The claim of the most risk averse investor ($i = n$) is strictly concave in W_t, and corresponds to the most senior preferred stock contract. For the lowest levels of W_t, an extra dollar for the firm accrues almost entirely to the most risk averse investor. As the firm accumulates more book equity, more junior preferred stock holders start sharing in the payouts. The claims of these other investors resemble preferred stock contracts for which the level of seniority increases in the degree of risk aversion, and for which the face value of the contract (achieved as $W_t \to +\infty$) decreases in the degree of risk aversion. The maximum value of each preferred stock contract is determined by the maximum payout rate c_{it} as $W_t \to +\infty$.

The variation in the shapes of the claim values is driven by differences in investors' coefficient of relative risk aversion. If the coefficient equals 1 for all investors, and the utility weights sum to one, then each investor's book equity stake is linear in the firm's total book equity and proportional to her utility weight λ_i (i.e. $\tilde{W}_{it} = \lambda_i W_t$). In this case, the utility weights equal constant equity shares, i.e. $\lambda_i = \phi_i$.

[19] More generally, whether payout to common equity is a strictly convex function depends on the firm's capital structure. With only debt and common equity (the classic textbook case) it is a convex function. However, with convertible securities (common in venture capital) it is concave for high firm values because in good times convertible preferred shares are converted to common shares, diluting existing common shareholders. We thank a referee for pointing this out.

[20] In practice, preferred stock contracts are senior to common stock in that dividends may not be paid to common stock, unless the dividend is paid on all preferred stock. The dividend rate on preferred stock is usually fixed at the time of issue. Preferred stock is junior to any debt claim. Unlike debt contracts, the firm is not in default when it suspends dividends to preferred stockholders.

5.3 Book equity stakes, book equity shares, and utility weights: A numerical example

Figure 1 numerically evaluates the investors' book equity stakes ($\tilde{W}_{it}$) and book equity shares ($\phi_{it} \equiv \tilde{W}_{it}/W_t$). The firm has 4 investors with coefficients of relative risk aversion $\gamma_1 = 0.8, \gamma_2 = 2, \gamma_3 = 3, \gamma_4 = 9$, and with initial endowments $W_{10} = W_{20} = W_{30} = W_{40} = 25$.[21] We assume lead investor 1 (the entrepreneur) can make take-it-or-leave-it offers to the co-investors (i.e. $s_1 = 1$; $s_2 = s_3 = s_4 = 0$) who each have reservation utility $I_i(W_{i0}|\beta)$ for $i = 2, 3$, and 4.

Panel A of Figure 1 plots investors' book equity stakes ($\tilde{W}_{it}$) as a function of the firm's total book equity. In bad times, more risk averse investors enjoy a higher equity stake at the expense of less risk averse investors. In good times, the least risk averse investor enjoys the upside, whereas all other agents' equity stake is capped. The cap decreases with investors' coefficient of risk aversion. The book equity stake of the least (most) risk averse investor is strictly convex (concave). For the other investors, the book equity stake is initially convex up to some inflection point, and concave thereafter, resembling preferred stock. Our model-generated claim values resemble those produced by existing valuation models. E.g. the preferred stock valuation model by Emanuel (1983) produces claim values for cumulative preferred stock and non-cumulative preferred stock that are graphically very similar to ours (see Figures 3 and 4 in Emanuel (1983)), even though his valuation formulas are very different.

[21] Risk preferences vary widely across the population (see Barsky et al. (1997)). The most commonly accepted measures of the coefficient of relative risk aversion lie between 1 and 3, but there is a wide range of estimates in the literature, from as low as 0.2 to 10 and higher. We set $r = 0.03$ in line with current practice in the US. We set the subjective discount rate $\rho = 0.1$, which is within the range commonly used in the corporate finance literature (for example, Bolton et al. (2011) and Lambrecht and Myers (2017)). We set $\mu = 0.6$ and $\sigma = 0.89$, in line with the estimates of the annualized volatility of venture capital investment returns, which reflects investment returns on startups (Korteweg and Sorensen (2010), Gornall and Strebulaev (2020)). The corresponding Sharpe ratio of the entrepreneur's investment opportunity, ξ, is 0.64. The Sharpe ratio of co-investors 2 to 4's outside opportunity is $\beta = 0.9\xi = 0.576$.

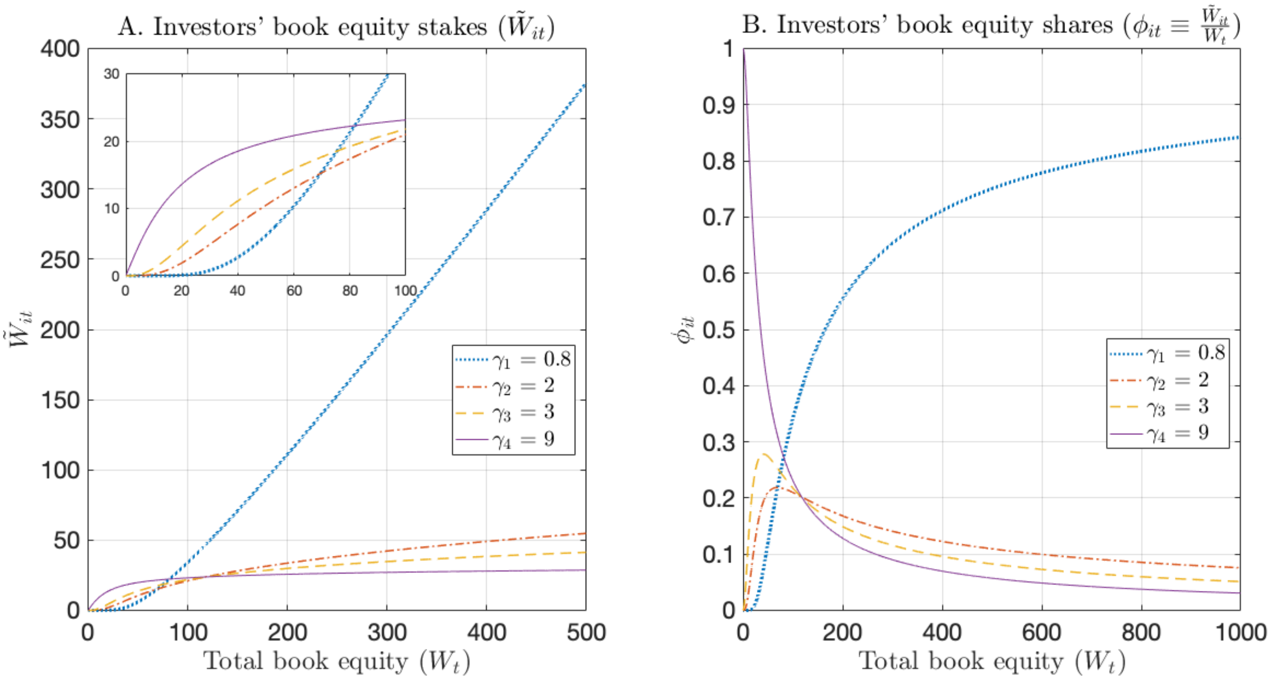

Figure 1: Investors' equity stakes and equity shares

This figure considers a startup with 4 investors with initial endowments, $W_{10} = W_{20} = W_{30} = W_{40} = 25$ and heterogeneous risk aversion, $\gamma_1 = 0.8, \gamma_2 = 2, \gamma_3 = 3$, and $\gamma_4 = 9$. Lead investor 1 has all bargaining power. Panels A and B plot, respectively, investors' book equity stakes ($\tilde{W}_{it}$), and book equity shares ($\phi_{it} \equiv \frac{\tilde{W}_{it}}{W_t}$) as functions of the firm's total book equity W_t. The small window inside Panel A zooms in on the individual book equity stakes when W_t is relatively small. The parameter values are: $r = 0.03$, $\mu = 0.6$, $\sigma = 0.89$, $\rho = 0.1$, $\xi = 0.64$, $\beta = 0.9\xi = 0.576$.

Panel B plots investors' book equity shares ϕ_{it} ($\equiv \tilde{W}_{it}/W_t$). As total book equity rises, less risk averse investors' share in the firm increases and, accordingly, also their influence over the firm's investment policy. This relation can be non-monotonic as illustrated by the book equity shares of investors 2 and 3, which first increase and then decrease in the firm's book equity. This highlights the complicated nature of group behavior.

We conclude this section by providing a numerical example that calculates investors' utility weights, book equity stakes, and life-time utilities. Table 1 considers the same firm as in Figure 1 with the same 4 investors and parameter values (see also footnote 21).

The table considers two polar cases for the Sharpe ratio of co-investors' outside investment opportunity: $\beta = 0$ (Panel A) and $\beta = \xi$ (Panel B). Panel A.1 considers the case where lead investor 1 has all bargaining power ($s_1 = 1$). In Panel A.2 all four investors

Table 1: Investors' utility weights, book equity stakes, and life-time utilities

			Panel A. $\beta = 0$									Panel B. $\beta = \xi$			
Investors			$I_i(\psi)$	A.1. Lead investor has all bargaining power				A.2. Equal bargaining power				$I_i(\psi)$	No synergy created		
i	γ_i	W_{i0}		s_i	λ_i	$\tilde{W}_{i0}$	J_i	s_i	λ_i	$\tilde{W}_{i0}$	J_i		λ_i	$\tilde{W}_{i0}$	J_i
1	0.8	25	99.19	1	0.56	66.86	120.76	0.25	0.03	35.47	106.37	99.19	0.02	25	99.19
2	2	25	-9.47	0	0.17	7.81	-9.47	0.25	0.09	18.28	-4.05	-2.96	0.16	25	-2.96
3	3	25	-5.27	0	0.19	9.90	-5.27	0.25	0.15	20.36	-1.25	-0.83	0.28	25	-0.83
4	9	25	-5.23	0	0.08	15.42	-5.23	0.25	0.73	25.89	-0.08	-0.11	0.54	25	-0.11
sum		100	79.22	1	1	100	100.79	1	1	100	101.00	95.29	1	100	95.29

The table presents investors' utility weights (λ_i), book equity stakes ($\tilde{W}_{i0}$), reservation utilities ($I_i(\psi)$) and life-time utilities (J_i) for different values of β and s_i. β denotes the Sharpe ratio of the co-investors' alternative investment opportunity. $\psi = \xi$ for investor 1 and $\psi = \beta$ for all other investors. s_i represents investor i's bargaining power. Each investor has the same initial capital contribution (W_{i0}) but a different coefficient of relative risk aversion (γ_i). The parameter values are: $r = 0.03, \rho = 0.1$, $\mu = 0.6$, $\sigma = 0.89$ and $\xi = 0.64$.

have equal bargaining power ($s_1 = s_2 = s_3 = s_4 = 0.25$). The following patterns leap out from the table. First, when $\beta < \xi$, co-investors' initial equity stake in the startup, $\tilde{W}_{i0}$ ($i = 2, 3, 4$), is strictly increasing in risk aversion. Less risk averse co-investors have a stronger preference for risky assets or projects than their more risk averse peers, and are therefore willing to sacrifice more of their endowment to join the startup. In return for a capital contribution of 25, co-investor 2 gets an initial equity stake in the firm equal to $\tilde{W}_{20} = 7.81$ and $\tilde{W}_{20} = 18.28$ for $s_2 = 0$ and $s_2 = 0.25$, respectively. In contrast, the corresponding equity stakes are higher and equal to 15.42 and 25.89 for co-investor 4.

Second, increasing the lead investor's bargaining power s_1 from 0.25 to 1 increases her utility weight from 0.03 to 0.56. A higher utility weight means that the lead investor (who might be the CEO or company director) gains more influence in the decision making and adopts a more aggressive investment policy, which increases her life-time utility from 106.37 to 120.76. As to be expected, the life-time utility of each co-investor falls. E.g. the life-time utility of the most risk averse co-investor (i.e. investor 4) drops from -0.08 to -5.23.

Panel B sets $\beta = \xi$, which means that no surplus is generated. Therefore investors' bargaining power becomes irrelevant and only their reservation utility matters. As to be expected there is no redistribution and each investor's book equity stake $\tilde{W}_{i0}$ equals 25. The utility weights are increasing in the coefficient of risk aversion, ranging from $\lambda_1 = 0.02$ to $\lambda_4 = 0.54$. This monotonicity result is not a general property. E.g. in Panel A.1 ($\beta = 0$ and $s_1 = 1$), co-investors' utility weights are an inverted U-shaped function of risk aversion. One can numerically show that, all else equal, for sufficiently low (high) levels of initial capital contribution, co-investors' utility weights decline (increase) in investors' coefficient of risk aversion. The inverted U-shape arises for medium levels of initial capital contribution by investors.

Increasing co-investors' Sharpe ratio from $\beta = 0$ to $\beta = \xi$ increases (decreases) co-investors' (the lead investor's) life-time utility. Having a better outside option benefits co-investors, but hurts the lead investor. Comparing the first-best case in Panel B with the case where the lead investor has all bargaining power in Panel A.1, we see that the lead investor's life-time utility increases from 99.19 to 120.76, whereas the utility of co-investor 2, 3 and 4 drops, respectively, from -2.96, -0.83 and -0.11 in the first-best case to -9.47, -5.27 and -5.23 for the case where the lead investor has all the bargaining power.

6 Extensions

6.1 Exit through IPO, takeover or liquidation

We now extend our base model in two ways. First, we introduce a future exit opportunity that enables investors to cash out their stake in the firm through an IPO, takeover, or liquidation. Second, we assume that investors get no payout prior to exit. The exit happens at some random future time τ (e.g. the advent of a "hot" IPO market, takeover bid, or liquidation trigger), which arrives according to a Poisson process with intensity π. Upon exit, each investor i sells her entire equity stake $\tilde{W}_{i\tau}$ $(i = 1, ..., n)$ for a cash amount $\theta\tilde{W}_{i\tau}$. The market to book value at exit, θ, is exogenously given and reflects a market or takeover premium ($\theta > 1$) or a liquidation discount ($\theta < 1$). The social planner maximizes a weighted average of investors' expected utility upon exit. We show that all propositions (except for Proposition 4) and all corollaries remain in essence the same (see Internet Appendix B for all solutions and proofs). We therefore only present a few key results here in the main text.

Consider first the single investor problem. In the absence of intermediate payouts the intertemporal budget constraint is:

$$dW_{it} = \left(\omega_{it}^d(\mu - r) + r\right) W_{it}\, dt + \sigma\, \omega_{it}^d\, W_{it}\, dB_t \quad for\ t < \tau \tag{29}$$

Investor i's maximum expected utility from running her own division until exit, $I_i(W_{it}|\xi)$, and her optimal investment policy ($\omega_{it}^d(W_{it})$) are given by:

$$\begin{aligned} I_i(W_{it}|\xi) &= \max_{\omega_{it}^d} E_t\left[e^{-\rho(\tau-t)}u_i(\theta W_{i\tau})\right] = \tilde{a}_i(\xi)^{\gamma_i}\frac{W_{it}^{1-\gamma_i}}{1-\gamma_i} \\ \omega_{it}^d(W_{it}|\xi) &= \frac{\xi}{\sigma\gamma_i} \quad \text{for} \;\; t < \tau, \text{ where } \; \tilde{a}_i(\xi) = \left[\frac{\pi\gamma_i\theta^{1-\gamma_i}}{r\gamma_i^2 + (\rho + \pi + \frac{\xi^2}{2} - r)\gamma_i - \frac{\xi^2}{2}}\right]^{\frac{1}{\gamma_i}} \end{aligned}$$

The solution is similar to the Merton (1969) solution (see Equations (4)-(6)), except that we do not have intertemporal payout and, as a result, the expression for the constant $\tilde{a}_i(\xi)$ is different from the earlier constant $a_i(\xi)$ (see Equation (6)).

The budget constraint for the group in the absence of intermediate payouts is:

$$dW_t = (\omega_t(\mu - r) + r)\, W_t\, dt + \sigma\, \omega_t\, W_t\, dB_t \quad for\ t < \tau \tag{30}$$

The social planner's maximization problem for given utility weights λ_i is now:

$$J(W_t) = \max_{\omega_t} E_t \left[e^{-\rho(\tau - t)} \sum_{i=1}^{n} \lambda_i u_i(\theta \tilde{W}_{i\tau}) \right] = \sum_{i=1}^{n} \lambda_i J_i(W_t) \tag{31}$$

subject to the transversality condition $\lim_{t\to\infty} [e^{-\rho t} J(W_t)] = 0$ and the budget constraint (30).

The startup's optimal investment policy (ω_t^*), payouts to investors upon exit ($\theta \tilde{W}_{i\tau}$), and the group's expected utility ($J(W)$) are now:

$$\omega_t^* = \frac{\xi}{\sigma} \frac{W'(z_t)}{W(z_t)} = \sum_{i=1}^{n} \frac{\tilde{W}_{it}}{W_t} \frac{\xi}{\sigma \gamma_i} \quad \text{for } t < \tau \text{ and } \theta \tilde{W}_{i\tau}(z_\tau) = \theta \tilde{a}_i(\xi) \, \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_\tau}{\gamma_i}} \quad \text{for } i = 1, ..., n$$

$$J(W(z_t)) = \sum_{i=1}^{n} \lambda_i \, J_i(z_t) = \sum_{i=1}^{n} \lambda_i \left[\frac{\tilde{a}_i(\xi)}{1 - \gamma_i} (\lambda_i e^{z_t})^{\frac{1}{\gamma_i} - 1} \right] \text{ where } W(z_t) = \sum_{i=1}^{n} \tilde{a}_i(\xi) \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}}$$

The solution is the same as in Proposition 1, except for the absence of intertemporal payout and the different expression for the constant $\tilde{a}_i(\xi)$. Without an opportunity to exit (i.e. $\pi = 0$, and hence $\tilde{a}_i = 0$), the startup is worthless because there are no payouts.

The payouts upon exit ($\theta \tilde{W}_{i\tau}$) as a function of W_τ have similar shapes as the payout c_{it}^* (see Proposition 2) and the book equity stakes $\tilde{W}_{it}$ (Corollary 4) in our baseline model with intermediate consumption. Payout to the least (most) risk averse investor is convex (concave) in the firm's total book equity value at exit, while the payouts for all the other investors are S-shaped in W_τ (see Internet Appendix B for proofs). Our non-linear payoffs approximate the piece-wise linear payoff structures we observe in practice.[22]

The above results are valid for any combination of positive utility weights. The par-

[22] Our non-linear payoffs share similarities with some real-life piece-wise linear payoffs. For example, the payoff to investors 1 and 4 resembles a long call with payoff $(W_\tau - K_1)^+$, and cash and a short put with payoff $K_4 - (K_4 - W_\tau)^+$, respectively. The S-shaped payoff for investors $i = 2$ and 3 could be approximated by a call spread with payoff $(W_\tau - K_{il})^+ - (W_\tau - K_{ih})^+$, where K_{il} and K_{ih} are strikes such that $K_{il} < K_{ih}$. We perform a more precise approximization following Carr and Madan (1998) in Internet Appendix B, in which the non-linear payoff can be decomposed into a portfolio of cash, equity, put and call options on the equity value of the firm. We thank a referee for pointing this out.

ticipation constraints pin down the utility weights. A meaningful comparison between joining the startup and investors' outside investment opportunity only exists if the latter neither allows for intertemporal consumption and is expected to have the same termination date as the startup. Furthermore, any market to book value parameter θ' and Sharpe ratio β for the outside investment must be such that investors prefer to join the startup, i.e. $I_i(W_{i0}|\beta, \theta', \pi) \leq I_i(W_{i0}|\xi, \theta, \pi)$. A sufficient condition for this to be the case is that $\beta \leq \xi$ and $\theta' \leq \theta$. One can show that under these assumptions our model with exit is a straightforward extension of our baseline model and all previous results hold (see Internet Appendix B for further details and proofs), with the exception of Proposition 4 which reports results specifically for the case where all investors have equal access to the firm's technology, i.e. $\beta = \xi$.

If we assume that $\beta = \xi$ and $\theta' = \theta$ then solving the social planner's problem (31) subject to the participation constraints $J(W_0) = I_i(W_{i0}|\xi)$ for $i = 1, .., n$ gives similar utility weights as before (compare with Equation (25)).

$$\lambda_i = \left(\frac{W_{i0}}{\tilde{a}_i(\xi)} \right)^{\gamma_i} e^{-z_0} \quad \text{for} \quad i = 1, .., n \tag{32}$$

Whereas previously $(W_{i0}/a_i(\xi))^{-\gamma_i} = u'(c^*_{i0}) = \frac{dI(W_{i0}(\xi))}{dW_{i0}}$ (see Equation (25)), this equality no longer applies here as there is no consumption at $t = 0$. More importantly, although at inception investors still achieve their first-best expected utility $I(W_{i0}|\xi)$, ex post they may be better or worse off because we can no longer claim that $\tilde{W}_{it} = W^d_{it}$ for all $0 < t \leq \tau$. One can show that without intermediate payouts W^d_{it} follows a geometric Brownian motion, whereas $\tilde{W}_{it}$ does not. In contrast for our baseline model with intermediate payouts both W^d_{it} and $\tilde{W}_{it}$ follow the same geometric Brownian motion given by Equation (9) if $\beta = \xi$. With intermediate payouts, the firm has n additional control variables (c_{it}) that can be used to smooth individual investors' intertemporal consumption in response to shocks. The intermediate payouts ensure that the evolution of each investor's marginal utility of wealth is independent of her risk preferences and the same across all investors. One can show that with intermediate payouts:

$$\frac{du'(W_{it}^d)}{u'(W_{it}^d)} = \frac{d(W_{it}^d)^{-\gamma}}{(W_{it}^d)^{-\gamma}} = (\rho - r)dt - \xi dB_t \quad for\ i = 1, .., n \tag{33}$$

Therefore, all investors experience the same relative changes in marginal utility over the firm's lifetime. In the absence of tailormade intermediate payouts, the evolution of investors' marginal utility of wealth differs across investors and depends on each investor's risk preference, such that ex post investors may receive a different payout upon exit compared to what they would have got under the first-best scenario.

6.2 Bankruptcy

Insolvency does not arise in our model thus far because the firm is able continuously to rebalance to its optimal asset to equity ratio in response to adverse economic shocks. We now introduce the possibility of bankruptcy by exposing the firm's risky asset base to an exogenous negative jump or "crash". Assume in this subsection that the return on the firm's risky assets is given by the jump-diffusion process $(\mu + \pi\kappa f)dt + \sigma dB_t - f dy_t$, where y_t is a pure Poisson jump process with intensity π and $E(dy_t) = \pi dt$. When a crash occurs the startup's risky asset base A_t drops by a fraction f $(0 \leq f \leq 1)$. The expected return on the risky assets is given by $(\mu + \pi f(\kappa - 1))dt$. We assume that $\kappa > 1$ and that $\pi f(\kappa - 1)$ captures an exogenous risk premium associated with the crash risk. For further details we refer to Lambrecht and Tse (2023), who adopt the same process.

Define the firm's net debt level as $D_t \equiv -L_t = W_t\,(\omega_t - 1)$. Lender's recovery rate in default following a crash is given by $A_t(1-f)/D_t = \omega_t(1-f)/(\omega_t - 1)$. Assuming that debt is priced competitively and lenders are risk neutral then the cost of the startup's debt, $R(\omega_t)$, is given by:

$$R(\omega_t) \;=\; r + \pi\left[1 - \frac{\omega_t(1-f)}{\omega_t - 1}\right] \;=\; r \,+\, \pi\left(\frac{f\omega_t - 1}{\omega_t - 1}\right) \tag{34}$$

The debt is risky under the firm's optimal policy if $f\omega_t > 1$. A sufficient condition for this to be the case at all times is $\gamma_n < \frac{\hat{\xi} f}{\sigma}$, where $\hat{\xi}$ is defined in Equation (37). The firm's intertemporal budget constraint with risky debt is:

$$dW_t \;=\; \left[\left((\mu + \pi\kappa f - R(\omega_t))\omega_t + R(\omega_t)\right) W_t \;-\; c_t\right] dt \;+\; \sigma\omega_t dB_t \quad for \quad t < \tau \tag{35}$$

where τ denotes the random arrival time of the crash. The startup's optimal policies with risky debt ($c^*_{1t}, ..., c^*_{nt}$ and ω^*_t) can be obtained as the solution to the following social planner problem.

$$J(W_t) = \max_{\{c_{1t},\ldots,c_{nt};\; \omega_t\}} E_t\left[\sum_{i=1}^{n} \lambda_i \int_t^{\tau} e^{-\rho(s-t)} u_i(c_{is})\, ds\right] = \sum_{i=1}^{n} \lambda_i J_i(W_t) \tag{36}$$

subject to the transversality condition $\lim_{t\to\infty}\left[e^{-\rho t} J(W_t)\right] \;=\; 0$, and the budget constraint (35). We also impose the condition $\gamma_n \;\leq\; \min[1, \frac{\hat{\xi} f}{\sigma}]$, which is sufficient to ensure that investors wish to adopt risky debt and $J(0) = 0$ for tractability. We omit the analysis on utility weights λ_i on this occasion. As discussed previously, they can be found as the solution to investors' participation constraints. Solving the social planner problem gives the following proposition.[23]

Proposition 5 *If the returns on the startup's assets are subject to a negative Poisson jump ("crash") large enough to cause insolvency, then the startup's optimal investment (ω^*_t) and payout (c^*_{it}) policies for any positive utility weights λ_i are as given in Proposition*

[23]The proof for result related to this extension (i.e. Section 6.2) is in Internet Appendix C.

1 provided that ξ *and* $a_i(\xi)$ *are, respectively, replaced by* $\hat{\xi}$ *and* $\hat{a}_i(\hat{\xi})$ *for all* i*, where:*

$$\hat{\xi} = \frac{\mu - r + (\kappa - 1)\pi f}{\sigma} \quad \textit{and} \tag{37}$$

$$\hat{a}_i(\hat{\xi}) = \frac{\gamma_i^2}{(r+\pi)\gamma_i^2 + (\rho + \frac{\hat{\xi}^2}{2} - r)\gamma_i - \frac{\hat{\xi}^2}{2}} \tag{38}$$

Crash risk affects the firm's investment and payout policies through the constants $\hat{\xi}$ and $\hat{a}_i(\hat{\xi})$, which determine the Sharpe ratio and investor's marginal propensity to consume. The effect of crash risk on the firm's policies depends on the interplay between the crash risk premium κ and the crash risk intensity π.

6.3 Equity issues

We now consider equity issues by m new investors who join the firm at time τ and each contribute $W_{i\tau}$ in capital (for $i = n+1, ..., n+m$), bringing the firm's total equity after the equity issue, W_τ^+, to $W_\tau^+ = W_\tau + \sum_{i=n+1}^{n+m} W_{i\tau}$. In what follows z^+ and λ_i^+, respectively, denote the auxiliary variable and utility weights after the equity issue.

Our analysis is similar to the one for group synergies in Section 5.1. Investors' participation constraint pins down their utility weights. E.g. if m new investors with reservation utility $I_i(W_{i\tau}|\beta)$ (for $i = n+1, ..., n+m$) supply capital competitively then their utility weights are determined by the participation constraints for $i = n+1, ..., n+m$:

$$J_i(W_\tau^+ | n + m\, investors) = I_i(W_{i\tau}|\beta) \Longleftrightarrow \lambda_i^+ = W_{i\tau}^{\gamma_i} \left(\frac{a_i(\beta)^{\gamma_i}}{a_i(\xi)} \right)^{\frac{\gamma_i}{1-\gamma_i}} e^{-z_\tau^+}$$

The surplus transfer from the new investors to the existing investors equals:

$$\Delta^+ \equiv \sum_{i=n+1}^{n+m} (W_{i\tau} - \underline{W}_{i\tau}) = \sum_{i=n+1}^{n+m} \left(1 - \left(\frac{a_i(\beta)}{a_i(\xi)} \right)^{\frac{\gamma_i}{1-\gamma_i}} \right) W_{i\tau} > 0 \tag{39}$$

If $(s_1^+, ..., s_n^+)$ is a sharing rule that distributes the surplus across all n existing investors, then their corresponding equity stakes and utility weights after the equity issue are:

$$\tilde{W}_{i\tau}^+ \equiv \tilde{W}_{i\tau} + s_i^+ \Delta^+ \quad with \quad \sum_{i=1}^{n} s_i^+ = 1 \quad for \quad i = 1, ..., n \tag{40}$$

$$\lambda_i^+ = \left(\frac{\tilde{W}_{i\tau}^+}{a_i(\xi)} \right)^{\gamma_i} e^{-z_\tau^+} \quad for \ i = 1, ..., n \tag{41}$$

The firm's optimal policies after the equity issue are as described in Proposition 1 but for the enlarged set of $n + m$ investors and with the new utility weights λ_i^+.

6.4 Subgroups

The model is sufficiently flexible to accommodate subgroups. Consider, for example, a firm with a majority group of n inside shareholders, and a minority group of outside shareholders. Assume that the latter are contractually entitled to a fraction φ of the firm's total payout, whereas the former controls and runs the firm and issues tailor-made contracts to insiders. Insiders' objective function attaches zero utility weight to outsiders' utility. Outsiders' payouts feature in the budget constraint, however. The solution method to this problem is as described in our main model, except that the firm's total payout in the budget constraint is now given by $(\sum_{i=1}^n c_{it})/(1 - \varphi)$ (and not $c_t \equiv \sum_{i=1}^n c_{it}$).

7 Empirical Implications and Conclusions

Our model predicts that startups are initially all-equity financed holding a net cash balance. A line of credit is gradually introduced to finance growth as retained earnings accumulate and the firm's book equity grows. Startups have different types of equity. Our model generates a capitalization table that includes different classes of preferred equity and a residual claim resembling common equity. The model predicts that the firm's claims are tailored to investors' risk preferences, with less (more) risk averse investors having claims that tend to be relatively more convex (concave) in the firm's book equity.[24]

There are, to our knowledge, no empirical studies on small, young startups. The existing literature focuses primarily on more mature venture capital backed startups. Cumming (2006) reports that VCs in all (non-US) countries around the world have consistently reported the use of a variety of securities, including common equity, preferred equity, convertible preferred equity, debt, convertible debt, and combinations. Kaplan and Strömberg (2003) find that convertible preferred equity is the most popular with VCs in the US, but Cumming (2006) finds this not to be the case in Canada where, for example, 76.9% of startups and 85.7% of high-tech startups use straight preferred equity for private independent limited partnership VC funds. Cumming argues that the US tax code creates a bias in favor of convertible preferred equity.

Our paper shows that the diversity in claims resolves (rather than causes) tensions between investors with heterogeneous preferences. An investor's optimal contract not only depends on her own level of risk aversion, but also on the risk preferences of the other investors. Consequently, cap tables can become unwieldy as more investors join the startup, which might explain why companies like Spotify and Slack go public even when private capital is available. Exit through an IPO may provide an opportunity for mature startups to simplify the firm's governance and capital structure.[25] These findings generate new empirical hypotheses.

Our model model may cast some light on findings in the empirical literature on group

[24]Crowdequity platforms such as SeedInvest illustrate the diversity of securities issued by startups and the wide choice of securities available to suit investors' personal preferences.

[25]However, the separation between ownership and control in public corporations may introduce a new complexity, i.e. a diverse set of managerial compensation schemes tailored to managers' preferences.

decisions, which tests two competing hypotheses for group behavior: the group shift hypothesis and the diversification of opinions hypothesis. Although the latter hypothesis appears to enjoy strong empirical support (see Bär et al. (2011)), it struggles to explain the puzzling observation that the average group is more (less) risk averse than the average individual in high (low) risk situations (Shupp and Williams (2008)). Our model provides a possible explanation by showing that group behavior is not merely a weighted average of individuals' preferred decisions but, crucially, the weights vary endogenously over time and correspond to investors' time-varying equity shares. More weight shifts towards the least risk averse agent as the firm grows during booms, whereas in busts increased weight shifts towards the most risk averse agent. The group's risk aversion is determined primarily by the most risk averse investor when equity falls to critically low levels. During "normal" times the member with the median level of risk aversion has most influence (consistent with the findings of the experimental study by Ambrus et al. (2015)). As such our model reconciles the diversification hypothesis with the group shift hypothesis. Although each investor has a constant coefficient of RRA, the group's level of RRA is a time-varying weighted average of investors' coefficient of risk aversion, reflecting the dynamic equity shares. This causes the group's risk aversion to be decreasing in the firm's equity. Our model may help explain the behavior of other group entities such as fund management teams, syndicates or partnerships.

Our model has several limitations that pose challenges for future research. First, we assume that the startup's assets are tradeable allowing investment to be continuously rebalanced in response to economic shocks. It remains an open question how the presence of intangible, non-tradeable assets (such as human capital) would affect the firm's optimal policies. Second, we assume that the firm's book equity is observable and verifiable, enabling contracts written on this state variable to be implemented. Future research could examine what happens if lack of transparency allows an insider (e.g. the company director or entrepreneur) to manipulate the firm's earnings and policies. Finally, we assume that investors cannot trade on their own account to hedge the startup's risk. Future research could explore how the presence of one or more wealthy and diversified co-investors affects the startup's optimal policies.

Appendix

Proof of Proposition 1

For any positive utility weights λ_i, the group's indirect value function $J(W_t)$ satisfies the following Hamilton–Jacobi–Bellman (HJB) equation

$$\rho J(W_t) = \max_{\{c_{it};\omega_t\}} \sum_{i=1}^{n} \lambda_i u_i(c_{it}) + \left[\left(\omega_t(\mu - r) + r\right) W_t - \sum_{i=1}^{n} c_{it} \right] J'(W_t) + \frac{1}{2}\sigma^2\omega_t^2 W_t^2 J''(W_t) \tag{42}$$

Substituting back the first-order conditions, $\omega_t^* = -\frac{\xi}{\sigma}\frac{J'(W_t)}{W_t J''(W_t)}$ and $c_{it}^* = \left(\frac{J'(W_t)}{\lambda_i}\right)^{-\frac{1}{\gamma_i}}$ gives

$$\sum_{i=1}^{n} \frac{\gamma_i}{1-\gamma_i} \lambda_i^{\frac{1}{\gamma_i}} J'(W_t)^{1-\frac{1}{\gamma_i}} + rW_t J'(W_t) - \frac{1}{2}\xi^2 \frac{J'(W_t)^2}{J''(W_t)} - \rho J(W_t) = 0 \tag{43}$$

This non-linear ordinary differential equation (ODE) cannot be solved analytically in W_t. We introduce an auxiliary state variable, $z_t \equiv -\ln(J'(W_t))$, to obtain closed-form solution. Substituting $J'(W_t) = e^{-z_t}$ and $J''(W_t) = -\frac{e^{-z_t}}{W'(z_t)}$ into Equation (43) gives:

$$\sum_{i=1}^{n} \frac{\gamma_i}{1-\gamma_i} \lambda_i^{\frac{1}{\gamma_i}} \left(e^{-z_t}\right)^{1-\frac{1}{\gamma_i}} + rW(z_t)e^{-z_t} + \frac{1}{2}\xi^2 W'(z_t)e^{-z_t} - \rho J(W(z_t)) = 0 \tag{44}$$

Differentiate this transformed ODE with respect to z_t, and then divide both sides by e^{-z_t}:

$$\sum_{i=1}^{n} \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} + \left(r - \frac{1}{2}\xi^2 - \rho\right) W'(z_t) + \frac{1}{2}\xi^2 W''(z_t) - rW(z_t) = 0 \tag{45}$$

The solution to this linear ODE can be written as $W_t = W_{pt} + W_{ct}$, where W_{pt} is the particular solution and W_{ct} is the complementary solution. One can conjecture and verify that $W_p(z_t) = \sum_{i=1}^{n} a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}}$, where $a_i(\xi)$ is defined in Equation (6). The complementary solution is of the form $W_c(z_t) = B_1 e^{b_+ z_t} + B_2 e^{b_- z_t}$, where B_1 and B_2 are constants to be determined and $b_\pm = \frac{\left(\rho + \frac{1}{2}\xi^2 - r\right) \pm \sqrt{(\rho+\frac{1}{2}\xi^2 - r)^2 + 2\xi^2 r}}{\xi^2}$, with $b_+ > 0$ and $b_- < -1$. The particular solution captures the expected lifetime utility under a particular payout and investment policy. The complementary solution captures the value from growth and abandonment options, which the firm and investors do not have. Therefore, $B_1 = B_2 = 0$, as otherwise, an exploding bubble component would be added when $z \to \pm\infty$. Thus, $W_t = W_p(z_t)$. Substituting W_t to Equation (44) and the first-order conditions gives the firm's indirect utility function (15) and optimal policies ((11) and (12)). The feasibility and transversality conditions are satisfied if $\gamma_1 > \gamma^* \equiv \frac{-(\rho + \frac{\xi^2}{2} - r) + \sqrt{(\rho + \frac{\xi^2}{2} - r)^2 + 2r\xi^2}}{2r} \in (0, 1)$.

To verify the second-order conditions, denote the right-hand side of (42) as $g(W_t)$. Given c_{it}^* and ω_t^*, $g_{\omega\omega} = -\frac{\sigma^2 W(z_t)^2}{W'(z_t)} e^{-z_t} < 0$, $g_{c_i c_i} = -\lambda_i \gamma_i (c_i^*)^{-1-\gamma_i} < 0$ and $g_{\omega c_i} = g_{c_i c_j} = 0$ for $i \neq j$. The determinants of the leading principle minors of the Hessian matrix are: $\det(H_1) = g_{\omega\omega}$ and $\det(H_k) = g_{\omega\omega} \prod_{i=1}^{k-1} g_{c_i c_i}$ for $k > 1$, and alternate in sign. ■

Proof of Proposition 2

For all i, one can show that $\lim_{z_t \to -\infty} c_{it}^* = \lim_{z_t \to -\infty} \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}} = 0$, and

$$\frac{\partial c_{it}^*}{\partial W_t} = \left[a_i(\xi) + \sum_{j=1}^{i-1} a_j(\xi) \frac{\gamma_i}{\gamma_j} \frac{\lambda_j^{\frac{1}{\gamma_j}}}{\lambda_i^{\frac{1}{\gamma_i}}} e^{z_t\left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i}\right)} + \sum_{j=i+1}^{n} a_j(\xi) \frac{\gamma_i}{\gamma_j} \frac{\lambda_j^{\frac{1}{\gamma_j}}}{\lambda_i^{\frac{1}{\gamma_i}}} e^{z_t\left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i}\right)} \right]^{-1} \tag{46}$$

Equation (46) implies $\frac{\partial c_{it}^*}{\partial W_t} > 0$ for all i, with $\lim_{z_t \to +\infty} \frac{\partial c_{1t}^*}{\partial W_t} = \frac{1}{a_1(\xi)}$ and $\lim_{z_t \to +\infty} \frac{\partial c_{it}^*}{\partial W_t} = 0$ for $i = 2, 3, ..., n$. The second derivative of the payout policy with respect to W_t is

$$\frac{\partial^2 c_{it}^*}{\partial W_t^2} = \frac{(\lambda_i e^{z_t})^{\frac{1}{\gamma_i}}}{\gamma_i W'(z_t)^3} \sum_{j=1}^{n} \frac{a_j(\xi)}{\gamma_j} (\lambda_j e^{z_t})^{\frac{1}{\gamma_j}} \left(\frac{1}{\gamma_i} - \frac{1}{\gamma_j} \right) \geq (\leq)\, 0$$

$$\Leftrightarrow \sum_{j=1}^{i-1} \frac{a_j(\xi)}{\gamma_j} \left(\lambda_j e^{z_t}\right)^{\frac{1}{\gamma_j}} \left(\frac{1}{\gamma_j} - \frac{1}{\gamma_i}\right) \leq (\geq) \sum_{j=i+1}^{n} \frac{a_j(\xi)}{\gamma_j} \left(\lambda_j e^{z_t}\right)^{\frac{1}{\gamma_j}} \left(\frac{1}{\gamma_i} - \frac{1}{\gamma_j}\right) \tag{47}$$

Therefore, $\frac{\partial^2 c_{1t}^*}{\partial W_t^2} \geq 0$ and $\frac{\partial^2 c_{nt}^*}{\partial W_t^2} \leq 0$. Both sides of (47) are positive and convex monotonically increasing functions of z_t. With $\gamma_1 < ... < \gamma_n$, the left-hand side of (47) is dominated by the right-hand side when $z_t \to -\infty$. For $z_t \to +\infty$, the opposite holds. Thus, the single-crossing property of two convex monotonically increasing functions implies that for $i = 2, ..., n-1$, there exists a $W_i^* \equiv W(z_i^*)$ such that $\frac{\partial^2 c_{it}^*}{\partial W_t^2} \geq (\leq)\, 0$ for $W_t \leq (\geq)\, W(z_i^*)$ and $W_{n-1}^* < ... < W_2^*$. $z_i^* \in \mathbb{R}$ is such that (47) is binding. Lastly, by setting $\gamma_1 = ... = \gamma_n = \gamma$ in (47), one can see that $\frac{\partial^2 c_{it}^*}{\partial W_t^2} = 0$ for all i. ■

Proof of Proposition 3

If the lead investor has all the bargining power, all co-investors' participation constraints are binding, i.e. $J_i(W_0) = \frac{a_i(\xi)}{1-\gamma_i}(\lambda_i e^{z_0})^{\frac{1}{\gamma_i}-1} = I_i(W_{i0}|\beta) = a_i(\beta)^{\gamma_i} \frac{W_{i0}^{1-\gamma_i}}{1-\gamma_i}$ for $i = 2, ..., n$. Solving for λ_i gives (24). Substituting (24) to the total book equity constraint gives λ_1:

$$W_0 = \sum_{i=1}^{n} a_i(\xi) \lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_0}{\gamma_i}} = a_1(\xi) \lambda_1^{\frac{1}{\gamma_1}} e^{\frac{z_0}{\gamma_1}} + \sum_{i=2}^{n} a_i(\xi) W_{i0} \left[\frac{a_i(\xi)}{a_i(\beta)^{\gamma_i}}\right]^{-\frac{1}{1-\gamma_i}} \tag{48}$$

$$\Rightarrow \lambda_1 = \left[W_{10} + \sum_{i=2}^{n} \left(1 - \left(\frac{a_i(\beta)}{a_i(\xi)}\right)^{\frac{\gamma_i}{1-\gamma_i}}\right) W_{i0}\right]^{\gamma_1} a_1(\xi)^{-\gamma_1} e^{-z_0} = \left(\frac{W_{10} + \Delta}{a_1(\xi)}\right)^{\gamma_1} e^{-z_0}$$

■

Proof of Proposition 4

Solving for λ_i from $J_i(W_0) = I_i(W_{i0}|\xi)$ gives Equation (25). Substituting λ_i into $\tilde{W}_{i0}$ yields $\tilde{W}_{i0} = W_{i0}$ and $W_0 = \sum_{i=1}^{n} W_{i0}$. By substituting λ_i into $c_{i0}^* = (\lambda_i e^{z_0})^{\frac{1}{\gamma_i}}$, one can show that $u'(c_{i0}^*) = \left(\frac{W_{i0}}{a_i(\xi)}\right)^{-\gamma_i} = \frac{dI_i(W_{i0}|\xi)}{dW_{i0}}$. We prove the rest of the proposition by first showing that the evolution of investor i's book equity stake $\tilde{W}_{it}$ in the firm is the same as the first-best outcome given by the stochastic differential equation (9). By substituting the group's optimal policies (Equations (11) and (12)) to Equation (2), one can get

$$dW_t = \left[\xi^2 W'(z_t) + rW(z_t) - \sum_{i=1}^{n} (\lambda_i e^{z_t})^{\frac{1}{\gamma_i}}\right] dt + \xi W'(z_t) dB_t \tag{49}$$

Applying Itô's Lemma to $z(W_t)$ gives

$$dz_t = \left[\frac{\xi^2 W'(z_t) + rW(z_t) - \sum_{i=1}^{n} c_{it}^*}{W'(z_t)} - \frac{1}{2}\xi^2 \frac{W''(z_t)}{W'(z_t)}\right] dt + \xi dB_t \equiv \mu_z dt + \sigma_z dB_t \tag{50}$$

Next, we apply Itô's Lemma to $\tilde{W}_{it} = a_i(\xi)\lambda_i^{\frac{1}{\gamma_i}} e^{\frac{z_t}{\gamma_i}}$ and show that

$$\frac{d\tilde{W}_{it}}{\tilde{W}_{it}} = \left[\frac{\mu_z}{\gamma_i} + \frac{1}{2}\frac{\sigma_z^2}{\gamma_i^2}\right] dt + \frac{\sigma_z}{\gamma_i} dB \equiv \alpha_{\tilde{W}_i} dt + \sigma_{\tilde{W}_i} dB_t \tag{51}$$

We then compare the process (51) with (9). They have the same volatility term $\frac{\xi}{\gamma_i}$. Denote the drift term in (9) as $\alpha_i(\xi)$:

$$\alpha_{\tilde{W}_i} = \alpha_i(\xi) \Leftrightarrow \frac{\xi^2}{\gamma_i} + \frac{rW(z_t) - \sum_{i=1}^{n} c_{it}^*}{\gamma_i W'(z_t)} - \frac{1}{2}\xi^2 \frac{W''(z_t)}{\gamma_i W'(z_t)} + \frac{1}{2}\frac{\xi^2}{\gamma_i^2} - \left(\frac{\xi^2}{\gamma_i} + r - \frac{1}{a_i(\xi)}\right) = 0$$

$$\Leftrightarrow \sum_{i=1}^{n} \left(r - \frac{\xi^2 - 2(\rho + \frac{1}{2}\xi^2 - r)\gamma_i - 2r\gamma_i^2}{-2\gamma_i^2} + \frac{\rho + \frac{1}{2}\xi^2 - r}{\gamma_i} - \frac{\xi^2}{2\gamma_i^2}\right) \tilde{W}_i(z_t) = 0$$

Therefore, the two processes have the same initial value ($W_{i0} = \tilde{W}_{i0}$), drift and volatility, implying $W_{it}^d = \tilde{W}_{it}$ for all t. Similarly, we can show $c_{it}^*(W_t) = c_{it}^d(W_{it}^d|\xi)$ for all t. ■

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