Earth Pressures Mobilised in Dry Sand with 
Active Rigid Retaining Wall Movement

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ABSTRACT

A series of centrifuge tests was conducted to explore the earth pressures mobilised in loose and dense sand for a complete set of active movement modes of a rigid retaining wall: rotation about the base, translation and rotation about the top. Earth pressures behind the wall were measured by a Tekscan pressure mapping system. This paper reveals that earth pressures at all depths decrease simultaneously with active wall rotation about the base, while those in shallow layers can increase with active wall translation and rotation about the top. A linkage between earth pressures and shear strains was built as a simplified constitutive law, together with deformation mechanisms, allowing designers to predict flexible wall deflections during construction as well as ultimate collapse.

KEYWORDS: centrifuge modelling, earth pressure, retaining walls, sands

INTRODUCTION

Terzaghi (1936) pointed out a fundamental fallacy in earth pressure computations as shown in Figure 1, showing earth pressures to distribute linearly for wall rotation about the base but irregularly for translation at the point when soil load reaches the active limit (Coulomb, 1776).
Figure 1. Earth pressure coefficient and distribution mobilised with (a) wall rotation about the base; 
(b) wall translation (adapted from Terzaghi (1936))

The earth pressure mobilisation with active rigid wall translation was subsequently researched 
analytically and found to distribute irregularly when considering soil arching (Handy, 1985; 
using the Discrete Element Method (DEM) and obtained an approximately linear pressure 
distribution for wall translation but an irregular profile for wall rotation about the top, 
consistent with the prediction proposed by Chang (1997). These behaviours have also been 
investigated through 1g physical model testing by Fang & Ishibashi (1986) and Khosravi et al. 
(2013), with the results being widely cited to validate analytical and numerical approaches. 
Wall heights and hence stress levels in conventional 1g tests are, however, much lower than 
those in practice, preventing earth pressures in the field from being well reproduced.

In order to explore in-situ earth pressure mobilisation, this paper presents centrifuge modelling 
of earth pressure mobilisation with active rigid retaining wall movement, allowing correct 
stress levels to exist within small-scale models.

**EXPERIMENTAL SET-UP**

A series of 40g centrifuge tests was conducted to explore the earth pressures mobilised in loose 
and dense sand with active rigid wall movement modes as shown in Table 1, with test 
procedures being depicted in detail by Deng & Haigh (2020a). The sand was dry in all six tests, 
however these results should be equally applicable to saturated sand under drained conditions.
Table 1. Summary of test configurations

<table>
<thead>
<tr>
<th>Test ID</th>
<th>Wall movement mode</th>
<th>Sand relative density</th>
<th>Sand unit weight (kN/m³)</th>
<th>Maximum wall displacement</th>
</tr>
</thead>
<tbody>
<tr>
<td>CD01</td>
<td>Rotation about the base</td>
<td>37% (Loose)</td>
<td>14.4</td>
<td>2.0% H</td>
</tr>
<tr>
<td>CD02</td>
<td>Rotation about the base</td>
<td>82% (Dense)</td>
<td>16.7</td>
<td>1.0% H</td>
</tr>
<tr>
<td>CD03</td>
<td>Translation</td>
<td>33% (Loose)</td>
<td>14.2</td>
<td>1.0% H</td>
</tr>
<tr>
<td>CD04</td>
<td>Translation</td>
<td>85% (Dense)</td>
<td>16.9</td>
<td>1.0% H</td>
</tr>
<tr>
<td>CD05</td>
<td>Rotation about the top</td>
<td>36% (Loose)</td>
<td>14.4</td>
<td>2.0% H</td>
</tr>
<tr>
<td>CD06</td>
<td>Rotation about the top</td>
<td>87% (Dense)</td>
<td>17.0</td>
<td>0.25% H</td>
</tr>
</tbody>
</table>

The models were constructed in a 780 mm wide, 560 mm high and 200 mm thick container as shown in Figure 2(a). The retaining wall has a bending stiffness of 2.4 GNm²/m at prototype scale and was found to act rigidly during the tests. Layers of props at two heights were used to control wall displacements, with individual layers of props being retracted to cause rotation and all props being retracted simultaneously to cause translation (Deng & Haigh, 2018). Details of the experimental set-up are presented by Deng (2020).

Figure 2. Schematic diagrams of (a) experimental set-up; (b) wall left side (units: mm)

Hostun sand with basic geotechnical properties reported by Deng & Haigh (2020a) and strength parameters obtained from a series of consolidated-drained triaxial compression tests (Table 2) was pluviated into the container using an automatic sand-pouring machine (Madabhushi et al., 2006). The peak friction angle measured in triaxial tests is smaller than that obtained in plane-strain conditions, with the differences being calculated as 1.1° and 5.6° for loose and dense sand, respectively (Bolton, 1986; Lau & Bolton, 2011). Similar to the mobilisation strain proposed by Vardanega & Bolton (2011), a reference strain, \( \gamma_{M=2} \), is defined to describe the shear strain at which half peak deviator stress is mobilised. Deng & Haigh (2020a) presented the wall-sand interface friction angles as 21.7° and 24.7° for loose and dense sand, respectively.
A Tekscan pressure mapping system was used to measure the earth pressures acting upon a 150 mm × 250 mm area at model scale as shown in Figure 2(b). Load cells (LC) were installed on the props with the corresponding measurements being found to be compatible with the stress distributions recorded by Tekscan. Detailed information on the methodology, application, calibration and validation of Tekscan is reported by Deng (2020).

EARTH PRESSURES WITH ACTIVE WALL MOVEMENT

Figures 3-5 show the measured earth pressures at the beginning of the test and for wall movements of 5%, 10%, 20%, 50% and 100% of the maximum displacements, at which the reduction of soil loads terminated and the active state was considered to be fully mobilised. Active earth pressures calculated using Equation 1 (Coulomb, 1776; Mayniel, 1808) are included for comparison, with the at-rest earth pressures predicted by Equation 2 (Jáky, 1948) also being illustrated for loose sand. It is worth noting that the vertical stress is assumed to be equal to the overburden sand weight with any effect of wall friction being ignored. Table 2
shows the sand friction angle to be stress-dependent and interpolation was thus conducted among the corrected plane-strain peak friction angles at the three confining pressures, causing active and at-rest earth pressure distributions to be curved.

\[ K_a = \frac{(\cos \phi)^2}{\cos \delta [1 + \sin(\delta + \phi) \sin \frac{\phi}{\cos \delta}]^2} \]  
\[ K_0 = 1 - \sin \phi \]

where \( K_a \) and \( K_0 \) are the theoretical active and at-rest earth pressure coefficients, \( \phi \) is the sand friction angle, and \( \delta \) is the wall-sand interface friction angle.

**Earth pressures with wall rotation about the base**

Figure 3 illustrates the measured earth pressures mobilised with wall rotation about the base, with the active state being fully generated at wall top displacements of 2.0%\( H \) and 1.0%\( H \) in loose and dense sand, respectively.

Figure 3. Earth pressures mobilised with active wall rotation about the base (a) in loose sand; (b) in dense sand

Figure 3(a) shows the initial earth pressure distribution in loose sand to be different from that predicted by Jáky (1948) with the pressures above a depth of 8 m being underestimated. Such a phenomenon was also observed in \( 1g \) tests (Khosravi et al., 2013) and explained as being the result of sand pouring. Settlement of sand particles during model preparation mobilises the wall-sand interface friction and consequently increases lateral earth pressures. Khosravi et al. (2013) also proposed the formation of local sand arching behind the wall base during sand pouring, causing earth pressure reduction at depth, validated by the centrifuge test measurements. While this is an experimental artefact, similar behaviours may well be seen in the field due to backfill placement and compaction.
Figure 3 shows that the earth pressures induced by loose and dense sand decrease with active wall rotation about the base with approximately linear distributions, validating the conclusion proposed by Terzaghi (1936) as shown in Figure 1(a). Conventional theory (Coulomb, 1776; Mayniel, 1808) is shown to slightly underestimate the active earth pressures induced by loose and dense sand.

Earth pressures with wall translation

The measured earth pressures mobilised with wall translation are shown in Figure 4, illustrating different final distributions in loose and dense sand even though total loads became constant at the same wall displacement of 1.0%H.

Figure 4. Earth pressures mobilised with active wall translation (a) in loose sand; (b) in dense sand

Figure 4(a) shows earth pressures below a depth of $\frac{1}{3}H$ to decrease with wall translation, with those above this depth behaving oppositely, causing the final distribution to be a vertical bow, symmetric about mid-depth. Similar behaviours were observed in $1g$ tests (Fang & Ishibashi, 1986; Khosravi et al., 2013) and analytical results (Handy, 1985; Paik & Salgado, 2003), with the mechanism being attributed to sand arching effects. No sand arches were, however, seen in shallow and middle layers behind the wall according to the corresponding deformation measurements (Deng & Haigh, 2020a), instead, showing a rigid sand block to form above a shear band and to cause highly-stressed zones. The measured limiting active earth pressures are well characterised by the apparent earth pressure diagram proposed by Tschebotarioff (1973), rather than a uniform pressure distribution suggested by Terzaghi & Peck (1967).
Figure 4(b) shows a similar but more rapidly mobilised earth pressure behaviour because of the lower compressibility of dense sand before the wall translates 0.1%\(H\), after which earth pressures above a depth of \(\frac{1}{4}H\) reduce. Such a behaviour is also the result of the low compressibility, causing the compression limit to be reached at a very small wall displacement, followed by collapse of highly-stressed zones with further wall translation.

Figure 4 shows the earth pressure distribution with wall translation to be irregular immediately after the total load becomes constant, similar to that suggested by Terzaghi (1936) as shown in Figure 1(b), subsequently becoming approximately linear with further wall displacements.

**Earth pressures with wall rotation about the top**

Figure 5 presents the measured earth pressures mobilised with wall rotation about the top, with total loads becoming constant at wall base displacements of 2.0\%\(H\) and 0.125\%\(H\) in loose and dense sand, respectively.

Figure 5. Earth pressures mobilised with active wall rotation about the top (a) in loose sand; (b) in dense sand

Similar earth pressure behaviours to those with wall translation were observed during wall rotation about the top, with earth pressures below a depth of \(\frac{2}{5}H\) decreasing while those above this level behaving oppositely. Figure 5, typically, shows the earth pressure distributions at wall base displacements of 0.2\%\(H\) in loose sand and 0.05\%\(H\) in dense sand to be very similar to that shown in Figure 1(b). While the wall movement mode of rotation about the top restricts sand deformation in shallow layers but promotes sand collapse at depth, causing more intensive
highly-stressed zones to be formed in shallow layers, validated by the full generation of the passive state above a depth of 2.3 m in loose sand as shown in Figure 5(a).

It is worth noting in Figure 5(b) that earth pressures behind the wall base remain constant at a very small value of $0.05\gamma_s H$, probably caused by retraction of the lower props during the centrifuge swinging up. Figure 5(b) shows earth pressures in shallow layers to increase more dramatically due to the lower compressibility of dense sand, with a symmetric pressure profile about mid-depth being generated at the wall base displacement of $0.125%H$. The total load then keeps constant with further wall rotation, with stresses continuing to increase in shallow layers to compensate pressure reduction at depth, interestingly, resulting in earth pressure redistribution behind the wall.

It can be concluded from Figures 4 and 5 that wall translation and rotation about the top can generate highly-stressed zones in shallow layers, causing the earth pressure increase with active wall movement. As shown in Figure 6, a series of Mohr’s circles describing the stress state of an element at a depth of 1.87 m in dense sand behind a rotating wall about the top visually explains the counterintuitive phenomenon of sand in shallow layers entering the intermediate passive state with active wall rotation.

Figure 6. Mohr’s circles of stress mobilised with active wall rotation about the top

**SIMPLIFIED LINKAGE BETWEEN EARTH PRESSURES AND SHEAR STRAINS**

Although the earth pressures mobilised with a complete set of active rigid wall movement modes have been illustrated and analysed, they cannot be directly used in design due to flexible
walls having more complex displacement modes than pure rotation or translation. In order to involve rigid wall results in flexible wall design, similar to the work for walls in clay (Deng et al., 2021), a linkage between earth pressures and shear strains is required.

To eliminate the effect of variable strength parameters and initial stress states with depth, $f_a(K)$ is defined using Equation 3 to describe the earth pressure mobilisation towards the active limit.

$$f_a(K) = \frac{K - K_{ini}}{|K_a - K_{ini}|}$$

where $K_{ini}$ is the measured initial earth pressure coefficient.

Deng & Haigh (2020a) presented peak values of maximum shear strain of sand, $\gamma_{max,peak}$, with $f(\gamma)$ being defined using Equation 4 to characterise strain mobilisation.

$$f(\gamma) = \frac{\gamma_{max,peak}}{\gamma_{M=2}}$$

Figure 7 shows the relationships between earth pressures and shear strains, with the counterintuitive earth pressure increase in shallow layers and small pressures at depth being excluded for simplicity. Figure 7(a) shows the earth pressures mobilised in loose sand with active wall translation to decrease to certain values and then stay constant, with $f_a(K)$ at the depths of 3.4 m, 4.3 m, 5.2 m, 6.1 m, 7.0 m and 7.9 m dropping to -0.20, -0.35, -0.49, -0.61, -0.74 and -0.91, respectively, showing the effect of highly-stressed zones to alleviate with depth. Similar behaviours are seen for those mobilised with wall rotation about the top. It is worth noting in Figure 7(b) that the results for wall translation are not included due to the corresponding huge shear strains (>100%) being caused by global collapse rather than material deformation (Deng & Haigh, 2020a).

Figure 7. Linkages between earth pressure and shear strain (a) in loose sand; (b) in dense sand
Two simplified laws with equations presented in Table 3 are proposed and shown in Figure 7 in order to approximately characterise active earth pressure mobilisation. The limiting active earth pressure coefficients can then be calculated as $0.91K_a + 0.09K_{ini}$ in loose and dense sand, with that in dense sand being mobilised at a lower strain. It should be pointed out that these results are particular for loose and dense Hostun sand, with more experimental exploration needed for full validation across soil types and densities.

Table 3. Simplified laws for active earth pressure mobilisation

<table>
<thead>
<tr>
<th>Sand density</th>
<th>Mathematical law</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loose</td>
<td>$f_a(K) = \begin{cases} -0.33 \times f(\gamma)^{0.4} &amp; f(\gamma) \leq 12.5 \ -0.91 &amp; f(\gamma) &gt; 12.5 \end{cases}$</td>
</tr>
<tr>
<td>Dense</td>
<td>$f_a(K) = \begin{cases} -0.50 \times f(\gamma)^{0.4} &amp; f(\gamma) \leq 4.5 \ -0.91 &amp; f(\gamma) &gt; 4.5 \end{cases}$</td>
</tr>
</tbody>
</table>

CONCLUSIONS

This paper has presented a centrifuge study of the earth pressures mobilised with a complete set of active rigid wall movement modes, with a linkage between earth pressures and shear strains being built. Earth pressures at all depths decrease simultaneously with active wall rotation about the base, while those in shallow layers can increase with active wall translation and rotation about the top. The earth pressures and deformation mechanisms (Deng & Haigh, 2020a) mobilised towards the active state, together with those towards the passive state (Deng & Haigh, 2020b), give the possibility of proposing a cost-benefit design method for retaining walls in sand, following mobilisable strength design for walls in clay (Deng et al., 2021).
ACKNOWLEDGEMENTS

The first author received a scholarship from the China Scholarship Council.

REFERENCES


To Assessor

All the page, line and figure numbers in blue are based on the old version used by reviewers, while those in black in the answers are based on the resubmitted version.

To Reviewer 1

The revised version of the paper has benefited from clarification and edits suggested by the reviewers. The following minor edits are suggested for the final version:

TEXT:
- Page 2, line 30: suggest changing ‘… those in practical, …’ for ‘… those in practice, …’
‘practical’ has been changed to ‘practice’ as shown in Line 30, Page 2.

FIGURES
- Figure 2: it is suggested to add 'LC: load cell'. Alternatively, indicate 'LC' in page 4, line 27.
‘LC’ has been added after ‘load cells’ in Line 27, Page 4.

REFERENCES:
- It is suggested to swap Deng & Hang (2020a) and Deng & Hang (2020b) in the main text and as required in references section. In the current text, Deng & Hang (2020a) is only mentioned at the end in the 'Conclusions' section. Additionally, with reference to current text, Deng & Hang (2020b) has been already published and makes sense to cite it first. This would also follow the order of citation in the main text.
Deng & Haigh (2020a) and Deng & Haigh (2020b) have been swapped in the main text and reference section.

- Change 2019 for 2020 for Deng et al. (2019) (in main text and references)? This because link indicates that paper was 'Published Online: December 14, 2020'
The reference to Deng et al. (2021) has been updated, with the paper being published in print in February 2021.

- DOIs are available for papers from most journals. Unless there is a policy to only include DOIs for ICE journals' papers, it is suggested to include that for all those available. It is a useful feature for the readership. Alternatively, just leave the DOIs for the two Géotechnique publications ahead of print?

The DOI has been added for all publications where these are known in References.
Figure 3

(a) and (b) show the relationship between depth (m) and earth pressure (kPa) for different scenarios. The diagrams illustrate the effects of various concentrations (0%, 0.1%, 0.2%, 0.4%, 1.0%, 2.0%, 0.05%, 0.1%, 0.2%, 0.5%, 1.0%) on the earth pressure profile. The lines represent different concentrations, with symbols indicating specific data points. The diagrams compare the earth pressure predictions for Coulomb and Jaky models.
Figure 6

Shear stress ($\tau$) and effective normal stress ($\sigma'$) diagram with failure envelopes and active wall rotation.

<table>
<thead>
<tr>
<th>Wall Movement</th>
<th>Normal Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$\sigma'_{h,1} = 17.29$ kPa</td>
</tr>
<tr>
<td>$0.0125% H$</td>
<td>$\sigma'_{h,2} = 19.45$ kPa</td>
</tr>
<tr>
<td>$0.025% H$</td>
<td>$\sigma'_{h,3} = 21.27$ kPa</td>
</tr>
<tr>
<td>$0.05% H$</td>
<td>$\sigma'_{h,4} = 27.76$ kPa</td>
</tr>
<tr>
<td>$0.125% H$</td>
<td>$\sigma'_{h,5} = 31.46$ kPa</td>
</tr>
<tr>
<td>$0.25% H$</td>
<td>$\sigma'_{h,6} = 37.31$ kPa</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>$30.19$ kPa</td>
</tr>
</tbody>
</table>