

Ultrafast helicity-dependent photocurrents in Weyl Magnet Mn_3Sn (Supplementary Information)

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SUPPLEMENTARY NOTE 1: SAMPLE CHARACTERISATION BY X-RAY DIFFRACTION ANALYSIS

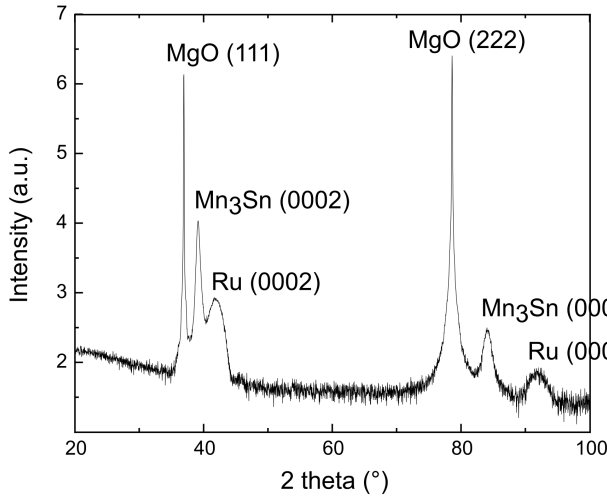


Fig. S 1 – X-ray diffraction of the MgO(111)(0.5 nm)/Ru(5 nm)/Mn₃Sn(50 nm)/Si(3 nm) sample.

SUPPLEMENTARY NOTE 2: TERAHERTZ TRANSIENTS MEASURED AT OPPOSITE PUMP HELICITIES

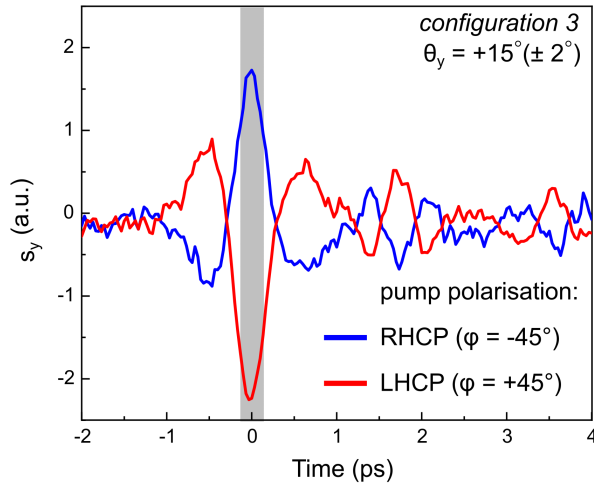


Fig. S 2 – Transients THz fields detected for different optical pump helicities: right-handed circular polarisation (RHCP) and left-handed circular polarisation (LHCP). The shaded region in the graph indicates the region over which $s_i(t)$ is integrated to obtain the amplitude S_i .

To improve the signal-to-noise ratio, we define the amplitude of the emission S_i as:

$$S_i = \int_{t=+0.15 \text{ ps}}^{t=-0.15 \text{ ps}} s_i(t) dt \quad (1)$$

obtained by integrating the time-domain THz field transient $s_i(t)$ around the maximum absolute value (set at $t = 0$) over a narrow time region of ~ 0.3 ps.

SUPPLEMENTARY NOTE 3: TERAHERTZ TRANSIENTS MEASURED AS A FUNCTION OF PUMP POLARISATION - CONFIGURATION 1

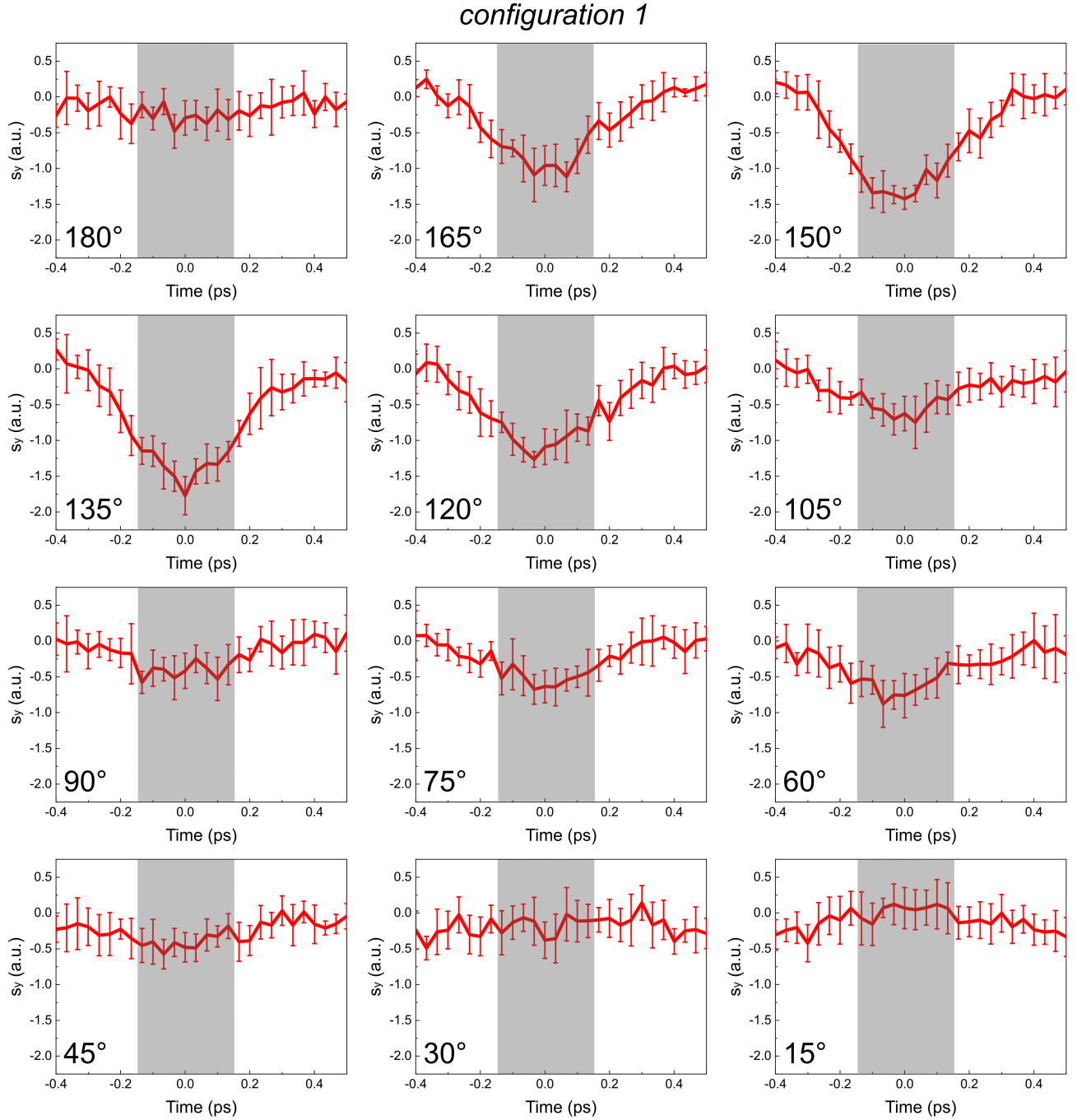


Fig. S 3 – Transients THz fields detected along y for different φ values, corresponding to different pump polarisations. The measurement was performed in *configuration 1*. The shaded region in the graph indicates the region over which $s_i(t)$ is integrated to obtain the amplitude S_i .

SUPPLEMENTARY NOTE 4: TERAHERTZ TRANSIENTS MEASURED AS A FUNCTION OF PUMP POLARISATION - CONFIGURATION 3

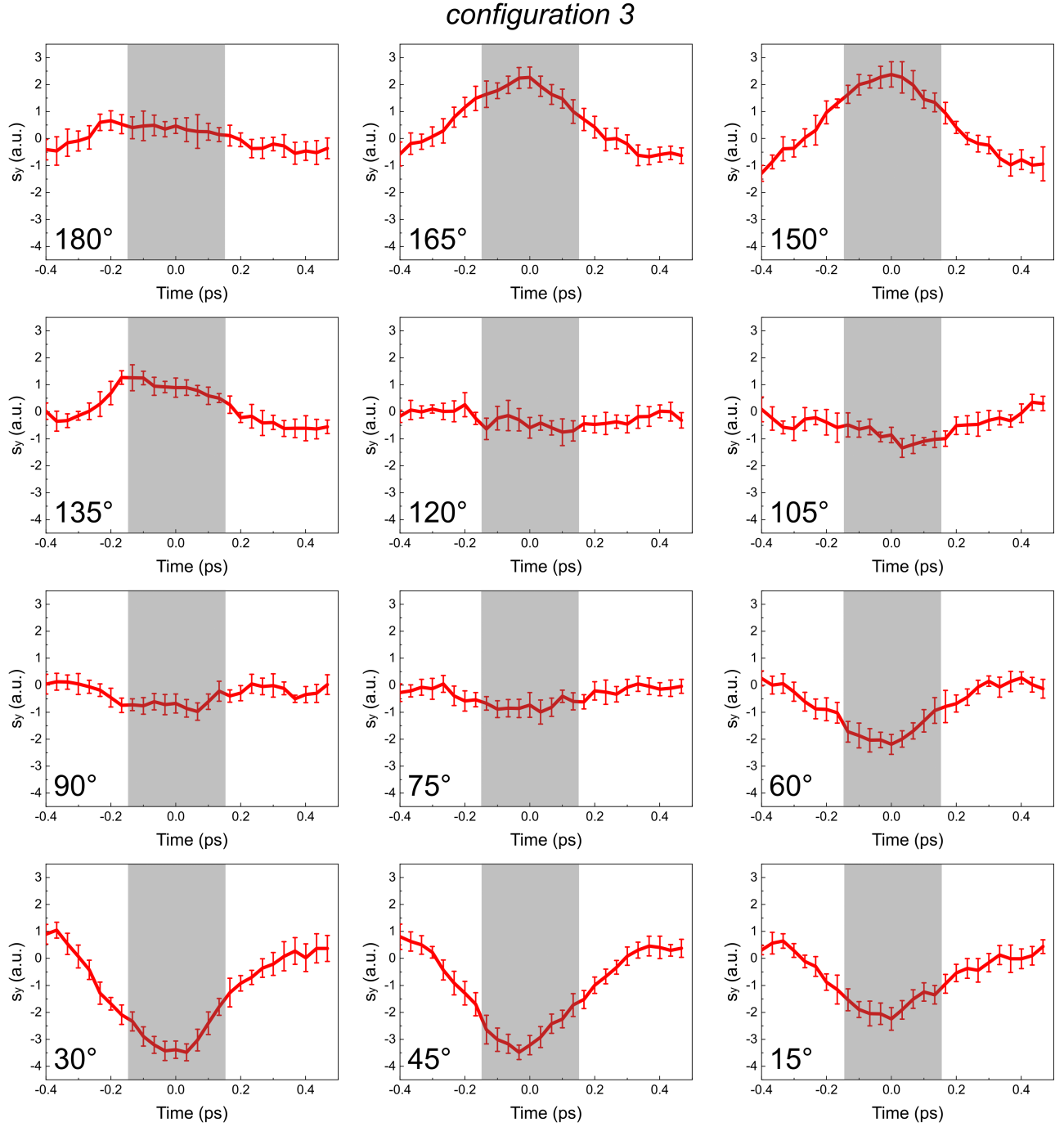


Fig. S 4 – Transients THz fields detected along y for different φ values, corresponding to different pump polarisations. The measurement was performed in *configuration 3*, $\theta_y = 30^\circ$. The shaded region in the graph indicates the region over which $s_i(t)$ is integrated to obtain the amplitude S_i .

SUPPLEMENTARY NOTE 5: COMPONENTS OF THZ SIGNAL AT NORMAL INCIDENCE

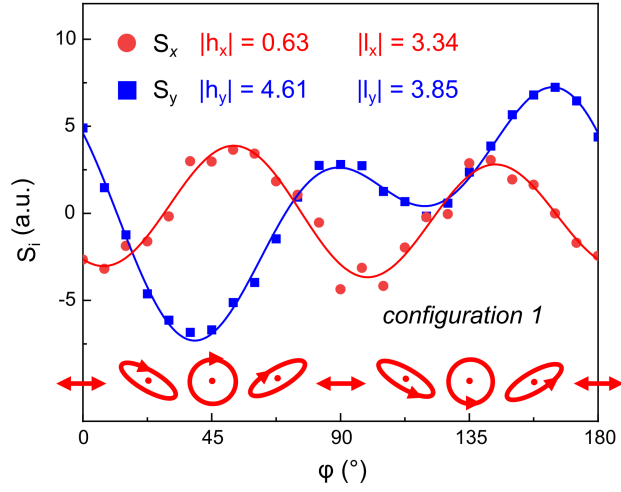


Fig. S 5 – THz components emitted at normal laser incidence. THz signals measured for two orthogonal THz polarisations (along x and along y) are plotted as a function of φ . The data sets were fitted with Eq. (1) in the main text to extract the h_i and l_i parameters.

SUPPLEMENTARY NOTE 6: COMPONENTS OF THZ SIGNAL AT DIFFERENT TILTING ANGLES IN CONFIGURATION 3

$\theta_y(^{\circ})$	$h_y(a.u.)$	$l_y(a.u.)$	$B_y(a.u.)$
0	4.4 ± 0.1	3.5 ± 0.1	6.0 ± 0.1
15	10.5 ± 0.2	6.6 ± 0.2	15.4 ± 0.1
30	19.1 ± 0.3	11.6 ± 0.3	5.8 ± 0.2

Table S 1: Components of THz signal at different tilting angles in configuration 3. The table displays h_y , l_y , and B_y with their errors for different tilting angles θ_y in *configuration 3*. The parameters were obtained by fitting $S_y(\varphi)$ curves displayed in Fig. 2(c) with Eq. (1): $S_y(\varphi) = h_y \sin(2\varphi + \varphi_1) + l_y \sin(4\varphi + \varphi_2) + B_y$.

SUPPLEMENTARY NOTE 7: EVEN AND ODD THZ RESPONSES MEASURED ALONG x

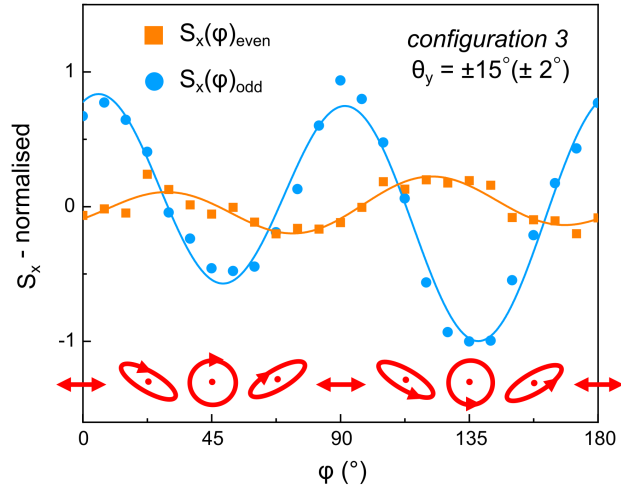


Fig. S 6 – Even and odd THz responses measured along x as a function of φ . The responses were obtained using Eq. (2) and Eq. (3) in the main text respectively. The experiment was performed in *configuration 3*.

SUPPLEMENTARY NOTE 8: THZ TRANSIENTS MEASURED AT DIFFERENT MAGNETIC FIELDS

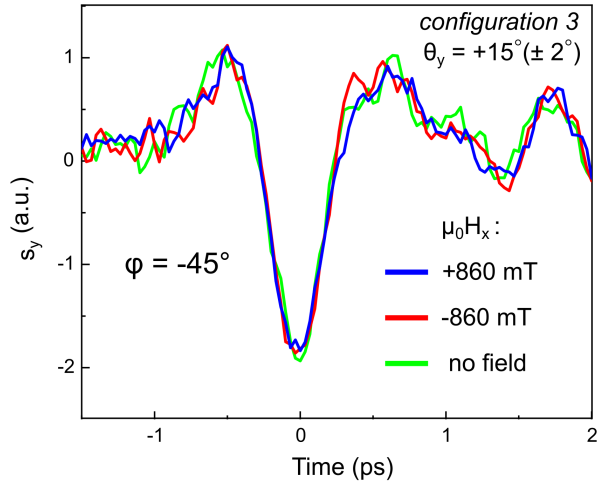


Fig. S 7 – Transients THz fields detected for different directions of magnetic field and with no field applied. The measurement was performed at room temperature at a fixed position of the quarter wave plate ($\varphi = 45^\circ$), corresponding to RHCP.

SUPPLEMENTARY NOTE 9: TEMPERATURE DEPENDENCE OF THz SIGNAL COMPONENTS

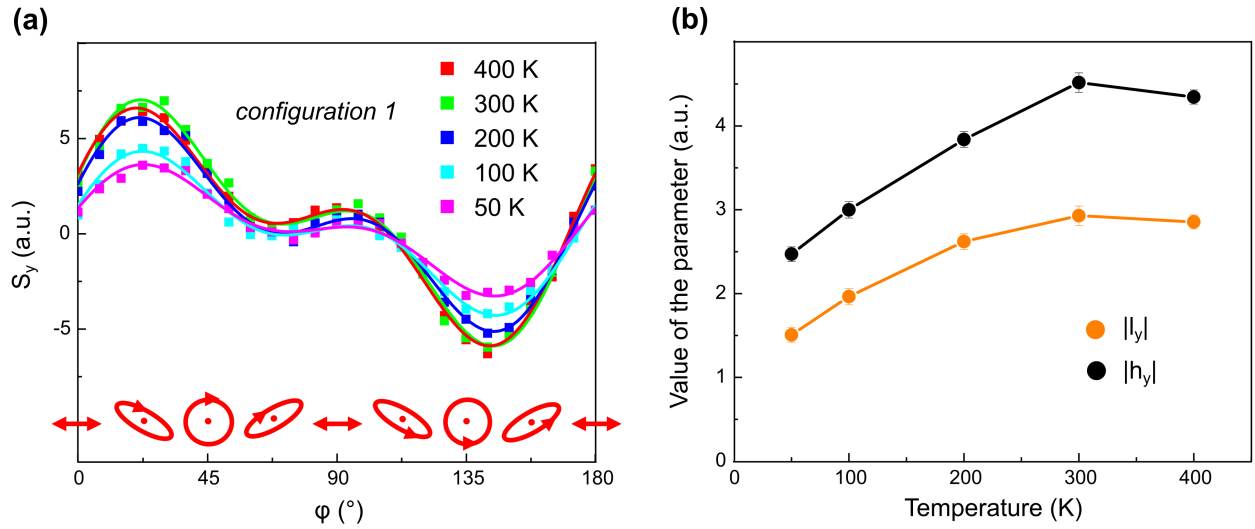


Fig. S 8 – Temperature dependence of THz signal components. (a) shows peak THz signals measured along y as a function of φ at different temperatures between 50 K and 400 K. The measurement was performed in *configuration 1* and with no magnetic field applied. The values of $|l_y|$ and $|h_y|$ were extracted from these data sets and are displayed in (b).

SUPPLEMENTARY NOTE 10: FURTHER THEORY DETAILS

A. Relevant space-groups

Mn₃Sn crystallizes in a layered hexagonal lattice. When ignoring the magnetic moments, the bulk space group is the non-magnetic (gray) group P6₃/mmc1' (No.194). Accounting for magnetism, the triangular antiferromagnetic phase stabilized between approximately 250 K and 420 K, is described by the magnetic space-group $Cm'cm'$ (No. 63.464 in the BNS convention) albeit in a non-standard setting as shown in Ref. [1].

For the tensor symmetry analysis, only the point-groups (PG) matter. The non-magnetic bulk PG is 6/mmm1', whereas the bulk magnetic PG is $m'm'm$. Because the photogalvanic effect (PGE) vanishes in the bulk, we also analyze the symmetry properties of the surface of the material. For all configurations, the light impinges on the surface perpendicular to the c -axis of the material. The non-magnetic PG associated with this surface is 3m1', whereas the magnetic PG is simply m' . Note that this assumes that there is no reordering of the surface. We will argue that the observed signal arises predominantly from a bulk effect and as such this is not a major limitation.

B. Symmetry analysis of non-linear optical tensors

This section discusses the full symmetry analysis for the optical response tensors considered in the main text. We begin by discussing the role of time-reversal symmetry, and then go on to discussing spatial symmetries.

1. Role of time-reversal symmetry

The role of time-reversal in tensor symmetry analysis requires special care as described in Refs. [2–4]. There are two distinct microscopic mechanisms (usually called "shift" and "injection" currents) [5] leading to induced currents in the PGE. These behave differently under time-reversal. As we are only interested in phenomenological expressions for the induced current, we do not consider the microscopic mechanisms. We therefore neglect antiunitary symmetries entirely when constraining our tensors, so that the analysis is agnostic to the underlying microscopic mechanism. The unitary PG in the non-magnetic case are 6/mmm in the bulk and 3m on the surface. In the magnetic case, the unitary part of the bulk PG is 2/m, whereas the unitary part of the surface PG is 1.

2. Role of spatial symmetries

To understand how the remaining unitary point-group symmetries constrain the tensors $\chi_{ijk}^{(2)}$ and $\chi_{ijk}^{(3)}$ in Eq. (4) of the main text, we first decompose the tensors describing the induced currents into symmetric and antisymmetric components with respect to the light field:

$$J_i = \chi_{ijk}^{(2),s} [E_j, E_k^*] + \chi_{ik}^{(2),a} P_{\text{circ}} \hat{e}_k + \chi_{ijk}^{(3),s} q_j [E_k, E_l^*] + \chi_{ijk}^{(3),a} q_j P_{\text{circ}} \hat{e}_k$$

Where the symmetric (s) and antisymmetric (a) terms corresponds to the linear/circular PGE/PDE respectively. Following notation from Ref. [6] we have defined:

$$[E_j, E_k^*] = \frac{E_j E_k^* + E_k E_j^*}{2} \quad (2)$$

$$P_{\text{circ}} \hat{e}_k = i(E \times E^*)_k \quad (3)$$

The permutation symmetry of the tensors can be accounted for by using Jahn's symbols as discussed in [4]. The Jahn's symbol for each tensor is respectively $\chi_{[ijk]}^{(2),s}$: V[V2], $\chi_{ik}^{(2),a}$: V{V2}, $\chi_{ij[kl]}^{(3),s}$: V2[V2] and $\chi_{ijk}^{(3),a}$: V2{V2}. To find all symmetry-allowed terms, we use the MTENSOR tool [4] hosted on the Bilbao Crystallographic Server (BCS) [7]. We write the resultant currents in the (a, b, c) Cartesian coordinate system of the material. The full expressions for the induced current for all combinations of magnetic/non-magnetic, PDE/PGE and surface/bulk are in Sec. C 4.

C. Dependence of signal on polarization and sample geometry

1. Dependence of incoming fields on angle and sample geometry

We assume the laser is linearly polarized before passing through the quarter-wave plate, corresponding to a Jones vector [8] of $\hat{x}$. After passing through a quarter-wave plate at angle φ , we find in the laboratory frame:

$$|E_x|^2 \propto \cos 4\varphi \quad (4)$$

$$|E_y|^2 \propto \cos 4\varphi \quad (5)$$

$$[E_x, E_y] \propto \sin 4\varphi \quad (6)$$

$$P_{\text{circ}} \hat{e}_k \propto [\sin 2\varphi] \hat{e}_k \quad (7)$$

We assume that the THz emission follows the same polarization dependence, and therefore associate components in the measured signal with frequency 4φ with the LPGE/LPDE and components with frequency 2φ with the CPGE/CPDE.

In the laboratory (x, y, z) Cartesian frame, $\mathbf{E} = (E_x, E_y, 0)$ and $\mathbf{q} = (0, 0, 1)$. Writing $\tilde{\mathbf{E}}, \tilde{\mathbf{q}}$ for the quantities measured in the material (a, b, c) Cartesian frame then gives for a rotation along the x -axis (*configuration 2*):

$$\tilde{\mathbf{E}} = (E_x, E_y \cos \theta_x, E_y \sin \theta_x) \quad (8)$$

$$\tilde{\mathbf{q}} = (0, q_z \sin \theta_x, q_z \cos \theta_x) \quad (9)$$

$$P_{\text{circ}} \hat{e}_a = 0 \quad (10)$$

Whereas rotating along the y -axis (*configuration 3*) gives:

$$\tilde{\mathbf{E}} = (E_x \cos \theta_y, E_y, E_x \sin \theta_y) \quad (11)$$

$$\tilde{\mathbf{q}} = (q_z \sin \theta_y, 0, q_z \cos \theta_y) \quad (12)$$

$$P_{\text{circ}} \hat{e}_b = 0 \quad (13)$$

2. Expressions at normal incidence, $\theta_x = 0, \theta_y = 0$ (*configuration 1*)

At normal incidence (*configuration 1*), we can ignore optical corrections from refraction. From the full expression in Sec. C4, we find for the induced bulk current when ignoring the magnetic ordering:

$$\begin{aligned} J_a &= 0 \\ J_b &= 0 \\ J_c &\propto q_z \cos 4\varphi \end{aligned} \quad (14)$$

Whereas when we include the magnetic ordering, we find the bulk current:

$$J_a \propto q_z \cos 4\varphi \quad (15)$$

$$J_b \propto q_z (\sin 4\varphi + C_1 \sin 2\varphi) \quad (16)$$

$$J_c \propto q_z \cos 4\varphi \quad (17)$$

The allowed surface current at normal incidence when ignoring magnetic ordering are given by:

$$J_a \propto \cos 4\varphi + C_2 q_z \cos 4\varphi \quad (18)$$

$$J_b \propto \sin 4\varphi + C_3 q_z \sin 4\varphi \quad (19)$$

$$J_c \propto \cos 4\varphi + C_4 q_z \cos 4\varphi \quad (20)$$

When taking into account the magnetic ordering of the surface, all terms are symmetry-allowed. Here C_i are constants that are independent of φ and $\mathbf{q}$. We note that the non-magnetic analysis cannot explain the appearance of a circular effect at normal incidence, even when taking the surface into account. This is discussed further in the next section.

3. Expressions away from normal incidence (configuration 2 & 3)

As can be seen from the full expressions in Sec. C4, away from normal incidence, all cases (magnetic/non-magnetic and surface/bulk) allow for a linear and a circular photocurrent, though all bulk contributions still arise exclusively from the PDE due to bulk inversion symmetry. Because the measured signal does not depend on the magnetic field (see Fig. 3 and discussion in the main text as well as Fig. S 7), we discuss only the non-magnetic symmetry settings in what follows.

As the detector is in a fixed position, the detected signal will display a purely geometric variation under rotation away from normal incidence, as various faces of the crystal are exposed. The generated photocurrents will induce a dipole of strength $\mathbf{d}$, and the associated emitted THz field can be written in terms of unit vectors as:

$$\mathbf{S}(\mathbf{r}) \propto \mathbf{E}^{\text{THz}}(\mathbf{r}) \propto \hat{\mathbf{r}} \times \hat{\mathbf{d}} \times \hat{\mathbf{r}} \quad (21)$$

Note that this ignores reflection and refraction effects, which will play a role away from normal incidence. As the sample is rotated by the same angles in both x and y , however, these effects should be irrelevant when comparing *configuration 2* and *configuration 3*. The measured signal in *configuration 1* is then:

$$S_x \propto J_a \quad (22)$$

$$S_y \propto J_b \quad (23)$$

Whereas in *configuration 2* it is :

$$S_x \propto J_a \quad (24)$$

$$S_y \propto J_b \cos \theta_x + J_c \sin \theta_x \quad (25)$$

An in *configuration 3*:

$$S_x \propto J_a \cos \theta_y + J_c \sin \theta_y \quad (26)$$

$$S_y \propto J_b \quad (27)$$

Where the induced currents $\mathbf{J}$ also depend on the angles $\theta_{x,y}$. This dependence can be written out explicitly, using the expressions in Sec. C1 and Sec. C4. This was shown for *configuration 1* in all symmetry settings in the previous section. For the non-magnetic bulk signal in *configuration 2*, we find:

$$S_x \propto \sin \theta_x \cos \theta_x (C_1 \sin 4\varphi + C_2 \sin 2\varphi) \quad (28)$$

$$S_y \propto (C_3 \cos \theta_x \sin \theta_x + C_4 \cos^3 \theta_x \sin \theta_x + C_5 \cos \theta_x \sin^3 \theta_x) \cos 4\varphi + (C_6 \cos^3 \theta_x \sin \theta_x + C_7 \cos \theta_x \sin^3 \theta_x) \sin 4\varphi \quad (29)$$

And in *configuration 3*:

$$S_x \propto (D_1 \cos \theta_y \sin \theta_y + D_2 \cos^3 \theta_y \sin \theta_y + D_3 \cos \theta_y \sin^3 \theta_y) \cos 4\varphi + (D_4 \cos^3 \theta_y \sin \theta_y + D_5 \cos \theta_y \sin^3 \theta_y) \sin 4\varphi \quad (30)$$

$$S_y \propto \sin \theta_y \cos \theta_y (D_6 \sin 4\varphi + D_7 \sin 2\varphi) \quad (31)$$

Turning to the non-magnetic surface effects, the lowest order surface effects arises from the photogalvanic effect. For the surface photogalvanic effect, the angular dependence in *configuration 2* is given by:

$$S_x \propto (C_1^s + C_2^s \cos^2 \theta_x) \cos 4\varphi + C_3^s \sin \theta_x \sin 4\varphi + C_4^s \sin \theta_x \sin 2\varphi \quad (32)$$

$$S_y \propto (C_5^s \sin \theta_x + C_6^s \cos^2 \theta_x \sin \theta_x + C_7^s \sin^3 \theta_x) \cos 4\varphi + (C_8^s \cos^2 \theta_x + C_9^s \cos^2 \theta_x \sin \theta_x) \sin 4\varphi \quad (33)$$

And in *configuration 3*:

$$S_x \propto (D_1^s \cos \theta_y + D_2^s \sin \theta_y + D_3^s \cos^3 \theta_y + D_4^s \sin^3 \theta_y + D_5^s \cos^2 \theta_y \sin \theta_y) \cos 4\varphi + D_6^s \cos^2 \theta_y \sin \theta_y \sin 4\varphi \quad (34)$$

$$S_y \propto (D_7^s \sin \theta_y + D_8^s \cos \theta_y) \cos 4\varphi + D_9^s \sin \theta_y \sin 2\varphi \quad (35)$$

In the above expressions, C_i and D_i (C_i^s and D_i^s) are constant independent of $\theta_{x,y}$ and φ associated with the generated bulk (surface) currents.

Note that there is no circular current being generated perpendicular to the rotation axis, independent of where the current is generated, which explains the large suppression in the circular current seen in Fig. 2 of the main text. We attribute the appearance of a very small circular effect perpendicular to the rotation axis to an imperfect sample alignment. All terms generated in the bulk are odd in the rotation angle, whereas some of the linear terms arising from the surface photogalvanic effect are even in rotation angle. By contrast, all circular terms are odd in rotation angle for both the surface and bulk effect. As shown in Fig. 2 in the main text and Fig. S 6, we see that the odd response dominates, even though the linear and circular components have comparable magnitudes. This suggests that the signal originates predominantly from the bulk photon drag effect.

At normal incidence, we would expect to not detect any signal from the bulk photon drag effect. As shown in Fig. 2(c) of the main text, the detected signal at normal incidence is very weak compared to the signal at larger rotation angles. This suggests that the measured signal arises from an imperfect sample alignment, resulting in a small non-zero rotation angle. This also explains the observed signal in Fig. S 5.

4. Effect of in-plane rotation, $\theta_c \neq 0$

Rotating by 90° in-plane (along the c -axis) keeps all optical parameters the same, so that all changes in signal arise as a result of the photocurrent generation mechanisms in the material. As shown in Fig. 3(b) in the main text, an in-plane rotation by 90° has negligible impact on the measured signal. Writing R_{xa} for the rotation matrix relating the lab (x, y, z) coordinates to the material (a, b, c) coordinates, the measured signals are given by:

$$\begin{aligned} S_x &\propto \sum_{i \in (a,b,c)} R_{xi}^{-1} J_i(\mathbf{R}\mathbf{E}, \mathbf{R}\mathbf{q}) \\ S_y &\propto \sum_{i \in (a,b,c)} R_{yi}^{-1} J_i(\mathbf{R}\mathbf{E}, \mathbf{R}\mathbf{q}) \end{aligned} \quad (36)$$

Inserting this into the equations found in Sec. C 4, we find that the non-magnetic bulk PDE is invariant under this transformation (in both the linear and circular component), whereas none of the other contributions are invariant under this transformation. This adds further credence to the result that our signal arises predominantly from a non-magnetic bulk PDE.

SUPPLEMENTARY NOTE 11: FULL EXPRESSION FOR NON-LINEAR OPTICAL TENSORS

Here we provide full expressions for the symmetry-allowed currents, written in the material (a, b, c) Cartesian coordinate system. These are found using the MTENSOR functionality [4] on the BCS [7].

D. Non-magnetic bulk

1. Linear photon drag effect

$$\begin{aligned} J_a &= q_a(\chi_{aaaa}^{(3),s}|E_a|^2 + \chi_{aabb}^{(3),s}|E_b|^2 + \chi_{aacc}^{(3),s}|E_c|^2) + \\ &\quad \frac{q_b}{2}(\chi_{aaaa}^{(3),s} - \chi_{aabb}^{(3),s})[E_a, E_b] + q_c\chi_{acac}^{(3),s}[E_a, E_c] \end{aligned} \quad (37)$$

$$\begin{aligned} J_b &= q_b(\chi_{aabb}^{(3),s}|E_a|^2 + \chi_{aaaa}^{(3),s}|E_b|^2 + \chi_{aacc}^{(3),s}|E_c|^2) + \\ &\quad \frac{q_a}{2}(\chi_{aaaa}^{(3),s} - \chi_{aabb}^{(3),s})[E_a, E_b] + q_c\chi_{acac}^{(3),s}[E_b, E_c] \end{aligned} \quad (38)$$

$$J_c = q_c(\chi_{ccaa}^{(3),s}|E_a|^2 + \chi_{ccaa}^{(3),s}|E_b|^2 + \chi_{cccc}^{(3),s}|E_c|^2) + q_a\chi_{caac}^{(3),s}[E_a, E_c] + q_b\chi_{caac}^{(3),s}[E_b, E_c] \quad (39)$$

For a total of 7 independent components.

2. Circular photon drag effect

$$J_a = q_b\chi_{abc}^{(3),a}P_{\text{circ}}\hat{e}_c - q_c\chi_{acb}^{(3),a}P_{\text{circ}}\hat{e}_b \quad (40)$$

$$J_b = q_c\chi_{acb}^{(3),a}P_{\text{circ}}\hat{e}_a - q_a\chi_{abc}^{(3),a}P_{\text{circ}}\hat{e}_c \quad (41)$$

$$J_c = q_b\chi_{cab}^{(3),a}P_{\text{circ}}\hat{e}_a - q_a\chi_{cab}^{(3),a}P_{\text{circ}}\hat{e}_b \quad (42)$$

For a total of 3 independent components

E. Magnetic bulk

1. Linear photon drag effect

$$J_a = q_a(\chi_{aaaa}^{(3),s}|E_a|^2 + \chi_{aabb}^{(3),s}|E_b|^2 + \chi_{aacc}^{(3),s}|E_c|^2) + q_a\chi_{aaac}^{(3),s}[E_a, E_c] + q_b(\chi_{abbc}^{(3),s}[E_b, E_c] + \chi_{abab}^{(3),s}[E_a, E_b]) + q_c(\chi_{acaa}^{(3),s}|E_a|^2 + \chi_{acbb}^{(3),s}|E_b|^2 + \chi_{accc}^{(3),s}|E_c|^2 + \chi_{acac}^{(3),s}[E_a, E_c]) \quad (43)$$

$$J_b = q_b(\chi_{bbaa}^{(3),s}|E_a|^2 + \chi_{bbbb}^{(3),s}|E_b|^2 + \chi_{bbcc}^{(3),s}|E_c|^2) + q_b\chi_{bbac}^{(3),s}[E_a, E_c] + q_a(\chi_{babc}^{(3),s}[E_b, E_c] + \chi_{baab}^{(3),s}[E_a, E_b]) + q_c(\chi_{bcbc}^{(3),s}[E_b, E_c] + \chi_{bcab}^{(3),s}[E_a, E_b]) \quad (44)$$

$$J_c = q_a(\chi_{caaa}^{(3),s}|E_a|^2 + \chi_{cabb}^{(3),s}|E_b|^2 + \chi_{cacc}^{(3),s}|E_c|^2) + q_a\chi_{caac}^{(3),s}[E_a, E_c] + q_b(\chi_{cbbc}^{(3),s}[E_b, E_c] + \chi_{cbab}^{(3),s}[E_a, E_b]) + q_c(\chi_{ccaa}^{(3),s}|E_a|^2 + \chi_{ccbb}^{(3),s}|E_b|^2 + \chi_{cccc}^{(3),s}|E_c|^2 + \chi_{ccac}^{(3),s}[E_a, E_c]) \quad (45)$$

For a total of 28 independent components.

2. Circular photon drag effect

$$J_a = -q_a\chi_{aab}^{(3),a}P_{\text{circ}}\hat{e}_b - q_c\chi_{acb}^{(3),a}P_{\text{circ}}\hat{e}_b + q_b(\chi_{abc}^{(3),a}P_{\text{circ}}\hat{e}_c + \chi_{aba}^{(3),a}P_{\text{circ}}\hat{e}_a) \quad (46)$$

$$J_b = q_a(\chi_{bac}^{(3),a}P_{\text{circ}}\hat{e}_c + \chi_{baa}^{(3),a}P_{\text{circ}}\hat{e}_a) + q_c(\chi_{bcc}^{(3),a}P_{\text{circ}}\hat{e}_c + \chi_{bca}^{(3),a}P_{\text{circ}}\hat{e}_a) - q_b\chi_{bbb}^{(3),a}P_{\text{circ}}\hat{e}_b \quad (47)$$

$$J_c = -q_a\chi_{cab}^{(3),a}P_{\text{circ}}\hat{e}_b - q_c\chi_{ccb}^{(3),a}P_{\text{circ}}\hat{e}_b + q_b(\chi_{cbc}^{(3),a}P_{\text{circ}}\hat{e}_c + \chi_{cba}^{(3),a}P_{\text{circ}}\hat{e}_a) \quad (48)$$

For a total of 13 independent coefficients.

F. Non-magnetic surface

1. Linear photogalvanic effect

$$J_a = \chi_{aaa}^{(2),s}(|E_a|^2 - |E_b|^2) + \chi_{aac}^{(2),s}[E_a, E_c] \quad (49)$$

$$J_b = \chi_{aac}^{(2),s}[E_b, E_c] - \chi_{aaa}^{(2),s}[E_a, E_b] \quad (50)$$

$$J_c = \chi_{caa}^{(2),s}(|E_a|^2 + |E_b|^2) + \chi_{ccc}^{(2),s}|E_c|^2 \quad (51)$$

For a total of 4 independent coefficients.

2. Circular photogalvanic effect

$$J_a = -\chi_{ab}^{(2),a} P_{\text{circ}} \hat{e}_b \quad (52)$$

$$J_b = \chi_{ab}^{(2),a} P_{\text{circ}} \hat{e}_a \quad (53)$$

$$J_c = 0 \quad (54)$$

With a single independent parameter.

3. Linear photon drag effect

$$\begin{aligned} J_a = & q_a(\chi_{aaaa}^{(3),s}|E_a|^2 + \chi_{aabb}^{(3),s}|E_b|^2 + \chi_{aac}^{(3),s}|E_c|^2) \\ & + q_a\chi_{aac}^{(3),s}[E_a, E_c] + \frac{q_b}{2}(\chi_{aaaa}^{(3),s} - \chi_{aabb}^{(3),s})[E_a, E_b] \\ & - q_b\chi_{aac}^{(3),s}[E_b, E_c] + q_c\chi_{acaa}^{(3),s}(|E_a|^2 - |E_b|^2) + q_c\chi_{acac}^{(3),s}[E_a, E_c] \end{aligned} \quad (55)$$

$$\begin{aligned} J_b = & q_b(\chi_{aabb}^{(3),s}|E_a|^2 + \chi_{aaaa}^{(3),s}|E_b|^2 + \chi_{aac}^{(3),s}|E_c|^2) \\ & - q_b\chi_{aac}^{(3),s}[E_a, E_c] - q_a\chi_{aac}^{(3),s}[E_b, E_c] \\ & + \frac{q_a}{2}(\chi_{aaaa}^{(3),s} - \chi_{aabb}^{(3),s})[E_a, E_b] + q_c(\chi_{acac}^{(3),s}[E_b, E_c] - \chi_{acaa}^{(3),s}[E_a, E_b]) \end{aligned} \quad (56)$$

$$\begin{aligned} J_c = & q_c(\chi_{ccaa}^{(3),s}(|E_a|^2 + |E_b|^2) + \chi_{cccc}^{(3),s}|E_c|^2) \\ & + q_a(\chi_{caaa}^{(3),s}(|E_a|^2 - |E_b|^2) + \chi_{caac}^{(3),s}[E_a, E_c]) \\ & + q_b(\chi_{caac}^{(3),s}[E_b, E_c] - \chi_{caaa}^{(3),s}[E_a, E_b]) \end{aligned} \quad (57)$$

For a total of 10 independent components.

4. Circular photon drag effect

$$J_a = q_b P_{\text{circ}}(\chi_{abc}^{(3),a} \hat{e}_c - \chi_{aab}^{(3),a} \hat{e}_a) - \chi_{aab}^{(3),a} q_a P_{\text{circ}} \hat{e}_b - \chi_{acb}^{(3),a} q_c P_{\text{circ}} \hat{e}_b \quad (58)$$

$$J_b = -q_a P_{\text{circ}}(\chi_{abc}^{(3),a} \hat{e}_c + \chi_{aab}^{(3),a} \hat{e}_a) + q_b P_{\text{circ}} \chi_{aab}^{(3),a} \hat{e}_b + q_c P_{\text{circ}} \chi_{acb}^{(3),a} \hat{e}_a \quad (59)$$

$$J_c = \chi_{cab}^{(3),a} P_{\text{circ}}(q_b \hat{e}_a - q_a \hat{e}_b) \quad (60)$$

For a total of 4 independent components.

G. Magnetic surface

As the unitary part of the symmetry group is 1 on the surface when considering magnetic symmetries, all terms are allowed.

SUPPLEMENTARY REFERENCE

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