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Top quark production at the Tevatron at NNLO[1]

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Abstract

We present results for top quark production at the Tevatron including next-to-next-to-leading order (NNLO) soft-gluon corrections. We show the stability of the cross section with respect to kinematics choice and scale when the NNLO corrections are taken into account.

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1 Introduction

Improved theoretical calculations of the top quark production cross sections and differential distributions are becoming more important as the top is observed at Tevatron Run II, where increasingly accurate measurements of the top mass and cross section are expected. The current state-of-the-art in the theory of top hadroproduction is calculations of next-to-next-to-leading order (NNLO) soft-gluon corrections to the double differential cross section [1, 2] from threshold resummation techniques [3, 4, 5]. Near threshold there is limited phase space for the emission of real gluons so that soft-gluon corrections dominate the cross section.

These soft corrections appear in the form of logarithmic "plus" distributions $[\ln^l(x_{\rm th})/x_{\rm th}]_+$, with $l \le 2n-1$ for the n-th order corrections in α_s, where $x_{\rm th}$ is a kinematical variable that measures distance from threshold. Calculations for top quark production until recently had been done through next-to-next-to-leading logarithmic (NNLL) accuracy, i.e. including logarithms with $l = 3, 2, 1$ at NNLO [6, 7]. This NNLO-NNLL calculation greatly diminished the factorization and renormalization scale dependence of the cross section. However, the dependence of the corrections on the choice of kinematics, single-particle-inclusive (1PI) or pair-invariant-mass (PIM), was substantial even near threshold. Away from threshold, where hard real gluonic radiation becomes non-negligible, the discrepancy between 1PI and PIM results is not surprising. However, near threshold the results should be the same if all the NNLO soft corrections are included. Thus subleading (beyond NNLL) contributions can still have an impact on the cross section.

We have recently calculated next-to-next-to-next-to-leading logarithmic (NNNLL) terms at NNLO in Ref. [1], i.e. terms with $l = 0$, following the unified approach of Ref. [2]. We find that the inclusion of NNNLL terms indeed eliminates the kinematics ambiguity near threshold.

2 Analytical results

We first discuss the analytical expressions for the corrections. We begin with the next-to-leading order (NLO) corrections. In the $\overline{\rm MS}$ scheme, the NLO soft and virtual corrections for $q\overline{q} \to t\overline{t}$ in 1PI kinematics are

$$s^2 \, \frac{d^2\hat{\sigma}^{(1)\,{\rm 1PI}}_{q\overline{q}}}{dt_1 \, du_1} = F^{B\;{\rm 1PI}}_{q\overline{q}} \frac{\alpha_s(\mu_R^2)}{\pi} \left\{ c^{\rm 1PI}_{3\,q\overline{q}} \left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + c^{\rm 1PI}_{2\,q\overline{q}} \left[\frac{1}{s_4}\right]_+ + c^{\rm 1PI}_{1\,q\overline{q}} \, \delta(s_4) \right\} , \tag{2.1}$$

where $F^{B\;{\rm 1PI}}_{q\overline{q}}$ is the Born term and μ_R the renormalization scale. Also $c^{\rm 1PI}_{3\,q\overline{q}} = 4C_F$ and

$$\begin{aligned} c^{\rm 1PI}_{2\,q\overline{q}} &= C_A \left[-3\ln\left(\frac{u_1}{t_1}\right) - \ln\left(\frac{m^2 s}{t_1 u_1}\right) + L'_\beta\right] \\ &\quad + 2C_F \left[4\ln\left(\frac{u_1}{t_1}\right) - \ln\left(\frac{t_1 u_1}{m^4}\right) - L'_\beta - 1 - \ln\left(\frac{\mu_F^2}{s}\right)\right] \end{aligned} \tag{2.2}$$

where μ_F is the factorization scale, $C_A = N_c = 3$ is the number of colors, $C_F = (N_c^2-1)/(2N_c)$, and $L'_\beta = [(1-2m^2/s)/\beta]\ln[(1-\beta)/(1+\beta)]$ with $\beta = \sqrt{1-4m^2/s}$. For use below, we write $c^{\rm 1PI}_{2\,q\overline{q}} \equiv T^{\rm 1PI}_{2\,q\overline{q}} - 2C_F\ln(\mu_F^2/s)$. Finally, $c^{\rm 1PI}_{1\,q\overline{q}} = \sigma^{(1)\,S+V\;{\rm 1PI}}_{q\overline{q}\,\delta}/[(\alpha_s/\pi)F^{B\;{\rm 1PI}}_{q\overline{q}}]$ where $\sigma^{(1)\,S+V\;{\rm 1PI}}_{q\overline{q}\,\delta}$ denotes the $\delta(s_4)$ terms in Eq. (4.7) of Ref. [8] with the definitions of t_1 and u_1 interchanged with respect to that reference. We also define $c_{1\,q\overline{q}}^{\rm 1PI} \equiv T_{1\,q\overline{q}}^{\rm 1PI} + C_F[-3/2 + \ln(t_1 u_1/m^4)] \ln(\mu_F^2/s) + (\beta_0/2)\ln(\mu_R^2/s)$, where $T_{1\,q\overline{q}}^{\rm 1PI}$ has no scale dependence.

We further define the constants $\zeta_2 = \pi^2/6$, and $\zeta_3 = 1.2020569\cdots$, $\beta_0 = (11C_A - 2n_f)/3$, and $K = C_A(67/18 - \pi^2/6) - 5n_f/9$ where n_f is the number of light quark flavors.

Following Ref. [2] we write the NNLO soft-plus-virtual corrections in 1PI kinematics as

$$
\begin{aligned}
s^2 \frac{d^2\hat{\sigma}_{q\overline{q}}^{(2)\,\rm 1PI}}{dt_1\,du_1} &= F_{q\overline{q}}^{B\,\rm 1PI} \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \frac{1}{2}\left(c_{3\,q\overline{q}}^{\rm 1PI}\right)^2 \left[\frac{\ln^3(s_4/m^2)}{s_4}\right]_+ + \left[\frac{3}{2} c_{3\,q\overline{q}}^{\rm 1PI}\, c_{2\,q\overline{q}}^{\rm 1PI} - \frac{\beta_0}{4} c_{3\,q\overline{q}}^{\rm 1PI}\right] \left[\frac{\ln^2(s_4/m^2)}{s_4}\right]_+ \right. \\
&+ \left[c_{3\,q\overline{q}}^{\rm 1PI}\, c_{1\,q\overline{q}}^{\rm 1PI} + \left(c_{2\,q\overline{q}}^{\rm 1PI}\right)^2 - \zeta_2 \left(c_{3\,q\overline{q}}^{\rm 1PI}\right)^2 - \frac{\beta_0}{2} T_{2\,q\overline{q}}^{\rm 1PI} + \frac{\beta_0}{4} c_{3\,q\overline{q}}^{\rm 1PI} \ln\left(\frac{\mu_R^2}{s}\right) \right. \\
&\quad \left. + 2C_F K + 8\frac{C_F}{C_A}\ln^2\left(\frac{u_1}{t_1}\right)\right] \left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ \\
&+ \left[c_{2\,q\overline{q}}^{\rm 1PI}\, c_{1\,q\overline{q}}^{\rm 1PI} - \zeta_2\, c_{2\,q\overline{q}}^{\rm 1PI}\, c_{3\,q\overline{q}}^{\rm 1PI} + \zeta_3 \left(c_{3\,q\overline{q}}^{\rm 1PI}\right)^2 - \frac{\beta_0}{2} T_{1\,q\overline{q}}^{\rm 1PI} + \frac{\beta_0}{4} c_{2\,q\overline{q}}^{\rm 1PI} \ln\left(\frac{\mu_R^2}{s}\right) + \mathcal{G}_{q\overline{q}}^{(2)} \right. \\
&\quad \left. + C_F \frac{\beta_0}{4}\ln^2\left(\frac{\mu_F^2}{s}\right) - C_F K \ln\left(\frac{\mu_F^2}{s}\right) - C_F K \ln\left(\frac{t_1 u_1}{m^4}\right) + 8\frac{C_F}{C_A}\ln^2\left(\frac{u_1}{t_1}\right)\ln\left(\frac{m^2}{s}\right)\right] \left[\frac{1}{s_4}\right]_+ \\
&\left. + R_{q\overline{q}}^{\rm 1PI}\,\delta(s_4)\right\} . \qquad (2.3)
\end{aligned}
$$

Here the term $\mathcal{G}_{q\overline{q}}^{(2)} = C_F C_A(7\zeta_3/2 + 22\zeta_2/3 - 299/27) + n_f C_F(-4\zeta_2/3 + 50/27)$ denotes a set of two-loop contributions universal for processes with $q\overline{q}$ initial states [2]. Process-dependent two-loop corrections are not included in $\mathcal{G}_{q\overline{q}}^{(2)}$ but their contribution is expected to be negligible. The virtual contribution $R_{q\overline{q}}^{\rm 1PI}$ is not fully known. However, we can determine certain terms in $R_{q\overline{q}}^{\rm 1PI}$ exactly. These exact terms involve the factorization and renormalization scales as well as the those terms (ζ terms) that arise from the inversion from moment to momentum space [1].

The analytical form of the NLO and NNLO corrections for the $q\overline{q}$ channel in PIM kinematics is similar, see Ref. [1]. The expressions for the gg channel in both 1PI and PIM kinematics are more involved since the gg color structure is more complex [1].

3 Numerical results

We now study the numerical impact of the corrections. The total partonic cross section may be expressed in terms of dimensionless scaling functions $f_{ij}^{(k,l)}$ that depend only on $\eta \equiv s/4m^2 - 1$ [7],

$$
\sigma_{ij} = \frac{\alpha_s^2(\mu)}{m^2} \sum_{k=0}^{\infty} \left(4\pi\alpha_s(\mu)\right)^k \sum_{l=0}^{k} f_{ij}^{(k,l)}(\eta)\ \ln^l\left(\frac{\mu^2}{m^2}\right) . \qquad (3.1)
$$

Here we have set $\mu \equiv \mu_F = \mu_R$.

In Fig. 1 we plot the $f_{ij}^{(2,0)}$ scaling functions, the most important contributions at NNLO and independent of μ. To demonstrate the effect of adding successive subleading contributions, we show the NNLL results in the upper plots, the scaling functions through NNNLL in the middle plots, and the results with the NNNLL and virtual ζ terms in the lower plots.

We first discuss the results for the $q\overline{q}$ channel. We note that to NNLL, the two kinematics choices give rather different results, even at low η. When the NNNLL terms are added the two results coincide near threshold. Adding the virtual ζ terms resulting from inversion improves

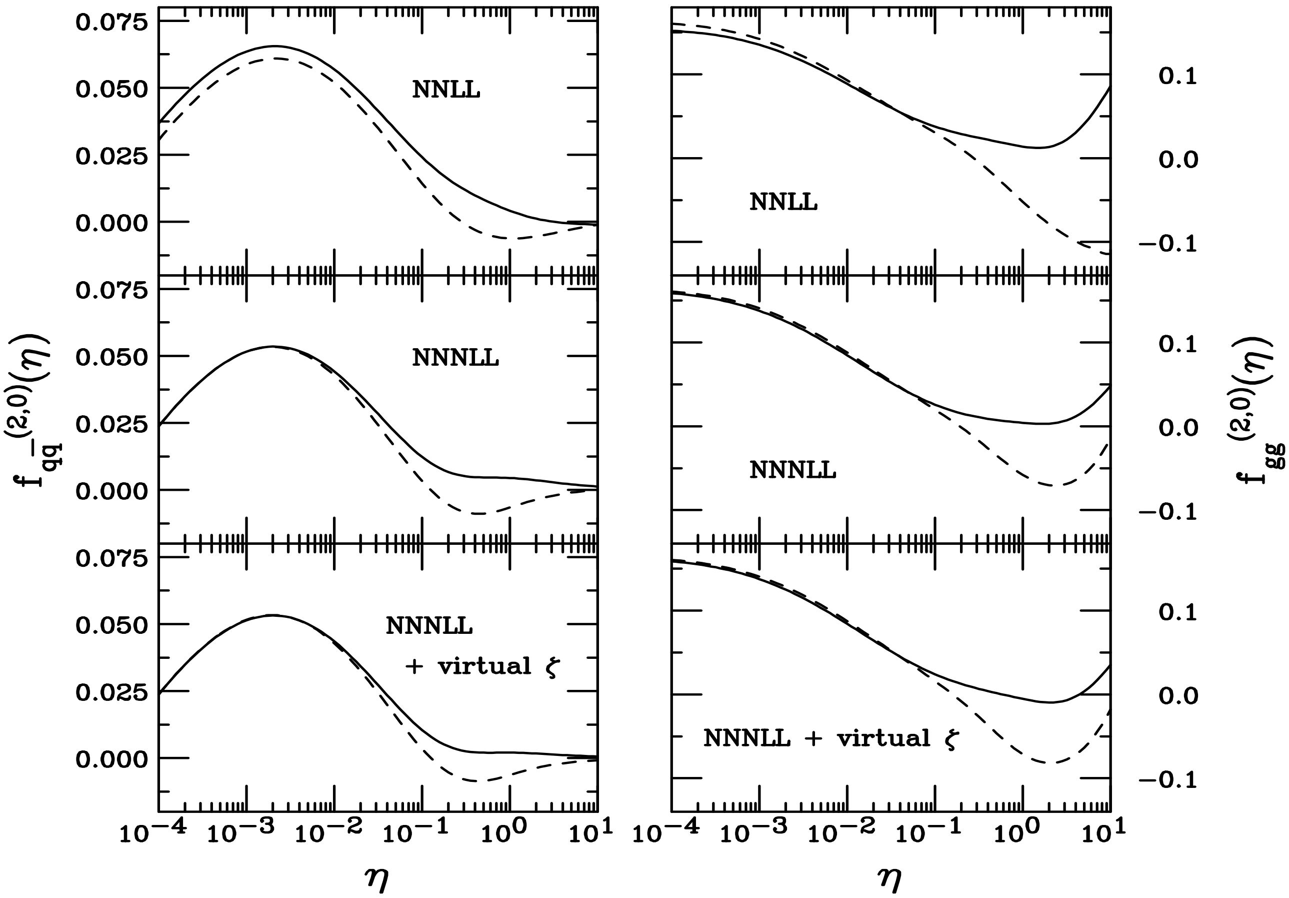

Figure 1: The $f_{ij}^{(2,0)}$ scaling functions in the $\overline{\rm MS}$ scheme for the $q\overline{q}$ (left) and gg (right) channels. The top plots show the NNLL result, the center plots the NNNLL results and the bottom plots give the results including the virtual ζ terms. The solid curves are for 1PI kinematics, the dashed for PIM kinematics.

the agreement between the 1PI and PIM kinematics further. This agreement also indicates that additional two-loop contributions not included in our expressions should be small. A similar trend is seen for the gg channel on the right-hand side of Fig. 1. The agreement between the NNLL 1PI and PIM scaling functions at low η is significantly better than in the $q\overline{q}$ channel. Note however the significant divergence at large η. Again, inclusion of the subleading contributions improves agreement over all η. The improvement at larger η, $\eta > 0.1$ is notable. We remark that the effect of the virtual ζ terms is numerically small for both channels, in agreement with the arguments in Section IIIC of Ref. [6].

We now turn to our calculations of the hadronic total cross sections. We use the recent MRST2002 NNLO (approximate) parton densities [9] with an NNLO evaluation of α_s. Our calculations use the exact LO and NLO cross sections with the soft NNNLL and virtual ζ corrections and the full soft-plus-virtual scale-dependent terms at NNLO. In addition we multiply the NNLO scaling functions by a damping factor, $1/\sqrt{1+\eta}$, as in Ref. [7], to lessen the influence of the large η region where the threshold approximation does not hold so well.

In Fig. 2, we present the NLO and approximate NNLO $t\bar{t}$ cross sections at $\sqrt{S} = 1.8$ TeV (left-hand side) and 1.96 TeV (right-hand side) as functions of top quark mass for $\mu = m$. We also show the average of the two kinematics results. Both the 1PI and PIM cross sections are

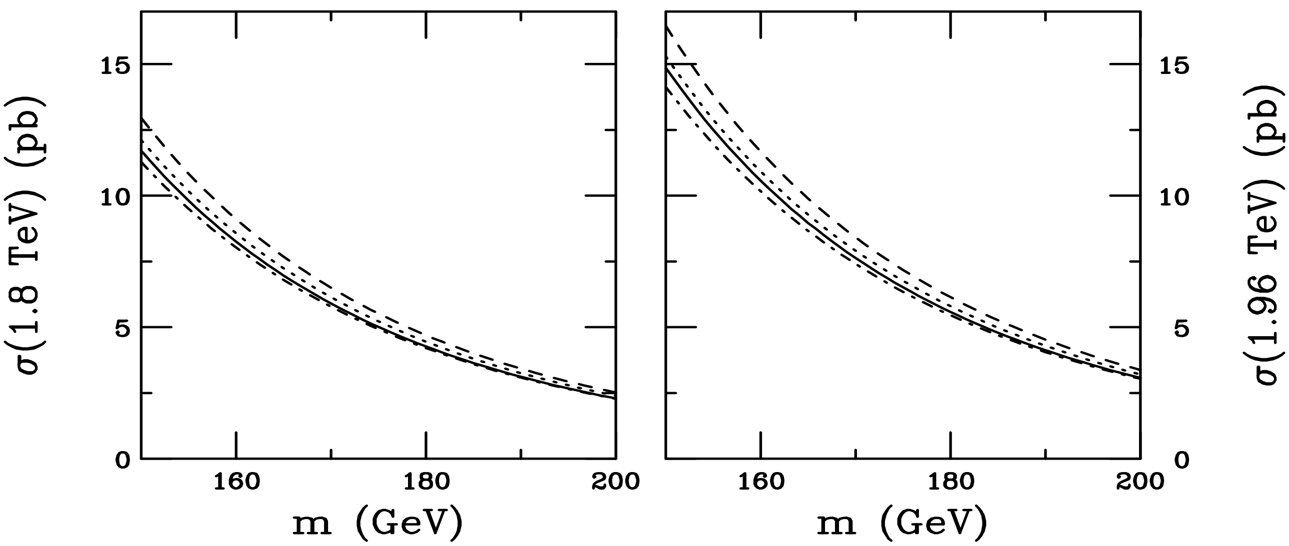

Figure 2: The $t\bar{t}$ total cross sections in $p\bar{p}$ collisions at $\sqrt{S} = 1.8$ TeV (left-hand side) and 1.96 TeV (right-hand side) as functions of m for $\mu = m$. The NLO (solid), and approximate NNLO 1PI (dashed), PIM (dot-dashed) and average (dotted) results are shown.

reduced due to the subleading terms. The average of the two kinematics results is just above the NLO cross sections for both energies.

Our results using the CTEQ6M NLO parton densities [10] are similar. At $\sqrt{S} = 1.8$ TeV, averaging over the NNLO 1PI and PIM results with the two sets of parton distributions at $\mu = m = 175$ GeV, our best estimate for the cross section is 5.24 ± 0.31 pb where the quoted uncertainty is due to the kinematics choice. At $\sqrt{S} = 1.96$ TeV our corresponding best estimate is 6.77 ± 0.42 pb. We note that differences with our previous estimates in [6, 7] have as much to do with the use of the new parton densities as with the inclusion of the new subleading terms. We also note that the scale dependence of the NNLO cross section is negligible [1].

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