EXPLORING THE ROLE OF PEDAGOGY IN MATHEMATICAL CREATIVITY VIA MULTIPLE SOLUTION TASKS: A COMPARATIVE STUDY OF TWO SCHOOLS IN CHINA

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This study aimed to explore the relationship between pedagogy and mathematical creativity by comparing the mathematical creativity of students who experienced more student-centred pedagogy (SCP) with that of students who experienced more teacher-centred pedagogy (TCP). In total, 163 Grade 9 students from two schools in China, each enacting primarily one of these forms of pedagogy, participated in the study. Multiple solution tasks (MST) were used to measure mathematical creativity in terms of fluency, flexibility, originality. The total mean scores for fluency, flexibility, and originality of the SCP school were all higher than the respective scores of the TCP school though the differences were not statistically significant. Implications for research and practice are discussed in light of these findings.

INTRODUCTION

Creativity plays a crucial role in the full cycle of advanced mathematical thinking. Giftedness in mathematics assessments does not necessarily imply mathematical creativity (Sriraman, 2005), which requires divergent thinking that might not be covered in a national mathematics assessment. An example of this would be Chinese students who attained high scores in many international mathematics assessments but are in need of improvement in mathematical creativity (OCED, 2014). Relatedly, Lu and Kaiser (2022) conducted an empirical study among 107 Chinese upper secondary students, and found low levels of fluency and originality among participants reflecting their difficulties with attempting diverse ways to solve tasks. Regarding ways to foster mathematical creativity, some researchers suggested that, in contrast with teacher-centred pedagogy (TCP), student-centred pedagogy (SCP) has such a potential (e.g., Silver, 1997). Yet the relationship between pedagogy and mathematical creativity has attracted little research attention thus far. This study takes a step towards addressing this need for research by focusing on the following research question: Are there differences in mathematical creativity between students who have experienced more SCP and students who have experienced more TCP?

THEORETICAL PERSPECTIVES

The perspectives on TCP, SCP, and mathematical creativity are described as follows.

In a learning environment with TCP, the teacher primarily communicates to students through lectures solely designed to impart knowledge. SCP, in contrast, provides a learning environment where students construct their understanding, and teachers act as facilitators to "guide on the side" and help students achieve goals. While TCP is

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based on the behaviorist theory in which external stimuli causes behaviour changes, SCP involves constructivist and democratic principles where much knowledge is socially constructed (Serin, 2018). In this study, students categorized as having experienced more SCP did not necessarily only experience SCP-based lessons but did learn in an environment with more SCP compared to the comparative group.

The mathematical creativity in this study refers to the relative creativity, or creativity in school mathematics, which differs from that of professional mathematicians in that relative creativity is evaluated with reference to students' previous experiences and to the performance of other students who have a similar educational history (Leikin, 2009). Therefore, in this study mathematical creativity is to generate novel/original mathematical ideas, which are new to the person or the performance of other students in the similar educational history, with respect to the mathematics they have learned by discerning acceptable mathematical problems and models. I measured creativity in terms of three cognitive outcomes: fluency, flexibility, and originality. Fluency refers to the total number of appropriate problems generated by a solver; flexibility refers to the total number of strategies generated; originality refers to the uniqueness of one's solutions compared to others' response across two schools (Leikin, 2009).

METHODOLOGY

Comparative case study

In this study, School X and School Y (pseudonym) were selected as cases to play the role of SCP and TCP, respectively. Four Grade 9 classrooms, two from each school, were then randomly selected. In total, 83 School X students and 80 School Y students participated in this study. Importantly, Grade 9 is the third year of Chinese secondary education, and participants from the same school have received the same school instruction since Grade 7.

The two schools were selected due to having comparable features but different pedagogical approaches. Regarding similarities, firstly, both schools are in rural towns under the same county of the same city, so the schools follow the same educational policies and have similar economic conditions, though Y town has slightly better economic development and a better geographic location. Secondly, both schools randomly divide students into classrooms rather than dividing them based on achievement. Thirdly, both are the only school in their respective towns, both of which require recruiting students only from within the district; thus, both schools have similar sources for students. These similarities provide some control over confounding variables and allow for a meaningful comparison. Regarding the differences, School Y is one of the best-performing schools among all 14 towns and employs TCP, while School X is the most popular school among all towns due to its reformed SCP. School X has been using a reformed pedagogy for more than two decades and exemplifies the result of Chinese compulsory education reform (Sun & Wang, 2011).

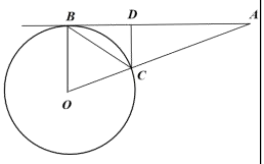
I used the RTOP observation protocol (Piburn et al., 2000), which was developed to evaluate the extent to which a classroom adopts reform-based pedagogy, to verify that School X's mathematical pedagogy was more student-centred than that of School Y. The total RTOP score ranges between 0 and 100, in which lower scores reflect TCP environments and higher scores represent SCP environments. Specifically, the RTOP results showed that School X scored 21.4 higher on average than School Y. In total, four School X teachers (11 lessons) and five School Y teachers (13 lessons) were observed. All four participated classrooms were observed for at least three consecutive lessons, and the rest non-participated classrooms were randomly selected from each grade (Grade 7, 8, 9) for observation.

Due to the schools each having a distinct pedagogy, the class schedule also significantly differed between them. Specifically, School X students received eight mathematics lessons per week (45 minutes/lesson), while School Y students received seven mathematics teaching lessons and six mathematics self-study lessons (40 minutes/lesson) each week. Self-study lessons refer to the periods that students independently work on the assigned problems while the teacher simply monitors. Thus, each week School Y students received 160 more minutes of compulsory mathematics discipline than School X students.

Design of the multiple solution task

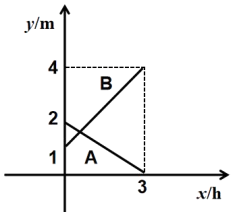
Solve the following problem in as many ways as possible.

Task A (Geometry)



The straight line *AB* is tangent to the circle with center *O* in point *B*. *OA* intersects the circle in point *C*. *D* is on *AB* so that *CD* is perpendicular to *AB (see figure)*. Prove that $\angle BCD = \angle BCO$.

Task B (Functional word problem)



A company has two cuboid reservoirs A and B. The water in reservoir A is injected into reservoir B at a speed of 6 m^3per hour. The functional relationship between water depth y (m) and injection time x (h) in reservoirs A and B is shown in the figure below. The functional relationship between the water depth y and the injection time x in the reservoirs A and B is: $y_A = -\frac{2x}{3} + 2$, $y_B = x+1$. After injecting $\frac{3}{5}$ hours, the depths of water in the two pools are the same. So, how long will the two reservoirs' water storage capacity be the same?

Figure 1. Tasks used in this study

This study used multiple solution tasks (MST) to indicate mathematical creativity. Two tasks, listed in Figure 1, were chosen for their coverage of geometry and functional word problems to assess multiple facets of students' conceptual and

procedural knowledge. Task A is at an easy level of difficulty and Task B is at a difficult level. Circle related geometry and linear function are both important contents examined in Zhongkao, the Chinese official Senior High School Entrance Examination held annually at the end of Grade 9.

Analysis method

Problems solved by students were first analysed based on appropriateness. The notion of appropriateness allows evaluating reasonable ways of solving a problem that potentially led to the correct solution outcome regardless of the minor mistakes made by a solver (Leikin, 2009). The data were then analysed based on fluency, flexibility, and originality. The detailed scoring scheme is explained in Table 1, which was adapted from Leikin (2009) in that students were given half credit for the partial correct procedure rather than an absolute zero or a full credit.

	Fluency	**Flexibility**	**Originality**	**Creativity**
Scores per solution	Flu_i=1 solution is appropriate Flu_i=0.5 solution is partial appropriate Flu_i=0	Flx_i=10 solutions from a different group of strategies Flx_i=1 similar strategy but a different representation Flx_i=0.1 same strategy, same representation If Flu_i=0.5, Flx_i=Flx_i/2	Or_i=10 (P<15%) Or_i=1 15%≤P<40 Or_i=0.1 (P≥40)	$Flx_i \times Or_i$
Total Score	$\sum_{i=1}^{n} Flu_i$	$\sum_{i=1}^{n} Flx_i$	$\sum_{i=1}^{n} Or_i$	$\sum_{i=1}^{n} Flx_i Or_i$

n is the total number of appropriate solutions, including partial appropriate solutions.

P=(m_j/n)·100%, where m_j is the number of students who used strategy j.

Table 1. Creativity scoring scheme, adapted from Leikin (2009)

FINDINGS

Fluency, Flexibility and Originality

For Task A, School Y participants generated 81 appropriate solutions and 5 partial appropriate solutions; School X students generated 108 appropriate solutions and 1 partial appropriate solution. For Task B, School X produced 7 and School Y produced 8 appropriate solutions. Only six students from each school generated an

appropriate solution(s) for task B, in which only one student from each school came up with two different strategies.

Table 2 and 3 demonstrate the number of students (n) generating the corresponding strategy, in which strategy A comprises of four different sub-strategies (A1, A2, A3, A4). For Task A, School X's solutions comprise of ten subcategories and School Y covers eight subcategories. For Task B, School X's solution comprise of two categories and School Y covers three categories. The value in the brackets represents the percentage of students within their schools' participants. For example, 14.4% School X participants and 6.3% School Y participants used strategy C. The corresponding originality is also described in both tables. In total, 21.7% School X and 15% School Y participants generated original solutions. School X have more participants who were able to generate original solutions among these two tasks.

# of students / Strategy	School X (n = 83)				School Y (n = 80)				Originality
Strategy A	A1	A2	A3	A4	A1	A2	A3	A4	0.1 for 1.1 to 1.3; 10 for 1.4
	59 (71%)	5 (6%)	2 (2.4%)	2 (2.4%)	41 (51%)	0	3 (3.8%)	0	
Strategy B	26 (31.3%)				31 (38.8%)				1
Strategy C	12 (14.4%)				5 (6.3%)				10
Strategy D	1 (1.2%)				2 (2.5%)				10
Strategy E	1 (1.2%)				1 (1.3%)				10
Strategy F	1 (1.2%)				1 (1.3%)				10
Strategy G	1 (1.2%)				0				10
Strategy H	0				1 (1.3%)				10

Table 2. Distribution of categories of solutions solved by participants (Task A)

	School X	School Y	Originality
Strategy I	5 (6%)	2 (2.5%)	0.1
Strategy II	2 (2.4%)	4 (5)	0.1
Strategy III	0	1 (1.3%)	10

Table 3. Distribution of categories of solutions solved by participants (Task B)

As indicated in Table 4, The total mean fluency, flexibility, and originality of School X participants were all higher than the respective scores of School Y students. However, the Mann-Whitney U test suggests such differences are not statistically

significant, with the p-value of 0.105, 0.115, 0.301 for fluency, flexibility, and originality, which attributes to the small sample size, the scoring scheme's nature, and the difficulty of Task B.

	Fluency	Flexibility	Originality	Creativity
	mean±SD	mean±SD	mean±SD	mean±SD
School X	1.38±1.17	13.00±10.86	2.68±5.35	22.38±43.17
School Y	1.14±1.21	10.68±10.62	1.93±4.63	17.34±42.44

Table 4. Mean and SD of Total Creativity for both tasks

Creativity

All participants were ranked based on their total creativity score. 28% School X and 35% School Y participants obtained a zero score, and thirteen out of the top twenty performers are School X students. Specifically, for Task A, eight out of the top ten performers are School X students. However, according to Mann-Whitney U test, such differences are not statistically significant (p-value: 0.248).

Level	**Characteristic of Creative Thinking Level**
Level 0 (Not Creative)	Students were not able to show any components of creativity ($Cr = 0$)
Level 1 (Almost Not Creative)	Students were able to show fluency without or with low originality and flexibility in solving problem ($Flu_i>0$, $Ori_i = 0.1$ and $Flx_{i\neq1}< 10$)
Level 2 (Quite Creative)	Students were able to show flexibility or originality in solving problem with low fluency ($0 < Flu_i\leq1$, $Ori_i > 0.1$ or $Flx_{i\neq1}= 10$)
Level 3 (Creative)	Students were fluent and then they were flexible or demonstrate originality ($Flu_i>1$, $Flx_{i\neq1}= 10$ or $Ori_i=10$)
Level 4 (Very Creative)	Students satisfied all components of creativity ($Flu_i > 1$, $Flx_{i\neq1}=10$ and $Ori_i=10$)

Table 5. Creative Thinking Level, Adapted from Tatag (2011).

To further analyse the two schools' performance, I grouped students' responses into Creative Thinking Level (CTL), which is the level I adapted from Tatag (2011) to classify students' creativity with relevance to the rest of the group. The indicators of CTL are explained in Table 5, and the CTL results are indicated in Table 6. The

following ordinal logistic regression model was employed: logit ($P(Y \leq k|S)$) = $\log_e(\frac{P(Y\leq k|S)}{1-P(Y\leq k|S)})$, where Y denotes the CTL (k = 0, 1, 2, 3, 4) and S denotes the school (0 for School Y, 1 for School X). The model suggests that the odds of School X students obtaining a higher CTL for Task A is 1.455 times, and for Task B is 0.959 times, as large as it is for School Y students. The estimated differences between the two schools are smaller for Task B than for Task A.

# of students	Level 0	Level 1	Level 2	Level 3	Level 4
			Task A		
School X	23 (27.7%)	26 (31.3%)	0	19 (22.9%)	15 (18.1%)
School Y	28 (35%)	21 (25%)	8 (10%)	16 (20%)	7 (8.8%)
			Task B		
School X	77 (92.7%)	5 (6%)	0	1 (1.2%)	0
School Y	74 (92.5)	5 (6.3%)	0	0	1 (1.3%)

Table 6. Distribution of CTL

Inter-rater Reliability

Two raters independently coded at least 12% of the student responses from each task of each sample. The inter-rater agreements were 96.4%, 91.1%, 96.4%, and the Cohen's Kappa was 0.819, 0.741, 0.819 for fluency, flexibility, and originality.

CONCLUSIONS AND DISCUSSION

The analysis of the MST performance showed that the SCP participants outperformed the TCP participants in terms of the total creativity; however, due to the small sample sizes and 93% zero-achievers in Task B, the differences are not statistically significant. School Y students receiving 160 minutes/week more compulsory mathematics learning time might narrow the differences between the schools' MST performance, which remains a possible avenue for future exploration. Overall, my study results implicated that integrating SCP into secondary mathematics might avoid excessively subjecting students to intense discipline while achieving a similar/better level of mathematical creativity. Moreover, 7% fewer zero achievers among School X participants indicates that SCP might help improve problem solving among the low performers.

Students' performance on Task B suggests two things. First, the benefit of SCP when solving easy-level MST might exceed the benefit from solving difficult MST. Second, when tasks are too difficult, students' mathematical creativity via MST can be restricted to their mathematical knowledge and problem-solving skills. As a result, the valid data received would be insufficient, and individual results could dominate the group results. I therefore suggest tasks aiming to indicate creativity be set at an

easy or moderate level so that students' divergent thinking is not submerged. Although this study cannot guarantee the definitive causal claim between pedagogy and mathematical creativity, which attributes to the limitation of comparative studies: the quandary of "many variables, small-N" (Lijphart, 1971), it blazes a new path in this direction and underscores the need for more inquiry into this line of research. Future studies should increase school cases and further control pedagogy as the main variable to robustly investigate the relationship between pedagogy and mathematical creativity.

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