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MRST partons and uncertainties

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We discuss uncertainties in the extraction of parton distributions from global analyses of DIS and related data. We present *conservative* sets of partons, at both NLO and NNLO, which are stable to x, Q^2, W^2 cuts on the data. We give the corresponding values of $\alpha_S(M_Z^2)$ and the cross sections for W production at the Tevatron.

The parton distributions of the proton, which are currently determined from global analyses of a wide range of DIS and related hard scattering data, are subject to many sources of uncertainty. There are uncertainties due to (i) the experimental errors on the data that are fitted in the global analysis, (ii) the choice of data cuts ($W_{\rm cut}, x_{\rm cut}, Q^2_{\rm cut}$), defined such that data with values of W, x or Q^2 below the cut are excluded from the global fit, (iii) the truncation of the DGLAP perturbation expansion, (iv) specific theoretical effects, such as $\ln 1/x$, $\ln(1-x)$, absorptive and higher-twist corrections, and (v) input assumptions, such as isospin invariance, the choice of parameterization, heavy target corrections and the form of the strange quark sea.

So far, attention has been focussed on the uncertainties arising from the experimental errors; see Refs. [1, 2] for estimates based on global NLO analyses. However, Ref. [3] concentrates on the remaining uncertainties, (ii)–(v); here we present some results from this forthcoming paper. In principle, if the DGLAP formalism is valid and the various data sets are compatible, then changing the data that are included in the global analysis should not move the predictions outside the error bands. In practice this is not the case. Consider, for instance, the effect of different choices of $x_{\rm cut}$ on the data that are fitted. Table 1 shows the values of χ^2 for NLO global analyses performed for different values of $x_{\rm cut}$, together with the number of data points fitted. Each column represents the χ^2 values corresponding to a fit performed with a different choice of the cut in x.

$x_{\rm cut}$:	0	0.0002	0.001	0.0025	0.005	0.01
# data points	2097	2050	1961	1898	1826	1762
$\chi^2(x > 0)$	2267					
$\chi^2(x > 0.0002)$	2212	2203				
$\chi^2(x > 0.001)$	2134	2128	2119			
$\chi^2(x > 0.0025)$	2069	2064	2055	2040		
$\chi^2(x > 0.005)$	2024	2019	2012	1993	1973	
$\chi^2(x > 0.01)$	1965	1961	1953	1934	1917	1916
Δ_i^{i+1}		0.19	0.10	0.24	0.28	0.02

Table 1: The measure of stability, Δ_i^{i+1}, to changing the choice of $x_{\rm cut}$.

To obtain a measure of the stability of the analysis to changes in the choice of $x_{\rm cut}$, we compare fits in adjacent columns, that is with $(x_{\rm cut})_{i+1}$ and $(x_{\rm cut})_i$. In particular, it is informative to compare the contributions to their respective χ^2 values from the subset of data with $x > (x_{\rm cut})_{i+1}$. If stability were achieved, then we would expect the difference $\Delta\chi^2$ between these two χ^2 contributions to be very small. We stress that these two χ^2 contributions describe the quality of the two fits to the *same* subset of the data. Thus a measure, Δ_i^{i+1}, of the stability of the analysis is $\Delta\chi^2$ divided by the number of data points omitted when going from the fit with $(x_{\rm cut})_i$ to the fit with $(x_{\rm cut})_{i+1}$. For example, if we raise the $x_{\rm cut}$ from 0.001 to 0.0025 then $\Delta\chi^2 = 2055 - 2040$ for the data with $x > 0.0025$, and the number of data points omitted is $1961 - 1898 = 63$. Thus the measure $\Delta_{0.001}^{0.0025} = 15/63 = 0.24$, as shown in the last row of Table 1.

Inspection of the values of Δ_i^{i+1} shows a significant improvement in the quality of the fit each time $x_{\rm cut}$ is raised until the final step when $x_{\rm cut}$ is increased from 0.005 to 0.01, when we see that there is no further improvement at all. In fact, raising $x_{\rm cut}$ from 0.01 to 0.02 confirms this stability. Hence we conclude that $x \simeq 0.005$ is a safe choice of $x_{\rm cut}$.

After similar studies of the effect of varying $Q^2_{\rm cut}$ and $W^2_{\rm cut}$, and combinations of these cuts, we find a restricted domain ($x_{\rm cut} > 0.005$, $Q^2_{\rm cut} > 10$ GeV^2, $W^2 > 15$ GeV^2) in which the obtained NLO parton distributions are stable to further cuts. We denote this conservative parton set by MRST03(cons). Their behaviour relative to MRST02 [2] is shown in Fig. 1.

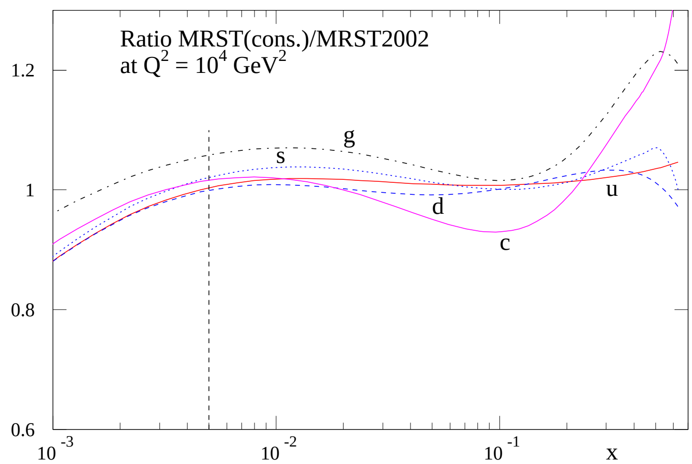

Figure 1: The *conservative* partons compared to MRST2002 [2].

Clearly there are uncertainties associated with truncation of the DGLAP evolution at NLO. Now, the DIS coefficient functions are known at NNLO [4]. Also valuable, almost complete, information has been obtained about the NNLO splitting functions [5]; indeed a range of compact analytic functions exist that are all compatible with this information [6]. We therefore study the effect of data cuts at NNLO. In going from NLO to NNLO the stable domain ($x_{\rm cut} > 0.005$, $Q^2_{\rm cut} > 7$ GeV^2, $W^2_{\rm cut} > 15$ GeV^2) has not increased as much as we might have hoped. Nevertheless there are advantages in going to NNLO [3].

To illustrate the value of having 'conservative' parton sets[1] at both NLO and NNLO we consider, as important examples, the determination of $\alpha_S(M_Z^2)$ from DIS data and the prediction of the cross section for W production, σ_W, at

[1] These sets are available at http://durpdg.dur.ac.uk/hepdata/mrs.html

the Tevatron. The values of α_S found in the NLO and NNLO global fits that produced the *conservative* sets of partons are given in Table 2 corresponding to the MRST03 entry, together with other recent determinations from DIS fits. The quoted errors reflect the tolerance $\Delta\chi^2$ used in the various analyses. Remarkably, the determinations of $\alpha_S(M_Z^2)$ have converged approximately to a common value, even though they are based on different selections of the data.

	$\Delta\chi^2$	$\alpha_S(M_Z^2)$	$\pm$ expt	$\pm$ theory	$\pm$ model
NLO					
CTEQ6	100	0.1165	$\pm\, 0.0065$		
ZEUS	50	0.1166	$\pm\, 0.0049$		$\pm\, 0.0018$
MRST03	5	0.1165	$\pm\, 0.002$	$\pm\, 0.003$	
H1	1	0.115	$\pm\, 0.0017$	$\pm\, 0.005$	${}^{+0.0009}_{-0.0005}$
Alekhin	1	0.1171	$\pm\, 0.0015$	$\pm\, 0.0033$	
NNLO					
MRST03	5	0.1153	$\pm\, 0.002$	$\pm\, 0.003$	
Alekhin	1	0.1143	$\pm\, 0.0014$	$\pm\, 0.0009$	

Table 2: The values of $\alpha_S(M_Z^2)$ found in NLO and NNLO fits to DIS data. The experimental errors quoted correspond to an increase $\Delta\chi^2$ from the best fit value of χ^2. CTEQ6 [7] and MRST03 are global fits. H1[8] fit only a subset of the F_2^{ep} data, while Alekhin [9] also includes F_2^{ed} and ZEUS [10] in addition include xF_3^{ν} data.

Fig. 2 shows values of the W production cross section (times the leptonic branching ratio $B_{l\nu} = 0.1068$) at the Tevatron energy $\sqrt{s} = 1.96$ TeV, obtained from NLO and NNLO global fits of data, subject to various values of $x_{\rm cut}$ and $Q^2_{\rm cut}$. We see that the NNLO predictions are much more stable to variations of $x_{\rm cut}$. Note that at NNLO the conservative parton set predicts a small decrease of 0.7% relative to the NNLO default prediction (with $x_{\rm cut} = 0$, $Q^2_{\rm cut} = 2$ GeV^2), while at NLO there is an increase. The conservative partons thus show greater convergence with increased perturbative order, than the default predictions. The NNLO conservative value $B_{l\nu}\sigma_W = 2.67$ nb, with an

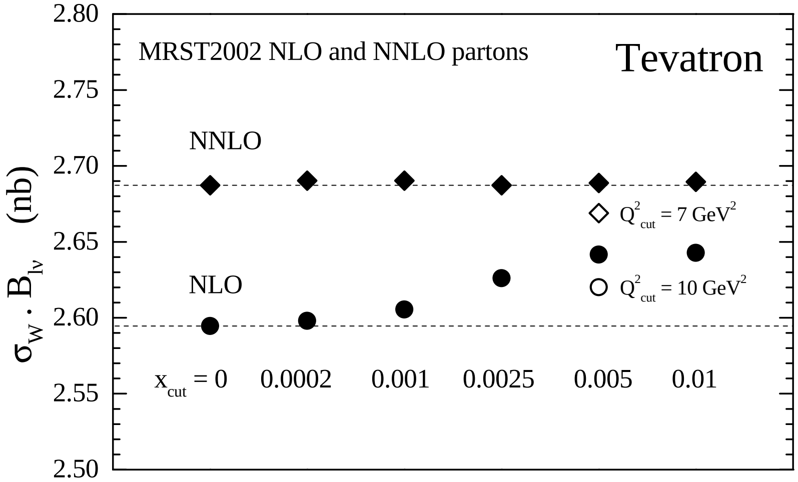

Figure 2: Predictions for W production at the Tevatron, $\sqrt{s} = 1.96$ TeV, for various values of x_{cut}, and for the conservative sets of partons (shown by the open symbols).

expected total theoretical and experimental uncertainty of about 2%, may act as a very good luminosity monitor at the Tevatron.

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