

Pinpointing Feshbach resonances and testing Efimov universalities in ^{39}K

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Using a combination of bound-state spectroscopy and loss spectroscopy, we pinpoint eight intrastate Feshbach resonances in ^{39}K , as well as six previously unexplored interstate ones. We perform a detailed characterization of four of the intrastate resonances and two of the interstate ones. We carry out coupled-channel scattering calculations and find good agreement with experiment. The combination of experiment and theory provides a faithful map of the scattering length a and permits precision measurements of the signatures of Efimov physics across four intermediate-strength intrastate resonances. We measure the modulation of the a^4 scaling of the three-body loss coefficient for both $a < 0$ and $a > 0$, as well as the many-body loss dynamics at unitarity (where a diverges). The absolute positions of the observed Efimov features confirm a ubiquitous breakdown of Efimov–van der Waals universality in ^{39}K , while their relative positions are in agreement with the universal Efimov ratios. The loss dynamics at unitarity are the same (within experimental uncertainties) for the three broadest Feshbach resonances, consistent with observing little variation in the widths of the corresponding Efimov features.

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I. INTRODUCTION

The exquisite ability to tune interatomic interactions using magnetic Feshbach resonances lies at the heart of many ultracold-atom experiments [1]. A single resonance can provide access to the full range of contact interactions, characterized by the s -wave scattering length a . Near resonance, strongly correlated phases of matter [2] can be realized, with highlights including unitary Bose [3–10] and Fermi [11–13] gases. Focusing on Bose gases, the weakly repulsive regime offers a textbook setting for exploring interacting Bose-Einstein condensates [14,15], while tuneable attractive interactions facilitate studies of condensate collapse [16–19], soliton formation [20–24], and negative absolute temperatures [25]. The zero crossing of a provides pristine conditions for simulating quantum phenomena in the absence of interactions (see, e.g., Refs. [26,27]). Furthermore, knowledge of the details of Feshbach resonances and the resulting map of scattering lengths across multiple atomic states facilitates studies of quantum mixtures [28] and many-body interferometry [7,31,32].

Feshbach resonances also provide a testbed for fundamental few-body physics, with the resonance positions B_{res} acting as invaluable benchmarks. Sufficiently close to resonance, the

structure of few-body bound states is particularly simple. A dimer of size a exists on the repulsive side ($a > 0$) and becomes unbound as the magnetic field $B \rightarrow B_{\text{res}}$; this provides a gateway into the realm of ultracold molecules [33–35], and a particularly accurate method for pinpointing B_{res} [1,36–38]. In Bose gases, the Efimov effect [39–43] leads to a spectrum of three-body bound states that persist even when the dimer is unbound.

A hallmark of the Efimov states is their discrete scaling symmetry, which manifests in log-periodic modulation of few-body observables, such as the three-body recombination rate [46–54] (see also Ref. [55]). While the functional form of these modulations is predicted by universal Efimov theory, their phase (captured by a_- , the value of a where the lowest Efimov state meets the continuum) depends on the details of the short-range interaction. Remarkably, for many Feshbach resonances, across spin states and atomic species, a_- was measured to be within 20% of $-9r_{\text{vdW}}$ [52,56,64], where r_{vdW} is the van der Waals length [1]. Theoretically, such an Efimov–van der Waals universality, with $a_- \approx -9.7r_{\text{vdW}}$, was understood for broad resonances; its origin was traced to quantum reflection off the van der Waals potential [65,66]. For narrow resonances, by contrast, it is expected that the resonance strength sets a_- [67,68]. For intermediate-strength resonances, the situation is more open [38,54,69–74]. Early experiments reported results consistent with Efimov–van der Waals universality deep into the intermediate-strength regime (see Ref. [56] for a summary). However, recent high-precision measurements for a single resonance at 33.6 G in ^{39}K [38,54] showed a clear deviation from this universality, finding $a_- = -14.05(17)r_{\text{vdW}}$ [38], in line with the latest theoretical predictions [38,72,74], while

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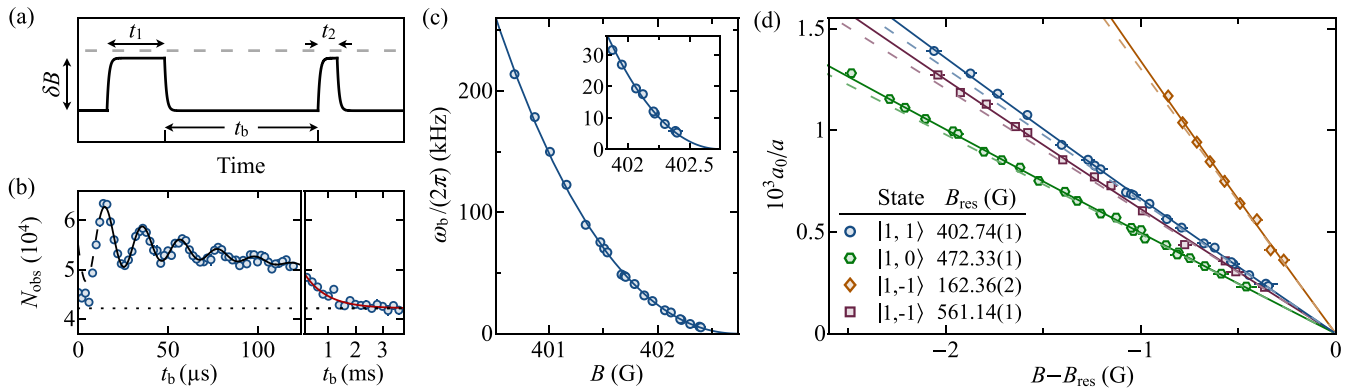


FIG. 1. High-precision bound-state spectroscopy in ^{39}K . (a) Quench protocol used to probe the Feshbach dimer binding energy E_b interferometrically. We perform two field pulses of magnitude δB towards B_{res} (dashed line), separated by a hold time t_b at the magnetic field of interest B . (b) Example of the evolution of the observed atom number $N_{\text{obs}}(t_b)$ at $B = 401.69(2)$ G in the $|1, 1\rangle$ state. We extract the oscillation frequency ω_b by fitting the early-time data (black line, see text for details). At long times, N_{obs} slowly decays due to molecular loss processes; the red line shows a guide to the eye and the dotted line shows the long-time asymptote. (c) Dependence of ω_b on magnetic field for the $|1, 1\rangle$ state Feshbach resonance near 402.7 G. The inset shows a zoom-in near resonance, and we use Eqs. (1) and (2) to extract B_{res} (see text for details). (d) Characterization of four intrastate resonances (see legend and Table I). Inverse scattering length $1/a$, calculated from the measured ω_b , vs detuning from resonance $B - B_{\text{res}}$. The solid lines show fits using Eqs. (1) and (2), while the dashed lines show the linear approximation valid close to resonance.

also observing Efimov ratios largely consistent with universal theory [54].

The Efimov effect is also expected to influence the many-body physics at unitarity, from setting the lifetime of the unitary gas [3,6] to predictions of a low-temperature superfluid of Efimov trimers [44,45]. So far, three-body correlations in a thermal unitary Bose gas have been observed [7], but in the degenerate case signatures are limited to experiments that convert the strongly correlated state into an atom-molecule mixture by sweeping from unitary to weak interactions [8,9].

In this paper, we use a combination of bound-state and loss spectroscopy to pinpoint and characterize eight intrastate Feshbach resonances in ^{39}K (first surveyed in Refs. [75–77]) and six previously unexplored interstate ones. We compare the properties of the resonances with the results of coupled-channel scattering calculations. Using the resultant accurate determination of the interaction landscape and leveraging the advantages of homogeneous samples [78,79], we map out Efimov loss extrema across four intermediate-strength intrastate Feshbach resonances. Our measurements show a ubiquitous breakdown of Efimov–van der Waals universality in ^{39}K , with a_- in the range $-13(1)r_{\text{vdW}}$. The Efimov ratios between features across the Feshbach resonance are in agreement with Efimov theory. We also explore the many-body loss dynamics in unitary gases for the three broadest Feshbach resonances in different spin states. We observe the same behavior (within experimental uncertainties) in all three cases, consistent with observing similar widths of the corresponding Efimov features.

II. BOUND-STATE SPECTROSCOPY

In the vicinity of an isolated Feshbach resonance, neglecting inelastic effects, a approximately follows [80]

$$a = a_{\text{bg}} \left(1 - \frac{\Delta}{B - B_{\text{res}}} \right), \quad (1)$$

where a_{bg} is a slowly varying background scattering length and Δ is the resonance width. Close to the pole, $a \approx -a_{\text{bg}}\Delta/(B - B_{\text{res}})$. If a_{bg} is constant across the entire resonance, a crosses zero at $B_{\text{res}} + \Delta$. More generally, however, the position B_{zero} of the zero crossing may differ from this value. In the presence of inelasticity, Eq. (1) breaks down close to resonance [81]. However, for the resonances used to test Efimov universalities here, this effect is insignificant more than 1 mG from B_{res} ; see Appendix A.

For large positive values of a close to resonance, the binding energy of the least-bound molecular state is [82]

$$E_b = \frac{\hbar^2}{m(a - \bar{a})^2}, \quad (2)$$

where m is the atom mass and $\bar{a} = 0.956r_{\text{vdW}}$, with $r_{\text{vdW}} = 64.6a_0$ [83] for ^{39}K , and a_0 is the Bohr radius.

For our bound-state spectroscopy we begin with a quasipure ^{39}K Bose-Einstein condensate confined in the uniform potential of a cylindrical optical box trap [19,78,79] of volume $V \approx 3 \times 10^{-14} \text{ m}^3$. We prepare weakly interacting repulsive spin-polarized samples in a hyperfine state of choice within the $F = 1$ manifold. Following Refs. [84,85], and as depicted in Fig. 1(a), we employ a Ramsey-like quench protocol. After tuning B to a value of interest over tens of ms [86] we perform two field pulses of magnitude δB towards B_{res} , separated by a time t_b [88]; our field changes occur in $\approx 2 \mu\text{s}$ [9,89]. The first pulse of duration t_1 on the order of tens of μs initiates a coherent superposition of atoms and molecules [90]. During the evolution time t_b a phase difference (set by E_b) accumulates, before the short second pulse with $t_2 = 10 \mu\text{s}$ projects this difference onto a population imbalance. Finally, we record a time-of-flight absorption image of the sample.

As shown in Fig. 1(b), the observed atom number N_{obs} exhibits decaying oscillations with t_b . Our imaging detects only free atoms, and we associate the first increase of N_{obs} at small t_b with the number of atoms transferred to the molecular

state. Once the oscillation has decayed, we are still able to revert a significant fraction of molecules to atoms using the second pulse, consistent with a long-lived molecular sample that eventually decays due to molecular loss processes (see, e.g., Ref. [54]).

To extract the oscillation frequency ω_b , which directly gives $E_b = \hbar\omega_b$, we fit $N_{\text{obs}}(t_b)$ with $N_0 - \alpha t_b + N_m \exp(-t_b/\tau) \sin(\omega_b t_b + \phi)$ [91]. While most extracted fit parameters are sensitive to the details of the quench protocol, crucially, ω_b is robust [84,85].

In Fig. 1(c) we show $\omega_b/(2\pi)$ versus B for the $|1, 1\rangle$ state near the Feshbach resonance at 402.7 G. The solid line shows a fit to the data using Eqs. (1) and (2), which yields $B_{\text{res}} = 402.76(3)$ G, $a_{\text{bg}}\Delta = 1600(70)a_0\text{G}$, and a relatively uncertain $a_{\text{bg}} = -60(30)a_0$. If we assume that $\Delta = B_{\text{zero}} - B_{\text{res}}$ and use the independently measured $B_{\text{zero}} = 350.4(1)$ G [19,92], we obtain a refined $B_{\text{res}} = 402.74(1)$ G and $a_{\text{bg}} = -29.3(3)a_0$, improving upon the high-precision measurement from Ref. [7].

In Fig. 1(d) we plot $1/a$ versus $B - B_{\text{res}}$, and also show analogous measurements for three other resonances. We show the constrained fits (solid lines) and their linear approximation valid near B_{res} (dashed lines).

III. BENCHMARKING LOSS SPECTROSCOPY

We next benchmark the more economical loss spectroscopy as an accurate albeit less precise method for measuring B_{res} . We focus on the $|1, 1\rangle$ state near $B_{\text{res}} = 402.74(1)$ G and prepare samples of density $n_0 \approx 1.3 \mu\text{m}^{-3}$ at a temperature $T_0 \approx 100$ nK; thermal samples are used to facilitate starting both above and below resonance. As depicted in Fig. 2(a), in both cases we then quench to the same final B and wait for a time t_1 , before quenching back to measure N_{obs} .

In Fig. 2(b) we plot the fractional change in atom number $\delta N/N_0 = N_{\text{obs}}/N_0 - 1$ after $t_1 = 400 \mu\text{s}$ as a function of $B - B_{\text{res}}$ for both quenches. We observe maximal loss near B_{res} , and see that the quench direction does not matter when t_1 is sufficiently long, as discussed in Appendix B. The central loss feature is asymmetric, with enhanced loss for $a < 0$ and a pronounced secondary feature, as expected from Efimov physics. We associate the secondary feature with the lowest-lying Efimov resonance, which occurs visibly closer to B_{res} than $a_- = -9.7r_{\text{vdW}}$ predicted from Efimov–van der Waals universality. Our measurements establish quench-based loss spectroscopy as an accurate tool to pinpoint Feshbach resonances up to the width of the loss feature [93].

IV. SUMMARY OF ^{39}K FESHBACH RESONANCES

In Table I we summarize our intrastate resonance measurements, including those from Fig. 1(d) and four based on loss spectroscopy. We also include the high-precision measurement from Ref. [38] and the high-field one from Ref. [76] (see also Ref. [94]). Moreover, we include our independent measurements of the associated zero crossings based on either the critical condition for collapse [16,19] or thermalization rates [97] (for details see Ref. [98]). Note that our measured values of $a_{\text{bg}}\Delta$ are those that characterize the strength of the pole

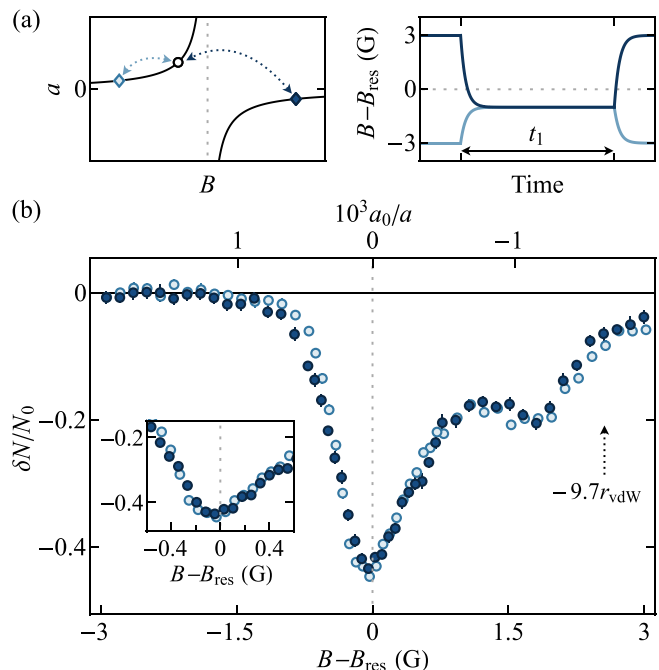


FIG. 2. Benchmarking quench-based loss spectroscopy. (a) Interaction landscape and quench protocol. We prepare thermal samples in the $|1, 1\rangle$ state ± 3 G away from $B_{\text{res}} = 402.74(1)$ G (dashed line). In both cases we quench to the same final B , wait for a time t_1 , before quenching back and probing the sample. (b) Fractional change in atom number $\delta N/N_0 = N_{\text{obs}}/N_0 - 1$ across the Feshbach resonance for $t_1 = 400 \mu\text{s}$. The asymmetric primary loss feature (see inset for a zoom-in) occurs at B_{res} , validating our loss spectroscopy, while the secondary feature arises due to Efimov physics.

in a , and are not necessarily consistent with independently measured values of B_{zero} and a_{bg} far from the pole.

In Table II we summarize our interstate resonance measurements, also including a high-precision one from Ref. [100]. By transposing loss spectroscopy to spin mixtures

TABLE I. Summary of intrastate Feshbach resonance measurements in ^{39}K : the resonance position B_{res} , effective width $a_{\text{bg}}\Delta$, and zero crossing B_{zero} . We also include the measurements from Refs. [38,76,92], as well as the corresponding atomic-state magnetic moments $\bar{\mu}$ (at B_{res}), which are relevant for experiments. Empty entries denote cases where the parameter has not been experimentally determined, while the solidus indicates those that do not occur.

$ F, m_F\rangle$	B_{res} (G)	$a_{\text{bg}}\Delta$ (a_0 G)	B_{zero} (G)	$\bar{\mu}$ (μ_B)
$ 1, 1\rangle$	25.91(6)			-0.605
$ 1, 1\rangle$	402.74(1)	1530(20)	350.4(1) [92]	-0.961
$ 1, 1\rangle$	752.3(1) [76]			-0.987
$ 1, 0\rangle$	58.97(12)			-0.337
$ 1, 0\rangle$	65.57(23)			-0.370
$ 1, 0\rangle$	472.33(1)	2040(20)	393.2(2)	-0.945
$ 1, 0\rangle$	491.17(7)	140(30) ^a	490.1(2)	-0.949
$ 1, -1\rangle$	33.5820(14) [38]	-1073 [38]	/	+0.324
$ 1, -1\rangle$	162.36(2)	760(20)	/	-0.489
$ 1, -1\rangle$	561.14(2)	1660(20)	504.9(2)	-0.959

^aSee Appendix C.

TABLE II. Summary of interstate Feshbach resonance measurements in ^{39}K , including one from Ref. [100]. We also include the corresponding atomic-state magnetic moments $\bar{\mu}_{1,2}$ (at B_{res}), relevant for experiments. Empty entries denote cases where the parameter has not been experimentally determined.

$ F, m_F\rangle_1 + F, m_F\rangle_2$	B_{res} (G)	$a_{\text{bg}}\Delta$ (a_0 G)	$\bar{\mu}_1(\mu_B)$	$\bar{\mu}_2(\mu_B)$
$ 1, 1\rangle + 1, 0\rangle$	25.81(6)		-0.605	-0.155
$ 1, 1\rangle + 1, 0\rangle$	39.81(6)		-0.651	-0.235
$ 1, 1\rangle + 1, 0\rangle$	445.42(3) ^a	1110(40) ^a	-0.967	-0.939
$ 1, 1\rangle + 1, -1\rangle$	77.6(4)		-0.747	+0.034
$ 1, 1\rangle + 1, -1\rangle$	501.6(3)		-0.973	-0.948
$ 1, 0\rangle + 1, -1\rangle$	113.76(1) [100]	715(7) [100]	-0.569	-0.215
$ 1, 0\rangle + 1, -1\rangle$	526.21(5) ^a	970(50) ^a	-0.956	-0.953

^aSee Appendix D.

we locate six previously predicted [101] but experimentally elusive interstate resonances. The resonances at 445.4 and 526.2 G are promising candidates for studies of quantum mixtures (e.g., Bose polarons [102–104]). In particular, the magnetic moments of the two states involved ($\bar{\mu}_1, \bar{\mu}_2$) are very similar (differing by only 3 and 0.3% near 445.4 and 526.2 G, respectively), which opens the door to forming magnetically levitated uniform Bose mixtures with tuneable interparticle interactions. In Appendix D we show details of

our characterization of these two resonances using rf molecular association spectroscopy [1, 105, 106].

In Appendix A we compare our measurements to coupled-channel scattering calculations using the most recent interaction potentials [95]. The calculated resonance positions differ from the experiment by no more than 0.25 G, even for the broadest resonances. The calculated values of $a_{\text{bg}}\Delta$ are in good agreement with the experiment within at most two standard deviations.

V. TESTING EFIMOV UNIVERSALITIES

Equipped with a high-precision map $a(B)$, we now turn to tests of Efimov universalities. The three-body loss rate of a homogeneous thermal Bose gas is given by

$$\dot{N}/N = -L_3 n^2, \quad (3)$$

which upon integration yields $1/N^2 = 2tL_3/V^2 + 1/N_0^2$. The scaling $L_3 \propto \hbar a^4/m$, expected on dimensional grounds (for $r_{\text{vdW}} \ll a \ll \lambda_T$, where λ_T is the thermal wavelength), is modulated by Efimov physics; zero-range zero-temperature theory predicts log-periodic deviations [40], captured by the dimensionless

$$\mathcal{Z}_3 \equiv \frac{mL_3}{3\hbar a^4}, \quad (4)$$

where

$$\mathcal{Z}_3 = \begin{cases} 4590 \frac{\sinh(2\eta_-)}{\sin^2[s_0 \ln(\frac{a}{a_-})] + \sinh^2(\eta_-)} & \text{for } a < 0 \\ 67.12e^{-2\eta_+} (\sin^2[s_0 \ln(\frac{a}{a_+})] + \sinh^2[\eta_+]) + 16.84(1 - e^{-4\eta_+}) & \text{for } a > 0, \end{cases} \quad (5)$$

with $s_0 = 1.00624$. For $a < 0$, loss resonances occur at $a_-e^{j\pi/s_0}$, where $j \in \{0, 1, \dots\}$, whereas for $a > 0$ the log-periodic sinusoidal modulation exhibits minima at $a_+e^{j\pi/s_0}$ and corresponding maxima at $a_p e^{j\pi/s_0}$, with $a_p = a_+e^{\pi/(2s_0)}$ and $a_-/a_p = -1$. Here η_{\pm} denotes the Efimov width parameter, which encodes the lifetime of the Efimov trimers and is thought to be essentially constant across B_{res} .

To study the loss dynamics we prepare box-trapped thermal gases with initial densities n_0 ranging from 0.08 to $3 \mu\text{m}^{-3}$. We use trap depths $U_{\text{td}}/k_B \approx 1 \mu\text{K}$ and restrict ourselves to temperatures $T \lesssim 100$ nK to mitigate nonzero- T effects; these are especially prominent for $a < 0$, where they systematically shift the peak in \mathcal{Z}_3 to less negative values [38, 107, 108]. For a given T and n_0 we initiate the loss measurements either by slowly (in tens of ms) ramping B to set a final a or by performing a spin flip at the field of interest [108]; see also Appendix B for quench-based loss measurements featuring rich molecular dynamics and high-density effects.

In Fig. 3(a) we show an example of the loss dynamics for a thermal gas at $a = -930(30)a_0$ in the $|1, 1\rangle$ state. Here T remains constant while N halves, owing to the fact that our uniform samples offer negligible antievaporation [109]; heating occurs only for high a when the mean free path $\ell = (8\pi n a^2)^{-1}$ becomes short, such that the loss products undergo

additional collisions and deposit some of their energy before leaving the trap. Here we restrict our analysis to times where T remains within $\approx 10\%$ of its initial value. The slope of the linear $1/N^2(t)$ [110] yields L_3/V^2 , from which we calculate \mathcal{Z}_3 [111].

In Figs. 3(b) and 3(c) we show the extracted $\mathcal{Z}_3(a)$ on both sides of the Feshbach resonance at 402.7 G. For $a < 0$ (b), we see an order-of-magnitude enhancement of \mathcal{Z}_3 signaling a_- , while for $a > 0$ (c) \mathcal{Z}_3 is consistent with a log-periodic oscillation. To extract the positions of the maxima of \mathcal{Z}_3 we fit each series with Eqs. (5), but also include an overall prefactor P to account for systematic uncertainties in density and the approximation $\langle n^2 \rangle = n^2$ made in Eq. (3); we find $P \approx 0.5$, consistent with 1 within our systematic uncertainties. The solid lines display the average fit across the series, while the dotted ones extend it outside the fit range.

In Table III we summarize our characterization of the Efimov features for measurements across four Feshbach resonances, as well as measurements from Refs. [38, 54]. In all cases we find values of $a_- = -13(1)r_{\text{vdW}}$, with the Efimov ratios $a_p/a_- \approx -1$. We also find little variation between the Efimov width parameters η_- (extracted from resonances at $a < 0$) for different states.

Table III also includes values of the effective range r_{eff} near B_{res} for each resonance, obtained from coupled-channel

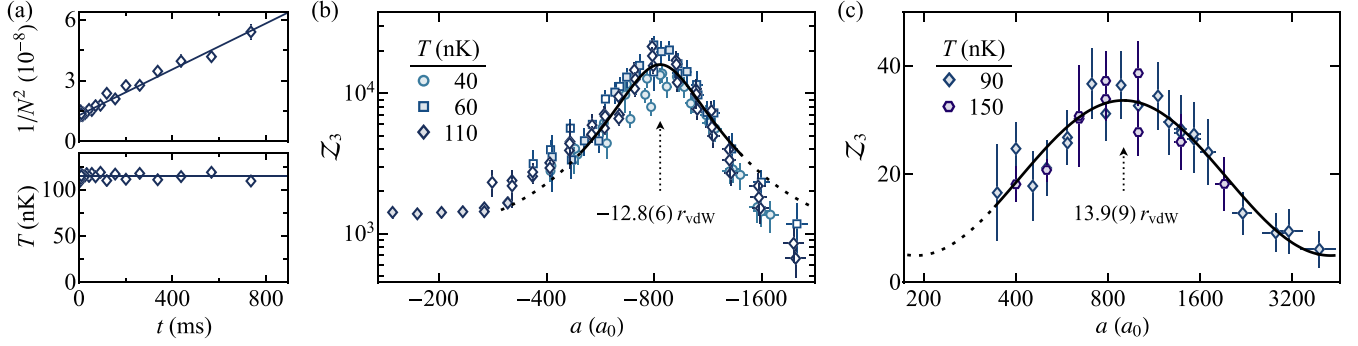


FIG. 3. Testing Efimov universalities in the $|1, 1\rangle$ state ($B_{\text{res}} = 402.74$ G). (a) Loss dynamics in a box-trapped thermal gas at $a = -930(30)a_0$, plotting the evolution of $1/N^2$ (top) and temperature T (bottom). As expected for three-body recombination in a uniform system, $1/N^2(t)$ is linear and $T(t)$ is constant. (b), (c) Modulation of the three-body loss coefficient $\mathcal{Z}_3 = mL_3/(3\hbar a^4)$ for $a < 0$ (b) and $a > 0$ (c). Different data series are grouped by their approximate T (see legend) and the symbol filling denotes the preparation protocol: field ramps (filled) and spin flips (open). Our error bars combine fit errors and the uncertainty in a , but exclude a systematic density uncertainty of $\lesssim 30\%$. We extract the positions of the Efimov features a_- (b) and a_p (c) with fits using Eqs. (5) and a free amplitude (solid lines). While the ratio $a_p/a_- = -1.08(9)$ is consistent with Efimov universality ($a_p/a_- = -1$), the absolute positions exclude Efimov–van der Waals universality ($a_- = -9.7r_{\text{vdW}}$).

calculations as described in Appendix A. For the four stronger resonances with $s_{\text{res}} \approx 2.6(2)$, r_{eff} is within a few percent of $134a_0$. This is about 25% less than the value of $\approx 2.8r_{\text{vdW}}$ expected for broad resonances [112] (see also Ref. [54]).

VI. LOSS DYNAMICS AT UNITARITY

Finally, we study the loss dynamics of initially degenerate gases quenched to unitarity at different resonances (following Ref. [9]). For both a thermal and a degenerate homogeneous Bose gas at unitarity one can define a dimensionless loss rate

$$\Gamma = -t_n \dot{N}/N, \quad (6)$$

where t_n is the density-set timescale $t_n = \hbar/E_n$, with $E_n = \hbar^2(6\pi^2 n)^{2/3}/(2m)$, both of which are global variables in a homogeneous system.

For a thermal unitary gas, to a good approximation [3,4,6],

$$\Gamma = (1 - e^{-4\eta_\infty}) \frac{18\sqrt{3}}{\pi^2} \left(\frac{E_n}{E}\right)^2, \quad (7)$$

where $E = (3/2)k_B T$ and η_∞ is the Efimov width parameter. Instead, for a degenerate gas, $\Gamma = A$, where A depends only on the Efimov physics [113]. Previously, $A = 0.28(3)$ was measured for $B_{\text{res}} = 402.7$ G in $|1, 1\rangle$ [9] and $A \approx 0.18$ estimated from a measurement in ^{85}Rb [8], where $\eta_- = 0.057(2)$ [53].

TABLE III. Extracted Efimov loss features across the four intermediate-strength Feshbach resonances from Fig. 1(d), as well as corresponding Efimov features near $B_{\text{res}} = 33.6$ G from Refs. [38,54]. Note that for Efimov–van der Waals universality $a_- = -9.7r_{\text{vdW}} \approx -630a_0$ for ^{39}K , while the universal Efimov ratio $a_p/a_- = -1$. For reference, we include the approximate values of s_{res} from Ref. [77] and values of the effective range r_{eff} (calculated close to B_{res}) from our coupled-channel calculations.

$ F, m_F\rangle$	B_{res} (G)	s_{res}	$r_{\text{eff}}(a_0)$	$a_-(a_0)$	a_-/r_{vdW}	η_-	$a_p(a_0)$	a_p/r_{vdW}	a_p/a_-
$ 1, 1\rangle$	402.74(1)	2.8	136	-830(40)	-12.8(6)	0.27(6)	900(60)	13.9(9)	-1.08(9)
$ 1, 0\rangle$	472.33(1)	2.8	137	-840(30)	-13.0(5)	0.26(5)	760(70)	11.8(11)	-0.90(9)
$ 1, -1\rangle$	33.5820(14) [38]	2.6	135	-908(11) [38]	-14.05(17) [38]	0.25(1) [38]	876(28) [54]	13.6(4) [54]	-0.96(3) [54]
$ 1, -1\rangle$	162.36(2)	1.1	59	-780(70)	-12.1(11)	0.5(1)			
$ 1, -1\rangle$	561.14(1)	2.5	132	-810(30)	-12.5(5)	0.33(6)	800(100)	12.4(15)	-0.99(13)

As in Ref. [9], we start with a quasipure Bose-Einstein condensate and quench it to unitarity. As the cloud decays and heats, from the time evolution of N and E we map out Γ as a function of E/E_n , and observe a crossover from degenerate-gas to thermal-gas behavior; this occurs at a crossover time t_c and corresponding remaining atom number N_c . In Fig. 4 we show $\Gamma(E/E_n)$ for the three broadest intrastate resonances (at 402.74, 472.33, and 561.14 G). In all cases degeneracy persists for $t_c \approx 4t_{n_0}$, with a remaining fraction $N_c/N_0 \approx 0.4$ [see Fig. 4(b)].

We find remarkably universal behavior of Γ across both degenerate-gas and thermal-gas regimes. This is consistent with observing similar values of η_- for the different Efimov loss resonances. However, the absolute value of η extracted from Γ in the thermal regime gives $\eta_\infty \approx 0.1$ (consistent with Refs. [4,9,89]), smaller than $\eta_- \approx 0.3$ extracted from the Efimov loss resonance widths (see Table III). This discrepancy is unexpected, but could arise due to systematic density uncertainties which affect the extraction of η_∞ (and also η_+).

VII. CONCLUSIONS AND OUTLOOK

In conclusion, our experiments provide a high-precision characterization of few-body physics in ^{39}K , offering invaluable input for modeling its interaction potential and for the ongoing theoretical efforts to understand complex

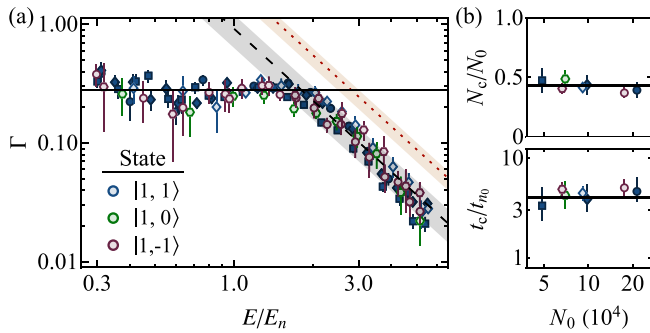


FIG. 4. Comparison of loss dynamics at unitarity for the three broadest intrastate resonances, each in a different internal state. The three states are indicated by the symbol color, while different symbol shapes indicate different initial atom numbers N_0 [see (b)]. The three filled symbols correspond to data for $|1, 1\rangle$ reproduced from Ref. [9]. All measurements were taken using the same box trap, with $V \approx 3.5 \times 10^{-14} \text{ m}^3$. (a) Dimensionless loss rate $\Gamma = -\dot{N}_c/N$ vs E/E_n . The solid horizontal line corresponds to $\Gamma = A = 0.28$ [9]. The dashed line corresponds to $\eta_\infty = 0.09(4)$ [4] in Eq. (7) and the dotted line corresponds to $\eta_\infty = 0.3(1)$ (equal to η_- obtained from \mathcal{Z}_3). (b) Remaining fraction of atoms at the degenerate- to thermal-gas crossover, N_c/N_0 (top) and the crossover time t_c/t_{n_0} (bottom).

multichannel effects in Efimov physics [38,72–74,95]. Our precise determination of $a(B)$ across multiple Feshbach resonances and spin states allows a comprehensive study of Efimov physics in ^{39}K . For its broadest intrastate resonances, which all feature a similar intermediate-strength character (captured by s_{res}), we find similar values of $a_- = -13(1)r_{\text{vdW}}$, suggesting a systematic breakdown of Efimov–van der Waals universality set by s_{res} . Moreover, the observed Efimov ratios uphold Efimov universality and we find that the loss dynamics, both at and away from unitarity, are remarkably universal.

Our paper points to many avenues for further research. It would be interesting to explore the effects of few-body physics on the many-body state at unitarity [10]. This includes surveying Feshbach resonances across atomic species to find cases where the unitary lifetime is longer, and understanding the intricate few-body loss channels (see, e.g., Ref. [45]) to devise schemes that could extend the lifetime (as, e.g., in molecular samples [114,115]). Finally, the rich $a(B)$ landscape of ^{39}K also offers exciting prospects for novel experiments. The pair of overlapping Feshbach resonances at 472.3 and 491.2 G gives access to a steep zero crossing for studies of the collapse of uniform condensates via modulational instability, while the two interstate resonances at 445.4 and 526.2 G are particularly promising for creating uniform Bose mixtures.

The supporting data for this paper are available from Ref. [116].

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APPENDIX A: COMPARISON OF FESHBACH RESONANCE PROPERTIES WITH COUPLED-CHANNEL CALCULATIONS

We have carried out coupled-channel scattering calculations for comparison with our measurements of the Feshbach resonance properties. The total wave function is expanded in a complete basis set of functions for electron and nuclear spins and end-over-end rotation, producing a set of coupled differential equations that are solved by propagation with respect to the internuclear distance R . The methods used are similar to those in Ref. [117], so only a brief outline is given here. The Hamiltonian for the interacting pair is

$$\hat{H} = \frac{\hbar^2}{2\mu} \left[-\frac{1}{R} \frac{d^2}{dR^2} R + \frac{\hat{L}^2}{R^2} \right] + \hat{H}_A + \hat{H}_B + \hat{V}(R), \quad (\text{A1})$$

where μ is the reduced mass and \hat{L} is the two-atom rotational angular momentum operator. The single-atom Hamiltonians \hat{H}_i contain the hyperfine couplings and the Zeeman interaction with the magnetic field. The interaction operator $\hat{V}(R)$ contains the two isotropic Born-Oppenheimer potentials, for the $X^1\Sigma_g^+$ singlet and $a^3\Sigma_u^+$ triplet states, and anisotropic spin-dependent couplings that arise from magnetic dipole-dipole and second-order spin-orbit coupling. Here we use the singlet and triplet interaction potentials of Tiemann *et al.* [95] (their model 3), with the second-order spin-orbit coupling function of Xie *et al.* [54].

The scattering calculations are carried out using the MOLSCAT package [118,119]. The wave function is expanded in a fully uncoupled basis set that contains all allowed electron and nuclear spin functions, and is limited by $L_{\text{max}} = 2$. Solutions are propagated from $R_{\text{min}} = 5a_0$ to $R_{\text{mid}} = 30a_0$ using the fixed-step symplectic log-derivative propagator of Manolopoulos and Gray [120] with an interval size of $0.002a_0$, and from R_{mid} to $R_{\text{max}} = 8,000a_0$ using the variable-step Airy propagator of Alexander and Manolopoulos [121].

Each scattering calculation produces the scattering matrix S for a single value of the collision energy E_{coll} and magnetic field B . The complex energy-dependent s -wave scattering length $a(k_0)$ is obtained from the diagonal element of S in the incoming channel, S_{00} , using the identity [81]

$$a(k_0) = \frac{1}{ik_0} \left(\frac{1 - S_{00}(k_0)}{1 + S_{00}(k_0)} \right), \quad (\text{A2})$$

where k_0 is the incoming wave number, related to the collision energy by $E_{\text{coll}} = \hbar^2 k_0^2 / (2\mu)$. Note that $a(k_0)$ becomes constant at sufficiently low E_{coll} , with limiting value a . In the present paper, s -wave scattering lengths are calculated at $E_{\text{coll}}/k_B = 100 \text{ pK}$, which is low enough to neglect the dependence on k_0 .

TABLE IV. Comparison of experimental resonance parameters with those from coupled-channel scattering calculations using the most recent interaction potentials [95]. Empty entries indicate cases where the parameter has not been determined, while the solidus indicates those that do not occur.

States $ F, m_F\rangle_1 + F, m_F\rangle_2$	Experiment			Theory					
	B_{res} (G)	$a_{\text{bg}} \Delta$ (a_0 G)	B_{zero} (G)	B_{res} (G)	$a_{\text{bg}} \Delta$ (a_0 G)	Δ (G)	a_{bg} (a_0)	Γ_{inel} (μG)	B_{zero} (G)
$ 1, 1\rangle + 1, 1\rangle$	25.91(6)			25.879	15.39	-0.465	-33.10	0	
$ 1, 1\rangle + 1, 1\rangle$	402.74(1)	1530(20)	350.4(1) [92]	402.554	1514	-51.291	-29.52	0	350.714
$ 1, 1\rangle + 1, 1\rangle$	752.3(1) [76]			752.255	14.01	-0.397	-35.30	0	
$ 1, 0\rangle + 1, 0\rangle$	58.97(12)			58.949	194.2	-6.648	-29.22	-5	
$ 1, 0\rangle + 1, 0\rangle$	65.57(23)			65.573	140.7	-3.361	-41.87	-183	
$ 1, 0\rangle + 1, 0\rangle$	472.33(1)	2040(20)	393.2(2)	472.118	1997	-117.78	-16.96	-8	393.635
$ 1, 0\rangle + 1, 0\rangle$	491.17(7)	140(30)	490.1(2)	490.930	139.5	-1.045	-133.43	-2	489.931
$ 1, -1\rangle + 1, -1\rangle$	33.5820(14) [38]	-1073 [38]	/	33.568	-1073	79.469	-13.50	100	/
$ 1, -1\rangle + 1, -1\rangle$	162.36(2)	760(20)	/	162.347	711.3	-60.628	-11.73	-180	/
$ 1, -1\rangle + 1, -1\rangle$	561.14(2)	1660(20)	504.9(2)	560.935	1611	-55.358	-29.10	-9	504.717
$ 1, 1\rangle + 1, 0\rangle$	25.81(6)			25.785	47.53	-1.345	-35.35	-5	
$ 1, 1\rangle + 1, 0\rangle$	39.81(6)			39.835	84.47	-2.061	-40.99	-56	
$ 1, 1\rangle + 1, 0\rangle$	445.42(3)	1110(40)		445.293	1140	-37.569	-30.35	-9	407.479
$ 1, 1\rangle + 1, -1\rangle$	77.6(4)			77.725	2618	-95.738	-27.35	-8738	/
$ 1, 1\rangle + 1, -1\rangle$	501.6(3)			501.480	897.8	-18.304	-49.05	-3364	486.845
$ 1, 0\rangle + 1, -1\rangle$	113.76(1) [100]	715(7) [100]		113.768	755.7	-19.215	-39.33	-295	97.793
$ 1, 0\rangle + 1, -1\rangle$	526.21(5)	970(50)		525.995	876.8	-28.249	-31.04	-9	497.648

When both atoms are in their lowest state $|1, 1\rangle$, only elastic scattering is possible. The scattering length is then real and approximately follows Eq. (2) in the vicinity of an isolated resonance. If either atom is excited, however, inelastic collisions may also occur. For most of the atomic states considered here, these are spin-relaxation collisions, mediated by the weak spin-spin and second-order spin-orbit couplings. The scattering length is then complex, $a(B) = \alpha(B) - i\beta(B)$, and the rate coefficient for two-body loss is approximately proportional to β [81]. Even if there is very little inelastic scattering away from resonance, $\beta(B)$ shows a narrow peak of height a_{res} near B_{res} and $\alpha(B)$ shows an oscillation of magnitude $\pm a_{\text{res}}/2$ instead of a pole. A feature of MOLSCAT is that it can converge automatically on both elastic and inelastic Feshbach resonances and characterize them to obtain their parameters, as described by Frye and Hutson [122]. For elastic scattering, the parameters extracted are B_{res} , Δ , and a_{bg} ; in the presence of weak inelastic scattering, these are supplemented by a_{res} , which can be recast as $\Gamma_{\text{inel}} = -2a_{\text{bg}}\Delta/a_{\text{res}}$ to assess the size of the region around B_{res} where $a(B)$ is not polelike [81, 122].

We have characterized theoretically all the resonances for which experimental properties are given in Tables I and II. The calculated properties are compared with experiment in Table IV. The agreement is generally good. The calculated resonance positions differ from experiment by up to 0.25 G, but that is comparable to the deviations between experiment and theory in Ref. [95]. Note that the position measured here for the resonance near 403 G is far closer to theory than the experimental value used in Ref. [95]. The calculated values of $a_{\text{bg}}\Delta$ are generally within one or two standard deviations of the measured ones, but the calculation breaks them down into their components a_{bg} and Δ , which the present experiments cannot do without assuming that Eq. (1) holds all the way to

B_{zero} . For the resonances at higher thresholds, the small values of Γ_{inel} confirm that the scattering length is polelike until very close to B_{res} .

MOLSCAT can also converge numerically upon fields B_{zero} where $\alpha(B)$ is zero. These may differ from the value $B_{\text{res}} + \Delta$ implied by the approximate Eq. (1) if a_{bg} varies significantly across the width of the resonance. These values are included in Table IV, and are close to the experimental values even in cases where B_{zero} differs significantly from $B_{\text{res}} + \Delta$.

We have also calculated values of the effective range near B_{res} from the expansion

$$\frac{1}{a(k_0)} = \frac{1}{a} - \frac{1}{2}r_{\text{eff}}k_0^2 + \dots \quad (\text{A3})$$

using the methods of Ref. [123]. The results are included in Table III.

APPENDIX B: PECULIARITIES OF QUENCH-BASED LOSS SPECTROSCOPY

Using field quenches for loss spectroscopy circumvents substantial atom loss in the time it takes to reach the magnetic field of interest. This leads to more narrow spectra and avoids systematic asymmetries due to the quench direction, allowing a reliable extraction of B_{res} . However, such quenches can also induce additional dynamics, primarily dictated by shallow molecular bound states, which we explore below.

In Fig. 5(a) we plot $N_{\text{obs}}(t_1)$ for the quench experiments from Fig. 2 at three different fields, below ($B < B_{\text{res}}$), on ($B \approx B_{\text{res}}$), and above ($B > B_{\text{res}}$) resonance. The early times reveal rich dynamics, while for late times ($t_1 = 400 \mu\text{s}$) we observe loss essentially independent of the quench direction.

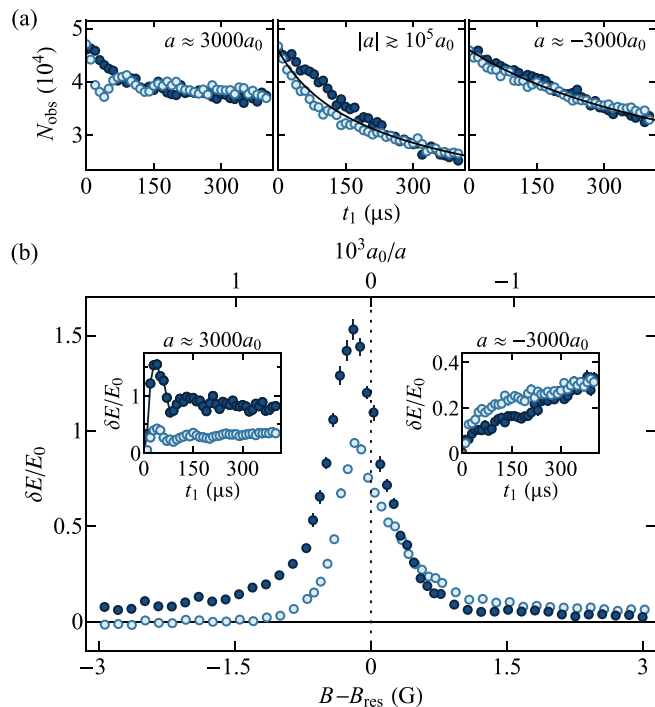


FIG. 5. On the perils of quench-based loss spectroscopy [see Fig. 2(a) for protocol]. (a) Evolution of $N_{\text{obs}}(t_1)$ for quenches below, on, and above resonance (left to right), with initial $N_0 = 4.6(1) \times 10^4$ atoms at $T \approx 100$ nK. While the early times display rich dynamics, the long-time loss is independent of the quench direction. (b) Fractional change in energy $\delta E/E_0$ for quench-based loss spectroscopy, measured using $t_1 = 400 \mu\text{s}$ [as in Fig. 2(b)]. Here substantial differences persist between the quench directions even at long times. We note that $\delta E/E_0$ does not peak at B_{res} , making it unsuitable for accurate measurements of B_{res} .

For $a \approx 3000a_0$, the first quench to B already creates a non-negligible superposition of atoms and molecules, leading to an oscillation in $N_{\text{obs}}(t_1)$ upon quenching back to $\approx 500 a_0$. Meanwhile, for quenches back to $\approx -500 a_0$, where the dimer state is unbound, $N_{\text{obs}}(t_1)$ decreases monotonically. However, the observed loss rate is still an order of magnitude larger compared to our adiabatically prepared samples [Fig. 3(c)]. Instead, for $a \approx -3000a_0$, we only see a minute difference between the quench directions, owing to the dimer state being unbound. On resonance, we observe a surprisingly slow initial loss rate when quenching back to $\approx -500a_0$, where the dimer state is unbound. This loss is inconsistent with the late-time behavior that is captured by the thermal unitary atom-loss scaling law $\dot{N}/N \propto N^{26/9}$ [9] (solid line, fit to $t_1 > 250 \mu\text{s}$).

In Fig. 5(b) we show the change in energy per particle $\delta E/E_0$ for $t_1 = 400 \mu\text{s}$ [analogous to $\delta N/N_0$ in Fig. 2(b)]. Here dramatic differences persist even at long times. For $B < B_{\text{res}}$, the injected δE is significantly higher for quenches back to $\approx -500a_0$, while for $B > B_{\text{res}}$ differences are less pronounced. The maximal δE occurs visibly below B_{res} , arising from a complex interplay between loss and molecular dynamics, which renders it unsuitable for accurately extracting B_{res} .

The robust behavior of the quench-based loss dynamics for $a < 0$ suggests that our quench experiments might provide an

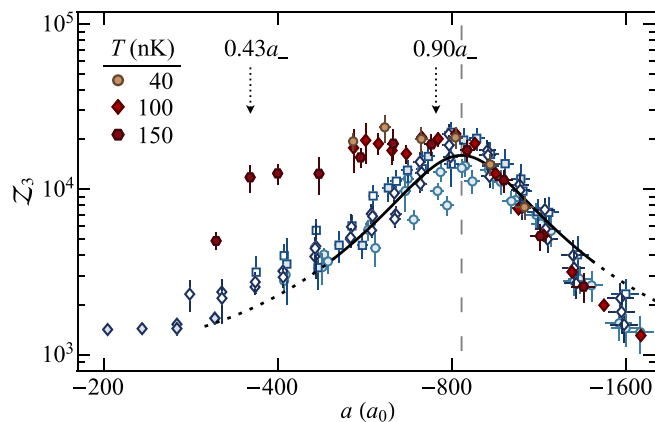


FIG. 6. Extracted Z_3 at $a < 0$ for quench-based loss spectroscopy of samples with high phase-space density, prepared with different T (red tones, see legend) compared to data from Fig. 3(b) (blue tones). We posit that the significant broadening to lower $|a|$ occurs due to tetramer bound states related to the ground-state Efimov trimer. The arrows indicate positions where these tetramer states are expected to merge with the four-atom continuum. Note that for the quench-based measurements here we scale Z_3 between series at different T to match the high- $|a|$ data from Fig. 3(b); we attribute the $\lesssim 30\%$ differences to slight changes in V .

accurate measurement of Z_3 for $a < 0$. In Fig. 6 we compare Z_3 extracted from such quench-based measurements (red tones, shown in legend) to the data from Fig. 3(b) (blue tones). Here we use gases with $n\lambda_T^3 \approx 1$ and resolve significantly shorter timescales. We observe significant broadening of the peak in Z_3 towards lower $|a|$. We posit that this might be a signature of tetramer states related to the ground-state Efimov trimer, which are predicted to merge with the four-atom continuum at $-0.43a_-$ and $-0.90a_-$ [124].

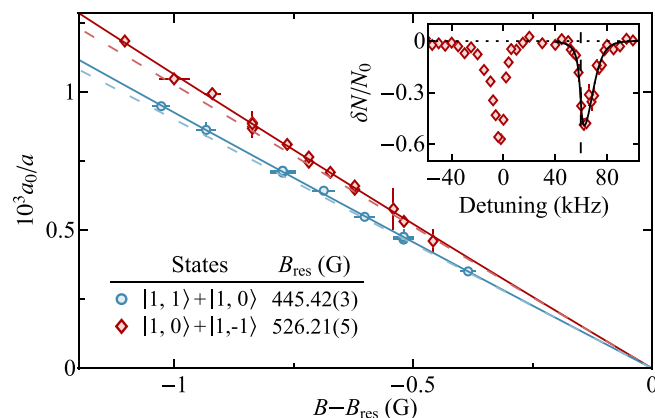


FIG. 7. Interstate rf association spectroscopy. Inverse scattering length calculated from the extracted E_b vs $B - B_{\text{res}}$. Solid lines show fits using Eqs. (1) and (2). The inset shows a typical rf spectrum of $\delta N/N_0$ vs rf detuning from the bare atomic transition; here the solid curve shows a fit of the dimer peak, with the extracted E_b/h indicated by the dashed line.

APPENDIX C: OVERLAPPING FESHBACH RESONANCES

For two overlapping resonances with widths of the same sign, Eq. (1) extends to the more general form [125,126]

$$a(B) = a_{\text{bg}} \left(1 - \frac{\Delta_1}{B - B_{\text{res},1}} \right) \left(1 - \frac{\Delta_2}{B - B_{\text{res},2}} \right), \quad (\text{C1})$$

where a_{bg} is the common background scattering length, $B_{\text{res},1(2)}$ is the first (second) resonance position, and we assume $\Delta_{1(2)} = B_{\text{zero},1(2)} - B_{\text{res},1(2)}$. For $\Delta_1/\Delta_2 \gg 1$, the effective width of the narrow resonance is $a'_{\text{bg}}\Delta_2 = a_{\text{bg}}[1 - \Delta_1/(B_{\text{res},2} - B_{\text{res},1})]\Delta_2$.

For the narrow intrastate resonance near 491 G we find $a'_{\text{bg}}\Delta_2 = 140(30)a_0\text{G}$ using our measurements (see Table I), while $a'_{\text{bg}}/a_{\text{bg}} \approx 5.2$. To calculate $a(B)$ for our measurements of a_- for the resonance near 162.4 G we include the small ($\approx 10a_0$) contribution from the one near 33.6 G.

APPENDIX D: INTERSTATE RF ASSOCIATION SPECTROSCOPY

Here we present measurements of E_b for two interstate resonances based on rf molecular association spectroscopy [1,105,106] [see Fig. 7, plotted akin to Fig. 1(d)]. To measure E_b we prepare thermal clouds spin polarized in one of the two states, and then apply a weak rf pulse up to tens of ms to associate the interstate Feshbach dimer, which enhances atom loss.

A typical rf spectrum is shown in the inset of Fig. 7, which exhibits features corresponding to both the bare atomic transition and the dimer. To extract E_b (dashed line) from the asymmetric dimer peak, we fit a Lorentzian convolved with a Maxwell-Boltzmann distribution (solid line) [106] (see also Ref. [100]); this asymmetry arises due to the initial kinetic energy of the associated atoms. Curiously, the atomic feature near zero detuning exhibits an asymmetric tail as well, which we also attribute to kinetic effects. We restrict ourselves to cases where the two peaks are well separated.

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