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Bending space and time

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Abstract. Albert Einstein showed how gravity could best be represented by a curved spacetime. This paper discusses how this works out both in the case of cosmology and of black holes, and how it allows the possibility of travel to the past in principle (but not in practice). The observable universe and the inflationary universe are also considered.

1. Spacetime

In Isaac Newton's time and the times before, the general perspective was that space is unchangeable and rigid – Newton called this absolute space – and that time is linear and steps forward moment by moment. Nothing that happens in absolute space can change it in any way. Thus, singularities or infinities would not destroy the space; they would instead be an infinite value of some quantity like luminosity or temperature. It should be noted that the space does not have to be Euclidian, although everyone always assumed that it was until the middle of the 19th century when various paradoxes of gravitational forces encouraged some people to explore the interesting consequences of choosing a fixed, non-Euclidian space like Lobachevsky space.

Hermann Minkowski developed spacetime diagrams which can explain many phenomena of space and time. For example, there are clear regions that one cannot reach from other regions in spacetime because the finiteness of the speed of light partitions space and time into accessible and inaccessible places. Minkowski advanced our understanding of space and time, and made the famous remark: “*Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows and only a kind of union of the two will preserve an independent reality*.” It must be remembered that Minkowski made this remark before the speed of light achieved its universal speed limit status. Indeed, around this time some French physicists were trying to calculate the speed of gravity compared to the speed of light.

The speed of light has units of length per unit time and its constant nature means that there is an invariant link between space and time that does not exist in Newtonian theory. Minkowski’s perspective forces one to consider the block of spacetime as fundamental rather than space and time separately. At any moment all of space can be treated as a sheet. As one steps forward in time, more and more sheets of space are stacked on top of the first such that the entire block of spacetime is built up – a stack of spaces sliced at different times. If the whole block of spacetime is fundamental, then it includes all the future and all the past. Therefore, is the future already laid out for us in this block? The relativity principle says that the spacetime block can be sliced in different ways by moving at high speeds. That is, moving at high speed is equivalent to slicing through the block at an angle and there is no limit to the number of different slices that can be made (figure 1 (left)). The spacetime block is invariant, but the slicing made and thus the observations taken of mass, length and time undergo the various relativistic

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transformations of Hendrik Lorentz and Henri Poincaré. Thus, spacetime being fundamental alters our perspective on what is and is not invariant.

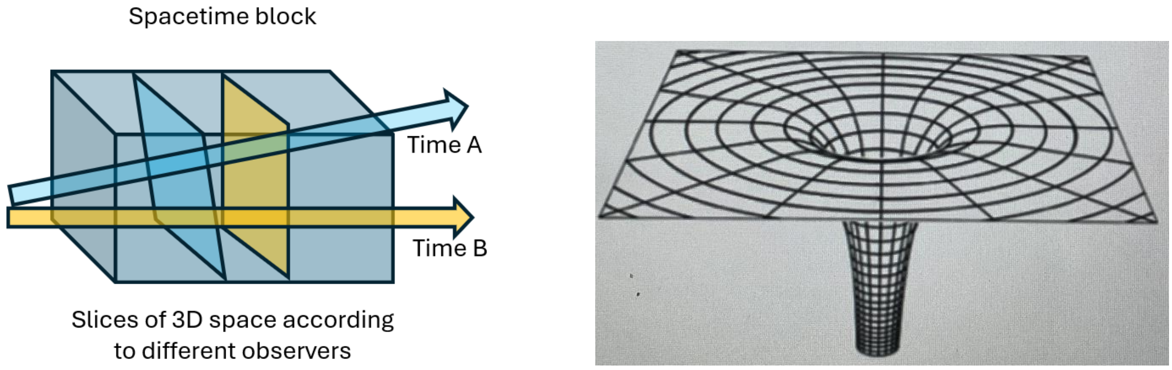

Figure 1. (left) The old perspective of spacetime as a block. An instant is a slice of the block and observers moving at different speeds will slice the block at different angles. (right) A depiction of Einstein's curved spacetime geometry.

Albert Einstein's general theory of relativity introduced a more sophisticated and unusual development: spacetime is not a collection of rigid, flat slices and instead the slices can be distorted by the presence of mass and energy in the spacetime (figure 1 (right)). This thinking marks a significant change in perspective from the absolute spacetime that is unchanged by what occurs in it to a spacetime with a geometry determined by the masses, energies and movement of all things in it. The usual analogy is that spacetime is like a rubber sheet where the motion of mass and energy determines the geometry on which the masses move, which then determines how they move in the sense that they will follow the shortest distance between two points on a curved surface. This change to spacetime has many unusual ramifications. For example, if the rubber sheet is moved up and down to create a ripple in one position, then one would expect it to affect something in a different part of the sheet in a way that does not occur in Newtonian theory. Similarly, if something is rotated in one position in Newtonian theory, it does not affect something somewhere else. However, if a rubber sheet is rotated then it will tend to drag things some distance away in the same direction. Indeed, all of these things happen in general relativity. Einstein's simple idea of allowing space and time to become dynamic is rather remarkable and it also means that the old interpretation of gravitational forces is unnecessary. For example, the Newtonian picture of a comet orbiting the Sun is that the comet is pulled towards the Sun and so its orbit is slightly deflected. In contrast the general relativity perspective is that the mass of the Sun causes a distortion in the geometry of spacetime, which makes the shortest distance between two points a curved path – a geodesic (figure 2). Similarly, for a closed orbit the orbiting body follows the shortest time path in a distorted picture of space and time.

2. Cosmology and black holes

The most dramatic consequences of general relativity are the equations that allow the geometry of space to be determined from any distribution of mass and energy. These equations are almost impossible to solve except in situations with high symmetry. Fortunately, the Universe is highly symmetric on very large scales and solutions of this situation can be found to a good approximation. Every solution of Einstein's equations describes an entire universe. Some of those universes contain a single mass and look different to our Universe but can be good approximations to the Solar System. Other solutions may describe an expanding cosmos of clusters of galaxies. Einstein's theory made cosmology possible. Before Einstein's theory of general relativity, cosmology was effectively a branch of art history in that one could imagine any universe one liked. There was no way to generate mathematical or physical

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descriptions of a universe and no way to know the consequences of each one. However, with Einstein's theory each solution of the equations is a possible universe. Now astronomical observations can be carried out to rule out some solutions and show that others match our observations quite accurately. Einstein enabled cosmology to pass from art history to astrophysical science.

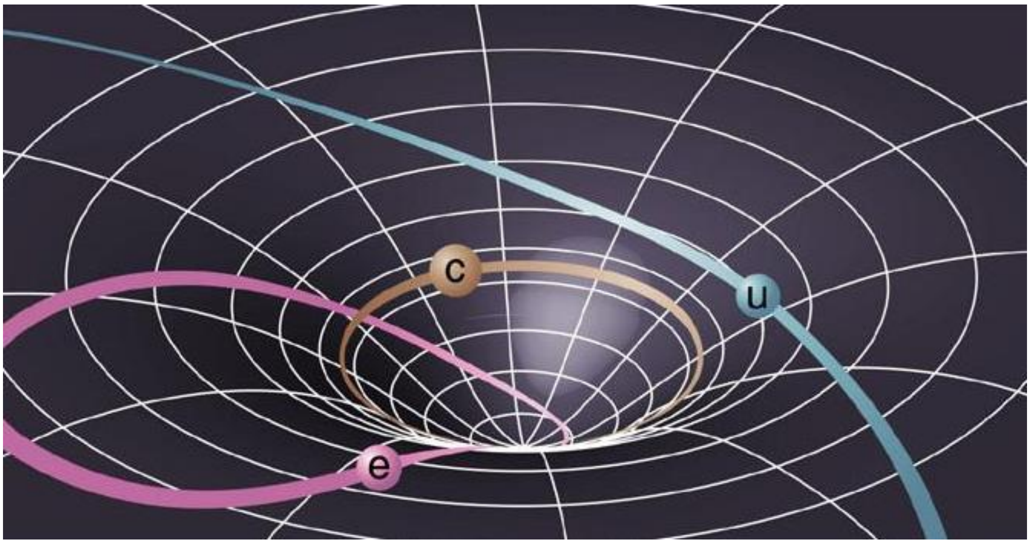

Figure 2. Schematic of different orbits in a curved spacetime. “c” and “e” represent circular and elliptic orbits respectively. “u” is an unbound object like a comet – its path appears to curve as it follows the straightest possible path in the curved spacetime.

There are situations where strong distortions of space will dramatically change how spacetime behaves in comparison to what Newtonian theory or special relativity would suggest. For example, black holes, wormholes, and situations with rotation and time travel are possible with general relativity. The first person to realise such strong distortions of spacetime were possible in Einstein's theory was Hermann Weyl in 1919. Weyl remarked in his book *Space – Time – Matter* that one could have hugely distorted regions of spacetime that were drawn back upon each other in a way that might allow time travel to be possible. He was right.

The idea of a black hole is attributed to the Cambridge Professor and vicar, John Michell. Michell was a Professor of Geology at Cambridge for many years and is considered by many as the greatest scientist in England in the century after Newton. He was responsible for many important developments in Earth Science and was the Rector of St Botolph's Church. He left Cambridge because he wanted to marry, so he became the Rector of a parish in Yorkshire and continued to write many fascinating papers. One such paper was about the idea of a star-like object in space, which was so dense that the escape velocity from its surface was equal to the speed of light. He formulated a Newtonian description of a black hole and determined that its radius would be given by

$$r = \frac{2GM}{c^2}.$$

Michell designed the apparatus for the Cavendish experiment and gave it to Henry Cavendish when he moved to Yorkshire, so he had a strong interest in gravity. Black holes in the general relativity framework differ from Michell's Newtonian conception. If there is a structure with an escape velocity of the speed of light, it does not mean one cannot escape from it. Instead it means that one cannot escape to infinity but can jump some distance off the surface. In general relativity the idea is much more

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restrictive as nothing can pass outwards through the radius *r*. Furthermore, a black hole is a region of spacetime and not a solid object. For example, the density of a black hole can be calculated as mass over volume, which is proportional to mass over radius cubed. The black hole radius is proportional to the mass, so the density is proportional to one over mass squared. Therefore as the mass gets larger, the density gets smaller. A billion solar mass black hole, such as those at the centre of many galaxies, would be less dense than air on average. Thus, one could be passing through the horizon of such a black hole and not notice anything odd, at least on a local scale. If one tried to escape, one would not be able to do so and would learn that the global spacetime in the vicinity was very strange. Relatively small black holes are extreme and create massive gravitational tidal forces, whereas massive black holes are relatively benign regions of spacetime with unusual global properties.

Rotating black holes add further strangeness. In 1963 New Zealand mathematician, Roy Kerr found an extraordinarily mathematically difficult solution to Einstein's equations that is now called the Kerr metric. Every rotating black hole in the Universe is described by Kerr's metric to our knowledge. Incidentally when Kerr found the solution he did not know he had found a black hole. He used other coordinates in which the solution looked like a complicated mathematical structure. Other scientists reduced and changed the coordinates to make the solution recognisable as a rotating black hole. One of these scientists was Robert Boyer, who was at Oxford for his doctorate and later at King's College London in his mid-20s. Boyer was tragically killed by the University of Texas tower shooter in Austin, Texas in 1966 as he headed back from lunch before a seminar he was to give to the Physics Department.

Kerr's solution describes a spinning black hole and the solution has an interesting structure (figure 3). There is a surface of no return called the event horizon, which is spherical and not distorted by the rotation as a planet would be. Outside the event horizon is another surface called the ergosphere and as the black hole is rotated faster, the ergosphere flattens and gets bigger. The ergosphere is a surface which cannot stand still and is always dragged in the same direction as the rotation of the black hole; however it is possible to escape. This effect is similar to the rubber sheet analogy where the rotation of one object can cause other objects to also rotate.

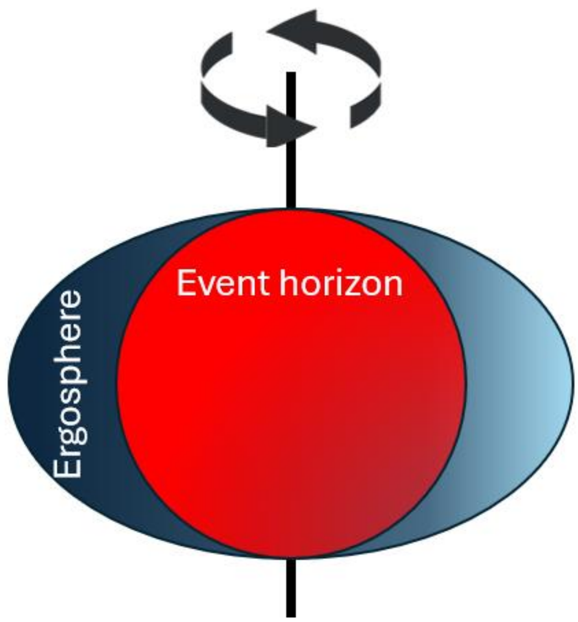

Figure 3. Diagram of a rotating black hole with the spherical event horizon and elliptic ergosphere.

In 2015 the first gravitational wave detections were made directly on Earth. It is believed that the waves originated from two spinning Kerr black holes merging to produce another spinning black hole with energy given out in gravitational waves. This event has a very particular signature. It is over in about one-fifth of a second and does not leave much debris because a black hole is produced. If it were

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instead a black hole merging with a neutron star, there would be much more debris. It has now been shown by gravitational wave detection several times that these rotating black holes can dramatically distort spacetime.

3. Rotation

The story of rotation in spacetime has a famous beginning. After the War, Einstein went to the Institute for Advanced Study in the US, which existed to surround him with people and an environment that would not disturb him. Kurt Gödel was among the people at the Institute. Einstein had been in Oxford at Christ Church as a research lecturer. It was Frederick Lindemann's hope that Einstein would stay in Oxford as the centrepiece of the Department of Physics. However, he did not remain despite Erwin Schrödinger and other great physicists from Europe staying for a time. Einstein instead went to Princeton, where he frequently interacted with Gödel.

Gödel startled Einstein by finding a solution of his equations that described a rotating spacetime, a rotating universe. Within this universe there were so-called closed timelike curves, which are paths that are closed loops in time. Supposedly time travel was allowed in this solution. This conclusion shocked Einstein, who had thought the laws of physics excluded time travel. However, this time travel is different to what is seen in the movies, so it is perhaps better to think of it as follows. Consider a group of soldiers marching one behind the other in a straight line. This situation is comparable to linear time as the soldiers can definitively say whether they are in front of or behind anyone else. Time ordering in ordinary cosmology is like this line of soldiers. Now suppose that the soldiers march in a circle. In this case, there is no clear ordering as they are both in front of and behind everyone else, and only a self-consistent order remains. This situation is like a closed timelike curve. A soldier cannot jump back down the line to change how somebody is marching. That is, the time paths are self-consistent histories and one cannot go into the past or change the past. Gödel’s solution does not describe our Universe because it is not expanding and the time travelling aspects are likely to be unstable. Therefore, the closed timelike curves do not violate the laws of physics but are unlikely to be seen in practice because the conditions necessary to realise them are extraordinarily improbable.

4. Speed of light and the observable Universe

The speed of light is finite, which links space and time as mentioned previously; however it also partitions the Universe. The Universe consists of everything that there is, whereas the observable universe is a section of the Universe with a radius nearly equal to the speed of light times the age of the Universe. It should be noted that an additional factor is required to account for the expansion of the Universe. The observable universe is finite by definition and can be studied by astronomical observations. In contrast, the larger Universe proper may be finite or infinite. Since all of it can never be seen, its properties cannot conclusively be known. Statements about the observable universe can be made, such as whether it appears to have a beginning. The same questions about the Universe can never be answered because the observational reach required is lacking. The concept of known unknowns and unknown unknowns is familiar. Cosmology extends these concepts because the partitioning of spacetime leads to unknowable unknowns.

The Minkowski spacetime diagrams can help with a better understanding of the partitioning of the Universe. Consider figure 4; as time passes, one moves upwards in the diagram and the path of light follows the yellow lines. Most astronomical observations of the Universe involve gathering photons; thus observations are made along the yellow lines. In cosmology one would see down a past null cone or experimentally confirm some events inside the past light cone. If things that move slower than the speed of light are captured, such as cosmic rays or debris, then they are inside the yellow lines. Other observations, such as geological evidence or materials brought back from the Moon, are located very close to us on the spacetime diagram. All observational astronomy and cosmology studies involve what happens on and inside the past light cone. If the past light cone is followed back to one million years from the apparent beginning of the observable universe, then this is the time when radiation stopped scattering off electrons and flew freely towards us. This is called the time of last scattering. If one goes

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back to when the observable universe is only about a second to 100 seconds old, then it was hot enough for nuclear reactions to take place. At one second from the start, neutrinos stopped colliding and started flying freely towards us. Hence, in photon astronomy a snapshot is obtained of what the Universe was like when it was about a million years old when the last scattering of photons took place. If neutrino astronomy could be done, a snapshot would be obtained of what the Universe looked like when it was one second old. If gravitational wave astronomy is done at the right frequencies, a snapshot might be obtained all the way back to 10^{-43} or 10^{-35} of a second.

Astronomy is about the observable part of spacetime. One is severely limited by what regions of spacetime can access be accessed, as one cannot see what goes on beyond our past light cone because that would require faster than light propagation. Instead it is often assumed that what occurs for galaxies beyond the past light cone looks like the future evolution of our own evolutionary path through spacetime. In this way one makes an unverifiable assumption and extrapolates outward. Cosmologists try to say things about the unreachable parts of the Universe because one might have a theory that says something about these regions leading to a logical or physical impossibility, allowing that theory to be ruled out without seeing what happens in these unreachable domains. Alternatively there may be a theory about something inside the light cone that implies something bizarre occurs beyond it.

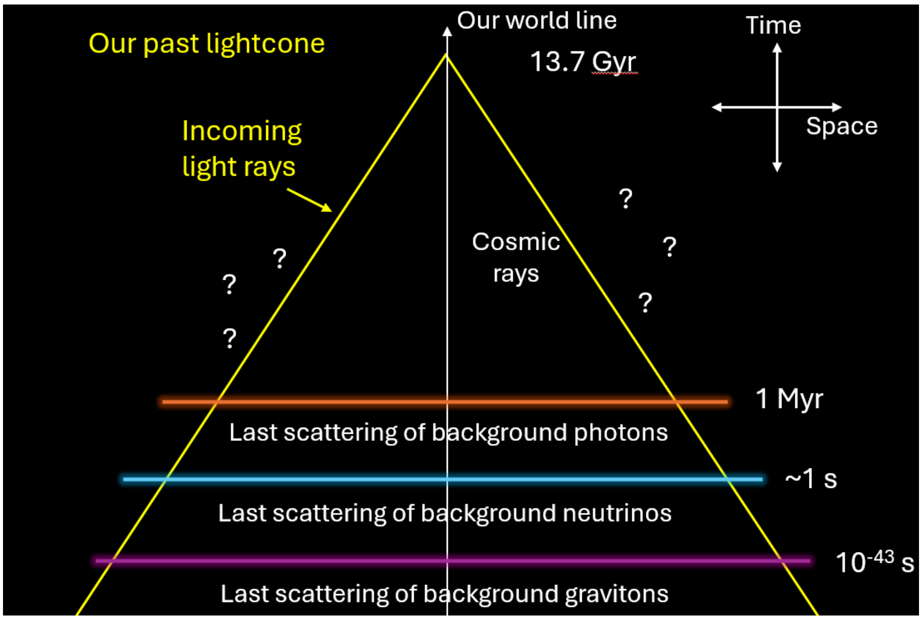

Figure 4. Our past light cone.

5. The inflationary Universe

The Universe is expanding as discovered in the late 1920s. This expansion is a consequence of Einstein's theory of general relativity, although Einstein did not make the discovery. Time is not only passing forward, but space is also stretching. We are not expanding, nor is Oxford, the Earth, the Solar System, the Galaxy or the local group of galaxies. Such things are held together by chemical or gravitational forces that are stronger than the effect of the expansion of the Universe on their dimension. The expansion of the Universe is felt at the scale of clusters of galaxies. In Arthur Eddington's famous analogy, the expansion is like putting raisins in a great lump of dough. The dough will rise in the oven and the raisins will move away from one another, but the raisins do not expand. The raisins are like the clusters of galaxies and the dough is like the space in between.

Space and time can be further linked by considering whether it is unexpected for the Universe to be so large. We are complicated chemical beings made of more complex elements than hydrogen and

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helium. Instead we are made up of carbon, nitrogen and oxygen, and perhaps we will be aided by or superseded by other things made of silicon or more complex elements. All elements heavier than helium form in stars and were not made in the Big Bang. Therefore, all the carbon nuclei in one's body at some stage of their existence were in a star. The nuclear reactions within stars convert hydrogen to helium, then helium makes beryllium, beryllium with another helium makes carbon and so on. The nuclear reactions within stars make all the biologically interesting elements but take billions of years to reach completion. This train of thought makes the age of the Universe less surprising; we could not exist in a universe that was significantly younger because it could not contain the building blocks for our complexity and information processing. Thus, we must find ourselves in a universe several billions of years old. If it takes 10 billion years for the Universe to contain the building blocks of life and the Universe is expanding, then the Universe must at present be at least 10 billion light years in spatial extent. We could not exist in a universe that was significantly smaller. A universe comparable in size to our Galaxy with only 100 billion stars and 100 billion planetary systems would be little more than a month old. The amount of time and expansion is also responsible for the darkness of the night sky, the low energy in the Universe and the huge distances between stars, galaxies and planets. To a first approximation, the Universe is empty. There is one atom on average in every cubic metre. One cannot make a better vacuum than that in any laboratory on Earth.

The first solutions of Einstein's equations in the 1920s remain the best descriptions of the Universe in its present age. The expansion of the Universe has two essentially different possible tracks: expansion that runs away at high velocity forever or expansion that starts slower than the escape speed, reaches a maximum and then collapses to a Big Crunch. A critical universe exists on the dividing line between these outcomes (figure 5).

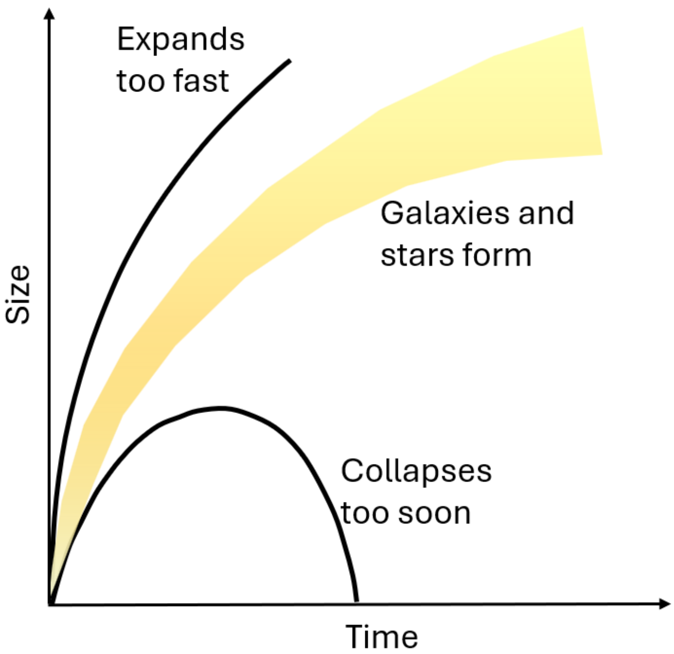

Figure 5. Diagram of the different possibilities for an expanding universe.

The picture of expansion is more sophisticated than it may appear. Expanding universes are not explosions and they are not expanding into anything. The Universe is everything there is and the geometry of spacetime for a specific type of expansion may not be flat and Euclidean. The geometry will depend on the expansion type (figure 6). The critical universe has a Euclidean geometry and is flat. In contrast the fast expansion universe has a hyperbolic geometry and the slow expansion one an elliptical geometry. If one were to draw a triangle with each pair of points joined by the shortest line on

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the surface, then in the elliptical geometry the triangle curves outwards (positive curvature) and in the hyperbolic geometry it would curve inwards (negative curvature).

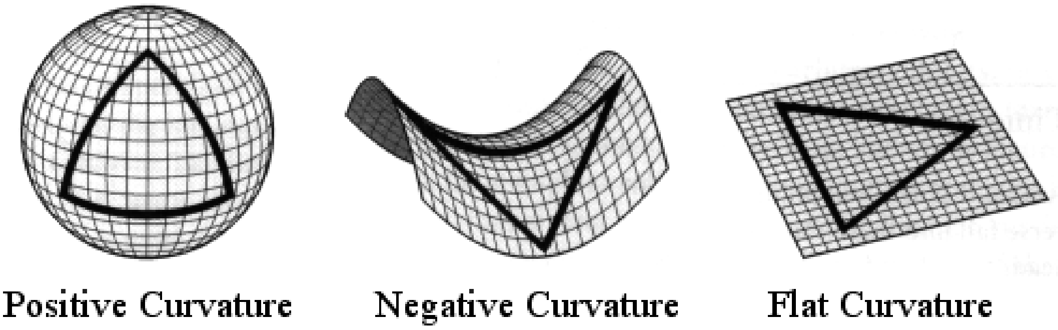

Figure 6. The different geometries of spacetime depending on the rate of expansion of the Universe.

The Universe is very flat, approximately within 2% of the critical compromise universe, which is quite mysterious. The starting conditions for the expansion must be very precise for this to happen or the expansion will veer further and further away from the critical path. Upon calculation, one finds that we are so close to the critical path that the start speed of expansion needed to be accurate to one part in 1 followed by 35 zeros in order to stay as close as we have for 14 billion years. This similarity appears strange and scientists struggled to explain why it was true all the way until 1980. In 1980/1981, a new idea appeared in cosmology that remains the standard today: the inflationary universe. This idea adds to the previous picture that there was a surge in the expansion – an acceleration - near the beginning of the Universe for an incredibly short time. There are good particle physics reasons why this might have occurred related to existence of elementary particle fields, such as with the Higgs boson, that produce an antigravitational effect before decaying and normal physics resumes. This surge in expansion was exponentially rapid and drove the expansion fantastically close to the critical divide, such that we can remain so close after 14 billion years.

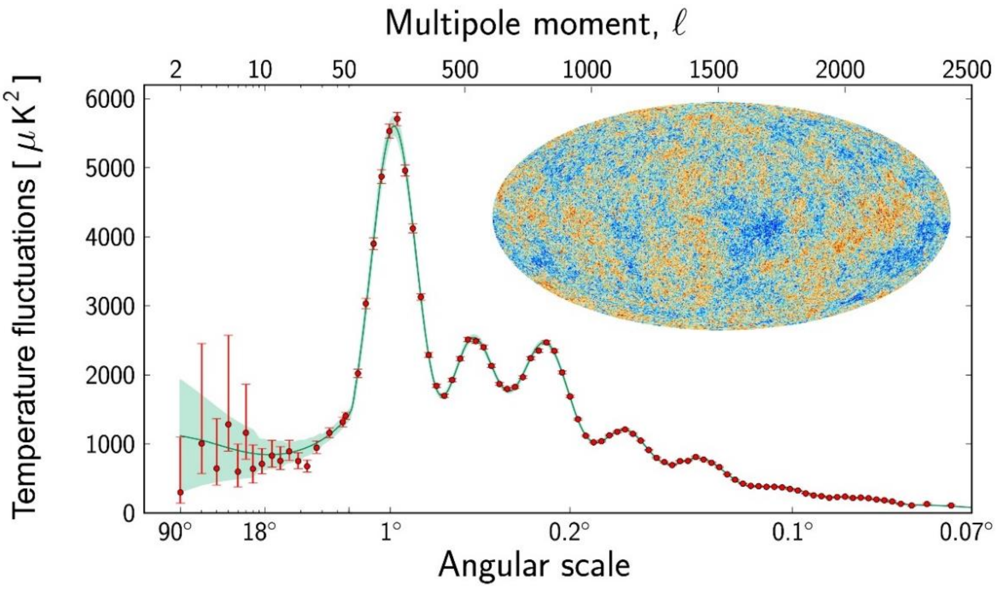

Figure 7. Temperature fluctuations in the Cosmic Microwave Background measured by the Planck satellite. Credit: ESA and the Planck Collaboration. Available at: https://sci.esa.int/web/planck/-/51555-planck-power-spectrum-of-temperature-fluctuations-in-the-cosmic-microwave-background.

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The initial patch of the Universe that expanded into everything had quantum fluctuations. These fluctuations spread across the Universe during the expansion and one can try to observe them. Three satellite missions, COBE, WMAP and most recently Planck (figure 7), have measured minute differences in the Cosmic Background Radiation temperature throughout the sky to test predictions of the inflationary universe. The solid line in figure 7 is the theoretical prediction and the red points are the observations. These observations are remarkable and provide observational confirmation that inflation did occur in the Universe when it was about 10^{-35} of a second old.

The idea of inflation has several strange consequences for the structure of spacetime. For example, at 10^{-35} seconds old the density, temperature and other quantities of the Universe could vary significantly from one point to another. One small patch of the Universe would have then accelerated within a relatively short period of time to produce a smooth, almost flat region with tiny fluctuations that we live within. Thus the inflation stretched out the curvature of spacetime beyond our horizon such that the geometry of space appears almost Euclidean to us. However, other regions of the Universe could have undergone different histories and may look very different. Hence, one of the predictions of the inflation theory is that the Universe should be complex beyond our visible universe. Furthermore, if a patch of the Universe accelerates and inflates to become almost Euclidian, then it creates the conditions for sub-regions to undergo further inflation. It is a self-reproducing fractal process and all that can be said is that we live in one of the regions that has undergone this accelerated expansion (figure 8). The expansion of our region had a beginning; however, it is not known whether the possibly infinite self-reproducing network had a beginning. Our current understanding of the physics of this process predicts that it will not have an end. Therefore the Universe is destined to continue forever.

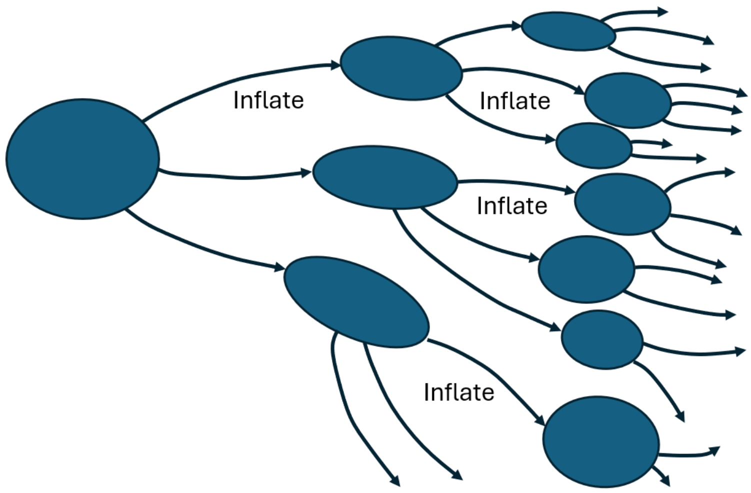

Figure 8. Self-reproducing regions of the Universe owing to inflation.

6. Evidence of expansion and the future

It might be asked whether there is evidence that the Universe will expand forever or if this is solely a theoretical prediction. At the end of the 1990s new astronomical observations became possible. These observations measured light from distant objects, similar to what Edwin Hubble and Milton Humason did in the 1920s, to determine their distance and speed of recession via the Doppler effect and so to deduce how fast the Universe is expanding. Historically the distance was difficult to calculate because it was determined by brightness of specific classes of objects with unknown intrinsic properties, which made it challenging to differentiate between bright objects that were distant and faint objects that were close. Brian Schmidt, Saul Perlmutter and their groups developed a method to work out the distance to supernovae that were almost on the edge of the visible universe. They used these observations to learn about the expansion of the Universe and they arrived at a dramatic conclusion which led to them

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receiving the Nobel Prize in Physics in 2011. They discovered that the expansion of the Universe began to accelerate about four or five billion years ago and has continued to accelerate today. This process appears similar to inflation, but it cannot be the same type of inflation as what occurred near the beginning of the Universe. Subsequent observations of microwave background fluctuations and oscillations of the matter distribution in space have independently reached the same conclusion. It is not known why the expansion of the Universe began to accelerate again, but there is an accurate description of the acceleration within Einstein's theory. It could be random chance or there could be some physical reason why acceleration onsets at this particular time.

The consequence of acceleration makes the outlook for the Universe quite bleak. Everything is moving away from everything else at an exponential speed, so all star formation, galaxy formation and so forth will stop. All stars will run out of nuclear fuel such that they will collapse into black holes or evaporate into elementary particles. Even the black holes will evaporate from Hawking radiation into elementary particles. In a sense the Universe is destined to become a great cemetery of dead stars.

It is a sobering thought to realise that we live at a propitious interval of cosmic history in which the requisite elements have had enough time to form, the stars are still alive and it is still possible to study and understand spacetime. Someone in the Universe after 100 billion years could only learn about its structure by reading old books. Such books would tell of a golden era where one could observe to the edge of the 100-billion-light year horizon of the Universe and see distant stars and galaxies before they had come to the inevitable end of their lifetimes. We live in this golden era of astronomy where it is possible to understand the structure of spacetime and test our theories with astronomical observation.