Supplementary Material, ZDM – Mathematics Education

PROOF AND PROVING IN SCHOOL AND UNIVERSITY MATHEMATICS EDUCATION RESEARCH: A SYSTEMATIC REVIEW

Gabriel J. Stylianides[1]
University of Oxford, Department of Education, UK, https://orcid.org/0000-0003-1770-8753

Andreas J. Stylianides
University of Cambridge, Faculty of Education, UK, https://orcid.org/0000-0002-3526-0342

Andreas Moutsios-Rentzos
National and Kapodistrian University of Athens, Department of Pedagogy and Primary Education, Greece, https://orcid.org/0000-0002-9614-2825

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[1] Corresponding author. Email address: gabriel.stylianides@education.ox.ac.uk.

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