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The epistemology of spacetime

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Abstract

How is it that the basic structures of space and time come to manifest themselves in physical theories and theorising, and in our empirical experience of the world? This question is central to an important field of the philosophy of physics: the epistemology of spacetime. In this article, we survey systematically the various responses which have been offered to this question, highlighting little-explored connections and open research questions.

1 | INTRODUCTION

How is it that the basic structures of space and time come to manifest themselves in physical theories and theorising, and in our empirical experience of the world? The project of attempting to answer this question is a venerable philosophical tradition, with its roots in Aristotle, its trunk in Descartes, Newton, and Leibniz, and its canopy in Helmholtz, Poincaré, Reichenbach, and Weyl. The profound questions raised by these latter authors include: from what minimal assumptions can one reconstruct the geometry of spacetime, and does such a reconstruction afford a means of gaining empirical access to spacetime? And: could it be that the structure of space and time is underdetermined by empirical experience—indeed, by the mathematical structure of our theories themselves—and is therefore merely a matter of convention? Questions such as these lie at the core of assessing our ability to gain epistemic and conceptual access to the structure of space and time.

As matter of fact, however, the philosophy of space and time has for decades been dominated by metaphysical issues: most notably, the substantivalism/relationalism debate, which regards the ontological status of space and time (as opposed to the epistemological question which is our concern here—*viz*., how can we come to *know* the nature of space and time?). Although great progress has been made in these fields, this has arguably been at the expense of work in the epistemology of space and time developing upon the above tradition, which, with the exception of developments on 'axiomatic constructivism' from the 1970s up to the early 1990s, and works by internationally-isolated groups such as the German school of 'proto-physics' (discussed in detail below), has arguably continued to be a largely-neglected field. (Of course, this is not to dismiss much excellent earlier works in this field—see inter alia, Grünbaum, 1973; Sklar, 1974; van Fraassen, 1970, for important contributions to the epistemology of spacetime.)

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This article thus seeks to review and revive the highly significant tradition of the epistemology of spacetime. It will do so in a systematic way by, firstly, returning to the Helmholtz-Lie 'Problem of Space', articulated in the late 19th Century (this being the question: "Which of the many candidate mathematical geometries represents the physical structure of space and time?") and thereby pursuing an historical study of that important debate; by, secondly, demonstrating that philosophers' and physicists' considerations of the Problem of Space led directly to four major themes within the epistemology of space and time (Carrier, 1994a): namely, (i) constructive axiomatisation, (ii) conventionalism, (iii) Kantian transcendentalism, and (iv) 'field interpretations' of space and time; by, thirdly, pointing out the relevance of contemporary methods in physics, mathematics, and philosophy, in order to provide modern-day appraisals of, and developments upon, strategies (i)-(iv)—several of which have been neglected for decades within the discipline. In making these points, we hope to encourage a re-awakening of what deserves to be a major sub-field of the philosophy of physics.

2 | THE HELMHOLTZ-LIE PROBLEM OF SPACE

For much of its history, geometry was regarded as the science of the actual structure of space. However, in the 19th Century this immediate and a priori link between the mathematical discipline of geometry and the physical science of space was severed, by two developments: (i) the discovery of non-Euclidean geometries by Bolyai and Lobachevsky, and (ii) the rise of the modern axiomatic understanding of geometry as describing abstract structures. This meant that a new and urgent question arose: what is it that could make a geometrical theory a theory of space, if being geometrical is no longer sufficient to guarantee spatial representation? This problem was addressed as the so-called Problem of Space: the question of which, among the infinitely many possible geometries (as ultimately described by Riemann, 1868), are candidates to represent physical space. The foundational philosophical idea behind this work was based upon forging a link between the epistemology and semantics of geometric representation: a geometry may be considered physical insofar as it would be coherent to describe physical operations of geometrical measurement within it, through the use of rigid bodies (the principle of free mobility). (For further discussion of Helmholtz's philosophy of science and of the Problem of Space, see DiSalle, 1993; Patton, 2018; for a more general history of the philosophy of geometry, see Torretti, 1978b.)

The solution to the Problem of Space in this 'classical' form was offered by von Helmholtz (1868, 1896) (in work that was later made rigorous by Lie [1890]), who then showed that the only geometries which permit the free mobility of rigid bodies are the geometries of constant curvature. Grounding the notion of 'physical geometry' in a principle of free mobility immediately raises the question of that principle's conceptual status. Depending on whether it is read as "a [empirically groundable] fact of nature, a convention, or a condition of the possibility of geometric measurement", Helmholtz's solution to the Problem of Space amounts to adopting one of three out of the four major stances on the status of physical geometry—viz., (i) empiricist constructivism, (ii) conventionalism, or (iii) transcendentalism about measurement conditions (Carrier, 1994a). This shows how these three stances on the status of physical geometry are in fact already foreshadowed by Helmholtz's work on the Problem of Space. The fourth major stance is the (iv) 'field interpretation' of physical geometry, which renders geometric structures on a par with non-geometrical fields such as the electromagnetic field: "Physical geometry—just like other physical fields—becomes manifest empirically in a multitude of indicator phenomena" (Carrier, 1994a, p. 289). (i)-(iv) constitute the focus of the sections 3-6 below; before that, though, we should say something on Weyl's analysis of the Problem of Space.

The Helmholtz-Lie solution to the Problem of Space—that the only candidate physical geometries are geometries of constant curvature—received a severe challenge with the publication of Einstein's General Theory of Relativity in 1915: for this theory argued that the actual physical geometry of the world was a variably curved geometry, of precisely the kind that the Helmholtz-Lie analysis excluded! Hence, the question arose of what it was that had gone wrong in that solution, and whether the Problem of Space could be reframed in a manner appropriate to the new theory. This question was soon taken up by the great mathematician, physicist and philosopher Hermann Weyl, when he was called upon to edit a new edition of Riemann's inaugural dissertation (Riemann & Weyl, 1919). In reflecting upon Helmholtz's discussion of which geometries could be physical, he came to the conclusion that Helmholtz's argument relied on the idea that the only way to operationalise geometry was through the notion of rigid bodies, whose congruence (or incongruence) could be compared at arbitrarily large finite distances. This, thought Weyl, reflected a vestigial attachment to the kind of 'Ferngeometrie' (distance geometry) that Einstein, using Riemann's tools, had shown to be inappropriate for describing physical reality. The solution, therefore, was to develop a version of Helmholtz's analysis that made use only of notions of infinitesimal congruence, and which was thereby appropriate for the 'local-infinitesimal' geometry that characterises General Relativity. However, the full articulation of Weyl's ideas took time. His discussion of the problem found its fullest expression in a series of lectures that he delivered in Madrid and Barcelona in late 1922, and which were later published as (Weyl, 1923).[1] For commentaries on Weyl's work, see Bernard (2018), Bernard and Lobo (2019), and Dewar and Eisenthal (2020).

3 | CONSTRUCTIVE AXIOMATICS

In 1921, Weyl showed that any two metrics which are both projectively equivalent (i.e., for which the metric-compatible derivative operators identify the same geodesics) and conformally equivalent (i.e., which agree on lightcone structure) are equal up to a constant factor. (By 'metric', Weyl in fact meant the structure of a Weyl geometry: see (Matveev & Scholz, 2020) for a contemporary discussion. The special case of projective and conformal structure fixing a Lorentizan metric is discussed in Malament (2012).) This mathematical observation acquires physical significance when it is observed that, on the usual interpretation, the conformal structure of a metric is determined by its causal structure (which points are causally connected with one another) and that its projective structure is determined by its inertial structure (which timelike curves are geodesics). Thus, Weyl's theorem suggests that the causal and inertial structure is what gives a Lorentzian metric its physical significance. It is not uncommon to find the claim that Weyl's theorem (or Malament's restricted version thereof) affords a means of gaining operational access to the metric field (i.e., spacetime structure) in general relativity—see, for example, Malament, 2012, §2.1. The thought is the following: light rays in relativity theory traverse null geodesics, and thereby afford access to the conformal structure of spacetime; massive test particles traverse timelike geodesics, and thereby afford access to the projective structure of spacetime. Together, by Weyl's theorem, tracking such a set of bodies allows one to reconstruct a Weyl metric field.

Notably, Weyl's theorem has in this manner even been made one of the centerpieces of a 'constructive' axiomatisation of general relativity à la Ehlers et al. (1972) and subsequent extensions: in what is known as the 'causal-inertial method', one takes pre-relativistic local observations on the behaviour of light and massive particles as axioms, and explicitly aims to define all relevant spacetime structures, including the metric field, therefrom. The motivation for such axiomatizations are harmonious with those presented in (Reichenbach, 1924): one desires to build up one's theories of physics from axioms, each of which has a direct empirical correlate; in this way, the theory may be empirically grounded. Although such motivations should be appealing to any empiricist, and such axiomatizations are mathematically fascinating, they remain under-studied in the philosophical literature; what is needed is a systematic engagement with this agenda. Desirable tasks to achieve here include the following: (i) a comparison of the specific Ehlers-Pirani-Schild axiomatisation with a number of ensuing variants of the causal-inertial method (see Linnemann & Read, 2021a, 2022b, and references therein); (ii) an assessment of the Coleman-Korté (1980) claim to derive inertial structure from differential-topological assumptions only, thereby saving the causal-inertial method from any charge of circularity (of the form: the Ehlers-Pirani-Schild approach presupposes that one can identify the free massive particles—but how can this be done without an antecedently-given conception of spacetime to underwrite those free motions? (Sklar, 1977)); (iii) an engagement with the physical significance of the causal-inertial method: can it indeed be used to recover the spacetime structure of general relativity, without additional chronometric assumptions? (For recent attempts to address these questions, see Linnemann & Read, 2021a, 2021b); for older—though nevertheless still excellent—discussion of the Ehlers-Pirani-Schild approach, see Carrier, 1994b.)

4 | CONVENTIONALISM

One of the richest strands of thought about geometrical measurement concerns the subtle interplay between matters of fact and matters of convention. First, there is a tradition—going back to Poincaré (1902)—of thinking that the use of one geometry rather than another to represent physical space cannot be fixed merely by empirical considerations alone, but includes in addition some element of conventional choice. In the work of the logical empiricists, such 'geometrical conventionalism' became a hallmark: indeed, Reichenbach's conversion to geometrical conventionalism, under the influence of Schlick, marks his transition from neo-Kantianism (on which see below) to logical empiricism. In the remainder of his life's work, such as (Reichenbach, 1928), he maintained that it is a matter of convention whether we employ a theory with curved spacetime but no gravitational force, or a theory with flat spacetime and gravitational force.

Although historical issues regarding this conventionality of spacetime geometry have been of continued interest to philosophers—see, for example, Ivanova (2015a, 2015b) and (DiSalle, 2002, 2008)—it is also important to recognise that the idea that space and time might manifest conventional aspects has led to the development of a number of further important contemporary threads. One notable such thread regards the conventionality of simultaneity in special relativity: an issue first presented in its modern form in (Reichenbach, 1928), but more recently associated in particular with the work of Grünbaum (1973). In essence, the issue is that the special theory of relativity, as presented in (Einstein, 1905), presupposes a particular means of synchronising spatially separated clocks (the so-called 'Einstein-Poincaré' synchrony convention); if other means are chosen, however, the resulting structure of space and time will be different. (See Winnie, 1970a, 1970b) for a formulation of 'synchrony-general' special relativity.) Malament (1977a) claimed that, in fact, simultaneity is not conventional in special relativity; however, the interpretation of his proof is controversial, and the debate on this front is still ongoing. (For further discussions of this issue, see Janis, 2018.)

In addition to the conventionality of simultaneity, there are many further threads regarding the conventionality of spacetime geometry which remain to be explored fully; here we will constrain ourselves to mentioning three. First: in the context of Newtonian theories, it is sometimes claimed that there is a conventional choice between geometrised Newtonian gravitation ('Newton-Cartan theory'), and non-geometrised Newtonian gravitation. In Weatherall and Manchak (2014), the authors argue that although such conventionalism is appropriate for this case, it is not appropriate for the relativistic case. However, in Read and Teh (2018) it is shown that a quite general method of 'teleparallelisation' links flat Newtonian gravitation with Newton-Cartan theory, and teleparallel gravity with general relativity, which suggests that these cases should be considered fully analogous. In addition to this, Dürr (2021a), questions the accuracy of the reading of Reichenbach by Weatherall and Manchak.

Second: philosophers of physics have recently heeded the call to explore the 'space of spacetime theories' (Lehmkuhl, 2017). Within this space, there are many apparent cases of the conventionality of spacetime geometry—for example: (a) between Nordström gravity and Einstein-Fokker gravity (Dürr, 2020); (b) between the Jordan and Einstein frames of Brans-Dicke theory; (c) between different versions of what is known as $f(R)$ gravity (Dürr, 2021b); and (d) between theories manifesting torsion, curvature, and/or non-metricity (Hehl, 2017).

Third: two *à la mode* topics in the philosophy of physics are (i) theoretical equivalence, and (ii) the existence of so-called 'dualities' in physics. In both of these cases, one has mappings between *prima facie* distinct theories, which preserve some salient features, but not others (see Le Bihan & Read, 2018; Weatherall, 2019a, 2019b, for respective reviews in this journal). Clearly, cases of the conventionality of geometry—in which one can map between theories with different spatiotemporal structures—seem to fit the mould of the above literature. Although some connections on this front have been made (see again, e.g., Dürr, 2021b), the full suite of links between the notions of conventionality, theoretical equivalence, and duality remains to be explored in a systematic manner.

Stepping back now from issues of the conventionality of spacetime geometry in the sense promulgated by Poincaré and Reichenbach, it is worth noting that issues of conventionality more broadly construed bear on some of the deepest questions in analytic philosophy. If there is a single point that one might wish to identify as the moment of transition between logical empiricism and contemporary analytic philosophy, it would be Quine's attack on the analytic/synthetic distinction in Quine (1951); key to this assault was his contention that "the lore of our fathers is … a pale grey lore, black with fact and white with convention. But I have found no substantial reasons for concluding that there are any quite black threads in it, or any white ones." Thus, understanding the nature of scientific convention is a precondition for understanding the birth of analytic philosophy as it is currently practised. (For further discussion of the role of conventionalism in the development of analytic philosophy, and how the more general notion of 'truth by convention' to be found in the analytic philosophy literature differs from the Reichenbachian conventionalism discussed in the above paragraphs, see Ben-Menahem, 2006.)

Finally, the working-out of the conventions of *measurement* continues to be a vital part of scientific practice. It was only in 2018 that the General Conference on Weights and Measures agreed that the kilogram should no longer be defined as being "equal to the mass of the international prototype of the kilogram", and should instead be "defined by taking the fixed numerical value of the Planck constant h to be $6.62607015 \times 10^{-34}$ when expressed in the unit [J s], which is equal to [kg m^2 s^{-1}]". By doing so, the committee made it (seemingly) an analytic truth that Planck's constant has the value that it has, rather than an empirically determined fact. In turn, this raises subtle and profound questions about authority over meanings, especially in the context of science: such a redefinition might profitably be compared with the International Astronomical Union's reclassification of Pluto, especially given the discrepancy in how controversial these two redefinitions were. (For recent work on units in physics, see Dewar, 2021; Wolff, 2020.)

5 | KANTIAN TRANSCENDENTALIST APPROACHES

We turn now to the third set of approaches to the epistemology of spacetime identified in (Carrier, 1994a) as arising out of the Helmholtz-Lie Problem of Space—namely, Kantian transcendentalist approaches. It is helpful to divide the literature here into two categories: first, (neo-)Kantian approaches according to which spacetime geometry is to be regarded as a precondition for the possibility of experience; second, more constructivist approaches (in the constructivist sense of Carnap's *Aufbau* (1926)), according to which the structure of space and time is built up linearly from accepted phenomenological inputs. We address these two approaches in two separate subsections in what follows, beginning with the former.

5.1 | The relativised a priori

In the wake of the development of general relativity, philosophers grappled with the problem of how it could be the case that the supposed non-Euclidean nature of physical geometry was reconcilable with the Kantian claim that the geometry of space and time—a precondition for the possibility of experience—can be known a priori to be Euclidean. One response to this, offered most famously by Reichenbach (1920) (before his conversion to conventionalism—see above), is that the Kantian synthetic a priori should be *relativised*, and can change, depending upon one's theoretical commitments (in particular, those which are construed as 'constitutive principles'). Thus, the geometrical framework of general relativity can equally well serve as such Kantian preconditions. In the philosophy of physics, such approaches are often identified as 'neo-Kantian'; towards the end of the 20th Century, they were revived by Friedman (2001), who also defended a relativised conception of the Kantian a priori.

As above, much work continues to be undertaken on this neo-Kantian approach to spacetime. Historically, one might ask whether authors' neo-Kantian convictions are cleanly separable from their conventionalist commitments: see, for example, Ivanova (2015b) and Friedman (1999). One might also ask whether Friedman's approach truly is best understood as a successor of the Reichenbachian relativised a priori, or, rather, whether it is better associated with the neo-Kantianism of Cassirer (on this question, see Everett, 2015; for primary literature, see Cassirer, 1921). Less historically, one might ask whether Friedman's approach is viable on its own terms. For example, Samaroo (2015) argues that what Friedman regards as being constitutive principles of the relativised a priori (e.g., the equivalence principle, in the case of general relativity) are in fact not so. (An interesting sufficient criterion for what should count as relativised a priori has been provided by Curiel (2016): kinematical constraints are relativised a priori statements in the precise sense that they are "precondition[s] for the appropriate application of a theory in modeling a kind of system". The values of the proposal of course hinges on the stability of the kinematical/dynamical distinction, which, however, Curiel sets out to defend.) Pitts (2018), on the other hand, objects to the constitutive a priori role of inputs (again such as the equivalence principle) leading to general relativity, in light of the availability of empirically equivalent alternatives to general relativity (in particular spin-2 gravity) which do not embrace such principles.

5.2 | Proto-physics

There is a separate approach to spacetime geometry which can be construed as a response to the Helmholtz-Lie Problem of Space in a Kantian transcendentalist fashion: the German school of proto-physics, developed by Lorenzen, Janich and their students in the late 1960s to early 1980s.[2] This approach aims at a non-circular, systematic account of how to set up basic measurement operations for those quantities that are essential to physical theorising—such as distance, time, and mass. Very crudely, its main motivation derives from a dissatisfaction with accounts of measurement à la options (i), (iii) and (iv), all of which suffer (the claim goes) from what proto-physicists call 'methodological circularity': to account for measurement, these options presuppose theories that themselves have to be confirmed under recourse to measurement. The proto-physicists' strategy to evade such circularity is to ground concepts in actions of one's lifeworld, that is semantics in everyday pragmatics (thus allowing one to bypass the circle which is claimed to arise if one were to proceed merely conceptually). The concept of 'lifeworld' is hereby best explicated as the qualitative realm of statements and actions from everyday practice and craftmanship which do not readily include quantitative statements.

The Problem of Space has, for instance, been approached by Janich (1997) as follows: A notion of 'plane' is to be idealised from the result of a realisation procedure of rubbing alternatingly two out of three objects of sufficient hardness against each other until they fit on top of one another. (This procedure called 'Dreiplattenverfahren' already goes back to Dingler (1907).) From the basic notion of 'plane', one then obtains further notions through further realisation procedures and the idealisation of their respective results; for instance, the next concept is that of 'straight line', which can be grounded in the idealisation of intersecting two surfaces arising from the 'Dreiplattenverfahren'. Ultimately—so Janich tries to establish—one arrives at a setup tantamount to Euclidean space. The degree to which Janich's or other protogeometric accounts of Inhetveen (1983) and Lorenzen (1984) indeed succeed with the reconstruction of Euclidean geometry is, however, subject to controversy.[3]

The distinction between lifeworld and quantitative realm of proper science gives rise to protophysics' transcendentalist character reminiscent of Reichenbach's neo-Kantianism: Euclidean space including a notion of rigidity (i.e., the assumption that rods retain their length when transported)—developed out of certain lifeworld practices—is seen as constitutive for quantitative measurement. Importantly, this does not mean that no other space than the Euclidean one could in principle be linked to lifeworld actions, nor that (advanced) physical theories featuring non-Euclidean space(time) are necessarily problematic. Rather, the gist of proto-geometry is that, when retracing the methodological order of how central spatial quantities are established prior to being used in proper physical theories, one learns that certain actions from the lifeworld uniquely pin down a certain spatial setup. Once this spatial setup is in place, physical theories can be formulated and tested in terms of spatial quantities—including theories that may work with other notions of space after all. What matters, however, is that some sense of spatial measurement has been made available to begin with.

For comparison: Dingler held the view that Euclidean space is determined under any proper recourse to lifeworld *as a matter of necessity*. It is this strong sense of necessity that puts Dingler's notion of 'production a priori' into vicinity with synthetic a priori judgements in the sense of Kant (see Schlaudt, 2014). At the same time, this sense of necessity may also seem to lie at the heart of Dingler's infamous opposition to modern physics.[4] But note that even Dingler's sense of ‘a priori’ is as such compatible with advanced physical theories which feature non-Euclidean space. The opposition of Dingler—and, arguably, also that of Lorenzen (1977)—to relativity is instead much more rooted in an understanding of proto-geometry as providing an account of what space is *tout court* rather than as an account of how first spatial measurements are possible (as the proto-physics mainstream has it by now).

Let us go beyond proto-geometry then. More generally, starting from pre-scientific experience, proto-physics (as opposed to mere proto-geometry) investigates (i) with what intentions basic measurement devices are built, (ii) what kind of pre-scientific concepts their construction relies on, and (iii) how consecutive pre-scientific operational steps can possibly be used to realise them in a non-circular fashion.

Even a modest variant of proto-physics has to count as an ambitious project—a systematic, linear approach to the genesis of basic measurements. This might simply be impossible, at least up to full completion. An alternative (and potentially more satisfactory) option for understanding the genesis of basic measurements may lie in the iterative picture of how measurement device and theory development influence one another (e.g., Carrier, 1990; Chang, 2004; Tal, 2014). Still, as a matter of fact, proto-physical accounts of space, time and mass measurements have been put forward and seriously defended. Given its high potential for increasing our understanding of how science literally becomes possible, the achievements and insights of the proto-physics program deserve not to continue to go unnoticed in contemporary philosophical communities. Future work needs to involve a comprehensive reconstruction and reappraisal of proto-physics approaches, including the leading proto-geometric accounts mentioned, and the proto-physical account of time by Janich (2012). Such systematic re-appraisals would provide clarity from a neutral vantage point, as well as a point of entry to the topic for non-German speakers (most extant work on proto-physics having been published in German). More concretely, the following questions are worthy of pursuit: (i) Which relevant philosophical questions are raised in the proto-physics program that have so far remained largely unnoticed in mainstream philosophy of science/physics?; (ii) Which relevant philosophical results and reasoning patterns of the proto-physics program can be obtained which do not depend on idiosyncratic commitments?; (iii) Can these be brought to fruition outside of the specific program of proto-physics?

6 | FIELD INTERPRETATIONS

In Helmholtz's approach to physical geometry, the characteristics of physical space are accounted for operationally via the behaviour of freely-moving rigid rods. Not surprisingly, the characteristics of spacetime can be afforded a similar operationalist grounding; indeed, ‘field interpretations’ of the general relativistic spacetime metric—such as the ‘dynamical’ approach to spacetime (Brown, 2005), and the ‘relationalist’ interpretation of general relativity due to Rovelli (1997)—seek to explicate the essence of spacetime through its dynamical implications for material structure. In both such accounts, a grounding of spacetime in operational procedures is motivated not merely on empiricist grounds, but also by the alleged explanatory advantages of such procedures. It has, however, been a matter of ongoing debate in what sense field interpretations build on presupposed geometric backgrounds, and whether the idea of a unidirectional explanatory arrow from fields to physical geometry is feasible (on these objections, see, respectively, Acuña, 2016; Norton, 2008).

There are also important technical questions which remain to be addressed. Notably: the ‘dynamical’ approach of Brown takes a different form in the context of special versus general relativity—for while in the former case this position is one of ontological reduction (spacetime is nothing more than a codification of the dynamical behaviour of physical fields), in the latter case the dynamical view reneges on said programme of ontological reduction (see Brown, 2005, ch. 9). Several authors have, however, questioned whether the same metaphysical moves of ontological reduction can be made in the context of general relativity. For example, Dewar (2020) has provided important technical tools needed in order to extend this aspect of the dynamical view to the dynamical spacetimes, while Stevens (2017) has shown that ‘super-Humeanism’—the view that both the laws of nature and spacetime structure are mere codifications of empirical regularities—is a promising metaphysical setting for the dynamical approach. Given these tools, it is a natural outstanding task to bring them together, in order to render for the first time the dynamical approach, qua metaphysical thesis (regarding the constitution of spacetime—rather than the epistemological thesis that the correct way to secure operational access to spacetime is by way of the behaviour of material fields), suitable for application to all spacetime theories.

In our view, the main tasks to be accomplished in this field are twofold: (i) compare various field interpretations of general relativity—contemporary and historical—in order to present a unified account of this approach to the foundations of physical geometry; and (ii) extend, following the procedure outlined above, the metaphysical aspects of the dynamical approach to spacetime theories to the context of general relativity.

7 | CONCLUSIONS

We have traced the various sub-schools of the epistemology of spacetime back to the Helmholz-Lie Problem of Space—this being the question: which, among the infinitely many possible geometries, are candidates to represent physical space? First: one might attempt to build up the structure of space and time from axioms which are empirically indubitable: this is the constructive axiomatic approach. Second: one might seek to argue that the geometry of spacetime is in fact (at least in some respects) a conventional matter. Third: one might attempt to argue that, in one way or another, spacetime constitutes part of a Kantian precondition for the possibility of experience. Fourth: one might seek to argue that the nature of spacetime is settled by the behaviour of certain relevant material configurations, such as rods and clocks. All of these approaches are epistemological, insofar as they pertain to the question of how we secure access to the structure of spacetime, and to how we delimit the range of spatiotemporal possibilities for the physical world. This should be obvious for the first and forth approaches above; on the second, one would maintain that one can access the structure of spacetime because this is merely a conventional matter; on the third, one would maintain that one can again access the structure of space and time, since this constitutes part of the Kantian a priori.

There are two further points which should be made about these approaches. The first is that they are not *exclusively* epistemological, but also have metaphysical components. For example, in the case of constructive axiomatics or of the field approach, one could say that one is giving an account of how one can use empirical inputs (whether those be, e.g., the behaviour of light rays and freely falling particles as on the EPS axiomatisation, or the behaviour of material rods and clocks as on the dynamical approach)—or, one could extend this to a metaphysical thesis, and maintain that spacetime is to be (in some sense to be expanded upon) ontologically reduced to such inputs. There is also a clear metaphysical component to the conventionalist and neo-Kantian approaches, insofar as they give a particular account not just of how we can access spacetime structure, but of what that spacetime structure *is*. This being said, in our view—and the views of many historical authors such as those mentioned above—the epistemological questions can be sufficiently clearly distinguished from the metaphysical questions to warrant the study of the epistemology of spacetime as an independent discipline.

Our second point is this: there are several senses in which the four reactions to the Helmholtz-Lie Problem of Space which we have articulated above are not independent of one another. For example, as we have already alluded to above, there are both conventionalist and neo-Kantian themes in the works of both Poincaré (1902) and Carnap (1921) (see Friedman, 1999; Ivanova, 2015b). Moreover, on studying the EPS constructive axiomsatisation in its original presentation, one can see clear conventional inputs, as well as appeal to the behaviours of physical clocks (see Linnemann & Read, 2021a, 2022b). And as a final example: one might (in our view correctly) take the distinction between constructive axiomatics and proto-physics to be one of degree, and principally a matter of the structures which one is willing to take as basic (in the former case, empirical inputs still couched in theoretical terms, e.g., the behaviour of light rays and freely falling particles; in the latter, more basic phenomenological inputs).

To close this article, we mention two further topics unrelated to the Helmholtz-Lie Problem of Space which arguably fall within the remit of the epistemology of spacetime. First: in 2009, Manchak (building on previous work by Malament [1977b]) demonstrates that reasoning on the basis of past light cones cannot determine the global geometrical structure of spacetime: clearly, this imposes fundamental limitations on our ability to gain empirical and operational access to the nature of the world (at least, on the assumption that general relativity is correct). Second: Maudlin (1993) has presented arguments to the effect that one can use the resources of indexicals to access the structure of space and time. Such arguments remain controversial: see Dasgupta (2015) for a critique, and Cheng and Read (2022) for recent discussion.

As already mentioned in the introduction, the metaphysics of space and time has made great strides in the past several decades. By contrast, the epistemology of spacetime—a venerable tradition—has by comparison languished. This paper has sought to expose to contemporary philosophers the central research questions in the epistemology of spacetime, as well as their origins in the Helmholtz-Lie Problem of Space. At all stages, we have sought to highlight important outstanding research questions—thereby passing the baton to the wider community. The time is ripe for us to return once more, dear friends!

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ENDNOTES

[1] Despite its significance for the foundations of spacetime physics, this work has never been fully translated into English.

[2] The proto-physics program is sometimes seen as a continuation of Hugo Dingler's foundationalism; such a presentation, however, fails to recognise decisive differences. See Schlaudt (2014) for a critical comparison, and Torretti (1978a) for a review of Dingler's philosophy of geometry.

[3] See Amiras (2003, 2006) for a critical overview and alternative take on proto-geometry as a functional as opposed to constructive account of the relationship between lifeworld pragmatics and Euclidean geometry.

[4] Dingler's strong antisemitism and association with the antisemitic German Physics movement ('Deutsche Physik') are well-known, as is the latter's corrupt agitation against relativity. As Wolters (1991) notes, however, Dingler's systematic views on measurement and thus parts of his opposition to relativity have developed earlier than his antisemitism.

REFERENCES

Acuña, P. (2016). Minkowski spacetime and Lorentz invariance: The cart and the horse or two sides of a single coin? *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, *55*, 1–12.

Amiras, L. (2003). *Die Grundlagen der Geometrie in der Protophysik: kritische Retrospektive und neue Perspektive*.

Amiras, L. (2006). *Protogeometrie. Elemente der Grundlagen der Geometrie als Theorie räumlicher Figuren, habilitation thesis at PH Weingarten*.

Ben-Menahem, Y. (2006). *Conventionalism*. Cambridge University Press.

Bernard, J. (2018). Riemann's and Helmholtz-Lie's problems of space from Weyl's relativistic perspective. *Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics*, *61*, 41–56.

Bernard J., & Lobo C. (Eds.). (2019). Weyl and the problem of space. From science to philosophy. *Studies in history and philosophy of science* (Vol. 49). Springer.

Brown, H. (2005). *Physical relativity: Spacetime structure from a dynamical perspective*. Oxford University Press.

Carnap, R. (1921). *Der Raum: Ein Beitrag zur Wissenschaftslehre*.

Carnap, R. (1926). *Der Logische Aufbau der Welt, habilitation thesis*. Felix Meiner Verlag.

Carrier, M. (1990). Constructing or completing physical geometry? On the relation between theory and evidence in accounts of space-time structure. *Philosophy of Science*, *57*(3), 369–394.

Carrier, M. (1994a). Geometric facts and geometric theory: Helmholtz and 20th-century philosophy of physical geometry. In L. Kürger (Ed.), *Universalgenie Helmholtz - Rückblick nach 100 Jahren*. Akademie Verlag.

Carrier, M. (1994b). *The Completeness of Scientific Theories*. Kluwer Academic Publishers.
Cassirer, E. (1921). *Zur Einsteinschen Relativitätstheorie*.
Chang, H. (2004). *Inventing temperature: Measurement and scientific progress*. Oxford University Press.
Cheng, B., & Read, J. (2022). Shifts and reference. In A. Vassallo (ed.), *The foundations of spacetime physics: Philosophical perspectives*. Routledge.
Coleman, R. A., & Korté, H. (1980). Jet bundles and path structures. *Journal of Mathematical Physics*, *21*(6), 1340–1351.
Curiel, E. (2016). *Kinematics, dynamics, and the structure of physical theory*. arXiv preprint arXiv:1603.02999.
Dasgupta, S. (2015). Inexpressible ignorance. *Philosophical Review*, *124*(4), 441–480.
Dewar, N. (2020). General-relativistic covariance. *Foundations of Physics*, *50*(4), 294–318.
Dewar, N. (2021). On absolute units. *The British Journal for the Philosophy of Science*.
Dewar, N., & Eisenthal, J. (2020). A Raum with a view: Hermann Weyl and the problem of space. In C. Beisbart, T. Sauer, & C. Wüthrich (Eds.), *Thinking about space and time* (pp. 111–132). Birkhäuser.
Dingler, H. (1907). *Beiträge zur Kenntnis der infinitesimalen deformation einer Fläche*.
DiSalle, R. (1993). Helmholtz's empiricist philosophy of mathematics: Between laws of perception and laws of nature. In D. Cahan (Ed.), *Hermann von Helmholtz and the foundations of nineteenth-century science* (pp. 498–521). University of California Press.
DiSalle, R. (2002). Conventionalism and modern physics: A re-assessment. *Conventionalism and modern physics: A re-assessment* (Vol. 36, pp. 169–200). Noûs.
DiSalle, R. (2008). *Understanding space-time: The philosophical development of physics from Newton to Einstein*. Cambridge University Press.
Dürr, P. (2020). Unweyling three mysteries of Nordström gravity. *Studies in History and Philosophy of Modern Physics*, *69*, 32–49.
Dürr, P. (2021a). *Revisiting geometric conventionalism*.
Dürr, P. (2021b). Theory (in-)equivalence and conventionalism in f(R) gravity. *Studies in History and Philosophy of Science*, *88*, 10–29.
Ehlers, J., Pirani, F., & Schild, A. (1972). The geometry of free fall and light propagation. In L. O'Raifeartaigh (Ed.), *General relativity: Papers in honour of J. L. Synge* (pp. 63–84). Clarendon Press.
Einstein, A. (1905). On the electrodynamics of moving bodies. *Annalen der Physik*, *17*, 891–921.
Everett, J. (2015). The constitutive a priori and the distinction between mathematical and physical possibility. *Studies in History and Philosophy of Modern Physics*, 52, 139–152
Friedman, M. (1999). *Reconsidering logical positivism*. Cambridge University Press.
Friedman, M. (2001). *Dynamics of reason*. Center for the Study of Language and Information.
Grünbaum, A. (1973). *Philosophical problems of space and time: Second enlarged edition*. *Boston studies in the philosophy and history of science*. Springer.
Hehl, F. W. (2017). Gauge theory of gravity and spacetime. In D. Lehmkuhl, G. Schimann, & E. Scholz (Eds.), *Towards a theory of spacetime theories*. *Einstein studies series* (Vol. 13). Birkhäuser.
Inhetveen, R. (1983). *Konstruktive Geometrie: eine formentheoretische Begründung der euklidischen Geometrie*. Bibliographisches Institut.
Ivanova, M. (2015a). Conventionalism about what? Where Duhem and Poincaré part ways. *Studies in history and philosophy of science* (Vol. 54).
Ivanova, M. (2015b). Conventionalism, structuralism and neo-Kantianism in Poincaré's philosophy of science. *Studies in history and philosophy of modern physics* (Vol. 52).
Janich, P. (1997). *Das Maß der Dinge. Protophysik von Raum, Zeit und Materie*. Suhrkamp Verlag.
Janich, P. (2012). *Protophysics of time: Constructive foundation and history of time measurement*. Springer Science & Business Media.
Janis, A. (2018). *Conventionality of simultaneity*. Stanford Encyclopedia of Philosophy.
Le Bihan, B., & Read, J. (2018). Duality and ontology. *Philosophy Compass*, *13*, e12555.
Lehmkuhl, D. (2017). Introduction. In D. Lehmkuhl, G. Schimann, & E. Scholz (Eds.), *Towards a theory of spacetime theories*. *Einstein studies series* (Vol. 13). Birkhäuser.
Lie, S. (1890). Über die Grundlagen der Geometrie. *Berichte über die Verhandlungen der Königlich Sächsichen Gesellschaft der Wissenschaften zu Leipzig*, *42*(284–321), 355–418.
Linnemann, N., & Read, J. (2021a). *Constructive axiomatics in physics, Part I: Walkthrough to the Ehlers-Pirani-Schild Axiomatisation*.
Linnemann, N., & Read, J. (2022b). *Constructive axiomatics in physics, Part II: The Ehlers-Pirani-Schild Axiomatisation in Context*.
Lorenzen, P. (1977). Relativistische Mechanik mit klassischer Geometrie und Kinematik. *Mathematische Zeitschrift*, *155*(1), 1–9.
Lorenzen, P. (1984). *Elementargeometrie: das Fundament der analytischen Geometrie*. Bibliographisches Institut.
Malament, D. B. (1977a). Causal theories of time and the conventionality of simultaneity. *Noûs*, *11*, 293–300.

Malament, D. B. (1977b). Observationally indistinguishable space-times. In J. Earman, C. Glymour, & J. Statchel (Eds.), *Foundations of space–time theories. Minnesota studies in the philosophy of science* (Vol. VIII, pp. 61–80). University of Minnesota Press.

Malament, D. B. (2012). *Topics in the foundations of general relativity and Newtonian gravitation theory*. University of Chicago Press.

Matveev, V., & Scholz, E. (2020). Light cone and Weyl compatibility of conformal and projective structures. *General Relativity and Gravitation*, *52*(7), 1–9.

Maudlin, T. (1993). Buckets of water and waves of space. *Philosophy of Science*, *60*(3), 183–203.

Norton, J. (2008). Why constructive relativity fails. *The British Journal for the Philosophy of Science*, *59*(4), 821–834.

Patton, L. (2018). Hermann von Helmholtz. In E. N. Zalta (Ed.), *The Stanford encyclopedia of philosophy*. https://plato.stanford.edu/entries/hermann-helmholtz/

Pitts, J. B. (2018). Kant, Schlick and Friedman on space, time and gravity in light of three lessons from particle physics. *Erkenntnis*, *83*.

Poincaré, H. (1902). *Science and hypothesis*. The Walter Scott Publishing Co.

Quine, W. V. O. (1951). Two dogmas of empiricism. *Philosophical Review*, *60*(1), 20–43.

Read, J., & Teh, N. J. (2018). The teleparallel equivalent of Newton–Cartan gravity. *Classical and Quantum Gravity*, *35*(18), 18LT01.

Reichenbach, H. (1920). *Relativitätstheorie und Erkenntnis Apriori*. Technische Hochschule Stuttgart.

Reichenbach, H. (1924). *Axiomatik der relativistischen Raum-Zeit-Lehre*. Fried. Vieweg & Sohn.

Reichenbach, H. (1928). *Philosophie der Raum-Zeit-Lehre*. Walter de Gruyter.

Riemann, B. (1868). *Über die Hypothesen, welche der Geometrie zu Grunde liegen* (Vol. 13). Abhandlungen Der Königlichen Gesellschaft Der Wissenschaften Zu Göttingen.

Riemann, B., & Weyl, H. (1919). *Über die Hypothesen, welche der Geometrie zu Grunde liegen*. Springer-Verlag. http://www.springer.com/de/book/9783662423165

Rovelli, C. (1997). Halfway through the woods: Contemporary research on space and time. In J. Earman & J. Norton (Eds.), *The cosmos of science* (pp. 180–223). University of Pittsburgh Press.

Samaroo, R. (2015). *Friedman's thesis*. *Studies in history and philosophy of modern physics* (Vol. 52).

Schlaudt, O. (2014). La réception de Hugo Dingler par l'École d'Erlangen. *Philosophia Scientiae*, *18*(2), 141–149.

Sklar, L. (1974). *Space, time, and spacetime*. University of California Press.

Sklar, L. (1977). Facts, conventions, and assumptions in the theory of space time. In J. Earman, C. Glymour, & J. Stachel (Eds), *Minnesota studies in philosophy of science* (pp. 206–274). University of Minnesota Press.

Stevens, S. (2017). Regularity Relationalism and the Constructivist Project. *The British Journal for the Philosophy of Science*. https://doi.org/10.1093/bjps/axx037

Tal, E. (2014). Making time: A study in the epistemology of measurement. *The British Journal for the Philosophy of Science*, *67*(1), 297–335.

Torretti, R. (1978a). Hugo Dinger's philosophy of geometry. *Diálogos*, *32*, 85–128.

Torretti, R. (1978b). *Philosophy of geometry from Riemann to Poincaré*. D. Reidel.

van Fraassen, B. (1970). *An introduction to the philosophy of time and space*. Random House.

von Helmholtz, H. (1868). Über die Tatsachen, die der Geometrie zum Grunde liegen. *Nachrichten von Der Königlichen Gesellschaft Der Wissenschaften Zu Göttingen*, *9*, 193–221.

von Helmholtz, H. (1896). Über den Ursprung und die Bedeutung der geometrischen Axiome. In *Vorträge und Reden* (4th ed., Vol. 2, pp. 1–31). Vieweg. https://archiv.ub.uni-heidelberg.de/volltextserver/13156/1/geo2.pdf

Weatherall, J. O. (2019a). Theoretical equivalence in physics, Part 1. *Philosophy Compass*, *14*(5), e12592.

Weatherall, J. O. (2019b). Theoretical equivalence in physics, Part 2. *Philosophy Compass*, *14*(5), e12591.

Weatherall, J. O., & Manchak, J. B. (2014). The geometry of conventionality. *Philosophy of Science*, *81*(2), 233–247.

Weyl, H. (1923). *Mathematische analyse des Raum problems*. Verlag von Julius Springer.

Winnie, J. (1970a). Special relativity without one-way velocity assumptions: Part I. *Philosophy of Science*, *37*, 81–99.

Winnie, J. (1970b). Special relativity without one-way velocity assumptions: Part II. *Philosophy of Science*, *37*, 223–238.

Wolff, J. E. (2020). *The metaphyiscs of quantities*. Oxford University Press.

Wolters, G. (1991). Opportunismus als Naturanlage: Hugo Dingler und das 'Dritte Reich'. In J. Peter (Ed.). *Entwicklungen der methodischen Philosophie*. Suhrkamp Verlag.

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