BOTTOM-UP MARKUP FLUCTUATIONS*

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Abstract

We study markup cyclicality in a granular macroeconomic model with oligopolistic competition. We first characterize how firm, sectoral, and aggregate markups comove with output at different levels of aggregation in response to firm-level shocks. We then quantify the model's ability to reproduce salient features of the cyclical properties of measured markups in French administrative firm-level data from the bottom (firm) level to the aggregate level. We document that (i) firm-level markups rise with market share and sector-level markups with concentration, (ii) the relationship between markups and sectoral output varies by firm size—negative for small firms but positive for large ones, (iii) sector-level markups move positively with sectoral output, and (iv) sectoral markups show no systematic relationship with aggregate output. Our model helps rationalize these seemingly conflicting patterns of markup cyclicality in the data.

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Introduction

A long tradition in the business-cycle literature evaluates models by how well they match salients data moments, such as the relative volatility and correlation with GDP of key macroeconomic aggregates. While there is broad consensus concerning, for example, the behavior of consumption, investment, or unemployment over the business cycle, disagreement lingers, both in terms of theory and measurement, over the cyclical behaviour of markups (see, e.g., **?** and **??**).[1]

In this paper, we re-examine this question by studying the cyclical properties of markups at the firm, sector, and aggregate level, both theoretically and empirically, based on French administrative data. We consider a model of oligopolistic competition in which granular firm-level shocks result not only in sector and economy-wide output changes, as in **?**, but also in markup dynamics. We characterize the model's implications for comovement between output and markups, that is, "markup cyclicality", at various levels of disaggregation from the bottom (firm) level up to the sector and aggregate levels, and show how this comovement is mediated by market structure within and across sectors. We then assess the ability of our granular oligopolistic setting to reproduce salient measures of the cyclical properties of markups in the French data at the firm, sector, and aggregate levels.

To model in a tractable way the determination and aggregation of markups in an economy featuring a large but finite number of sectors with a discrete number of firms, we use a nested CES demand structure studied in **?**.[2] Firms compete under flexible prices, setting markups that are increasing in within-sector sales shares.[3] Firm-level shocks follow a random growth process that generates empirically plausible firm dynamics, firm-size distributions, and granular sectoral and aggregate fluctuations (**?**). The model yields predictions for the joint behavior of within-sector market shares, markups, and output following exogenous changes in firm-level shifters. Furthermore, the model's convenient equilibrium aggregation yields simple sectoral and aggregate counterparts to many of these firm-level objects.

Our first theoretical contribution is to provide simple analytic expressions showing how the sign of markup cyclicality depends on the level of aggregation, the market structure within and

[1] Other studies contributing to the active debate on the sign and magnitude of markup cyclicality include **?**, **?**, **?**, and **?**. Additionally, a growing literature documents lower-frequency trends in markups, such as **?** and **?**.

[2] A similar framework has been used in a number of macro applications to quantify the welfare costs of market power (**?** and **?**), trends in market power (**?**), optimal product market policy (**?**), managerial compensation (**?**), wage inequality (**?**), pro-competitive gains from trade (**?**), exchange-rate pass-through (**?**), and granularity in trade (**?**). Other prominent work featuring fluctuations in market power in macroeconomic models include **?**, **?**, **?**, and **?**.

[3] Much of the literature on markup cyclicality is motivated by the implications of models with nominal rigidities (e.g., **?** for a comprehensive early survey), which depend on the nature of nominal rigidities (prices vs. wages) and on the source of aggregate shocks (e.g., monetary vs. productivity). By contrast, we examine how far a model with flexible prices and granular firm-level shocks can go in accounting for observed patterns of markup cyclicality at different levels of disaggregation. See **?** and **?** for recent analyses of money non-neutrality in an oligopolistic model like ours with price rigidities.

across sectors, and the set of shocked firms.[4] We show that sectoral output and markups comove positively in response to shocks to large firms in the sector, whereas they comove negatively in response to shocks to small firms. In turn, the effect of such shocks on the aggregate markup depends on the distribution of sector-level markups and sectoral expenditure shares. Under the additional assumption that shocks are uncorrelated across firms, we provide sufficient conditions for a positive asymptotic correlation between markups and output at the sectoral and aggregate levels.

Second, we compare theoretically the implications of our model to an alternative specification in which firm-level markups are heterogeneous but constant in response to shocks (i.e., complete pass-through) so that sectoral and aggregate markups only change due to between-firm reallocation and not within-firm markup changes. We show that, although within-firm markup changes account for half of sectoral markup fluctuations in the variable markup model, changes in sectoral and aggregate markups can be larger or smaller than in the constant markup model, depending on parameter values, because the extent of between-firm reallocation falls with incomplete pass-through. Additionally, we provide asymptotic formulas for sectoral and aggregate output volatility, which generalize those in **?** to an oligopolistic setting with variable markups. We show how the introduction of variable markups dampens granular aggregate volatility, acting in a similar way to a decline in the Herfindahl index. Intuitively, when pass-through rates are lower for larger firms the weight of large firm shocks in the price index is effectively reduced.

Our empirical analysis and calibration of the model is based on French administrative firm-level annual data over 26 years (1994-2019) covering approximately 400,000 firms per year. We compute empirical distributions of firm market shares, sectoral concentration and output over time. To obtain a measure of firm-level markups, which we then aggregate at the sector and economy-wide levels, we follow the methodology in **?**, **?** and **?**.[5] This approach requires output elasticities for some flexible input, which we obtain by estimating production functions at the 2-digit sector level for a subset of firms and years where quantity information — rather than revenues only — is available and then applying these estimates to the full sample of firms. By using firm-level quantities instead of revenues, our markup measures circumvent some of the biases discussed in **?** and **?**. We use our measure of markups to calculate several metrics of markup cyclicality that we compare with model-generated data over short (25-year) samples.

We start by analyzing a basic mechanism in our oligopolistic setting: within a narrowly de-

[4]**?** studies the role of input-output linkages and endogenous markups in shaping comovement of sector-level variables, providing an analytical characterization of the impact of microeconomic shocks on aggregate output using an approximation with respect to the deep parameters of the model. Our analytic results make use of a different approximation with respect to firm-level idiosyncratic shocks, similar to the one used in, for example, **?**, **?**, and **?** in the context of exchange rate shocks.

[5]Given the level of aggregation in our baseline dataset, we do not measure markups at the level of geographic regions, as in **?**, or products, as in **?**. For consideration of pricing with multi-product firms in a similar framework to ours, see **?**.

fined sector, a firms' market power is increasing in its market share. This relationship implies a positive correlation between firm-level markup and firm-level market share, both in the cross section and over time. Moreover, aggregating firm-level outcomes implies the same correlation between sectoral markups and sectoral concentration. We find support for these predicted correlations in the French data, both at the firm and at the sector levels, in the cross section, and over time.[6] Moreover, changes in firm-level markups account for a sizable portion of changes in sectoral markups both in our model (50%) and in the data (59% for the median sector).[7]

Second, we examine in the data and calibrated model three measures of markup cyclicality. We start by examining how firm-level markups covary with sector-level output. We find that markups are "countercyclical" for the average firm but this relation switches sign for large firms. This is consistent with findings in **?** that markups are more countercyclical for small firms than for large firms. Our model reproduces this heterogeneity in markup cyclicality because the typical sectoral expansion is driven by shocks to large firms and these firms increase markups as their market share rises. Consistent with this, we show that market shares of large firms are procyclical with respect to sectoral output.

We then proceed to evaluate notions of sector-level markup cyclicality. Following **?** (the working paper version of **?**), we ask whether sector markups comove with sector output over the business cycle. Like **?** for the US, we find evidence for a positive systematic comovement between the two measures, or "procyclicality", in the French data. The model simulations also reproduce this fact, as anticipated in our theoretical discussion. Also consistently with the model mechanism, we find that in the data, sectoral expansions tend to be associated with an increase in sectoral concentration.

Finally, we follow the work of **?**, who investigate yet another notion of cyclicality: the extent to which sector level markups comove with aggregate output. While sectoral markups in our data display robust procyclical patterns within their own sectors, their relationship with aggregate output is less robust both in terms of sign (ranging from acyclical to countercyclical) and significance. Simulations of our model imply that this comovement measure is not statistically different from zero.

Overall, we document seemingly conflicting patterns of markup cyclicality across different layers of aggregation despite using a single dataset and measure of firm-level markups. Hence, by exploiting different reduced-form measures of markup cyclicality, two researchers may arrive at opposing conclusions even within a single dataset. Our model can reproduce qualitatively, and sometimes quantitatively these different reduced-form measures of markup cyclicality.

Finally, we examine the model's implications for fluctuations in aggregate markups and out-

[6] Relatedly, **?** and **?** provide evidence of a positive relation between market shares and markups in the time series using firm-level data from China and Belgium, respectively.

[7] This is consistent with Figure 3 in **?**, showing that within-firm changes in markups in the US are quantitatively important in accounting for high frequency movements in aggregate markups.

put. Our baseline calibration features only granular firm-level shocks and abstracts from aggregate shocks that leave the firm-size distribution unchanged because, in our model, they do not affect markups. We use this quantitative laboratory featuring realistic heterogeneity within and across sectors to quantitatively assess the contribution of idiosyncratic shocks to aggregate output and markup fluctuations. Importantly, compared to prior work on the granular origin of business cycles (e.g. **?** **?** and **?** **?**), our model features movements in desired markups that can partly offset the impact of own firm-level shocks or magnify the impact from shocks to competing firms.[8] Our quantitative model generates roughly 25% (on average, across 25-year simulated samples) of the volatility of aggregate output in the French data. To put this number in perspective, it is slightly lower than the 30% ratio reported in **?** for an heterogeneous firm model featuring granular but price-taking firms calibrated to US data. Furthermore, our model additionally yields a ratio of markup volatility relative to output volatility of 0.36, close to the 0.40 ratio observed in data.

Turning to aggregate markup cyclicality, our model implies a counterfactually large and positive point estimate for the correlation between aggregate output and markups relative to the data. However, there is substantial variation in point estimates across 25-year simulated samples. This is because, as our analytic expressions show, the extent of markup cyclicality depends on the set of shocked firms, which vary across small samples. Moreover, superimposing aggregate productivity shocks to account for the overall aggregate volatility reduces this correlation significantly. Finally, the magnitude and cyclicality of aggregate markups in our model is not very different to the case in which we counterfactually fix markups at their initial, heterogeneous equilibrium level. Of course, rather than exogenously fixing markups, our model provides a unified theory of both markup (level) heterogeneity across firms and endogenous markup changes.

The paper is organized as follows. In Section I, we present our granular oligopolistic setup and describe the equilibrium from the bottom (firm) level to the aggregate level. In Section II, we characterize analytically various measures of markup cyclicality at various aggregation levels. In Section III, we discuss our French administrative firm data, the markup-estimation strategy, and the model calibration. In Section IV, we explore the relationship between markups and market shares in the model and data. In Section V, we compare a host of markup-cyclicality

[8] **?** study the granular origins of a country's comparative advantage in an oligopolistic framework that is similar to ours. For work on the granular origins of business-cycle fluctuations — but featuring either perfect competition or constant markups — see **?** on the evolution of business-cycle volatility over time and across countries and **?** or **?** for granular settings linking trade, aggregate volatility, and cross-country comovement. **?** provide an empirical benchmark for the role of granularity in aggregate fluctuations. **?** examine aggregate granular fluctuations in a multi-sector model allowing for changes in markups due to nominal price rigidities. Our emphasis on the micro origins of aggregate fluctuations is also related to the literature on production networks. See **?** for an initial benchmark, and **?** for an analysis of how market power distorts the propagation of shocks along input linkages. **?** provide a very general characterization of the impact of microeconomic shocks on aggregate productivity and output in a large class of models in which productivities and wedges (e.g., markups) are exogenous primitives. **?** study the role of variable markups in shaping the aggregate implications of changes in market size.

metrics in the French data and model-generated data. In Section VI, we examine fluctuations in aggregate markups and output implied by the model. Section VII concludes. In the Online Appendix we provide proofs, details on markup estimation, additional results, and robustness checks.

I Model

Our model consists of a representative household that supplies labor and consumes a fixed set of goods. These goods are produced by a discrete number of flexible-price firms that compete oligopolistically and are owned by the representative household. In this section, we describe the model and characterize the equilibrium, first within a sector and then at the aggregate level.

I.A Preferences and technologies

Households have intratemporal preferences at time t over consumption of a final composite good, Y_t, and labor, L_t, represented by the utility function

$$U\left(Y_t, L_t\right) = \frac{1}{1-\eta} Y_t^{1-\eta} - \frac{f_0}{1+f^{-1}} L_t^{1+f^{-1}},$$

where $f \geq 0$ is the Frisch elasticity of labor supply, and $\eta \leq 1$ is the constant relative risk aversion (which in our model shapes income effects in labor supply). Households choose consumption and labor to maximize utility subject to the constraint that consumption expenditures must not exceed the sum of wage payments and aggregate profits.

The final good aggregates output of N sectors according to a constant-elasticity-of-substitution (CES) aggregator:

$$Y_t = \left[\sum_{k=1}^{N} A_k^{\frac{1}{\sigma}} Y_{kt}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where Y_{kt} denotes sector k output, A_k is a demand shifter for sector k which we assume is constant over time, and $\sigma \geq 1$ is the elasticity of substitution across sectors.

Each sector k is itself a CES aggregator of the output of N_k individual firms given by

$$Y_{kt} = \left[\sum_{i=1}^{N_k} A_{kit}^{\frac{1}{\varepsilon}} Y_{kit}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where Y_{kit} denotes the output of firm i in sector k, A_{kit} is a firm-quality shifter, and ε is the elasticity of substitution between the output of firms in sector k.[9]

[9]The model's implications for markups, market shares, and concentration measures are unchanged if A_{kit} is a taste shock. However, measures of aggregate output calculated using chain-weighted deflators are path-dependent

We assume $\sigma \leq \varepsilon$, so that goods are more substitutable within sectors than across sectors. With a finite number of sectors and a discrete number of firms per sector, firm-level shocks can generate aggregate fluctuations as in **?**. By contrast, with a continuum of sectors, as in **?**, firm-level shocks would not generate aggregate fluctuations.

Firm i in sector k produces output according to the constant-returns-to-scale technology:

$$Y_{kit} = Z_{kit} L_{kit}, \tag{1}$$

where Z_{kit} denotes the productivity of firm i in sector k and L_{kit} is a variable input, employment, that is perfectly mobile across firms.[10] Labor market clearing requires that the sum of employment across all firms equals aggregate labor, L_t.[11] We assume that the number of firms per sector, N_k, is exogenously given.[12]

We introduce assumptions about the stochastic process of firm-level shocks A_{kit} and Z_{kit} in Section II for our asymptotic results and in Section III for our calibration strategy.

I.B Market structure and sector equilibrium

We now describe the equilibrium in a sector. Firm i in sector k setting a non-quality adjusted price P_{kit} faces demand $Y_{kit} = A_k A_{kit} \left(P_{kit}\right)^{-\varepsilon} \left(P_{kt}\right)^{\varepsilon-\sigma} P_t^{\sigma} Y_t$, where the sector k price is

$$P_{kt} = \left[\sum_{i=1}^{N_k} A_{kit} P_{kit}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}, \tag{2}$$

and the aggregate price is

$$P_t = \left[\sum_{k=1}^{N} A_k P_{kt}^{1-\sigma}\right]^{\frac{1}{1-\sigma}}.$$

The markup for firm i in sector k, which we characterize below, is defined as the ratio of price

in the presence of taste shocks (i.e. growth between t and t' depends on the sequence of shocks between t and t'); see e.g. **?**. For this reason, we abstract from taste shocks.

[10] In Appendix A.G we provide analytic results allowing for decreasing returns to scale at the firm level.

[11] Our results in Section II on firm-level and sectoral-level outcomes are unchanged if the variable input, L_{kit}, is a composite of multiple inputs (e.g., labor, intermediate goods, and capital) that is common across firms in the sector. The specific assumptions on the composition of this variable input matter only for the aggregate response of the economy to given firm-level shocks. When estimating markups in Section III, we assume the input L_{kit} is a translog combination of labor, capital, materials, and services inputs with parameters that vary by sector. We then compare measures of cyclicality of estimated markups in the data with measures of cyclicality implied by our model in Section V.

[12] Endogenizing the number of firms per sector via a free-entry condition is computationally challenging in our model with oligopolistic competition, granular firm-level shocks, and realistic sectoral heterogeneity, requiring auxiliary assumptions to select among multiple equilibria with different sets of entering firms (see e.g. **? ?** and **? ?**). However, if entering and exiting firms have low market shares, changes over time in the number of firms would have small effects on sectoral and aggregate markups. Whereas in our data the number of firms changes year to year, in Appendix D we show robustness of our main empirical results to restricting the sample each period to continuing firms.

to marginal cost,

$$\mu_{kit} \equiv \frac{Z_{kit} P_{kit}}{W_t}, \tag{3}$$

where W_t is the price of the variable input (i.e., the wage). This markup determines how the firm's revenues are split into labor payments and profits, such that

$$L_{kit} W_t = \mu_{kit}^{-1} P_{kit} Y_{kit}, \qquad \text{and} \qquad \Pi_{kit} = \left(1 - \mu_{kit}^{-1}\right) P_{kit} Y_{kit}.$$

The market share of firm i in sector k, $s_{kit} \equiv \frac{P_{kit} Y_{kit}}{P_{kt} Y_{kt}}$, can be expressed in terms of markups and *firm shifters*, which are defined as a composite of quality and productivity shifters, $V_{kit} \equiv A_{kit} Z_{kit}^{\varepsilon-1}$. Specifically,

$$s_{kit} = \frac{V_{kit} \mu_{kit}^{1-\varepsilon}}{\sum_{i'=1}^{N_k} V_{ki't} \mu_{ki't}^{1-\varepsilon}}. \tag{4}$$

One can consider two alternative market structures. Firms maximize profits by choosing price, taking other firms' prices as given (Bertrand competition), or by choosing quantity, taking other firms' quantities as given (Cournot competition). In both cases, firms take into account that they are non-atomistic in their sector, and hence their choices affect sectoral output and prices. We assume, however, that individual firms behave as if the sector they produce in is atomistic in the aggregate economy (as in the case of a continuum of sectors).[13]

Under these assumptions, equilibrium markups and market shares in each sector k solve the non-linear system of equations given by the market share equation (4) and

$$\mu_{kit} = \begin{cases} \frac{\varepsilon}{\varepsilon-1} \left[1 - \left(\frac{\varepsilon/\sigma - 1}{\varepsilon - 1}\right) s_{kit}\right]^{-1} & \text{under Cournot,} \\ \frac{\varepsilon}{\varepsilon-1} \left[\frac{1-\left(\frac{\varepsilon-\sigma}{\varepsilon}\right) s_{kit}}{1-\left(\frac{\varepsilon-\sigma}{\varepsilon-1}\right) s_{kit}}\right] & \text{under Bertrand.} \end{cases} \tag{5}$$

Under both formulations, since $\varepsilon > \sigma$, markups are increasing in market shares,[14] with $\lim_{s_{kit} \to 0} \mu_{kit} = \dfrac{\varepsilon}{\varepsilon - 1}$ and $\lim_{s_{kit} \to 1} \mu_{kit} = \dfrac{\sigma}{\sigma - 1}$. If $\varepsilon = \sigma$, markups are common across firms and constant over time as in the standard monopolistically competitive model. In our analytic and

[13] In Appendix A.H we solve for markups in the case in which each firm maximizes real profits internalizing the effect of their individual choice of output or prices on aggregate output and the real wage, thus relaxing our baseline behavioral assumption. Firms do not internalize, however, the impact that changes in profits have on the welfare of the firm's owner (**?**). We show that markups depend not only on the firm's sectoral sales share, but also on the firm's sectoral employment share as well as the firm's economy-wide share in sales and employment, which increases the computational burden of solving the model. Applying the new formula using the sales and employment shares in our baseline calibration results has a negligible impact on markup levels compared to our baseline. This is because most sectors in our data are quite small.

[14] The property is satisfied in a variety of models with variable elasticity of demand (see, e.g., the reviews in **?** and **?**)

quantitative results, we focus on the case of Cournot competition. It generates more markup variation than Bertrand and is thus better able to match estimates of incomplete pass-through and markup-size relationship. In Appendix A, we provide analytic results under Bertrand.

Two remarks are in order about firm shifters. First, firm-level market shares and markups in sector k depend only on relative firm shifters across firms within this sector. This result implies that market shares and markups in sector k do not vary in response to proportional changes in shifters to all firms in sector k (including sectoral demand shifters A_k), shocks in other sectors, or changes in the aggregate wage. It follows that aggregate shocks to firms in all sectors generate fluctuations in aggregate output but not in aggregate markups. For this reason, in our baseline quantitative analysis we abstract from standard aggregate productivity shocks.

Second, the split of firm shifters into quality and productivity does not matter for the model implications on markups, concentration and output (for the latter, this is as long as deflators use quality-adjusted prices) at the firm, sector, or aggregate levels. In practice, price deflators used by statistical agencies typically do not incorporate high-frequency changes in quality. Therefore, in order to compare output in the model and data, we only consider firm-level productivity shocks and abstract from quality shocks.

I.C Sectoral outcomes

We now describe how the model aggregates outcomes from the firm level to the sector level. We define sectoral markup as the ratio of sectoral revenues to labor payments,

$$\mu_{kt} \equiv \frac{P_{kt}Y_{kt}}{W_t L_{kt}}, \tag{6}$$

where sectoral employment is $L_{kt} = \sum_{i=1}^{N_k} L_{kit}$. Sectoral markups can be expressed as an harmonic mean (weighted by market shares) of firm-level markups,

$$\mu_{kt} = \left[\sum_{i=1}^{N_k} \mu_{kit}^{-1} s_{kit}\right]^{-1}. \tag{7}$$

Substituting the markup-market-share relationship (equation 5) under Cournot competition, we can express the sectoral markup, μ_{kt}, as a simple function of the sector's Herfindahl-Hirschman index, $HHI_{kt} = \sum_{i=1}^{N_k} s_{kit}^2$:[15]

$$\mu_{kt} = \frac{\varepsilon}{\varepsilon - 1}\left[1 - \left(\frac{\varepsilon/\sigma - 1}{\varepsilon - 1}\right) HHI_{kt}\right]^{-1}. \tag{8}$$

[15]The HHI is an average of market shares, weighted by market shares themselves, and hence ranges between 0 and 1.

Note the positive relationship between sectoral markup and HHI takes the same form as the firm-level relationship between markup and market share in equation (5). In the same way that a firm with a large market share charges a higher markup, a sector with a large average market share, that is, a high HHI, has a high sectoral markup as long as $\varepsilon > \sigma$.[16]

Sectoral markups can be expressed as the standard ratio between sectoral price and marginal cost (i.e., the ratio of wage to sectoral productivity), $\mu_{kt} = P_{kt}Z_{kt}/W_t$. Sectoral productivity, $Z_{kt} \equiv Y_{kt}/L_{kt}$, can be expressed in terms of firm-level markups and firm shifters as

$$Z_{kt} = \frac{\left(\sum_{i=1}^{N_k} V_{kit}\mu_{kit}^{1-\varepsilon}\right)^{\frac{\varepsilon}{\varepsilon-1}}}{\sum_{i=1}^{N_k} V_{kit}\mu_{kit}^{-\varepsilon}}. \tag{9}$$

I.D Aggregate outcomes

We now describe how the model aggregates outcomes from the sector level to the aggregate level. We define aggregate markup as the ratio of aggregate revenues and labor payments,

$$\mu_t \equiv \frac{P_tY_t}{W_tL_t} = \left[\sum_{k=1}^{N} s_{kt}\mu_{kt}^{-1}\right]^{-1}. \tag{10}$$

As indicated by the second equality, aggregate markups can be expressed as a harmonic weighted average of sectoral markups, where sectoral expenditure shares are determined by sectoral markups and sectoral shifters $V_{kt} \equiv A_k Z_{kt}^{\sigma-1}$,

$$s_{kt} \equiv \frac{P_{kt}Y_{kt}}{P_tY_t} = \frac{V_{kt}\left(\mu_{kt}\right)^{1-\sigma}}{\sum_{k'} V_{k't}\left(\mu_{k't}\right)^{1-\sigma}}. \tag{11}$$

Alternatively, under Cournot we can express the aggregate markup as a simple function of average sectoral HHI (weighted by sectoral expenditure shares) that mirrors the expressions for firm-level and sector-level markups in equations (5) and (8), respectively:

$$\mu_t = \frac{\varepsilon}{\varepsilon-1}\left[1 - \left(\frac{\varepsilon/\sigma - 1}{\varepsilon - 1}\right)\sum_{k=1}^{N} s_{kt}HHI_{kt}\right]^{-1}.$$

The weighted average of sectoral HHIs is equal to the average market share across firms weighted by firms' expenditure share in the whole economy.[17] When this weighted-average economy-wide market share is high, the aggregate markup is high.

Aggregate markups can also be expressed as the standard ratio between aggregate price and

[16] A similar mapping between sectoral markups and concentration indices can be obtained under Bertrand competition (see **?**).

[17] Specifically, $\sum_{k=1}^{N} s_{kt}HHI_{kt} = \sum_{k=1}^{N}\sum_{i=1}^{N_k} s_{kt}s_{kit}^2 = \sum_{k=1}^{N}\sum_{i=1}^{N_k} \frac{P_{kt}Y_{kt}}{P_tY_t}\frac{P_{kit}Y_{kit}}{P_{kt}Y_{kt}} s_{kit} = \sum_{k=1}^{N}\sum_{i=1}^{N_k} \frac{P_{kit}Y_{kit}}{P_tY_t} s_{kit}$.

aggregate marginal cost, $\mu_t = P_t Z_t / W_t$, where aggregate productivity, $Z_t \equiv Y_t / L_t$, can be expressed in terms of sectoral markups and sectoral shifters as

$$Z_t = \frac{\left(\sum_{k=1}^{N} V_{kt} \mu_{kt}^{1-\sigma}\right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{k=1}^{N} V_{kt} \mu_{kt}^{-\sigma}\right)}. \tag{12}$$

Finally, aggregate output and labor are given by

$$Y_t^{\eta+\frac{1}{f}} = \frac{Z_t^{1+\frac{1}{f}}}{f_0 \mu_t} \qquad \text{and} \qquad L_t = \frac{Y_t}{Z_t} = \frac{Z_t^{\frac{1-\eta}{\eta+\frac{1}{f}}}}{(f_0 \mu_t)^{\frac{1}{\eta+\frac{1}{f}}}}. \tag{13}$$

The aggregate markup μ_t distorts the leisure/consumption choice relative to the optimal allocation.

I.E Summary of equilibrium

Our model aggregates outcomes in a very parsimonious manner from the firm level to the sector level, and from the sector level to the aggregate level. Here we summarize how to solve for prices and quantities as a function of time t of firm shifters, $\{V_{kit}\}$, and sectoral demand shifters, $\{A_k\}$.

Equilibrium firm-level markups and market shares, μ_{kit} and s_{kit}, are the solution to equations (4) and (5). Sectoral markups and productivities, μ_{kt} and Z_{kt}, are solved from equations (7) and (9), respectively, and sectoral expenditure shares, s_{kt}, from equation (11).

Aggregate markup, productivity, output, and employment, μ_t, Z_t, Y_t, and L_t, are solved from equations (10), (12), and (13). Setting $W_t = \bar{W}$ as the numeraire, sectoral and aggregate price levels, P_{kt} and P_t, are given by $P_{kt} = \mu_{kt} W_t / Z_{kt}$ and $P_t = \mu_{kt} W_t / Z_t$. Sectoral output is solved from

$$Y_{kt} = A_k P_{kt}^{-\sigma} P_t^{\sigma} Y_t,$$

and sectoral employment using $L_{kt} = Y_{kt}/Z_{kt}$. Firm-level expenditures and employment, $P_{kit} Y_{kit}$ and L_{kit}, are solved from $P_{kit} Y_{kit} = s_{kit} P_{kt} Y_{kt}$ and equation (6), respectively. Finally, given a split of firm shifters V_{kit} into productivity $\{Z_{kit}\}$ and quality $\{A_{kit}\}$, firm-level output Y_{kit} and price P_{kit} are solved from equations (1) and (3), respectively.

In the following section, we use a first-order approximation to characterize the equilibrium response to firm-level shocks at the firm, sectoral, and aggregate levels.

II Analytic Results

In this section, we characterize, up to a first-order approximation, the equilibrium response of markups, prices, and output to firm-level shocks at the firm, sectoral, and aggregate levels.

We first introduce a first-order approximation to solve for changes in firm-level markups and market shares in a sector. We then develop expressions for changes in prices, markups, and output in response to firm-level shocks, first at the sector level and then at the aggregate level. We provide expressions for asymptotic covariances between markup and output changes at different aggregation levels under the additional assumption that firm-level shocks are i.i.d. across firms and over time with variance $\sigma_v^2 \equiv \mathbb{V}ar\left[\widehat{V}_{kit}\right]$. We focus on the case of Cournot competition, and present results under Bertrand in the appendix. We highlight the role of variable markups versus constant markups in shaping markup cyclicality, as well as the impact of variable markups on aggregate output volatility.

II.A Firm-level outcomes

Consider an initial equilibrium in sector k with market shares $\{s_{ki}\}$ and markups $\{\mu_{ki}\}$ where, for simplicity, we omit time subscripts in the initial equilibrium. Taking a first-order approximation of the expressions for market share (equation 4) and firm-level markup (equation 5), changes in the equilibrium market shares and markups are the solutions to the following linear system of equations

$$\widehat{s}_{kit} = \widehat{V}_{kit} + (1-\varepsilon)\,\widehat{\mu}_{kit} - \sum_{i'=1}^{N_k} s_{ki'}\left(\widehat{V}_{ki't} + (1-\varepsilon)\,\Gamma_{ki'}\widehat{s}_{ki't}\right), \tag{14}$$

$$\widehat{\mu}_{kit} = \Gamma_{ki}\widehat{s}_{kit}. \tag{15}$$

Variables with hats denote log differences at time t relative to the initial equilibrium, e.g. $\widehat{V}_{kit} \equiv \log V_{kit} - \log V_{ki}$, and Γ_{ki} denotes the *markup elasticity* with respect to market share for firm i in sector k evaluated at the initial equilibrium.

Markup elasticities under Cournot are, by equation (5),

$$\Gamma_{ki} \equiv \frac{\partial \log \mu_{ki}}{\partial \log s_{ki}} = \frac{\left(\frac{\varepsilon}{\sigma} - 1\right) s_{ki}}{\varepsilon - 1 - \left(\frac{\varepsilon}{\sigma} - 1\right) s_{ki}}.$$

As discussed above, if $\varepsilon > \sigma$, markups are increasing in market shares. That is, $\Gamma_{ki} \geq 0$, with strict inequality if $s_{ki} > 0$. Moreover, markup elasticities are increasing in market shares. This property is satisfied in a variety of models of demand with variable elasticity, as discussed in **?** and **?**.

We now introduce pass-through elasticities, which are not required to solve for sectoral market shares and markups but, nevertheless, we use in our analytic results that follow. Changes in firm-level prices are given by $\widehat{P}_{kit} = -\widehat{Z}_{kit} + \widehat{\mu}_{kit}$ where we used that the wage is the numeraire. Combined with equations (15) and $\widehat{s}_{kit} = \widehat{A}_{kit} + (1-\varepsilon)\left(\widehat{P}_{kit} - \widehat{P}_{kt}\right)$, we obtain

$$\widehat{P}_{kit} = \alpha_{ki}\left(-\widehat{Z}_{kit} + \Gamma_{ki}\widehat{A}_{kit}\right) + (1-\alpha_{ki})\,\widehat{P}_{kt}, \tag{16}$$

where α_{ki} is the *pass-through rate* governing how firm-level prices respond to idiosyncratic shocks (given changes in sectoral price), given by

$$\alpha_{ki} = \frac{1}{1+(\varepsilon-1)\,\Gamma_{ki}}. \tag{17}$$

Conversely, $1-\alpha_{ki}$ governs how prices respond to changes in sectoral price (given changes in marginal cost). Because markup elasticities are increasing in market shares (if $\varepsilon > \sigma$), pass-through rates are decreasing in market shares.[18] To isolate the role of changes in markups in response to shocks, we consider an alternative case in which markups are fixed at the initial equilibrium levels, imposing $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$.

II.B Sectoral outcomes

In this section, we characterize how sectoral prices, markups, and output respond to firm-level shocks, and provide expressions for variances and covariances of markup and output changes over long realizations of shocks.

Sectoral prices As a first step in understanding changes in sectoral output, we solve for changes in sectoral prices (relative to the the numeraire, i.e., wage), which are related to sectoral output by CES demand, $Y_{kt} = A_k P_{kt}^{-\sigma} P_t^{\sigma} Y_t$.

Taking a first-order approximation of the sectoral price definition (2) and using firm-level price changes (16), log changes in sectoral prices can be expressed as a weighted average of firm shifters,

$$\widehat{P}_{kt} = -\frac{1}{\varepsilon-1}\frac{\sum_{i=1}^{N_k} s_{ki}\alpha_{ki}\widehat{V}_{kit}}{\sum_{i=1}^{N_k} s_{ki}\alpha_{ki}}, \tag{18}$$

where the weights are given by the product of market shares, s_{ki}, and pass-through rates, α_{ki}. Because $\varepsilon \geq 1$, sectoral prices fall in response to an increase in firm shifter.

To understand how sectoral price changes are shaped by pass-through rates, note that if $\alpha_{ki} =$

[18] We can further solve for $\widehat{P}_{kit}$ using $\widehat{P}_{kt} = s_{ki}\widehat{P}_{kit} + (1-s_{ki})\widehat{P}_{k-it}$, where $\widehat{P}_{k-it}$ is the competitors' price index defined in **?**. We can rewrite (16) as $\widehat{P}_{kit} = \tilde{\alpha}_{ki}\left(-\widehat{Z}_{ki} + \Gamma_{ki}\widehat{A}_{kit}\right) + (1-\tilde{\alpha}_{ki})\,\widehat{P}_{k-it}$, where $\tilde{\alpha}_{ki} = \frac{\alpha_{ki}}{1-(1-\alpha_{ki})s_{ki}}$, which is a U-shaped function of market shares s_{ki}.

α_k, $\widehat{P}_{kt}$ is independent of α_k for given market shares s_{ki} in the initial equilibrium. That is, the response in sectoral price is identical to that if markups are fixed at their initial level ($\alpha_{ki} = 1$). Intuitively, as pass-through α_k falls, the larger markup change by a firm to an own shock is exactly offset by a larger change in markup, in the opposite direction, of its competitors.

With heterogeneity in pass-through rates, because α_{ki} is decreasing in s_{ki}, there is a cutoff market share $\bar{s}_k^p$ such that a positive shock to firm i with $s_{ki} > \bar{s}_k^p$ results in a smaller reduction in sectoral prices than if markups are fixed at their initial level. Intuitively, firm i's increase in markup more than offsets the markup decrease of its competitors. Conversely, a positive shock to firm i with $s_{ki} < \bar{s}_k^p$ results in a larger reduction in sectoral prices than if markups were fixed at their initial level.[19]

From equation (18), the asymptotic variance of price changes in sector k assuming firm-level shifters are i.i.d. with common variance σ_v^2 is

$$\mathbb{V}ar\left[\widehat{P}_{kt}\right] = \left(\frac{\sigma_v}{\varepsilon - 1}\right)^2 \sum_{i=1}^{N_k} \left(\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}\right)^2. \tag{19}$$

If markups are fixed at their initial level (or, more generally, if $\alpha_{ki} = \alpha_k$), this variance is proportional to the sectoral HHI, as in **?**: $\mathbb{V}ar\left[\widehat{P}_{kt}\right] = \left(\frac{\sigma_v}{\varepsilon-1}\right)^2 \sum_{i=1}^{N_k} s_{ki}^2$. Comparing this expression with (19), we note $\mathbb{V}ar\left[\widehat{P}_{kt}\right]$ is lower under variable markups than under constant markups if and only if the variance of $\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}$ is lower than the variance of s_{ki}. Because α_{ki} is decreasing in s_{ki}, this condition is satisfied if $s_{ki}\alpha_{ki}$ is increasing in s_{ki} (see condition A7 in Appendix A.C). Intuitively, under this condition, pass-through rates are lower for larger firms, effectively reducing the weight of large firm shocks in the price index (with similar effects on volatility as a decline in the HHI).

Sectoral markups Changes in sectoral markups, defined in equation (7), can be decomposed into changes in markups *within* firms and the reallocation of expenditures *between* firms with heterogeneous markups:

$$\widehat{\mu}_{kt} = \sum_{i=1}^{N_k} s_{ki} \frac{\mu_k}{\mu_{ki}} \left(\widehat{\mu}_{kit} - \widehat{s}_{kit}\right). \tag{20}$$

[19] The threshold $\bar{s}_k^p$ is defined implicitly by $\alpha_k(\bar{s}_k^p) = \sum_{i=1}^{N_k} s_{ki}\alpha_{ki}$.

In Appendix A, we derive the following expression for changes in sectoral markups:[20]

$$\widehat{\mu}_{kt} = 2\left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right)\mu_k \sum_{i=1}^{N_k} s_{ki}\alpha_{ki}\left[s_{ki} - \frac{\sum_{i'} s_{ki'}^2\alpha_{ki'}}{\sum_{i'} s_{ki'}\alpha_{ki'}}\right]\widehat{V}_{kit}. \tag{21}$$

The following proposition states that a positive shock to firm i results in an increase in the sectoral markup if and only if firm i is sufficiently large in its sector.

Proposition 1 *Consider a positive shock to firm i in sector k, $\widehat{V}_{kit} > 0$. Then, under Cournot competition, sector k markup increases, $\widehat{\mu}_{kt} > 0$, if and only if $s_{ki} > \sum_{i'} s_{ki'}^2\alpha_{ki'} / \sum_{i'} s_{ki'}\alpha_{ki'}$.*

Intuitively, recall from equation (20) that changes in sectoral markups reflect changes in firm-level markups (within term) and between-firm reallocation (between term). Consider first the within term. A positive shock to firm i raises firm i's markup and reduces it for competing firms. The former dominates if firm i is large, whereas the latter dominates if firm i is small. Consider now the between term. A positive shock to firm i reallocates market shares towards firm i, increasing the sectoral markup if firm i's markup is sufficiently high (or, equivalently, if its market share is sufficiently large). Therefore, the within and between terms push the sectoral markup in the same direction.

The "2" in front of (21) reflects the fact that the magnitude of the within term is equal to the magnitude of the between term (and hence each accounts for 50% of changes in sectoral markups). A change in parameters (e.g., an increase in $\varepsilon - \sigma$) that increases the sensitivity of markups to firm-level shocks (increasing the within term) also increases the dispersion of markups across firms (increasing the between term). In Appendix A.A we show this 50-50 within/between decomposition of changes in sectoral markups under Cournot competition holds globally not only up to a first order.

How do changes in sectoral markups compare in the specification with variable markups versus the specification with constant markups in which sectoral markups change only due to between-firm reallocation? If firm-level markups are fixed at their initial level (by imposing $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$), changes in sectoral markups in equation (20) are:

$$\widehat{\mu}_{kt} = \sum_{i=1}^{N_k} s_{ki}\left(1 - \frac{\mu_k}{\mu_{ki}}\right)\widehat{V}_{kit}. \tag{22}$$

In response to a positive shock to firm i, sectoral markups increase if and only if $\mu_{ki} > \mu_k$.

In general, we cannot easily compare (21) and (22). To make analytic progress, in Appendix A.B we restrict the extent of ex-ante firm heterogeneity to two types and provide a simple sufficient

[20] Ex-ante firm heterogeneity is a necessary condition for sectoral markups to change in response to firm-level shocks. To see this, if s_{ki} and μ_{ki} are equal across all firms in sector k, equations (15), (20), and $\sum_{i=1}^{N_k} s_{ki}\widehat{s}_{ki} = 0$ imply that $\widehat{\mu}_{kt} = 0$.

condition for sectoral markups to change by more (and display a higher variance) under variable markups than under constant markups. Intuitively, changes in sectoral markups can be smaller under variable markups than under constant markups because the larger response of sectoral markups due to changes in firm-level markups is more than offset by a smaller extent of between-firm reallocation due to incomplete pass-through.

To summarize, even though changes in sectoral markups under variable markups are twice as large as the between-firm reallocation term for any firm-level shocks, variable markups do not necessarily magnify changes in sectoral markups relative to the model specification with constant markups, because incomplete pass-through mutes the extent of between-firm reallocation

Covariance between sectoral prices and sectoral markups Recall from previous results that in response to a positive shock to firm i in sector k, the sectoral price falls, whereas the sectoral markup can increase or decrease depending on the firm's initial markup. In finite samples, comovement can be positive or negative depending on which firms are shocked. We now calculate the asymptotic covariance between sectoral price and markup changes, which shapes the covariance between sectoral output and markup that we examine below.

To build intuition, consider first the case of constant markups,

$$\mathbb{C}ov\left[\widehat{\mu}_{kt},\widehat{P}_{kt}\right]=-\frac{1}{\varepsilon-1}\sum_{i=1}^{N_k}s_{ki}^2\left[1-\frac{\mu_k}{\mu_{ki}}\right]\times\sigma_v^2. \tag{23}$$

Thus, sectoral markups and prices are negatively correlated as long as large firms within sector charge higher markups. Intuitively, shocks to small firms induce a positive comovement, whereas shocks to large firms induce a negative comovement. Overall, comovement is negative because shocks to large firms induce larger changes in sectoral price than shocks to small firms.

With variable markups, using equations (18) and (21) we obtain

$$\mathbb{C}ov\left[\widehat{\mu}_{kt},\widehat{P}_{kt}\right]=-\left(\frac{2\mu_k}{\varepsilon-1}\right)\left(\frac{1}{\sigma}-\frac{1}{\varepsilon}\right)\sum_{i'=1}^{N_k}s_{ki'}^2\alpha_{ki'}\sum_{i=1}^{N_k}\left[\frac{s_{ki}^2\alpha_{ki}}{\sum_{i'=1}^{N_k}s_{ki'}^2\alpha_{ki'}}-\frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\right]\frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\times\sigma_v^2. \tag{24}$$

When $\varepsilon>\sigma$, sectoral prices and markups comove negatively in long samples if and only if

$$\sum_{i=1}^{N_k}\left[\frac{s_{ki}^2\alpha_{ki}}{\sum_{i'=1}^{N_k}s_{ki'}^2\alpha_{ki'}}-\frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\right]\frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}>0. \tag{25}$$

If firms are ex-ante homogeneous, equation (25) holds with equality and sectoral markups are constant over time. If firms are heterogeneous in the initial equilibrium, inequality (25) may or may not hold. The following proposition, proven in Appendix A.C, states that if pass-through rates do not fall too strongly with market shares, inequality (25) holds, so sectoral prices and markups comove negatively.

Proposition 2 *Under Cournot competition, if firms are ex-ante heterogeneous and $s_{ki}\alpha_{ki}$ is increasing in s_{ki}, sectoral markup and price comove negatively, $\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] < 0$.*

In Appendix A.B we show that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} provided that market shares are not too large. Intuitively, the condition that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} implies, by equation (18), that sectoral prices are more responsive to large firm shocks than to small firm shocks (i.e., the lower pass-through rate by large firms does not fully offset their higher weight in the price index). The fact that sectoral markups increase in response to large firm shocks and decrease in response to small firm shocks implies a negative covariance between sectoral price and markup, as in the case of constant markups.

Covariance between sectoral output and markups Changes in sector k output in response to sector k shocks, derived in Appendix A.D, are:

$$\widehat{Y}_{kt} = -\left[\sigma\left(1-s_k\right) + \left(\frac{f+1}{f\eta+1} + \left(\frac{\sigma-1}{f\eta+1}\right)\left(1-\frac{\mu}{\mu_k}\right)\right)s_k\right]\widehat{P}_{kt} + \frac{s_k\mu}{\mu_k}\frac{\widehat{\mu}_{kt}}{f\eta+1}. \tag{26}$$

A sufficient condition for sectoral output and price to move in opposite directions is that sector k is small in the aggregate ($s_k \to 0$) or that disutility of labor is linear ($f \to \infty$). In this case, the previous results on sectoral price apply immediately to sectoral output (with the opposite sign).[21] Specifically, in response to a positive shock to firm i in sector k, sectoral output increases whereas sectoral markup can increase or decrease depending on the firm's initial markup.

Taking into account a long sequence of firm shocks in sector k, the covariance between changes in sectoral output and sectoral markup is:

$$\mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kt}\right] = -\left[\sigma\left(1-s_k\right) + \frac{f+1+(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)}{f\eta+1}s_k\right]\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] + \frac{s_k\mu}{\mu_k}\frac{1}{f\eta+1}\mathbb{V}ar\left[\widehat{\mu}_{kt}\right],$$

where $\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right]$ is defined above. The following proposition provides sufficient conditions for procyclical sectoral markups with respect to sectoral output.

[21] If f finite and sector k is sufficiently large in the aggregate, it is *possible* that sectoral output and price both fall in response to positive sector k firm level shocks if sectoral markup μ_k is very low relative to the aggregate markup and/or if sector k markup falls substantially when sectoral price falls.

Proposition 3 *Under the conditions of Proposition 2, sectoral markup and sectoral output comove positively,* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right]>0$*, if at least one of the following conditions holds: (i)* $s_k\to 0$*, (ii)* $f\to\infty$*, or (iii)* $\sigma\to 1$*. If (i)-(iii) are violated, then* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right]>0$ *as long as sectoral markup* μ_k *is not too low relative to the aggregate markup.*

In our empirical analysis, we also examine the cyclicality between sector output and firm-level markups. For the case of $f\to\infty$, the covariance between changes in firm i markup and sector k output, derived in Appendix A.D, we show that

$$\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kit}\right]=\left(\sigma\left(1-s_k\right)+\eta^{-1}s_k\right)\frac{\alpha_{ki}\Gamma_{ki}}{\left(\epsilon-1\right)\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\left[s_{ki}\alpha_{ki}-\frac{\sum_{i'=1}^{N_k}\left(s_{ki'}\alpha_{ki'}\right)^2}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\right]\times\sigma_v^2. \tag{27}$$

The following proposition states that firm-level markups are procyclical for large firms and countercyclical for small firms:

Proposition 4 *If* $s_{ki}\alpha_{ki}$ *is increasing in* s_{ki} *and* $f\to\infty$*, firm-level markups and sectoral output comove positively,* $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kit}\right]>0$*, if and only if* $s_{ki}>\bar{s}_k^{\mu}$*, and comove negatively if and only if* $s_{ki}<\bar{s}_k^{\mu}$*, where* $\bar{s}_k^{\mu}$ *is defined by the condition that the square bracket in (27) is equal to 0.*

Intuitively, firm-level markups are positively correlated with sectoral output in response to own-shocks and negatively correlated in response to competitors' shocks. Because large firms have a disproportionate impact on sectoral price and output (if $s_{ki}\alpha_{ki}$ is increasing in s_{ki}), firm-level markups are procyclical for large firms and countercyclical for small firms.[22]

Before presenting the aggregate results, we briefly discuss our model's implications for changes in markups when some prices are nominally rigid.

Discussion of firm and sector-level markups with rigid prices Whereas in this paper we study markup fluctuations under flexible prices, markups can also fluctuate if costs change and prices are nominally rigid. Consider the specification under Bertrand competition, and suppose that the price of firm i in sector k at time t is stuck at $\bar{P}_{kit}$ before the shocks hit.[23] For a sticky price firm, the markup is $\mu_{kit}=Z_{kit}\bar{P}_{kit}/W_t$. For flexible price firms (which do not anticipate that that their price may be stuck in the future), markups are given by (5). Market shares for all firms are determined by the system of equations (4).

In response to productivity shocks to flexible price firms, the sign of the change in sectoral markups depends on the relative markup level of the shocked firms, as in the baseline model. Consider now an increase in productivity of a rigid-price firm and suppose that the change

[22]The cutoff $\bar{s}_k^{\mu}$ differs from the cutoff defined in Proposition 1 because the condition in Proposition 1 is based on a shock to one firm only, whereas the asymptotic covariance in Proposition 4 takes into account shocks to all firms in the sector.

[23]Here for simplicity we take $\bar{P}_{kit}$ as given and do not study how firms choose their reset price. For a detailed analysis of sticky prices in a dynamic environment with oligopolistic competition, see **?** and **?**.

in nominal wage is negligible. The markup of the shocked rigid-price firm rises mechanically, while markups of other firms remain unchanged (since prices and thus market shares do not change). Hence, the sectoral markup rises irrespective of whether the shocked firm is small or large. This force strengthens procyclical sectoral markups in comparison to the flexible-price model.

Consider now a uniform decline in marginal costs for all firms (productivity rises relative to the nominal wage). For firms with rigid price, markups rise. For firms with flexible price, markups also rise since these firms lower their price and increase market share relative to sticky price firms. Hence, markups rise for all firms. There are additional compositional effects on the sectoral markup as market shares shift towards flexible price firms. This composition effect increases the sectoral markup if flexible price firms charge higher markups.[24] Whether the increase in markups is procyclical or countercyclical depends on the source of the movement in marginal cost. In response to an increase in productivity for all firms, markups and output rise. This force provides another reason for procyclical markups relative to the flexible price baseline where, recall, sectoral shocks leave markups unchanged. In response to contractionary monetary policy that reduces marginal cost for all firms, markups rise but output falls, resulting in countercyclical markups. Our flexible-price baseline abstracts from this well-studied source of countercyclical markups.

II.C Aggregate outcomes

In this section, we characterize changes in aggregate price (i.e., the inverse of the real wage, given that the wage is the numeraire), markup, productivity, and output. We provide expressions for the variance of aggregate output and for sectoral and aggregate markup cyclicality with respect to aggregate output, which we consider in our empirical analysis.

Up to a first order, changes in the aggregate price are $\widehat{P}_t = \sum_k s_k \widehat{P}_{kt}$. Based on our results above, any positive firm-level shock in sector k reduces the corresponding sectoral price and therefore reduces the aggregate price (or increases the real wage) proportionately to the share in expenditures of sector k. Whether the real wage increases more or less under variable markups relative to constant markups depends, as discussed above, on the shocked firm's relative size in its sector.

Changes in aggregate markup can be decomposed into a within-sector markup term and a reallocation term, analogous to the decomposition of sectoral markups in equation (20):

[24] For evidence that prices are more flexible for large firms (which in our model charge higher markups), see **?** and **?**.

$$\widehat{\mu}_t = \sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} + (1-\sigma) \sum_k s_k \left(1 - \frac{\mu}{\mu_k}\right) \widehat{P}_{kt}. \tag{28}$$

In response to a positive shock to a firm in sector k, aggregate markup can increase or decrease. The first (within) term in (28) is positive if the shocked firm is relatively large (and sets a higher markup) in sector k. The second (between) term in (28) is positive, when $\sigma > 1$, if sector k has a relatively high markup relative to the aggregate markup.

Changes in aggregate productivity, using $\widehat{Z}_t = \widehat{\mu}_t - \widehat{P}_t$, can be expressed in terms of changes in sectoral markups and prices as

$$\widehat{Z}_t = \sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} - \sum_k s_k \left[1 + (\sigma - 1)\left(1 - \frac{\mu}{\mu_k}\right)\right] \widehat{P}_{kt}. \tag{29}$$

Finally, by equation (13), changes in aggregate output are

$$\widehat{Y}_t = \left(f^{-1} + \eta\right)^{-1} \left[f^{-1} \widehat{Z}_t - \widehat{P}_t\right]. \tag{30}$$

With linear disutility of labor ($f \to \infty$), the aggregate productivity term drops, so $\widehat{Y}_t = -\eta^{-1} \sum_k s_k \widehat{P}_{kt}$.

A positive firm-level shock in sector k reduces the sectoral price and increases aggregate output. Based on the discussion above on the role of variable markups for the response of sectoral prices, the increase in aggregate output is smaller under variable markups compared to constant markups if and only if the shocked firm has a high market share.

Variance of aggregate output The variance of aggregate output (when $f \to \infty$) is

$$\mathbb{V}ar\left[\widehat{Y}_t\right] = \eta^{-2} \sum_k s_k^2 \mathbb{V}ar\left[\widehat{P}_{kt}\right] = \frac{\sigma_v^2}{\eta^2 (\varepsilon - 1)^2} \sum_k s_k^2 \sum_{i=1}^{N_k} \left(\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}\right)^2, \tag{31}$$

where the second equality used equation (19). Based on the discussion following equation (19), aggregate output is less volatile under variable markups than under constant markups when pass-through rates are decreasing in size, effectively reducing the weight of large firms in the price index (with similar effects on volatility as a reduction in market-share concentration).

In Appendix A.F we provide an expression for the variance of aggregate output without imposing $f \to \infty$, as well as for the variance of the aggregate markup, which we use in our quantitative analysis.

Covariance between aggregate output and markup We first calculate the covariance between aggregate output and sector k markup, which is one of the measures of cyclicality in our empirical analysis. When calculating this covariance, we use the fact that sector k markups are affected only by shocks to sector k firms and not by shocks to firms in other sectors. We can thus express this covariance as

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] = \mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kt}\right] + \sigma\left(1 - s_k\right)\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right]. \tag{32}$$

The following proposition, proven in Appendix A.E, provides conditions under which the covariance between aggregate output and sector k markups is positive:[25]

Proposition 5 *Under the conditions of Proposition 3,* $\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] > 0$.

Finally, the covariance between aggregate output and aggregate markup (when $f \to \infty$) is:

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_t\right] = -\frac{\mu}{\eta}\sum_k \frac{s_k^2}{\mu_k}\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] + \frac{\sigma - 1}{\eta}\sum_k s_k^2\left(1 - \frac{\mu}{\mu_k}\right)\mathbb{V}ar\left[\widehat{P}_{kt}\right]. \tag{33}$$

The first term in (33) is positive if sectoral markups and sectoral prices comove negatively, which we discussed above. The second term in (33) is positive unless larger sectors have relatively lower markups.

So far, we have calculated measures of markup cyclicality considering only i.i.d firm-level shocks. In our quantitative analysis, we also allow for aggregate productivity shocks to firms in all sectors. In our model, in which firm-level markups are functions of market shares, markups do not respond to aggregate shocks. Therefore, incorporating aggregate shocks leaves the covariance of aggregate markups and output unchanged but decreases the correlation, because the volatility of aggregate output increases with these shocks.

From these theoretical results, we see the sign of markup cyclicality depends on the level of aggregation, market structure within and across all industries, and the set of shocked firms. Moreover, the sign and magnitude of covariances in finite samples may differ from those of the asymptotic covariances we derived.

In what follows we calibrate the model to match salient features of the French firm-level data. We use our calibrated model for two purposes. First, we evaluate quantitatively its implications for different measures of markup cyclicality and compare these to their data counterparts. Second, we quantify aggregate fluctuations in output and markups in response to idiosyncratic

[25] Note also that by equation (32), $\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] \leq \mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kt}\right]$, where the inequality holds strictly if the economy has more than one sector (i.e. $s_k < 1$). The fact that the covariance between sectoral markups and aggregate output is lower than that between sectoral markups and sectoral output does not immediately extend to correlations because, for some sectors, the variance of aggregate output is smaller than the variance of sectoral output.

firm-level shocks. Relative to the analytic results in this section, we relax some of the assumptions imposed in our propositions, solve the model non-linearly and calculate our moments in finite samples.

III Data, Estimation, and Calibration

For the remainder of this paper, we use the model above as a data-generating process from which we simulate firm-level outcomes, which are then aggregated into sector and aggregate time series. We then proceed to compare the resulting model-implied moments to their empirical counterparts. This section describes how we use French administrative firm-level data to obtain empirical moments of interest and to parameterize our model. We first describe the data and how we estimate markups, and then how we calibrate the model. Appendix B provides additional information on the data and the procedures to estimate production functions and markups.

III.A Data

Our empirical analysis deploys French firm-level data between 1994 and 2019. We use administrative sources for income statements and balance sheets, complemented by a firm survey with information on quantities.

Firm-level income statements and balance sheet data are obtained from two administrative datasets: the FICUS data covering the period 1994-2007 and the FARE dataset covering the period 2008-2019. These datasets cover the universe of French firms and originate from the French tax administration that collects yearly tax statements for each firm, including income statements, balance-sheet, and demographic information. The Institut National de la Statistique et des Etudes Economiques (INSEE) uses these statements to construct the FICUS-FARE datasets. We assign firms to sectors according to the Nomenclature d'Activités Française (NAF2008) 5-digit classification, a French industry classification similar to the 4-digit NACE Rev 2 classification. We keep firms that were government-owned earlier in our sample since during the period we consider most of them switched to private ownership.[26]

We use a subset of the variables available in the FICUS-FARE dataset: total firm revenues, wage

[26] In 2008, the NACE and NAF industry classification changed. To construct a panel of firms between 1994 and 2019 with a consistent industry classification over time, we proceed as follows. For firms for which the old and new industry codes are observed, we apply the new code to all years. For firms for which we only observe one industry code on either side of 2008, we assign the code that is most frequently associated with the observed industry code (using the sample of firms where we do observe the two sectoral codes). Finally, information about government-owned firms can be found in the variable APPGR of FICUS-FARE, which is available only before 2009. Government-owned firms represented 0.12% of the total number of firms in 1994 and 0.05% in 2008. Over the same period, their share of revenue fell from 7.2% to 4.2%. We thank Isabelle Mejean for sharing the computer code to merge the FICUS and FARE datasets.

bill (the sum of wages and social security payments), capital (measured by fixed assets), and expenditures on both material and service inputs. Our baseline measure of materials (which is our choice of variable input in the estimation of markups) is the sum of expenditures on materials and merchandises (variables ACHAMPR and ACHAMAR, respectively) net of changes in stocks (VARSTMP for materials and VARSTMA for merchandise).[27] We consider as a separate input expenditures on service inputs (AUTACH), including research expenditures, outsourcing costs, and external personnel cost (including temporary workers). We use GDP deflators and 2-digit sector-price indices provided by EU-KLEMS.

We construct a measure of firm-level quantity using the survey Enquête Annuelle de Production (EAP) conducted by INSEE. Specifically, this survey covers the universe of large firms (with at least 20 employees or 5 million euros in annual revenues) and a representative sample of small firms for a subset of 2-digit sectors, mainly in manufacturing, during the period 2009-2019.[28] For each firm, this dataset provides information on revenues and quantities by product, where a product is a combination of an 8-digit code and a unit of account.[29] We drop around one-third of firm-products without quantity data. We calculate price by product for each firm as the ratio of revenues to quantity sold. We follow **?** and **?** in standardizing prices by dividing them by the quantity-weighted average price of the same product across firms in the same year. We do this because firms produce goods in different units (e.g., kilograms and liters), and aggregation requires homogenous units. Our measure of firm-level price in a given year is the revenue-weighted average of standardized prices across the products the firm sells. Finally, firm-level quantity is defined as the ratio of firm-level revenues to firm-level price.

For the rest of the paper, it will be useful to distinguish two samples: (i) the sample of firms with quantity information in the EAP dataset, which we employ to estimate 2-digit sector-level production functions; this will form the basis of our *estimation sample*, and (ii) the sample of all firms in FICUS-FARE that belong to sectors covered by the EAP dataset — including those with revenues but no quantity data — and that is used to calculate our measures of markup cyclicality; and which we treat henceforth as our *baseline* sample.

For the first sample, which covers the period 2009-2019, we keep firms with more than two employees and with positive value added, revenue, materials, services expenditure, wage bill, cap-

[27]The variable ACHAMPR is defined as "everything that the firm purchases in order to be transformed," and ACHAMAR is defined as "everything that the firm purchases to be sold as is." VARSTMP and VARSTMA are defined analogously for changes in stocks.

[28]The EAP survey is used by EUROSTAT to produce statistics such as Structural Business Statistics (SBS) and Production Statistics (PRODCOM). According to INSEE, roughly 35,000 unique firm identifiers are included in this survey, covering 90% of French industrial production. The 2-digit NACE rev 2 sectors covered by our EAP sample are 08, 13, 14, 15, 16, 17, 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 43 and 46, where the last two are non-manufacturing sectors ("specialized construction activities" and "wholesale trade, except of motor vehicles and motorcycles" respectively).

[29]Examples of units of accounts are kilograms, tonnes, or pieces. We define a product as a combination of a unit of account and product code because firms that use different units of accounts for the same product code might produce relatively heterogeneous goods.

ital, and price. We winsorize these variables by sector at the 1% level. We end up with $220,733$ firm-year observations across 11 years and 22 sectors at the 2-digit level. Despite the smaller number of observations and shorter time-span, the estimation sample represents about 38% of value-added in the baseline sample. For the second sample, which covers the period 1994-2019, we keep firms with positive value added, revenue, materials, services expenditure, wage bill, and capital. We further restrict the sample to firms with positive estimated gross markups (estimated as described below).[30] This yields a firm-level panel with $9,383,228$ observations, covering 26 years, 22 sectors at the 2-digit level, and 275 sectors at the 5-digit level. Table A1 in Appendix B.A displays further summary statistics for both the estimation sample (Panel A) and the baseline sample used to measure markup cyclicality (Panel B). Firms in the estimation sample are larger than firms in the baseline sample since the EAP survey focuses on large firms. The average market share across all firms and years is low, at about 0.07%. However, the distribution of market shares is highly skewed, with the top 0.01% attaining a market share above 38%.

Although this dataset is very rich, it misses some important information that limits the extent of our analysis. First, we do not use information on imports and exports in the corresponding sector. Specifically, when we compute market share as the ratio of a firm's revenue relative to the sum of all French firms' revenue in this sector, we do not take into account the sales of foreign firms in this market. Moreover, when we estimate markups we do not exclude sales to foreign countries because we do not know the share of expenditures on inputs this is accounted for by exports.

Second, because firm-level revenues in our dataset are reported at the national level, we do not have information on revenues at the local level. This limitation is important for non-tradeable goods, whose markets are most likely local.[31] Because our definition of a market is at the national level, for non-tradable goods we likely underestimate the concentration in the local market relevant for the firm.

III.B Markup estimation

We estimate markups for two purposes. First, we calculate measures of markup cyclicality in the data, which we compare with markup dynamics implied by our model. Second, when we calibrate the model, we target the relationship between sectoral markups and HHI.

Our empirical framework to estimate markups in the data is more general than our theoretical framework described above, where labor was the only factor of production. Specifically, we also

[30] Market shares (defined as the share of firm-level revenues in the corresponding 5-digit sector), sectoral aggregates, and HHIs are calculated prior to restricting the sample.

[31] See **?** for a study of diverging local and national market-concentration trends or **?** for the evolution and consequences of market-concentration trends in the US retail sector.

include materials, capital, and services as inputs to production.[32] We assume that materials, M, are chosen statically (i.e. they are not subject to adjustment costs). Following **?** and **?**, the first-order condition for M in the cost-minimization problem by firm i in sector k implies

$$\mu_{kit} = \theta^M_{kit} \frac{P_{kit} Y_{kit}}{P^M_{kit} M_{kit}}, \tag{34}$$

where $P^M_{kit} M_{kit}$ denotes expenditure on materials by firm i in sector k, $P_{kit} Y_{kit}$ is revenues, and θ^M_{kit} is the output elasticity with respect to materials. To measure markups at the firm level, we require the ratio of material expenditures to revenues — which is available for all firms in our baseline sample — and the output elasticity with respect to materials.

To estimate output elasticities, we assume a flexible translog production function with the four inputs described previously. All firms in the same 2-digit sector share the same production-function parameters and — constrained by the availability of 2-digit-level price indices for intermediate inputs — inputs are homogeneous across firms within sector. The output elasticity with respect to materials of firm i in market k at time t is equal to $\theta^M_{kit} = \beta_m + 2\beta_{mm} m_{kit} + \beta_{ml} l_{kit} + \beta_{mo} o_{kit} + \beta_{mk} k_{kit}$ where the β_{xy} are the parameter of the production functions and $m_{kit}, l_{kit}, o_{kit}$ and k_{kit} are respectively the firm-level log materials, labor, service and capital usage. Note that output elasticities differ across firms within 2-digit sectors even if these firms have the same production function parameters β.

We estimate the parameters in the production function implementing a two-stage iterative generalized method of moments (GMM) following **?** which builds on **?**, **?**, and **??**. Here we provide an overview of our approach, and in Appendix B.B we provide additional details. The first stage of this method controls for unobserved productivity by using conditional demand of material input. As discussed in **?** and **?**, under imperfect competition of the form considered in this paper, this first-stage requires additional controls, namely market share and firm-level price. In the second stage, we implement a dynamic panel estimator using GMM. Importantly, in both stages we use quantity data as our output measure, thus we can only implement this approach in our estimation sample of firms.

Finally, assuming (i) that all firms in the same 2-digit sector (including those with and without quantity information) share the same production function parameters and (ii) that these parameters are stable over time, allows us to apply the parameters obtained in the estimation sample to *all* FICUS-FARE firm-year observations in the corresponding sectors and compute output elasticities in the longer and larger baseline sample."

Equipped with output elasticities and material-to-revenue ratios, we calculate markups in the baseline sample using expression (34). Table A1 provides descriptive statistics of our firm-level markup estimates. The distribution of markups is quite skewed, with a median of 1.21, a mean

[32] We maintain the assumption that firms are price takers in input markets. **?** relaxes this assumption to estimate markdowns on inputs.

of 1.39 and a top quartile of 1.77.

Given our use of quantity as a measure of output, our estimates are not subject to biases that may emerge when revenues are used, as discussed in e.g. **?**, **?** and **?**. Nevertheless, in Section V.D we provide sensitivity analysis on the use of quantity, revenue or accounting data to compute markups and measure markup cyclicality.

III.C Calibration

We now describe how we parameterize the model to match salient features of the French data. We first introduce the firm-level productivity stochastic process. We then describe our calibration strategy, which targets the size and concentration of each of the 275 sectors at the 5-digit level together with other moments from our data and the literature.

Firm-level productivity process

We assume that firm-level quality shifters, A_{kit}, are fixed over time so that the firm-shifter V_{kit} is driven only by productivity shocks.[33] Following **?**, we assume that firm-level productivity, Z_{ikt}, follows the discretized random growth process introduced by **?**. Specifically, firm productivity in sector k evolves on an evenly spaced log grid, $\Phi_k = \{1, \varphi_k, (\varphi_k)^2, \dots, (\varphi_k)^S\}$, where $\varphi_k > 1$ and S is an integer. A firm's productivity follows a Markov chain on this grid with associated transition matrix $\{P^{(k)}_{n,m}\}_{n,m\in[|1,S|]}$ such that $P^{(k)}_{n,n} = 1 - a_k - c_k$, $P^{(k)}_{n,n-1} = a_k$, $P^{(k)}_{n+1,n} = c_k$, $P^{(k)}_{1,1} = a_k + b_k$, and $P^{(k)}_{S,S} = b_k + c_k$ with $1 > a_k, b_k, c_k > 0$ and $a_k + b_k + c_k = 1$. The three parameters φ_k, δ_k, and c_k characterize this productivity process in sector k.

As shown in **?** and **?**, this process implies Gibrat's law for productivity. That is, away from the boundaries, productivity growth is independent of its current level. Additionally, this process generates a stationary Pareto distribution of productivity (**?**) with a tail index equal to $\delta_k = \log(\frac{a_k}{c_k})/\log(\varphi_k)$. Note, however, that in our environment this does not immediately imply that firm size satisfies these properties due to the finite number of firms within sectors and variable markups.

In Section VI, we consider an exercise where we add aggregate TFP shocks to target the volatility of annual changes in aggregate output.

[33]Although our analytic results do not take a stand on the importance of productivity versus quality-shifter firm-level shocks, in the data we construct sectoral output deflating nominal value-added by industry price indices. The latter typically do not take into account high-frequency changes in quality shifters. Therefore, for consistency, in the data we abstract from shocks to quality shifters.

Table I: CALIBRATION TARGETS

Panel A: Economy Wide targets

Moment	Source	Data	Model
Slope of $\Delta\mu_{t,k}^{-1}$ on $\Delta HHI_{t,k}$	Table V	-0.37	-0.36
Constant of volatility on market share	Table A5	0.27	0.27

Panel B: Sectoral targets

Moment	Source	Data	Model
Number of firms N_k $^{(*)}$	Baseline sample	1453	1453
Revenue share $^{(*)}$	Baseline sample	0.16%	0.16%
HHI (See Fig I) $^{(*)}$	Baseline sample	0.115	0.115

NOTE: Rows with $^{(*)}$ refer to 275 moments (one per 5-digit sector). We report averages across the 275 sectors.

Calibration strategy

We now describe how we assign values to the model's parameters: the two macro parameters η and f, the two demand elasticities ε and σ, the number of firms N_k, the demand shifter A_k, and the productivity parameters φ_k, δ_k, and c_k for each of our 275 sectors. Table I and Figure I summarize data targets and model fit, while Table II displays parameter values.

We set the relative risk aversion to 1 (log utility) and the Frisch labor-supply elasticity to 1, both of which are standard values in the business-cycle literature. We assume that in all sectors, firms compete à la Cournot. We set the within-sector elasticity to $\varepsilon = 5$.[34] We calibrate the between sector elasticity σ to target the slope of the regression in first-differences (over time) of the inverse sectoral markup on the HHI (constructed as described in Section IV). In the data, the coefficient of this regression is -0.37, as reported in column (2) of the Table V discussed in Section IV. In the model, taking first-differences of equation (8) implies $\Delta\mu_{tk}^{-1} = -\frac{\left(\frac{\varepsilon}{\sigma}-1\right)}{\varepsilon}\Delta HHI_{tk}$. Given our choice of ε, we set $\sigma = 1.8$, which implies a slope of -0.36. The two demand elasticities σ and ε shape own-cost pass-through rates $\tilde{\alpha}_{ki}$, defined in footnote 18. Our baseline choices imply an own-cost pass-through rate of 0.63 for large firms (those with a market share of 57% or higher), which lies within confidence intervals in **?** for large Belgian firms.[35]

We now discuss how we assign parameter values that vary across our 275 sectors to match

[34] In Appendix F we provide sensitivity analysis for alternative values of ε ranging between 4 and 7 while recalibrating the remaining model parameters. Our quantitative results on markup cyclicality are fairly stable, while aggregate output volatility is increasing in ε.

[35] About 360 firm-year observations have a market share above 57%, representing approximatively the top 0.004% of the market-share distribution. Our model implies pass-through rates that are on the high-end of estimates in **?** for Belgian firms and on the low-end of estimates in **?** for French exporters, and is consistent with findings in both papers that pass-through rates are decreasing in firm size. For alternative values of ε reported in Appendix F, pass-through rates are lower (e.g., 0.55 for large firms when $\varepsilon = 7$) and hence closer to the point estimates in **?**.

Table II: BASELINE CALIBRATION

Panel A: Economy Wide parameters

Parameters	Value	Description
η	1	relative risk aversion
f	1	Frisch elasticity of labor supply
f_0	1	labor disutility parameter
ε	5	substitution across firms
σ	1.8	substitution across sectors

Panel B: Sectoral Parameters

Parameters	Value	Description
S	70	number of productivity bins per sector
φ_k	1.091	median firm-level productivity process
a_k, c_k	0.348, 0.250	median firm-level productivity process
A_k	0.0015	median sectoral preference shifter

salient features of our data in the period 1994-2019. We set the number of firms per sector, N_k, to the average number of firms in sector k observed in our data. We calibrate the constant sector-level demand shifter, A_k, to target the average revenue share of each of our 275 sectors in the data. For each sector k, we choose the tail parameter of the stationary distribution, δ_k, to match the average HHI in the data. Figure I reports HHI in the data against the model counterpart. The fit is good, as revealed by the fact that all dots lie close to the 45-degree line.

[36] The grid parameter φ_k determines the range of values that the HHI can take as we vary δ_k. We choose the lowest φ_k such that this range of values contains the value of the HHI in the data for this sector.[37] Finally, we set the remaining parameter of the productivity process, c_k, such that in each sector the conditional volatility of market-share for a hypothetical infinitesimal firm is equal to the constant in the regression of market-share volatility on market-share estimated across all sectors in the data and reported in Table A5.[38]

In what follows we use the calibrated model as a data-generating process to simulate firm-level, sector-level, and aggregate-level time series. We use the simulated panels to run the corresponding regressions that we run on actual data. We also compute aggregate business-cycle

[36] Given a guess of δ_k, we draw 1,000 samples of N_k firm-level productivities from the Pareto distribution characterized by δ_k. For each of these samples, we solve for firm-level market shares and compute the implied HHI. We then calculate the median HHI across the 1,000 samples, and iterate over δ_k until we match the HHI for a given sector in the data. We repeat this procedure for each of the 275 sectors.

[37] We choose the number of productivity bins $S = 70$. Higher values of S have a minor impact on our results.

[38] For each firm in our baseline sample, we calculate the time-series standard deviation of the firm's market share growth rate. Column (1) of Table A5 in the appendix reports a regression of this volatility measure on average market share and a constant term. In column (2) we report an alternative specification using the standard deviation of the growth rate of market share in the cross-section, as explained in the notes to Table A5. Both specifications yield a precisely estimated constant of approximately 0.27.

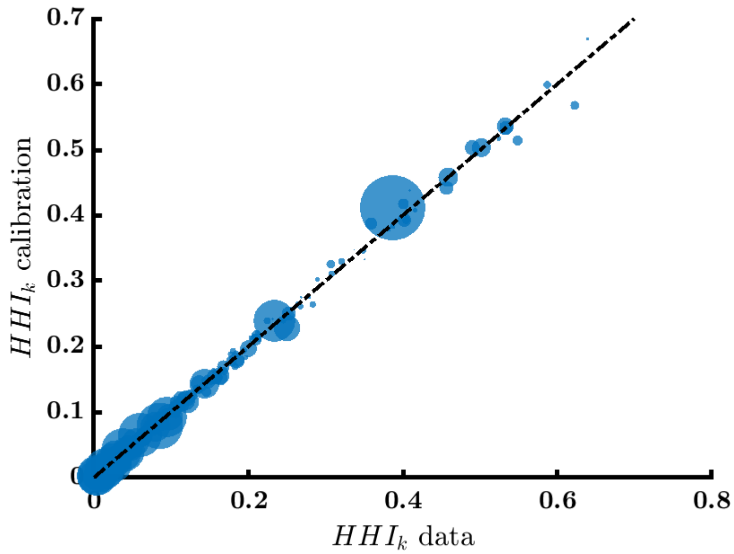

Figure I: Model Fit

NOTE: For each sector, this figure show the HHI in the data (x-axis) against the median HHI computed over 1,000 samples drawn from the baseline calibration (y-axis). The size of each dot is proportional to the sector's average revenue share between 1994 and 2019.

statistics using the simulated aggregate time-series, which we then compare with counterparts in the data.

IV Inspecting the Mechanism

In this section, we examine in the data some basic implications of our model. We analyze the relation between firm-level markups, marginal costs, and market shares within sectors, as well as that between sectoral markups and measures of concentration across sectors. We also quantify the contribution of changes in firm-level markups for sector-level markup changes.

IV.A Firm-level evidence

Hardwired into our model is a key micro-level relationship between markups and concentration. At the firm-level, and following the discussion in Section I.B, markups increase with a firm's market share. In turn, this immediately gives rise to a notion of markup procyclicality at the micro-level: a firm's markup increases whenever its market share increases.

Taking the inverse of equation (5) and applying first differences yields a simple linear relation between the firm's market share and its inverse-markup,

$$\Delta \mu_{kit}^{-1} = -\frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon} \Delta s_{kit}, \tag{35}$$

where $\Delta \mu_{kit}^{-1}$ is the first-difference of the inverse (gross) markup of firm i in sector k at time t and Δs_{kit} is the first-difference of its market share. This motivates the following simple empirical specification,

$$\Delta \mu_{kit}^{-1} = \alpha_t + \beta \Delta s_{kit} + \epsilon_{kit}, \tag{36}$$

where β is the coefficient of interest, which the model predicts to be negative. We allow for year fixed-effects α_t to control for unobserved markup shifters that are common across all firms. In alternative specifications we further allow for sector-year fixed effects, α_{kt}, thus absorbing markup variation that is common across all firms in a sector, in a given year. While these fixed effects are not present in our theoretical model they nevertheless allow us to empirically control for flexible aggregate and sector-specific markup trends in the data and serve as a robustness check.

We start by inspecting these firm-level relations in the French data. We have estimates of firm-level markups over the period 1994-2019, as described in the previous section. We calculate firm-level market shares as the ratio of firm-level revenues to total revenues in the corresponding 5-digit sector. Taking first-differences yields time series for $\Delta \mu_{kit}$ and Δs_{kit} for 955,657 unique firms over the period 1995-2019.

Table III: INVERSE MARKUP AND MARKET SHARE

	(1)	(2)	(3)
Dependent Variable:		$\Delta \mu_{kit}^{-1}$	
Δs_{kit}	-0.268	-0.268	-0.293
	(0.092)	(0.093)	(0.099)
Year FE	N	Y	N
Sector $\times$ Year FE	N	N	Y
Observations	8,051,767	8,051,767	8,051,767

NOTE: $\Delta \mu_{kit}^{-1}$ is the first difference of the inverse of firm i sector k gross markup between t and $t-1$, and Δs_{kit} gives the first difference of market share of firm i in sector k. Columns (1)-(3) report baseline empirical estimates for the FICUS-FARE (1995-2019) data. Column (1) reports pooled estimates while columns (2) and (3) report estimates that further control for year or sector$\times$year fixed effects, respectively. Sector-year fixed effects are defined at the 5-digit NAF sector classification. Standard errors (in parentheses) are two-way clustered at the firm and year level. $\Delta \mu_{kit}^{-1}$ is winsorized at the 3% level.

Table III displays our estimates and the associated two-way (at firm and year level) cluster-robust standard errors. Column (1) displays the firm-level relation in first-differences, obtained by pooling all firm-level data (across sectors and years) for a total of over 8 million observations. This yields a negative and statistically significant coefficient, as theory predicts. Further, the empirical estimates remain stable and significant when additionally controlling for year (column 2) and sector-year (column 3) fixed effects. Finally, as a further robustness check, in Table A6 of Appendix E.B we report estimates for an alternative specification that regresses firm markups on firm market shares in levels, allowing for firm fixed effects. The estimates are again similar to those in first-differences in columns (1) and (2) of Table III.

The data is therefore consistent with changes in a firm's market share acting as a proximate driver of its markup dynamics, as predicted by theory. Notice however that, ultimately, in the model a firm's market share and markup are jointly determined in equilibrium by exogenous firm-level technology (and/or quality) shifters. All else constant, a decrease in firm's marginal cost relative to that of its competitors will increase its competitiveness in the product market and, hence, its market share (and therefore its markup, as above). We now turn to assessing this relation in the data.

To do so, recall from Section III.B that, for the estimation sample, we can obtain both firm-level price data, P_{kit}, and markup estimates, μ_{kit}. Given this, for firms in this smaller estimation sample we can exploit the relation $P_{kit}/\mu_{kit} = mc_{kit}$ to back out an empirical proxy of firm-level marginal costs. We can thus inspect the model-implied predictions regarding marginal costs, market shares and markups, albeit in a significantly smaller sample. Table IV reports empirical estimates of simple OLS regressions of firm-level market share and markup growth rates on the growth rates of our firm-level marginal cost proxy.

Table IV: MARKUPS, MARKET SHARES AND MARGINAL COSTS

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable:		$\Delta \log s_{kit}$			$\Delta \log \mu_{kit}$	
$\Delta \log mc_{it}$	-0.022	-0.022	-0.023	-0.091	-0.091	-0.096
	(0.003)	(0.003)	(0.003)	(0.008)	(0.009)	(0.009)
Year FE	N	Y	N	N	Y	N
Sector × Year FE	N	N	Y	N	N	Y
Observations	178,368	178,368	178,368	178,368	178,368	178,368

NOTE: $\Delta \log \mu_{kit}$ is the first-difference of (log) gross markup of firm i sector k at time t, $\Delta \log s_{kit}$ is the first-difference of (log) market share, and $\Delta \log mc_{it}$ is the first-difference of (log) marginal cost when the latter is defined as the difference between (log) price and (log) markup of firm i in sector k at time t. Columns (1)-(6) report empirical estimates for the estimation sample FARE (2009-2019) data. Columns (1) and (4) report pooled estimates while columns (2), (3), (5) and (6) report estimates that further control for year or sector×year fixed effects. Sector-year fixed effects are defined at the 5-digit NAF sector classification level. Standard errors (in parentheses) are two-way clustered at the firm and year level. $\Delta \log \mu_{kit}$ and $\Delta \log mc_{it}$ are winsorized at the 3% level.

Starting with the simple bivariate relation between marginal cost growth and market share growth, our estimates in column (1) — where we pool data across all sectors and years — imply that a one percent year-on-year increase in firm-level marginal costs is associated with a small but significant 0.02 percent decrease of a firm's market share growth. This estimate is robust to additionally controlling for average economy-wide marginal cost dynamics (i.e. the year fixed effects specification in column 2) or the average marginal cost growth across competitors in a given firm's sector (i.e. the sector-year fixed effects specification in column 3). The second panel of Table IV completes the argument by inspecting the relation between firm-level marginal cost growth and firm-level markups. Again, we observe that year-on-year increases in marginal costs result, as our model with incomplete pass-through predicts, in lower markups both unconditionally (in column 4) and when conditioning on year or sector-year fixed effects in columns (5) and (6), respectively. Finally, in Tables A7 and A8 of Appendix E.B we verify that these empirical estimates are robust to considering an alternative specification in levels (rather than growth rates) with firm fixed-effects.

The negative correlation between markups and marginal cost reported in Table IV may be spurious if markups are subject to measurement error, since marginal costs are measured as the ratio of prices to markups.[39] To address this concern, in Appendix E.C we construct a proxy for marginal cost based on energy prices and a firm's reliance on energy as an intermediate input. Following **?**, the rationale for this proxy is that energy price changes are exogenous to the firm but are nevertheless relevant for the evolution of a firm's marginal cost, depending on a firm's energy intensity. Using this proxy as an instrument for marginal cost, Table A10 confirms that higher marginal cost is associated both with lower markup and lower market share, with larger point estimates than in our OLS specification.

Taken together, we conclude that our data is consistent with the basic qualitative firm-level predictions of our model.

IV.B Sector-level evidence

As discussed in Section I.C, equilibrium aggregation of firm-level outcomes yields additional predictions at the sector-level. First, note that, by the same logic as above, taking the inverse of equation (8) and then first-differences yields the following relation between inverse sectoral markup and a sector's HHI:

$$\Delta\mu_{kt}^{-1} = -\frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon}\Delta HHI_{kt}, \tag{37}$$

where $\Delta\mu_{kt}^{-1}$ is the first-difference of the inverse (gross) markup of sector k at time t and ΔHHI_{kt} is the first-difference of its HHI. This yields a sector-level counterpart to equation (35) where

[39] Unlike this specification, other regressions in the paper include markups in the left hand side — rather than as an independent variable — so measurement-error bias is less of a concern.

now inverse sectoral markups comove linearly with sectoral concentration. To assess this relationship in the data, we consider the following empirical specification:

$$\Delta\mu_{kt}^{-1} = \alpha_t + \beta\Delta HHI_{kt} + \epsilon_{kt}, \tag{38}$$

where β is the coefficient of interest. We allow for year fixed effects α_t and additionally consider robustness to the inclusion of broad 2-digit sector-year fixed effects. Finally, in Appendix E.B, we report estimates based on a levels specification and sector-level fixed effects.

To construct sectoral markups in the data for our 275 narrowly defined 5-digit sectors, we aggregate firm-level markups to their sector-level counterparts by taking an harmonic market-share weighted average of firm-level markups — as indicated by the model equation (7). For each 5-digit sector, we construct the HHI by summing-up the square of firm-level market shares. We then take first-differences across time periods of both the sector-level markup and HHI series. This results in a balanced panel of 275 sectors at the 5-digit level across 25 years for a total of 6,875 observations.

Table V: SECTOR INVERSE MARKUP AND SECTOR HHI

	(1)	(2)	(3)
Dependent Variable:		$\Delta\mu_{kt}^{-1}$	
ΔHHI_{kt}	-0.374	-0.370	-0.378
	(0.177)	(0.178)	(0.180)
Year FE	N	Y	N
Sector(2D)×Year FE	N	N	Y
Number of Sectors	275	275	275
Observations	6,875	6,875	6,875

NOTE: $\Delta\mu_{kt}^{-1}$ is the first difference of sector k (inverse) markup in year t, ΔHHI_{kt} is the first difference of HHI in sector k. Columns (1)-(3) report empirical estimates for the FICUS-FARE (1995-2019) data, aggregated to the 5-digit NAF sector level. Column (1) reports pooled estimates while columns (2) and (3) report estimates that further control for year or broad 2-digit-sector×year fixed effects, respectively. Standard errors (in parentheses) are two-way clustered at the sector and year level. Underlying firm-level inverse markups are winsorized at 3%.

Column (1) of Table V displays estimates of a pooled regression across all sectors and years. Our estimates indicate a negative and significant relation between the change in concentration and the change in the inverse of sector markups. The estimates remain stable when additionally considering year or broader 2-digit sector-year fixed effects in columns (2) and (3). The point estimate is about -0.37 which, recall, is the target in our calibration. Table A9 in Appendix E.B reports alternative specifications in levels rather than growth rates.

Note that our model additionally imposes cross-equation restrictions. Comparing equations (35) and (37), the slope coefficients of these two relations — that is, the slope of the inverse of firm markup on market share and the slope of the inverse sector markup on sector HHI —

should coincide. Comparing point estimates across Tables III and V suggests that the implied slopes are indeed similar: focusing on the more demanding sector-year fixed effects specifications, we obtain slopes of -0.293 and -0.378, respectively, with both estimates falling within (less than) a one standard-error of each other.

Finally, recall that in our model, changes in sectoral markups reflect two forces. First, for given firm market shares, the evolution of endogenous firm-level markups may lead to changes in aggregated, sector-level markups. Second, for given heterogeneous firm-level markups, equilibrium reallocation of market shares also impact sector markup dynamics. Specifically, note that following equation (7), the change in sectoral markups between two time periods can be written as

$$\Delta\mu_{kt}^{-1} = \sum_{i=1}^{N_k} \Delta\mu_{kit}^{-1}\, \bar{s}_{kit} + \sum_{i=1}^{N_k} \Delta s_{kit}\, \overline{\mu}_{kit}^{-1},$$

where Δ denotes the year-on-year difference and bars denote averages over two consecutive years. This yields a standard within-between decomposition of sectoral markup changes. The first term on the right-hand-side gives the within term, measuring the change in (inverse) sectoral markup due to changes in firm-level markups, evaluated at the average market share of each firm. The second term on the right-hand-side is the between or reallocation term: it measures the change in the (inverse) sectoral markup due to the changes in firm market share, evaluated at a firm's average (inverse) markup. As discussed in Section II.B and shown in Appendix A.A, in the model under Cournot competition the within and between terms are equal to each other in every sector. From this result, it follows that, under Cournot, the contribution of the within and between/reallocation terms are each predicted to be equal to 50%.

Given time series of firm-level markups and market shares in the data, the within-between decomposition above can be readily computed. To do this, for each sector we regress the within term over time on changes in sector-level markup. The coefficient of this regression is the contribution of the within term to the evolution of sector-level markups.[40] We find that for the median sector in the French data the within term accounts for 59% of the changes in sector markups, close to the model prediction 50%. While there is heterogeneity in the data, for half of the sectors in France the contribution of the within term lies between 34% and 81%. Overall, taking firm and sector evidence together, the data is consistent with key predictions of the model.

[40] In the data, the reallocation term (defined as the difference between the change in (inverse) sectoral markup and the within term) captures not only changes in market share across continuing firms but also churn as some firms enter and exit the market each year.

V Model Meets Data: Reduced-Form Varieties of Markup Cyclicality

Our theoretical framework yields a simple relation between markups and size: the level of a firm's markup is determined by its market share within a sector. In turn, both markups and market shares are driven by firm-level marginal cost shifters. The aggregation of within-firm endogenous markup changes and reallocation of market shares across firms determines sectoral markups and sectoral concentration, yielding a linear relation between a sector's (inverse) markup and its concentration. As we have seen, the data broadly supports these model-implied relations.

By contrast, a large applied literature investigates different definitions of "markup cyclicality." The evidence is mixed on whether markups are procyclical or countercyclical. In this section, we argue that these conflicting empirical results can be at least partly ascribed to the alternative reduced-form exercises and definitions of markup cyclicality being pursued. We show that our model with firm-level shocks only can go a long away in accounting for these seemingly conflicting reduced-form relations in the data.

V.A Firm-level markup cyclicality with sector output

We start by analyzing a firm-level notion of reduced-form markup cyclicality and ask how do firm markups covary with the respective sector-level output.

Before going to the data, recall that our setting is a granular one in which extensive within-sector heterogeneity in the firm-size distribution enables large firm dynamics to lead the sectoral business cycle. In particular, in our setting with idiosyncratic firm-level shocks — and if pass-through rates do not fall too strongly with market shares — sector-output fluctuations are necessarily led by shocks to very large firms. To make matters concrete, consider a reduction in marginal cost for a large market-share firm. Given the granular nature of the economy, the corresponding sector output will typically increase (see equation 26). In addition, the large shocked firm will increase its market share and markup. This implies, as indicated in Proposition 4, that markups of large firms should comove positively with sector output.

By the same token, the average (small) firm in a given sector loses market share to the very largest firms: if sector-level output expansions are led by large firms, the latter will increase their market share whereas the average firm loses competitiveness - as evaluated by its market share within the sector. Again, due to the markup - market share relation in our setting, this implies the average firm-markup is expected to comove negatively with sector output, as summarized by Proposition 4.

To evaluate this implication of the model, we implement the following reduced-form regression, both in the data and in our model-simulated data:

$$\log(\mu_{kit}) = \alpha_i + \gamma_t + \beta_1 \log Y_{kt} + \beta_2 \log Y_{k,t} \times s_{kit} + \epsilon_{it}, \tag{39}$$

where $\log(\mu_{kit})$ is firm i sector k (log) gross markup in year t, $\log Y_{kt}$ is sector k's (log) value-added in year t and s_{kit} is firm-level market share for firm i in sector k at year t.[41] Given the specification in log-levels, α_i is a firm fixed effect, which controls for time-invariant firm-level unobservables determining the average level of a firm's markup, while γ_t is a year fixed effect.[42] In this specification, β_1 captures the average correlation between firm markups and their respective sector output, and coefficient β_2, in the interaction term, captures heterogeneity in this relation.[43] For robustness, we additionally consider a specification in first differences of (log) firm markups, $\Delta\log(\mu_{kit})$, and (log) sectoral value added, $\Delta \log Y_{kt}$ and no firm-level fixed effects. Finally, in Appendix E.D we consider alternative measures of sector output.

Table VI: FIRM MARKUP AND SECTOR OUTPUT

	(1) Data	(2) Data	(3) Model	(4) Model
Dependent variable:	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$
Y_{kt}	-0.073		-0.001	
	(0.008)			
$\log Y_{kt} * s_{kit}$	0.574		0.265	
	(0.044)			
$\Delta \log Y_{kt}$		-0.024		-0.001
		(0.009)		
$\Delta \log Y_{kt} * s_{kit}$		0.280		0.247
		(0.040)		
Firm FE	Y	N	Y	N
Year FE	Y	N	Y	N
Number of Observations	9,039,476	8,051,767	-	-

NOTE: μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t and $\log Y_{kt}$ sector k's (log) value-added in year t. $\Delta\log(\mu_{kit})$ is the first-difference of (log) gross markup in year t for firm i sector k, s_{kit} gives the market share of firm i in sector k, year t and $\Delta \log Y_{kt}$ is the first-difference of sector k (log) value-added in year t. Columns (1) and (2) report empirical estimates for the FICUS-FARE (1995-2019) data. Standard errors are two-way clustered at the sector×year level. Columns (3) and (4) report estimates based on model-simulated data. Log and first-difference of log markup are winsorized at the 3% level.

Before proceeding, note that **?** estimates a similar version of this regression with aggregate output Y_t in place of sectoral output Y_{kt}, using data for four large European countries. For these

[41] To obtain sector value-added, we sum firm-level nominal value-added to the NAF 5-digit level and deflate using EU-KLEMS sectoral price deflators.

[42] We drop the year 1994 to have the same yearly coverage as in Section IV. For this reason, the number of observations is lower than what is reported in Table A1. Including the year 1994 gives very similar results.

[43] According to our model, given the parameters ε and σ, the market share suffices to determine the markup (equation 5). For this reason, we estimate equation (39) without further controls.

data, **?** obtains a negative β_1 and a positive β_2 estimate, concluding that (i) in the data "markups are countercyclical" and (ii) that there is "substantial heterogeneity in markup cyclicality across firms, with small firms having significantly more countercyclical markups than large firms."[44]

Columns (1) and (2) of Table VI summarize the estimates of the reduced-form regression (39) on our French firm-level data, in both levels (with firm fixed effects) and first differences of logged variables. In both cases, point estimates β_1 are negative and significant : the markup of the average firm is "countercyclical" with respect to own-sector output. Although the point estimates vary across empirical specifications, the average firm's markup behavior qualitatively aligns with our model's predictions. Further, there is substantial heterogeneity in markup cyclicality across firms. In particular, the estimates for the interaction term — in both specifications — imply that markups of firms with market shares roughly above 10% (in the top 0.1% of the market share distribution) are procyclical with respect to sectoral output. However, the vast majority of firms have a countercyclical markup with sectoral output since the average firm has a very low market share (roughly 0.07%).[45]

Columns (3) and (4) of Table VI implement these reduced-form regressions on model-simulated data for 399,520 firms distributed across 275 sectors at the 5-digit level.[46] The model is able to reproduce the qualitative patterns observed in the data. In line with Proposition 4, markups are countercyclical with respect to own-sector output for the average firm and procyclical for large firms. Furthermore, point estimates for the implied large-firm procyclicality are of the same order of magnitude in the model and the data, and particularly close for the first-differences specification. In Appendix E.D, we show that these results are robust to alternative definitions of sectoral output dynamics, by considering log-deviations of sector real value added from its trend (defined by either a Hodrick-Prescott or **? ?** filters).

As discussed above, underlying the prediction of the model for heterogeneous cyclicality of firm-level markups is the fact that, in our granular environment, large firms' market shares are correlated positively with sector output whereas small firms' market shares are countercyclical. To assess this mechanism in the data, we estimate the following regression:

$$\log(s_{kit}) = \alpha_i + \gamma_t + \beta \log Y_{kt} + \epsilon_{kit},$$

where $\log(s_{kit})$ is the (log of) market share of firm i in sector k, $\log Y_{k,t}$ is the (log of) sector k real value-added, α_i is a firm-level fixed effect, and γ_t is a year fixed effect. We additionally consider a first-difference specification (without firm fixed effects). In either specification, β measures

[44] When we consider regression (39) using aggregate output, Y_t, rather than sectoral output, Y_{kt}, we find the same qualitative results as in **?**.

[45] We also consider an alternative specification where sector value-added is interacted with an indicator for market-shares over 50%. We find a coefficient of 0.240 (0.044) on the interaction term for the log-level specification and of 0.026 (0.013) for the first-difference specification.

[46] The number of firms is smaller in the simulation than in the data as our model abstracts from entry and exit and targets the yearly average number of firms in the economy over our period.

the relation between market share and sector value-added. To assess the predicted heterogeneous behavior of firm-level market shares with respect to sector output, we implement this regression (i) on the whole sample of firms, (ii) on the subsample of firms whose average market share is lower than 50%, and (iii) on the subsample of firms whose average market share is above 50%.

Columns (1) and (4) in Table VII report estimates of β on the French data and on model-simulated data. Both in the data and in the model — and for both level and first-difference specifications — the average firm's market share is countercyclical. Columns (2) and (5) report estimates for the subsample of firms whose market share is lower than 50%. Estimates of β are negative both in the data and in the model. Columns (3) and (6) report estimates for the subsample of firms whose market share is above 50%: consistently with our argument, estimates of β are now positive, both in the data and in the model, although the magnitude is smaller in the data.[47] In Appendix E.D we confirm that these findings are robust to alternative detrending techniques for sectoral output dynamics. Taken together, these results provide support for a key implication of our granular model with firm-level shocks. The average firm's market share and markup are countercyclical with respect to its own sector value-added, whereas large firms' market share and markups are procyclical.

V.B Sector-level markup cyclicality with sector output

We now explore sector-level notions of markup cyclicality. We first ask how sector markups covary with own-sector output. Recall that in our granular setting, sectoral business cycles are driven by large firm dynamics, and that shocks to large firms induce changes in both sector-level output and markups. According to Proposition 3, we should expect a positive correlation between sector markup and sector output.

To assess this relationship in the model and in the data, we follow **?** and consider the following sector-level specification:

$$\Delta \log \mu_{kt} = \alpha_k + \gamma_t + \beta \Delta \log Y_{kt} + \epsilon_{kt}, \tag{40}$$

where $\Delta \log \mu_{kt}$ denotes the first-difference of sector k's (log) markup, and $\Delta \log Y_{kt}$ denotes the first difference of sector k's (log) real value-added. Sector-level markups are aggregated from firm-level estimates according to a harmonic weighted average, as indicated by the model equation (7).[48] We measure sector value-added in the data as in the previous section. We follow

[47] In our model, changes in sectoral output are driven by independent firm-level shocks. Adding sector-level productivity or demand shifters that affect all firms symmetrically would reduce the estimate of β, bringing it more in line with our empirical findings.

[48] **?** measure sectoral markup using various measures of the inverse of the labor share at the sectoral level. We construct sector-level markup from the aggregation of firm-level markup based on equation (34) with material as a variable input.

Table VII: Firm Market Share and Sector Output

	(1) Data (all data)	(2) Data ($\bar{s}_{ki} < 0.50$)	(3) Data ($\bar{s}_{ki} > 0.50$)	(4) Model (all data)	(5) Model ($\bar{s}_{ki} < 0.50$)	(6) Model ($\bar{s}_{ki} > 0.50$)
Dependent variable:	$\log s_{kit}$					
$\log Y_{kt}$	-0.594	-0.595	0.144	-2.613	-2.621	0.535
	(0.009)	(0.009)	(0.060)			
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Number of Obs.	9,039,476	9,039,036	440	-	-	-
Dependent variable:	$\Delta \log s_{kit}$					
$\Delta \log Y_{kt}$	-0.488	-0.488	0.091	-2.585	-2.591	0.274
	(0.018)	(0.018)	(0.037)			
Firm FE	N	N	N	N	N	N
Year FE	N	N	N	N	N	N
Number of Obs.	8,251,767	8,251,340	427	-	-	-

NOTE: s_{kit} gives the market share of firm i in sector k, year t, and $\log Y_{kt}$ is the deviation of sector k (log) value-added in year t from its mean. $\Delta \log s_{kit}$ gives the first-difference of (log) market share of firm i in sector k, year t, and $\Delta \log Y_{kt}$ is the first-difference of sector k (log) value-added in year t. $\bar{s}_{ki}$ is the average market share of firm i in market k. Column (1-3) reports empirical estimates for the FICUS-FARE (1995-2019) data. Sectors are defined at the 5-digit NAF sector classification level. First-difference in log market shares are winsorized at the 3% level. Standard errors in the data are two-way clustered at the sector×year level. Column (4-6) reports estimates based on model-simulated data.

? and include sector and year fixed effects to control for possibly heterogeneous trends in sector level growth rates. For robustness, we also consider an alternative specification where we use sectoral variables in log deviations from trend (rather than first differences) where the trend is estimated following **?**. Finally, in Appendix E.E we consider additional fixed-effect specifications for variables in levels and alternative detrending procedures.

? estimate a positive and significant β in US sectoral data using a similar specification, concluding that "markups are generally procyclical (...) hitting troughs during recessions and reaching peaks in the middle of expansions."

Table VIII reports estimates of β in our French data using both first-difference (Column 1) and trend-deviation specifications (Column 2). Sector markups comove positively and significantly with sector output, which is consistent with the findings in **?** despite differences regarding the country of analysis, sample period, and the methods deployed to estimate markups. Appendix E.E further confirms the robustness of this correlation to alternative empirical specifications and detrending procedures.

Columns (3) and (4) in Table VIII report estimates of β in model-simulated data, applying the same empirical specifications as in the French data. We report the median and standard deviation of β estimates over 5,000 samples of 25 years each. The model implies a positive correlation

Table VIII: SECTOR MARKUP AND SECTOR OUTPUT

	(1) Data	(2) Data	(3) Model	(4) Model
Dependent variable:	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$
$\Delta \log Y_{kt}$	0.160		0.110	
	(0.040)		(0.040)	
$\log \widehat{Y}_{kt}$		0.139		0.117
		(0.057)		(0.035)
Sector FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Sectors	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log) change (columns 1 and 3), and **?** trend deviation of markup (columns 2 and 4), $\Delta \log \mu_{kt}$ and $\log \widehat{\mu}_{kt}$ respectively on sector value-added $\Delta \log Y_{kt}$ and $\log \widehat{Y}_{kt}$ respectively. Column (1-2) reports empirical estimates for the FICUS-FARE (1995-2019) data, and standard errors (in parentheses) are clustered at the sector level. Sectors are defined at the 5-digit NAF sector classification level. To construct sector-level markup the underlying firm-level inverse markups are winsorized at 3%. Columns (3-4) report estimates based on model-simulated data. The point estimates for these columns give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

between sector markup and sector output, yielding point estimates that are only slightly lower than their empirical counterpart.

To further understand the previous result, recall that sector-level markups in our model are linked to sector-level concentration as measured by the HHI, a relationship we explored empirically in Section IV.B. Therefore, underlying the cyclicality of sector-level markup is the cyclicality of the sectoral HHI. In our granular environment, in a typical sectoral expansion a few large firms expand by increasing their market share while smaller firms lose market share, resulting in higher concentration. To assess this mechanism in the data, we estimate a similar specification to equation (40), with sectoral HHI rather than sectoral markup on the left hand side:

$$\Delta \log HHI_{kt} = \alpha_k + \gamma_t + \beta \Delta \log Y_{kt} + \epsilon_{kt}.$$

Here, $\Delta \log HHI_{kt}$ is the first-difference of sector k's (log) HHI and $\Delta \log Y_{kt}$ denotes the first difference of sector k's (log) output. We include sector and year fixed effect as in the markup cyclicality specification, equation (40). As above, we consider an alternative specification where we use log deviations from trend computed as in **?** and present further robustness exercises in Appendix E.E.

Columns (1) and (2) in Table IX report estimates of β in our French data for both first-difference and trend-deviation specifications. Sector concentration comoves positively and significantly with sector output. We next apply the same empirical specification to model simulated data,

Table IX: SECTOR CONCENTRATION AND SECTOR OUTPUT

	(1) Data	(2) Data	(3) Model	(4) Model
Dependent variable:	$\Delta \log HHI_{kt}$	$\log \widehat{HHI}_{kt}$	$\Delta \log HHI_{kt}$	$\log \widehat{HHI}_{kt}$
$\Delta \log Y_{kt}$	0.332		0.533	
	(0.067)		(0.235)	
$\log \widehat{Y}_{kt}$		0.281		0.726
		(0.049)		(0.288)
Sector FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Sectors	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log) change (columns 1 and 3), and **?** trend deviation of HHI (columns 2 and 4) $\Delta \log HHI_{kt}$ and $\log \widehat{HHI}_{kt}$ respectively on sector value-added $\Delta \log Y_{kt}$ and $\log \widehat{Y}_{kt}$ respectively. Column (1-2) reports empirical estimates for the FICUS-FARE (1995-2019) data, and standard errors (in parentheses) are clustered at the sector level. Sectors are defined at the 5-digit NAF sector classification level. Columns (3-4) report estimates based on model-simulated data. The point estimates for these columns give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

and we calculate the median and standard deviation of β estimates over 5,000 samples of 25 years each. Columns (3) and (4) in Table IX show positive and significant estimates of β that are comparable in magnitude to those in the data. Table A14 in Appendix E.E confirms the robustness of this correlation to alternative empirical specifications and detrending procedures.

V.C Sector-level markup cyclicality with aggregate output

The work by **?** explores yet another reduced-form notion of markup cyclicality. Unlike the previous specification which relates changes in sectoral markups and changes in sectoral output, **?** measure cyclical comovement between sectoral markup and aggregate real GDP. More recently, **?** explore a similar notion of cyclicality when assessing markup dynamics in retail sectors.

To understand this form of comovement in the context of our model, note that sector markups only react to within-sector firm shocks. Over long samples, under the conditions of Proposition 5, the model implies (i) positive comovement of a sector's markup with aggregate GDP and (ii) that this comovement is nevertheless weaker than that between a sector's markup and its output (see footnote 25). Over a given cyclical episode — or more generally, in small samples — the model prediction is ambiguous. A positive comovement is expected if the fluctuation in aggregate economic activity is driven by large firms in the same sector. However, aggregate output movements also reflect shocks hitting other sectors in the economy. If a sector comoves negatively with aggregate output, a negative correlation of that sector's markup with aggregate output will obtain. Overall, we should expect a weaker relationship between the average sector's markup and aggregate GDP fluctuations than between sectoral markups and sectoral output.

To explore this notion of cyclicality, we implement the following regression:

$$\Delta \log \mu_{kt} = \alpha_k + \beta \Delta \log Y_t + \epsilon_{kt}, \tag{41}$$

where $\Delta \log \mu_{kt}$ is the first-difference of sector k's log markup in year t, $\Delta \log Y_t$ is the first difference of (log) aggregate value-added in year t, and α_k is a sector fixed effect that controls for sector-specific trends in markups. Sector-level markups are computed as above by taking a weighted harmonic mean of firm-level markups and aggregate value-added is computed by summing firm-level value-added deflated by the respective EU-KLEMS sector-level deflator. For robustness, we again consider specifications in log deviations from a **?** trend rather than growth rates. In Appendix E.E we consider an alternative HP-filter detrending procedure.

? consider a version of this regression based on US data.[49] They conclude that "the price markup is estimated to be highly countercyclical" with the possible exception of service industries, for which they find evidence favoring procyclicality. In the same vein, **?** study the cyclicality of markup growth of one particular sector — US retail — with respect to GDP growth. They report a positive but statistically insignificant relation, leading them to conclude that markups in retail are "roughly acyclical or mildly procyclical."

Columns (1) and (2) of Table X summarize our estimates based on French data. While the point estimates are negative, the standard errors are large. The coefficient in column (1) is marginally significant (t-stat of 2.06) while that in column (2) is insignificant (t-stat of 1.53). As shown in Table A15 of Appendix E.E and in Table A3 of Appendix C, both the sign and significance of this reduced-form relation are sensitive to the specific detrending choice, sample definition, and the measure of markup used.

We now explore the relation between sector-specific markups and aggregate output implied by our model. Table X present median estimates (along with their respective standard errors) of β over 5,000 samples of 25 year length. For the baseline calibration, reported in columns (3) and (4), our model implies a positive point estimate. However, the model simulations point to considerable variation in this relation across small samples, with a point estimate that is not statistically different from zero.

As we will discuss in Section VI, our model with only idiosyncratic productivity shocks understates aggregate output volatility. In order to match the observed volatility of aggregate output, we consider an extension with aggregate productivity shocks. Firm-level productivity is given

[49] **?** measure markup using intermediate input share computed from sector level data from KLEMS. Here, we construct sector-level markup by the aggregation of firm-level markup based on equation (7) and we use material as a variable input to estimate firm-level markup using equation (34).

by $\widetilde{Z}_t \times Z_{ikt}$, where $\widetilde{Z}_t$ is normally distributed and independent across periods with volatility set to match the volatility of aggregate output in the data.[50]

For this alternative parameterization, reported in columns (5) and (6), our model implies a point estimate that is roughly equal to zero. Of the 5,000 samples, about 20 to 30% (depending on how we filter the model-generated data) display sectoral markups that are countercyclical with respect to aggregate output. Intuitively, aggregate productivity shocks enhance aggregate output volatility but do not affect the relative firm-level productivity and therefore do not affect the market-share and markup distributions. It follows that aggregate shocks do not affect sector-level markups. Point estimates with aggregate shocks are therefore smaller than without aggregate productivity shocks and closer to its empirical counterpart. We conclude that the model can generate a weaker comovement between sectoral markups and aggregate output in comparison to the more robust positive relation between sectoral markups and sectoral output.

V.D Robustness of empirical results to alternative samples and markup measures

As explained in Section III.A, our measure of markups is constructed by combining (i) estimates of the production function at the 2-digit level on a restricted sample of firms with revenue and quantity data, referred to as the estimation sample, and (ii) expenditure shares on materials from a larger sample of firms, referred to as the baseline sample. This procedure allows us to construct firm-level markups for about 9.4 million firm-year observations from 1994 to 2019. In this section, we discuss the sensitivity of our empirical results to using the restricted estimation sample or alternative measures of markups. We provide additional information and robustness exercises in Appendix C.

We begin by re-calculating our results in the smaller estimation sample, which contains 220,733 observations over the shorter period 2009-2019. The results are collected in column (7) of Table A3 in Appendix C.F and can be compared with our baseline estimates in column (1). The qualitative conclusions remain similar, though with reduced statistical significance. This reflects both a smaller sample size—fewer time periods and firms—and an underrepresentation of small firms in the population. By extending the sample of firms beyond those with quantity data, our baseline sample addresses these two issues.

We next calculate markups using output elasticities estimated using revenue rather than quantity data. Revenue-based markups are more widely used in the literature than quantity-based markups because revenue data is more readily availability, but are prone to the biases discussed in e.g. **?** and **?**. **?** show sizable differences in markup levels estimated using quantity or revenues data for a large US and Canadian retailer. We obtain in our data similar differences

[50] We set the standard deviation of $\log \widetilde{Z}_t$ to 2.20% (resp. 2.04%) for the specification in first-differences (resp. in deviation from trend)

Table X: SECTOR MARKUP AND AGGREGATE OUTPUT

	(1)	(2)	(3)	(4)	(5)	(6)
	Data		Model without Aggr. Shocks		Model with Aggr. Shocks	
Dependent variable:	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$
$\Delta \log Y_t$	-0.239		0.165		0.008	
	(0.116)		(0.101)		(0.042)	
$\log \widehat{Y}_t$		-0.145		0.169		0.017
		(0.095)		(0.119)		(0.044)
Share negative coefficients	-	-	0.02	0.02	0.29	0.21
Sector FE	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector k's markup in year t in first-differences ($\Delta \log \mu_{kt}$, in columns 1, 3 and 5) and **?** trend deviation ($\log \widehat{\mu}_{kt}$, in columns 2, 4 and 6) on (log) aggregate real value-added in year t in either first-differences or **?** trend deviation ($\Delta \log Y_t$ and $\log \widehat{Y}_t$, resp.). Columns (1) and (2) report empirical estimates for the FICUS-FARE (1995-2019) data. Regressions are weighted by average sectoral value-added. To construct sector-level markup the underlying firm-level inverse markups are winsorized at 3%. Standard errors (in parentheses) are clustered at the sector level. Columns (3) and (4) report estimates based on model-simulated data. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples. Columns (5) and (6) report estimates based on model-simulated data with aggregate TFP shocks. Point estimates for this column are computed as for columns (3) and (4). The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of aggregate output (either measured as a deviation from trend or a log first-difference) in France. The line "Share negative coefficients" gives the share of simulation with negative estimated coefficients in regression on the model-based simulations.

in the average level of quantity and revenue-based markups to those reported in **?**, as shown in Table A2 of Appendix B.C. Nevertheless, when we re-estimate our central empirical specifications using revenue-based markup (see column 5 of Table A3) the results are qualitatively similar to our baseline, with only a few exceptions: the relation between sector markups and concentration retains the baseline sign but loses significance, while the relation between sectoral markups and aggregate output gains significance albeit with point estimates that are now closer to zero. The fact that many of the patterns on markup cyclicality do not change much when using revenue- or quantity-based markups is consistent with results in **?**. While revenue-based markups *levels* are biased, **?** provide conditions and quantitative examples under which they are positively correlated (across firm or/and over time) with quantity-based markups.

In the same vein, in Appendix C.A we run our baseline specifications using accounting markups computed with the Lerner-index, without involving any production function estimation. While immediate to compute, these accounting-based measures only recover markups if firms operate a constant-return-to-scale technology and costs are correctly accounted for. The results are displayed in column (2) of Table A3. Results are qualitatively similar to our baseline.

Table A3 displays also shows that our empirical results are largely robust to individual perturbations of our benchmark procedure, such as the treatment of outliers, multi-product firms, and time coverage.

VI Fluctuations in Aggregate Markup and Output

In this final section, we turn our attention to the volatility and cyclicality of aggregate markup and output fluctuations in the data and in the quantitative model. We first consider our model with only idiosyncratic firm-level shocks that follow the Markov process introduced above. We then introduce aggregate productivity shocks to fully account for aggregate output volatility in our data.

Using our FICUS-FARE data, we construct aggregate markups, μ_t, as a weighted harmonic mean of firm-level markups, and aggregate GDP, Y_t, as described in Section V.C. We detrend these variables using the **?** filter. Using our calibrated model, we simulate 5,000 samples of 25-year firm-level histories. We implement the same procedure to construct detrended time-series of model simulated aggregate output and markups. For robustness, we also consider a first-differences specification. Table XI presents data- and model-based estimates of the correlation and standard deviation of aggregate output and markups.

We first consider aggregate markup cyclicality. Recall from expression (33) that our model implies a positive comovement between aggregate output and aggregate markups unless larger sectors have lower markups or, for finite samples, if a particular expansionary episode is driven by a sector with a sufficiently low markup, in which case negative comovement may obtain. That is, whereas according to our model we should observe positive comovement over sufficiently long samples, in any given short sample comovement may be absent or negative depending on sectors driving the aggregate fluctuations.

In Table XI, we can see that both in the data and in the model the correlation between aggregate markup and aggregate output is positive. Our model predicts, however, much higher correlation than that observed in the data: the correlation is, at most, 0.06 in the data and 0.91 (computed as the median correlation across shock realizations) in the model.[51]

Next, we examine the magnitude of aggregate fluctuations in output and markups implied by our model. The literature quantifying the granular origin of business cycles (see e.g. **? ?** and **? ?**) has largely abstracted from considerations of oligopolistic competition and incomplete pass-

[51] Our model predicts large variation in the correlation coefficient and in the relative volatility of aggregate markups and output across small samples, depending on which sectors are driving aggregate dynamics and the relative levels of their markups. To see this variation at play, Figure A1 in Appendix E plots the histogram of correlation coefficients $\rho(\Delta \log \mu_t, \Delta \log Y_t)$ and relative standard deviations $\sigma(\Delta \log \mu_t)/\sigma(\Delta \log Y_t)$ across our 5,000, 25-year samples.

Table XI: AGGREGATE MARKUP AND AGGREGATE OUTPUT

	(1) Data			(2) Model without aggr. shocks			(3) Model with aggr. shocks		
	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$
$\log \widehat{Y}_t$	3.16	1	1	0.83	1	1	3.16	1	1
$\log \widehat{\mu}_t$	1.27	0.40	0.03	0.30	0.36	0.91	0.30	0.09	0.29
$\Delta \log Y_t$	3.28	1	1	0.72	1	1	3.28	1	1
$\Delta \log \mu_t$	1.63	0.50	0.06	0.26	0.36	0.91	0.26	0.08	0.27

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for deviations from trend computed as in **?** of (log) aggregate output $\log \widehat{Y}_t$ and (log) aggregate markup $\log \widehat{\mu}_t$, and, for log first-difference of aggregate output $\Delta \log Y_t$ and aggregate markup $\Delta \log \mu_t$. Column (1) reports empirical estimates for the FICUS-FARE (1995-2019) data. Column (2) reports the median over 5,000 simulated samples, each of 25 years. Column (3) reports the average over 5,000 simulated samples of 25 years from a model with aggregate TFP shocks. The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of aggregate output (either measured as a deviation from trend or a log first-difference) in France.

through. Recall from our analytic results in Section II.C that incomplete pass-through weakens the response of aggregate output to firm-level shocks relative to a specification of the model with heterogeneous but constant markups.

Table XI shows that, for the detrended specification, the standard deviation of aggregate output is 3.16% in our French data and 0.83% in our model (median across samples). That is, the volatility of aggregate output in our purely granular model is 26% of the volatility in the data (we obtain very similar results if we calculate volatilities using first-differences). This is only slightly smaller than the comparable number in **?** who report, in their perfectly competitive granular environment, a volatility of model GDP relative to (US) data of 30%. How large are granular movements in aggregate markups? In the data, the relative volatility of aggregate markup to aggregate output ranges from 40% to 50%, depending on the detrending method. Our calibrated model generates a relative volatility of 36% (median across samples).

Although our model with firm-level shocks generates non-negligible fluctuations in aggregate output and markups, it only accounts for a fraction of aggregate fluctuations in the data. Moreover, as we discussed above, the correlation of markups and output is much higher than that in the data. In what follows we show that if we superimpose on our calibrated model aggregate productivity shocks — in order to match aggregate output volatility in the data — the procyclicality of markups is much lower and closer to the data.

As described in the previous section, we assume that firm-level productivity is given by $\widetilde{Z}_t \times Z_{ikt}$, where $\widetilde{Z}_t$ is normally distributed and independent across periods. We choose the standard deviation of $\widetilde{Z}_t$ to match the volatility of aggregate output in the data. Column (3) of Table XI shows the implied business-cycle moments for the median 25-year sample. As discussed in Section I, aggregate shocks do not affect firm market shares and markups, and hence the volatility of aggregate markups is unchanged relative to the model with only firm-level shocks. Because movements in output driven by aggregate productivity shocks are uncorrelated with markups, the correlation between aggregate markup and output falls to 0.27. In 16% of our 25-year samples, aggregate markups are countercyclical.[52]

Role of changes in firm-level markups for aggregate results The within/between decomposition in Section IV demonstrates the importance of changes in firm-level markups to account for changes in sectoral markups in the model and data, as discussed in Section II. However, this does not imply that movements in aggregate output and markups would be very different if firm-level markups were heterogeneous but fixed over time.

To examine this counterfactual, we use the first-order approximation expressions derived in Section II. In Appendix A.F, we provide expressions for correlations and volatilities under variable markups versus constant markups, given market shares and markups in the initial equilibrium. Because in our model the distribution of firms by sector changes every period, we consider 1,000 independent samples drawn from the equilibrium in our quantitative model.[53]

Consider first movements in aggregate output. We compare the standard deviation of aggregate output under variable markups with that under heterogeneous but constant markups, given the same initial firm-level expenditure shares and markups and assuming the same volatility of firm-level shocks.[54] For the median sample, the standard deviation of aggregate output under variable markups is 0.87 times that under heterogeneous but constant markups (the 95% confidence interval calculated across samples is 0.82-0.97). As explained in Section II, incomplete pass-through (given pass-through rates that are decreasing in size) reduces aggregate output

[52] We also considered second-moment shocks to firm-level productivity as in **?**. An increase in the dispersion of firm-level productivity shocks reallocates market shares toward large firms, increasing the aggregate markup, but also raise aggregate output (Oi-Hartman-Abel effect). That is, in our model second moment shocks increase the correlation between aggregate markup and output.

[53] The magnitudes of correlations and ratios of standard deviations based on the first-order approximations are remarkably close to those in our quantitative non-linear results. For the median sample, the standard deviation of aggregate markup relative to output is 0.42 (vs. 0.36 in our quantitative non-linear results) and the correlation between aggregate markup and output is 0.87 (vs. 0.91 in our quantitative non-linear results).

[54] Market shares of large firms are less volatile under variable markups than under constant markups. One could compare aggregate volatilities under these two specifications after adjusting the size of firm-level shocks to keep the same average volatility of market-shares (which, recall, is a target in our baseline calibration). If we match an unweighted average of these market-share volatilities, our results remain roughly unchanged. If we target a weighted average of these market-share volatilities, aggregate volatility is slightly higher under variable markups. In all cases, variable markups have a limited impact on reducing aggregate output volatility.

volatility in a similar way that a decline in firm concentration does.[55]

Consider next movements in aggregate markups. For the median sample, their standard deviation under variable markups is 1.08 times that under heterogeneous but constant markups (the 95% confidence interval is 1.00-1.18). The median volatility of aggregate markups relative to output is 0.42 under variable markups and 0.34 under heterogeneous but constant markups. The median correlation between markups and output is 0.87 under variable markups and 0.92 under constant markups (the 95% confidence interval for the difference in correlations is [0,0.07]).

Overall, the magnitude and cyclicality of aggregate markups in our model are not substantially different when we counterfactually fix markups at their initial, heterogeneous level. Of course, rather than exogenously fixing markups our model provides a unified theory of both markup (level) heterogeneity across firms and endogenous markup changes which is consistent with a number of observations about markup changes in the data.

VII Conclusion

In this paper, we examine markup cyclicality through the lens of a simple oligopolistic macroeconomic model with rich implications for the behavior of markups at the firm, sector, and aggregate levels. Working with administrative firm data for France, we show the model can reproduce qualitatively, and many times quantitatively, an array of markup-related empirical moments at various levels of disaggregation. Within our framework and measure of markups, we can reconcile seemingly conflicting variants of "markup-cyclicality" that have been considered in the literature. Finally, our granular oligopolistic setting produces non-negligible aggregate fluctuations, both in output and markups.

One obvious route for future work is to superimpose in our model alternative shocks and frictions. Along this line, prime candidates would be to consider price setting and customer-accumulation frictions (see, e.g., **?**, **?** or **?**), as well as aggregate monetary and financial shocks. Relatedly, we have focused on static, intra-temporal markup decisions in which movements in markups are the result of changes in the shape of the demand curve in response to firm-level shocks. These forces would remain relevant even if one were to extend the model to allow for richer inter-temporal dynamics that result in more complex dynamic markup strategies (see

[55]Whereas variable markups reduce the volatility of aggregate output, markup heterogeneity per se contributes to higher aggregate volatility. By equation (30), markup heterogeneity under constant markups does not affect output changes with linear disutility of labor ($f \to \infty$). However, with finite labor disutility, reallocation of economic activity across heterogeneous markup firms is an additional source of output fluctuations, as studied in **?**. In our model, the median standard deviation under heterogeneous and constant markups is 1.18 times that under homogeneous and constant markups (the 95% confidence interval is 1.13-1.22). Combining both results, the standard deviation under variable and heterogeneous markups is 1.02 times that under homogeneous and constant markups (the 95% confidence interval is 0.99-1.14). That is, output volatility under variable markups is only slightly higher than under constant and homogeneous markups.

e.g., **?**). Bringing the resulting firm, sector, and aggregate dynamics to data - and comparing them against the forces in our static benchmark - would then render possible an assessment of the empirical merits of this more general approach.

Finally, extensions to more realistic and richer product and market structures would allow us to more accurately map model objects to the increasingly detailed micro data available to researchers. Such extensions include multi-product firms, interlinked through intermediate-inputs, with some firms competing only locally in spatially segmented (product and factor) markets and others doing so nationwide and/or internationally.

Online Appendix (not for publication)

Bottom-up Markup Fluctuations

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A Additional Information on Theoretical Results

A.A Global between / within decomposition of changes in sectoral markups

The change in the inverse of the sectoral markup between two time periods is, by equation (7),

$$\frac{1}{\mu_{kt'}} - \frac{1}{\mu_{kt}} = \sum_{i=1}^{N_k} \left(\frac{s_{kit'}}{\mu_{kit'}} - \frac{s_{kit}}{\mu_{kit}} \right)$$

This change in sectoral markups can be decomposed into a within term (i.e., changes in firm-level markups evaluated at firms' expenditure share averaged over both time periods) and a between term (i.e., changes in expenditure shares evaluated at firm-level markups averaged over both time periods) as follows:

$$\text{(A1)} \qquad \frac{1}{\mu_{kt'}} - \frac{1}{\mu_{kt}} = \sum_{i=1}^{N_k} \frac{1}{2} \left[(s_{kit'} + s_{kit}) \left(\frac{1}{\mu_{kit'}} - \frac{1}{\mu_{kit}} \right) + \left(\frac{1}{\mu_{kit'}} + \frac{1}{\mu_{kit}} \right) (s_{kit'} - s_{kit}) \right]$$

Note that if markups are equal across firms (as is the case with $\sigma = \varepsilon$), then all terms in (A1) are equal to zero.

It is straightforward to show that, by equation (5) under Cournot competition, the within and the between terms in (A1) are equal to

$$\frac{1}{2} \sum_{i=1}^{N_k} (s_{kit'} - s_{kit}) (s_{kit'} + s_{kit}) \left(\frac{1}{\sigma} + \frac{1}{\varepsilon} \right).$$

Therefore, under Cournot competition, the contribution in changes in sectoral markups of the between and the within terms is 50% each, irrespective of the values of σ and ε (as long as $\sigma \neq \varepsilon$). If σ is close to ε, firm-level markups are less responsive to shocks (reducing the within term), but firm-level markups are also less heterogeneous across firms (reducing the between term).

With Bertrand competition, the within/between decomposition is not constant at 50-50.

A.B Changes in sectoral markups

Substituting equations (15), (16), (18), and

(A2) $$\widehat{s}_{kit} = \widehat{A}_{kit} + (1-\varepsilon)\left(\widehat{P}_{kit} - \widehat{P}_{kt}\right),$$

into equation (20) yields

(A3) $$\widehat{\mu}_{kt} = \mu_k \sum_{i=1}^{N_k} s_{ki}\alpha_{ki} \left[\left(\frac{\Gamma_{ki}-1}{\mu_{ki}}\right) - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'} \left(\frac{\Gamma_{ki'}-1}{\mu_{ki'}}\right)}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \right] \widehat{V}_{kit}.$$

Setting $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$, we obtain the expression for changes in sectoral markups under constant markups, (22).

Under Cournot competition, markup elasticities satisfy

(A4) $$\frac{\Gamma_{ki}-1}{\mu_{ki}} = \frac{\varepsilon-1}{\varepsilon} - \frac{2}{\mu_{ki}}.$$

The term $(\Gamma_{ki}-1)/\mu_{ki}$ is increasing in markup μ_{ki} and hence also in market share s_{ki}. Substituting equation (A4) into (A3), we obtain expression (21).

Under Bertrand competition, markup elasticities Γ_{ki} are given by

$$\Gamma_{ki} \equiv \frac{\partial \log \mu_{ki}}{\partial \log s_{ki}} = \left[\varepsilon\left(\frac{\mu_{ki}-1}{\mu_{ki}}\right) - 1\right](\mu_{ki}-1),$$

and $(\Gamma_{ki}-1)/\mu_{ki}$ by

(A5) $$\frac{\Gamma_{ki}-1}{\mu_{ki}} = \varepsilon\left(\frac{\mu_{ki}-1}{\mu_{ki}}\right)^2.$$

Both Γ_{ki} and $\frac{\Gamma_{ki}-1}{\mu_{ki}}$ are increasing in markups and market shares. Changes in sectoral markups under Bertrand competition are

$$\widehat{\mu}_{kt} = \mu_k \varepsilon \sum_{i=1}^{N_k} s_{ki}\alpha_{ki} \left[(\varepsilon - s_{ki}(\varepsilon-\sigma))^{-2} - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'} (\varepsilon - s_{ki'}(\varepsilon-\sigma))^{-2}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \right] \widehat{V}_{kit}.$$

As under Cournot, a positive shock to firm i results in an increase in sectoral markup if and only if firm i is sufficiently large in its sector.

To compare analytically changes in sectoral markups under constant markups (equation 15) and variable markups (equation 21), we restrict the extent of *ex-ante firm heterogeneity*. Specifically, we assume that sector k contains N_k^A type A firms and $N_k^B = N_k - N_k^A$ type B firms, and in the initial equilibrium, firms within each type have equal demand/productivity composite, V_{kit}. In the initial equilibrium, each firm of type $g = A, B$ has market share s_k^g, markup μ_k^g, and markup elasticity Γ_k^g. Firms of type A are indexed by $i = 1, ..., N_k^A$ and firms of type B are indexed by $N_k^A + 1, ..., N_k$. In this case, equation (A3) under Cournot competition can be written as

$$\widehat{\mu}_{kt} = \frac{2}{1 + (\varepsilon - 1)\widetilde{\Gamma}_k} \left[s_k^A \left(1 - \frac{\mu_k}{\mu_k^A}\right) \sum_{i=1}^{N_k^A} \widehat{V}_{kit} + s_k^B \left(1 - \frac{\mu_k}{\mu_k^B}\right) \sum_{i=N_k^A+1}^{N_k} \widehat{V}_{kit} \right], \tag{A6}$$

where

$$\widetilde{\Gamma}_k = N_k^B s_k^B \Gamma_k^A + N_k^A s_k^A \Gamma_k^B.$$

The term in square brackets in equation (A6) corresponds to the change in the sectoral markup under fixed markups as expressed above. Therefore, given the same firm-level shocks, sectoral markups change by more (and the variance is higher) under variable markups than under constant markups if and only if the term in front of the square brackets in equation (A6) is higher than 1, which is the case if $(\varepsilon - 1)\widetilde{\Gamma}_k < 1$. This condition is violated if σ is sufficiently low and/or ε sufficiently high.

A.C Proof of Proposition 2

Define $f(s)$ and $g(s)$ as probability density functions defined over market shares in sector k, $s = s_{k1}, ..., s_{kN_k}$, given by $f(s) = \frac{s\alpha(s)}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}$ and $g(s) = sf(s)a$ with $a = \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}{\sum_{i'=1}^{N_k} s_{ki'}^2\alpha_{ki'}} > 1$ and $\alpha(s)$ is defined in equation (17). Because the likelihood ratio $g(s)/f(s) = sa$ is increasing in s, $g(.)$ first-order stochastically dominates $f(.)$. If $s_{ki}\alpha_{ki}$ is increasing in s_{ki}, $f(s)$ is increasing in s. It then follows that $\sum_{i=1}^{N_k} [g(s_{ki}) - f(s_{ki})] f(s_{ki}) > 0$, which corresponds to inequality (25). Note that if $s_{ki}\alpha_{ki}$ is decreasing in s_{ki}, inequality (25) is reversed.□

Under what conditions is $s_{ki}\alpha_{ki}$ increasing in market shares, as required by Proposition 2? Under Cournot competition,

$$s_{ki}\alpha_{ki} = \frac{\left(1 - \frac{1}{\varepsilon}\right) s_{ki} - \left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right) s_{ki}^2}{1 - \frac{1}{\varepsilon} + (\varepsilon - 2)\left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right) s_{ki}},$$

which is increasing in s_{ki} if and only if

$$2\left(\frac{\varepsilon - 1}{\varepsilon}\right) s_{ki} + \left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right)(\varepsilon - 2)\, s_{ki}^2 < \frac{\sigma\,(\varepsilon - 1)^2}{\varepsilon\,(\varepsilon - \sigma)}. \tag{A7}$$

Because the left-hand side of this equation is increasing in s_{ik} (for $s_{ik} \leq 1$), this inequality holds for $s_{ki} \leq \tilde{s}_k$, where $\tilde{s}_k$ is a function of σ and ε. This implies that inequality (25) is satisfied if all market shares in sector k are less than or equal to $s_{ki} \leq \tilde{s}_k$. Note the condition that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} is sufficient but not necessary for inequality (25) to hold. In particular, inequality (25) may hold (so that sectoral markups and prices comove negatively) even if $s_{ki}\alpha_{ki}$ is increasing in some range of the distribution of market shares in a sector but decreasing at the upper tail of the distribution.

A.D Changes in sectoral and aggregate output

Substituting (28) and (29) into (30), changes in aggregate output can be expressed in terms of changes in sectoral markup and price as

$$\text{(A8)} \qquad \widehat{Y}_t = (1+f\eta)^{-1} \sum_k s_k \left[-\left(f+1+(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)\right) \widehat{P}_{kt} + \frac{s_k \mu}{\mu_k} \widehat{\mu}_{kt} \right]$$

In response to sector k shocks only, changes in aggregate output are

$$\text{(A9)} \qquad \widehat{Y}_t = (1+f\eta)^{-1} s_k \left[-\left(f+1+(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)\right) \widehat{P}_{kt} + \frac{s_k \mu}{\mu_k} \widehat{\mu}_{kt} \right]$$

and change in aggregate price by $\widehat{P}_t = s_k \widehat{P}_{kt}$.

Changes in sectoral output are given by

$$\text{(A10)} \qquad \widehat{Y}_{kt} = -\sigma \widehat{P}_{kt} + \sigma \widehat{P}_t + \widehat{Y}_t.$$

In response to sector k shocks only, substituting changes in aggregate output and price using the expressions above, changes in sectoral output are given by equation (26).

The covariance between changes in firm i markup and sector k output, expression (27), is obtained as follows. First, changes in firm-level market shares are, combining (A2), (16), and (18),

$$\text{(A11)} \qquad \widehat{s}_{kit} = \alpha_{ki} \left[\widehat{V}_{kit} - \frac{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'} \widehat{V}_{ki't}}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}} \right].$$

Changes in firm-level markups are, combining (A11) and (15),

$$\text{(A12)} \qquad \widehat{\mu}_{kit} = \Gamma_{ki} \alpha_{ki} \left[\widehat{V}_{kit} - \frac{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'} \widehat{V}_{ki't}}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}} \right].$$

Changes in sectoral output when $f \to \infty$ are, by equations (18) and (26),

$$\text{(A13)} \qquad \widehat{Y}_{kt} = -\left[\sigma(1-s_k) + \eta^{-1} s_k\right] \widehat{P}_{kt} = \frac{\left[\sigma(1-s_k) + \eta^{-1} s_k\right]}{\varepsilon - 1} \frac{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki} \widehat{V}_{kit}}{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki}}.$$

Calculating $\mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kit}\right]$ in the presence of shocks to all firms (only those in sector k are relevant), we obtain expression (27).

A.E Proof of Proposition 5

Write equation (32) as

$$\mathbb{C}ov\left[\widehat{Y}_t,\widehat{\mu}_{kt}\right]=-s_k\left[\frac{f+1+(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)}{f\eta+1}\right]\mathbb{C}ov\left[\widehat{P}_{kt},\widehat{\mu}_{kt}\right]+\frac{s_k\mu}{\mu_k}\frac{1}{f\eta+1}\mathbb{V}ar\left[\widehat{\mu}_{kt}\right].$$

Under the conditions of Proposition 3, $\mathbb{C}ov\left[\widehat{P}_{kt},\widehat{\mu}_{kt}\right]\leq 0$. Moreover, the term $s_k(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)$ is non-negative if $\mu_k\geq\mu$ and otherwise approaches zero if either (i) $s_k\to 0$, (ii) $f\to\infty$, or (iii) $\sigma\to 1$.

A.F Volatility and covariance of aggregate markups and output

In this section, we provide expressions for the variance of and covariance between aggregate markups and aggregate output. We do not impose $f\to\infty$, as we do in the main text. We use these expressions in Section VI.

The covariance between sector prices and markups, $\mathbb{C}ov\left[\widehat{\mu}_{kt},\widehat{P}_{kt}\right]$, is given by (24) under variable markups and (23) under constant markups.

The variance of sectoral prices is given by (19). Under constant markups, $\Gamma_{ki}=0$ and $\alpha_{ki}=1$. The variance of the aggregate price, using $\widehat{P}_t=\sum_k s_k\widehat{P}_{kt}$, is

$$\mathbb{V}ar\left[\widehat{P}_t\right]=\sum_k s_k^2\mathbb{V}ar\left[\widehat{P}_{kt}\right].\tag{A14}$$

The variance of sectoral markups, using (A3), is

$$\mathbb{V}ar\left[\widehat{\mu}_{kt}\right]=\mu_k^2\sum_{i=1}^{N_k}s_{ki}^2\alpha_{ki}^2\left[\left(\frac{\Gamma_{ki}-1}{\mu_{ki}}\right)-\frac{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}\left(\frac{\Gamma_{ki'}-1}{\mu_{ki'}}\right)}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\right]^2\sigma_v^2.\tag{A15}$$

Using (28), the variance of aggregate markups is

(A16)

$$\mathbb{V}ar\left[\widehat{\mu}_t\right]=\sum_k s_k^2\left[\left(\frac{\mu}{\mu_k}\right)^2\mathbb{V}ar\left[\widehat{\mu}_{kt}\right]+(1-\sigma)^2\left(1-\frac{\mu}{\mu_k}\right)^2\mathbb{V}ar\left[\widehat{P}_{kt}\right]+2\left(1-\sigma\right)\frac{\mu}{\mu_k}\left(1-\frac{\mu}{\mu_k}\right)\mathbb{C}ov\left[\widehat{\mu}_{kt},\widehat{P}_{kt}\right]\right],$$

and the covariance between aggregate price and markup is

$$\mathbb{C}ov\left[\widehat{P}_t,\widehat{\mu}_t\right]=\mu\sum_k\frac{s_k^2}{\mu_k}\mathbb{C}ov\left[\widehat{P}_{kt},\widehat{\mu}_{kt}\right]+(1-\sigma)\sum_k s_k^2\left(1-\frac{\mu}{\mu_k}\right)\mathbb{V}ar\left[\widehat{P}_{kt}\right].\tag{A17}$$

Using (30) and $\widehat{Z}_t = \widehat{\mu}_t - \widehat{P}_t$, the variance of aggregate output is

$$\text{(A18)} \qquad \mathbb{V}ar\left[\widehat{Y}_t\right] = \left(\frac{1}{1+\eta f}\right)^2 \mathbb{V}ar\left[\widehat{\mu}_t\right] + \left(\frac{1+f}{1+\eta f}\right)^2 \mathbb{V}ar\left[\widehat{P}_t\right] - \frac{2\left(1+f\right)}{\left(1+\eta f\right)^2}\mathbb{C}ov\left[\widehat{P}_t, \widehat{\mu}_t\right],$$

and the covariance between aggregate output and markup is

$$\text{(A19)} \qquad \mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_t\right] = \left(\frac{1}{1+\eta f}\right) \mathbb{V}ar\left[\widehat{\mu}_t\right] - \left(\frac{1+f}{1+\eta f}\right) \mathbb{C}ov\left[\widehat{P}_t, \widehat{\mu}_t\right].$$

A.G Decreasing returns to scale

The production function is now given by

$$\text{(A20)} \qquad Y_{kit} = Z_{kit} L_{kit}^{\beta}.$$

where $\beta \leq 1$. Marginal cost is

$$\text{(A21)} \qquad MC_{kit} = \beta^{-1} W_t \left(Y_{kit}\right)^{(1-\beta)/\beta} \left(Z_{kit}\right)^{-1/\beta},$$

or, using $P_{kit}Y_{kit} = s_{kit}P_{kt}Y_{kt}$,

$$\text{(A22)} \qquad MC_{kit} = \beta^{-1} W_t \mu_{kit}^{\beta-1} \left(P_{kt}Y_{kt}s_{kit}\right)^{(1-\beta)} \left(Z_{kit}\right)^{-1}.$$

The firm-level markup, μ_{kit}, is defined as the ratio of price to marginal cost, and is related to expenditure shares by equation (5), which does not depend on β.

Labor payments of firm i in sector k are

$$L_{kit}W_t = \beta\mu_{kit}^{-1}P_{kit}Y_{kit},$$

and profits (revenues minus labor payments) are

$$\Pi_{kit} = \left(1 - \beta\mu_{kit}^{-1}\right) P_{kit}Y_{kit}.$$

We define the sectoral markup as the ratio of sectoral revenues to labor payments,

$$\text{(A23)} \qquad \mu_{kt} \equiv \frac{P_{kt}Y_{kt}}{W_t L_{kt}},$$

which can be expressed as a function of firm-level markups and expenditure shares,

$$\text{(A24)} \qquad \mu_{kt}^{-1} = \beta \sum_{i=1}^{N_k} \mu_{kit}^{-1} s_{kit}.$$

The 50-50 between/within decomposition of changes in sectoral markups under Cournot competition derived in Appendix A.A holds irrespectively of the value of β.

The expenditure share of firm i in sector k, using $P_{kit} = \mu_{kit} MC_{kit}$, satisfies

$$\text{(A25)} \qquad s_{kit} = \frac{V_{kit}\left(\mu_{kit}^{\beta} s_{kit}^{1-\beta}\right)^{1-\varepsilon}}{\sum_{i'=1}^{N_k} V_{ki't}\left(\mu_{ki't}^{\beta} s_{ki't}^{1-\beta}\right)^{1-\varepsilon}}.$$

Equilibrium firm-level expenditure shares and markups are the solution to equations (5) and (A25).

Log-linearizing (A25) and using $\widehat{\mu}_{kit} = \Gamma_{ki}\widehat{s}_{kit}$, we obtain the analog to equation (14):

$$\text{(A26)} \qquad \widehat{s}_{kit} = \widehat{V}_{kit} + (1-\varepsilon)\,\Lambda_{ki}\widehat{s}_{kit} - \sum_{i'=1}^{N_k} s_{ki'}\left(\widehat{V}_{ki't} + (1-\varepsilon)\,\Lambda_{ki'}\widehat{s}_{ki't}\right),$$

where $\Lambda_{ki} = \beta\Gamma_{ki} + 1 - \beta$. When $\beta < 1$, $\Gamma_{ki} < \Lambda_{ki}$ if and only if $\Gamma_{ki} < 1$.

We can follow similar steps to obtain expressions for changes in sectoral markups and prices to firm-level shocks, as well as the implied variances and covariances.

A.H Markups when firms internalize impact on aggregates

In our baseline model we assume that when a firm chooses quantity, it does not take into account that its choice impacts aggregate output and the wage. This is a behavioral assumption since, with a discrete number of sectors and a discrete number of firms by sector, a firm's choice does has a non-zero effect on aggregates. Here we solve for markups relaxing this assumption.

The inverse demand for firm i in sector k (omitting time subscripts) is

$$p_i \equiv \frac{P_{ki}}{P} = Y_{ki}^{-\frac{1}{\varepsilon}} \left(Y_k\right)^{\frac{1}{\varepsilon}-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}}.$$

Differentiating p_{ki} with respect to Y_{ki}, taking other firms' quantities as given (but not sectoral or aggregate output), we obtain

$$\text{(A27)} \qquad \frac{d\log p_{ki}}{d\log Y_{ki}} = -\frac{1}{\varepsilon}\left(1 - s_{ki}\right) - \frac{1}{\sigma}s_{ki} + \frac{1}{\sigma}s_k s_{ki}$$

where we used

$$\frac{d\log Y_k}{d\log Y_{ki}} = \frac{P_{ki}Y_{ki}}{P_k Y_k} = s_{ki}$$

and

$$\frac{d\log Y}{d\log Y_k} = \frac{P_k Y_k}{PY} = s_k.$$

The last term in (A27) is zero if we calculate this derivative taking Y as given. From labor supply choice, we can express the real wage as

$$w \equiv \frac{W}{P} = f_0 Y^{\eta} L^{\frac{1}{f}}$$

Differentiating w with respect to Y_{ki} and using the fact that $d\log Y_{ki} = d\log L_{ki}$ (since $Y_{ki} = Z_{ki}L_{ki}$), we obtain

$$\frac{d\log w}{d\log Y_{ki}} = \eta s_k s_{ki} + \frac{1}{f} s_k^n s_{ki}^n, \tag{A28}$$

where $s_k^n = \sum_{i\in k} L_{ki}/L$ and $s_{ki}^n = L_{ki}/L_k$.

Firm i chooses output Y_{ki} to maximize real profits (i.e. profits relative to the aggregate price index), $Y_{ki} \times \left[p_{ki}\left(Y_{ki}, Y_{-ki}\right) - \frac{w}{Z_{ki}}\left(Y_{ki}, Y_{-ki}\right)\right]$, taking output choices by other firms, Y_{-ki}, as given. We do not take into account the effect that changes in profits have on consumption and leisure of the firm's owner (**?**). The first order condition is

$$p_{ki} - \frac{w}{Z_{ki}} + Y_{ki}\left(\frac{dp_{ki}}{dY_{ki}} - \frac{dw}{dY_{ki}}\frac{1}{Z_{ki}}\right) = 0$$

which can be re-arranged as

$$p_{ki} = \frac{w}{Z_{ki}}\left(\frac{1 + \frac{d\log w}{d\log Y_{ki}}}{1 + \frac{d\log p_{ki}}{d\log Y_{ki}}}\right).$$

Substituting the expressions for $\frac{d\log p_{ki}}{d\log Y_{ki}}$ and $\frac{d\log w}{d\log Y_{ki}}$, (A27) and (A28), we obtain

$$P_{ki} = \frac{W}{Z_{ki}}\left(\frac{1 + \eta s_k s_{ki} + \frac{1}{f} s_k^n s_{ki}^n}{1 - \frac{1}{\varepsilon}\left(1 - s_{ki}\right) - \frac{1}{\sigma} s_{ki} + \frac{1}{\sigma} s_k s_{ki}}\right) \tag{A29}$$

Since markups now depend on economy-wide sales and employment shares, $s_k s_{ki}$ and $s_k^n s_{ki}^n$, rather than on the shares within sectors, we must solve for markups in all sectors simultaneously rather than sector by sector in our baseline model, which is more intensive computationally.

If $s_k \to 0$, then (A29) becomes

$$P_{ki} = \frac{W}{Z_{ki}}\left(\frac{1}{1 - \frac{1}{\varepsilon}\left(1 - s_{ki}\right) - \frac{1}{\sigma} s_{ki}}\right). \tag{A30}$$

This is the expression for prices in the baseline model, in which we assumed that $\frac{d\log w}{d\log Y_{ki}} = \frac{d\log Y}{d\log Y_{ki}} = 0$ when firms choose quantity.

Markups in expressions (A29) and (A30) differ for two reasons. First, a unilateral increase in Y_{ki}, raises Y, implying a smaller decline in price p_{ki} compared to the case in which individual firms take Y as given. This implies a higher effective demand elasticity, lowering markups. This effect is captured by the term $\frac{1}{\sigma} s_k s_{ki}$ in the denominator of (A29).

Second, an increase in Y_{ki} raises w. This reduces the profit maximizing quantity compared to the case in which w is taken as given by individual firms. This effect is smaller the higher is the Frish elasticity f and the less sensitive is the marginal utility of consumption to aggregate output. This effect is zero if labor disutility is independent of aggregate labor (e.g. for perfectly elastic labor supply $f = \infty$) and if the marginal utility of consumption is independent of aggregate consumption (e.g. for linear utility in consumption $\eta = 0$).

Applying (A29) using sales and employment shares in our baseline calibration has a negligible impact on markup levels compared to those based on (A30) our baseline. For example, across the two alternatives, the implied level of markups levels for the highest markup firms (where the effect would be largest) differ only at the third decimal place.

B Additional Information on Data and Estimation

B.A Descriptive statistics

In Table A1, we report descriptive statistics for the estimation sample used in the estimation of the production function and for the baseline sample used to compute markups in our empirical exercise.

Table A1: Descriptive Statistics

Panel A: Estimation Sample						
	#obs	mean	median	25th per.	75th per.	95th per.
Sales (log)	220,733	8.14	7.97	6.98	9.08	11.04
Quantity (log)	220,733	7.53	7.45	6.29	8.76	10.94
Price (log)	220,733	0.61	0.28	-.005	.90	2.95
Wage bill (log)	220,733	6.83	6.67	5.82	7.61	9.41
Capital (log)	220,733	6.92	6.74	5.59	8.08	10.37
Services (log)	220,733	6.72	6.61	5.55	7.73	9.60
Materials (log)	220,733	6.94	6.83	5.62	8.15	10.26

Panel B: Baseline Sample						
	#obs	mean	median	25th per.	75th per.	95th per.
Sales (log)	9,383,228	5.85	5.58	4.69	6.77	8.86
Wage bill (log)	9,383,228	4.32	4.32	3.06	5.49	7.21
Capital (log)	9,383,228	4.00	3.84	2.93	4.92	7.09
Services (log)	9,383,228	4.20	3.90	3.01	5.13	7.26
Materials (log)	9,383,228	4.69	4.44	3.40	5.78	8.22
Markup	9,383,228	1.39	1.21	0.82	1.77	3.30
Elast. Materials	9,383,228	0.45	0.40	0.25	0.59	1.02
Sales/Materials	9,383,228	3.63	2.92	2.09	4.35	8.95
Local RTS	9,383,228	0.95	0.96	0.84	1.09	1.26
	#obs	mean	median	top 1%	top 0.1%	top 0.01%
Market Share (pp)	9,383,228	0.07	0.003	0.93	9.12	38.40

NOTE: Panel A (estimation sample) gives statistics for the sample of firms in the EAP survey from 2009 to 2019 with quantity information. Data is winsorized by 2-digit sectors at 1%. Panel B (baseline sample) gives statistics of for the sample of firms in FICUS-FARE from 1994 to 2019 as described in the main text. Local Return to Scale (Local RTS) is defined as the sun of the material, labor, capital and service elasticities. Markup, material elasticity, ratio of sales to materials and local RTS are winsorized at the 3% level.

B.B Production function and markup estimation

In this appendix, we describe the empirical framework we use to estimate production functions and firm-level markups. We also discuss its implementation in the FICUS-FARE French firm census data. This framework is based on the so-called production approach and builds on the methodology in **?** and **??** as discussed and implemented in **?**.

We assume all firms within 2-digit sectors have a common production function, up to a firm-specific Hicks-neutral TFP. We further assume that this firm-specific TFP Z_{it} follows an AR(1) process in logs, that is, $\log Z_{it} = \rho \log Z_{it-1} + \xi_{it}$. For simplicity, in what follows, we omit sector notation. The production function of firm i is

$$Y_{it} = Z_{it} F\left(L_{it}, K_{it}, M_{it}, O_{it}\right), \tag{A31}$$

where Z_{it} denotes TFP, L_{it} denotes labor, K_{it} denotes capital, M_{it} denotes materials, and O_{it} denotes services. These inputs are homogenous across firms within sectors and traded in competitive markets. In our estimation of markups, we do not impose that F is constant returns to scale.

B.B.1 Calculating markups

When minimizing costs, we assume that materials are a variable input that is not subject to any adjustment cost or any intertemporal decision. Under these assumptions, the first-order-condition of the firms' cost-minimization problem for materials M_{it} can be rewritten as

$$P_t^M = \lambda_{it} Z_{it} \frac{\partial F}{\partial M} \quad \Leftrightarrow \quad \mu_{it} = \frac{P_{it} Y_{it}}{P_t^M M_{it}} \frac{Z_{it} \frac{\partial F}{\partial M}}{Y_{it}/M_{it}},$$

where $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$; that is output price is equal to the markup over marginal cost. We denote by $\theta_{it}^M = \frac{Z_{it} \frac{\partial F}{\partial M}}{Y_{it}/M_{it}}$ the elasticity of the production function with respect to material input M_{it}. Markup is equal to the product of the ratio of sales to materials and the elasticity of the production function with respect to materials:

$$\mu_{it} = \frac{P_{it} Y_{it}}{P_t^M M_{it}} \theta_{it}^M. \tag{A32}$$

We calculate the ratio of sales to materials using the FICUS-FARE data (Panel B Table A1) on sales and input expenditures, and we estimate the production function as discussed in the next section.

B.B.2 Production-function estimation

In this section, we sketch the production-function estimation procedure. We implement a two-stage procedure using a control-function approach, as introduced by **??**, but adapted to an oligopolistic competition environment following **?**.

We implement the procedure described below at the 2-digit sector level. Given our assumptions that inputs are homogeneous and that firms are price takers in the input markets, we deflate input expenditures by sector-level price indices to recover inputs' quantities.

Below, we denote with small capital letters the logarithm of large capital letters: $z_{it} = \log Z_{it}$, $p_{it} = \log P_{it}$, $l_{it} = \log L_{it}$, $k_{it} = \log K_{it}$, $m_{it} = \log M_{it}$ and $o_{it} = \log O_{it}$. We assume that firms, in a given sector, produce by combining their inputs using a translog production function:

$$y_{it} = z_{it} + \sum_{u \in \{l,k,m,o\}} \beta_u u_{it} + \sum\sum_{\{u,v\} \in \{l,k,m,o\}} \beta_{uv} u_{it} v_{it} = z_{it} + X'_{it}\beta$$

where, in the last equality, we collect all the terms in the vector of data $X'_{it} = (l_{it}, k_{it}, m_{it}, o_{it}, l^2_{it}, k^2_{it}, m^2_{it}, o^2_{it}, l_{it}k_{it}, l_{it}m_{it}, l_{it}o_{it}, k_{it}m_{it}, k_{it}x_{it}, , m_{it}x_{it})$ and the vector of parameters to be estimated $\beta' = (\beta_l, \beta_k, \beta_m, \beta_o, \beta_{l^2}, \beta_{k^2}, \beta_{m^2}, \beta_{o^2}, \beta_{lk}, \beta_{lm}, \beta_{lo}, \beta_{km}, \beta_{ko}, \beta_{mo})$. Finally, we assume that quantity is observed with some measurement error ϵ_{it}, that is, observed quantity $\widetilde{y}_{it}$ differs from actual quantity y_{it} such that

$$\widetilde{y}_{it} = y_{it} + \epsilon_{it} = X'_{it}\beta + z_{it} + \epsilon_{it}.$$

The estimation consists of two stages. First, we purge the observed quantity from the measurement errors ϵ_{it}. Second, we construct a dynamic panel GMM estimator to estimate the vector of parameters β.

The empirical counterpart of each variable is discussed in Section III.A and descriptive statistics are given in Panel A of Table A1.

First-stage. The first stage of this procedure consists of separating measurement error from the true quantity using the fact that firms observe their productivity z_{it} when deciding the amount of inputs. The demand for the variable input, here materials m_{it}, can be expressed as a function of productivity: $m_{it} = m(z_{it}, \Xi_{it})$, where Ξ_{it} is a vector of all variables that determine m_{it} other than productivity. This function is often called the control function, as introduced by **?** and later extended in **?** and **?**. Under the assumption that m_{it} rises monotonically in z_{it}, the demand function can be inverted, such that $z_{it} = m^{-1}(m_{it}, \Xi_{it})$. Substituting this function in the production function gives

$$\widetilde{y}_{it} = y_{it} + \epsilon_{it} = X'_{it}\beta + m^{-1}(m_{it}, \Xi_{it}) + \epsilon_{it}.$$

The fitted values of a non-parametric regression of $\widetilde{y}_{it}$ on the variables in X'_{it}, m_{it} and Ξ_{it} therefore identify ϵ_{it}, as long as the variables in Ξ_{it} that determine the demand for m_{it} are correctly specified.

To construct this control function, we use the first-order-condition with respect to the static input materials in the cost-minimization problem (as in equation A32)

$$P_t^M = \frac{P_{it}}{\mu_{it}} Z_{it} \frac{\partial F}{\partial M}. \tag{A33}$$

Using the fact that $\frac{\partial F}{\partial M}$ is a function of input usage, $\frac{\partial F}{\partial M}(L_{it}, K_{it}, M_{it}, X_{it})$, equation (A33) implicitly defines M_{it} as a function of productivity, Z_{it}, conditional on other inputs' usage L_{it}, K_{it}, X_{it}, material price, output price p_{it}, and markup. Furthermore, following **?**, we assume the markup is a function of market share, $\mu_{it} = \mu_t(s_{it})$, as is the case under the nested CES demand system in our model, under either Cournot or Bertrand competition. In the data, this market share is defined as the ratio between firm-level sales and the sum of the sales of all firms in the same 5-digit NAF sector.

Equipped with this control function, we run a non-parametric regression of $\widetilde{y}_{it}$ on inputs' usage and their interaction in X_{it}, market share s_{it} to control for markups, output price p_{it}, and a time-fixed effect to control for input price.[A1] The fitted values of this regression identify measurement errors ϵ_{it} and allow recovering quantity y_{it}.

Second-stage. In the second stage, as in **?**, we build a dynamic panel estimator in the spirit of **?**, where identification is achieved through an instrument. Specifically, we use past values of input usage as instruments for current values. Following **?**, the GMM-based asymptotic estimator we use is defined as:

Definition 1 *The GMM estimator is $\widehat{\beta} \in \mathbb{R}^{14}$ and $\widehat{\rho} \in \mathbb{R}$ such that the moments $\mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right]$ and $\mathbb{E}\left[\widehat{z}_{it-1}\widehat{\xi}_{it}\right]$ are equal to zero where $\widehat{z}_{it} = y_{it} - X'_{it}\widehat{\beta} = X'_{it}(\beta - \widehat{\beta}) + z_{it}$ and $\widehat{\xi}_{it} = \widehat{z}_{it} - \widehat{\rho}\widehat{z}_{it-1} = (X_{it} - \rho X_{it-1})'(\beta - \widehat{\beta}) + X'_{it-1}(\beta - \widehat{\beta})(\rho - \widehat{\rho}) + z_{it-1}(\rho - \widehat{\rho}) + \xi_{it}$*

In the remainder of this appendix, we follow **?** and discuss the conditions under which the above estimator admits a solution. To this end, consider the following system of equations, which defines the estimator and whose unknowns are $\widehat{\beta}$ and $\widehat{\rho}$:

$$\begin{cases} \mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right] = 0 \\ \mathbb{E}\left[\widehat{z}_{it-1}\widehat{\xi}_{it}\right] = \mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right]'(\beta - \widehat{\beta}) + \mathbb{E}\left[z_{it-1}\widehat{\xi}_{it}\right] = 0 \end{cases} \iff \begin{cases} \mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right] = 0 \\ \mathbb{E}\left[z_{it-1}\widehat{\xi}_{it}\right] = 0 \end{cases} \iff$$

$$\begin{cases} \mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right](\beta - \widehat{\beta}) + \mathbb{E}\left[X_{it-1}X'_{it-1}\right](\beta - \widehat{\beta})(\rho - \widehat{\rho}) + \mathbb{E}\left[X_{it-1}z_{it-1}\right](\rho - \widehat{\rho}) = 0 \\ \mathbb{E}\left[z_{it-1}\widetilde{X}'_{it}\right](\beta - \widehat{\beta}) + \mathbb{E}\left[z_{it-1}X'_{it-1}\right](\beta - \widehat{\beta})(\rho - \widehat{\rho}) + \mathbb{E}\left[z^2_{it-1}\right](\rho - \widehat{\rho}) = 0 \end{cases},$$

[A1] In practice such non-parametric regression is performed by regressing the observed quantity on a third-order polynomial of the variables.

where we use $\mathbb{E}[X_{it-1}\xi_{it}] = 0$ and $\mathbb{E}[z_{it-1}\xi_{it}] = 0$, and where we denote $\widetilde{X}_{it} = X_{it} - \rho X_{it-1}$. Note that the first line of the above system of equations corresponds to 14 equations, while the second line is just a scalar equation. We have $14+1$ equations with unknown $(\widehat{\beta}', \widehat{\rho}) \in \mathbb{R}^{14+1}$. In general, this system of equations has multiple solutions. However, when $(\widehat{\beta}', \widehat{\rho})$ is not too far from the true value (β', ρ), the terms in $(\beta - \widehat{\beta})(\rho - \widehat{\rho})$ are of second order. Ignoring these terms leads to the following reduced system which can be written in matrix form as:

$$\begin{cases} \mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right](\beta - \widehat{\beta}) + \mathbb{E}\left[X_{it-1}z_{it-1}\right](\rho - \widehat{\rho}) = 0 \\ \mathbb{E}\left[z_{it-1}\widetilde{X}'_{it}\right](\beta - \widehat{\beta}) + \mathbb{E}\left[z^2_{it-1}\right](\rho - \widehat{\rho}) = 0 \end{cases} \iff \begin{pmatrix} \mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[X_{it-1}z_{it-1}\right] \\ \mathbb{E}\left[z_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[z^2_{it-1}\right] \end{pmatrix} \begin{pmatrix} \beta - \widehat{\beta} \\ \rho - \widehat{\rho} \end{pmatrix} = 0$$

which admits a unique solution $(\widehat{\beta}, \widehat{\rho}) = (\beta, \rho)$ as long as the (15×15) matrix $\begin{pmatrix} \mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[X_{it-1}z_{it-1}\right] \\ \mathbb{E}\left[z_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[z^2_{it-1}\right], \end{pmatrix}$ is invertible. From here, **?** conclude that the GMM estimator is locally identified and consistent.

B.C Comparison with ?

Table 13 in **?** compares estimates of output elasticities and markups under different approaches for a large US and Canadian retailer. They conclude that using quantity or revenues makes a difference for the level of estimated markups.

Table A2, discussed in Section V.D, reports a similar analysis in our French data. Specifically, we consider our baseline quantity-based markups, accounting markups (Lerner index, calculated using net operating profits), and revenue-based markups, which are similar to the measures in **?**.

The top three rows of Table A2 report the median and inter-quartile range (in brackets) for (i) output (or revenue) elasticities with respect to the flexible input, (ii) markups, and (iii) ratio of the corresponding markup measure and the baseline quantity-based markup.

The median output elasticity in our data is 0.4 using quantities and 0.36 using revenues. The estimates in **?** are higher (close to 1) because the variable input is defined to be costs of goods sold, which accounts for a large share of variable cost of retailers. In spite of the differences in output elasticities, the ratio of median revenue-based markups and quantity-based markups is very similar (close to 0.85) in our data and in **?**. Furthermore, the ratio of Lerner-based markup to quantity-based markup is 0.93 in our data and close to 1 in **?** (our Lerner measure is based on net operating profits while **?** uses gross margin).

In Section C we discuss the robustness of our empirical results on markup cyclicality to these alternative markup measures.

Table A2: Comparison of Flexible-input Elasticities and Markup Estimates

	(1) Quantity	(2) Lerner	(3) Revenues
Median firm-level estimated flexible-input elasticity	0.40 [0.35]	N/A	0.36 [0.18]
Median firm-level markup	1.21 [0.95]	1.10 [0.25]	1.03 [0.33]
Markup relative to quantity-based markup	1 [0]	0.93 [0.54]	0.85 [0.75]

NOTE: Statistics for the sample of firms in FICUS-FARE from 1994 to 2019. The first three rows report median and interquartile range (in squared bracket) across the 9,383,228 observations. Underlying firm-level data are winsorized at the 3% level. Column (1) gives estimates for the baseline quantity-based markup. Column (2) gives estimates for accounting markup (Lerner index) computed from net operating profit margin as described in Appendix C.A. Column (3) gives estimates for revenues-based markup as described in Appendix C.D.

C Robustness of Empirical Results

In this appendix, we discuss further robustness exercises. We consider the following variations of our baseline choices: using accounting (Lerner index) markups (Section C.A), restricting the sample to the period covered by price data (Section C.B), alternative outlier treatment (Section C.C), computing markups using revenue data only (Section C.D), estimating production functions for single product firms (Section C.E), restricting the sample to the estimation sample (Section C.F), and focusing on manufacturing firms (Section C.G). The empirical results for these robustness exercises are collected in Table A3. For convenience, column (1) displays the baseline results.

C.A Accounting markups (Lerner Index)

Our baseline estimates and several of our robustness checks are based on the production function approach to recover markups, as described in Section III.B and Appendix B.B. In this robustness exercise, we instead compute markups using the "accounting approach" based on accounting profits.

Specifically, we compute the Lerner index of firm i at time t as $Lerner_{it} \equiv \frac{P_{it}Y_{it}-TC_{it}}{P_{it}Y_{it}}$ where TC_{it} is the total cost measured as the sum of labor, capital, material and service expenditures, and $P_{it}Y_{it}$ is total revenues of the firm i at time t.[A2] With constant returns to scale, $MC_{it} = TC_{it}/Y_{it}$, and $Lerner_{it} = \frac{P_{it}-\frac{TC_{it}}{Y_{it}}}{P_{it}} = \frac{P_{it}-MC_{it}}{P_{it}}$. We calculate accounting markups as $\mu_{it}^{Lerner} = (1-Lerner_{it})^{-1} = \frac{P_{it}Y_{it}}{TC_{it}}$.

The results of this exercise are collected in the column "Lerner" of Table A3. The results are

[A2]The expenditure on capital is computed assuming capital return net of depreciation of 4%. Here we abstract from risk or sector heterogeneity in depreciation rates.

Table A3: Robustness Table

	(1) Baseline	(2) Lerner	(3) 2009-2019	(4) Winsorized at 1%	(5) Revenues	(6) Single-Product	(7) Estimation Sample	(8) Manufacturing
Firm-Level Markup and Market Share								
First-Diff.	−0.268	−0.961	−0.459	−0.350	−0.818	−0.121	−0.063	−0.223
	(0.092)	(0.073)	(0.175)	(0.151)	(0.068)	(0.056)	(0.030)	(0.065)
Sector-Level Markup and HHI								
First-Diff.	−0.374	−0.093	−0.163	−0.672	−0.0341	−0.104	−0.093	−0.0225
	(0.177)	(0.047)	(0.379)	(0.276)	(0.0433)	(0.121)	(0.071)	(0.131)
Within Contribution to Sector Markup Change								
Median	0.586	0.863	0.646	0.565	0.689	0.569	0.665	0.751
Standard Deviation	0.290	0.172	0.334	0.287	0.276	0.268	0.323	0.250
Firm-Level Markup and Sector Output								
$\Delta \log Y_{kt}$	−0.024	0.033	0.021	−0.021	0.019	0.006	0.001	0.012
	(0.009)	(0.004)	(0.012)	(0.010)	(0.004)	(0.006)	(0.005)	(0.011)
$\Delta \log Y_{kt} * s_{kit}$	0.280	0.127	−0.002	0.355	0.152	0.226	0.067	0.145
	(0.041)	(0.030)	(0.096)	(0.050)	(0.029)	(0.034)	(0.043)	(0.055)
Firm-Level Market Share and Sector Output								
All firms	−0.488	−0.484	−0.507	−0.520	−0.487	−0.503	−0.501	−0.507
	(0.018)	(0.018)	(0.022)	(0.019)	(0.018)	(0.022)	(0.035)	(0.018)
$s_{kit} < 0.5$	−0.489	−0.485	−0.507	−0.521	−0.488	−0.504	−0.504	−0.508
	(0.018)	(0.018)	(0.022)	(0.019)	(0.018)	(0.022)	(0.035)	(0.018)
$s_{kit} > 0.5$	0.091	0.022	0.103	0.093	0.048	0.094	−0.214	−0.006
	(0.037)	(0.059)	(0.020)	(0.038)	(0.051)	(0.033)	(0.022)	(051)
Sector-Level Markup and Sector Output								
$\Delta \log Y_{kt}$	0.160	0.0788	0.121	0.166	0.0696	0.0997	0.0186	0.0828
	(0.0396)	(0.014)	(0.0526)	(0.0404)	(0.0181)	(0.0508)	(0.0218)	(0.0.0271)
Sector-Level HHI and Sector Output								
$\Delta \log Y_{kt}$	0.332	0.317	0.385	0.332	0.317	0.338	0.00842	0.298
	(0.067)	(0.0672)	(0.0723)	(0.067)	(0.0672)	(0.0711)	(0.0439)	(0.0933)
Sector-Level Markup and Aggregate Output								
$\Delta \log Y_t$	−0.239	0.0186	−0.179	−0.312	−0.0761	−0.115	0.0231	−0.140
	(0.116)	(0.0125)	(0.150)	(0.169)	(0.022)	(0.107)	(0.140)	(0.055)

NOTE: This table reproduces our baseline estimates for various robustness checks. Column (1) collects our baseline estimates discussed in the main text (in the same order as they appear: Tables III, V, Section IV.B, Tables VI, VII, VIII, IX and X. Each other column represents a robustness exercise described in this appendix. Specifically, column (2) is discussed in Appendix C.A, column (3) in C.B, column (4) in C.C, column (5) in C.D, column (6) in C.E, column (7) in C.F, and column (8) in C.G.

qualitatively consistent with our baseline results.

C.B Period 2009-2019

In this robustness exercise, we restrict our sample to the period 2009-2019. Recall that the subsample used to estimate the production function elasticity starts only in 2009, as firm-level quantities are not available for earlier years. In our baseline, in order to maximize the sample of markups available for our exercises, we assumed that the estimated production functions are stable and extend to the earlier period 1994-2009. The results of our empirical exercises are collected in the column "2009-2019" of Table A3. The results are qualitatively similar to our baseline specification. However, the coefficient of the regression of firm-level markup on sector output and the coefficient of the regression of sector markup on sector level concentration are no longer significant, likely due to the lower number of observations in our sector panel in this significantly shorter subsample.

C.C Outlier treatment

In this robustness exercise, we deploy a different outlier treatment relative to our baseline. Specifically, in our baseline specification we winsorize the firm-level markup distribution at the 3% level while in column "Winsorize at 1%" we report results for a winsorization at the 1% level. We also have explored 2% and 5% levels of winsorization. The results are barely affected by these alternative outlier treatments.

C.D Revenue-based markups

In this robustness exercise, we run our empirical specification on markups calculated with output elasticities estimated without quantity data. The results for this markup specification are collected in the column (5) "Revenues" of Table A3. Again, the results are qualitatively similar to our baseline specification in column (1).

However, the sector-markup on sector-concentration relationship — while retaining the baseline sign — loses significance while the relation between sector markups and aggregate output gains significance but with point estimates that are smaller and closer to zero.

C.E Single-product firms

The EAP database, as described in Section III.A, gives quantity and revenue information at the product level which we then aggregate at the firm-level. One concern is our aggregation from products to firms — in an environment where large firms tend to produce several products. In this robustness exercise, we restrict the estimation sample to single-product firms in order to address this concern. Results are collected in the column " Single-Product" of Table A3 and are qualitatively similar to the baseline estimates (with the exception of the coefficient of sector-level inverse markup on sector concentration, which becomes insignificant) even if the sample of firms used to estimate the production function drops to only 117,737 observations.

C.F Estimation sample

In this robustness exercise, we focus on the estimation sample only. Specifically, each regression and aggregation from firm to sector-level is carried out on the same sample used to estimate the production function, that is, over only 220,733 observations on the period 2009-2019. Relative to our first robustness exercise above, note that we now also lose a large number of firms as the EAP estimation sample is only a representative survey for smaller firms. The results are collected in the column "Estimation Sample" of Table A3. For this exercise, while we do obtain similar sign patterns across the different regressions, the statistical significance of several coefficients is reduced. This is likely because the sample size is much smaller — both in terms of the number of periods and number of firms — and does not adequately account for the population of small firms.

C.G Manufacturing firms

In this robustness exercise, we focus on the subset of sectors in manufacturing comprising of 2-digit sectors 13, 14, 15 ,16 ,17 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, and 33. Results are collected in the column "Manufacturing" of Table A3. Results are qualitatively similar, but since we lose many 5-digit sectors, some of the results lack statistical power, as for instance, the coefficient of inverse markup on concentration.

D Role of Entry and Exit

Consistently with the model, our baseline empirical specifications treat entrants, exiters and continuing firms symmetrically. Irrespective of its status, if a firm is present at time t in our dataset, it is included in all firm-level regressions (in levels) and its contribution to the respective sectoral aggregate is recorded in the corresponding sectoral variable at time t. The latter also implies that sectoral growth rates naturally reflect the contribution of entry and exit.[A3]

In this section, we investigate the role firm entry and exit for our main empirical results.[A4] In particular, we address the concern that our empirical results are driven by the particular markup

[A3]The only exception to this symmetric treatment of entering, exiting and continuing firms are specifications involving firm-level regressions in first differences where, to calculate growth rates, we can only include continuing firms each period.

[A4]In our data, the share of entrants and exiters in aggregate revenues is roughly 3 percent each on average over time.

dynamics of entrants and exiters. To do this, we re-estimate our key empirical specifications examining the cyclicality of markups — at the firm and sector levels — for a sample of continuing firms only.[A5]

Specifically, we re-estimate our firm-level markup regressions in first-differences restricting the sample such that for each firm we drop the first year observation (the growth rate in the first year after the firm enters) and the last year observation (the growth rate in the last year before the firm exits). That is, we drop the first and last observed markup change of a firm. We also re-estimate our sector-level markup regressions computing first differences (at period t) in sector-level markup for the set of continuing firms that are present both at time t and $t-1$. Throughout, we keep all covariates involving market shares (be it firm-level market shares or sector level sums of squared market shares) unchanged relative to the baseline sample. This is because the markup set by firms in the model depends on the whole distribution of firms in a given market rather than just the continuing firms' subset. Similarly, we use the same measures of overall sector output and aggregate value added as in our baseline specification.

Results are collected in column (2) of Table A4. Most results are very similar to our baseline — both with respect to the signs and sizes of the coefficients — with two exceptions. First, the coefficient of sectoral markup on sectoral output, while maintaining the predicted sign and significance, is now larger. Second, the coefficient of sector markup on aggregate output is now smaller and very close to zero (though, consistently with our discussion, it remains statistically undistinguishable from zero).

Thus, as in our baseline results for all firms, the markup dynamics of continuing firms reflect the evolution of their market shares, and the typical continuing firm markup growth rate is "countercyclical" with respect to its own-sector output growth. Aggregating markups over continuing firms in a given sector shows that these "continuing-firm" sectoral markups behave much like overall sectoral markups: they track the evolution of sectoral concentration in very similar fashion, comove positively with sectoral output, and are "acyclical" with respect to aggregate value added in the economy.

[A5]We do not consider a balanced sample for our entire 26-year period because this would significantly reduce the sample size and induce severe survivorship bias.

Table A4: Role of entry and exit

	(1) Baseline	(9) Continuing Firms Sample
Firm-Level Markup and Market Share		
First-Diff.	−0.268 (0.092)	−0.236 (0.101)
Sector-Level Markup and HHI		
First-Diff.	−0.374 (0.177)	−0.456 (0.2283)
Firm-Level Markup and Sector Output		
$\Delta \log Y_{kt}$	−0.024 (0.009)	−0.0157 (0.009)
$\Delta \log Y_{kt} * s_{kit}$	0.280 (0.041)	0.267 (0.0502)
Sector-Level Markup and Sector Output		
$\Delta \log Y_{kt}$	0.160 (0.0396)	0.375 (0.0397)
Sector-Level Markup and Aggregate Output		
$\Delta \log Y_t$	−0.239 (0.116)	−0.009 (0.105)

NOTE: This table reports empirical estimates for the continuing firms sample, as described in Section D.

E Additional Figures and Tables

E.A Relation between market share and volatility

Table A5: Market Share and Market Share Volatility

Dependent Variable	$\sigma_{kit}^{\overline{g_s}^i}$	$\sigma_{ki}^{\overline{g_s}^t}$
Coefficient	(1)	(2)
s_{kit}	-0.536 (0.001)	
$\overline{s}_{ki}$		-0.839 (0.026)
Constant	0.274 (0.000)	0.271 (0.000)
Observations	9,358,228	833,285

NOTE: $\sigma_{kit}^{\overline{g_s}^i}$ is, for a firm i in sector k at time t, the standard deviation of the growth rate of market share across firms in the same market share percentile. $\sigma_{ki}^{\overline{g_s}^t}$ is the standard deviation of the growth rate of market share of firm i in sector k across time. Column (1) reports the regression of $\sigma_{kit}^{\overline{g_s}^i}$ on market share of firm i at time t, s_{kit}. Column (2) reports the regression of $\sigma_{ki}^{\overline{g_s}^t}$ on average market share of firm i across time, $\overline{s}_{ki}$.

E.B Inspecting the mechanism: alternative specifications

Table A6: Firm Inverse Markup and Market Share: Level

	Dependent Variable: μ_{kit}^{-1}					
Coefficient	(1)	(2)	(3)	(4)	(5)	(6)
s_{kit}	-1.366	-1.382	-1.17	-0.469	-0.508	-0.297
	(0.112)	(0.113)	(0.132)	(0.133)	(0.137)	(0.146)
Year FE	N	Y	N	N	Y	N
Firm FE	N	N	N	Y	Y	Y
Market * Year FE	N	N	Y	N	N	Y
Observations	9,089,750	9,089,750	9,089,750	9,039,476	9,039,476	9,039,476

NOTE: μ_{kit}^{-1} is the inverse of firm i sector k gross markup in year t, and s_{kit} gives the market share of firm i in sector k. Columns (1)-(4) report empirical estimates for the FICUS-FARE (1995-2016) data. Standard errors (in parentheses) are clustered at the firm and year level. Inverse markups are winsorized at the 3% level.

Table A7: Market Share and Marginal Cost

	Dependent Variable: $\log s_{kit}$					
Coefficient	(1)	(2)	(3)	(4)	(5)	(6)
$\log mc_{it}$	-0.152	-0.153	-0.033	-0.009	-0.009	-0.008
	(0.013)	(0.013)	(0.011)	(0.002)	(0.002)	(0.002)
Year FE	N	Y	N	N	Y	N
Firm FE	N	N	N	Y	Y	Y
Market * Year FE	N	N	Y	N	N	Y
Observations	212,459	212,459	212,459	212,184	212,184	212,184

NOTE: $\log s_{kit}$ is the (log) firm i sector k market share, and $\log mc_{it} = \log p_{it} - \log \mu_{kit}$ is the (log) marginal cost defined as the difference between (log) price and (log) markup of of firm i in sector k at time t. Columns (1)-(4) report empirical estimates for the estimation sample FARE (2009-2019) data. We drop observations with negative markup. Standard errors (in parentheses) are clustered at the firm and year level. Variables are winsorized at the 3% level.

Table A8: Markup and Marginal Cost

	Dependent Variable: $\log \mu_{kit}$					
Coefficient	(1)	(2)	(3)	(4)	(5)	(6)
$\log mc_{it}$	-0.169	-0.169	-0.149	-0.093	-0.093	-0.096
	(0.007)	(0.007)	(0.006)	(0.007)	(0.007)	(0.008)
Year FE	N	Y	N	N	Y	N
Firm FE	N	N	N	Y	Y	Y
Market * Year FE	N	N	Y	N	N	Y
Observations	212,459	212,459	212,459	212,184	212,184	212,184

NOTE: $\log \mu_{kit}$ is the (log) firm i sector k gross markup, and $\log mc_{it} = \log p_{it} - \log \mu_{kit}$ is the (log) marginal cost defined as the difference between (log) price and (log) markup of of firm i in sector k at time t. Columns (1)-(4) report empirical estimates for the estimation sample FARE (2009-2019) data. We drop observations with negative markup. Standard errors (in parentheses) are clustered at the firm and year level. Variables are winsorized at the 3% level.

Table A9: Sector Inverse Markup and Sector HHI: Level

	Dependent Variable: μ_{kt}^{-1}				
Coefficient	(1)	(2)	(3)	(4)	(5)
HHI_{kt}	-1.203	-1.208	-0.266	-0.286	-0.168
	(0.219)	(0.219)	(0.207)	(0.206)	(0.162)
Year FE	N	Y	N	Y	N
Sector FE	N	N	Y	Y	N
Sector (2-digit) * Year FE	N	N	N	N	Y
Number of Sectors	275	275	275	275	275
Observations	6,875	6,875	6,875	6,875	6,875

NOTE: μ_{kt}^{-1} is sector k (inverse) markup in year t, HHI_{kt} is the HHI in sector k. Columns (1)-(4) report empirical estimates for the FICUS-FARE (1995-2016) data, aggregated to the sector level. Standard errors (in parentheses) are clustered at the sector and year level.

E.C Inspecting the mechanism: instrumental variable approach

In this appendix, we provide further evidence on the firm-level mechanism relating marginal cost, market share, and markups, based on an instrument for the marginal cost. Recall that the OLS regression in the main text (Table IV) should not be interpreted as causal and reflects a correlation, possibly subject to endogeneity concerns stemming from measurement error. Here we follow **?** and **?** and pursue an instrumentation strategy that exploits arguably exogenous energy price-driven changes in firm-level marginal costs.

Our instrument leverages two sources of industry-level heterogeneity in the exposure to energy price changes. First, we exploit the fact that there is heterogeneity in the energy-input mix used by different sectors. This heterogeneous exposure to the cost of particular energy inputs, in turn, induces industry-specific energy price variation over time, despite the fact that producers face common, nation-wide prices for each individual energy input. Second, even holding the energy input mix constant across industries, some industries may still be more exposed to energy price variation than others simply because they are more energy-intensive, i.e. they have a larger cost share of (all forms of) energy in intermediate inputs. Our instrument combines both sources of industry heterogeneity — which we take as an exogenous technological characteristic — with firm-level variation in intermediate input shares, to obtain a firm-level instrument that predicts marginal costs.

To construct our instrument we rely on energy consumption data in the EACEI survey (Enquête sur les consommations d'énergie dans l'industrie) provided by INSEE from 2008 onwards. This is an establishment-level survey providing data on both the quantity and expenditure on energy, by type of energy, whose purpose is to provide aggregate energy consumption statistics at the industry and regional levels. We aggregate this data at the 2-digit sector level and combine it with firm-level data on intermediate input shares in our estimation panel. In particular, we construct the instrument for firm i at time t as follows:

$$\log {mc_{it}}^{IV} \equiv \frac{P^M_{it-1} M_{it-1}}{P_{it-1} Y_{it-1}} \times \alpha^E_{s(i),t-1} \times \log P^E_{s(i),t} \tag{A34}$$

where $\frac{P^M_{it-1} M_{it-1}}{P_{it-1} Y_{it-1}}$ is the ratio of material expenditures to revenues of firm i at time $t-1$,[A6] $\alpha^E_{s(i),t-1}$ is the ratio of energy expenditure to material expenditure in the 2-digit sector $s(i)$ at time $t-1$, and where $P^E_{s(i),t}$ is the average cost of one unit of energy in the 2-digit sector $s(i)$ at time t. The latter is defined as the ratio of expenditure on energy, $Exp^E_{s,t}$, and the total consumption of energy, $Cons^E_{s,t}$, in the sector s at time t that is $P^E_{s,t} = \frac{Exp^E_{s,t}}{Cons^E_{s,t}}$. Importantly, energy consumption, $Cons^E_{s,t}$, is the sum of energy consumed in the form of electricity, steam, natural gas, network gas, coal, lignite, coal coke, petroleum coke, butane-propane, heavy fuel oil, and domestic fuel oil, all expressed in Petroleum Equivalent Tons (TEP) units. Finally, we lag both the firm-level

[A6] We also considered a specification in which intermediate input shares are defined over variable costs (no capital) rather than revenues. First stage statistics are lower (yet above 10 unless we drop all fixed effects, as in our baseline) and the point estimates are very similar.

share of intermediate inputs and sector-level energy cost shares in intermediate inputs to address endogeneity concerns.

Table A10 of this appendix shows the results of our instrumental variables exercise, where we use yearly changes in $\log {mc_{it}}^{IV}$ to instrument for changes in firm-level marginal costs and allow for year or sector $\times$ year fixed effects. When allowing for fixed effects, the first-stage F-statistics suggest that our instrument is strong. Our second stage results suggest that the qualitative predictions under OLS are robust to this instrumentation strategy: higher marginal cost growth driven by (heterogeneous exposure to) energy price growth is associated with lower market share and markup growth. Point estimates are larger than in our OLS specification.

Table A10: Markups, Market Shares and Marginal Costs: Instrumental Variable Approach

IV-Specification: energy price						
	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable:	$\Delta \log s_{kit}$			$\Delta \log \mu_{kit}$		
$\Delta \log mc_{it}$	-0.870	-0.386	-0.274	-1.235	-0.668	-0.951
	(0.634)	(0.199)	(0.038)	(0.482)	(0.192)	(0.110)
Year FE	N	Y	N	N	Y	N
Sector $\times$ Year FE	N	N	Y	N	N	Y
Kleibergen-Paap 1st Stage F-Stat	3.963	20.196	57.006	3.963	20.196	57.006
Observations	173,297	173,297	173,239	173,297	173,297	173,239

NOTE: $\Delta \log \mu_{kit}$ is the first-difference of (log) gross markup of firm i sector k at time t, $\Delta \log s_{kit}$ is the first-difference of (log) market share, and $\Delta \log mc_{it}$ is the first-difference of (log) marginal cost when the latter is defined as the difference between (log) price and (log) markup of firm i in sector k at time t. Marginal cost is instrumented by firm-level measures of energy price. Columns (1)-(6) report empirical estimates for the estimation sample FARE (2009-2019) data. Columns (1) and (4) report pooled estimates while columns (2), (3), (5) and (6) report estimates that further control for year or sector$\times$year fixed effects. Sector-year fixed effects are defined at the 5-digit NAF sector classification level. Standard errors (in parentheses) are two-way clustered at the firm and year level. $\Delta \log \mu_{kit}$ and $\Delta \log mc_{it}$ are winsorized at the 3% level.

E.D Firm-level evidence

Table A11: Firm Markup and Sector Output

	(1) Data	(2) Data	(3) Model	(4) Model
	Dependent variable: $\log(\mu_{kit})$			
$\log \widehat{Y}_{kt}$	0.010		-0.001	
	(0.014)			
$\log \widehat{Y}_{kt} * s_{kit}$	0.158		0.190	
	(0.065)			
$\log \widehat{Y}^{HP}_{kt}$		-0.022		-0.002
		(0.021)		
$\log \widehat{Y}^{HP}_{kt} * s_{kit}$		0.413		0.425
		(0.114)		
Firm FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Obs.	8,361,273	9,039,476	-	-

NOTE: μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t. $\log \widehat{Y}_{kt}$ (resp. $\log \widehat{Y}^{HP}_{kt}$) is log value-added of sector k at time t detrended following **?** (resp. using a HP-filter). Columns (1) and (2) report empirical estimates for the FICUS-FARE (1995-2019) data. Standard errors are two-way clustered at the sector×year level. Columns (3) and (4) report estimates based on model-simulated data. Log markup are winsorized at the 3% level. Note that the number of observations for the deviation from a **?** trend is lower, as we lose a few periods due to the filtering.

Table A12: Firm Market Share and Sector Output

	(1) Data (all data)	(2) Data ($\bar{s}_{ki} < 0.50$)	(3) Data ($\bar{s}_{ki} > 0.50$)	(4) Model (all data)	(5) Model ($\bar{s}_{ki} < 0.50$)	(6) Model ($\bar{s}_{ki} > 0.50$)
Dependent variable:			$\log s_{kit}$			
$\log \widehat{Y}_{kt}$	-0.486	-0.488	0.143	-1.377	-1.381	0.336
	(0.027)	(0.027)	(0.052)			
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Number of Obs.	8,361,273	8,360,864	440	-	-	-
Dependent variable:			$\log s_{kit}$			
$\log \widehat{Y}_{kt}^{HP}$	-0.829	-0.831	0.143	-3.469	-3.477	0.343
	(0.046)	(0.046)	(0.061)			
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Number of Obs.	9,039,476	9,039,036	440	-	-	-

NOTE: $\log s_{kit}$ gives the (log) market share of firm i in sector k, year t. $\log \widehat{Y}_{kt}$ (resp. $\log \widehat{Y}_{kt}^{HP}$) is log value-added of sector k at time t detrended following **?** (resp. using a HP-filter). $\bar{s}_{ki}$ is the average market share of firm i in market k. Column (1-3) reports empirical estimates for the FICUS-FARE (1995-2019) data. Sectors are defined at the 5-digit NAF sector classification level. Column (4-6) reports estimates based on model-simulated data. Standard errors in the data are two-way clustered at the sector×year level. First-difference in log market share are winsorized at the 3% level. Note that the number of observations for the deviation from a **?** trend is lower, as we lose a few periods due to the filtering.

E.E Sector-level evidence

Table A13: Sector Markup and Sector Output

	(1)	(2)	(3)	(4)
	Data		Model	
Dependent variable:	$\log \mu_{kt}$	$\log \widehat{\mu}_{kt}^{HP}$	$\log \mu_{kt}$	$\log \widehat{\mu}_{kt}^{HP}$
$\log Y_{kt}$	0.116		0.145	
	(0.042)		(0.023)	
$\log \widehat{Y}_{kt}^{HP}$		0.157		0.110
		(0.038)		(0.040)
Sector FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Sectors	275	275	275	275
Number of Obs.	6,875	6,875	6,875	6,875

NOTE: Regression of sector-level log level and HP-trend deviation of markup ($\log \mu_{kt}$, $\log \widehat{\mu}_{kt}^{HP}$ resp.) on sector log value-added ($\log Y_{kt}$, $\log \widehat{Y}_{kt}^{HP}$ resp.). Column (1-2) reports empirical estimates for the FICUS-FARE (1995-2019) data, and standard errors (in parentheses) are clustered at the sector level. Columns (3-4) reports estimates based on model-simulated data. The point estimate for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A14: Sector Concentration and Sector Output

	(1) Data	(2) Data	(3) Model	(4) Model
Dependent variable:	$\log HHI_{kt}$	$\log\widehat{HHI}_{kt}^{HP}$	$\log HHI_{kt}$	$\log\widehat{HHI}_{kt}^{HP}$
$\log Y_{kt}$	0.094		1.258	
	(0.046)		(0.292)	
$\log \widehat{Y}_{kt}^{HP}$		0.330		0.554
		(0.064)		(0.241)
Sector FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Sectors	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log markup on sector log value-added in level and HP-trend deviation ($\log HHI_{kt}$, $\log Y_{kt}$ and $\log\widehat{HHI}_{kt}^{HP}$, $\log \widehat{Y}_{kt}^{HP}$ resp.). Column (1-2) reports empirical estimates for the FICUS-FARE (1995-2019) data, and standard errors (in parentheses) are clustered at the sector level. Columns (3-4) reports estimates based on model-simulated data. The point estimate for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A15: Sector Markup and Aggregate Output

	(1) Data	(2) Model without Aggr. Shocks	(3) Model with Aggr. Shocks
Dependent variable:	$\log\widehat{\mu}_{kt}^{HP}$	$\widehat{\log}\mu_{kt}^{HP}$	$\log\widehat{\mu}_{kt}^{HP}$
$\log \widehat{Y}_{t}^{HP}$	-0.259	0.165	0.015
	(0.119)	(0.108)	(0.044)
Share negative coefficients	-	0.02	0.20
Sector FE	Y	Y	Y
Number of Sectors	275	275	275
Number of Obs.	6,875	6,875	6,875

NOTE: Regression of sector k's markup in year t in HP trend deviation $\log\widehat{\mu}_{kt}^{HP}$ on (log) aggregate real value-added in year t in HP trend deviation $\log \widehat{Y}_{t}^{HP}$. Columns (1) report empirical estimates for the FICUS-FARE (1995-2019) data. Standard errors (in parentheses) are clustered at the sector level. Columns (2) report estimates based on model-simulated data. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same samples. Column (3) reports estimates based on model-simulated data with aggregate TFP shocks. Point estimates and standard deviation for this column is computed as for columns (2). The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of aggregate output measured in deviation from HP trend in France. Regression are weighted by average sector value-added.

E.F Aggregate-level results

Table A16: Aggregate Markup and Aggregate Output

	(1) Data			(2) Model			(3) Model with Aggr. Shock		
	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$
$\log \widehat{Y}_t^{HP}$	1.81	1	1	0.54	1	1	1.81	1	1
$\log \widehat{\mu}_t^{HP}$	0.96	0.53	0.14	0.19	0.36	0.90	0.24	0.13	0.38

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for log aggregate output $\log \widehat{Y}_t^{HP}$ and log aggregate markup $\log \widehat{\mu}_t^{HP}$ in deviations from their HP trend. Column (1) reports empirical estimates for the FICUS-FARE (1995-2019) data. Column (2) reports the median over 5,000 independent simulated samples, each of 25 years. Column (3) reports the average over 5,000 simulated samples of 25 years from a model with aggregate TFP shocks. The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of aggregate output measured in deviation from HP trend in France.

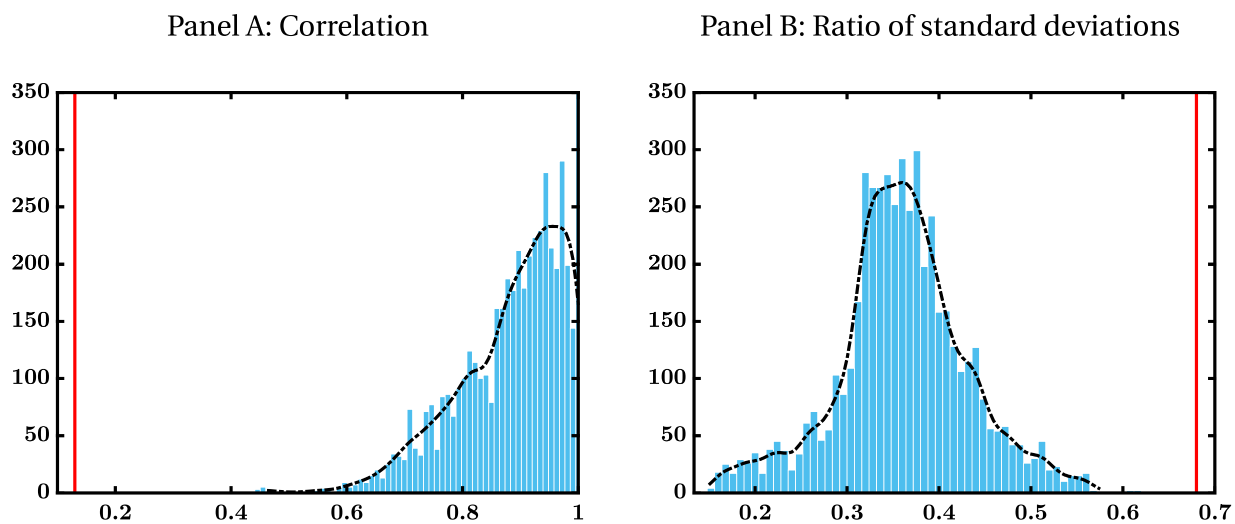

Figure A1: Histogram of Correlation and Relative Standard Deviations of Aggregate Markups and Output in Model-Simulated Data

NOTE: Kernel density of $\rho(\Delta \log \mu_t, \Delta \log Y_t)$, the correlation coefficient between aggregate markups and aggregate output, and $\sigma(\Delta \log \mu_t)/\sigma(\Delta \log Y_t)$, the ratio of standard deviation of aggregate markups and aggregate output, on model-simulated data based on 5,000 repetitions of 25 period samples. Vertical redlines show the empirical estimates.

F Alternative Calibration Results

In this section, we reproduce the quantitative results for various alternative calibration of the preference parameters ε. For each calibration, we choose the remaining parameters to match the same targets of Table I as in our baseline calibration with $\varepsilon = 5$.

Table A17: Firm Markup and Sector Output

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	($\sigma = 2.01$ and $\varepsilon = 7$)		($\sigma = 1.92$ and $\varepsilon = 6$)		($\sigma = 1.80$ and $\varepsilon = 5$)		($\sigma = 1.66$ and $\varepsilon = 4$)	
Dependent variable:	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$
$\log Y_{kt}$	-0.001		-0.001		-0.001		-0.001	
$\log Y_{kt} * s_{kit}$	0.272		0.236		0.265		0.264	
$\Delta \log Y_{kt}$		-0.001		-0.001		-0.001		-0.001
$\Delta \log Y_{kt} * s_{kit}$		0.281		0.227		0.247		0.248
Firm FE	Y	N	Y	N	Y	N	Y	N
Year FE	Y	N	Y	N	Y	N	Y	N

NOTE: μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t and $\log Y_{kt}$ sector k's (log) value-added in year t. $\Delta\log(\mu_{kit})$ is the first-difference of (log) gross markup in year t for firm i sector k , s_{kit} gives the market share of firm i in sector k, year t and $\Delta \log Y_{kt}$ is the first-difference of sector k (log) value-added in year t. All columns report estimates based on model-simulated data for various choices of elasticities σ and ε.

Table A18: Firm Market Share and Sector Output

	(1) ($\sigma = 2.01$ and $\varepsilon = 7$)			(2) ($\sigma = 1.92$ and $\varepsilon = 6$)			(3) ($\sigma = 1.8$ and $\varepsilon = 5$)			(4) ($\sigma = 1.66$ and $\varepsilon = 4$)		
	all	small	large	all	small	large	all	small	large	all	small	large
Dep. var.	$\log s_{kit}$											
$\log Y_{kt}$	-3.404	-3.419	0.583	-2.890	-2.900	0.273	-2.613	-2.621	0.535	-1.977	-1.979	0.146
Firm FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Dep. var.	$\Delta\log s_{kit}$											
$\Delta \log Y_{kt}$	-3.404	-3.412	0.355	-2.925	-2.932	0.253	-2.585	-2.591	0.274	-1.952	-1.956	0.283
Firm FE	N	N	N	N	N	N	N	N	N	N	N	N
Year FE	N	N	N	N	N	N	N	N	N	N	N	N

NOTE: s_{kit} gives the market share of firm i in sector k, year t, and $\log Y_{kt}$ is the deviation of sector k (log) value-added in year t from its mean. $\Delta \log s_{kit}$ gives the first-difference of (log) market share of firm i in sector k, year t, and $\Delta \log Y_{kt}$ is the first-difference of sector k (log) value-added in year t. $\bar{s}_{ki}$ is the average market share of firm i in market k. All columns report estimates based on model-simulated data for various choices of elasticities σ and ε.

Table A19: Sector Markup and Sector Output

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	($\sigma = 2.01$ and $\varepsilon = 7$)		($\sigma = 1.92$ and $\varepsilon = 6$)		($\sigma = 1.8$ and $\varepsilon = 5$)		($\sigma = 1.66$ and $\varepsilon = 4$)	
Dependent variable:	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$
$\Delta \log Y_{kt}$	0.091		0.105		0.110		0.103	
	(0.035)		(0.041)		(0.040)		(0.046)	
$\log \widehat{Y}_{kt}$		0.096		0.110		0.117		0.120
		(0.032)		(0.037)		(0.035)		(0.039)
Sector FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log) change (columns 1, 3, 5 and 7), and **?** trend deviation of markup (columns 2, 4, 6 and 8), ($\Delta \log \mu_{kt}$, $\log \widehat{\mu}_{kt}$ resp.) on sector value-added ($\Delta \log Y_{kt}$, $\log \widehat{Y}_{kt}$ resp.). All columns report estimates based on model-simulated data for various choices of elasticities σ and ε. The point estimates for these column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A20: Sector Concentration and Sector Output

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	($\sigma = 2.01$ and $\varepsilon = 7$)		($\sigma = 1.92$ and $\varepsilon = 6$)		($\sigma = 1.8$ and $\varepsilon = 5$)		($\sigma = 1.66$ and $\varepsilon = 4$)	
Dependent variable:	$\Delta \log HHI_{kt}$	$\log \widehat{HHI}_{kt}$	$\Delta \log HHI_{kt}$	$\log \widehat{HHI}_{kt}$	$\Delta \log HHI_{kt}$	$\log \widehat{HHI}_{kt}$	$\Delta \log HHI_{kt}$	$\log \widehat{HHI}_{kt}$
$\Delta \log Y_{kt}$	0.431		0.455		0.533		0.548	
	(0.193)		(0.213)		(0.235)		(0.346)	
$\log \widehat{Y}_{kt}$		0.530		0.565		0.726		0.737
		(0.214)		(0.259)		(0.288)		(0.356)
Sector FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log) change (columns 1, 3, 5 and 7), and **?** trend deviation of HHI (columns 2, 4, 6 and 8), ($\Delta \log HHI_{kt}$, $\log \widehat{HHI}_{kt}$ resp.) on sector value-added ($\Delta \log Y_{kt}$, $\log \widehat{Y}_{kt}$ resp.). All columns report estimates based on model-simulated data for various choices of elasticities σ and ε. The point estimates for these column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A21: Sector Markup and Aggregate Output

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	($\sigma = 2.01$ and $\varepsilon = 7$)		($\sigma = 1.92$ and $\varepsilon = 6$)		($\sigma = 1.8$ and $\varepsilon = 5$)		($\sigma = 1.66$ and $\varepsilon = 4$)	
Dependent variable:	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$
$\Delta \log Y_t$	0.140		0.138		0.165		0.169	
	(0.104)		(0.102)		(0.101)		(0.095)	
$\log \widehat{Y}_t$		0.144		0.146		0.169		0.171
		(0.107)		(0.106)		(0.119)		(0.099)
Sector FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector k's markup in year t in first-differences ($\Delta \log \mu_{kt}$, in columns 1, 3, 5 and 7) and **?** trend deviation ($\log \widehat{\mu}_{kt}$, in columns 2, 4, 6 and 8) on (log) aggregate real value-added in year t in either first-differences or **?** trend deviation ($\Delta \log Y_t$ and $\log \widehat{Y}_t$, resp.). All columns report estimates based on model-simulated data for various choices of elasticities σ and ε. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A22: Sector Markup and Aggregate Output with Aggregate Shocks

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	($\sigma = 2.01$ and $\varepsilon = 7$)		($\sigma = 1.92$ and $\varepsilon = 6$)		($\sigma = 1.8$ and $\varepsilon = 5$)		($\sigma = 1.66$ and $\varepsilon = 4$)	
Dependent variable:	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$	$\Delta \log \mu_{kt}$	$\log \widehat{\mu}_{kt}$
$\Delta \log Y_t$	0.005		0.006		0.008		0.012	
	(0.012)		(0.022)		(0.042)		(0.035)	
$\log \widehat{Y}_t$		0.010		0.010		0.017		0.022
		(0.016)		(0.027)		(0.044)		(0.036)
Sector FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector k's markup in year t in first-differences ($\Delta \log \mu_{kt}$, in columns 1, 3, 5 and 7) and **?** trend deviation ($\log \widehat{\mu}_{kt}$, in columns 2, 4, 6 and 8) on (log) aggregate real value-added in year t in either first-differences or **?** trend deviation ($\Delta \log Y_t$ and $\log \widehat{Y}_t$, resp.). All columns report estimates based on model-simulated data for various choices of elasticities σ and ε. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) with aggregate productivity shocks chosen to match the aggregate volatility of output in the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A23: Aggregate Markup and Aggregate Output

	(1)			(2)			(3)			(4)		
	($\sigma = 2.01$ and $\varepsilon = 7$)			($\sigma = 1.92$ and $\varepsilon = 6$)			($\sigma = 1.8$ and $\varepsilon = 5$)			($\sigma = 1.66$ and $\varepsilon = 4$)		
	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$
$\log \widehat{Y}_t$	0.71	1	1	0.77	1	1	0.83	1	1	1.04	1	1
$\log \widehat{\mu}_t$	0.25	0.35	0.93	0.30	0.39	0.93	0.30	0.36	0.91	0.36	0.35	0.90
$\Delta \log Y_t$	0.63	1	1	0.64	1	1	0.69	1	1	0.86	1	1
$\Delta \log \mu_t$	0.21	0.33	0.95	0.24	0.38	0.94	0.25	0.36	0.91	0.30	0.35	0.91

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for deviations from trend computed as in **?** of (log) aggregate output $\log \widehat{Y}_t$ and (log) aggregate markup $\log \widehat{\mu}_t$, and, for log first-difference of aggregate output $\log \Delta Y_t$ and aggregate markup $\log \Delta \mu_t$. Column (1-4) reports the median over 5,000 simulated samples of 25 years for each alternative calibration.

Table A24: Aggregate Markup and Aggregate Output with Aggregate Shocks

	(1) ($\sigma = 2.01$ and $\varepsilon = 7$)			(2) ($\sigma = 1.92$ and $\varepsilon = 6$)			(3) ($\sigma = 1.8$ and $\varepsilon = 5$)			(4) ($\sigma = 1.66$ and $\varepsilon = 4$)		
	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$
$\log \widehat{Y}_t$	3.16	1	1	3.16	1	1	3.16	1	1	3.16	1	1
$\log \widehat{\mu}_t$	0.25	0.08	0.25	0.25	0.08	0.22	0.30	0.09	0.29	0.36	0.11	0.35
$\Delta \log Y_t$	3.28	1	1	3.28	1	1	3.28	1	1	3.28	1	1
$\Delta \log \mu_t$	0.21	0.06	0.21	0.25	0.08	0.22	0.26	0.08	0.27	0.30	0.09	0.29

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for deviations from trend computed as in **?** of (log) aggregate output $\log \widehat{Y}_t$ and (log) aggregate markup $\log \widehat{\mu}_t$, and, for log first-difference of aggregate output $\Delta \log Y_t$ and aggregate markup $\Delta \log \mu_t$. Column (1-4) reports the median over 5,000 simulated samples of 25 years for each alternative calibration with aggregate productivity shocks chosen to match the aggregate volatility of output in the French data.