# Contents

# S1 Nomenclature

### Acronyms and Abbreviations

| | |
|---|---|
| AAC | Aerodynamic Aerosol Classifier |
| AMS | Aerosol Mass Spectrometer |
| APS | Aerodynamic Particle Sizer |
| CMD | Count Median Diameter |
| CPC | Condensation Particle Counter |
| CPD | Classes per Decade |
| DAPS | Differential Aerodynamic Particle Sizer |
| DMA | Differential Mobility Analyzer |
| DMPS | Differential Mobility Particle Sizer |
| DOS | Bis(2-Ethylhexyl) Sebacate |
| E-SPART | Electrical Single Particle Aerodynamic Relaxation Time |
| ELPI | Electrical Low Pressure Impactor |
| FWHM | Full Width at Half Maximum |
| GSD | Geometric Standard Deviation |
| HEPA | High Efficiency Particulate Arrestance |
| HF | High-flow ($Q_\mathrm{a} = 1.5$ and $Q_\mathrm{sh} = 15$ L/min) |
| LF | Low-flow ($Q_\mathrm{a} = 0.3$ and $Q_\mathrm{sh} = 3$ L/min) |
| LT | Limited Trajectory |
| MOUDI | Micro-Orifice Uniform Deposit Impactor |
| PS | Particle Streamline |
| PSL | Polystyrene Latex |
| SI | Supplemental Information |
| SMPS | Scanning Mobility Particle Sizer |
| SPART | Single Particle Aerodynamic Relaxation Time |
| TOF | Time-of-Flight |
| UDA | Up-Down Agreement |

### Classifier Properties

| | |
|---|---|
| $\dot{\omega}$ | Instantaneous angular acceleration/deceleration of AAC classifier (rad s$^{-2}$) |
| $\omega$ | Instantaneous angular speed of AAC classifier (rad s$^{-1}$) |
| $t_\mathrm{f}$ | Resident time of particles in classifier assuming plug flow (s) |
| $E$ | Force field within classifier (N kg$^{-1}$ for centrifugal) |
| $F$ | Force within classifier (N) |
| $K$ | Time integral of the angular speed function of the AAC classifier (s$^{-1}$) |

| | |
|---|---|
| $L$ | Classification length of classifier (m) |
| $Q$ | Flow rate (m$^3$ s$^{-1}$) |
| $r$ | Radial position within classifier (m) |
| $r_1$ | Inner radius of classifier (m) |
| $r_2$ | Outer radius of classifier (m) |
| $U$ | Mean speed of gas flow in classifier (m/s) |
| $z$ | Axial position within classifier (m) |

**Detector Properties**

| | |
|---|---|
| $\eta$ | Non-dimensional counting efficiency of particle detector |
| $t_\mathrm{c}$ | Counting time of particle detector (s) |
| $t_\mathrm{det}$ | Response time of particle detector, including the transport time of the particles within detector (s) |

**Transfer Function Properties**

| | |
|---|---|
| $\bar{\Omega}$ | Non-dimensional, average transfer function of classifier over counting time of particle detector |
| $\bar{\tau}^*$ | Average particle relaxation time setpoint of AAC over counting time of particle detector (s) |
| $\beta$ | Non-dimensional flow parameter of classifier |
| $\beta^*$ | Non-dimensional deconvolution parameter of transfer function |
| $d_\mathrm{a}^*$ | Particle aerodynamic diameter setpoint of AAC (m) |
| $\lambda_\Omega$ | Non-dimensional transmission efficiency of classifier |
| $\mu_\Omega$ | Non-dimensional width factor of transfer function |
| $N_\mathrm{det}$ | Particle number concentration that passes through the classifier (cm$^{-3}$) |
| $\Omega$ | Non-dimensional transfer function of classifier |
| $\tau^*$ | Particle relaxation time setpoint of AAC (s) |
| $\tau_x$ | Particle relaxation time of $\Omega_\mathrm{AAC,sc}$ boundary $x$ (s) |
| $\tau_\mathrm{max}$ | Maximum particle relaxation time corresponding to non-zero value of transfer function (s) |
| $\tau_\mathrm{min}$ | Minimum particle relaxation time corresponding to non-zero value of transfer function (s) |
| $\tau_{x,\mathrm{tc}}$ | Particle relaxation time of $\Omega_\mathrm{AAC,sc}$ boundary $x$ shifted by $t_\mathrm{c}$ (s) |
| $c_\mathrm{sc}$ | Constant portion of $K_\mathrm{sc}$ when in terms of $t_\mathrm{m}$ (s$^{-1}$) |
| $c_{\tau x,\mathrm{B}}$ | Grouped constant of boundary $x$ for $\Omega_\mathrm{AAC,sc,PS,B}$ (s) |
| $c_{\tau x}$ | Grouped constant of boundary $x$ for $\Omega_\mathrm{AAC,sc,LT}$ (s) |
| $c_{Bx}$ | Constant $x$ of $\Omega_\mathrm{AAC,sc,PS,B}$ (Non-dimensional or s$^{-1}$ depending on the constant) |
| $c_{fx}$ | Constant $x$ of $\Omega_\mathrm{AAC,sc,LT}$ (Non-dimensional or s$^{-1}$ depending on the constant) |
| $f_x$ | Non-dimensional component $x$ of transfer function based on limited trajectory theory |

**Math Properties**

| | |
|---|---|
| $\delta\left(x-a\right)$ | Dirac delta function |
| $\mathcal{EI}\left(x\right)$ | Exponential integral function |
| $\gamma$ | Euler's Constant (0.5772156649) |

$\mathcal{H}(x-a)$ Heaviside function

$\mathcal{F}(x)$ Generalized hypergeometric function

$\mathcal{J}(x)$ Integral of $j(x)$ with respect to $x$

$j(x)$ Generic mathematical function that is only a function of $x$

$a$ Generic constant for integral forms or definition of functions

$b$ Generic constant for integral forms

$C$ Constant that is any real number and greater than one

$p$ Constant that is any real number

$u$ Generic variable for substitutions or integration by parts

$v$ Generic variable for integration by parts

$w$ Generic variable for common equation forms or integration by parts

$x$ Generic variable for generic functions or for subscripts of boundary constants of transfer function

$y$ Generic variable for common equation forms or integration by parts or subscripts of boundary constants of transfer function

**Scanning Properties**

$\tau_{\mathrm{sc}}$ Time constant of scan (s)

$t_{\mathrm{d}}$ Delay time for particles to travel from classifier outlet to particle detector and be detected (s)

$t_{\mathrm{in}}$ Inlet time of particles during scan (s)

$t_{\mathrm{m}}$ Measurement time of particles (i.e. time exit classifier) during scan (s)

$t_{\mathrm{r}}$ Time the particle detector reports its concentration measurement relative to the start of the scan (s)

$t_{\mathrm{sc}}$ Scan time (s)

$\omega_{\mathrm{E}}$ Angular speed of AAC classifier at end of scan (rad $\mathrm{s}^{-1}$)

$\omega_{\mathrm{S}}$ Angular speed of AAC classifier at start of scan (rad $\mathrm{s}^{-1}$)

$n_{\mathrm{sc}}$ Non-dimensional number of measurements collected during scan

$t$ Time (s)

$t_{\mathrm{t}}$ Residence time of classified particles in the tubing between the classifier outlet and detector inlet (s)

**Other Instrument Properties**

$\bar{V}_{\mathrm{t}}$ Average flow velocity within the tube between classifier outlet and inlet of particle detector (m/s)

$\mathrm{d_t}$ Inner diameter of tube between classifier outlet and inlet of particle detector (m)

$L_{\mathrm{t}}$ Length of tube between classifier outlet and inlet of particle detector (m)

**Aerosol Properties**

$\alpha_{c_c}$ Non-dimensional coefficient to calculate the Cunningham slip correction factor (2.33)

$\beta_{c_c}$ Non-dimensional coefficient to calculate the Cunningham slip correction factor (0.966)

$d_{\mathrm{a}}$ Aerodynamic diameter of particle (m)

$d_{\mathrm{m}}$ Mobility diameter of particle (m)

$\frac{\mathrm{d}N}{\mathrm{d}\log d_{\mathrm{a}}}$ Spectral density of aerosol in terms of particle aerodynamic diameter ($\mathrm{cm}^{-3}$)

| | |
|---|---|
| $\frac{\mathrm{d}N}{\mathrm{d}\log\tau}$ | Spectral density of aerosol in terms of particle relaxation time ($\mathrm{cm}^{-3}$) |
| $\gamma_{c_c}$ | Non-dimensional coefficient to calculate the Cunningham slip correction factor (0.4985) |
| $\lambda$ | Mean free path of the surrounding gas (m) |
| $\mu$ | Viscosity of surrounding gas (Pa s or kg $\mathrm{s}^{-1}$ $\mathrm{m}^{-1}$) |
| $N_{\mathrm{tot}}$ | Total number concentration of particles ($\mathrm{cm}^{-3}$) |
| $\rho_{\mathrm{eff}}$ | Effective density of particle (kg $\mathrm{m}^{-3}$) |
| $\rho_{\mathrm{o}}$ | Unit density (1000 kg $\mathrm{m}^{-3}$) |
| $\tau$ | Relaxation time of particle (s) |
| $B$ | Mechanical mobility of particle (s $\mathrm{kg}^{-1}$) |
| $C_c$ | Non-dimensional Cunningham slip correction factor |
| $m$ | Mass of particle (kg) |
| $N$ | Particle number concentration of the aerosol as a function of particle property, such as $\tau$ or $d_{\mathrm{a}}$ ($\mathrm{cm}^{-3}$) |
| $P$ | Pressure of surrounding gas (Pa) |
| $T$ | Temperature of surrounding gas (K) |

**Frequent Subscripts, Superscripts and Accents**

| | |
|---|---|
| $'$ | Notation to distinguish variable of integration from similar variable in integral limits |
| $*$ | Denotes particle property corresponding to set point of classifier (i.e. the peak of its transfer function) |
| $\bar{}$ | Denotes average of parameter over counting time ($t_{\mathrm{c}}$) of particle detector |
| $\dot{}$ | Denotes derivative of parameter with respect to time |
| tc | Shifted by counting time of particle detector |
| $\tau x$ | Boundary $x$ of transfer function in terms of particle relaxation time |
| $c_c$ | Cunningham Slip |
| $i$ | Particular instant of an parameter |
| + | Assumes $\tau_{\mathrm{sc}} > 0$ |
| - | Assumes $\tau_{\mathrm{sc}} < 0$ |
| a | Aerodynamic or Aerosol |
| B | Balanced classifier flows |
| c | Centrifugal or Counting or Critical |
| d | Drag or Delay |
| det | Particle detector |
| down | Down scan |
| E | End of scan |
| exh | Exhaust |
| F$x$ | Form $x$ of equation/integral |
| I | Idealized |
| in | Classifier inlet |

| | |
|---|---|
| L | Lower |
| LT | Limited Trajectory |
| m | Mobility or Measurement |
| max | Maximum |
| min | Minimum |
| NI | Non-idealized |
| out | Classifier outlet |
| PS | Particle Streamline |
| r | Radial direction |
| S | Start of scan |
| s | Sample |
| sc | Scanning operation |
| sh | Sheath |
| ss | Steady-state/stepping operation |
| t | Tube between classifier outlet and inlet of particle detector |
| U | Upper |
| UB | Unbalanced classifier flows |
| up | Up scan |

## S1.1 Simplified Notation of Parameter Subscripts

All of the parameters in each SI section and its subsections apply to a particular instance of AAC operation and theory. These different instances are categorized as follows:

Table S1.1: Categorized instances of AAC operation and theory.

| AAC Category | Instance | Subscript Notation | Description |
|---|---|---|---|
| Operation | Steady-State | ss | AAC at a constant classifier angular speed and sheath flow (i.e. constant setpoint) |
| | Scanning | sc | Angular speed of the AAC classifier varies continuously with time (i.e. varying setpoint) |
| Transfer Function Theory | Limited Trajectory | LT | Determines the minimum and maximum particle and classifier properties that allow particles to pass through the AAC classifier |
| | Particle Streamline | PS | Determines the centrifugal flux function to estimate the transfer function of the AAC |
| Classification | Idealized | I | Neglects particle losses and broadening of AAC transfer function |
| | Non-Idealized | NI | Considers particle losses and broadening of AAC transfer function |
| Classifier Flows | Balanced | B | $Q_{\mathrm{sh}} = Q_{\mathrm{exh}}$ and $Q_{\mathrm{a}} = Q_{\mathrm{s}}$ |
| | Unbalanced | UB | $Q_{\mathrm{sh}} \neq Q_{\mathrm{exh}}$ and $Q_{\mathrm{a}} \neq Q_{\mathrm{s}}$ |

For readability, the title and footer of each SI section are used to clarify the common instances considered within the section and its subsections. This approach allows only the subscripts required to differentiate multiple instances of the same parameter within the same section to be used.

For example, consider the transfer function of the AAC denoted as $\Omega_{\mathrm{AAC}}$. If only one combination of instances is included within a section, such as the AAC operating at steady-state (ss) considering idealized (I), limited trajectory (LT) theory, the AAC transfer function within this section will be denoted as $\Omega_{\mathrm{AAC}}$ rather than $\Omega_{\mathrm{AAC,ss,LT,I}}$. To avoid the ambiguity of this reduced notation, the title and footer of this section would include steady-state (ss), idealized (I) and limited trajectory (LT).

However, if different instances are considered within the same section, such as the AAC operating at steady-state (ss) or scanning (sc), with both operating modes considering idealized (I), limited trajectory (LT) theory, the AAC transfer functions within this section will be denoted as $\Omega_{\mathrm{AAC,ss}}$ and $\Omega_{\mathrm{AAC,sc}}$, respectively, rather than $\Omega_{\mathrm{AAC,ss,LT,I}}$ and $\Omega_{\mathrm{AAC,sc,LT,I}}$, respectively. Similarly, to avoid the ambiguity of this reduced notation, the title and footer of this section would include idealized (I) and limited trajectory (LT). Finally, if the parameter applies to both options of a particular instance, for example it represents both balanced (B) and unbalanced (UB) classifier flows, no differentiation for this instance is included within the subscript of the parameter or section title/footer.

# S2 Angular Speed Profile of AAC Classifier during Scanning

Note: Similar to the other sections in the SI, this section and its subsections use simplified notation of the parameter subscripts, as outlined in Section S1.1, based on the common AAC operation and theory instances included within the title and footer of this section.

This section expands the scanning DMA theory developed by Wang and Flagan (1990) for the electrostatic field generated by the changing voltage of the DMA classifier to the centrifugal force field generated by the changing angular speed ($\omega$) of the AAC classifier. First, consider that the angular speed of the AAC classifier varies continuously with time ($t$) as follows:

$$\omega = \omega(t). \tag{S2.1}$$

The centrifugal force field ($E_\mathrm{c}$) generated inside the AAC classifier at time $t$ and radial position $r$ can then be calculated by:

$$E_\mathrm{c}(t,\, r) = \omega^2(t)\, r. \tag{S2.2}$$

The angular speed function ($\omega(t)$) of the classifier must produce a proportional change in the centrifugal force field ($E_\mathrm{c}$) over time $t$, and the resulting critical particle trajectories it induces, that are independent of the times that the particles arrive at the classifier inlet ($t_\mathrm{in}$) as follows:

$$\frac{E_\mathrm{c}(t_\mathrm{in} + t,\, r)}{E_\mathrm{c}(t_\mathrm{in},\, r)} = j(t), \tag{S2.3}$$

where $j$ is a generic function that is only a function of $t$. As an initial guess, define the angular speed function ($\omega(t)$) of the classifier as:

$$\omega^2(t) = \omega_\mathrm{S}^2\, C^{\frac{t}{\tau_\mathrm{sc}}}, \tag{S2.4}$$

where $\omega_\mathrm{S}$ is the classifier speed at the start of the scan (i.e. $t = 0$), $C$ is a constant that is any real number greater than one, and $\tau_\mathrm{sc}$ is the time constant of the scan. To check if this speed profile (Equation S2.4) satisfies the criteria, substitute it into Equation S2.3, and simplify as follows:

$$\frac{E_\mathrm{c}(t_\mathrm{in} + t,\, r)}{E_\mathrm{c}(t_\mathrm{in},\, r)} = \frac{r\, \omega_\mathrm{S}^2\, C^{\frac{t_\mathrm{in}+t}{\tau_\mathrm{sc}}}}{r\, \omega_\mathrm{S}^2\, C^{\frac{t_\mathrm{in}}{\tau_\mathrm{sc}}}} = C^{\frac{t}{\tau_\mathrm{sc}}} = j(t). \tag{S2.5}$$

Therefore, this speed profile (Equation S2.4) satisfies Equation S2.3 and results in a ratio of centrifugal force fields that is independent of $t_\mathrm{in}$. Over the scan time ($t_\mathrm{sc}$), the classifier speed must change from $\omega_\mathrm{S}$ to $\omega_\mathrm{E}$. Substituting these values into the speed profile (Equation S2.4) and solving for the time constant ($\tau_\mathrm{sc}$) of the scan:

$$\omega_\mathrm{E}^2 = \omega_\mathrm{S}^2\, C^{\frac{t_\mathrm{sc}}{\tau_\mathrm{sc}}} \rightarrow \boldsymbol{\tau_\mathrm{sc}} = \frac{\boldsymbol{t_\mathrm{sc}\ln(C)}}{\boldsymbol{2\ln\left(\frac{\omega_\mathrm{E}}{\omega_\mathrm{S}}\right)}}. \tag{S2.6}$$

Furthermore, substituting this scan constant (Equation S2.6) into the speed profile (Equation S2.4), and simplifying:

$$\omega^2(t) = \omega_\mathrm{S}^2\, C^{\frac{t}{\frac{t_\mathrm{sc}\ln(C)}{2\ln\left(\frac{\omega_\mathrm{E}}{\omega_\mathrm{S}}\right)}}} \rightarrow 2\ln\left(\frac{\omega(t)}{\omega_\mathrm{S}}\right) = \frac{t}{\frac{t_\mathrm{sc}\ln(C)}{2\ln\left(\frac{\omega_\mathrm{E}}{\omega_\mathrm{S}}\right)}}\ln(C) \rightarrow \boldsymbol{\omega(t) = \omega_\mathrm{S}\exp\left(\frac{t\ln\left(\frac{\omega_\mathrm{E}}{\omega_\mathrm{S}}\right)}{t_\mathrm{sc}}\right)}. \tag{S2.7}$$

Therefore, the required angular speed of the classifier during a scan is independent of the actual value of the positive, real number constant $C$. It is also interesting to note that defining this angular speed function to the power of $p$, as follows, simplifies to the same result as the time constant ($\tau_\mathrm{sc}$) of the scan compensates for this change by scaling by a factor of $1/p$:

$$\omega_\mathrm{E}^p = \omega_\mathrm{S}^p\, C^{\frac{t_\mathrm{sc}}{\tau_\mathrm{sc}}} \rightarrow \boldsymbol{\tau_\mathrm{sc} = \frac{t_\mathrm{sc}\ln(C)}{p\ln\left(\frac{\omega_\mathrm{E}}{\omega_\mathrm{S}}\right)}}, \tag{S2.8}$$

where $p$ is a constant that is any real number. For consistency with the previous inversion studies of the scanning DMA and the centrifugal force field generated in the AAC, this study will use $C = e$ and $p = 2$, as follows:

$$\omega^2(t) = \omega_\mathrm{S}^2 \exp\left(\frac{t}{\tau_\mathrm{sc}}\right), \tag{S2.9}$$

and

$$\tau_\mathrm{sc} = \frac{t}{2\ln\left(\frac{\omega(t)}{\omega_\mathrm{S}}\right)} = \frac{t_\mathrm{sc}}{2\ln\left(\frac{\omega_\mathrm{E}}{\omega_\mathrm{S}}\right)}. \tag{S2.10}$$

Therefore, the AAC completing an up scan (i.e. $\omega_\mathrm{E} > \omega_\mathrm{S}$) is reflected in a positive scan time constant (i.e. $\tau_\mathrm{sc} > 0$), which accelerates the classifier and classifies increasingly smaller particle relaxation times over the scan duration. A down scan (i.e. $\omega_\mathrm{E} < \omega_\mathrm{S}$) is reflected in a negative scan time constant (i.e. $\tau_\mathrm{sc} < 0$), which decelerates the classifier and classifies increasingly larger particle relaxation times over the scan duration. The required acceleration/deceleration profile during a scan is the derivative of the speed profile (Equation S2.9) as follows:

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = \dot{\omega} = \frac{\omega_\mathrm{S}}{2\tau_\mathrm{sc}}\exp\left(\frac{t}{2\tau_\mathrm{sc}}\right). \tag{S2.11}$$

The *required speed* ($\omega(t)$) and *required acceleration* ($\dot{\omega}(t)$) profiles for the AAC classifier over an example up or down scan is shown in Figure S2.1, while the *acceleration/deceleration capacity* of multiple AACs (i.e. the maximum acceleration/deceleration the classifier can actually achieve) and the variation between them is shown in Figure S2.2. As discussed in the main text, the acceleration/deceleration capacity varies slightly between different AACs due to small differences in drive belt tension, as well as changes in bearing friction and motor efficiency over the lifetime of the instrument. To account for this variation, the acceleration/deceleration capacities of each AAC should be periodically updated by measuring the maximum acceleration and deceleration the classifier can achieve as a function of its angular speed, such as those shown in Figure S2.2.

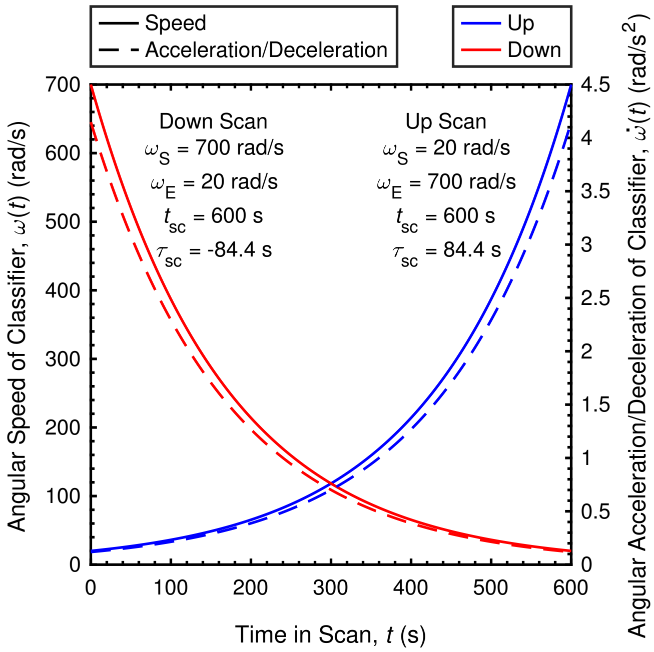


Figure S2.1: Example of the required speed and acceleration profile for the AAC classifier during a 600 s scan from 20 rad/s to 700 rad/s.

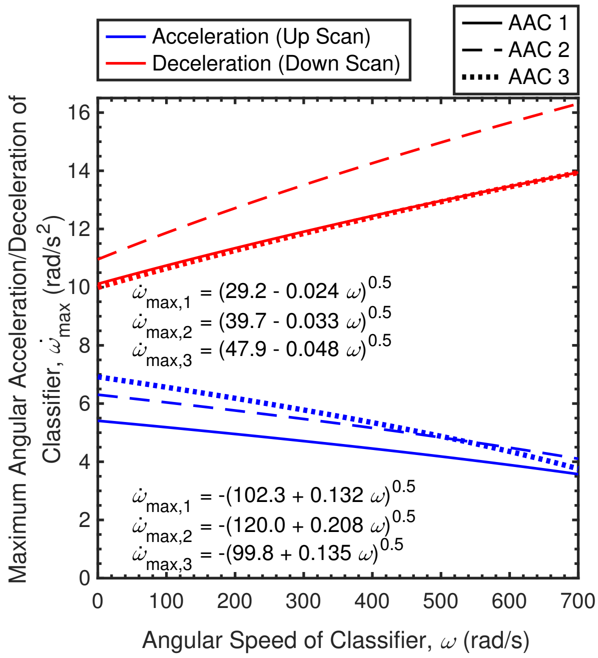


Figure S2.2: Acceleration/deceleration capacity of multiple AACs and the variation between them.

The speed of the AAC during maximum acceleration or deceleration (i.e. maximum motor or electrical brake current) is captured by a quadratic relationship with respect to time. Therefore, the maximum acceleration or deceleration the AAC achieved as a function of its angular speed (i.e. its *acceleration/deceleration capacity*) is determined by isolating time in the quadratic equation and substituting it into the derivative (with respect to time) of the quadratic equation.

## S2.1 Minimum Scan Time ($t_{\mathrm{sc,min}}$)

The scan time ($t_{\mathrm{sc}}$) of the AAC is limited by the angular acceleration the classifier can achieve as a function of its angular speed (i.e. acceleration/deceleration capacity of the classifier). The relationship between these parameters can be determined by substituting the required speed profile over a scan ($\omega(t)$ defined by Equation S2.9) into the corresponding required acceleration profile ($\dot{\omega}(t)$ defined by Equation S2.11) as follows:

$$\dot{\omega}(t) = \frac{\omega(t)}{2\tau_{\mathrm{sc}}}. \tag{S2.12}$$

Therefore, the required angular acceleration of the classifier ($\dot{\omega}(t)$) is linearly proportional to its required angular speed ($\omega(t)$) by a factor of $1/(2\tau_{\mathrm{sc}})$, while the acceleration capacity of the classifier is proportional to the square root of its angular speed as shown in Figure S2.2. This factor for required acceleration (i.e. $1/(2\tau_{\mathrm{sc}})$) can be related to the scan time ($t_{\mathrm{sc}}$) by substituting the definition of $\tau_{\mathrm{sc}}$ (Equation S2.10) into Equation S2.12 and rearranging as follows:

$$\dot{\omega}(t) = \frac{\omega(t)}{2\frac{t_{\mathrm{sc}}}{2\ln\left(\frac{\omega_{\mathrm{E}}}{\omega_{\mathrm{S}}}\right)}} \rightarrow \boldsymbol{t_{\mathrm{sc}} = \frac{\omega(t)}{\dot{\omega}(t)}\ln\left(\frac{\omega_{\mathrm{E}}}{\omega_{\mathrm{S}}}\right)}. \tag{S2.13}$$

Therefore, the minimum scan time ($t_{\mathrm{sc,min}}$) is based on the maximum angular acceleration or deceleration ($\dot{\omega}_{\mathrm{max}}$) the AAC can achieve during an up or down scan, respectively, as follows:

$$t_{\mathrm{sc,min}} = \frac{\omega(t)}{\dot{\omega}_{\mathrm{max}}\left(\omega(t)\right)}\ln\left(\frac{\omega_{\mathrm{E}}}{\omega_{\mathrm{S}}}\right). \tag{S2.14}$$

$\dot{\omega}_{\mathrm{max}}$ is a function of classifier speed ($\omega(t)$) to reflect both the rotational energy and friction of the classifier are related to its speed and thus dominates the maximum acceleration/deceleration capacity of the AAC. An example of these acceleration and deceleration capacities for multiple AACs and the variation between them is shown in Figure S2.2.

As shown in Figure S2.1, the required acceleration over an up scan increases. However, the friction of the spinning classifier also increases with its speed. Therefore, the acceleration capacity of the AAC decreases as an up scan progresses (as shown in Figure S2.2), while the required acceleration increases. Therefore, the minimum scan time of an up scan is the intersection of these two converging acceleration curves at the end of the up scan (i.e. $t = t_{\mathrm{sc}}$), where the required acceleration from the scan profile is the highest and the maximum acceleration capacity of the AAC is the lowest (i.e. at highest classifier speed during scan, $\omega = \omega_{\mathrm{E}}$). Substituting these values into Equation S2.14, the minimum scan time of an up scan ($t_{\mathrm{sc,min,up}}$) is:

$$t_{\mathrm{sc,min,up}} = \frac{\omega_{\mathrm{E}}}{\dot{\omega}_{\mathrm{max}}\left(\omega_{\mathrm{E}}\right)}\ln\left(\frac{\omega_{\mathrm{E}}}{\omega_{\mathrm{S}}}\right), \tag{S2.15}$$

where $\dot{\omega}_{\mathrm{max}}$ is the acceleration capacity of the AAC classifier at speed $\omega_{\mathrm{E}}$.

In contrast, as shown in Figure S2.1, the required deceleration over a down scan decreases. The deceleration capacity of the AAC also decreases over the down scan (as shown in Figure S2.2) as the friction decreases with classifier speed. Therefore, the minimum scan time of the down scan is based on two factors. First, the maximum deceleration capacity of the AAC must meet or exceed the maximum deceleration required by the scan profile. Secondly, the maximum deceleration capacity of the AAC cannot decrease faster than the deceleration required by the scan profile. However, the required deceleration decreases linearly with classifier speed and has a zero intercept (as shown by Equation S2.12), while the maximum deceleration capacity of the AAC has a nonzero intercept due to its active electric brake (as shown in Figure S2.2). Since the deceleration required over a down scan is a strictly increasing function with classifier speed, both factors can be satisfied by checking the deceleration capacity of the AAC at the start of the down scan (i.e. $t = 0$), where both the required deceleration and deceleration capacity of the AAC are the highest. Substituting these values into Equation S2.14, the minimum scan time of a down scan ($t_{\mathrm{sc,min,down}}$) is:

$$t_{\mathrm{sc,min,down}} = \frac{\omega_{\mathrm{S}}}{\dot{\omega}_{\max}\left(\omega_{\mathrm{S}}\right)} \ln\left(\frac{\omega_{\mathrm{E}}}{\omega_{\mathrm{S}}}\right), \tag{S2.16}$$

where $\dot{\omega}_{\max}$ is the deceleration capacity of the AAC classifier at speed $\omega_{\mathrm{S}}$.

# S3 Transfer Function of Steady-State or Scanning AAC: Limited Trajectory, Idealized and Uniform Axial Flow

Note: Similar to the other sections in the SI, this section and its subsections use simplified notation of the parameter subscripts, as outlined in Section S1.1, based on the common AAC operation and theory instances included within the title and footer of this section.

Based on limited trajectory theory, this section completes the following two tasks in parallel:

- Rederives/verifies the limited trajectory theory developed by Tavakoli and Olfert (2013) to describe the transfer function of the AAC at a constant classifier angular speed and sheath flow (i.e. one aerodynamic diameter setpoint); and

- Expands the limited trajectory theory developed by Wang and Flagan (1990) to derive the transfer function for the scanning DMA to the scanning AAC (i.e. considers that the angular speed $\omega$ of the AAC classifier varies continuously with time $t$).

As derived in Sections 2.1 to 2.5 of the main text, a particle with relaxation time $\tau$ will migrate from radial position $r_{\text{in}}$ to $r_{\text{out}}$ over its residence time in the classifier ($t_{\text{f}}$) as follows (restatement of Equation 14 in main text):

$$\tau = \frac{1}{K} \ln \left( \frac{r_{\text{out}}}{r_{\text{in}}} \right). \tag{S3.1}$$

For steady-state operation $K$ equals (restatement of Equation 16 in main text):

$$K_{\text{ss}} = \omega^2 \, t_{\text{f}}, \tag{S3.2}$$

while for scanning operation $K$ equals (restatement of Equation 19 in main text):

$$K_{\text{sc}} = \omega_{\text{S}}^2 \, \tau_{\text{sc}} \exp \left( \frac{t_{\text{m}}}{\tau_{\text{sc}}} \right) \left[ 1 - \exp \left( \frac{-t_{\text{f}}}{\tau_{\text{sc}}} \right) \right], \tag{S3.3}$$

where $t_{\text{f}}$ is the particle residence time along the critical trajectory in the classifier, $\tau_{\text{sc}}$ is the time constant of the scan and $t_{\text{m}}$ is the measurement time during the scan (defined by Equations 13, 10 and 18 in main text, respectively). Equation S3.1 can be rearranged by isolating $r_{\text{out}}$ as follows:

$$r_{\text{out}} = r_{\text{in}} \exp \left( K \tau \right). \tag{S3.4}$$

Based on the simplified schematic of the AAC classifier geometry shown in Figure S3.1, the particle with the largest relaxation time ($\tau_{\text{max}}$) that will pass through the classifier will start at the inner radius of the classifier (i.e. $r_{\text{in}} = r_1$) and reach its outer radius (i.e. $r_{\text{out}} = r_2$) after migration time $t_{\text{f}}$.

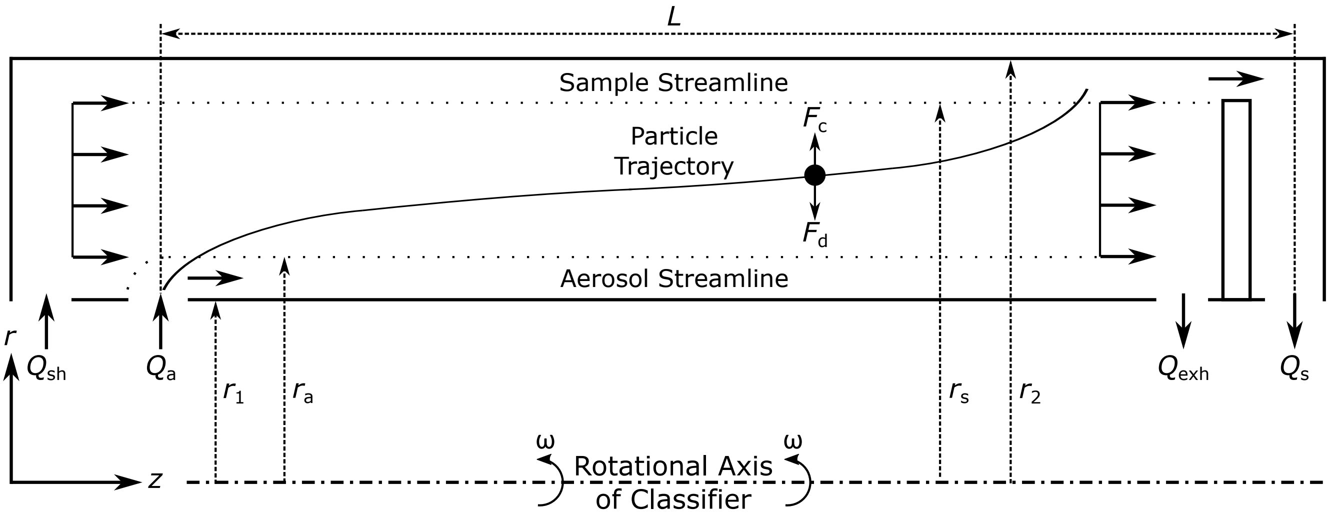


Figure S3.1: Simplified schematic of AAC classifier geometry.

Substituting these values (i.e. $r_{\mathrm{in}} = r_1$ and $r_{\mathrm{out}} = r_2$) into Equation S3.1:

$$\tau_{\max} = \frac{1}{K} \ln\left(\frac{r_2}{r_1}\right). \tag{S3.5}$$

Based on the definitions of $t_{\mathrm{f}}$ and $K_{\mathrm{ss}}$ (Equation 13 of main text and Equation S3.2, respectively), this solution (Equation S3.5) agrees with Equation 5 from Tavakoli and Olfert (2013) for the steady-state AAC. Since the axial flow is uniform, $r_{\mathrm{a}}$ and $r_{\mathrm{s}}$ (i.e. the radial positions within the AAC classifier at the outer edge of the aerosol streamlines and inner edge of the sample streamlines, respectively, as shown in Figure S3.1) can be derived in terms of the classifier dimensions and flows by rearranging the definition of uniform axial velocity (i.e. Equation 12 of the main text) as follows:

$$\begin{gathered}\frac{Q_{\mathrm{a}} + Q_{\mathrm{sh}}}{\pi(r_2^2 - r_1^2)} = \frac{Q_{\mathrm{sh}}}{\pi(r_2^2 - r_{\mathrm{a}}^2)} \rightarrow \\ r_{\mathrm{a}} = \sqrt{\frac{r_2^2\left[Q_{\mathrm{a}} + Q_{\mathrm{sh}} - Q_{\mathrm{sh}}\left(1 - r_1^2/r_2^2\right)\right]}{Q_{\mathrm{a}} + Q_{\mathrm{sh}}}},\end{gathered} \tag{S3.6}$$

and

$$\begin{gathered}\frac{Q_{\mathrm{a}} + Q_{\mathrm{sh}}}{\pi(r_2^2 - r_1^2)} = \frac{Q_{\mathrm{s}}}{\pi(r_2^2 - r_{\mathrm{s}}^2)} \rightarrow \\ r_{\mathrm{s}} = \sqrt{\frac{r_2^2\left[Q_{\mathrm{a}} + Q_{\mathrm{sh}} - Q_{\mathrm{s}}\left(1 - r_1^2/r_2^2\right)\right]}{Q_{\mathrm{a}} + Q_{\mathrm{sh}}}}.\end{gathered} \tag{S3.7}$$

Based on the simplified schematic of the AAC classifier geometry shown in Figure S3.1, the particle with the smallest relaxation time ($\tau_{\min}$) that will pass through the classifier will start at $r_{\mathrm{in}} = r_{\mathrm{a}}$ and reach $r_{\mathrm{out}} = r_{\mathrm{s}}$ after migration time $t_{\mathrm{f}}$. Substituting these values (i.e. Equations S3.6 and S3.7) into Equation S3.1, and simplifying:

$$\tau_{\min} = \frac{1}{K}\ln\left(\frac{r_{\mathrm{s}}}{r_{\mathrm{a}}}\right) = \frac{1}{K}\ln\left(\frac{\sqrt{\frac{r_2^2}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}}\left[Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{s}}\left(1-r_1^2/r_2^2\right)\right]}}{\sqrt{\frac{r_2^2}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}}\left[Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{sh}}\left(1-r_1^2/r_2^2\right)\right]}}\right)$$
$$\tau_{\min} = \frac{1}{2K}\ln\left(\frac{Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{s}}\left(1-r_1^2/r_2^2\right)}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{sh}}\left(1-r_1^2/r_2^2\right)}\right). \tag{S3.8}$$

Based on the definitions of $t_{\mathrm{f}}$ and $K_{\mathrm{ss}}$ (Equation 13 of main text and Equation S3.2, respectively), this solution (Equation S3.8) agrees with Equation 7 from Tavakoli and Olfert (2013) for the steady-state AAC. Therefore, the center of the AAC transfer function can be determined from[1]:

$$\tau^* = \frac{\tau_{\min}+\tau_{\max}}{2}. \tag{S3.9}$$

For particles with relaxation times less than $\tau_{\min}$ or greater than $\tau_{\max}$, the probability they will pass through the AAC classifier is zero. For particles with relaxation time greater than $\tau_{\min}$ and less than $\tau_{\max}$, only a fraction of them will pass through the AAC classifier, with the magnitude depending on their starting position ($r_{\mathrm{in}}$). For particles with a relaxation time greater than $\tau_{\min}$, only particles with initial position $r_{\mathrm{c,min}} < r_{\mathrm{in}} < r_{\mathrm{a}}$ will pass through the classifier. Therefore, assuming the aerosol is uniformly distributed between the inner radius and aerosol streamline ($r_1, r_{\mathrm{in}} < r_{\mathrm{a}}$, as shown in Figure S3.1), the fraction of particles between $r_{\mathrm{c,min}}$ and $r_{\mathrm{a}}$ is:

$$f_1 = \frac{r_{\mathrm{a}}^2 - r_{\mathrm{c,min}}^2}{r_{\mathrm{a}}^2 - r_1^2}. \tag{S3.10}$$

To determine $r_{\mathrm{c,min}}$, assume $r_{\mathrm{in}} = r_{\mathrm{c,min}}$ and reaches $r_{\mathrm{out}} = r_{\mathrm{s}}$ after migration time $t_{\mathrm{f}}$. Substituting these values into Equation S3.4, and simplifying:

$$r_{\mathrm{s}} = r_{\mathrm{c,min}}\exp\left(\tau K\right) \rightarrow \boldsymbol{r_{\mathrm{c,min}} = \frac{r_{\mathrm{s}}}{\exp\left(\tau K\right)}}. \tag{S3.11}$$

Substituting the definitions of $r_{\mathrm{a}}$, $r_{\mathrm{s}}$ and $r_{\mathrm{c,min}}$ (Equations S3.6, S3.7 and S3.11, respectively) into the definition of $f_1$ (Equation S3.10), and simplifying:

$$f_1 = \frac{r_{\mathrm{a}}^2 - r_{\mathrm{c,min}}^2}{r_{\mathrm{a}}^2 - r_1^2} = \frac{\frac{r_2^2}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}}\left[Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{sh}}\left(1-r_1^2/r_2^2\right)\right] - \frac{\frac{r_2^2}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}}\left[Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{s}}\left(1-r_1^2/r_2^2\right)\right]}{\exp\left(\tau K\right)^2}}{\frac{r_2^2}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}}\left[Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{sh}}\left(1-r_1^2/r_2^2\right)\right] - r_1^2}$$
$$f_1 = \frac{Q_{\mathrm{a}}+Q_{\mathrm{sh}}\,r_1^2/r_2^2 - \exp\left(-2\tau K\right)\left[Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{s}}\left(1-r_1^2/r_2^2\right)\right]}{Q_{\mathrm{a}}\left(1-r_1^2/r_2^2\right)}. \tag{S3.12}$$

[1]This equation assumes the transfer function is symmetric about $\tau^*$. While this is true for the AAC transfer function derived by particle streamline theory, the AAC transfer function derived by limited trajectory is slightly skewed to the smaller particle relaxation times. This skewness is negligible for most AAC applications, however it does become noticeable as the counting time of the particle detector increases (>2 s). This difference manifests in the average transfer function of the scanning AAC derived later in Sections S5 and S6.

Based on the definitions of $t_\mathrm{f}$ and $K_\mathrm{ss}$ (Equation 13 of main text and Equation S3.2, respectively), this solution (Equation S3.12) agrees with Equation 9 from Tavakoli and Olfert (2013) for the steady-state AAC. Similarly, assuming the aerosol is uniformly distributed between the inner radius and aerosol streamline ($r_1 < r_\mathrm{in} < r_\mathrm{a}$, as shown in Figure S3.1), the fraction of particles between $r_1$ and $r_\mathrm{c,max}$ is:

$$f_2 = \frac{r_\mathrm{c,max}^2 - r_1^2}{r_\mathrm{a}^2 - r_1^2}. \tag{S3.13}$$

To determine $r_\mathrm{c,max}$, assume $r_\mathrm{in} = r_\mathrm{c,max}$ and reaches $r_\mathrm{out} = r_2$ after migration time $t_\mathrm{f}$. Substituting these values into Equation S3.4, and simplifying:

$$r_2 = r_\mathrm{c,max} \exp{(\tau K)} \rightarrow \boldsymbol{r_\mathrm{c,max} = \frac{r_2}{\exp{(\tau K)}}}. \tag{S3.14}$$

Substituting the definitions of $r_\mathrm{a}$ and $r_\mathrm{c,max}$ (Equations S3.6 and S3.14, respectively) into the definition of $f_2$ (Equation S3.13), and simplifying:

$$\begin{aligned} f_2 &= \frac{r_\mathrm{c,max}^2 - r_1^2}{r_\mathrm{a}^2 - r_1^2} = \frac{\frac{r_2^2}{\exp{(\tau K_\mathrm{sc})^2}} - r_1^2}{\frac{r_2^2}{Q_\mathrm{a}+Q_\mathrm{sh}}\left[Q_\mathrm{a} + Q_\mathrm{sh} - Q_\mathrm{sh}\left(1 - r_1^2/r_2^2\right)\right] - r_1^2} \\ f_2 &= \frac{Q_\mathrm{a} + Q_\mathrm{sh}}{Q_\mathrm{a}}\left(\frac{\exp{(-2\tau K)} - r_1^2/r_2^2}{1 - r_1^2/r_2^2}\right). \end{aligned} \tag{S3.15}$$

Based on the definitions of $t_\mathrm{f}$ and $K_\mathrm{ss}$ (Equation 13 of main text and Equation S3.2, respectively), this solution (Equation S3.15) agrees with Equation 11 from Tavakoli and Olfert (2013) for the steady-state AAC. Finally, if the sample flowrate ($Q_\mathrm{s}$) is smaller than the aerosol flowrate ($Q_\mathrm{a}$), the transfer function ($\Omega_\mathrm{AAC}$) cannot be larger than:

$$f_3 = \frac{Q_\mathrm{s}}{Q_\mathrm{a}}. \tag{S3.16}$$

Therefore, the transfer function of the steady-steady or scanning AAC ($\Omega_\mathrm{AAC}$) based on limited trajectory and assuming plug flow is:

$$\Omega_\mathrm{AAC} = \max[0,\ \min(f_1,\ f_2,\ f_3,\ 1)], \tag{S3.17}$$

where $f_1, f_2$ and $f_3$ are defined in Equations S3.12, S3.15 and S3.16, respectively. This function reflects that the transfer function of the AAC cannot be greater than one (i.e. minimum function includes one) and that the transfer function outside the ranges of particle relaxation times corresponding to $f_1, f_2$ and $f_3$ is zero (i.e. maximum function includes zero). Therefore, the steady-state and scanning AAC transfer functions have the same form (i.e. same shape) independent of the measurement time. The steady-state transfer function has a constant value ($K_\mathrm{ss}$), while the scanning transfer function has a constant value ($K_\mathrm{sc}$) that changes based on the time the particles arrive at the classifier inlet. Please see Table 1 in the main text for a summary of these similar transfer functions.

## S3.1 Instantaneous Setpoint of Scanning AAC

The simplified form of the setpoint of the steady-state or scanning AAC can be found by substituting the definitions of $\tau_{\mathrm{min}}$ and $\tau_{\mathrm{max}}$ (Equations S3.8 and S3.5, respectively) into the definition of $\tau^*$ (Equation S3.9) as follows:

$$\tau^* = \frac{\tau_{\mathrm{min}} + \tau_{\mathrm{max}}}{2}$$
$$\tau^* = \frac{1}{2K}\left[\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2}\ln\left(\frac{Q_\mathrm{a} + Q_\mathrm{sh} - Q_\mathrm{s}\left(1 - r_1^2/r_2^2\right)}{Q_\mathrm{a} + Q_\mathrm{sh} - Q_\mathrm{sh}\left(1 - r_1^2/r_2^2\right)}\right)\right]. \tag{S3.18}$$

where $K = K_\mathrm{ss}$ or $K = K_\mathrm{sc}$ for the steady-state or scanning AAC, respectively. This equation determines the instantaneous setpoint of the scanning AAC as it does not account for the shift of its transfer function over the counting time of the downstream particle detector. This effect on the setpoint of the scanning AAC is considered later in Section S7.1.

Furthermore, the instantaneous setpoint offset between steady-state and scanning operation can be found by taking the ratio of $\tau^*_\mathrm{sc}$ to $\tau^*_\mathrm{ss}$ (Equation S3.18), while using the $K$ constants corresponding to the steady-state and scanning AAC transfer functions as follows:

$$\frac{\tau^*_\mathrm{sc}}{\tau^*_\mathrm{ss}} = \frac{K_\mathrm{ss}}{K_\mathrm{sc}}. \tag{S3.19}$$

This equation, based on the definitions of $K_\mathrm{ss}$ and $K_\mathrm{sc}$ (i.e. Equations S3.2 and S3.3, respectively), expands to:

$$\frac{\tau^*_\mathrm{sc}}{\tau^*_\mathrm{ss}} = \frac{\omega^2\, t_\mathrm{f}}{\omega_\mathrm{S}^2\, \tau_\mathrm{sc} \exp\left(\frac{t_\mathrm{m}}{\tau_\mathrm{sc}}\right)\left[1 - \exp\left(\frac{-t_\mathrm{f}}{\tau_\mathrm{sc}}\right)\right]}. \tag{S3.20}$$

## S3.2 Condensed Representation of Transfer Function for Scanning AAC

To simplify derivations in the later SI sections, the mathematical representation of the transfer function of the scanning AAC based on limited trajectory theory (Equation S3.17) as derived in Section S3 can be simplified by grouping the constants and reorganizing particular terms. Firstly, this transfer function can also be represented by a piecewise equation (as shown in Figure S3.2) as follows:

$$\Omega_{\mathrm{AAC}} = \begin{cases} f_1 & \text{if } \tau_{\min} \leq \tau \leq \tau_{13} \\ c_{f31} & \text{if } \tau_{13} < \tau < \tau_{23} \\ f_2 & \text{if } \tau_{23} \leq \tau \leq \tau_{\max} \\ 0 & \text{elsewhere} \end{cases}, \tag{S3.21}$$

where $c_{f31}$ is a constant based on $f_3$ (Equation S3.16):

$$c_{f31} = \min\left(f_3,\ 1\right) = \min\left(\frac{Q_{\mathrm{s}}}{Q_{\mathrm{a}}},\ 1\right), \tag{S3.22}$$

and $\tau_{13}$ and $\tau_{23}$ are the particle relaxation times that correspond to the intercepts of $f_1$ and $f_2$ with the middle component of the transfer function, respectively. As shown in Figure S3.2, these intercepts occur at $\tau^*$ for an AAC operating with balanced classifier flows and at some offset from $\tau^*$ for an AAC operating with unbalanced classifier flows. The offset intercept (denoted as $\tau_1$) of $f_1$ and $c_{f31}$ (Equation S3.22) can be found by solving the simplified representation of $f_1$ (Equation S3.37 as shown in Table S3.1) as follows:

$$\begin{aligned} c_{f31} &= c_{f11} \exp\left[c_{f12}\, \tau_1 \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] + c_{f13} \rightarrow \\ \tau_1 &= \frac{1}{c_{f12}} \ln\left(\frac{c_{f31} - c_{f13}}{c_{f11}}\right) \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right). \end{aligned} \tag{S3.23}$$

Similarly, the offset intercept (denoted as $\tau_2$) of $f_2$ and $c_{f31}$ (Equation S3.22) can be found by solving the simplified representation of $f_2$ (Equation S3.41 as shown in Table S3.1) as follows:

$$\begin{aligned} c_{f31} &= c_{f21} \exp\left[c_{f22}\, \tau_2 \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] + c_{f23} \rightarrow \\ \tau_2 &= \frac{1}{c_{f22}} \ln\left(\frac{c_{f31} - c_{f23}}{c_{f21}}\right) \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right). \end{aligned} \tag{S3.24}$$

Therefore considering both balanced or unbalanced classifier flows, the intercepts of the transfer function of the scanning AAC (denoted as $\tau_{13}$ and $\tau_{23}$) are:

$$\tau_{13} = \min\left(\tau_1,\ \tau^*\right), \tag{S3.25}$$

$$\tau_{23} = \max\left(\tau_2,\ \tau^*\right). \tag{S3.26}$$

Note that all of the boundary conditions (i.e. $\tau_{\min}$, $\tau_{13}$, $\tau^*$, $\tau_{23}$, $\tau_{\max}$) of the transfer function of the scanning AAC based on limited trajectory are of the form:

$$\tau_x = c_{\tau x} \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right), \tag{S3.27}$$

where $c_{\tau x}$ represents the grouped constants of each boundary $x$ shown in Figure S3.2. Therefore, the transfer function of the scanning AAC based on limited trajectory can then be simplified, as shown in Table S3.1 on the next page.

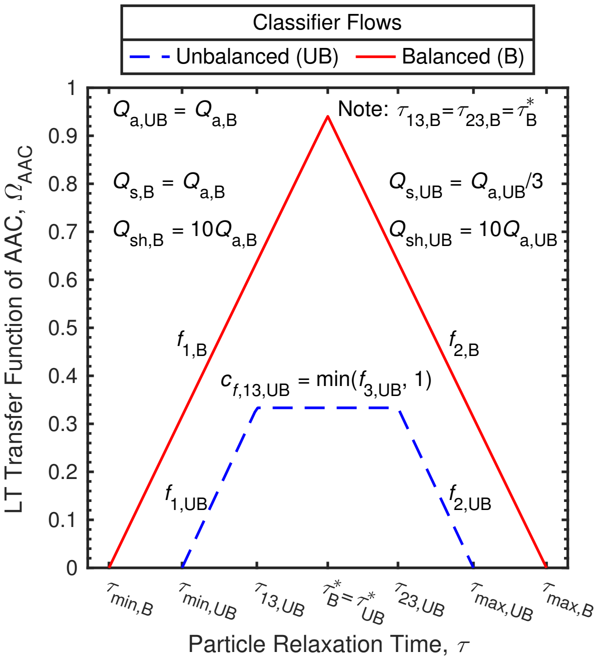


Figure S3.2: Transfer function of AAC based on limited trajectory theory with balanced (B) or unbalanced (UB) classifier flows.

Table S3.1: Constants and Boundaries of the Simplified Mathematical Representation of the Transfer Function of the Scanning AAC based on Limited Trajectory Theory.

| Original Equation | Simplified Form | | | |
|---|---|---|---|---|
| | Equation | | Constants | |
| S3.3 | $K_{\text{sc}}(t_{\text{m}}) = c_{\text{sc}} \exp\left(\frac{t_{\text{m}}}{\tau_{\text{sc}}}\right)$ | (S3.28) | $c_{\text{sc}} = \omega_{\text{S}}^2\, \tau_{\text{sc}} \left[1 - \exp\left(\frac{-t_{\text{f}}}{\tau_{\text{sc}}}\right)\right]$ | (S3.29) |
| S3.5 | $\tau_{\max} = c_{\tau\max} \exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)$ | (S3.30) | $c_{\tau\max} = \frac{1}{c_{\text{sc}}} \ln\left(\frac{r_2}{r_1}\right)$ | (S3.31) |
| S3.8 | $\tau_{\min} = c_{\tau\min} \exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)$ | (S3.32) | $c_{\tau\min} = \frac{1}{2c_{\text{sc}}} \ln\left(\frac{Q_{\text{a}} + Q_{\text{sh}} - Q_{\text{s}}\left(1 - r_1^2/r_2^2\right)}{Q_{\text{a}} + Q_{\text{sh}} - Q_{\text{sh}}\left(1 - r_1^2/r_2^2\right)}\right)$ | (S3.33) |
| S3.9 | $\tau^* = c_{\tau*} \exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)$ | (S3.34) | $c_{\tau*} = \frac{c_{\tau\min} + c_{\tau\max}}{2}$ | (S3.35) |
| | | | $c_{\tau*} = \frac{1}{2c_{\text{sc}}}\left[\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2}\ln\left(\frac{Q_{\text{a}} + Q_{\text{sh}} - Q_{\text{s}}\left(1 - r_1^2/r_2^2\right)}{Q_{\text{a}} + Q_{\text{sh}} - Q_{\text{sh}}\left(1 - r_1^2/r_2^2\right)}\right)\right]$ | (S3.36) |
| S3.12 | $f_1 = c_{f11} \exp\left[c_{f12}\, \tau \exp\left(\frac{t_{\text{m}}}{\tau_{\text{sc}}}\right)\right] + c_{f13}$ | (S3.37) | $c_{f11} = \frac{-\left[Q_{\text{a}} + Q_{\text{sh}} - Q_{\text{s}}\left(1 - r_1^2/r_2^2\right)\right]}{Q_{\text{a}}\left(1 - r_1^2/r_2^2\right)}$ | (S3.38) |
| | | | $c_{f12} = -2c_{\text{sc}}$ | (S3.39) |
| | | | $c_{f13} = \frac{Q_{\text{a}} + Q_{\text{sh}}\, r_1^2/r_2^2}{Q_{\text{a}}\left(1 - r_1^2/r_2^2\right)}$ | (S3.40) |
| S3.15 | $f_2 = c_{f21} \exp\left[c_{f22}\, \tau \exp\left(\frac{t_{\text{m}}}{\tau_{\text{sc}}}\right)\right] + c_{f23}$ | (S3.41) | $c_{f21} = \frac{Q_{\text{a}} + Q_{\text{sh}}}{Q_{\text{a}}\left(1 - r_1^2/r_2^2\right)} = (c_{f13} - 1)\left(\frac{r_2^2}{r_1^2}\right)$ | (S3.42) |
| | | | $c_{f22} = -2c_{\text{sc}} = c_{f12}$ | (S3.43) |
| | | | $c_{f23} = \frac{-(Q_{\text{a}} + Q_{\text{sh}})\left(r_1^2/r_2^2\right)}{Q_{\text{a}}\left(1 - r_1^2/r_2^2\right)} = -c_{f13} + 1$ | (S3.44) |
| S3.23 | $\tau_1 = c_{\tau1} \exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)$ | (S3.45) | $c_{\tau1} = \frac{1}{c_{f12}} \ln\left(\frac{c_{f31} - c_{f13}}{c_{f11}}\right)$ | (S3.46) |
| S3.24 | $\tau_2 = c_{\tau2} \exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)$ | (S3.47) | $c_{\tau2} = \frac{1}{c_{f22}} \ln\left(\frac{c_{f31} - c_{f23}}{c_{f21}}\right)$ | (S3.48) |
| S3.25 | $\tau_{13} = c_{\tau13} \exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)$ | (S3.49) | $c_{\tau13} = \min\left(c_{\tau1},\ c_{\tau^*}\right)$ | (S3.50) |
| S3.26 | $\tau_{23} = c_{\tau23} \exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)$ | (S3.51) | $c_{\tau23} = \max\left(c_{\tau2},\ c_{\tau^*}\right)$ | (S3.52) |

# S4 Transfer Function of Scanning AAC: Particle Streamline, Non-Idealized, Balanced Flows and Uniform Axial Flow

Note: Similar to the other sections in the SI, this section and its subsections use simplified notation of the parameter subscripts, as outlined in Section S1.1, based on the common AAC operation and theory instances included within the title and footer of this section.

Based on limited trajectory theory and that the assumption of uniform axial flow is valid (i.e. sufficiently long scan times), the transfer functions of the steady-state AAC and scanning AAC have the same form, as outlined in SI Section S3 and summarized in Table 1 of the main text. Therefore, the transfer function of the steady-state AAC based on particle streamline theory with balanced (B) flow, developed by Tavakoli and Olfert (2013) and parameterized by Johnson et al. (2018) to capture the non-idealized particle and flow behaviour, can also describe the scanning AAC as follows:

$$\Omega_{\mathrm{AAC,B}} = \frac{\lambda_\Omega\, \mu_\Omega^2}{2\beta} \left[ \left| \frac{\tau}{\tau^*_{\mathrm{sc,B}}} - \left(1 + \frac{\beta}{\mu_\Omega}\right) \right| + \left| \frac{\tau}{\tau^*_{\mathrm{sc,B}}} - \left(1 - \frac{\beta}{\mu_\Omega}\right) \right| - 2 \left| \frac{\tau}{\tau^*_{\mathrm{sc,B}}} - 1 \right| \right], \tag{S4.1}$$

where $\lambda_\Omega$ is the AAC transmission efficiency, $\mu_\Omega$ is the width factor of the transfer function and $\beta$ is non-dimensional classifier flow parameter as follows (restatement of Equation 33 in main text):

$$\beta = \frac{Q_\mathrm{s} + Q_\mathrm{a}}{Q_\mathrm{sh} + Q_\mathrm{exh}}. \tag{S4.2}$$

Russell et al. (1995) used a similar approach to adapt the transfer function of the steady-state DMA developed by Knutson and Whitby (1975) based on particle streamline theory to the SMPS.

## S4.1 Instantaneous Setpoint of Scanning AAC

The instantaneous setpoint of the scanning AAC operating with balanced classifier flows can be found by rearranging the $\tau^*_{\mathrm{sc}}/\tau^*_{\mathrm{ss}}$ ratio (i.e. Equation S3.20) as follows:

$$\begin{aligned}\tau^*_{\mathrm{sc,B}} &= \tau^*_{\mathrm{ss,B}} \frac{\omega_{\mathrm{S}}^2\, t_{\mathrm{f}}}{\omega_{\mathrm{S}}^2\, \tau_{\mathrm{sc}} \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\left[1-\exp\left(\frac{-t_{\mathrm{f}}}{\tau_{\mathrm{sc}}}\right)\right]} \\ \tau^*_{\mathrm{sc,B}} &= \tau^*_{\mathrm{ss,B}} \frac{t_{\mathrm{f}}}{\tau_{\mathrm{sc}} \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\left[1-\exp\left(\frac{-t_{\mathrm{f}}}{\tau_{\mathrm{sc}}}\right)\right]}.\end{aligned} \tag{S4.3}$$

Similar to Section S3.1, $\tau^*_{\mathrm{sc,B}}$ represents the instantaneous setpoint of the scanning AAC as it does not account for the shift of its transfer function over the counting time of the downstream particle detector. This effect on the setpoint of the scanning AAC is considered later in Section S7.1. Furthermore, the steady-state AAC setpoint derived by Tavakoli and Olfert (2013) using particle streamline theory (i.e. Equation 19) while assuming balanced classifier flows (i.e. $Q_{\mathrm{a}} = Q_{\mathrm{s}}$ and $Q_{\mathrm{sh}} = Q_{\mathrm{exh}}$) simplifies to:

$$\tau^*_{\mathrm{ss,B}} = \frac{2Q_{\mathrm{sh}}}{\pi\omega^2(r_1+r_2)^2 L}. \tag{S4.4}$$

Therefore, the expanded solution of $\tau^*_{\mathrm{sc,B}}$ can be found by substituting the definitions of $t_{\mathrm{f}}$, $\beta$ and $\tau^*_{\mathrm{ss,B}}$ (Equations 13 of main text, S4.2 and S4.4, respectively) at the start of the scan ($\omega = \omega_{\mathrm{S}}$) into Equation S4.3 as follows:

$$\begin{aligned}\tau^*_{\mathrm{sc,B}} &= \frac{2Q_{\mathrm{sh}}}{\pi\omega_{\mathrm{S}}^2(r_1+r_2)^2 L} \frac{\frac{\pi L\left(r_2^2-r_1^2\right)}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}}}{\tau_{\mathrm{sc}} \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\left[1-\exp\left(\frac{-t_{\mathrm{f}}}{\tau_{\mathrm{sc}}}\right)\right]} \\ \tau^*_{\mathrm{sc,B}} &= \frac{1}{\omega_{\mathrm{S}}^2\, \tau_{\mathrm{sc}}\left[1-\exp\left(\frac{-t_{\mathrm{f}}}{\tau_{\mathrm{sc}}}\right)\right]} \frac{2(r_2^2-r_1^2)}{(\beta+1)(r_1+r_2)^2} \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right).\end{aligned} \tag{S4.5}$$

## S4.2 Condensed Representation of Transfer Function for Scanning AAC

To simplify derivations in the later SI sections, the mathematical representation of the transfer function of the scanning AAC based on particle streamline theory (Equation S4.1) as derived in Section S4 can be simplified by grouping the constants and reorganizing particular terms. Firstly, this transfer function can also be represented by a piecewise equation (as shown in Figure S4.1) as follows:

$$\Omega_{\mathrm{AAC,B}} = \begin{cases} \lambda_\Omega\,\mu_\Omega\left[1+\frac{\mu_\Omega}{\beta}\left(\frac{\tau}{\tau^*_{\mathrm{sc,B}}}-1\right)\right] & \text{if } \tau_{\mathrm{min,B}} \le \tau \le \tau^*_{\mathrm{sc,B}} \\ \lambda_\Omega\,\mu_\Omega\left[1+\frac{\mu_\Omega}{\beta}\left(1-\frac{\tau}{\tau^*_{\mathrm{sc,B}}}\right)\right] & \text{if } \tau^*_{\mathrm{sc,B}} < \tau \le \tau_{\mathrm{max,B}} \\ 0 & \text{elsewhere} \end{cases}, \tag{S4.6}$$

where

$$\tau_{\mathrm{min,B}} = \left(1-\frac{\beta}{\mu_\Omega}\right)\tau^*_{\mathrm{sc,B}}, \tag{S4.7}$$

$$\tau_{\mathrm{max,B}} = \left(1+\frac{\beta}{\mu_\Omega}\right)\tau^*_{\mathrm{sc,B}}. \tag{S4.8}$$

Note that all of the boundary conditions (i.e. $\tau_{\mathrm{min,B}}$, $\tau^*_{\mathrm{ss,B}}$, $\tau_{\mathrm{max,B}}$) of the transfer function of the scanning AAC based on particle streamline theory are of the form:

$$\tau_{x,\mathrm{B}} = c_{\tau x,\mathrm{B}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right) \tag{S4.9}$$

where $c_{\tau x,\mathrm{B}}$ represents the grouped constants of each boundary $x$ shown in Figure S4.1. Therefore, the transfer function of the scanning AAC based on particle streamline can then be simplified, as shown in Table S4.1 on the next page.

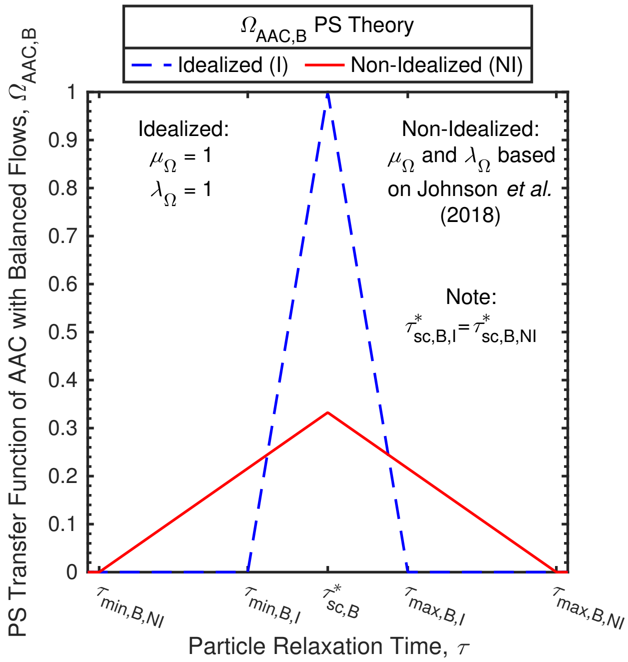


Figure S4.1: Transfer function of AAC based on particle streamline theory considering idealized (I) or non-idealized (NI) particle behaviour.

Table S4.1: Boundaries of the Simplified Mathematical Representation of the Transfer Function of the Scanning AAC based on Particle Streamline Theory.

| Original Equation | Simplified Form | |
|---|---|---|
| | Equation | Constants |
| S4.5 | $\tau^*_{\mathrm{sc,B}} = c_{\tau*,\mathrm{B}} \exp\left(\frac{-t_\mathrm{m}}{\tau_\mathrm{sc}}\right)$ (S4.10) | $c_{\tau*,\mathrm{B}} = \frac{1}{c_\mathrm{sc}} \frac{2\left(r_2^2 - r_1^2\right)}{(\beta+1)(r_1+r_2)^2}$ (S4.11)<br>$c_\mathrm{sc} = \omega_\mathrm{S}^2\, \tau_\mathrm{sc} \left[1 - \exp\left(\frac{-t_\mathrm{f}}{\tau_\mathrm{sc}}\right)\right]$ (S4.12) |
| S4.7 | $\tau_{\mathrm{min,B}} = c_{\tau\mathrm{min,B}} \exp\left(\frac{-t_\mathrm{m}}{\tau_\mathrm{sc}}\right)$ (S4.13) | $c_{\tau\mathrm{min,B}} = \left(1 - \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}$ (S4.14) |
| S4.8 | $\tau_{\mathrm{max,B}} = c_{\tau\mathrm{max,B}} \exp\left(\frac{-t_\mathrm{m}}{\tau_\mathrm{sc}}\right)$ (S4.15) | $c_{\tau\mathrm{max,B}} = \left(1 + \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}$ (S4.16) |

Where $c_\mathrm{sc}$ is the same for the transfer function of the scanning AAC based on limited trajectory or particle streamline theory (i.e. Equation S3.29 or S4.12, respectively.)

# S5 Average Transfer Function of Scanning AAC over Counting Time of Particle Detector: Limited Trajectory, Idealized and Uniform Axial Flow

Note: Similar to the other sections in the SI, this section and its subsections use simplified notation of the parameter subscripts, as outlined in Section S1.1, based on the common AAC operation and theory instances included within the title and footer of this section.

To determine the average transfer function of the scanning AAC over the counting time of downstream particle detector (i.e. solve the integral shown in Equation 34 of the main text), the piecewise notation of the transfer function based on limited trajectory theory (Equation S3.21), as derived in Section S3.2, can be represented as one equation using the Heaviside function ($\mathcal{H}\left(x-a\right)$, defined by Equation 35 of main text) as follows:

$$\begin{aligned}\Omega_{\mathrm{AAC}}(\tau) = & \Big[\mathcal{H}\left(\tau-\tau_{\mathrm{min}}\right)-\mathcal{H}\left(\tau-\tau_{13}\right)\Big] f_1 \\ & + \Big[\mathcal{H}\left(\tau-\tau_{13}\right)-\mathcal{H}\left(\tau-\tau_{23}\right)\Big] c_{f31} \\ & + \Big[\mathcal{H}\left(\tau-\tau_{23}\right)-\mathcal{H}\left(\tau-\tau_{\mathrm{max}}\right)\Big] f_2,\end{aligned} \tag{S5.1}$$

where the values ($f_1$, $c_{f31}$ and $f_2$) and relaxation time boundaries ($\tau_{\mathrm{min}}$, $\tau_{13}$, $\tau_{23}$ and $\tau_{\mathrm{max}}$) are defined in Table S3.1 and shown in Figure S3.2. All of the relaxation time boundaries of this transfer function are of the form $\tau_x = c_{\tau x}\exp\left(-t_{\mathrm{m}}/\tau_{\mathrm{sc}}\right)$ (i.e. Equation 36 of main text or Equation S3.27). This consistent form results in $\mathcal{H}\left(\tau-\tau_x\right) = \mathcal{H}\Big(\left(\tau_{\mathrm{sc}}/|\tau_{\mathrm{sc}}|\right)\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(c_{\tau x}/\tau\right)\right)\Big)$ (Equation 40 of the main text), and the limits of $\Omega_{\mathrm{AAC}}$ (Equation S5.1) can be converted from the particle relaxation time ($\tau$) to measurement time ($t_{\mathrm{m}}$) domain as follows:

$$\begin{aligned}\Omega_{\mathrm{AAC}}(t_{\mathrm{m}}) = & \left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau 13}}{\tau}\right)\right)\right)\right] f_1 \\ & + \left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau 13}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau 23}}{\tau}\right)\right)\right)\right] c_{f31} \\ & + \left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau 23}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max}}}{\tau}\right)\right)\right)\right] f_2.\end{aligned} \tag{S5.2}$$

## S5.1 Solving Integral of $\Omega_{\mathrm{AAC}}$ over $t_{\mathrm{c}}$ to determine $\bar{\Omega}_{\mathrm{AAC}}$

This section determines the average transfer function of the AAC ($\bar{\Omega}_{\mathrm{AAC}}$) over the counting time ($t_{\mathrm{c}}$) of the particle detector by substituting Equation S5.2 into Equation 34 of the main text and solving. To reduce the analysis, note that $f_1$ (Equation S3.37) and $f_2$ (Equation S3.41) have the same form as follows:

$$f_y = c_{fy1}\exp\left[c_{fy2}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] + c_{fy3}. \tag{S5.3}$$

where the corresponding coefficients for $f_1$ and $f_2$ are summarized in Table S5.1.

Table S5.1: Coefficients of the Transfer Function of the Scanning AAC based on Limited Trajectory Theory (i.e. for $f_1$ and $f_2$).

| Constant | $f_1$ | $f_2$ |
|---|---|---|
| $c_{fy1}$ | $c_{f11}$ | $c_{f21}$ |
| $c_{fy2}$ | $c_{f12}$ | $c_{f22}$ |
| $c_{fy3}$ | $c_{f13}$ | $c_{f23}$ |

Expanding Equation S5.2, considering its first term generically, then substituting in Equation S5.3 is equivalent to:

$$\begin{aligned}
&\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right) f_y \\
&= \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right)\left[c_{fy1}\exp\left[c_{fy2}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]+c_{fy3}\right] \\
&= \underbrace{\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right) c_{fy1}\exp\left[c_{fy2}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]}_{\text{Term Form: }\Omega_{\mathrm{AAC,F1}}(c_{\tau x},\, c_{fy1},\, c_{fy2},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})} + \underbrace{\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right) c_{fy3}}_{\text{Term Form: }\Omega_{\mathrm{AAC,F2}}(c_{\tau x},\, c_{fy3},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})}\,.
\end{aligned} \tag{S5.4}$$

Therefore, substituting the two term forms (i.e. $\Omega_{\mathrm{AAC,F1}}$ and $\Omega_{\mathrm{AAC,F2}}$) identified in Equation S5.4 into Equation S5.2, then into the definition of $\bar{\Omega}_{\mathrm{AAC}}$ (Equation 34 of the main text) simplifies to the following:

$$\begin{aligned}
\bar{\Omega}_{\mathrm{AAC}} &= \frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC}}(\tau, t'_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}} \\
\bar{\Omega}_{\mathrm{AAC}} &= \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F1}}(c_{\tau\mathrm{min}},\, c_{f11},\, c_{f12},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F1}}(c_{\tau x},\, c_{fy1},\, c_{fy2},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})} + \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F2}}(c_{\tau\mathrm{min}},\, c_{f13},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F2}}(c_{\tau x},\, c_{fy3},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})} \\
&\quad - \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F1}}(c_{\tau 13},\, c_{f11},\, c_{f12},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F1}}(c_{\tau x},\, c_{fy1},\, c_{fy2},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})} - \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F2}}(c_{\tau 13},\, c_{f13},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F2}}(c_{\tau x},\, c_{fy3},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})} \\
&\quad + \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F2}}(c_{\tau 13},\, c_{f31},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F2}}(c_{\tau x},\, c_{fy3},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})} - \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F2}}(c_{\tau 23},\, c_{f31},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F2}}(c_{\tau x},\, c_{fy3},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})} \\
&\quad + \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F1}}(c_{\tau 23},\, c_{f21},\, c_{f22},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F1}}(c_{\tau x},\, c_{fy1},\, c_{fy2},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})} + \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F2}}(c_{\tau 23},\, c_{f23},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F2}}(c_{\tau x},\, c_{fy3},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})} \\
&\quad - \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F1}}(c_{\tau\mathrm{max}},\, c_{f21},\, c_{f22},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F1}}(c_{\tau x},\, c_{fy1},\, c_{fy2},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})} - \underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F2}}(c_{\tau\mathrm{max}},\, c_{f23},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,F2}}(c_{\tau x},\, c_{fy3},\, \tau,\, \tau_{\mathrm{sc}},\, t_{\mathrm{m}},\, t_{\mathrm{c}})}\,.
\end{aligned} \tag{S5.5}$$

All of the terms in Equation S5.5 are of the form $\bar{\Omega}_{\mathrm{AAC,F1}}$ or $\bar{\Omega}_{\mathrm{AAC,F2}}$, which are solved in Sections S5.1.1 and S5.1.2, respectively. Therefore, substituting these solutions (i.e. Equations S5.22 and S5.25) into Equation S5.5:

$$
\begin{aligned}
&\bar{\Omega}_{\mathrm{AAC}} = \\
&\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\Bigg\{\left[\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau\min}\,c_{f12}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\min}}{\tau}\right)\right)\right) \\
&\qquad-\left[\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau\min}\,c_{f12}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\min}}{\tau}\right)\right)\right)\Bigg\} \\
&+\frac{c_{f13}}{t_{\mathrm{c}}}\Bigg[\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\min}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\min}}{\tau}\right)\right)\right) \\
&\qquad-\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\min}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\min}}{\tau}\right)\right)\right)\Bigg] \\
&-\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\Bigg\{\left[\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau13}\,c_{f12}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right) \\
&\qquad-\left[\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau13}\,c_{f12}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right)\Bigg\} \\
&-\frac{c_{f13}}{t_{\mathrm{c}}}\Bigg[\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right) \\
&\qquad-\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right)\Bigg] \\
&+\frac{c_{f31}}{t_{\mathrm{c}}}\Bigg[\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right) \\
&\qquad-\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right)\Bigg] \\
&-\frac{c_{f31}}{t_{\mathrm{c}}}\Bigg[\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right) \\
&\qquad-\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right)\Bigg] \\
&+\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\Bigg\{\left[\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau23}\,c_{f22}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right) \\
&\qquad-\left[\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau23}\,c_{f22}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right)\Bigg\} \\
&+\frac{c_{f23}}{t_{\mathrm{c}}}\Bigg[\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right) \\
&\qquad-\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right)\Bigg] \\
&-\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\Bigg\{\left[\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau\max}\,c_{f22}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\max}}{\tau}\right)\right)\right) \\
&\qquad-\left[\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau\max}\,c_{f22}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\max}}{\tau}\right)\right)\right)\Bigg\} \\
&-\frac{c_{f23}}{t_{\mathrm{c}}}\Bigg[\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\max}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\max}}{\tau}\right)\right)\right) \\
&\qquad-\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\max}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\max}}{\tau}\right)\right)\right)\Bigg],
\end{aligned}
\tag{S5.6}
$$

and grouping by common time intervals, simplifies to:

$$
\begin{aligned}
&\bar{\Omega}_{\mathrm{AAC}}(t_{\mathrm{m}}) = \\
&\left[\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau\mathrm{min}}\,c_{f12}\right) - \frac{c_{f13}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min}}}{\tau}\right)\right)\right] \\
&\quad\cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min}}}{\tau}\right)\right)\right) - \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min}}}{\tau}\right)\right)\right)\right] \\
&+\left[-\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\right]\cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min}}}{\tau}\right)\right)\right) - \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right)\right] \\
&+\left[\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right) + c_{f13}\right] \\
&\quad\cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min}}}{\tau}\right)\right)\right) - \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right)\right] \\
&+\left[-\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau13}\,c_{f12}\right) + \left(\frac{c_{f13}}{t_{\mathrm{c}}} - \frac{c_{f31}}{t_{\mathrm{c}}}\right)\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right] \\
&\quad\cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right) - \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right)\right] \\
&+c_{f31}\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right) - \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right)\right] \\
&+\left[\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau23}\,c_{f22}\right) + \left(\frac{c_{f31}}{t_{\mathrm{c}}} - \frac{c_{f23}}{t_{\mathrm{c}}}\right)\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right] \\
&\quad\cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right) - \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right)\right] \\
&+\left[\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right) + c_{f23}\right] \\
&\quad\cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right) - \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max}}}{\tau}\right)\right)\right)\right] \\
&+\left[-\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\right]\cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right) - \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max}}}{\tau}\right)\right)\right)\right] \\
&+\left[-\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau\mathrm{max}}\,c_{f22}\right) + \frac{c_{f23}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max}}}{\tau}\right)\right)\right] \\
&\quad\cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max}}}{\tau}\right)\right)\right) - \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max}}}{\tau}\right)\right)\right)\right].
\end{aligned}
\tag{S5.7}
$$

Finally, Equation S5.7 can converted from the measurement time ($t_{\mathrm{m}}$) back to the particle relaxation time ($\tau$) domain using $\mathcal{H}\left(\tau - \tau_x\right) = \mathcal{H}\left(\left(\tau_{\mathrm{sc}}/|\tau_{\mathrm{sc}}|\right)\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(c_{\tau x}/\tau\right)\right)\right)$ (Equation 40 of the main text) as follows:

$$
\begin{aligned}
&\bar{\Omega}_{\mathrm{AAC}}(\tau) = \\
&\quad \left[\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau\min}\,c_{f12}\right) - \frac{c_{f13}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\min}}{\tau}\right)\right)\right]\left[\mathcal{H}\left(\tau - \tau_{\min}\right) - \mathcal{H}\left(\tau - \tau_{\min,\mathrm{tc}}\right)\right] \\
&+ \left[-\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\right]\left[\mathcal{H}\left(\tau - \tau_{\min}\right) - \mathcal{H}\left(\tau - \tau_{13}\right)\right] \\
&+ \left[\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}} + t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right) + c_{f13}\right]\left[\mathcal{H}\left(\tau - \tau_{\min,\mathrm{tc}}\right) - \mathcal{H}\left(\tau - \tau_{13,\mathrm{tc}}\right)\right] \\
&+ \left[-\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau 13}\,c_{f12}\right) + \left(\frac{c_{f13}}{t_{\mathrm{c}}} - \frac{c_{f31}}{t_{\mathrm{c}}}\right)\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau 13}}{\tau}\right)\right)\right]\left[\mathcal{H}\left(\tau - \tau_{13}\right) - \mathcal{H}\left(\tau - \tau_{13,\mathrm{tc}}\right)\right] \\
&+ c_{f31}\left[\mathcal{H}\left(\tau - \tau_{13,\mathrm{tc}}\right) - \mathcal{H}\left(\tau - \tau_{23,\mathrm{tc}}\right)\right] \\
&+ \left[\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau 23}\,c_{f22}\right) + \left(\frac{c_{f31}}{t_{\mathrm{c}}} - \frac{c_{f23}}{t_{\mathrm{c}}}\right)\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau 23}}{\tau}\right)\right)\right]\left[\mathcal{H}\left(\tau - \tau_{23}\right) - \mathcal{H}\left(\tau - \tau_{23,\mathrm{tc}}\right)\right] \\
&+ \left[\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\mathrm{m}} + t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right) + c_{f23}\right]\left[\mathcal{H}\left(\tau - \tau_{23,\mathrm{tc}}\right) - \mathcal{H}\left(\tau - \tau_{\max,\mathrm{tc}}\right)\right] \\
&+ \left[-\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\right]\left[\mathcal{H}\left(\tau - \tau_{23}\right) - \mathcal{H}\left(\tau - \tau_{\max}\right)\right] \\
&+ \left[-\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau\max}\,c_{f22}\right) + \frac{c_{f23}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\max}}{\tau}\right)\right)\right]\left[\mathcal{H}\left(\tau - \tau_{\max}\right) - \mathcal{H}\left(\tau - \tau_{\max,\mathrm{tc}}\right)\right],
\end{aligned}
\tag{S5.8}
$$

where $\tau_x$ is defined by Equation S3.27 (restatement of Equation 36 in main text) and:

$$
\tau_{x,\mathrm{tc}} = c_{\tau x}\exp\left(\frac{-(t_{\mathrm{m}} + t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right). \tag{S5.9}
$$

This equation (S5.9) determines the $t_{\mathrm{c}}$ shift on the boundary conditions ($\tau_x \rightarrow \tau_{x,\mathrm{tc}}$) of the transfer function of the scanning AAC based on limited trajectory theory, and is a restatement of Equation 41 of the main text. Examples of this average transfer function (i.e. based on Equation S5.8) are shown in Figure 3 of the main text.

### S5.1.1 Analytical Solution for $\bar{\Omega}_{\mathrm{AAC,F1}}$ of Equation S5.5

Based on integral form 1 (i.e. $\Omega_{\mathrm{AAC,F1}}$) shown in Equation S5.4:

$$\begin{aligned}\bar{\Omega}_{\mathrm{AAC,F1}} &= \frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F1}}\,\mathrm{d}t'_{\mathrm{m}} \\ &= \frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t'_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right) c_{fy1}\exp\left[c_{fy2}\,\tau\exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\mathrm{d}t'_{\mathrm{m}}.\end{aligned} \tag{S5.10}$$

Assuming $\tau_{\mathrm{sc}}$ is greater than zero (denoted by $\bar{\Omega}_{\mathrm{AAC,F1+}}$), Equation S5.10 becomes:

$$\bar{\Omega}_{\mathrm{AAC,F1+}} = \frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \mathcal{H}\left(t'_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right)\right) c_{fy1}\exp\left[c_{fy2}\,\tau\exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\mathrm{d}t'_{\mathrm{m}}. \tag{S5.11}$$

Applying the following substitution to Equation S5.11:

$$\begin{aligned} u = c_{fy2}\,\tau\exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right) &\rightarrow t'_{\mathrm{m}} = \tau_{\mathrm{sc}}\ln\left(\frac{u}{c_{fy2}\,\tau}\right) \\ \mathrm{d}u = \frac{c_{fy2}\,\tau}{\tau_{\mathrm{sc}}}\exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\mathrm{d}t'_{\mathrm{m}} &\rightarrow \mathrm{d}t'_{\mathrm{m}} = \frac{\tau_{\mathrm{sc}}}{c_{fy2}\,\tau\exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}\mathrm{d}u = \frac{\tau_{\mathrm{sc}}}{u}\mathrm{d}u.\end{aligned} \tag{S5.12}$$

Based on this substitution (i.e. Equation S5.12), the inequality relationship defined by the Heaviside function within Equation S5.11 (i.e $\mathcal{H}\left(t'_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(c_{\tau x}/\tau\right)\right)$ then becomes:

$$t'_{\mathrm{m}} > \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right) \rightarrow \tau_{\mathrm{sc}}\ln\left(\frac{u}{c_{fy2}\,\tau}\right) > \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right) \rightarrow \boldsymbol{u > c_{\tau x}\,c_{fy2}}, \tag{S5.13}$$

as the base of the natural logarithm ($e$) is great than 1, $\ln(a) > \ln(b)$ is equivalent to $a > b$, and therefore,

$$\mathcal{H}\left(t'_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right)\right) = \mathcal{H}\left(u - c_{\tau x}\,c_{fy2}\right). \tag{S5.14}$$

Substituting Equations S5.12 and S5.14 into Equation S5.11 and temporarily treating the integral as indefinite:

$$\bar{\Omega}_{\mathrm{AAC,F1+}} = \frac{c_{fy1}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\int \mathcal{H}\left(u - c_{\tau x}\,c_{fy2}\right)\frac{\exp(u)}{u}\mathrm{d}u. \tag{S5.15}$$

Apply integration by parts to Equation S5.15 as follows:

$$
\begin{aligned}
w &= \mathcal{H}\left(u - c_{\tau x}\, c_{fy2}\right) \rightarrow \mathrm{d}w = \delta\left(u - c_{\tau x}\, c_{fy2}\right) \mathrm{d}u \\
\mathrm{d}y &= \frac{\exp\left(u\right)}{u} \mathrm{d}u \rightarrow y = \mathcal{EI}\left(u\right),
\end{aligned}
\tag{S5.16}
$$

where $\mathcal{EI}$ is the Exponential Integral Function as follows:

$$
\mathcal{EI}\left(x\right) = \int_{-\infty}^{x} \frac{\exp\left(x'\right)}{x'} \mathrm{d}x',
\tag{S5.17}
$$

and $\delta$ is the Dirac delta function as follows:

$$
\delta\left(x - a\right) = \begin{cases} 0 & \text{if } x \neq a \\ \infty & \text{if } x = a \end{cases}.
\tag{S5.18}
$$

Therefore, using integration by parts based on Equation S5.16, Equation S5.15 becomes:

$$
\bar{\Omega}_{\mathrm{AAC,F1+}} = \frac{c_{fy1}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \left[\mathcal{H}\left(u - c_{\tau x}\, c_{fy2}\right) \mathcal{EI}\left(u\right) - \int \mathcal{EI}\left(u\right) \delta\left(u - c_{\tau x}\, c_{fy2}\right) \mathrm{d}u\right].
\tag{S5.19}
$$

Furthermore, noting that $\delta\left(u - c_{\tau x}\, c_{fy2}\right)$ vanishes everywhere except $u = c_{\tau x} c_{fy2}$, Equation S5.19 becomes:

$$
\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,F1+}} &= \frac{c_{fy1}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \left[\mathcal{H}\left(u - c_{\tau x}\, c_{fy2}\right) \mathcal{EI}\left(u\right) - \mathcal{EI}\left(c_{\tau x}\, c_{fy2}\right) \int \delta\left(u - c_{\tau x}\, c_{fy2}\right) \mathrm{d}u\right] \\
\bar{\Omega}_{\mathrm{AAC,F1+}} &= \frac{c_{fy1}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \left[\mathcal{EI}\left(u\right) - \mathcal{EI}\left(c_{\tau x}\, c_{fy2}\right)\right] \mathcal{H}\left(u - c_{\tau x}\, c_{fy2}\right).
\end{aligned}
\tag{S5.20}
$$

Substituting Equations S5.12 and S5.14 into Equation S5.20 and applying the integration limits from the definition of $\bar{\Omega}_{\mathrm{AAC,F1+}}$ (i.e. $t_{\mathrm{m}}$ to $t_{\mathrm{m}} + t_{\mathrm{c}}$ in Equation S5.11) simplifies to:

$$
\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,F1+}} =& \frac{c_{fy1}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \left[\mathcal{EI}\left(c_{fy2}\, \tau \exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right) - \mathcal{EI}\left(c_{\tau x}\, c_{fy2}\right)\right] \mathcal{H}\left(t'_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\Bigg|_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \\
\bar{\Omega}_{\mathrm{AAC,F1+}} =& \frac{c_{fy1}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \Bigg\{ \left[\mathcal{EI}\left(c_{fy2}\, \tau \exp\left(\frac{t_{\mathrm{m}} + t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right) - \mathcal{EI}\left(c_{\tau x}\, c_{fy2}\right)\right] \mathcal{H}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right) \\
& - \left[\mathcal{EI}\left(c_{fy2}\, \tau \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right) - \mathcal{EI}\left(c_{\tau x}\, c_{fy2}\right)\right] \mathcal{H}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right) \Bigg\}.
\end{aligned}
\tag{S5.21}
$$

As shown in Section S5.1.3, this solution (Equation S5.21) can be expanded to consider both $\tau_{\mathrm{sc}} > 0$ and $\tau_{\mathrm{sc}} < 0$ based on Equation S5.37 (i.e. only differ by the inequality sign represented within their Heaviside functions), and thus both cases can be represented using one equation as follows:

$$\begin{aligned}\bar{\Omega}_{\mathrm{AAC,F1}} =& \frac{c_{fy1}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left\{\left[\mathcal{EI}\left(c_{fy2}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau x}\,c_{fy2}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right)\right. \\ &\left.-\left[\mathcal{EI}\left(c_{fy2}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)-\mathcal{EI}\left(c_{\tau x}\,c_{fy2}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right)\right\}. \end{aligned} \tag{S5.22}$$

### S5.1.2 Analytical Solution for $\bar{\Omega}_{\mathrm{AAC,F2}}$ of Equation S5.5

Based on integral form 2 (i.e. $\Omega_{\mathrm{AAC,F2}}$) shown in Equation S5.4:

$$\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,F2}} =& \frac{1}{t_{\mathrm{c}}} \int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,F2}} \, \mathrm{d}t'_{\mathrm{m}} \\
\bar{\Omega}_{\mathrm{AAC,F2}} =& \frac{1}{t_{\mathrm{c}}} \int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t'_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right) c_{fy3} \, \mathrm{d}t'_{\mathrm{m}}.
\end{aligned} \tag{S5.23}$$

Assuming $\tau_{\mathrm{sc}}$ is greater than zero (denoted by $\bar{\Omega}_{\mathrm{AAC,F2+}}$), Equation S5.23 becomes:

$$\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,F2+}} =& \frac{1}{t_{\mathrm{c}}} \int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \mathcal{H}\left(t'_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right) c_{fy3} \, \mathrm{d}t'_{\mathrm{m}} \\
=& \frac{c_{fy3}}{t_{\mathrm{c}}} \left(t'_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right) \mathcal{H}\left(t'_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right) \Bigg|_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \\
\bar{\Omega}_{\mathrm{AAC,F2+}} =& \frac{c_{fy3}}{t_{\mathrm{c}}} \left[\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right) \mathcal{H}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right. \\
& \left. - \left(t_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right) \mathcal{H}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right].
\end{aligned} \tag{S5.24}$$

As shown in Section S5.1.3, this solution (Equation S5.24) can be expanded to consider both $\tau_{\mathrm{sc}} > 0$ and $\tau_{\mathrm{sc}} < 0$ based on Equation S5.37 (i.e. only differ by the inequality sign represented within their Heaviside functions), and thus both cases can be represented using one equation as follows:

$$\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,F2}} =& \frac{c_{fy3}}{t_{\mathrm{c}}} \left[\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right) \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} + t_{\mathrm{c}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right)\right. \\
& \left. - \left(t_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right) \mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}} \ln\left(\frac{c_{\tau x}}{\tau}\right)\right)\right)\right].
\end{aligned} \tag{S5.25}$$

### S5.1.3 Reflecting an Up or Down Scan within the Heaviside Limits of $\Omega_{\mathrm{AAC,sc}}$

As mentioned in the main text, the AAC completing an up scan (i.e. $\omega_{\mathrm{E}} > \omega_{\mathrm{S}}$) is reflected in a positive scan time constant (i.e. $\tau_{\mathrm{sc}} > 0$), while a down scan (i.e. $\omega_{\mathrm{E}} < \omega_{\mathrm{S}}$) is reflected in a negative scan time constant (i.e. $\tau_{\mathrm{sc}} < 0$). This change affects the inequality that allows the Heaviside functions of the transfer function equation to be converted from the particle relaxation time ($\tau_x$) to the measurement ($t_{\mathrm{m}}$) as shown by Equation 40 of the main text. To quantify this effect, consider the integral of a generic function $\jmath(x)$ times a Heaviside function with a flipping inequality sign as follows:

$$I = \int_{x_1}^{x_2} \jmath(x)\, \mathcal{H}\left(\frac{a}{|a|}(x-a)\right) \mathrm{d}x. \tag{S5.26}$$

**Case 1: $a > 0$ (i.e. similar to $t_{\mathrm{sc}} > 0$)**

If $a > 0$, Equation S5.26 becomes:

$$I_+ = \int_{x_1}^{x_2} \jmath(x)\, \mathcal{H}\left(x-a\right) \mathrm{d}x. \tag{S5.27}$$

Apply integration by parts to Equation S5.27 as follows:

$$\begin{aligned} u = \mathcal{H}\left(x-a\right) &\rightarrow \mathrm{d}u = \delta\left(x-a\right)\mathrm{d}x \\ \mathrm{d}v = \jmath(x)\,\mathrm{d}x &\rightarrow v = \int \jmath(x)\,\mathrm{d}x = \mathcal{J}(x), \end{aligned} \tag{S5.28}$$

where $\mathcal{J}(x)$ is the integral of the generic function $\jmath(x)$ with respect to $x$. Therefore, using integration by parts based on Equation S5.28, Equation S5.27 becomes:

$$I_+ = \mathcal{H}\left(x-a\right)\mathcal{J}(x)\Big|_{x_1}^{x_2} - \int_{x_1}^{x_2} \mathcal{J}(x)\,\delta\left(x-a\right)\mathrm{d}x. \tag{S5.29}$$

Furthermore, noting that $\delta\left(x-a\right)$ vanishes everywhere except $x = a$, Equation S5.29 becomes:

$$\begin{aligned} I_+ =& \mathcal{H}\left(x-a\right)\mathcal{J}(x)\Big|_{x_1}^{x_2} - \mathcal{J}(a)\int_{x_1}^{x_2} \delta\left(x-a\right)\mathrm{d}x \\ =& \left[\mathcal{J}(x) - \mathcal{J}(a)\right]\mathcal{H}\left(x-a\right)\Big|_{x_1}^{x_2} \\ I_+ =& \left[\mathcal{J}(x_2) - \mathcal{J}(a)\right]\mathcal{H}\left(x_2-a\right) - \left[\mathcal{J}(x_1) - \mathcal{J}(a)\right]\mathcal{H}\left(x_1-a\right). \end{aligned} \tag{S5.30}$$

**Case 2: $a < 0$ (i.e. similar to $t_{\mathrm{sc}} < 0$)**

If $a < 0$, Equation S5.26 becomes:

$$I_{-} = \int_{x_1}^{x_2} j(x)\,\mathcal{H}\left(a - x\right)\mathrm{d}x. \tag{S5.31}$$

Apply integration by parts to Equation S5.31 as follows:

$$\begin{aligned} u = \mathcal{H}\left(a - x\right) &\rightarrow \mathrm{d}u = -\delta\left(x - a\right)\mathrm{d}x \\ \mathrm{d}v = j(x)\,\mathrm{d}x &\rightarrow v = \int j(x)\,\mathrm{d}x = \mathcal{J}(x), \end{aligned} \tag{S5.32}$$

where $\mathcal{J}(x)$ is the integral of the generic function $j(x)$ with respect to $x$. Therefore, using integration by parts based on Equation S5.32, Equation S5.31 becomes:

$$I_{-} = \mathcal{H}\left(a - x\right)\mathcal{J}(x)\Big|_{x_1}^{x_2} + \int_{x_1}^{x_2} \mathcal{J}(x)\,\delta\left(x - a\right)\mathrm{d}x. \tag{S5.33}$$

Furthermore, noting that $\delta\left(a - x\right)$ vanishes everywhere except $x = a$, Equation S5.33 becomes:

$$\begin{aligned} I_{-} &= \mathcal{H}\left(a - x\right)\mathcal{J}(x)\Big|_{x_1}^{x_2} + \mathcal{J}(a)\int_{x_1}^{x_2} \delta\left(x - a\right)\mathrm{d}x \\ I_{-} &= \left[\mathcal{H}\left(a - x\right)\mathcal{J}(x) + \mathcal{J}(a)\,\mathcal{H}\left(x - a\right)\right]\Big|_{x_1}^{x_2}. \end{aligned} \tag{S5.34}$$

However, the relationship between Heaviside function with opposite inequalities (i.e. $x > a$ versus $x < a$) is:

$$\mathcal{H}\left(x - a\right) = 1 - \mathcal{H}\left(a - x\right). \tag{S5.35}$$

Substituting this relationship (Equation S5.35) into Equation S5.34 and simplifying, as follows:

$$\begin{aligned} I_{-} &= \left[\mathcal{H}\left(a - x\right)\mathcal{J}(x) + \mathcal{J}(a)\left(1 - \mathcal{H}\left(a - x\right)\right)\right]\Big|_{x_1}^{x_2} \\ &= \Big[\mathcal{J}(a) + \big(\mathcal{J}(x) - \mathcal{J}(a)\big)\,\mathcal{H}\left(a - x\right)\Big]\Big|_{x_1}^{x_2} \\ I_{-} &= \left[\mathcal{J}(x_2) - \mathcal{J}(a)\right]\mathcal{H}\left(a - x_2\right) - \left[\mathcal{J}(x_1) - \mathcal{J}(a)\right]\mathcal{H}\left(a - x_1\right). \end{aligned} \tag{S5.36}$$

**Case 1 or 2:** $a > 0$ **or** $a < 0$

Since the solution for $I$ (Equation S5.26) for either $a > 0$ or $a < 0$ (i.e. denoted as $I_+$ and $I_-$ in Equations S5.30 and S5.36, respectively) have the same form that only vary by the inequality sign represented by its Heaviside functions, either case ($a < 0$ or $a > 0$) can be represented by:

$$\begin{aligned} I &= \int_{x_1}^{x_2} j(x)\,\mathcal{H}\left(\frac{a}{|a|}(x-a)\right)\mathrm{d}x \\ I &= \left[\mathcal{J}(x_2) - \mathcal{J}(a)\right]\mathcal{H}\left(\frac{a}{|a|}(x_2-a)\right) - \left[\mathcal{J}(x_1) - \mathcal{J}(a)\right]\mathcal{H}\left(\frac{a}{|a|}(x_1-a)\right). \end{aligned} \tag{S5.37}$$

# S6 Average Transfer Function of Scanning AAC over Counting Time of Particle Detector: Particle Streamline, Non-Idealized, Balanced Flows and Uniform Axial Flow

Note: Similar to the other sections in the SI, this section and its subsections use simplified notation of the parameter subscripts, as outlined in Section S1.1, based on the common AAC operation and theory instances included within the title and footer of this section.

To determine the average transfer function of the scanning AAC over the counting time of downstream particle detector (i.e. solve the integral shown in Equation 34 of the main text), the piecewise notation of the transfer function based on particle streamline theory (Equation S4.6), as derived in Section S4.2, can be represented as one equation using the Heaviside function ($\mathcal{H}(x-a)$, defined by Equation 35 of main text) as follows:

$$\begin{aligned}\Omega_{\mathrm{AAC,B}}(\tau) =& \lambda_\Omega\, \mu_\Omega \left[1+\frac{\mu_\Omega}{\beta}\left(\frac{\tau}{\tau^*_{\mathrm{sc,B}}}-1\right)\right]\left[\mathcal{H}\left(\tau-\tau_{\mathrm{min,B}}\right)-\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B}}\right)\right] \\ &+ \lambda_\Omega\, \mu_\Omega \left[1+\frac{\mu_\Omega}{\beta}\left(1-\frac{\tau}{\tau^*_{\mathrm{sc,B}}}\right)\right]\left[\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B}}\right)-\mathcal{H}\left(\tau-\tau_{\mathrm{max,B}}\right)\right],\end{aligned} \tag{S6.1}$$

where the relaxation time boundaries ($\tau_{\mathrm{min,B}}$, $\tau^*_{\mathrm{sc,B}}$ and $\tau_{\mathrm{max,B}}$) are defined in Table S4.1 and shown in Figure S4.1. All of the relaxation time boundaries of this transfer function are of the form $\tau_x = c_{\tau x}\exp\left(-t_{\mathrm{m}}/\tau_{\mathrm{sc}}\right)$ (i.e. Equation 36 of main text or Equation S4.9). Based on this consistent form, Equation S6.1 becomes:

$$\begin{aligned}\Omega_{\mathrm{AAC,B}}(\tau,t_{\mathrm{m}}) =& \lambda_\Omega\, \mu_\Omega \left[1+\frac{\mu_\Omega}{\beta}\left(\frac{\tau}{c_{\tau *,\mathrm{B}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}-1\right)\right]\left[\mathcal{H}\left(\tau-\tau_{\mathrm{min,B}}\right)-\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B}}\right)\right] \\ &+ \lambda_\Omega\, \mu_\Omega \left[1+\frac{\mu_\Omega}{\beta}\left(1-\frac{\tau}{c_{\tau *,\mathrm{B}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}\right)\right]\left[\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B}}\right)-\mathcal{H}\left(\tau-\tau_{\mathrm{max,B}}\right)\right].\end{aligned} \tag{S6.2}$$

This consistent form also results in $\mathcal{H}\left(\tau-\tau_{x,\mathrm{B}}\right) = \mathcal{H}\left(\left(\tau_{\mathrm{sc}}/|\tau_{\mathrm{sc}}|\right)\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(c_{\tau x,\mathrm{B}}/\tau\right)\right)\right)$ (Equation 40 of the main text), and the limits of $\Omega_{\mathrm{AAC,B}}$ (Equation S6.2) can be converted from the particle relaxation time ($\tau$) to the measurement time ($t_{\mathrm{m}}$) domain as follows:

$$
\begin{aligned}
\Omega_{\mathrm{AAC,B}}(t_{\mathrm{m}}) =& \lambda_\Omega\, \mu_\Omega \left[1+\frac{\mu_\Omega}{\beta}\left(\frac{\tau}{c_{\tau*,\mathrm{B}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}-1\right)\right] \\
&\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)\right] \\
&+\lambda_\Omega\, \mu_\Omega \left[1+\frac{\mu_\Omega}{\beta}\left(1-\frac{\tau}{c_{\tau*,\mathrm{B}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}\right)\right] \\
&\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right)\right] \qquad \text{(S6.3)}\\
\Omega_{\mathrm{AAC,B}}(t_{\mathrm{m}}) =& \lambda_\Omega\, \mu_\Omega \left[c_{\mathrm{BL}}+c_{\mathrm{B1}}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] \\
&\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)\right] \\
&+\lambda_\Omega\, \mu_\Omega \left[c_{\mathrm{BU}}-c_{\mathrm{B1}}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] \\
&\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right)\right],
\end{aligned}
$$

where the constants are defined in Table S6.1.

Table S6.1: Constants of the Simplified Mathematical Representation of the Transfer Function of the Scanning AAC based on Particle Streamline Theory.

| Parameter | Equation | |
|---|---|---|
| $c_{\mathrm{BL}}$ | $1-\frac{\mu_\Omega}{\beta}$ | (S6.4) |
| $c_{\mathrm{BU}}$ | $1+\frac{\mu_\Omega}{\beta}$ | (S6.5) |
| $c_{\mathrm{B1}}$ | $\frac{\mu_\Omega}{\beta c_{\tau*,\mathrm{B}}}$ | (S6.6) |

## S6.1 Solving Integral of $\Omega_{\mathrm{AAC,B}}$ over $t_{\mathrm{c}}$ to determine $\bar{\Omega}_{\mathrm{AAC,B}}$

This section determines the average transfer function of the AAC ($\bar{\Omega}_{\mathrm{AAC,B}}$) over the counting time ($t_{\mathrm{c}}$) of the particle detector by substituting Equation S6.3 into Equation 34 of the main text and solving. To reduce the analysis, note that the two terms in Equation S6.3 have the same form. Expanding Equation S6.3 based on its Heaviside function terms and considering its first term generically as follows:

$$\begin{aligned}
&\lambda_\Omega\,\mu_\Omega\left[c_{\mathrm{B}y}+c_{\mathrm{B}w}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right)\\
&=\underbrace{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{B}y}\,\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right)}_{\text{Term Form: }\Omega_{\mathrm{AAC,B,F1}}(c_{\tau x,\mathrm{B}},\,c_{\mathrm{B}y},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)}+\underbrace{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{B}w}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right)}_{\text{Term Form: }\Omega_{\mathrm{AAC,B,F2}}(c_{\tau x,\mathrm{B}},\,c_{\mathrm{B}w},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)}.
\end{aligned}\tag{S6.7}$$

Therefore, substituting the two term forms (i.e. $\Omega_{\mathrm{AAC,B,F1}}$ and $\Omega_{\mathrm{AAC,B,F1}}$) identified in Equation S6.7 into Equation S6.3, then into the definition of $\bar{\Omega}_{\mathrm{AAC}}$ (Equation 34 of the main text) simplifies to the following:

$$\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,B}}&=\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\Omega_{\mathrm{AAC,B}}(\tau,\,t'_{\mathrm{m}})\,\mathrm{d}t'_{\mathrm{m}}\\
\bar{\Omega}_{\mathrm{AAC,B}}&=\underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\Omega_{\mathrm{AAC,B,F1}}(c_{\tau\mathrm{min,B}},\,c_{\mathrm{BL}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,B,F1}}(c_{\tau\mathrm{min,B}},\,c_{\mathrm{BL}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega,\,t_{\mathrm{c}})}\\
&+\underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\Omega_{\mathrm{AAC,B,F2}}(c_{\tau\mathrm{min,B}},\,c_{\mathrm{B1}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,B,F2}}(c_{\tau\mathrm{min,B}},\,c_{\mathrm{B1}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega,\,t_{\mathrm{c}})}\\
&-\underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\Omega_{\mathrm{AAC,B,F1}}(c_{\tau*,\mathrm{B}},\,c_{\mathrm{BL}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,B,F1}}(c_{\tau*,\mathrm{B}},\,c_{\mathrm{BL}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega,\,t_{\mathrm{c}})}\\
&-\underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\Omega_{\mathrm{AAC,B,F2}}(c_{\tau*,\mathrm{B}},\,c_{\mathrm{B1}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,B,F2}}(c_{\tau*,\mathrm{B}},\,c_{\mathrm{B1}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega,\,t_{\mathrm{c}})}\\
&+\underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\Omega_{\mathrm{AAC,B,F1}}(c_{\tau*,\mathrm{B}},\,c_{\mathrm{BU}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,B,F1}}(c_{\tau*,\mathrm{B}},\,c_{\mathrm{BU}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega,\,t_{\mathrm{c}})}\\
&+\underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\Omega_{\mathrm{AAC,B,F2}}(c_{\tau*,\mathrm{B}},\,-c_{\mathrm{B1}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,B,F2}}(c_{\tau*,\mathrm{B}},\,-c_{\mathrm{B1}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega,\,t_{\mathrm{c}})}\\
&-\underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\Omega_{\mathrm{AAC,B,F1}}(c_{\tau\mathrm{max,B}},\,c_{\mathrm{BU}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,B,F1}}(c_{\tau\mathrm{max,B}},\,c_{\mathrm{BU}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega,\,t_{\mathrm{c}})}\\
&-\underbrace{\frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\Omega_{\mathrm{AAC,B,F2}}(c_{\tau\mathrm{max,B}},\,-c_{\mathrm{B1}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega)\,\mathrm{d}t'_{\mathrm{m}}}_{\text{Term Form: }\bar{\Omega}_{\mathrm{AAC,B,F2}}(c_{\tau\mathrm{max,B}},\,-c_{\mathrm{B1}},\,\tau,\,\tau_{\mathrm{sc}},\,t_{\mathrm{m}},\,\lambda_\Omega,\,\mu_\Omega,\,t_{\mathrm{c}})}.
\end{aligned}\tag{S6.8}$$

All of the terms in Equation S6.8 are of the form $\bar{\Omega}_{\mathrm{AAC,B,F1}}$ or $\bar{\Omega}_{\mathrm{AAC,B,F2}}$, which are solved[2] in Sections S6.1.1 and S6.1.2, respectively. Therefore, substituting these solutions (i.e. Equations S6.15 and S6.21) into Equation S6.8:

$$\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,B}} =& \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}}}{t_\mathrm{c}} \Bigg[ \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right) \\
&- \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right) \Bigg] \\
&+ \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{B1}}\, \tau_\mathrm{sc}}{t_\mathrm{c}} \tau \Bigg[ \left( \exp\left(\frac{t_\mathrm{m} + t_\mathrm{c}}{\tau_\mathrm{sc}}\right) - \frac{c_{\tau\mathrm{min,B}}}{\tau}\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right) \\
&- \left( \exp\left(\frac{t_\mathrm{m}}{\tau_\mathrm{sc}}\right) - \frac{c_{\tau\mathrm{min,B}}}{\tau}\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right) \Bigg] \\
&- \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}}}{t_\mathrm{c}} \Bigg[ \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right)\right) \\
&- \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right)\right) \Bigg] \\
&- \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{B1}}\, \tau_\mathrm{sc}}{t_\mathrm{c}} \tau \Bigg[ \left( \exp\left(\frac{t_\mathrm{m} + t_\mathrm{c}}{\tau_\mathrm{sc}}\right) - \frac{c_{\tau*\mathrm{,B}}}{\tau}\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right)\right) \\
&- \left( \exp\left(\frac{t_\mathrm{m}}{\tau_\mathrm{sc}}\right) - \frac{c_{\tau*\mathrm{,B}}}{\tau}\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right)\right) \Bigg] \\
&+ \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BU}}}{t_\mathrm{c}} \Bigg[ \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right)\right) \\
&- \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right)\right) \Bigg] \\
&- \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{B1}}\, \tau_\mathrm{sc}}{t_\mathrm{c}} \tau \Bigg[ \left( \exp\left(\frac{t_\mathrm{m} + t_\mathrm{c}}{\tau_\mathrm{sc}}\right) - \frac{c_{\tau*\mathrm{,B}}}{\tau}\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right)\right) \\
&- \left( \exp\left(\frac{t_\mathrm{m}}{\tau_\mathrm{sc}}\right) - \frac{c_{\tau*\mathrm{,B}}}{\tau}\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau*\mathrm{,B}}}{\tau}\right)\right)\right) \Bigg] \\
&- \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BU}}}{t_\mathrm{c}} \Bigg[ \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right) \\
&- \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right) \Bigg] \\
&+ \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{B1}}\, \tau_\mathrm{sc}}{t_\mathrm{c}} \tau \Bigg[ \left( \exp\left(\frac{t_\mathrm{m} + t_\mathrm{c}}{\tau_\mathrm{sc}}\right) - \frac{c_{\tau\mathrm{max,B}}}{\tau}\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} + t_\mathrm{c} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right) \\
&- \left( \exp\left(\frac{t_\mathrm{m}}{\tau_\mathrm{sc}}\right) - \frac{c_{\tau\mathrm{max,B}}}{\tau}\right) \mathcal{H}\left( \frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|} \left( t_\mathrm{m} - \tau_\mathrm{sc} \ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right) \Bigg].
\end{aligned} \tag{S6.9}$$

[2]Assumes $\lambda_\Omega$ and $\mu_\Omega$ are constant over the width of the scanning AAC transfer function (i.e. the AAC is operated with sufficiently high resolution and the particle detector is operated with sufficiently low counting time).

Substituting back in the definition of $c_{\mathrm{B1}}$ (Equation S6.6) into Equation S6.9, then grouping the terms by common time intervals and simplifying:

$$\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,B}}(t_{\mathrm{m}}) = & \left[-\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)+\frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right] \\
& \cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right)\right] \\
& +\left[-\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] \\
& \cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)\right] \\
& +\left[\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}+\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right] \\
& \cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)\right] \\
& +\left[\frac{\lambda_\Omega\,\mu_\Omega}{t_{\mathrm{c}}}\left(c_{\mathrm{BL}}-c_{\mathrm{BU}}\right)\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)-2c_{\mathrm{B2}}\right] \\
& \cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)\right] \\
& +\left[\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BU}}-\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right] \\
& \cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right)\right] \\
& +\left[\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] \\
& \cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right)\right] \\
& +\left[\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BU}}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)+\frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{max,B}}}{c_{\tau*,\mathrm{B}}}\right] \\
& \cdot\left[\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right)-\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)\right)\right],
\end{aligned} \tag{S6.10}$$

where

$$c_{\mathrm{B2}} = \frac{\lambda_\Omega\,\mu_\Omega^2\,\tau_{\mathrm{sc}}}{\beta\,t_{\mathrm{c}}}. \tag{S6.11}$$

Finally, Equation S6.10 can converted from the measurement time ($t_\mathrm{m}$) back to the particle relaxation time ($\tau$) domain using $\mathcal{H}\left(\tau-\tau_x\right)=\mathcal{H}\left(\left(\tau_\mathrm{sc}/|\tau_\mathrm{sc}|\right)\left(t_\mathrm{m}-\tau_\mathrm{sc}\ln\left(c_{\tau x}/\tau\right)\right)\right)$ (Equation 40 of the main text) as follows:

$$
\begin{aligned}
\bar{\Omega}_\mathrm{AAC}(\tau) = &\left[-\frac{\lambda_\Omega\,\mu_\Omega\,c_\mathrm{BL}}{t_\mathrm{c}}\left(t_\mathrm{m}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau\,\mathrm{min},\mathrm{B}}}{\tau}\right)\right)+\frac{c_\mathrm{B2}\,c_{\tau\,\mathrm{min},\mathrm{B}}}{c_{\tau*,\mathrm{B}}}\right]\left[\mathcal{H}\left(\tau-\tau_{\mathrm{min},\mathrm{B}}\right)-\mathcal{H}\left(\tau-\tau_{\mathrm{min},\mathrm{B,tc}}\right)\right] \\
&+\left[-\frac{c_\mathrm{B2}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_\mathrm{m}}{\tau_\mathrm{sc}}\right)\right]\left[\mathcal{H}\left(\tau-\tau_{\mathrm{min},\mathrm{B}}\right)-\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B}}\right)\right] \\
&+\left[\lambda_\Omega\,\mu_\Omega\,c_\mathrm{BL}+\frac{c_\mathrm{B2}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_\mathrm{m}+t_\mathrm{c}}{\tau_\mathrm{sc}}\right)\right]\left[\mathcal{H}\left(\tau-\tau_{\mathrm{min},\mathrm{B,tc}}\right)-\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B,tc}}\right)\right] \\
&+\left[\frac{\lambda_\Omega\,\mu_\Omega}{t_\mathrm{c}}\left(c_\mathrm{BL}-c_\mathrm{BU}\right)\left(t_\mathrm{m}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)-2c_\mathrm{B2}\right]\left[\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B}}\right)-\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B,tc}}\right)\right] \\
&+\left[\lambda_\Omega\,\mu_\Omega\,c_\mathrm{BU}-\frac{c_\mathrm{B2}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_\mathrm{m}+t_\mathrm{c}}{\tau_\mathrm{sc}}\right)\right]\left[\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B,tc}}\right)-\mathcal{H}\left(\tau-\tau_{\mathrm{max},\mathrm{B,tc}}\right)\right] \\
&+\left[\frac{c_\mathrm{B2}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_\mathrm{m}}{\tau_\mathrm{sc}}\right)\right]\left[\mathcal{H}\left(\tau-\tau^*_{\mathrm{sc,B}}\right)-\mathcal{H}\left(\tau-\tau_{\mathrm{max},\mathrm{B}}\right)\right] \\
&+\left[\frac{\lambda_\Omega\,\mu_\Omega\,c_\mathrm{BU}}{t_\mathrm{c}}\left(t_\mathrm{m}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau\,\mathrm{max},\mathrm{B}}}{\tau}\right)\right)+\frac{c_\mathrm{B2}\,c_{\tau\,\mathrm{max},\mathrm{B}}}{c_{\tau*,\mathrm{B}}}\right]\left[\mathcal{H}\left(\tau-\tau_{\mathrm{max},\mathrm{B}}\right)-\mathcal{H}\left(\tau-\tau_{\mathrm{max},\mathrm{B,tc}}\right)\right]
\end{aligned}
\tag{S6.12}
$$

where $\tau_x$ and $\tau_{x,\mathrm{tc}}$ are the same as those derived based on limited trajectory theory (defined by Equations S3.27 and S5.9, respectively) and are restatements of Equations 36 and 41 of the main text, respectively. Examples of this average transfer function (i.e. based on Equation S6.12) are shown in Figure 3 of the main text.

#### S6.1.1 Analytical Solution for $\bar{\Omega}_{\mathrm{AAC,B,F1}}$ of Equation S6.8

Based on integral form 1 (i.e. $\Omega_{\mathrm{AAC,B,F1}}$) shown in Equation S6.7:

$$\begin{aligned}\bar{\Omega}_{\mathrm{AAC,B,F1}} &= \frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,B,F1}}\,\mathrm{d}t'_{\mathrm{m}}\\ \bar{\Omega}_{\mathrm{AAC,B,F1}} &= \frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \lambda_{\Omega}\,\mu_{\Omega}\,c_{\mathrm{B}y}\,\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right)\mathrm{d}t'_{\mathrm{m}}.\end{aligned} \tag{S6.13}$$

Assuming $\tau_{\mathrm{sc}}$ is greater than zero (denoted by $\bar{\Omega}_{\mathrm{AAC,B,F1+}}$), Equation S6.13 becomes:

$$\begin{aligned}\bar{\Omega}_{\mathrm{AAC,B,F1+}} &= \frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \lambda_{\Omega}\,\mu_{\Omega}\,c_{\mathrm{B}y}\,\mathcal{H}\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathrm{d}t'_{\mathrm{m}}\\ &= \frac{\lambda_{\Omega}\,\mu_{\Omega}\,c_{\mathrm{B}y}}{t_{\mathrm{c}}}\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathcal{H}\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\Bigg|_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}}\\ \bar{\Omega}_{\mathrm{AAC,B,F1+}} &= \frac{\lambda_{\Omega}\,\mu_{\Omega}\,c_{\mathrm{B}y}}{t_{\mathrm{c}}}\Bigg[\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathcal{H}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\\ &\qquad -\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathcal{H}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\Bigg].\end{aligned} \tag{S6.14}$$

As shown in Section S5.1.3, this solution (Equation S6.14) can be expanded to consider both $\tau_{\mathrm{sc}} > 0$ and $\tau_{\mathrm{sc}} < 0$ based on Equation S5.37 (i.e. only differ by the inequality sign represented within their Heaviside functions), and thus both cases can be represented using one equation as follows:

$$\begin{aligned}\bar{\Omega}_{\mathrm{AAC,B,F1}} = \frac{\lambda_{\Omega}\,\mu_{\Omega}\,c_{\mathrm{B}y}}{t_{\mathrm{c}}}\Bigg[&\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}+t_{\mathrm{c}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right)\\ &-\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right)\Bigg].\end{aligned} \tag{S6.15}$$

### S6.1.2 Analytical Solution for $\bar{\Omega}_{\mathrm{AAC,B,F2}}$ of Equation S6.8

Based on integral form 2 (i.e. $\Omega_{\mathrm{AAC,B,F2}}$) shown in Equation S6.7:

$$
\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,B,F2}} &= \frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \Omega_{\mathrm{AAC,B,F2}}\,\mathrm{d}t'_{\mathrm{m}} \\
\bar{\Omega}_{\mathrm{AAC,B,F2}} &= \frac{1}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \lambda_{\Omega}\,\mu_{\Omega}\,c_{\mathrm{B}w}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\mathcal{H}\left(\frac{\tau_{\mathrm{sc}}}{|\tau_{\mathrm{sc}}|}\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right)\mathrm{d}t'_{\mathrm{m}}.
\end{aligned}
\tag{S6.16}
$$

Assuming $\tau_{\mathrm{sc}}$ is greater than zero (denoted by $\bar{\Omega}_{\mathrm{AAC,B,F2+}}$), Equation S6.16 becomes:

$$
\bar{\Omega}_{\mathrm{AAC,B,F2+}} = \frac{\lambda_{\Omega}\,\mu_{\Omega}\,c_{\mathrm{B}w}\,\tau}{t_{\mathrm{c}}}\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\mathcal{H}\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathrm{d}t'_{\mathrm{m}}.
\tag{S6.17}
$$

Apply integration by parts to Equation S6.17 as follows:

$$
\begin{aligned}
u = \mathcal{H}\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right) &\rightarrow \mathrm{d}u = \delta\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathrm{d}t'_{\mathrm{m}} \\
\mathrm{d}v = \exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\mathrm{d}t'_{\mathrm{m}} &\rightarrow v = \tau_{\mathrm{sc}}\exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right),
\end{aligned}
\tag{S6.18}
$$

where $\delta$ Dirac delta function defined in Equation S5.18. Therefore, using integration by parts based on Equation S6.18, Equation S6.17 becomes:

$$
\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,B,F2+}} = \frac{\lambda_{\Omega}\,\mu_{\Omega}\,c_{\mathrm{B}w}\,\tau}{t_{\mathrm{c}}}\Bigg[&\tau_{\mathrm{sc}}\exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\mathcal{H}\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\Bigg|_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \\
&-\int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \tau_{\mathrm{sc}}\exp\left(\frac{t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\delta\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathrm{d}t'_{\mathrm{m}}\Bigg].
\end{aligned}
\tag{S6.19}
$$

Furthermore, noting that $\delta\left(t'_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)$ vanishes everywhere except $t'_{\mathrm{m}} = \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)$, Equation S6.19 becomes:

$$
\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,B,F2+}} &= \frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{B}w}\,\tau}{t_\mathrm{c}}\left[\tau_\mathrm{sc}\exp\left(\frac{t'_\mathrm{m}}{\tau_\mathrm{sc}}\right)\mathcal{H}\left(t'_\mathrm{m}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\Bigg|_{t_\mathrm{m}}^{t_\mathrm{m}+t_\mathrm{c}}\right.\\
&\quad\left.-\,\tau_\mathrm{sc}\exp\left(\frac{\tau_\mathrm{sc}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)}{\tau_\mathrm{sc}}\right)\int_{t_\mathrm{m}}^{t_\mathrm{m}+t_\mathrm{c}}\delta\left(t'_\mathrm{m}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\mathrm{d}t'_\mathrm{m}\right]\\
&= \frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{B}w}\,\tau}{t_\mathrm{c}}\left(\tau_\mathrm{sc}\exp\left(\frac{t'_\mathrm{m}}{\tau_\mathrm{sc}}\right)-\frac{\tau_\mathrm{sc}\,c_{\tau x,\mathrm{B}}}{\tau}\right)\mathcal{H}\left(t'_\mathrm{m}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\Bigg|_{t_\mathrm{m}}^{t_\mathrm{m}+t_\mathrm{c}}\\
\bar{\Omega}_{\mathrm{AAC,B,F2+}} &= \frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{B}w}\,\tau_\mathrm{sc}}{t_\mathrm{c}}\tau\left[\left(\exp\left(\frac{t_\mathrm{m}+t_\mathrm{c}}{\tau_\mathrm{sc}}\right)-\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\mathcal{H}\left(t_\mathrm{m}+t_\mathrm{c}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right.\\
&\quad\left.-\left(\exp\left(\frac{t_\mathrm{m}}{\tau_\mathrm{sc}}\right)-\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\mathcal{H}\left(t_\mathrm{m}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right].
\end{aligned}
\tag{S6.20}
$$

As shown in Section S5.1.3, this solution (Equation S6.20) can be expanded to consider both $\tau_\mathrm{sc} > 0$ and $\tau_\mathrm{sc} < 0$ based on Equation S5.37 (i.e. only differ by the inequality sign represented within their Heaviside functions), and thus both cases can be represented using one equation as follows:

$$
\begin{aligned}
\bar{\Omega}_{\mathrm{AAC,B,F2}} = \frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{B}w}\,\tau_\mathrm{sc}}{t_\mathrm{c}}\tau&\left[\left(\exp\left(\frac{t_\mathrm{m}+t_\mathrm{c}}{\tau_\mathrm{sc}}\right)-\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\mathcal{H}\left(\frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|}\left(t_\mathrm{m}+t_\mathrm{c}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right)\right.\\
&\left.-\left(\exp\left(\frac{t_\mathrm{m}}{\tau_\mathrm{sc}}\right)-\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\mathcal{H}\left(\frac{\tau_\mathrm{sc}}{|\tau_\mathrm{sc}|}\left(t_\mathrm{m}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau x,\mathrm{B}}}{\tau}\right)\right)\right)\right].
\end{aligned}
\tag{S6.21}
$$

# S7 Parameters of Scanning AAC: Uniform Axial Flow

Note: Similar to the other sections in the SI, this section and its subsections use simplified notation of the parameter subscripts, as outlined in Section S1.1, based on the common AAC operation and theory instances included within the title and footer of this section.

## S7.1 Average Setpoint of Scanning AAC over $t_{\mathrm{c}}$

Similar to the definition of the average transfer function of the scanning AAC (i.e. Equation 34 of the main text), the average particle relaxation setpoint of the AAC ($\bar{\tau}^*_{\mathrm{sc}}$) over the counting time ($t_{\mathrm{c}}$) of the downstream particle detector can be found by[1] (restatement of Equation 44 in main text):

$$\bar{\tau}^*_{\mathrm{sc}} = \frac{1}{t_{\mathrm{c}}} \int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \tau^*_{\mathrm{sc}}(t'_{\mathrm{m}})\, \mathrm{d}t'_{\mathrm{m}}. \tag{S7.1}$$

Furthermore, the form of $\tau^*_{\mathrm{sc}}$ based on limited trajectory or particle streamline theory is the same ($c_{\tau*} \exp\left(-t_{\mathrm{m}}/\tau_{\mathrm{sc}}\right)$) as shown by Equations S3.34 and S4.10, respectively, and summarized in Table 2 of the main text. Therefore, substituting this consistent form into Equation S7.1:

$$\bar{\tau}^*_{\mathrm{sc}} = \frac{1}{t_{\mathrm{c}}} \int_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} c_{\tau*} \exp\left(\frac{-t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right) \mathrm{d}t'_{\mathrm{m}}, \tag{S7.2}$$

and letting:

$$u = \frac{-t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}} \rightarrow \mathrm{d}u = \frac{-1}{\tau_{\mathrm{sc}}} \mathrm{d}t'_{\mathrm{m}} \rightarrow \mathrm{d}t'_{\mathrm{m}} = -\tau_{\mathrm{sc}}\, \mathrm{d}u, \tag{S7.3}$$

becomes:

$$\begin{aligned} \bar{\tau}^*_{\mathrm{sc}} &= \frac{-\tau_{\mathrm{sc}}\, c_{\tau*}}{t_{\mathrm{c}}} \int \exp(u)\, \mathrm{d}u = \frac{-\tau_{\mathrm{sc}}\, c_{\tau*}}{t_{\mathrm{c}}} \exp\left(\frac{-t'_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\Bigg|_{t_{\mathrm{m}}}^{t_{\mathrm{m}}+t_{\mathrm{c}}} \\ \bar{\tau}^*_{\mathrm{sc}} &= \frac{-c_{\tau*}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right) - \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]. \end{aligned} \tag{S7.4}$$

As previously stated, this solution (i.e. Equation S7.4) applies to both limited trajectory and particle streamline theory as the form of $\tau^*_{\mathrm{sc}}$ based on either theory is the same ($c_{\tau*} \exp\left(-t_{\mathrm{m}}/\tau_{\mathrm{sc}}\right)$).

## S7.2 Classes per Decade of AAC Scan

The definition of $\mathrm{CPD_{sc}}$ (i.e. Equation 45 of main text) can be simplified by considering the particle relaxation setpoints ($\bar{\tau}^*_{\mathrm{sc},1}$ and $\bar{\tau}^*_{\mathrm{sc},2}$) between two consecutive measurements (i.e. $t_{\mathrm{m2}} = t_{\mathrm{m1}} + t_{\mathrm{c}}$) as follows:

$$\mathrm{CPD_{sc}} = \frac{2-1}{\left|\log\left(\bar{\tau}^*_{\mathrm{sc},2}\right) - \log\left(\bar{\tau}^*_{\mathrm{sc},1}\right)\right|} = \frac{1}{\left|\log\left(\frac{\bar{\tau}^*_{\mathrm{sc},2}}{\bar{\tau}^*_{\mathrm{sc},1}}\right)\right|}. \tag{S7.5}$$

Therefore, the CPD of the measurements collected by the scanning AAC can be determined by the ratio of particle relaxation setpoints between two consecutive measurements. Furthermore, the form of $\bar{\tau}^*_{\mathrm{sc}}$ based on limited trajectory or particle streamline theory is the same as shown in Section S7.1 and summarized in Table 4 of the main text. Therefore, the ratio of $\bar{\tau}^*_{\mathrm{sc}}$ at consecutive measurement times can be found based on the definition of $\bar{\tau}^*_{\mathrm{sc}}$ (i.e. Equation S7.4) as follows:

$$\begin{aligned}
\frac{\bar{\tau}^*_{\mathrm{sc},2}}{\bar{\tau}^*_{\mathrm{sc},1}} &= \frac{\frac{-c_{\tau*}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[\exp\left(\frac{-(t_{\mathrm{m2}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right) - \exp\left(\frac{-t_{\mathrm{m2}}}{\tau_{\mathrm{sc}}}\right)\right]}{\frac{-c_{\tau*}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[\exp\left(\frac{-(t_{\mathrm{m1}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right) - \exp\left(\frac{-t_{\mathrm{m1}}}{\tau_{\mathrm{sc}}}\right)\right]} \\
&= \frac{\left[\exp\left(\frac{-(t_{\mathrm{m1}}+t_{\mathrm{c}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right) - \exp\left(\frac{-(t_{\mathrm{m1}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)\right]}{\left[\exp\left(\frac{-(t_{\mathrm{m1}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right) - \exp\left(\frac{-t_{\mathrm{m1}}}{\tau_{\mathrm{sc}}}\right)\right]} \\
\frac{\bar{\tau}^*_{\mathrm{sc},2}}{\bar{\tau}^*_{\mathrm{sc},1}} &= \exp\left(\frac{-t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right).
\end{aligned} \tag{S7.6}$$

Substituting Equation S7.6 into Equation S7.5, the classes per decade of the measurements collected by the scanning AAC is:

$$\mathrm{CPD_{sc}} = \frac{1}{\left|\log\left(\exp\left(\frac{-t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)\right|}. \tag{S7.7}$$

As previously stated, this solution (i.e. Equation S7.7) applies to both limited trajectory and particle streamline theory as the the form of $\bar{\tau}^*_{\mathrm{sc}}$ based on either theory is the same as shown in Section S7.1 and summarized in Table 4 of the main text.

## S7.3 Range of AAC Scan

The range of particle relaxation times measured by the scanning AAC is smaller than the steady-state AAC due to the residence time of particles in the classifier ($t_\mathrm{f}$). Measurements cannot be collected until the particles at the classifier inlet at the start of the scan pass through the classifier (i.e. $t_\mathrm{m} = t_\mathrm{f}$). Furthermore, measurements can only be collected for particles that experience the changing centrifugal force field during their entire residence time within the classifier. Therefore, measurements cannot be collected for particles at the classifier outlet after the scan duration (i.e $t_\mathrm{m} = t_\mathrm{sc}$).

As shown in the following subsections (Sections S7.3.1 and S7.3.2), the ratio of $\bar{\tau}^*_\mathrm{sc}$ to $\tau^*_\mathrm{ss}$ based on limited trajectory or particle streamline theory is the same (Equations S7.12 or S7.16, respectively) as follows:

$$\frac{\bar{\tau}^*_\mathrm{sc}}{\tau^*_\mathrm{ss}} = \frac{-\omega^2\, t_\mathrm{f}}{\omega_\mathrm{S}^2\, t_\mathrm{c}\left[1-\exp\left(\frac{-t_\mathrm{f}}{\tau_\mathrm{sc}}\right)\right]}\left[\exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_\mathrm{sc}}\right)-\exp\left(\frac{-t_\mathrm{m}}{\tau_\mathrm{sc}}\right)\right]. \tag{S7.8}$$

At the start of the scan, $\omega = \omega_\mathrm{S}$ and $t_\mathrm{m} = t_\mathrm{f}$, substituting these values into Equation S7.8 to determine the offset between the setpoints of the scanning and steady-state AAC:

$$\begin{aligned}\left.\frac{\bar{\tau}^*_\mathrm{sc}}{\tau^*_\mathrm{ss}}\right|_\mathrm{S} &= \frac{-\omega_\mathrm{S}^2\, t_\mathrm{f}}{\omega_\mathrm{S}^2\, t_\mathrm{c}\left[1-\exp\left(\frac{-t_\mathrm{f}}{\tau_\mathrm{sc}}\right)\right]}\left[\exp\left(\frac{-(t_\mathrm{f}+t_\mathrm{c})}{\tau_\mathrm{sc}}\right)-\exp\left(\frac{-t_\mathrm{f}}{\tau_\mathrm{sc}}\right)\right]\\ \left.\frac{\bar{\tau}^*_\mathrm{sc}}{\tau^*_\mathrm{ss}}\right|_\mathrm{S} &= \frac{-t_\mathrm{f}}{t_\mathrm{c}\left[\exp\left(\frac{t_\mathrm{f}}{\tau_\mathrm{sc}}\right)-1\right]}\left[\exp\left(\frac{-t_\mathrm{c}}{\tau_\mathrm{sc}}\right)-1\right].\end{aligned} \tag{S7.9}$$

At the end of the scan, $\omega = \omega_\mathrm{E}$ and $t_\mathrm{m} = t_\mathrm{sc}$, substituting these values into Equation S7.8 to determine the offset between the setpoints of the scanning and steady-state AAC:

$$\begin{aligned}\left.\frac{\bar{\tau}^*_\mathrm{sc}}{\tau^*_\mathrm{ss}}\right|_\mathrm{E} &= \frac{-\omega_\mathrm{E}^2\, t_\mathrm{f}}{\omega_\mathrm{S}^2\, t_\mathrm{c}\left[1-\exp\left(\frac{-t_\mathrm{f}}{\tau_\mathrm{sc}}\right)\right]}\left[\exp\left(\frac{-(t_\mathrm{sc}+t_\mathrm{c})}{\tau_\mathrm{sc}}\right)-\exp\left(\frac{-t_\mathrm{sc}}{\tau_\mathrm{sc}}\right)\right]\\ \left.\frac{\bar{\tau}^*_\mathrm{sc}}{\tau^*_\mathrm{ss}}\right|_\mathrm{E} &= \frac{-\omega_\mathrm{E}^2\, t_\mathrm{f}}{\omega_\mathrm{S}^2\, t_\mathrm{c}\exp\left(\frac{t_\mathrm{sc}-t_\mathrm{f}}{\tau_\mathrm{sc}}\right)\left[\exp\left(\frac{t_\mathrm{f}}{\tau_\mathrm{sc}}\right)-1\right]}\left[\exp\left(\frac{-t_\mathrm{c}}{\tau_\mathrm{sc}}\right)-1\right].\end{aligned} \tag{S7.10}$$

### S7.3.1 Limited Trajectory

Taking the ratio of $\bar{\tau}^*_{\mathrm{sc}}$ (Equation S7.4) to $\tau^*_{\mathrm{ss}}$ (Equation S3.18) for the scanning and steady-state AAC setpoints based on limited trajectory theory, respectively, then substituting in the definition for $c_{\tau*}$ (i.e. Equation S3.36) becomes:

$$\frac{\bar{\tau}^*_{\mathrm{sc}}}{\tau^*_{\mathrm{ss}}} = \frac{\frac{-\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right) - \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\frac{1}{2c_{\mathrm{sc}}}\left[\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2}\ln\left(\frac{Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{s}}\left(1-r_1^2/r_2^2\right)}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{sh}}\left(1-r_1^2/r_2^2\right)}\right)\right]}{\frac{1}{2K_{\mathrm{ss}}}\left[\ln\left(\frac{r_2}{r_1}\right) + \frac{1}{2}\ln\left(\frac{Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{s}}\left(1-r_1^2/r_2^2\right)}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}-Q_{\mathrm{sh}}\left(1-r_1^2/r_2^2\right)}\right)\right]}. \tag{S7.11}$$

Finally, substituting the definitions of $K_{\mathrm{ss}}$ and $c_{\mathrm{sc}}$ (i.e. Equations S3.2 and S3.29, respectively) into Equation S7.11, and simplifying:

$$\begin{aligned}
\frac{\bar{\tau}^*_{\mathrm{sc}}}{\tau^*_{\mathrm{ss}}} &= \frac{\omega^2\, t_{\mathrm{f}}}{\omega_{\mathrm{S}}^2\, \tau_{\mathrm{sc}}\left[1-\exp\left(\frac{-t_{\mathrm{f}}}{\tau_{\mathrm{sc}}}\right)\right]}\,\frac{-\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right) - \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] \\
\frac{\bar{\tau}^*_{\mathrm{sc}}}{\tau^*_{\mathrm{ss}}} &= \frac{-\omega^2\, t_{\mathrm{f}}}{\omega_{\mathrm{S}}^2\, t_{\mathrm{c}}\left[1-\exp\left(\frac{-t_{\mathrm{f}}}{\tau_{\mathrm{sc}}}\right)\right]}\left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right) - \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right].
\end{aligned} \tag{S7.12}$$

### S7.3.2 Particle Streamline

Similar to the derivation based on limited trajectory theory in the previous Section (S7.3.1), taking the ratio of $\bar{\tau}^*_{\mathrm{sc,B}}$ (Equation S7.4) to $\tau^*_{\mathrm{ss,B}}$ (Equation S4.4) for the scanning and steady-state AAC setpoints based on particle streamline theory, respectively:

$$\frac{\bar{\tau}^*_{\mathrm{sc,B}}}{\tau^*_{\mathrm{ss,B}}} = \frac{\frac{-c_{\tau*,\mathrm{B}}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)-\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]}{\frac{2Q_{\mathrm{sh}}}{\pi\omega^2(r_1+r_2)^2L}}. \quad \text{(S7.13)}$$

Substituting the definition of $c_{\tau*,\mathrm{B}}$ (Equation S4.11) into Equation S7.13:

$$\begin{aligned}
\frac{\bar{\tau}^*_{\mathrm{sc,B}}}{\tau^*_{\mathrm{ss,B}}} &= \frac{\frac{-\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)-\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\frac{1}{c_{\mathrm{sc}}}\frac{2(r_2^2-r_1^2)}{(\beta+1)(r_1+r_2)^2}}{\frac{2Q_{\mathrm{sh}}}{\pi\omega^2(r_1+r_2)^2L}} \\
\frac{\bar{\tau}^*_{\mathrm{sc,B}}}{\tau^*_{\mathrm{ss,B}}} &= \frac{-\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)-\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\frac{\pi\omega^2(r_2^2-r_1^2)L}{c_{\mathrm{sc}}\,Q_{\mathrm{sh}}(\beta+1)}.
\end{aligned} \quad \text{(S7.14)}$$

Substituting the definitions of $\beta$ and $c_{\mathrm{sc}}$ (i.e. Equations S4.2 and S4.12, respectively) into Equation S7.14, and simplifying:

$$\begin{aligned}
\frac{\bar{\tau}^*_{\mathrm{sc,B}}}{\tau^*_{\mathrm{ss,B}}} &= \frac{-\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)-\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\frac{1}{\omega_{\mathrm{S}}^2\,\tau_{\mathrm{sc}}\left[1-\exp\left(\frac{-t_{\mathrm{f}}}{\tau_{\mathrm{sc}}}\right)\right]}\frac{\pi\omega^2(r_2^2-r_1^2)L}{Q_{\mathrm{sh}}(\frac{Q_{\mathrm{a}}}{Q_{\mathrm{sh}}}+1)} \\
\frac{\bar{\tau}^*_{\mathrm{sc,B}}}{\tau^*_{\mathrm{ss,B}}} &= \frac{-\omega^2}{\omega_{\mathrm{S}}^2\,t_{\mathrm{c}}\left[1-\exp\left(\frac{-t_{\mathrm{f}}}{\tau_{\mathrm{sc}}}\right)\right]}\left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)-\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\frac{\pi(r_2^2-r_1^2)L}{Q_{\mathrm{a}}+Q_{\mathrm{sh}}}.
\end{aligned} \quad \text{(S7.15)}$$

Finally, substituting in the definition of $t_{\mathrm{f}}$ (Equation 13 of main text) into Equation S7.15:

$$\frac{\bar{\tau}^*_{\mathrm{sc,B}}}{\tau^*_{\mathrm{ss,B}}} = \frac{-\omega^2\,t_{\mathrm{f}}}{\omega_{\mathrm{S}}^2\,t_{\mathrm{c}}\left[1-\exp\left(\frac{-t_{\mathrm{f}}}{\tau_{\mathrm{sc}}}\right)\right]}\left[\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)-\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]. \quad \text{(S7.16)}$$

# S8 Inversion of Transfer Function of Scanning AAC

Note: Similar to the other sections in the SI, this section and its subsections use simplified notation of the parameter subscripts, as outlined in Section S1.1, based on the common AAC operation and theory instances included within the title and footer of this section.

Similar to the process developed by Stolzenburg and McMurry (2008) for the steady-state DMA, by Wang and Flagan (1990) for the scanning DMA or by Johnson et al. (2018) for the steady-state AAC, the particle number concentration ($N_{\mathrm{det}}$) that passes through the scanning AAC at time $t_i$ can be determined from:

$$N_{\mathrm{det}}(t_i) = \int \eta(\tau)\, \bar{\Omega}_{\mathrm{AAC}}(\tau,\, t_i)\, \mathrm{d}N(\tau), \tag{S8.1}$$

where $\eta(\tau)$ is the counting efficiency of the downstream particle detector and $N(\tau)$ is the particle number concentration of the aerosol source at particle relaxation time $\tau$, while $\bar{\Omega}(\tau,\, t_i)$ is the average transfer function of the scanning AAC over the counting time of the particle detector at time $t_i$. Applying the chain rule to the integrand from Equation S8.1:

$$\frac{\mathrm{d}N(\tau)}{\mathrm{d}\tau} = \frac{\mathrm{d}N(\tau)}{\mathrm{d}\log\tau}\frac{\mathrm{d}\log\tau}{\mathrm{d}\tau}. \tag{S8.2}$$

Furthermore, the derivative of $\log(\tau)$ with respect to $\tau$ is equivalent to:

$$\frac{\mathrm{d}\log(\tau)}{\mathrm{d}\tau} = \frac{1}{\ln(10)}\frac{1}{\tau}. \tag{S8.3}$$

Therefore, substituting Equation S8.3 into Equation S8.2, and isolating $\mathrm{d}N$:

$$\frac{\mathrm{d}N(\tau)}{\mathrm{d}\tau} = \frac{\mathrm{d}N(\tau)}{\mathrm{d}\log\tau}\frac{1}{\ln(10)}\frac{1}{\tau} \rightarrow \boldsymbol{\mathrm{d}N(\tau) = \mathrm{d}\tau\frac{\mathrm{d}N(\tau)}{\mathrm{d}\log\tau}\frac{1}{\ln(10)}\frac{1}{\tau}}. \tag{S8.4}$$

Substituting Equation S8.4 into Equation S8.1, and assuming $\eta$ and $\mathrm{d}N(\tau)/\mathrm{d}\log\tau$ are constant[3] over the width of $\bar{\Omega}_{\mathrm{AAC}}$ simplifies to:

$$\begin{aligned} N_{\mathrm{det}}(t_i) &= \int \eta(\tau)\, \bar{\Omega}_{\mathrm{AAC}}(\tau,\, t_i)\frac{\mathrm{d}N(\tau)}{\mathrm{d}\log\tau}\frac{1}{\ln(10)}\frac{1}{\tau}\mathrm{d}\tau \\ N_{\mathrm{det}}(t_i) &= \eta(\bar{\tau}^*_{\mathrm{sc},i})\frac{\mathrm{d}N(\tau)}{\mathrm{d}\log\tau}\bigg|_{\bar{\tau}^*_{\mathrm{sc},i}}\frac{1}{\ln(10)}\int\frac{\bar{\Omega}_{\mathrm{AAC}}(\tau,\, t_i)}{\tau}\mathrm{d}\tau. \end{aligned} \tag{S8.5}$$

Since the AAC transfer function is centered around $\bar{\tau}^*_{\mathrm{sc},i}$ at time $t_i$ in the scan, this value is used to estimate the constants (i.e. $\eta$ and $\mathrm{d}N(\tau)/\mathrm{d}\log\tau$). Isolating the spectral density in terms of particle relaxation time (i.e. $\mathrm{d}N/\mathrm{d}\log\tau$) classified at time $t_i$ by the scanning AAC from Equation S8.5:

[3] This assumption that the aerosol concentration ($N$) is constant over the width of the AAC transfer function is valid if the AAC classification is operated with a sufficiently high resolution and the particle detector is operated with a sufficiently low counting time.

$$\left.\frac{\mathrm{d}N(\tau)}{\mathrm{d}\log\tau}\right|_{\bar{\tau}^{*}_{\mathrm{sc},i}} = \frac{N_{\mathrm{det}}(t_i)\ln(10)}{\eta(\bar{\tau}^{*}_{\mathrm{sc},i})\,\beta^{*}_{\mathrm{sc},i}}, \tag{S8.6}$$

where $\beta^{*}_{\mathrm{sc},i}$ is the non-dimensional deconvolution parameter of the transfer function of the scanning AAC at time $t_i$ as follows:

$$\beta^{*}_{\mathrm{sc},i} = \int \frac{\bar{\Omega}_{\mathrm{AAC}}(\tau,\, t_i)}{\tau}\mathrm{d}\tau. \tag{S8.7}$$

The spectral density of the aerosol can also be found in terms of aerodynamic diameter (i.e. $\mathrm{d}N/\mathrm{d}\log d_{\mathrm{a}}$) by applying the chain rule to Equation S8.6 as follows:

$$\frac{\mathrm{d}N}{\mathrm{d}\log d_{\mathrm{a}}} = \frac{\mathrm{d}N}{\mathrm{d}\log\tau}\frac{\mathrm{d}\log\tau}{\mathrm{d}\log d_{\mathrm{a}}}, \tag{S8.8}$$

where $\mathrm{d}\log\tau/\mathrm{d}\log d_{\mathrm{a}}$ was previously determined by Johnson et al. (2018):

$$\frac{\mathrm{d}\log\tau}{\mathrm{d}\log d_{\mathrm{a}}} = \frac{1}{d_{\mathrm{a}}\,C_{\mathrm{c}}\,(d_{\mathrm{a}})}\left[2d_{\mathrm{a}} + \alpha_{c_c}\,\lambda + \beta_{c_c}\,\lambda\exp\left(-\gamma_{c_c}\frac{d_{\mathrm{a}}}{\lambda}\right)\left(1-\frac{\gamma_{c_c}\,d_{\mathrm{a}}}{\lambda}\right)\right], \tag{S8.9}$$

where $\lambda$ is the mean free path of the surrounding gas and Kim et al. (2005) determined the factors, $\alpha_{c_c} = 2\times1.165 = 2.33$, $\beta_{c_c} = 2\times0.483 = 0.966$ and $\gamma_{c_c} = 0.997/2 = 0.4985$, to estimate the Cunningham slip correction.

# S9 Deconvolution Parameter ($\beta^*_{\text{sc,LT}}$) of Scanning AAC: Limited Trajectory, Idealized and Uniform Axial Flow

Note: Similar to the other sections in the SI, this section and its subsections use simplified notation of the parameter subscripts, as outlined in Section S1.1, based on the common AAC operation and theory instances included within the title and footer of this section.

Substituting the definition of $\bar{\Omega}_{\text{AAC}}$ based on limited trajectory theory (Equation S5.8) into the definition of the $\beta^*_{\text{sc}}$ (Equation S8.7) and organizing into separate integrals by the ranges of particle relaxation times becomes:

$$
\begin{aligned}
\beta^*_{\text{sc,LT}} &= \int \frac{\bar{\Omega}_{\text{AAC,LT}}(\tau,\, t)}{\tau} \mathrm{d}\tau \\
\beta^*_{\text{sc,LT}} &= \underbrace{\int_{\tau_{\text{min}}}^{\tau_{\text{min,tc}}} \frac{1}{\tau}\left[\frac{c_{f11}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{\tau\text{min}}\,c_{f12}\right) - \frac{c_{f13}}{t_{\text{c}}}\left(t_{\text{m}} - \tau_{\text{sc}}\ln\left(\frac{c_{\tau\text{min}}}{\tau}\right)\right)\right]\mathrm{d}\tau}_{\beta^*_{\text{sc,LT,1}}} \\
&+ \underbrace{\int_{\tau_{\text{min}}}^{\tau_{13}} \frac{1}{\tau}\left[-\frac{c_{f11}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\text{m}}}{\tau_{\text{sc}}}\right)\right)\right]\mathrm{d}\tau}_{\beta^*_{\text{sc,LT,2}}} \\
&+ \underbrace{\int_{\tau_{\text{min,tc}}}^{\tau_{13,\text{tc}}} \frac{1}{\tau}\left[\frac{c_{f11}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\text{m}}+t_{\text{c}}}{\tau_{\text{sc}}}\right)\right) + c_{f13}\right]\mathrm{d}\tau}_{\beta^*_{\text{sc,LT,3}}} \\
&+ \underbrace{\int_{\tau_{13}}^{\tau_{13,\text{tc}}} \frac{1}{\tau}\left[-\frac{c_{f11}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{\tau13}\,c_{f12}\right) + \left(\frac{c_{f13}}{t_{\text{c}}} - \frac{c_{f31}}{t_{\text{c}}}\right)\left(t_{\text{m}} - \tau_{\text{sc}}\ln\left(\frac{c_{\tau13}}{\tau}\right)\right)\right]\mathrm{d}\tau}_{\beta^*_{\text{sc,LT,4}}} \\
&+ \underbrace{\int_{\tau_{13,\text{tc}}}^{\tau_{23,\text{tc}}} \frac{c_{f31}}{\tau}\mathrm{d}\tau}_{\beta^*_{\text{sc,LT,5}}} \\
&+ \underbrace{\int_{\tau_{23}}^{\tau_{23,\text{tc}}} \frac{1}{\tau}\left[\frac{c_{f21}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{\tau23}\,c_{f22}\right) + \left(\frac{c_{f31}}{t_{\text{c}}} - \frac{c_{f23}}{t_{\text{c}}}\right)\left(t_{\text{m}} - \tau_{\text{sc}}\ln\left(\frac{c_{\tau23}}{\tau}\right)\right)\right]\mathrm{d}\tau}_{\beta^*_{\text{sc,LT,6}}} \\
&+ \underbrace{\int_{\tau_{23,\text{tc}}}^{\tau_{\text{max,tc}}} \frac{1}{\tau}\left[\frac{c_{f21}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\text{m}}+t_{\text{c}}}{\tau_{\text{sc}}}\right)\right) + c_{f23}\right]\mathrm{d}\tau}_{\beta^*_{\text{sc,LT,7}}} \\
&+ \underbrace{\int_{\tau_{23}}^{\tau_{\text{max}}} \frac{1}{\tau}\left[-\frac{c_{f21}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\text{m}}}{\tau_{\text{sc}}}\right)\right)\right]\mathrm{d}\tau}_{\beta^*_{\text{sc,LT,8}}} \\
&+ \underbrace{\int_{\tau_{\text{max}}}^{\tau_{\text{max,tc}}} \frac{1}{\tau}\left[-\frac{c_{f21}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{\tau\text{max}}\,c_{f22}\right) + \frac{c_{f23}}{t_{\text{c}}}\left(t_{\text{m}} - \tau_{\text{sc}}\ln\left(\frac{c_{\tau\text{max}}}{\tau}\right)\right)\right]\mathrm{d}\tau}_{\beta^*_{\text{sc,LT,9}}}\,.
\end{aligned}
\tag{S9.1}
$$

All of the terms in Equation S9.1 consist of different combinations of the following three integral forms:

$$\beta^*_{\text{sc,LT,F1}} = \int_{\tau_{\text{L}}}^{\tau_{\text{U}}} \frac{a}{\tau}\mathrm{d}\tau, \tag{S9.2}$$

$$\beta^{*}_{\mathrm{sc,LT,F2}} = \int_{\tau_{\mathrm{L}}}^{\tau_{\mathrm{U}}} \frac{a \ln\left(\frac{b}{\tau}\right)}{\tau} \mathrm{d}\tau, \tag{S9.3}$$

$$\beta^{*}_{\mathrm{sc,LT,F3}} = \int_{\tau_{\mathrm{L}}}^{\tau_{\mathrm{U}}} \frac{a\, \mathcal{EI}\left(b\tau\right)}{\tau} \mathrm{d}\tau, \tag{S9.4}$$

where $a$ and $b$ are constants. These integral forms $\beta^{*}_{\mathrm{sc,LT,F1}}$, $\beta^{*}_{\mathrm{sc,LT,F2}}$ and $\beta^{*}_{\mathrm{sc,LT,F3}}$, are solved in Section S9.1 (specifically Sections S9.1.1, S9.1.2 and S9.1.3, respectively). Based on the solutions for these integral forms, the terms of Equation S9.1 (i.e. $\beta^{*}_{\mathrm{sc,LT,1}}$ to $\beta^{*}_{\mathrm{sc,LT,9}}$) are solved in Section S9.2 (specifically Sections S9.2.1 to S9.2.5). Therefore substituting the solutions for terms $\beta^{*}_{\mathrm{sc,LT,1}}$ to $\beta^{*}_{\mathrm{sc,LT,9}}$ (Equations S9.26, S9.30, S9.34, S9.38, S9.42, S9.39, S9.35, S9.31 and S9.27, respectively) into Equation S9.1 and simplifying:

$$
\begin{aligned}
\beta^*_{\mathrm{sc,LT}} &= \int \frac{\bar{\Omega}_{\mathrm{AAC,LT}}(\tau,\, t)}{\tau} \mathrm{d}\tau \\
\beta^*_{\mathrm{sc,LT}} &= - c_{f11}\, \mathcal{EI}\left(c_{\tau \mathrm{min}} c_{f12}\right) - \frac{t_{\mathrm{c}}\, c_{f13}}{2\tau_{\mathrm{sc}}} \\
&\quad - \frac{c_{f11}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \ln\left(\frac{c_{\tau 13}}{c_{\tau \mathrm{min}}}\right) \left[\gamma + \frac{1}{2} \ln\left(\left|c_{f12}^2\, c_{\tau 13}\, c_{\tau \mathrm{min}}\right|\right)\right] \\
&\quad - \frac{c_{f11}\, c_{f12}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \Big[ c_{\tau 13}\, {}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\}, c_{f12}\, c_{\tau 13}\right) \\
&\qquad - c_{\tau \mathrm{min}}\, {}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\}, c_{f12}\, c_{\tau \mathrm{min}}\right) \Big] \\
&\quad + \frac{c_{f11}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \ln\left(\frac{c_{\tau 13}}{c_{\tau \mathrm{min}}}\right) \left[\gamma + \frac{1}{2} \ln\left(\left|c_{f12}^2\, c_{\tau 13}\, c_{\tau \mathrm{min}}\right|\right)\right] \\
&\quad + \frac{c_{f11}\, c_{f12}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \Big[ c_{\tau 13}\, {}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\}, c_{f12}\, c_{\tau 13}\right) \\
&\qquad - c_{\tau \mathrm{min}}\, {}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\}, c_{f12}\, c_{\tau \mathrm{min}}\right) \Big] + c_{f13} \ln\left(\frac{c_{\tau 13}}{c_{\tau \mathrm{min}}}\right) \\
&\quad + c_{f11}\, \mathcal{EI}\left(c_{\tau 13} c_{f12}\right) + \frac{t_{\mathrm{c}}\left(c_{f13} - c_{f31}\right)}{2\tau_{\mathrm{sc}}} \\
&\quad + c_{f31} \ln\left(\frac{c_{\tau 23}}{c_{\tau 13}}\right) \\
&\quad - c_{f21}\, \mathcal{EI}\left(c_{\tau 23} c_{f22}\right) + \frac{t_{\mathrm{c}}\left(c_{f31} - c_{f23}\right)}{2\tau_{\mathrm{sc}}} \\
&\quad + \frac{c_{f21}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \ln\left(\frac{c_{\tau \mathrm{max}}}{c_{\tau 23}}\right) \left[\gamma + \frac{1}{2} \ln\left(\left|c_{f22}^2\, c_{\tau \mathrm{max}}\, c_{\tau 23}\right|\right)\right] \\
&\quad + \frac{c_{f21}\, c_{f22}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \Big[ c_{\tau \mathrm{max}}\, {}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\}, c_{f22}\, c_{\tau \mathrm{max}}\right) \\
&\qquad - c_{\tau 23}\, {}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\}, c_{f22}\, c_{\tau 23}\right) \Big] + c_{f23} \ln\left(\frac{c_{\tau \mathrm{max}}}{c_{\tau 23}}\right) \\
&\quad - \frac{c_{f21}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \ln\left(\frac{c_{\tau \mathrm{max}}}{c_{\tau 23}}\right) \left[\gamma + \frac{1}{2} \ln\left(\left|c_{f22}^2\, c_{\tau \mathrm{max}}\, c_{\tau 23}\right|\right)\right] \\
&\quad - \frac{c_{f21}\, c_{f22}\, \tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \Big[ c_{\tau \mathrm{max}}\, {}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\}, c_{f22}\, c_{\tau \mathrm{max}}\right) \\
&\qquad - c_{\tau 23}\, {}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\}, c_{f22}\, c_{\tau 23}\right) \Big] \\
&\quad + c_{f21}\, \mathcal{EI}\left(c_{\tau \mathrm{max}} c_{f22}\right) + \frac{t_{\mathrm{c}}\, c_{f23}}{2\tau_{\mathrm{sc}}} \\
\beta^*_{\mathrm{sc,LT}} &= c_{f11} \left[\mathcal{EI}\left(c_{\tau 13}\, c_{f12}\right) - \mathcal{EI}\left(c_{\tau \mathrm{min}}\, c_{f12}\right)\right] \\
&\quad + c_{f13} \ln\left(\frac{c_{\tau 13}}{c_{\tau \mathrm{min}}}\right) + c_{f31} \ln\left(\frac{c_{\tau 23}}{c_{\tau 13}}\right) + c_{f23} \ln\left(\frac{c_{\tau \mathrm{max}}}{c_{\tau 23}}\right) \\
&\quad + c_{f21} \left[\mathcal{EI}\left(c_{\tau \mathrm{max}}\, c_{f22}\right) - \mathcal{EI}\left(c_{\tau 23}\, c_{f22}\right)\right].
\end{aligned}
\tag{S9.5}
$$

## S9.1 Solving Integral Forms ($\beta^*_{\mathrm{sc,LT,F}x}$) of Deconvolution Parameter Integral

### S9.1.1 Analytical Solution for $\beta^*_{\mathrm{sc,LT,F1}}$ (Equation S9.2)

Based on Equation S9.2:

$$\begin{aligned}\beta^*_{\mathrm{sc,LT,F1}} &= \int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \frac{a}{\tau}\mathrm{d}\tau = a\Big[\ln(\tau_\mathrm{U}) - \ln(\tau_\mathrm{L})\Big] \\ \beta^*_{\mathrm{sc,LT,F1}} &= a\ln\left(\frac{\tau_\mathrm{U}}{\tau_\mathrm{L}}\right).\end{aligned} \tag{S9.6}$$

### S9.1.2 Analytical Solution for $\beta^*_{\mathrm{sc,LT,F2}}$ (Equation S9.3)

Letting:

$$u = \ln\left(\frac{b}{\tau}\right) \rightarrow \mathrm{d}u = -\frac{1}{\tau}\mathrm{d}\tau \rightarrow \mathrm{d}\tau = -\tau\,\mathrm{d}u, \tag{S9.7}$$

and substituting it into Equation S9.3 as follows:

$$\begin{aligned}\beta^*_{\mathrm{sc,LT,F2}} &= \int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \frac{a\ln\left(\frac{b}{\tau}\right)}{\tau}\mathrm{d}\tau = a\int -u\,\mathrm{d}u \\ \beta^*_{\mathrm{sc,LT,F2}} &= -\frac{a}{2}\left[\ln^2\left(\frac{b}{\tau_\mathrm{U}}\right) - \ln^2\left(\frac{b}{\tau_\mathrm{L}}\right)\right].\end{aligned} \tag{S9.8}$$

Furthermore noting that $(x^2 - y^2) = (x+y)(x-y)$, Equation S9.8 simplifies to the following:

$$\begin{aligned}\beta^*_{\mathrm{sc,LT,F2}} &= -\frac{a}{2}\left[\ln\left(\frac{b}{\tau_\mathrm{U}}\right) + \ln\left(\frac{b}{\tau_\mathrm{L}}\right)\right]\left[\ln\left(\frac{b}{\tau_\mathrm{U}}\right) - \ln\left(\frac{b}{\tau_\mathrm{L}}\right)\right] \\ \beta^*_{\mathrm{sc,LT,F2}} &= -\frac{a}{2}\ln\left(\frac{b^2}{\tau_\mathrm{U}\,\tau_\mathrm{L}}\right)\ln\left(\frac{\tau_\mathrm{L}}{\tau_\mathrm{U}}\right).\end{aligned} \tag{S9.9}$$

### S9.1.3 Analytical Solution for $\beta^{*}_{\mathrm{sc,LT,F3}}$ (Equation S9.4)

Based on Equation S9.4:

$$\beta^{*}_{\mathrm{sc,LT,F3}} = \int_{\tau_{\mathrm{L}}}^{\tau_{\mathrm{U}}} \frac{a\,\mathcal{EI}\left(b\,\tau\right)}{\tau} \mathrm{d}\tau. \tag{S9.10}$$

Based Equation 6.6.1 of NIST (2019), the Exponential Integral ($\mathcal{EI}\left(x\right)$) can be represented as:

$$\mathcal{EI}\left(x\right) = \gamma + \ln\left(x\right) + \sum_{n=1}^{\infty} \frac{x^{n}}{n\,n!}, \tag{S9.11}$$

for $x > 0$ and where $\gamma$ is Euler's Constant (0.5772156649). To consider negative input values $\mathcal{EI}\left(x\right)$ can be approximated as:

$$\mathcal{EI}\left(x\right) = \gamma + \ln\left(|x|\right) + \sum_{n=1}^{\infty} \frac{x^{n}}{n\,n!}, \tag{S9.12}$$

for $x \geq -16.452$. This approximation has been verified to agree within 1% of both the values calculated by the Exponential Integral function within Matlab and numerical integration of $\exp\left(x\right)/x$ over this range with 0.001 resolution. To check if the scanning AAC theory falls within this range, consider all of the $\mathcal{EI}\left(x\right)$ inputs within Equation S9.1 over their respective integral limits. Only the extreme limits need to be considered. Therefore, any input terms to $\mathcal{EI}\left(x\right)$ containing $\tau_{13}$ or $\tau_{23}$ can be ignored as $\tau_{\mathrm{min}} < \tau_{13} < \tau_{23} < \tau_{\mathrm{max}}$.

Therefore based on the definitions of $\tau_{\mathrm{min}}$, $c_{\tau\mathrm{min}}$, $c_{f12}$ and $\tau_{x,\mathrm{tc}}$ (Equations S3.32, S3.33, S3.39 and S5.9, respectively), the lower integral limits of the $\mathcal{EI}\left(x\right)$ inputs within Equation S9.1 all simplify to the same constant as follows:

$$c_{\tau\mathrm{min}}\,c_{f12} = c_{f12}\,\tau_{\mathrm{min}} \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right) = c_{f12}\,\tau_{\mathrm{min,tc}} \exp\left(\frac{t_{\mathrm{m}} + t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right) = \boldsymbol{-\ln\left(\frac{Q_{\mathrm{a}} + Q_{\mathrm{sh}} - Q_{\mathrm{s}}\left(1 - r_1^2/r_2^2\right)}{Q_{\mathrm{a}} + Q_{\mathrm{sh}} - Q_{\mathrm{sh}}\left(1 - r_1^2/r_2^2\right)}\right)}. \tag{S9.13}$$

Similarly, based on the definitions of $\tau_{\mathrm{max}}$, $c_{\tau\mathrm{max}}$, $c_{f22}$ and $\tau_{x,\mathrm{tc}}$ (Equations S3.30, S3.31, S3.43 and S5.9, respectively), the upper integral limits of the $\mathcal{EI}\left(x\right)$ inputs within Equation S9.1 all simplify to the same constant as follows:

$$c_{\tau\mathrm{max}}\,c_{f22} = c_{f22}\,\tau_{\mathrm{max}} \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right) = c_{f22}\,\tau_{\mathrm{max,tc}} \exp\left(\frac{t_{\mathrm{m}} + t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right) = \boldsymbol{-2\ln\left(\frac{r_2}{r_1}\right)}. \tag{S9.14}$$

Therefore, if $c_{\tau\mathrm{min}}\,c_{f12} \geq -16.452$ and $c_{\tau\mathrm{max}}\,c_{f22} \geq -16.452$ are satisfied, the approximation for the Exponential Integral Function (Equation S9.12) is valid for the theory inputs of the scanning AAC. This criteria is satisfied over the entire operating range of the commercial AAC from Cambustion ($r_1$ = 56 mm, $r_2$ = 60 mm, $Q_{\mathrm{a}}$ = 0.3 to 1.5 L/min, $Q_{\mathrm{s}}$ = 0.3 to 1.5 L/min and $Q_{\mathrm{sh}}$ = 1.5 to 15 L/min) with $c_{\tau\mathrm{min}}\,c_{f12} = -0.13$ to $-0.02$ and $c_{\tau\mathrm{max}}\,c_{f22} = -0.14$. Therefore, substituting this approximation for $\mathcal{EI}\left(x\right)$ (Equation S9.12) into Equation S9.10:

$$\beta^*_{\mathrm{sc,LT,F3}} = a \int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \frac{1}{\tau} \left[ \gamma + \ln\left(|b\,\tau|\right) + \sum_{n=1}^{\infty} \frac{(b\,\tau)^n}{n\,n!} \right] \mathrm{d}\tau$$
$$\beta^*_{\mathrm{sc,LT,F3}} = a \Bigg[ \underbrace{\int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \frac{\gamma}{\tau} \mathrm{d}\tau}_{\beta^*_{\mathrm{sc,LT,F31}}} + \underbrace{\int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \frac{\ln\left(|b\,\tau|\right)}{\tau} \mathrm{d}\tau}_{\beta^*_{\mathrm{sc,LT,F32}}} + \underbrace{\int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \sum_{n=1}^{\infty} \frac{b^n\,\tau^{n-1}}{n\,n!} \mathrm{d}\tau}_{\beta^*_{\mathrm{sc,LT,F33}}} \Bigg]. \tag{S9.15}$$

Solving the first term (i.e. $\beta^*_{\mathrm{sc,LT,F31}}$) of Equation S9.15 as follows:

$$\beta^*_{\mathrm{sc,LT,F31}} = \int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \frac{\gamma}{\tau} \mathrm{d}\tau = \boldsymbol{\gamma \ln\left(\frac{\tau_\mathrm{U}}{\tau_\mathrm{L}}\right)}. \tag{S9.16}$$

Solving the second term (i.e. $\beta^*_{\mathrm{sc,LT,F32}}$) of Equation S9.15 by letting:

$$u = \ln\left(|b\,\tau|\right) \rightarrow \mathrm{d}u = \frac{1}{\tau}\mathrm{d}\tau, \tag{S9.17}$$

and substituting it into the equation as follows:

$$\beta^*_{\mathrm{sc,LT,F32}} = \int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \frac{\ln\left(|b\,\tau|\right)}{\tau} \mathrm{d}\tau = \int u\,\mathrm{d}u = \frac{u^2}{2} = \left.\frac{\ln^2\left(|b\,\tau|\right)}{2}\right|_{\tau_\mathrm{L}}^{\tau_\mathrm{U}}$$
$$\beta^*_{\mathrm{sc,LT,F32}} = \frac{1}{2}\left[\ln^2\left(|b\,\tau_\mathrm{U}|\right) - \ln^2\left(|b\,\tau_\mathrm{L}|\right)\right]. \tag{S9.18}$$

Furthermore noting that $(x^2 - y^2) = (x+y)(x-y)$, Equation S9.18 simplifies to the following:

$$\beta^*_{\mathrm{sc,LT,F32}} = \frac{1}{2} \ln\left(|b^2\,\tau_\mathrm{U}\,\tau_\mathrm{L}|\right) \ln\left(\left|\frac{\tau_\mathrm{U}}{\tau_\mathrm{L}}\right|\right). \tag{S9.19}$$

Solving the third term (i.e. $\beta^*_{\mathrm{sc,LT,F33}}$) of Equation S9.15 as follows:

$$\beta^*_{\mathrm{sc,LT,F33}} = \int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \sum_{n=1}^{\infty} \frac{b^n\,\tau^{n-1}}{n\,n!} \mathrm{d}\tau = \left.\sum_{n=1}^{\infty} \frac{b^n\,\tau^n}{n^2\,n!}\right|_{\tau_\mathrm{L}}^{\tau_\mathrm{U}}. \tag{S9.20}$$

However, based on Equation 15.2.1 of NIST (2019):

$$\sum_{n=1}^{\infty} \frac{b^n\,\tau^n}{n^2\,n!} = b\,\tau\,{}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\},b\,\tau\right), \tag{S9.21}$$

where $\mathcal{F}$ is the generalized hypergeometric function. Therefore, Equation S9.20 becomes:

$$\beta^{*}_{\mathrm{sc,LT,F33}} = b\Big[\tau_{\mathrm{U}}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},b\,\tau_{\mathrm{U}}\right) - \tau_{\mathrm{L}}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},b\,\tau_{\mathrm{L}}\right)\Big]. \tag{S9.22}$$

Therefore, substituting the solutions for $\beta^{*}_{\mathrm{sc,LT,F31}}$, $\beta^{*}_{\mathrm{sc,LT,F32}}$ and $\beta^{*}_{\mathrm{sc,LT,F33}}$ (Equations S9.16, S9.19 and S9.22, respectively) into the definition of $\beta^{*}_{\mathrm{sc,LT,F3}}$ (Equation S9.15):

$$\begin{aligned}\beta^{*}_{\mathrm{sc,LT,F3}} =& a\,\gamma\ln\left(\frac{\tau_{\mathrm{U}}}{\tau_{\mathrm{L}}}\right) + \frac{a}{2}\ln\left(\left|b^{2}\,\tau_{\mathrm{U}}\,\tau_{\mathrm{L}}\right|\right)\ln\left(\left|\frac{\tau_{\mathrm{U}}}{\tau_{\mathrm{L}}}\right|\right)\\ &+ a\,b\Big[\tau_{\mathrm{U}}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},b\,\tau_{\mathrm{U}}\right) - \tau_{\mathrm{L}}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},b\,\tau_{\mathrm{L}}\right)\Big].\end{aligned} \tag{S9.23}$$

## S9.2 Solving Terms ($\beta^*_{\mathrm{sc,LT},x}$) of Deconvolution Parameter

### S9.2.1 Analytical Solutions for $\beta^*_{\mathrm{sc,LT,1}}$ and $\beta^*_{\mathrm{sc,LT,9}}$ of Equation S9.1

Based on term 1 (i.e. $\beta^*_{\mathrm{sc,LT,1}}$) shown in Equation S9.1:

$$\begin{aligned}
\beta^*_{\mathrm{sc,LT,1}} &= \int_{\tau_{\min}}^{\tau_{\min,\mathrm{tc}}} \frac{1}{\tau}\left[\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau\min}\,c_{f12}\right) - \frac{c_{f13}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\min}}{\tau}\right)\right)\right]\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,LT,1}} &= \int_{\tau_{\min}}^{\tau_{\min,\mathrm{tc}}} \left[\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau\min}\,c_{f12}\right) - \frac{c_{f13}\,t_{\mathrm{m}}}{t_{\mathrm{c}}}\right]\frac{1}{\tau}\mathrm{d}\tau + \int_{\tau_{\min}}^{\tau_{\min,\mathrm{tc}}} \frac{c_{f13}}{t_{\mathrm{c}}}\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\min}}{\tau}\right)\frac{1}{\tau}\mathrm{d}\tau.
\end{aligned} \tag{S9.24}$$

Substituting the integral solutions of $\beta^*_{\mathrm{sc,LT,F1}}$ and $\beta^*_{\mathrm{sc,LT,F2}}$ (i.e. Equations S9.6 and S9.9) into Equation S9.24 as follows:

$$\beta^*_{\mathrm{sc,LT,1}} = \left[\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau\min}\,c_{f12}\right) - \frac{c_{f13}\,t_{\mathrm{m}}}{t_{\mathrm{c}}}\right]\ln\left(\frac{\tau_{\min,\mathrm{tc}}}{\tau_{\min}}\right) - \frac{\frac{c_{f13}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}}{2}\ln\left(\frac{c^2_{\tau\min}}{\tau_{\min,\mathrm{tc}}\,\tau_{\min}}\right)\ln\left(\frac{\tau_{\min}}{\tau_{\min,\mathrm{tc}}}\right). \tag{S9.25}$$

Substituting the definitions of $\tau_x$ and $\tau_{x,\mathrm{tc}}$ (Equations S3.27 and S5.9, respectively) into Equation S9.25 to convert from the particle relaxation time ($\tau$) to the measurement time ($t_{\mathrm{m}}$) domain:

$$\begin{aligned}
\beta^*_{\mathrm{sc,LT,1}} =& \left[\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau\min}\,c_{f12}\right) - \frac{c_{f13}\,t_{\mathrm{m}}}{t_{\mathrm{c}}}\right]\ln\left(\frac{c_{\tau\min}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}{c_{\tau\min}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}\right) \\
&- \frac{c_{f13}\,\tau_{\mathrm{sc}}}{2t_{\mathrm{c}}}\ln\left(\frac{c^2_{\tau\min}}{c_{\tau\min}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)c_{\tau\min}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}\right)\ln\left(\frac{c_{\tau\min}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}{c_{\tau\min}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}\right) \\
\beta^*_{\mathrm{sc,LT,1}} =& -c_{f11}\,\mathcal{EI}\left(c_{\tau\min}\,c_{f12}\right) - \frac{t_{\mathrm{c}}\,c_{f13}}{2\tau_{\mathrm{sc}}}.
\end{aligned} \tag{S9.26}$$

Following the same approach, term 9 (i.e. $\beta^*_{\mathrm{sc,LT,9}}$) shown in Equation S9.1 is equivalent to:

$$\begin{aligned}
\beta^*_{\mathrm{sc,LT,9}} &= \int_{\tau_{\max}}^{\tau_{\max,\mathrm{tc}}} \frac{1}{\tau}\left[-\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{\tau\max}\,c_{f22}\right) + \frac{c_{f23}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\max}}{\tau}\right)\right)\right]\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,LT,9}} &= c_{f21}\,\mathcal{EI}\left(c_{\tau\max}\,c_{f22}\right) + \frac{t_{\mathrm{c}}\,c_{f23}}{2\tau_{\mathrm{sc}}}.
\end{aligned} \tag{S9.27}$$

### S9.2.2 Analytical Solutions for $\beta^{*}_{\mathrm{sc,LT,2}}$ and $\beta^{*}_{\mathrm{sc,LT,8}}$ of Equation S9.1

Based on term 2 (i.e. $\beta^{*}_{\mathrm{sc,LT,2}}$) shown in Equation S9.1:

$$\beta^{*}_{\mathrm{sc,LT,2}} = \int_{\tau_{\mathrm{min}}}^{\tau_{13}} \frac{1}{\tau}\left[-\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\right]\mathrm{d}\tau. \tag{S9.28}$$

Substituting the integral solution of $\beta^{*}_{\mathrm{sc,LT,F3}}$ (i.e. Equation S9.23) into Equation S9.28 as follows:

$$\begin{aligned}\beta^{*}_{\mathrm{sc,LT,2}} =& -\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\gamma\ln\left(\frac{\tau_{13}}{\tau_{\mathrm{min}}}\right) + \frac{-\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}}{2}\ln\left(\left|\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)^{2}\tau_{13}\,\tau_{\mathrm{min}}\right|\right)\ln\left(\left|\frac{\tau_{13}}{\tau_{\mathrm{min}}}\right|\right)\\ &+ -\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\left[\tau_{13}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\tau_{13}\right)\right.\\ &\left.-\tau_{\mathrm{min}}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\tau_{\mathrm{min}}\right)\right].\end{aligned} \tag{S9.29}$$

Substituting the definitions of $\tau_x$ (Equation S3.27) into Equation S9.29 to convert from the particle relaxation time ($\tau$) to the measurement time ($t_{\mathrm{m}}$) domain:

$$\begin{aligned}\beta^{*}_{\mathrm{sc,LT,2}} =& -\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\gamma\ln\left(\frac{c_{\tau13}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}{c_{\tau\mathrm{min}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}\right)\\ &-\frac{c_{f11}\,\tau_{\mathrm{sc}}}{2t_{\mathrm{c}}}\ln\left(\left|\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)^{2}c_{\tau13}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)c_{\tau\mathrm{min}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right|\right)\ln\left(\left|\frac{c_{\tau13}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}{c_{\tau\mathrm{min}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}\right|\right)\\ &-\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}c_{f12}\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\left[c_{\tau13}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right){}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},c_{f12}\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)c_{\tau13}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\right.\\ &\left.-c_{\tau\mathrm{min}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right){}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},c_{f12}\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)c_{\tau\mathrm{min}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\right]\\ \beta^{*}_{\mathrm{sc,LT,2}} =& -\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\ln\left(\frac{c_{\tau13}}{c_{\tau\mathrm{min}}}\right)\left[\gamma+\frac{1}{2}\ln\left(\left|c_{f12}^{2}\,c_{\tau13}\,c_{\tau\mathrm{min}}\right|\right)\right]\\ &-\frac{c_{f11}\,c_{f12}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[c_{\tau13}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},c_{f12}\,c_{\tau13}\right)\right.\\ &\left.-c_{\tau\mathrm{min}}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},c_{f12}\,c_{\tau\mathrm{min}}\right)\right].\end{aligned} \tag{S9.30}$$

Following the same approach, term 8 (i.e. $\beta^{*}_{\mathrm{sc,LT,8}}$) shown in Equation S9.1 is equivalent to:

$$\beta^{*}_{\mathrm{sc,LT},8} = \int_{\tau_{23}}^{\tau_{\max}} \frac{1}{\tau} \left[ -\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \mathcal{EI}\left( c_{f22}\,\tau \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right)\right] \mathrm{d}\tau$$

$$\begin{aligned}\beta^{*}_{\mathrm{sc,LT},8} = &-\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}} \ln\left(\frac{c_{\tau\max}}{c_{\tau23}}\right)\left[\gamma + \frac{1}{2}\ln\left(\left|c_{f22}^{2}\,c_{\tau\max}\,c_{\tau23}\right|\right)\right] \\ &-\frac{c_{f21}\,c_{f22}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\Big[c_{\tau\max}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},c_{f22}\,c_{\tau\max}\right) \\ &\quad - c_{\tau23}\,{}_{3}\mathcal{F}_{3}\left(\{1,1,1\},\{2,2,2\},c_{f22}\,c_{\tau23}\right)\Big].\end{aligned} \tag{S9.31}$$

### S9.2.3 Analytical Solutions for $\beta^*_{\mathrm{sc,LT},3}$ and $\beta^*_{\mathrm{sc,LT},7}$ of Equation S9.1

Based on term 3 (i.e. $\beta^*_{\mathrm{sc,LT},3}$) shown in Equation S9.1:

$$\begin{aligned}
\beta^*_{\mathrm{sc,LT},3} &= \int_{\tau_{\mathrm{min,tc}}}^{\tau_{13,\mathrm{tc}}} \frac{1}{\tau}\left[\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)+c_{f13}\right]\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,LT},3} &= \int_{\tau_{\mathrm{min,tc}}}^{\tau_{13,\mathrm{tc}}} \frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\frac{\mathcal{EI}\left(c_{f12}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)}{\tau}\mathrm{d}\tau + \int_{\tau_{\mathrm{min,tc}}}^{\tau_{13,\mathrm{tc}}}\frac{c_{f13}}{\tau}\mathrm{d}\tau.
\end{aligned} \tag{S9.32}$$

Substituting the integral solutions of $\beta^*_{\mathrm{sc,LT,F1}}$ and $\beta^*_{\mathrm{sc,LT,F3}}$ (i.e. Equations S9.6 and S9.23) into Equation S9.32 as follows:

$$\begin{aligned}
\beta^*_{\mathrm{sc,LT},3} =& \frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\,\gamma\ln\left(\frac{\tau_{13,\mathrm{tc}}}{\tau_{\mathrm{min,tc}}}\right)+\frac{\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}}{2}\ln\left(\left|\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)^2\tau_{13,\mathrm{tc}}\,\tau_{\mathrm{min,tc}}\right|\right)\ln\left(\left|\frac{\tau_{13,\mathrm{tc}}}{\tau_{\mathrm{min,tc}}}\right|\right) \\
&+\frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)\left[\tau_{13,\mathrm{tc}}\,{}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\},\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)\tau_{13,\mathrm{tc}}\right)\right. \\
&\left.-\tau_{\mathrm{min,tc}}\,{}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\},\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)\tau_{\mathrm{min,tc}}\right)\right]+c_{f13}\ln\left(\frac{\tau_{13,\mathrm{tc}}}{\tau_{\mathrm{min,tc}}}\right).
\end{aligned} \tag{S9.33}$$

Substituting the definition of $\tau_{x,\mathrm{tc}}$ (Equation S5.9) into Equation S9.33 to convert from the particle relaxation time ($\tau$) to the measurement time ($t_{\mathrm{m}}$) domain:

$$\begin{aligned}
\beta^*_{\mathrm{sc,LT},3} =& \frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\gamma \ln\left(\frac{c_{\tau 13}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}{c_{\tau \min}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}\right) \\
&+ \frac{c_{f11}\,\tau_{\mathrm{sc}}}{2t_{\mathrm{c}}}\ln\left(\left|\left(c_{f12}\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)^2 c_{\tau 13}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)c_{\tau \min}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)\right|\right) \\
&\ln\left(\left|\frac{c_{\tau 13}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}{c_{\tau \min}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}\right|\right) + \frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}c_{f12}\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right) \\
&\left[c_{\tau 13}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right){}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\},c_{f12}\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)c_{\tau 13}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)\right)\right. \\
&\left. - c_{\tau \min}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right){}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\},c_{f12}\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)c_{\tau \min}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)\right)\right] \\
&+ c_{f13}\ln\left(\frac{c_{\tau 13}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}{c_{\tau \min}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}\right) \\
\beta^*_{\mathrm{sc,LT},3} =& \frac{c_{f11}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\ln\left(\frac{c_{\tau 13}}{c_{\tau \min}}\right)\left[\gamma + \frac{1}{2}\ln\left(\left|c_{f12}^2\,c_{\tau 13}\,c_{\tau \min}\right|\right)\right] \\
&+ \frac{c_{f11}\,c_{f12}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[c_{\tau 13}\,{}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\},c_{f12}\,c_{\tau 13}\right)\right. \\
&\quad\left. - c_{\tau \min}\,{}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\},c_{f12}\,c_{\tau \min}\right)\right] + c_{f13}\ln\left(\frac{c_{\tau 13}}{c_{\tau \min}}\right).
\end{aligned} \tag{S9.34}$$

Following the same approach, term 7 (i.e. $\beta^*_{\mathrm{sc,LT},7}$) shown in Equation S9.1 is equivalent to:

$$\begin{aligned}
\beta^*_{\mathrm{sc,LT},7} =& \int_{\tau_{23,\mathrm{tc}}}^{\tau_{\max,\mathrm{tc}}}\frac{1}{\tau}\left[\frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\mathcal{EI}\left(c_{f22}\,\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right)+c_{f23}\right]\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,LT},7} =& \frac{c_{f21}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\ln\left(\frac{c_{\tau \max}}{c_{\tau 23}}\right)\left[\gamma+\frac{1}{2}\ln\left(\left|c_{f22}^2\,c_{\tau \max}\,c_{\tau 23}\right|\right)\right] \\
&+ \frac{c_{f21}\,c_{f22}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\left[c_{\tau \max}\,{}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\},c_{f22}\,c_{\tau \max}\right)\right. \\
&\quad\left. - c_{\tau 23}\,{}_3\mathcal{F}_3\left(\{1,1,1\},\{2,2,2\},c_{f22}\,c_{\tau 23}\right)\right] + c_{f23}\ln\left(\frac{c_{\tau \max}}{c_{\tau 23}}\right).
\end{aligned} \tag{S9.35}$$

#### S9.2.4 Analytical Solutions for $\beta^*_{\text{sc,LT},4}$ and $\beta^*_{\text{sc,LT},6}$ of Equation S9.1

Based on term 4 (i.e. $\beta^*_{\text{sc,LT},4}$) shown in Equation S9.1:

$$\begin{aligned}\beta^*_{\text{sc,LT},4} &= \int_{\tau_{13}}^{\tau_{13,\text{tc}}} \frac{1}{\tau}\left[-\frac{c_{f11}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{\tau 13}\,c_{f12}\right) + \left(\frac{c_{f13}}{t_{\text{c}}} - \frac{c_{f31}}{t_{\text{c}}}\right)\left(t_{\text{m}} - \tau_{\text{sc}}\ln\left(\frac{c_{\tau 13}}{\tau}\right)\right)\right]\text{d}\tau \\ &= \int_{\tau_{13}}^{\tau_{13,\text{tc}}} \left[-\frac{c_{f11}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{\tau 13}\,c_{f12}\right) + \frac{t_{\text{m}}(c_{f13}-c_{f31})}{t_{\text{c}}}\right]\frac{1}{\tau}\text{d}\tau \\ &\quad + \int_{\tau_{13}}^{\tau_{13,\text{tc}}} \frac{-\tau_{\text{sc}}(c_{f13}-c_{f31})}{t_{\text{c}}}\frac{\ln\left(\frac{c_{\tau 13}}{\tau}\right)}{\tau}\text{d}\tau.\end{aligned} \tag{S9.36}$$

Substituting the integral solutions of $\beta^*_{\text{sc,LT,F1}}$ and $\beta^*_{\text{sc,LT,F2}}$ (i.e. Equations S9.6 and S9.9) into Equation S9.36 as follows:

$$\begin{aligned}\beta^*_{\text{sc,LT},4} =&\left[-\frac{c_{f11}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{\tau 13}\,c_{f12}\right) + \frac{t_{\text{m}}(c_{f13}-c_{f31})}{t_{\text{c}}}\right]\ln\left(\frac{\tau_{13,\text{tc}}}{\tau_{13}}\right) \\ &+ \frac{\frac{\tau_{\text{sc}}(c_{f13}-c_{f31})}{t_{\text{c}}}}{2}\ln\left(\frac{c_{\tau 13}^2}{\tau_{13,\text{tc}}\,\tau_{13}}\right)\ln\left(\frac{\tau_{13}}{\tau_{13,\text{tc}}}\right).\end{aligned} \tag{S9.37}$$

Substituting the definitions of $\tau_x$ and $\tau_{x,\text{tc}}$ (Equations S3.27 and S5.9, respectively) into Equation S9.37 to convert from the particle relaxation time ($\tau$) to the measurement time ($t_{\text{m}}$) domain:

$$\begin{aligned}\beta^*_{\text{sc,LT},4} =&\left[-\frac{c_{f11}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{\tau 13}\,c_{f12}\right) + \frac{t_{\text{m}}(c_{f13}-c_{f31})}{t_{\text{c}}}\right]\ln\left(\frac{c_{\tau 13}\exp\left(\frac{-(t_{\text{m}}+t_{\text{c}})}{\tau_{\text{sc}}}\right)}{c_{\tau 13}\exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)}\right) \\ &+ \frac{\tau_{\text{sc}}(c_{f13}-c_{f31})}{2t_{\text{c}}}\ln\left(\frac{c_{\tau 13}^2}{c_{\tau 13}\exp\left(\frac{-(t_{\text{m}}+t_{\text{c}})}{\tau_{\text{sc}}}\right)c_{\tau 13}\exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)}\right)\ln\left(\frac{c_{\tau 13}\exp\left(\frac{-t_{\text{m}}}{\tau_{\text{sc}}}\right)}{c_{\tau 13}\exp\left(\frac{-(t_{\text{m}}+t_{\text{c}})}{\tau_{\text{sc}}}\right)}\right) \\ \beta^*_{\text{sc,LT},4} =& c_{f11}\,\mathcal{EI}\left(c_{\tau 13}\,c_{f12}\right) + \frac{t_{\text{c}}\left(c_{f13}-c_{f31}\right)}{2\tau_{\text{sc}}}.\end{aligned} \tag{S9.38}$$

Following the same approach, term 6 (i.e. $\beta^*_{\text{sc,LT},6}$) shown in Equation S9.1 is equivalent to:

$$\begin{aligned}\beta^*_{\text{sc,LT},6} &= \int_{\tau_{23}}^{\tau_{23,\text{tc}}} \frac{1}{\tau}\left[\frac{c_{f21}\,\tau_{\text{sc}}}{t_{\text{c}}}\mathcal{EI}\left(c_{\tau 23}\,c_{f22}\right) + \left(\frac{c_{f31}}{t_{\text{c}}} - \frac{c_{f23}}{t_{\text{c}}}\right)\left(t_{\text{m}} - \tau_{\text{sc}}\ln\left(\frac{c_{\tau 23}}{\tau}\right)\right)\right]\text{d}\tau \\ \beta^*_{\text{sc,LT},6} &= -\,c_{f21}\,\mathcal{EI}\left(c_{\tau 23}\,c_{f22}\right) + \frac{t_{\text{c}}\left(c_{f31}-c_{f23}\right)}{2\tau_{\text{sc}}}.\end{aligned} \tag{S9.39}$$

### S9.2.5 Analytical Solution for $\beta^*_{\mathrm{sc,LT,5}}$ of Equation S9.1

Based on term 5 (i.e. $\beta^*_{\mathrm{sc,LT,5}}$) shown in Equation S9.1:

$$\beta^*_{\mathrm{sc,LT,5}} = \int_{\tau_{13,\mathrm{tc}}}^{\tau_{23,\mathrm{tc}}} \frac{c_{f31}}{\tau} \mathrm{d}\tau. \tag{S9.40}$$

Substituting the integral solution of $\beta^*_{\mathrm{sc,LT,F1}}$ (i.e. Equation S9.6) into Equation S9.40 as follows:

$$\beta^*_{\mathrm{sc,LT,5}} = c_{f31} \ln\left(\frac{\tau_{23,\mathrm{tc}}}{\tau_{13,\mathrm{tc}}}\right). \tag{S9.41}$$

Substituting the definition of $\tau_{x,\mathrm{tc}}$ (Equation S5.9) into Equation S9.41 to convert from the particle relaxation time ($\tau$) to the measurement time ($t_\mathrm{m}$) domain:

$$\begin{aligned}\beta^*_{\mathrm{sc,LT,5}} =& c_{f31} \ln\left(\frac{c_{\tau 23} \exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_\mathrm{sc}}\right)}{c_{\tau 13} \exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_\mathrm{sc}}\right)}\right)\\ \beta^*_{\mathrm{sc,LT,5}} =& c_{f31} \ln\left(\frac{c_{\tau 23}}{c_{\tau 13}}\right).\end{aligned} \tag{S9.42}$$

# S10 Deconvolution Parameter ($\beta^*_{\mathrm{sc,PS,B}}$) of Scanning AAC: Particle Streamline, Non-Idealized, Balanced Flows and Uniform Axial Flow

Substituting the definition of $\bar{\Omega}_{\mathrm{AAC}}$ based on particle streamline theory (Equation S6.12) into the definition of the $\beta^*_{\mathrm{sc}}$ (Equation S8.7) and organizing into separate integrals by the ranges of particle relaxation times becomes:

$$
\begin{aligned}
\beta^*_{\mathrm{sc,PS,B}} &= \int \frac{\bar{\Omega}_{\mathrm{AAC,B}}(\tau,\,t)}{\tau}\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,PS,B}} &= \underbrace{\int_{\tau_{\mathrm{min,B}}}^{\tau_{\mathrm{min,B,tc}}} \frac{1}{\tau}\left[-\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right) + \frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right]\mathrm{d}\tau}_{\beta^*_{\mathrm{sc,PS,B1}}} \\
&+ \underbrace{\int_{\tau_{\mathrm{min,B}}}^{\tau^*_{\mathrm{sc,B}}} \frac{1}{\tau}\left[-\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\mathrm{d}\tau}_{\beta^*_{\mathrm{sc,PS,B2}}} \\
&+ \underbrace{\int_{\tau_{\mathrm{min,B,tc}}}^{\tau^*_{\mathrm{sc,B,tc}}} \frac{1}{\tau}\left[\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}} + \frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right]\mathrm{d}\tau}_{\beta^*_{\mathrm{sc,PS,B3}}} \\
&+ \underbrace{\int_{\tau^*_{\mathrm{sc,B}}}^{\tau^*_{\mathrm{sc,B,tc}}} \frac{1}{\tau}\left[\frac{\lambda_\Omega\,\mu_\Omega}{t_{\mathrm{c}}}\left(c_{\mathrm{BL}} - c_{\mathrm{BU}}\right)\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right) - 2c_{\mathrm{B2}}\right]\mathrm{d}\tau}_{\beta^*_{\mathrm{sc,PS,B4}}} \\
&+ \underbrace{\int_{\tau^*_{\mathrm{sc,B,tc}}}^{\tau_{\mathrm{max,B,tc}}} \frac{1}{\tau}\left[\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BU}} - \frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_{\mathrm{m}}+t_{\mathrm{c}}}{\tau_{\mathrm{sc}}}\right)\right]\mathrm{d}\tau}_{\beta^*_{\mathrm{sc,PS,B5}}} \\
&+ \underbrace{\int_{\tau^*_{\mathrm{sc,B}}}^{\tau_{\mathrm{max,B}}} \frac{1}{\tau}\left[\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau\exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right]\mathrm{d}\tau}_{\beta^*_{\mathrm{sc,PS,B6}}} \\
&+ \underbrace{\int_{\tau_{\mathrm{max,B}}}^{\tau_{\mathrm{max,B,tc}}} \frac{1}{\tau}\left[\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BU}}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}} - \tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right) + \frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{max,B}}}{c_{\tau*,\mathrm{B}}}\right]\mathrm{d}\tau}_{\beta^*_{\mathrm{sc,PS,B7}}}\,.
\end{aligned}
\tag{S10.1}
$$

All of the terms in Equation S10.1 consist of different combinations of the following three integral forms:

$$\beta^*_{\mathrm{sc,PS,F1}} = \int_{\tau_{\mathrm{L}}}^{\tau_{\mathrm{U}}} \frac{a}{\tau}\mathrm{d}\tau, \tag{S10.2}$$

$$\beta^*_{\mathrm{sc,PS,F2}} = \int_{\tau_{\mathrm{L}}}^{\tau_{\mathrm{U}}} \frac{a\ln\left(\frac{b}{\tau}\right)}{\tau}\mathrm{d}\tau, \tag{S10.3}$$

$$\beta^*_{\mathrm{sc,PS,F3}} = \int_{\tau_{\mathrm{L}}}^{\tau_{\mathrm{U}}} a\mathrm{d}\tau, \tag{S10.4}$$

where $a$ and $b$ are constants. These integral forms $\beta^*_{\mathrm{sc,PS,F1}}$, $\beta^*_{\mathrm{sc,PS,F2}}$ and $\beta^*_{\mathrm{sc,PS,F3}}$, are solved in Section S10.1 (specifically Sections S10.1.1, S10.1.2 and S10.1.3, respectively). Based on the solutions for these integral forms, the terms of Equation S10.1 (i.e. $\beta^*_{\mathrm{sc,PS,B1}}$ to $\beta^*_{\mathrm{sc,PS,B7}}$) are solved in Section S10.2 (specifically Sections S10.2.1 to S10.2.4). Therefore substituting the solution for terms $\beta^*_{\mathrm{sc,PS,B1}}$ to $\beta^*_{\mathrm{sc,PS,B7}}$ (Equations S10.13, S10.17, S10.21, S10.25, S10.22, S10.18, and S10.14, respectively) into Equation S10.1 as follows:

$$\begin{aligned}
\beta^*_{\mathrm{sc,PS,B}} &= \int \frac{\bar{\Omega}_{\mathrm{AAC,B}}(\tau,\, t)}{\tau}\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,PS,B}} &= -\frac{c_{\mathrm{B2}}\, c_{\tau\mathrm{min,B}}\, t_{\mathrm{c}}}{c_{\tau*,\mathrm{B}}\, \tau_{\mathrm{sc}}} - \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}}\, t_{\mathrm{c}}}{2\tau_{\mathrm{sc}}} \\
&\quad - c_{\mathrm{B2}}\left(1 - \frac{c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right) \\
&\quad + \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}} \ln\left(\frac{c_{\tau*,\mathrm{B}}}{c_{\tau\mathrm{min,B}}}\right) + c_{\mathrm{B2}}\left(1 - \frac{c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right) \\
&\quad + \frac{2c_{\mathrm{B2}}\, t_{\mathrm{c}}}{\tau_{\mathrm{sc}}} + \frac{\lambda_\Omega\, \mu_\Omega\, t_{\mathrm{c}}}{2\tau_{\mathrm{sc}}}\left(c_{\mathrm{BL}} - c_{\mathrm{BU}}\right) \\
&\quad + \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BU}} \ln\left(\frac{c_{\tau\mathrm{max,B}}}{c_{\tau*,\mathrm{B}}}\right) + c_{\mathrm{B2}}\left(1 - \frac{c_{\tau\mathrm{max,B}}}{c_{\tau*,\mathrm{B}}}\right) \\
&\quad + c_{\mathrm{B2}}\left(\frac{c_{\tau\mathrm{max,B}}}{c_{\tau*,\mathrm{B}}} - 1\right) \\
&\quad - \frac{c_{\mathrm{B2}}\, c_{\tau\mathrm{max,B}}\, t_{\mathrm{c}}}{c_{\tau*,\mathrm{B}}\, \tau_{\mathrm{sc}}} + \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BU}}\, t_{\mathrm{c}}}{2\tau_{\mathrm{sc}}}.
\end{aligned} \tag{S10.5}$$

Substituting the definitions of $c_{\tau\mathrm{min,B}}$ (Equation S4.14) and $c_{\tau\mathrm{max,B}}$ (Equation S4.16) into Equation S10.5 and simplifying:

$$\begin{aligned}
\beta^*_{\mathrm{sc,PS,B}} &= -\frac{c_{\mathrm{B2}}\left(1 - \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}\, t_{\mathrm{c}}}{c_{\tau*,\mathrm{B}}\, \tau_{\mathrm{sc}}} - \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}}\, t_{\mathrm{c}}}{2\tau_{\mathrm{sc}}} \\
&\quad - c_{\mathrm{B2}}\left(1 - \frac{\left(1 - \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}}{c_{\tau*,\mathrm{B}}}\right) \\
&\quad + \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}} \ln\left(\frac{c_{\tau*,\mathrm{B}}}{\left(1 - \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}}\right) + c_{\mathrm{B2}}\left(1 - \frac{\left(1 - \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}}{c_{\tau*,\mathrm{B}}}\right) \\
&\quad + \frac{2c_{\mathrm{B2}}\, t_{\mathrm{c}}}{\tau_{\mathrm{sc}}} + \frac{\lambda_\Omega\, \mu_\Omega\, t_{\mathrm{c}}}{2\tau_{\mathrm{sc}}}\left(c_{\mathrm{BL}} - c_{\mathrm{BU}}\right) \\
&\quad + \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BU}} \ln\left(\frac{\left(1 + \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}}{c_{\tau*,\mathrm{B}}}\right) + c_{\mathrm{B2}}\left(1 - \frac{\left(1 + \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}}{c_{\tau*,\mathrm{B}}}\right) \\
&\quad + c_{\mathrm{B2}}\left(\frac{\left(1 + \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}}{c_{\tau*,\mathrm{B}}} - 1\right) \\
&\quad - \frac{c_{\mathrm{B2}}\left(1 + \frac{\beta}{\mu_\Omega}\right) c_{\tau*,\mathrm{B}}\, t_{\mathrm{c}}}{c_{\tau*,\mathrm{B}}\, \tau_{\mathrm{sc}}} + \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BU}}\, t_{\mathrm{c}}}{2\tau_{\mathrm{sc}}} \\
\beta^*_{\mathrm{sc,PS,B}} &= \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BU}} \ln\left(1 + \frac{\beta}{\mu_\Omega}\right) - \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}} \ln\left(1 - \frac{\beta}{\mu_\Omega}\right).
\end{aligned} \tag{S10.6}$$

Furthermore, substituting the definitions of $c_{\mathrm{BL}}$ (Equation S6.4) and $c_{\mathrm{BU}}$ (Equation S6.5) into Equation S10.6 and simplifying:

$$\begin{aligned} \beta^{*}_{\mathrm{sc,PS,B}} =& \lambda_{\Omega}\,\mu_{\Omega}\left(1+\frac{\mu_{\Omega}}{\beta}\right)\ln\left(1+\frac{\beta}{\mu_{\Omega}}\right)-\lambda_{\Omega}\,\mu_{\Omega}\left(1-\frac{\mu_{\Omega}}{\beta}\right)\ln\left(1-\frac{\beta}{\mu_{\Omega}}\right) \\ \beta^{*}_{\mathrm{sc,PS,B}} =& \lambda_{\Omega}\,\mu_{\Omega}\left[\ln\left(\frac{1+\frac{\beta}{\mu_{\Omega}}}{1-\frac{\beta}{\mu_{\Omega}}}\right)+\frac{\mu_{\Omega}}{\beta}\ln\left(1-\left(\frac{\beta}{\mu_{\Omega}}\right)^{2}\right)\right]. \end{aligned} \tag{S10.7}$$

This solution (i.e. Equation S10.7) assumes $\lambda_{\Omega}$ and $\mu_{\Omega}$ are constant over the width of the scanning AAC transfer function (i.e. the AAC is operated with sufficiently high resolution, and the particle detector is operated with sufficiently low counting time).

## S10.1 Solving Integral Forms ($\beta^*_{\mathrm{sc,PS,F}x}$) of Deconvolution Parameter Integral

### S10.1.1 Analytical Solution for $\beta^*_{\mathrm{sc,PS,F1}}$ (Equation S10.2)

$\beta^*_{\mathrm{sc,PS,F1}}$ (Equation S10.2) is the same integral form as $\beta^*_{\mathrm{sc,LT,F1}}$ (Equation S9.2), therefore the solution (Equation S9.6) derived in Section S9.1.1 applies as follows:

$$\beta^*_{\mathrm{sc,PS,F1}} = \int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \frac{a}{\tau} \mathrm{d}\tau = \boldsymbol{a} \ln \left( \frac{\boldsymbol{\tau}_\mathbf{U}}{\boldsymbol{\tau}_\mathbf{L}} \right). \tag{S10.8}$$

### S10.1.2 Analytical Solution for $\beta^*_{\mathrm{sc,PS,F2}}$ (Equation S10.3)

$\beta^*_{\mathrm{sc,PS,F2}}$ (Equation S10.3) is the same integral form as $\beta^*_{\mathrm{sc,LT,F2}}$ (Equation S9.3), therefore the solution (Equation S9.9) derived in Section S9.1.2 applies as follows:

$$\beta^*_{\mathrm{sc,PS,F1}} = \int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} \frac{a \ln\left(\frac{b}{\tau}\right)}{\tau} \mathrm{d}\tau = -\frac{\boldsymbol{a}}{\mathbf{2}} \ln \left( \frac{\boldsymbol{b}^2}{\boldsymbol{\tau}_\mathbf{U}\, \boldsymbol{\tau}_\mathbf{L}} \right) \ln \left( \frac{\boldsymbol{\tau}_\mathbf{L}}{\boldsymbol{\tau}_\mathbf{U}} \right). \tag{S10.9}$$

### S10.1.3 Analytical Solution for $\beta^*_{\mathrm{sc,PS,F3}}$ (Equation S10.4)

Based on Equation S10.4:

$$\beta^*_{\mathrm{sc,PS,F3}} = \int_{\tau_\mathrm{L}}^{\tau_\mathrm{U}} a \, \mathrm{d}\tau = \boldsymbol{a} \left( \boldsymbol{\tau}_\mathbf{U} - \boldsymbol{\tau}_\mathbf{L} \right). \tag{S10.10}$$

## S10.2 Solving Terms ($\beta^*_{\mathrm{sc,PS,B}x}$) of Deconvolution Parameter

### S10.2.1 Analytical Solutions for $\beta^*_{\mathrm{sc,PS,B1}}$ and $\beta^*_{\mathrm{sc,PS,B7}}$ of Equation S10.1

Based on term 1 (i.e. $\beta^*_{\mathrm{sc,PS,B1}}$) shown in Equation S10.1:

$$
\begin{aligned}
\beta^*_{\mathrm{sc,PS,B1}} &= \int_{\tau_{\mathrm{min,B}}}^{\tau_{\mathrm{min,B,tc}}} \frac{1}{\tau}\left[-\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)\right)+\frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right]\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,PS,B1}} &= \int_{\tau_{\mathrm{min,B}}}^{\tau_{\mathrm{min,B,tc}}}\left[-\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}\,t_{\mathrm{m}}}{t_{\mathrm{c}}}+\frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right]\frac{1}{\tau}\mathrm{d}\tau+\int_{\tau_{\mathrm{min,B}}}^{\tau_{\mathrm{min,B,tc}}}\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}\frac{\ln\left(\frac{c_{\tau\mathrm{min,B}}}{\tau}\right)}{\tau}\mathrm{d}\tau.
\end{aligned}
\tag{S10.11}
$$

Substituting the integral solutions of $\beta^*_{\mathrm{sc,PS,F1}}$ and $\beta^*_{\mathrm{sc,PS,F2}}$ (i.e. Equations S10.8 and S10.9) into Equation S10.11 as follows:

$$
\begin{aligned}
\beta^*_{\mathrm{sc,PS,B1}} = &\left[-\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}\,t_{\mathrm{m}}}{t_{\mathrm{c}}}+\frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right]\ln\left(\frac{\tau_{\mathrm{min,B,tc}}}{\tau_{\mathrm{min,B}}}\right) \\
&-\frac{\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}\,\tau_{\mathrm{sc}}}{t_{\mathrm{c}}}}{2}\ln\left(\frac{c^2_{\tau\mathrm{min,B}}}{\tau_{\mathrm{min,B,tc}}\,\tau_{\mathrm{min,B}}}\right)\ln\left(\frac{\tau_{\mathrm{min,B}}}{\tau_{\mathrm{min,B,tc}}}\right).
\end{aligned}
\tag{S10.12}
$$

Substituting the definitions of $\tau_x$ and $\tau_{x,\mathrm{tc}}$ (Equations S3.27 and S5.9, respectively) into Equation S10.12 to convert from the particle relaxation time ($\tau$) to the measurement time ($t_{\mathrm{m}}$) domain:

$$
\begin{aligned}
\beta^*_{\mathrm{sc,PS,B1}} = &\left[-\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}\,t_{\mathrm{m}}}{t_{\mathrm{c}}}+\frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right]\ln\left(\frac{c_{\tau\mathrm{min,B}}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}{c_{\tau\mathrm{min,B}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}\right) \\
&-\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}\,\tau_{\mathrm{sc}}}{2t_{\mathrm{c}}}\ln\left(\frac{c^2_{\tau\mathrm{min,B}}}{c_{\tau\mathrm{min,B}}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)c_{\tau\mathrm{min,B}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}\right)\ln\left(\frac{c_{\tau\mathrm{min,B}}\exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)}{c_{\tau\mathrm{min,B}}\exp\left(\frac{-(t_{\mathrm{m}}+t_{\mathrm{c}})}{\tau_{\mathrm{sc}}}\right)}\right) \\
\beta^*_{\mathrm{sc,PS,B1}} = &-\frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{min,B}}\,t_{\mathrm{c}}}{c_{\tau*,\mathrm{B}}\,\tau_{\mathrm{sc}}}-\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BL}}\,t_{\mathrm{c}}}{2\tau_{\mathrm{sc}}}.
\end{aligned}
\tag{S10.13}
$$

Following the same approach, term 7 (i.e. $\beta^*_{\mathrm{sc,LT,7}}$) shown in Equation S10.1 is equivalent to:

$$
\begin{aligned}
\beta^*_{\mathrm{sc,PS,B7}} &= \int_{\tau_{\mathrm{max,B}}}^{\tau_{\mathrm{max,B,tc}}}\frac{1}{\tau}\left[\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BU}}}{t_{\mathrm{c}}}\left(t_{\mathrm{m}}-\tau_{\mathrm{sc}}\ln\left(\frac{c_{\tau\mathrm{max,B}}}{\tau}\right)\right)+\frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{max,B}}}{c_{\tau*,\mathrm{B}}}\right]\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,PS,B7}} &= -\frac{c_{\mathrm{B2}}\,c_{\tau\mathrm{max,B}}\,t_{\mathrm{c}}}{c_{\tau*,\mathrm{B}}\,\tau_{\mathrm{sc}}}+\frac{\lambda_\Omega\,\mu_\Omega\,c_{\mathrm{BU}}\,t_{\mathrm{c}}}{2\tau_{\mathrm{sc}}}.
\end{aligned}
\tag{S10.14}
$$

#### S10.2.2 Analytical Solutions for $\beta^*_{\mathrm{sc,PS,B2}}$ and $\beta^*_{\mathrm{sc,PS,B6}}$ of Equation S10.1

Based on term 2 (i.e. $\beta^*_{\mathrm{sc,PS,B2}}$) shown in Equation S10.1:

$$\begin{aligned}\beta^*_{\mathrm{sc,PS,B2}} &= \int_{\tau_{\mathrm{min,B}}}^{\tau^*_{\mathrm{sc,B}}} \frac{1}{\tau}\left[-\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] \mathrm{d}\tau \\ \beta^*_{\mathrm{sc,PS,B2}} &= \int_{\tau_{\mathrm{min,B}}}^{\tau^*_{\mathrm{sc,B}}} -\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}} \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right) \mathrm{d}\tau.\end{aligned} \tag{S10.15}$$

Substituting the integral solution of $\beta^*_{\mathrm{sc,PS,F3}}$ (i.e. Equation S10.10) into Equation S10.15 as follows:

$$\beta^*_{\mathrm{sc,PS,B2}} = -\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}} \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\left(\tau^*_{\mathrm{sc,B}} - \tau_{\mathrm{min,B}}\right). \tag{S10.16}$$

Substituting the definition of $\tau_x$ (Equation S3.27) into Equation S10.16 to convert from the particle relaxation time ($\tau$) to the measurement time ($t_{\mathrm{m}}$) domain:

$$\begin{aligned}\beta^*_{\mathrm{sc,PS,B2}} &= -\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}} \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\left(c_{\tau*,\mathrm{B}} \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right) - c_{\tau\mathrm{min,B}} \exp\left(\frac{-t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right) \\ \beta^*_{\mathrm{sc,PS,B2}} &= -c_{\mathrm{B2}}\left(1 - \frac{c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right).\end{aligned} \tag{S10.17}$$

Following the same approach, term 6 (i.e. $\beta^*_{\mathrm{sc,LT,6}}$) shown in Equation S10.1 is equivalent to:

$$\begin{aligned}\beta^*_{\mathrm{sc,PS,B6}} &= \int_{\tau^*_{\mathrm{sc,B}}}^{\tau_{\mathrm{max,B}}} \frac{1}{\tau}\left[\frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau \exp\left(\frac{t_{\mathrm{m}}}{\tau_{\mathrm{sc}}}\right)\right] \mathrm{d}\tau \\ \beta^*_{\mathrm{sc,PS,B6}} &= c_{\mathrm{B2}}\left(\frac{c_{\tau\mathrm{max,B}}}{c_{\tau*,\mathrm{B}}} - 1\right).\end{aligned} \tag{S10.18}$$

### S10.2.3 Analytical Solutions for $\beta^*_{\mathrm{sc,PS,B3}}$ and $\beta^*_{\mathrm{sc,PS,B5}}$ of Equation S10.1

Based on term 3 (i.e. $\beta^*_{\mathrm{sc,PS,B3}}$) shown in Equation S10.1:

$$\begin{aligned}
\beta^*_{\mathrm{sc,PS,B3}} &= \int_{\tau_{\mathrm{min,B,tc}}}^{\tau^*_{\mathrm{sc,B,tc}}} \frac{1}{\tau}\left[\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}} + \frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau \exp\left(\frac{t_\mathrm{m}+t_\mathrm{c}}{\tau_{\mathrm{sc}}}\right)\right]\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,PS,B3}} &= \int_{\tau_{\mathrm{min,B,tc}}}^{\tau^*_{\mathrm{sc,B,tc}}} \frac{\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}}}{\tau}\mathrm{d}\tau + \int_{\tau_{\mathrm{min,B,tc}}}^{\tau^*_{\mathrm{sc,B,tc}}} \frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}} \exp\left(\frac{t_\mathrm{m}+t_\mathrm{c}}{\tau_{\mathrm{sc}}}\right)\mathrm{d}\tau.
\end{aligned} \tag{S10.19}$$

Substituting the integral solutions of $\beta^*_{\mathrm{sc,PS,F1}}$ and $\beta^*_{\mathrm{sc,PS,F3}}$ (i.e. Equations S10.8 and S10.10) into Equation S10.19 as follows:

$$\beta^*_{\mathrm{sc,PS,B3}} = \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}} \ln\left(\frac{\tau^*_{\mathrm{sc,B,tc}}}{\tau_{\mathrm{min,B,tc}}}\right) + \frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}} \exp\left(\frac{t_\mathrm{m}+t_\mathrm{c}}{\tau_{\mathrm{sc}}}\right)\left(\tau^*_{\mathrm{sc,B,tc}} - \tau_{\mathrm{min,B,tc}}\right). \tag{S10.20}$$

Substituting the definition of $\tau_{x,\mathrm{tc}}$ (Equation S5.9) into Equation S10.20 to convert from the particle relaxation time ($\tau$) to the measurement time ($t_\mathrm{m}$) domain:

$$\begin{aligned}
\beta^*_{\mathrm{sc,PS,B3}} &= \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}} \ln\left(\frac{c_{\tau*,\mathrm{B}} \exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_{\mathrm{sc}}}\right)}{c_{\tau\mathrm{min,B}} \exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_{\mathrm{sc}}}\right)}\right) \\
&+ \frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}} \exp\left(\frac{t_\mathrm{m}+t_\mathrm{c}}{\tau_{\mathrm{sc}}}\right)\left(c_{\tau*,\mathrm{B}} \exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_{\mathrm{sc}}}\right) - c_{\tau\mathrm{min,B}} \exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_{\mathrm{sc}}}\right)\right) \\
\beta^*_{\mathrm{sc,PS,B3}} &= \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BL}} \ln\left(\frac{c_{\tau*,\mathrm{B}}}{c_{\tau\mathrm{min,B}}}\right) + c_{\mathrm{B2}}\left(1 - \frac{c_{\tau\mathrm{min,B}}}{c_{\tau*,\mathrm{B}}}\right).
\end{aligned} \tag{S10.21}$$

Following the same approach, term 5 (i.e. $\beta^*_{\mathrm{sc,LT,5}}$) shown in Equation S10.1 is equivalent to:

$$\begin{aligned}
\beta^*_{\mathrm{sc,PS,B5}} &= \int_{\tau^*_{\mathrm{sc,B,tc}}}^{\tau_{\mathrm{max,B,tc}}} \frac{1}{\tau}\left[\lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BU}} - \frac{c_{\mathrm{B2}}}{c_{\tau*,\mathrm{B}}}\tau \exp\left(\frac{t_\mathrm{m}+t_\mathrm{c}}{\tau_{\mathrm{sc}}}\right)\right]\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,PS,B5}} &= \lambda_\Omega\, \mu_\Omega\, c_{\mathrm{BU}} \ln\left(\frac{c_{\tau\mathrm{max,B}}}{c_{\tau*,\mathrm{B}}}\right) + c_{\mathrm{B2}}\left(1 - \frac{c_{\tau\mathrm{max,B}}}{c_{\tau*,\mathrm{B}}}\right).
\end{aligned} \tag{S10.22}$$

### S10.2.4 Analytical Solution for $\beta^*_{\mathrm{sc,PS,B4}}$ of Equation S10.1

Based on term 4 (i.e. $\beta^*_{\mathrm{sc,PS,B4}}$) shown in Equation S10.1:

$$\begin{aligned}
\beta^*_{\mathrm{sc,PS,B4}} &= \int_{\tau^*_{\mathrm{sc,B}}}^{\tau^*_{\mathrm{sc,B,tc}}} \frac{1}{\tau}\left[\frac{\lambda_\Omega\,\mu_\Omega}{t_\mathrm{c}}\left(c_\mathrm{BL}-c_\mathrm{BU}\right)\left(t_\mathrm{m}-\tau_\mathrm{sc}\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)\right)-2c_\mathrm{B2}\right]\mathrm{d}\tau \\
\beta^*_{\mathrm{sc,PS,B4}} &= \int_{\tau^*_{\mathrm{sc,B}}}^{\tau^*_{\mathrm{sc,B,tc}}} \left[\frac{\lambda_\Omega\,\mu_\Omega\,t_\mathrm{m}}{t_\mathrm{c}}\left(c_\mathrm{BL}-c_\mathrm{BU}\right)-2c_\mathrm{B2}\right]\frac{1}{\tau}\mathrm{d}\tau \\
&+\int_{\tau^*_{\mathrm{sc,B}}}^{\tau^*_{\mathrm{sc,B,tc}}} -\frac{\lambda_\Omega\,\mu_\Omega\,\tau_\mathrm{sc}}{t_\mathrm{c}}\left(c_\mathrm{BL}-c_\mathrm{BU}\right)\frac{\ln\left(\frac{c_{\tau*,\mathrm{B}}}{\tau}\right)}{\tau}\mathrm{d}\tau.
\end{aligned} \tag{S10.23}$$

Substituting the integral solutions of $\beta^*_{\mathrm{sc,PS,F1}}$ and $\beta^*_{\mathrm{sc,PS,F2}}$ (i.e. Equations S10.8 and S10.9) into Equation S10.23 as follows:

$$\begin{aligned}
\beta^*_{\mathrm{sc,PS,B4}} &= \left[\frac{\lambda_\Omega\,\mu_\Omega\,t_\mathrm{m}}{t_\mathrm{c}}\left(c_\mathrm{BL}-c_\mathrm{BU}\right)-2c_\mathrm{B2}\right]\ln\left(\frac{\tau^*_{\mathrm{sc,B,tc}}}{\tau^*_{\mathrm{sc,B}}}\right) \\
&+\frac{\frac{\lambda_\Omega\,\mu_\Omega\,\tau_\mathrm{sc}}{t_\mathrm{c}}\left(c_\mathrm{BL}-c_\mathrm{BU}\right)}{2}\ln\left(\frac{c^2_{\tau*,\mathrm{B}}}{\tau^*_{\mathrm{sc,B,tc}}\,\tau^*_{\mathrm{sc,B}}}\right)\ln\left(\frac{\tau^*_{\mathrm{sc,B}}}{\tau^*_{\mathrm{sc,B,tc}}}\right).
\end{aligned} \tag{S10.24}$$

Substituting the definitions of $\tau_x$ and $\tau_{x,\mathrm{tc}}$ (Equations S3.27 and S5.9, respectively) into Equation S10.24 to convert from the particle relaxation ($\tau$) to the measurement time ($t_\mathrm{m}$) domain:

$$\begin{aligned}
\beta^*_{\mathrm{sc,PS,B4}} &= \left[\frac{\lambda_\Omega\,\mu_\Omega\,t_\mathrm{m}}{t_\mathrm{c}}\left(c_\mathrm{BL}-c_\mathrm{BU}\right)-2c_\mathrm{B2}\right]\ln\left(\frac{c_{\tau*,\mathrm{B}}\exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_\mathrm{sc}}\right)}{c_{\tau*,\mathrm{B}}\exp\left(\frac{-t_\mathrm{m}}{\tau_\mathrm{sc}}\right)}\right) \\
&+\frac{\lambda_\Omega\,\mu_\Omega\,\tau_\mathrm{sc}}{2t_\mathrm{c}}\left(c_\mathrm{BL}-c_\mathrm{BU}\right)\ln\left(\frac{c^2_{\tau*,\mathrm{B}}}{c_{\tau*,\mathrm{B}}\exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_\mathrm{sc}}\right)c_{\tau*,\mathrm{B}}\exp\left(\frac{-t_\mathrm{m}}{\tau_\mathrm{sc}}\right)}\right)\ln\left(\frac{c_{\tau*,\mathrm{B}}\exp\left(\frac{-t_\mathrm{m}}{\tau_\mathrm{sc}}\right)}{c_{\tau*,\mathrm{B}}\exp\left(\frac{-(t_\mathrm{m}+t_\mathrm{c})}{\tau_\mathrm{sc}}\right)}\right) \\
\beta^*_{\mathrm{sc,PS,B4}} &= \frac{2c_\mathrm{B2}\,t_\mathrm{c}}{\tau_\mathrm{sc}}+\frac{\lambda_\Omega\,\mu_\Omega\,t_\mathrm{c}}{2\tau_\mathrm{sc}}\left(c_\mathrm{BL}-c_\mathrm{BU}\right).
\end{aligned} \tag{S10.25}$$

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