

1 **Date:** 22nd November 2013

2 **Title:** Design methods for flexibly formed concrete beams

3 Dr J.J. Orr, MEng (hons), PhD

4 Lecturer in Civil Engineering, Department of Architecture and Civil Engineering, University of Bath, Bath,
5 BA2 7AY

6
7 j.j.orr@bath.ac.uk; 01225 383 412

8
9 Dr Antony Darby, BSc(hons), PhD, CEng, MStructE

10 Reader in Structural Engineering, Department of Architecture and Civil Engineering, University of Bath

11
12 Professor Tim Ibell, FEng, BSc(Eng), PhD, CEng, FStructE, FICE, FHEA

13 Professor of Civil Engineering, Department of Architecture and Civil Engineering, University of Bath

14
15 Dr Mark Evernden, MEng, PhD, FHEA

16 Lecturer in Civil Engineering, Department of Architecture and Civil Engineering, University of Bath

17
18 Number of Words: 4745

19 Number of Figures: 19

20

21 **Abstract**

22 This paper presents an iterative method for the design of structurally optimised,
23 simply supported, flexibly formed, concrete beams. The design and construction
24 method using flexible formwork has been shown to facilitate material savings of up to
25 40%, providing significant opportunities for low-carbon concrete design.

26 The proposed method considers both ultimate and serviceability limit states for the
27 optimised beam. The iterative nature of the design process, and its interdependency
28 with the construction method, is demonstrated. Further considerations such as
29 reinforcement detailing, methods to support the flexible mould, and alternative support
30 conditions, are also discussed.

31 It is seen that replacing conventional orthogonal moulds with a flexible system
32 comprised primarily of high strength, low cost fabric sheets, utilises concrete to create
33 extraordinary possibilities for highly optimised, low carbon, architecturally interesting,
34 building forms.

35 **Keywords:** Concrete structures, Research and development, Sustainability, Design
36 methods and aids.

37

Notation	
f_{cd}	Design value of concrete compressive strength (N/mm ²)
f_y	Yield strength of steel reinforcement (N/mm ²)
$y_{b,i}$	Distance to reinforcing bar i (mm)
z	The lever arm between tension and compression forces (mm)
A_s	Area of flexural reinforcement (mm ²)
D	Beam depth (mm)
E_d	Design effect of an action
$F_{b,i}$	Horizontal component of force in reinforcing bar i (N)
F_{cd}	Design value of the concrete compression force (N)
$G_{kj,sup}$	Upper characteristic value of permanent action j
M_{Ed}	Design bending moment (Nmm)
V	Shear force (N)
Q_k	Characteristic value of a single variable action
R_d	Design resistance
α_i	The inclination of reinforcing bar i (Figure 5)
$\gamma_{G,j}$	Partial factor for permanent action j
γ_Q	Partial factor for variable actions
η	Factor defining the effective strength of concrete
λ	Factor defining the effective height of the compression zone
σ_c	Compressive stress (N/mm ²)

41 **1 Introduction**

42 There is an increasing global focus on sustainability in the design process, backed
43 up by stringent carbon dioxide emissions reduction targets. The relatively low
44 embodied energy of concrete by volume is offset by current rates of consumption
45 which make the manufacture of cement accountable for some 5% of global CO₂
46 emissions (WRI, 2005).

47 Concrete is a fluid that offers the opportunity to economically create structures of
48 almost any geometry. Yet this unique fluidity is seldom capitalised on, with concrete
49 instead being cast into rigid prismatic moulds to create high material use structures
50 with large carbon footprints. The ubiquitous use of orthogonal moulds as concrete
51 formwork has resulted in a well-established vocabulary of prismatic forms for concrete
52 structures, yet such rigid formwork systems must resist considerable fluid pressures,
53 may consume significant amounts of material and can be expensive to construct.
54 Moreover, the resulting structural member requires more material and has a greater
55 self-weight than one cast with a variable cross section.

56 Simple optimisation routines (Orr *et al.*, 2011, Kostova *et al.*, 2012, Garbett *et al.*,
57 2010) may be used to design a variable cross section member in which the flexural and
58 shear capacity at any point along the element length reflects the requirements of the
59 loading envelope applied to it. However, the construction of structures with complex
60 non-orthogonal geometries is often perceived to be both difficult and expensive.

61 By replacing conventional moulds with a flexible system comprised primarily of low
62 cost fabric sheets, flexible formwork takes advantage of the fluidity of concrete to
63 create highly optimised, architecturally interesting, building forms. In doing so, material
64 savings of up to 40% can be achieved (Orr *et al.*, 2012a). The optimised section

65 provides ultimate limit state capacity while reducing embodied carbon, thus improving
66 the life cycle performance of the entire structure.

67 Control of the flexibly formed beam cross section is key to achieving low-material
68 use design. The basic assumption is that a sheet of flexible, permeable fabric is held in
69 a system of falsework before reinforcement and concrete are added. By varying the
70 geometry of the fabric mould with distance along the beam, the optimised shape is
71 created.

72 Flexible formwork therefore has the potential to facilitate the change in design and
73 construction philosophy that will be required for a move towards a less material
74 intensive, more sustainable, construction industry. Its potential is further demonstrated
75 in work by Lee (2010) and in Figure 1 - Figure 3.

76 **2 Design**

77 A design process for steel reinforced fabric formed concrete beams is set out below
78 and an example beam design for a single span element is shown. Flexural and shear
79 capacities are assessed, and the close link between design and construction
80 processes is described.

81 Flexural design typically relies on the assumption that plane sections before bending
82 remain plane after bending. Work by Davies *et al* (1973) suggests that such an
83 approach remains valid for non-prismatic beams. However the influence of the non-
84 prismatic section on shear behaviour is less certain, and care must be taken to prevent
85 brittle shear failures, as described in §2.2.

86 The design presented below is to intended to be undertaken with due consideration
87 of the current relevant national standard. The entire design process is summarised in
88 Figure 4.

89 **2.1 Design considerations**

90 In the design of a fabric formed beam that has a variable cross section (for example
91 Figure 3), the following items should be considered:

92 1. Material properties

- 93 a. Fabric type, manufacturer and mechanical properties;
- 94 b. Cement type, additives and design concrete strength;
- 95 c. Flexural and transverse reinforcement type (fibre reinforced polymer
96 (FRP), Steel) and mechanical properties.

97 2. Dimensional requirements

- 98 a. Beam span, support type and dimensions;
- 99 b. Cover requirements, durability class;
- 100 c. Local detailing rules and/or methods;
- 101 d. Internal reinforcement detailing to avoid conflicts with the fabric mould;
- 102 e. Aesthetic considerations and architectural intent.

103 3. Construction processes

- 104 a. Decision on in-situ versus precast construction;
- 105 b. Methods to account for quality of workmanship, and checking procedures
106 to verify the cast element geometry.

107 4. Design

- 108 a. Design code and appropriate standards to conform to;

109 b. Load conditions and loading envelope;

110 c. Checks on all actions acting on the element (bending, shear, torsion).

111 Research at the University of Bath has shown that for tapering elements loaded
112 predominantly in shear the use of inclined tension reinforcement to provide both
113 flexural and transverse capacity according to the variable angle truss model (BSI,
114 2004) can result in unconservative beam designs due to the manner in which yielding
115 of the steel reinforcement is considered (Orr *et al.*, 2012b).

116 However, the variable angle truss model has been applied successfully to the
117 design of 4m span 'T' and 'Double-T' beams under uniformly distributed loading that did
118 not rely on the inclined tension reinforcement for shear capacity (Orr *et al.*, 2012a).
119 This method, which follows BS EN 1992-1-1 (2004), is used here in the design of an
120 optimised fabric formed beam. The results are then compared to a design using the
121 compressive force path method (Kotsovos and Pavlovic, 1999), which has been shown
122 to be applicable and conservative for both uniformly distributed and point loaded beam
123 elements (Lee, 2010, Orr *et al.*, 2012b).

124 **2.2 Design process**

125 Following the general process described by the flowchart shown in Figure 4, a beam
126 whose span, material properties, loading and design code (the 'inputs' in Figure 4)
127 have been decided, may be designed in shear and flexure in the following manner:

- 128 1. Divide the length of the beam into equally spaced transverse sections along the
129 beam length (x-axis). At each section determine from the loading envelope the
130 maximum values of shear and bending that the section must resist. Make a
131 preliminary estimate of the maximum beam depth using span/depth ratios. Using

132 this depth, estimate the required area of yielding longitudinal tension and
133 compression steel ($A_s f_y$) using the stress block shown in Figure 5;

134 2. Using the stress block for flexure shown in Figure 5 calculate the lever arm (z)
135 required at each x-axis position to provide the design flexural capacity, using the
136 value for A_s estimated in (1). Note that the beam depth (D) will vary with the
137 design moment (M_{Ed}) and thus the yielding steel provides a horizontal component
138 of force ($F_{b,i}$) due to its angle of inclination, α ;

139 a. Note that in positions of low moment very small effective depths will be
140 required; these will be adjusted later in the design process to account for
141 shear and anchorage;

142 3. Using the compressive force path method (Kotsovos and Pavlovic, 1999), a strut
143 and tie model, or other suitable method, the shear capacity at each x-axis
144 position should be checked against the design envelope ($R_d \geq E_d$). The
145 contribution to shear capacity of any inclined tension reinforcement should be
146 limited or ignored (Orr *et al.*, 2012b);

147 4. The consideration of shear forces in (3) is likely to increase section depths in
148 some positions. Where the beam depth increases, the tension reinforcement
149 lever arm will also increase. The result of this will be an increase in flexural
150 capacity in the end zones. Such changes may mean that assumptions of steel
151 yielding in the end zone at their design moment made in (1) and (2) are no longer
152 valid. The effect of this on the ductility of the section can be critical, and should
153 be carefully considered. Strut and tie models can be useful for assessing this
154 aspect of the design. Alterations to the value of A_s from (1) can also be made to
155 improve constructability, to curtail bars, or to change the section depths;

- 156 5. Any transverse steel should be of sufficient length to be both fully anchored and
157 be able to develop its full tensile strength, if this full tensile strength is to be relied
158 upon for shear capacity. Minimum section depths, support widths, and detailing
159 clauses from the relevant national design code should be applied. If modifications
160 to the beam geometry are made, the shear and flexural capacity at each section
161 should be re-calculated by returning to (2);
- 162 6. Anchorage and development of longitudinal flexural reinforcement should be
163 verified such that the stress development required to satisfy the loading envelope
164 can be achieved. Anchorage may be provided using conventional design
165 methods, or for sections with limited space for conventional bends innovative
166 techniques such as splayed bars (Darby *et al.*, 2007) could be used. If
167 modifications to the beam geometry are made, the shear and flexural capacity at
168 each section should be re-calculated by returning to (2);
- 169 7. Detailing of the beam for construction constraints (for example the inclusion of
170 precasting joints or sockets, the use of end plates for lifting and/or support)
171 should be included and checked for clashes with designed reinforcement (Figure
172 6). If modifications to the beam geometry are made, the shear and flexural
173 capacity at each section should be re-calculated by returning to (2);
- 174 8. Load deflection predictions for serviceability may be made of the optimised beam
175 using techniques described in the literature (Orr *et al.*, 2011). If modifications to
176 the beam geometry are made, the shear and flexural capacity at each section
177 should be re-calculated by returning to (2);
- 178 9. The final beam design (in elevation) is thus determined and may be plotted.
179 Cover to the outer layer of reinforcement should be provided perpendicular to the
180 line of the steel reinforcement. The beam shape in three dimensions may be

181 plotted using the section prediction techniques described by Orr *et al.* (2011),
182 Veenendaal (2008), or by using other computational methods. A three-
183 dimensional view of the beam can be created to ensure reinforcement clashes do
184 not occur. Alternatively visualisations of the beam using a plaster model at a
185 suitable scale can be undertaken using a scaled fabric and timber mould to
186 produce accurate representations of the beam geometry. Return to step (2) if
187 geometrical changes are desired.

188 It is evident that most of the steps described above are well suited to automation by
189 computational analysis and work in this area is under development at the University of
190 Bath with the aim of producing software for integration into commercial software
191 packages.

192 **2.3 Example beam design**

193 The design process described above is applied to a 7m span simply supported beam.
194 The method has been applied to single span elements but future work may consider
195 the design of continuous beams, which introduces new challenges, including moment
196 redistribution and areas of combined high shear and high moment at support zones.

197 A simplified loading envelope, along with shear and bending moment diagrams for
198 the sample beam design, are shown in Figure 7. Partial factors for the loads are taken
199 as $\gamma_G = 1.35$ and $\gamma_Q = 1.5$ (NA to BS EN 1990 (2005)). The moment and shear
200 envelopes for design are determined along the length of the beam by taking the
201 maximum and minimum values of shear and moment at each point.

202 The iterative design process will result in a changing beam profile, from one
203 optimised only for flexure to one including full shear design and requirements of
204 transverse and longitudinal reinforcement detailing. The example beam has initially

205 been given constant top and bottom reinforcement (as shown in Figure 7(right))
206 although this could also be varied in the design process as described above.

207 Shown in Figure 8 is a summary of changes to the beam geometry that arise as the
208 design process progresses. Figure 8 illustrates how the cross-section variations react
209 to the changing tension and compression paths along the length of the element. The
210 changing cross sectional geometry is achieved by holding the flexible formwork in such
211 a way that the cast structure takes up the desired geometry, as described in §2.4.

212 The resulting design can be not only aesthetically pleasing, but illustrates the
213 optimisation processes that have been undertaken.

214 The process described above could be extended to a continuous beam as shown in
215 Figure 9 (including hogging, sagging and support zones). The bending moment
216 envelope for the indeterminate beam should be iteratively adjusted through the design
217 process to account for changes in bending stiffness. Additional challenges of moment
218 redistribution and support zone shear behaviour have been highlighted previously as
219 areas requiring further work.

220 A beam that has been designed and constructed using the procedure described in
221 §2.2 and §2.3 is shown in Figure 10.

222 **2.3.1 Profile predictions**

223 It is possible to estimate the shape of the concrete filled fabric formwork using iterative
224 computational approaches, as described by Orr *et al.* (2011). Such an approach can be
225 indicative of the final element geometry. The flexible mould is initially assumed to have
226 a hydrostatic form (Figure 5) but this can be optimised further through the construction
227 process, as described in Table 1.

228 Alternatively, plaster modelling at a suitable scale can provide accurate
229 representations of the geometrical form that the full size element will have (provided
230 the same construction process is used), and this technique has been successfully
231 applied by researchers at the University of Manitoba on a number of occasions (West
232 and Araya, 2012).

233 **2.4 Construction**

234 The method of construction for the flexibly formed element must be considered
235 throughout the design process. Typically, flexible formwork is held in place during
236 curing using a suitable system of falsework, as shown in Figure 10, Figure 11 and
237 Figure 12. Tensioning the fabric is key to avoiding surface wrinkles, and this is easily
238 achieved either by hand or using mechanical devices.

239 In the beam example given above, the optimisation process defines the initial
240 geometry. This shape then informs the falsework design that will support the fabric
241 during casting. Subsequent details such as the fabric material properties and the
242 prestress applied to the fabric then control the final surface geometry. There is thus a
243 detailed and iterative link between the design intent, the falsework and the flexible
244 mould that must be managed by the designer to achieve the desired outcome.

245 These links are the subject of current research at the University of Bath that aims to
246 fully computerise the design and analysis of flexible formwork. Various mould options
247 are described by both Orr *et al* (2011) and West (2006). Some methods for controlling
248 aspects of the beam geometry are summarised in Table 1.

249

250

251

Geometrical aspect	Method	Description	Graphic
Beam elevation and cross section	Hanging mould	A fabric sheet is draped between two timber supports, and pinned in position to define the beam elevation. Reinforcement and concrete are then added.	<i>Insert - Table 1 Row 1</i>
Beam elevation and cross section	Spline mould	As above, but using a metal or FRP spline to tension the fabric and provide greater geometrical control.	<i>Insert - Table 1 Row 2</i>
Beam elevation and cross section	Keel mould	The keel mould uses a piece of timber or a metal spline, cut to the precise geometry that has been defined by the optimisation process, to hold two sheets of fabric in position during casting. The process could feasibly be used in both pre-cast and on-site fabrication.	<i>Insert - Table 1 Row 3</i>
Beam cross section	Pinch / Side formers	Externally pushing the fabric formwork together to reduce the beam cross-section width at suitable locations. This can be achieved using timber/steel formers, or by stitching the fabric together.	<i>Insert - Table 1 Row 4</i>
Surface finish	Fabric prestress	Fabric tensioned in two directions to remove wrinkles. Level of prestress can be measured using gauges. Method can be achieved using rope (shown) or by hand before stapling the fabric to the timber-forming table.	<i>Insert - Table 1 Row 5</i> Photo © Mark West 2013
Reinforcement position	Hanging reinforcement	Reinforcement cage hung from the top of a mould, does not touch surrounding fabric	<i>Insert - Table 1 Row 6</i>

Reinforcement position	Fabric spacers	Reinforcement separated from fabric using flat-bottomed plastic spacers, shown right (suitable for below ground installations). The loading should be checked against BS 7973-1 (2001).	<i>Insert - Table 1 Row 7</i> Photo © CB Shaw 2013
------------------------	----------------	---	---

253

254 **2.5 Formwork design**

255 In common with the design of all formwork and falsework for concrete structures, the
 256 quality of the workmanship will greatly influence the resulting element. A number of
 257 areas of design must be considered for flexibly formed construction, as outlined below:

258 1. **The fabric properties.** The fabric should have sufficient tensile strength to resist
 259 the concrete fluid pressures, adequate flexibility to achieve the design shape, a
 260 low creep modulus to prevent shape changes during curing, a pore size that
 261 allows water but not cement sand or aggregate to escape during casting, and a
 262 low adherence to the cured concrete surface to facilitate easy formwork removal.
 263 All these specifications can be verified by casting trial elements, or by reference
 264 to previous projects (see for example Brennan *et al* (2013) and projects
 265 referenced in Orr *et al* (2012c))

266 2. **Fluid pressures.** Beams of modest spans, which tend to have relatively shallow
 267 depths, do not generally create very high fluid pressures in the formwork.
 268 However, where tall columns, or deep beams, or any element with a significant
 269 head of fluid concrete is to be cast, care should be taken to ensure that the fabric
 270 is able to resist the resulting fluid pressures.

271 3. **Timber falsework.** This paper has shown how falsework can be used to support
 272 sheets of fabric in which concrete is cast to create optimised structural elements.

273 The falsework used to support the fabric should be verified to have sufficient
274 capacity to not only support the weight of wet concrete, but to not deform beyond
275 the required tolerances for construction of the final element during the casting
276 process.

277 **4. Modelling and trial elements.** Construction of complex elements (trusses,
278 continuous beams or non prismatic columns for example) can be aided by the
279 construction of trial elements to verify the behaviour of both the fabric and the
280 construction process. These may be undertaken at small-scale or as full size
281 prototypes.

282 **3 Detailing**

283 **3.1 Anchorage**

284 One of the initial concerns with flexibly formed elements had been the adequate
285 anchorage of longitudinal reinforcement in sections with shallow end zones. Previous
286 work focused on using welded end plates to provide anchorage (Garbett, 2008, Bailiss,
287 2006). More recent work (Kostova *et al.*, 2012, Orr *et al.*, 2012b) has demonstrated
288 how looped or splayed bars may be used as an effective anchorage method.

289 **3.1.1 Loops**

290 In the anchorage zone a looped bar can be used (Figure 13). Simple design checks
291 can be undertaken to determine the force in the loop and the required anchorage
292 length of that loop to develop its capacity at the support. More conventional methods
293 are also possible provided there is sufficient end zone space for the anchorage (Figure
294 14).

295 It is recommended that a strut and tie model be used to estimate the forces in the
296 bar. If the bar is yielding at the position from which anchorage is required, then the full

297 capacity of the bar must be generated. The looped bar must be designed such that the
 298 stress σ_c (Figure 13) does not lead to splitting failures in the end zone (see Bruggeling
 299 and Huyghe (1991) for details).

300 3.1.2 Splayed bars

301 Preliminary work has shown that the splayed bar (Figure 15) has the potential to
 302 provide full anchorage to both steel and fibre reinforced polymer (FRP) longitudinal
 303 reinforcement (Kostova *et al.*, 2012). The method is ideally suited to FRP bars where
 304 local bends or welds are not a feasible construction method. The addition of a confining
 305 splay can prevent premature concrete splitting. The advantages and disadvantages of
 306 various anchorage methods are provided in Table 2.

307 **Table 2: Summary of anchorage methods for fabric formed beam**

Method	Comments	Advantages	Disadvantages
Bent bars	Use with steel only.	Simple, well established design methods. Understood on site. Suitable for larger elements.	Can require deeper end zone, may influence optimal design.
Welded end plates	Welded connection for steel bars only.	Simple and easy to build. Provides node position for connection of elements in prefabricated construction	Requires careful control to prevent reductions in ductility; Welding generally avoided in site work.
Loops	Use with steel only	Easy to build and can achieve full anchorage. Understood on site.	Multiple bar anchorages could be problematic
Splayed bar	Use with both FRP and steel reinforcement, with or without confinement provided by steel or FRP helices.	Easy to build; Full anchorage achieved and shown to be effective in preliminary tests.	Limited test data.

308 **3.2 Support zones**

309 The majority of structural testing to date on flexibly formed concrete structures has
310 been undertaken on simply supported beams. Such conditions regularly occur in
311 precast frame construction, where the elements may be supported in a number of
312 ways, such as those suggested in Figure 16 and Figure 17.

313 Support zones may also be detailed as fixed connections, and here additional
314 considerations should be made. Two examples of fixed connections, made between
315 non-prismatic precast sections and columns are shown in Figure 17. Additional design
316 guidance on such approaches is given in the literature (see for example Bruggeling
317 and Huyghe (1991)).

318 **3.3 Reinforcement**

319 The positioning of longitudinal bars in a flexibly formed beam is crucial to their
320 behaviour in both flexure and shear. It is further important to ensure that any changes
321 in direction of the tension reinforcement are properly considered in the design process.

322 This potential concern is illustrated in Figure 18, where a simply supported beam
323 has been designed with longitudinal reinforcement whose contribution to the shear
324 capacity of the section is disregarded. The 'ideal profile' for bending results in a
325 theoretical effective depth at the support zone of zero (Figure 18(a)). Consideration of
326 shear capacity requires additional support zone depth; in turn this affects the flexural
327 reinforcement location. A doubly curved bar, if used (Figure 18(b)), could result in
328 unwanted additional forces near the support zone (Figure 18(c)); these should be
329 avoided by retaining a single direction of bar curvature (Figure 18(d)).

330 The practical provision of transverse reinforcement is a further important
331 consideration, with options including bent steel bars and fibre reinforced polymer

332 reinforcement. Detailing of the transverse reinforcement in a fabric formed structure is
333 extremely important and sensible bar bending and arrangement schedules will be
334 required. Although the transverse reinforcement sizes and shapes may be
335 complicated, prefabricated reinforcement cages may provide a cost effective solution,
336 the quality of which can be effectively monitored and assured. Practical examples of
337 how fabric formed beams can be designed, reinforced, and constructed, are provided
338 by Bailiss (2006), Lee (2010), Garbett *et al* (2010), and Orr *et al* (2011).

339 Research at the University of Bath has shown how continuous FRP fibres can be
340 wound into shear reinforcement cages for prismatic concrete beams (Figure 19); such
341 an approach could easily be adapted for the rapid production of lightweight, efficient,
342 reinforcement cages that can take up any optimised geometry.

343 **4 Conclusions**

344 This paper provides designers with methods required to create efficient, fabric formed
345 concrete beams. Within this paper, the knowledge developed during research at the
346 University of Bath has been summarised, with opportunities for further work
347 highlighted.

348 The process of design has been highlighted for a simple beam, proceeding from the
349 calculation of moment and shear envelopes, through capacity predictions, to the design
350 detailing of the steel reinforced concrete element. Construction processes that have
351 applied in work at the University of Bath are summarised with potential site applications
352 for fabric formwork shown.

353

354

References

- 356 Bailliss J (2006) *Fabric-formed concrete beams: Design and analysis*. MEng Thesis, University
357 of Bath, Bath, UK.
- 358 Brennan J, Pedreschi R, Walker P, and Ansell M (2013) The potential of advanced textiles for
359 fabric formwork. *Proceedings of the ICE Construction Materials* **166 (CM4)**: 229-237
- 360 Bruggeling, ASG. and Huyghe, GF (1991) *Prefabrication with concrete*, Balkema, Rotterdam
- 361 BSI (2001) BS7973-1: Spacers and chairs for steel reinforcement and their specification.
362 Product performance requirements. BSI, London, UK
- 363 BSI (2004) BS EN 1992-1-1: Eurocode 2: Design of concrete structures - Part 1-1: General
364 rules and rules for buildings. BSI, London, UK
- 365 BSI (2005) BS EN 1990:2002+A1: Eurocode - Basis of structural design. BSI, London, UK
- 366 Darby AP, Ibell TJ, Tallis S, and Winkle C (2007) End Anchorage for Internal FRP
367 reinforcement. In *FRPRCS-8* (Triantafillou, TC (ed)). University of Patras, Greece.
- 368 Davies G, Lamb RS, and Snell C (1973) Stress distribution in beams of varying depth. *The*
369 *Structural Engineer* **51(11)**: 421-434.
- 370 Garbett J (2008) *Bone growth analogy for optimising flexibly formed concrete beams*. MEng
371 Thesis, University of Bath, Bath, UK.
- 372 Garbett J, Darby AP and Ibell TJ (2010) Optimised beam design using innovative fabric-formed
373 concrete. *Advances in Structural Engineering* **13(5)**: 849-860.
- 374 Kostova K, Ibell T, Darby AP and Evernden M (2012) Advanced composite reinforcement for
375 fabric formed structural elements. In *Second International Conference on Flexible*
376 *Formwork* (Orr JJ, Darby AP, Evernden M and Ibell T. (eds)). University of Bath, Bath,
377 UK
- 378 Kotsovos, MD and Pavlovic MN (1999) *Ultimate limit state design of concrete structures, a new*
379 *approach*. Thomas Telford, London
- 380 Lee, DSH (2010) *Study of construction methodology and structural behaviour of fabric formed*
381 *form-efficient reinforced concrete beam*. PhD Thesis, University of Edinburgh, Edinburgh.
- 382 Orr JJ, Darby AP, Ibell TJ and Evernden M (2012a) Optimisation and durability in fabric cast
383 'Double T' beams. In *The Second International Conference on Flexible Formwork* (Orr
384 JJ, Darby AP, Evernden M and Ibell T. (eds)). University of Bath, Bath, UK
- 385 Orr JJ, Darby AP, Ibell TJ, *et al* (2011) Concrete structures using fabric formwork. *The*
386 *Structural Engineer* **89(8)**: 20-26.
- 387 Orr JJ, Darby AP, Ibell TJ, *et al*. (2012b) The shear behaviour of non-prismatic concrete beams
388 determined using digital image correlation. In *5th International Conference on Optical*
389 *Measurement Techniques for Structures and Systems 2012* (Dirckx J. (ed)). Shaker
390 Publishing, Maastricht, Netherlands
- 391 Orr JJ, Evernden M, Darby AP and Ibell TJ, (eds) (2012c). *Proceedings of the Second*
392 *International Conference on Flexible Formwork*. University of Bath, Bath.
- 393 Veenendaal D. (2008) *Evolutionary optimisation of fabric formed structural elements*. MSc
394 Thesis, University of Delft.
- 395 West M (2006). Flexible Gewebeschalungen Für Fertigteilebinder (Flexible Fabric Molds for
396 Precast Trusses (in German and English). *Betonwerk + Fertigteil-Technik (Concrete*
397 *Plant + Precast Technology)*, **72(10)**: 46-52.
- 398 West M and Araya R (2012) Recent fabric formwork construction projects. In *Second*
399 *International Conference on Flexible Formwork* (Orr JJ, Darby AP, Evernden M and
400 Ibell T. (eds)). University of Bath, Bath, UK

401
402
403
404
405
406

WRI (2005) *Carbon Dioxide Emissions by Source 2005*. Earthtrends Data Tables: Climate and Atmosphere, Available online from <http://tinyurl.com/ybdohdn>

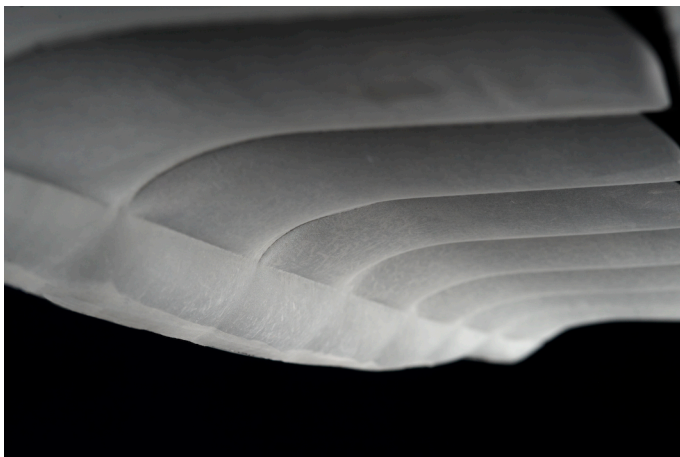
407

Figures



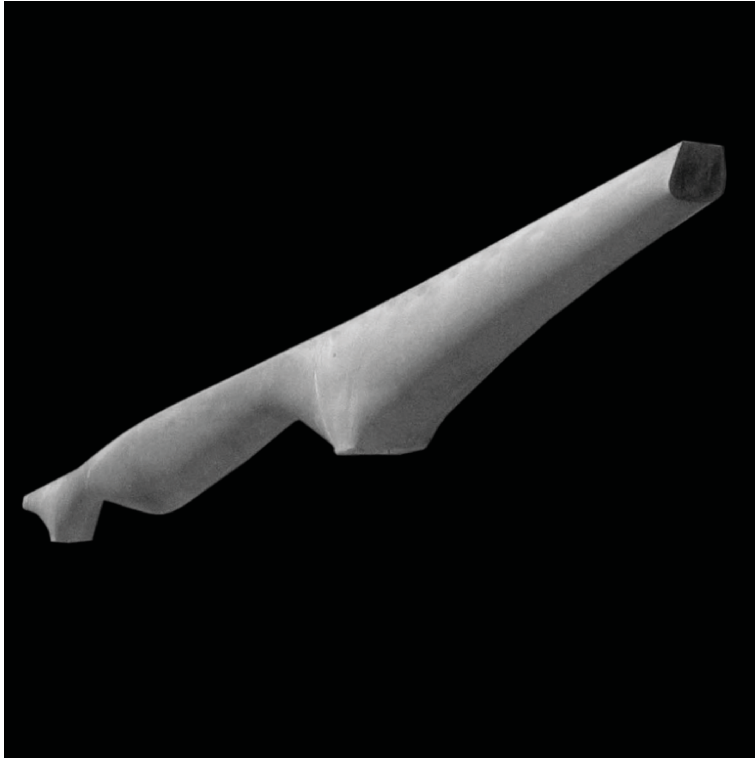
408

409 **Figure 1: Flexibly formed concrete walls (cast by Professor Mark West, University of**
410 **Manitoba**



411

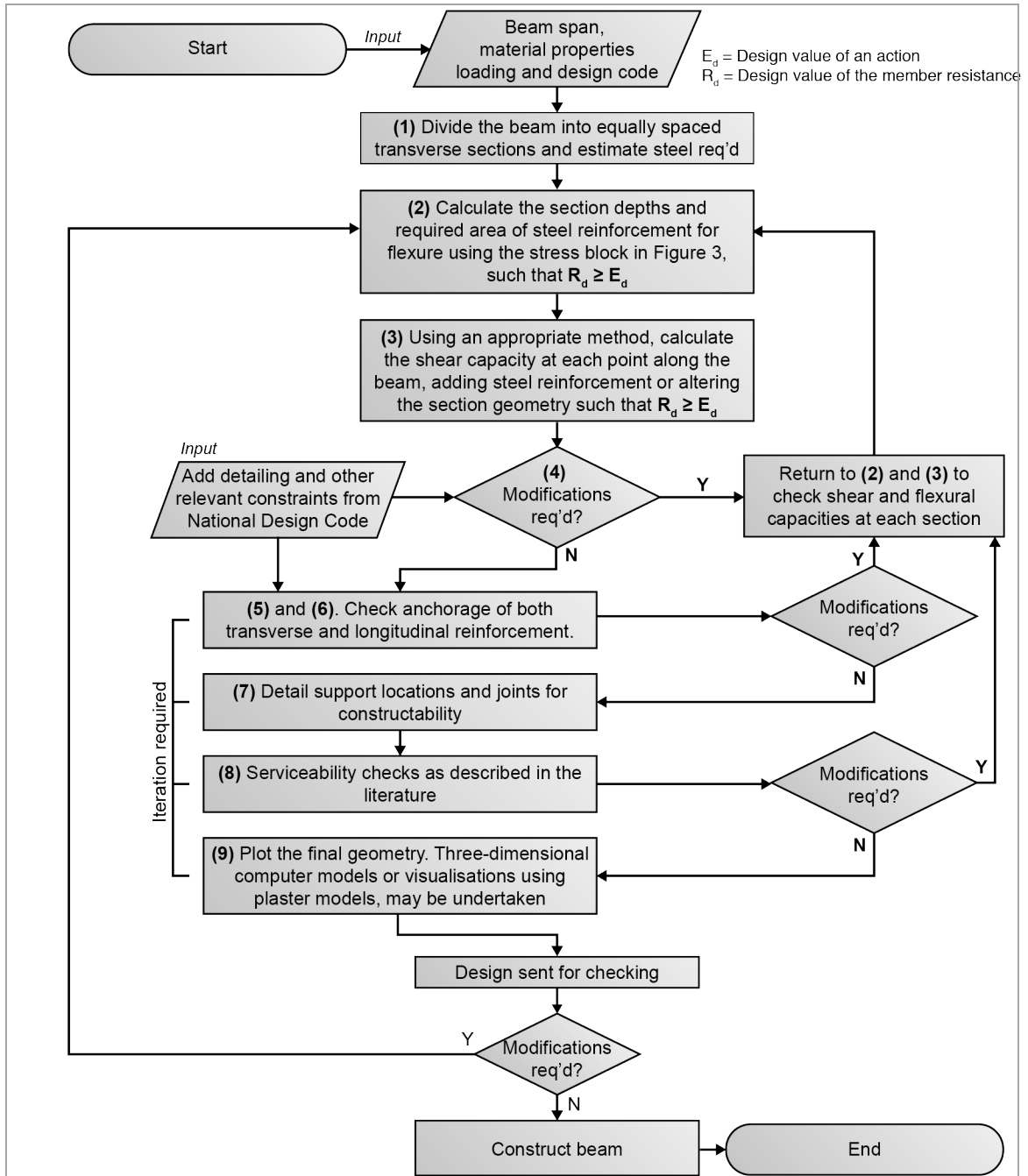
412 **Figure 2: Flexibly formed UHPFRC beam by the authors**



413

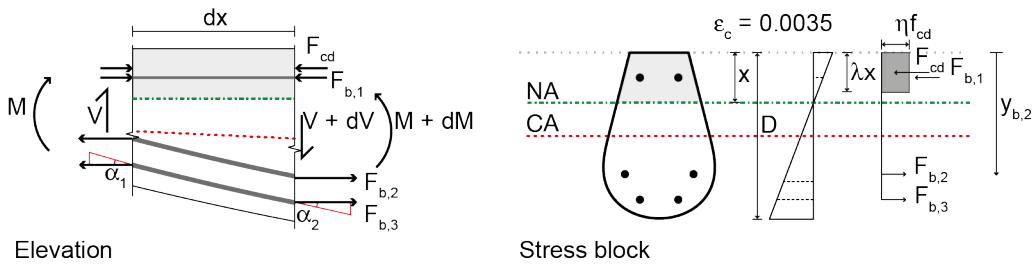
414

Figure 3: Flexibly formed 12m long beam (cast by Professor West).



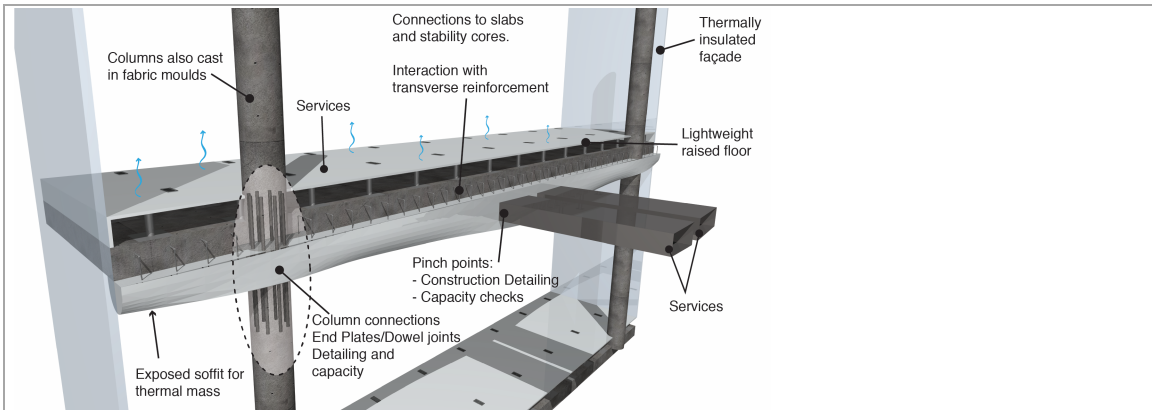
415

416 **Figure 4: Design process flow chart summary**



417

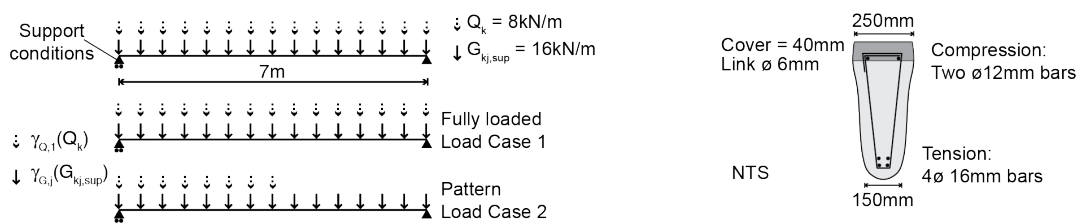
418 **Figure 5: Stress strain profile for flexural capacity**



419

420

Figure 6: Construction joints, end zone detailing and other considerations

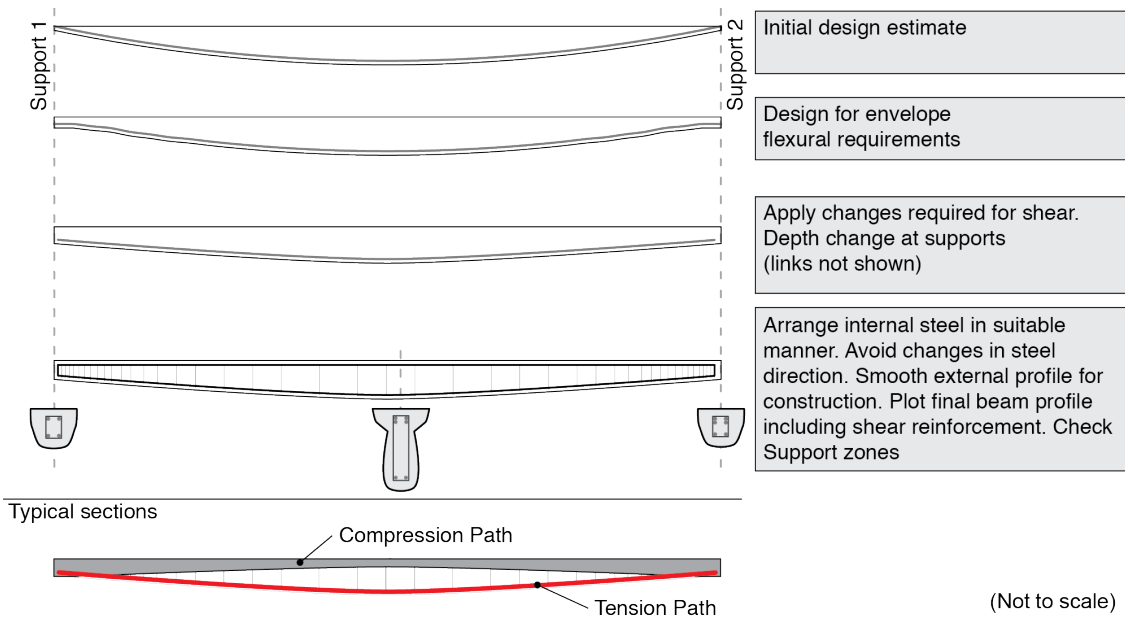


421

422

423

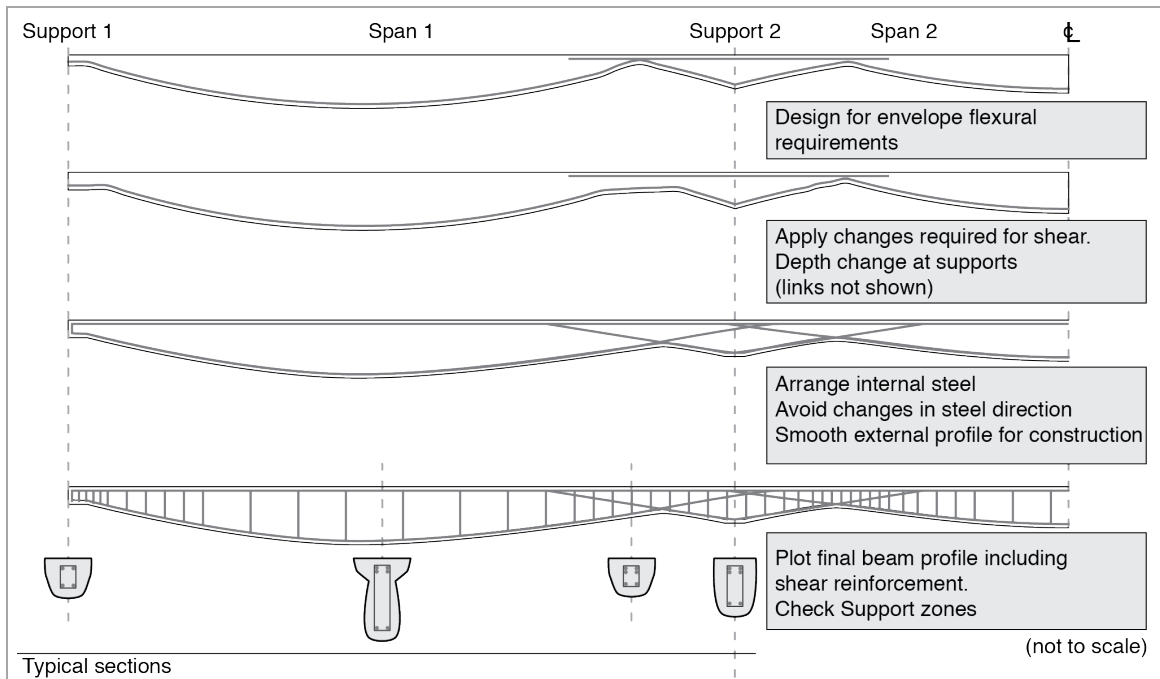
Figure 7: Beam layout and load envelope (left), cross section and reinforcement assumed in design process (right)



424

425

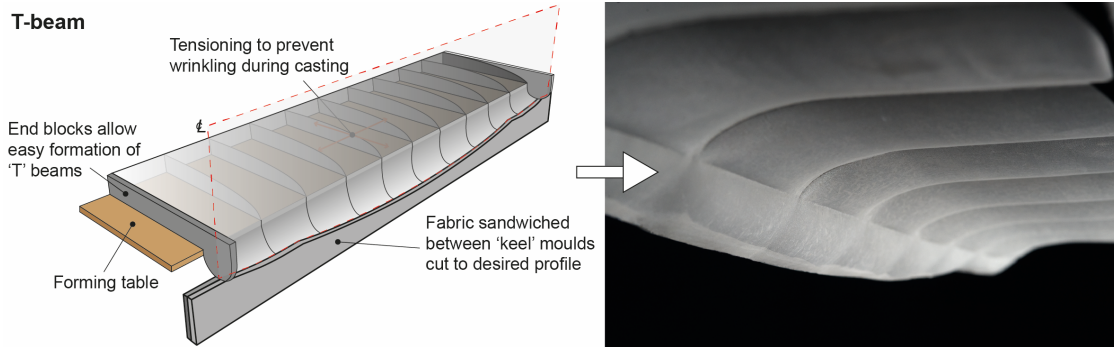
Figure 8: Progression of beam section design



426

427

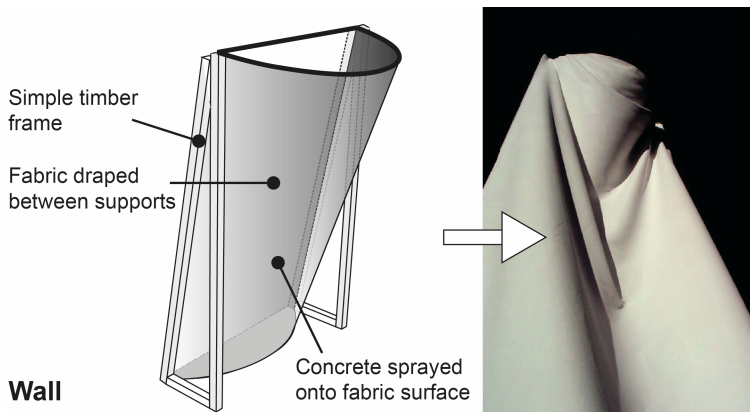
Figure 9: Indicative progression of beam section design for a continuous beam



428

429

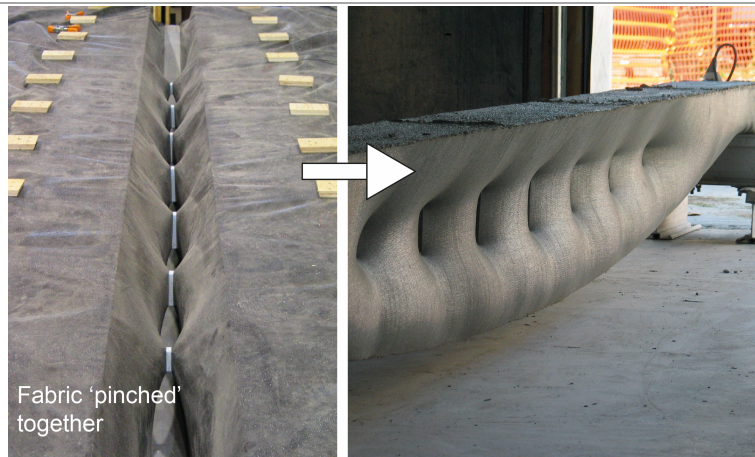
Figure 10: Example formwork system - 'T' beam cast at the University of Bath



430

431

Figure 11: Example formwork system - wall or shell (image courtesy Mark West)

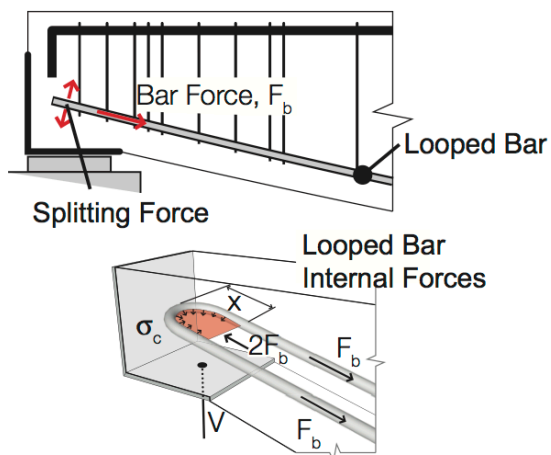


432

433

434

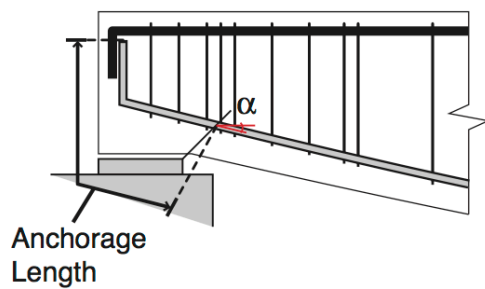
Figure 12: Example formwork system - adding detail to a beam (image courtesy Mark West).



435

436

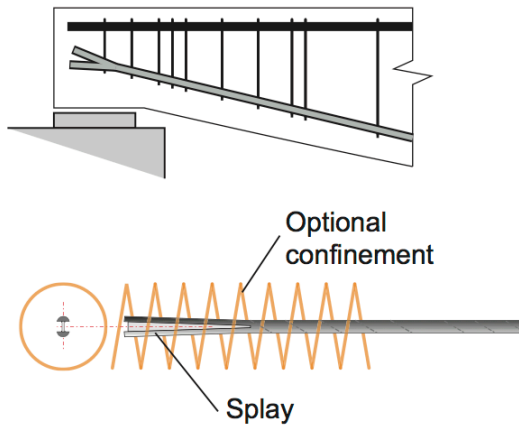
Figure 13: Anchorage using a looped bar.



437

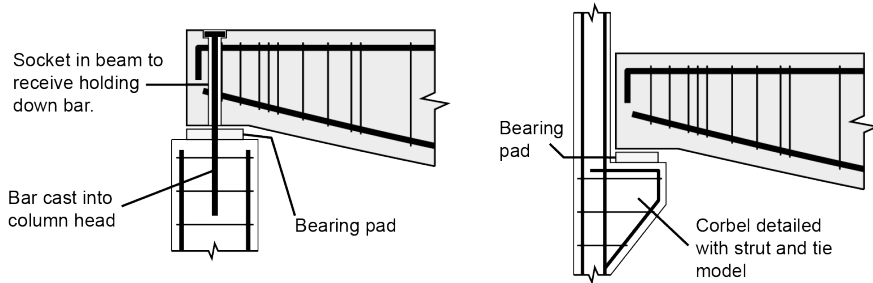
438

Figure 14: Anchorage using an 'L' bar.



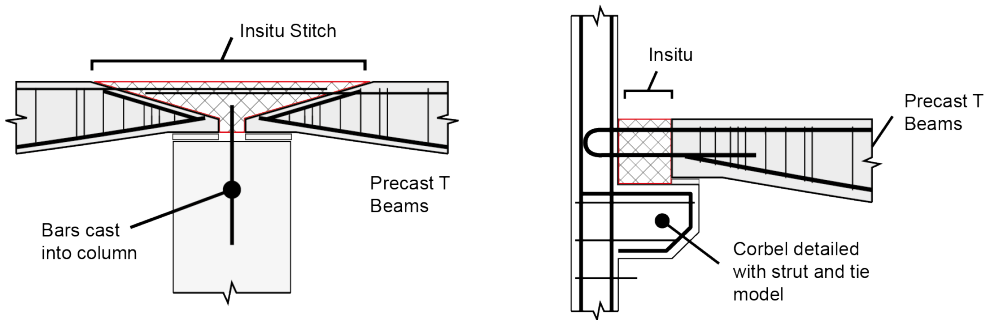
439

440 **Figure 15: Anchorage using a splayed bar.**



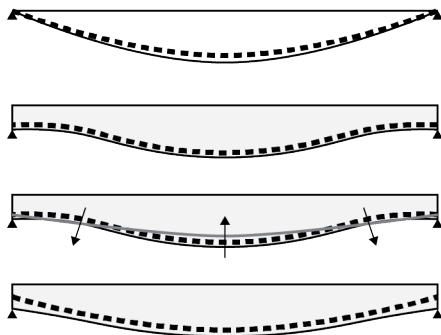
441

442 **Figure 16: Detailing of simple supports.**



443

444 **Figure 17: Detailing of fixed supports.**



(a) 'Ideal' profile for flexure under uniform load, zero support depth.

(b) Shear design increases support depth

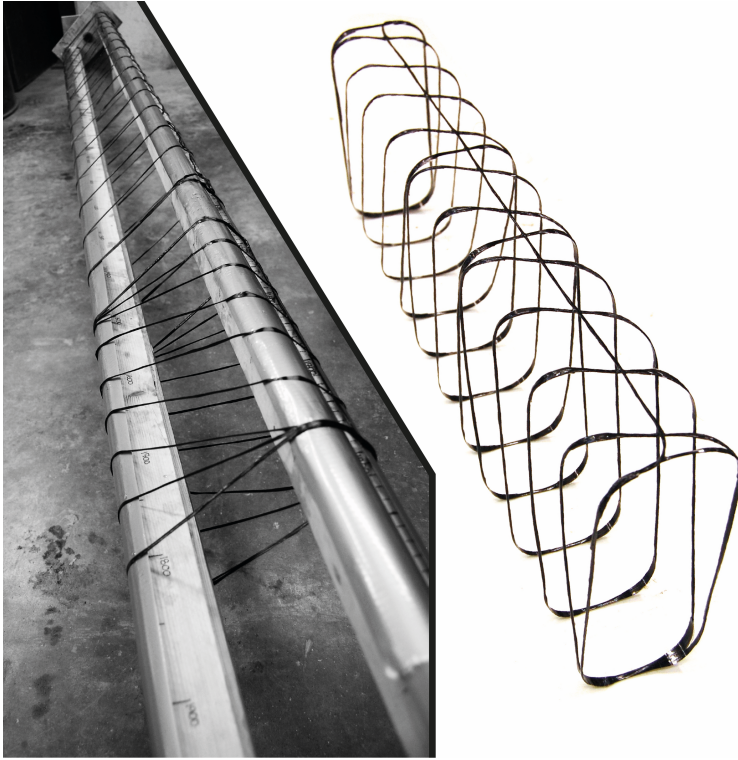
(c) Under loading a doubly curved bar wants to straighten and may result in a vertical force applied close to the support zone.

(d) Ideal profile, single direction of bar curvature.

445

446

Figure 18: Reinforcement considerations (a)-(d)



447

448

Figure 19: FRP filament winding and a completed reinforcement cage

449

450 **Table Captions**

451 **Table 1: Methods for geometrical control in construction**

452 **Table 2: Summary of anchorage methods for fabric formed beam**