# Profits and Fractal Properties: Notes on Marx, Countertendencies and Simulation Models

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**ABSTRACT**

There are new reasons for revisiting Marx's elaboration on the rate of profit because contemporary debates provide findings from the MEGA Project, long-term data on the rate of profit, and tools for dealing with complexity and non-equilibrium systems. This article proposes that the interplay between the tendency and the countertendencies of the rate of profit to fall can be translated into a simple system of equations, one based on each chapter of Section III of *Capital*—as if Marx sought to mathematically formalise his insights. This article reviews previous debates, presents data and runs a simulation model, showing that the rate of profit behaves as fractals.

## 1. Introduction

This article explores the relationship between the rate of profit and the long-term dynamics of capitalist economies, based on Marx's insights of the movements of the rate of profit. The latter were developed in his manuscripts for the third volume of *Das Kapital*, written in 1863-5 (MEGA2 II.4.2), and later edited by Engels to be transformed into three well-known chapters: 13, 14 and 15 (Marx 1894). Those insights are very polemical, inspiring long controversies over at least three rounds of debate between students and critics of Marx. They might, nevertheless, provide a reference for investigating the long-term dynamics of capitalism, for the profit rate may be understood as a synthetic variable whose movements summarise the workings of capitalist economies.

The last round of debates on the rate of profit, triggered by Michel Heinrich's (2013a) researches on Volume III, repositions Marx's elaboration. Some important implications are as follows. First, Marx had only general ideas, truly insights, to be developed later. Heinrich concludes that Engels's edition delivered a more finished version of those insights. Second, given the overall stage of elaboration of Volume III—a draft, as Heinrich suggests—today's researchers may use those insights according to their agenda, but with a clear understanding that it is an unfinished, unedited topic. Nobody can be sure, furthermore, of what Marx would have done if he had prepared a final version of Volume III.

The post-MEGA2 literature puts into perspective Marx's elaboration, stressing how unfinished, how incomplete it was. Probably, the post-MEGA2 literature suggests how much was left for later generations of researchers to develop. It could not be otherwise: capitalism is a complex system, always undergoing changes and metamorphoses, and hence demanding accompanying transformations of the theories that seek to understand and transform it.

Marx's unfinished agenda on the rate of profit and his clues about the complexity of capitalism invite us to revisit the rate of profit. Vis-à-vis what was available to Marx, our generation has two advantages: (i) two centuries of capitalist economic history and data (Duménil and Levy 2015); and, (ii) new tools to investigate complex dynamics (Goldenfeld and Kadanoff 1999).

Revisiting Marx's elaboration on the rate of profit, this article highlights how the long-term dynamics of capitalism may be interpreted as a struggle between the tendency of the

rate of profit to fall and its countertendencies. This insight from his writings of 1863-5 can now be tested against evidence on the behaviour of the rate of profit in the last 150 years.

The contribution of this article is the suggestion that the interplay between the tendency and the countertendencies (Callinicos 2014, p. 270) can be translated into a very simple system of equations. This system of equations is an attempt to model the long-term behaviour of the rate of profit. More specifically, each chapter of Section III of *Das Kapital* may be transformed into one equation, as if Marx was preparing to mathematically formalise his insights. This system of equations will push us to a non-linear world. Accordingly, this article applies tools of non-linear mathematics and complexity to, through simulations, try to replicate and evaluate these long-term dynamics.

A short literature review on the debates related to the profit rate is the subject of the second section of this article. This review indicates the specific contribution of Marx vis-à-vis classical economists, summarises the three rounds of debate regarding the rate of profit in the history of economic thought, and shows how this subject permeates modern economics. This second section hints at elements of complexity in the behaviour of the profit rate. The third section then summarises a short history of complexity and non-linear mathematics. This gives the reader some familiarity with the necessary tools to describe the long-term behaviour of the profit rate. A fourth section discusses the role of countertendencies in the history of capitalism and empirically evaluates the long-term behaviour of the profit rate, based on data for the last 150 years of the United States (US). It also conducts a preliminary analysis of the non-linear properties of this behaviour. The fifth section combines the two strands of analysis reviewed in sections Two and Three, suggesting a translation of Marx's insights on the interplay between the tendency for the rate of profit to fall and its counter-tendencies into a system of equations. This system of equations feeds a simulation model that is implemented to test the ability of those equations to replicate the long-term dynamics of the rate of profit in a capitalist economy. A final section presents the implications of this analysis and evaluates the results.

## 2. The rate of profit in the history of economic thought

### 2.1 Before Marx

Since classical economists, the behaviour of the rate of profit has been a key subject of academic investigations. A consensus can be found among them: namely, over time, the profit rate falls. Adam Smith (1776, I. 9.1) proposes that '[t]he increase of stock, which raises wages, tends to lower profit' (Smith 1776, I.9.2). A tendency to lower profits is identified, for different reasons, by Ricardo (1821) and John Stuart Mill (1848). Marx did try a synthesis of these debates about what he, at least then, thought was the 'most important law of political economy' (Rosdolsky 1968, p. 319).

Nonetheless, classical economists also identified factors that raised profits. Adam Smith might be the first to have indicated such counteracting factors: 'The acquisition of new territory, or of new branches of trade, may sometimes raise the profits of stock, and with them the interest of money, even in a country which is fast advancing in the acquisition of riches' (Smith 1776, I.9.12).

Ricardo (1821) mentions both the 'tendency' of profits to fall and the role of 'improvements in machinery' to 'check' 'this tendency, this gravitation':

[t]he natural tendency of profits then is to fall; for in the progress of society and wealth, the additional quantity of food required is obtained by the sacrifice of more and more labour. This tendency, this gravitation as it were of profits, is happily checked at repeated intervals by the improvements in machinery, connected with the production of necessaries, as well as by discoveries in the science of agriculture which enable us to relinquish a portion of labour before required, and therefore to lower the price of the prime necessary of the labourer (Ricardo 1821, 6.29).

John Stuart Mill (1848), in turn, is original in his explicit introduction of a special role for 'counteracting circumstances' (IV.4.16), 'counter-agencies' (IV.4.19) or 'counter-forces which check the downward tendency of profits' (IV.4. 25). Those 'counteracting circumstances' are 'waste of capital in periods of over-trading and rash speculation' (IV.4.17); 'improvements in production' (IV.4.19); 'cheap commodities from foreign countries' (IV.4.21); and 'the perpetual overflow of capital into colonies or foreign countries' (IV.4. 25).

*2.2 Marx's elaboration*

Marx, in the *Grundrisse*, discusses the 'Tendenz der Profitrate zu fallen' (MEGA II.1, p. 435, 532, 621-625, 627, 629, 635 and 636) and the factors blocking such a tendency—reductions in existing taxes (the state is here), a decrease in ground rents, new branches of production (p. 624) and monopolies. In the *Theories of Surplus-Value*, Marx reviews the classical economists' elaboration of the tendency of the rate of profit to fall, with a particular focus on Ricardo's theory (MEGA II.3.3, p. 1063-1093).

Petith (2005) explores the early efforts made by Marx to describe the tendency of profit to fall. In the *Manuscript of 1861-1863* (MEGA II/3.5, p. 1632ff), Marx wrote a first comprehensive draft for what would become the third volume.

In the manuscripts for *Capital* Volume III (MEGA II.4.2, p. 285-340), Marx presents factors that define the tendency of the rate of profit to fall. He later contrasts those with a list of six counteracting factors (the numbers are from Marx): '1) *Erhöhung des Exploitationsgrad der Arbeit*' (MEGA II.4.2, p. 302) (title introduced by Engels: 'more intense exploitation of labour'); '2) *Herunterdrücken des Arbeitslohns unter seinen Werth*' (MEGA II.4.2, p. 305) (Engels's title: 'reduction of wages below their value'); Marx's topic 3 has no title, the main subject perhaps summarized in the second paragraph, which connects 'der *Entwicklung der Industrie*' with '*Depreciation des vorhandnen Capitals*' (MEGA II.4.2, lines 32-33, p. 305) (Engels's title: 'cheapening of the elements of constant capital'); '4) *Die relative Surpluspopulation*' (MEGA II.4.2, p. 305) (Engels' title: 'the relative surplus population'); Marx's topic 5 also has no title, but begins as follows: '5) *Soweit der auswärtige Handel*' (MEGA II.4.2, p. 306) (Engels' title: 'foreign trade'); finally, in topic 6, with no title from Marx (MEGA II.4.2, p. 309), Marx deals with 'share capital', without using this word (Engels' title: 'the increase in share capital').[1]

Marx distinguishes his approach from Ricardo's, taking the tendency and countertendency as a starting point:

> these two aspects involved in the accumulation process cannot just be considered as existing quietly side by side, which is how Ricardo treats them; they contain a

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[1] Engels inserted the expression 'so-called dividends' in topic 6. However, Marx mentions 'dividends' (line 14, p. 502) in the topic on '*Die Rolle des Kredits in der kapitalischen Produktion.*' (MEGA II.4.2, pp. 501-505). In a subtopic—'*III- Bildung der Aktiengesellschaften*'—of this part, Marx explicitly identifies their role as a countertendency (MEGA II.4.2, p. 502). This is one connection between the movements of the profit rate and the credit system (Callinicos 2014, pp. 277-278).

contradiction, and this is announced by the appearance of contradictory tendencies and phenomena (Marx's words, without editorial interference from Engels: MEGA II.4.2, p. 323, translation Marx, 1894, p. 357).

Those 'contradictory tendencies' are also a step forward vis-à-vis Mill's counteracting factors—remember that the latter's final analysis leads to a stationary state (Mill, 1848, IV.6.3). Marx's explorations, with this interplay and struggle between the tendency and the countertendency, are his major contribution.

### 2.3 Perceptions of the movements of the rate of profit after Marx

The decline of the profit rate is a problem not only for classical economists and Marx. Schumpeter (1911) suggests a dynamic whereby the introduction of a successful innovation generates profits, then is eroded by competition from imitators. He is an author that connects the classical economists, Marx and contemporary theories, especially the evolutionary approach (Nelson and Winter 1982) and modern industrial economics (Bain 1951; Caves and Porter 1977; Caves 1998; Chandler 1992). The microeconomics of large firms and market structures show how leading firms struggle to preserve profits from existing and potential competitors.

Contemporary transnational corporations have grown using the two routes discussed by Adam Smith in 1776 (Chandler 1992, p. 83). For Chandler, organisational capabilities 'accounted for the long-term persistence of profits by the same players over the decades. Such capabilities and the resulting retained earnings became the basis for their continued growth' (1992, p. 83). In microeconomic terms, it is a significant problem for firms to avoid falling profits; 'the long-term persistence of profits' is not an easy task, and not all firms are able to keep them or even survive: the forces of 'creative destruction' are restless (Schumpeter 1942). Capitalist macro-dynamics have microfoundations in the behaviour of the profit rate.

### 2.4 Three rounds of a debate

A chapter of the history of economic thought may be written about the long debate on the Marxian elaboration about the tendency of the rate of profit to fall. At least three rounds of this debate may be identified, two of which came before MEGA2.

For the purpose of this article, the first debate centred on the tendency of the profit rate to fall was triggered by Sweezy's 1942 critique. This author argues that the very process behind the falling rate of profit (the rising organic composition of capital), predicated upon labour-saving technical change and hence growing unemployment, is coterminous with a rising rate of exploitation. The latter would thus not be a countertendency, but rather an integral element of the law. With no grounds to abstractly ascertain the prevalence of either force, only in particular conjunctures could the final effect be known. In light of this, Sweezy proposes a theory that, integrating the state, allows for crises to spring from the rising value composition of capital, decreasing rates of exploitation, disproportionality between sectors or, as he would later highlight, underconsumption.[2]

The second round starts with Okishio (1961), who seeks to break the connection between the introduction of innovations profitable for individual capitalists and a declining average rate of profit. In other words, under the conditions proposed it is suggested that a rising organic composition of capital cannot lead to a declining profit rate, which is a pillar of the law. Although the best-known proof is in the Okishio theorem (1961), other authors had obtained similar results (e.g. Bortkiewicz 1907; Moszkowska 1935; Shibata 1939). Some dispute the theorem over interpretations of value-theory (e.g. Carchedi 2009), and the validity of the hypotheses is controversial, as the author himself admits (Okishio 2000). Its impact on the debate has, nevertheless, been tremendous— even motivating obituaries of the law (Parijs 1980).

The findings of the MEGA2 Project help promote a better understanding of Marx (Cerqueira 2014), casting new light on Volume III of *Das Kapital*. Engels had given numerous clues about the state of the manuscripts for Volume III and how difficult his editorial work was for publishing this volume in 1894. Volumes II.4.2 and II.14 of MEGA2 show the problems Engels confronted in his edition. Further forays into other unpublished Notebooks about the crisis of 1866, expected in Volume IV.19 of MEGA2, will also be very helpful. The end result might be a much more nuanced, problematic, open-ended Marx: *human, all-too-human*.

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[2] A later interpretation along these lines is found in Fine and Harris (1979 p. 64), who suggest it would be better called 'the law of the tendency of the rate of profit to fall and its counteracting influences'. Reuten (2004) suggests this leans towards a cyclical view of the rate of profit. Mariolis (2014) considers that any model should include counteracting influences to describe economic cycles.

The third round of this debate (Carchedi and Roberts 2013; Kliman *et al.* 2013; Heinrich 2013a, 2013b; Harvey 2015) is a product of MEGA2. Heinrich stresses that Marx, after his 1863-1865 Manuscripts (MEGA II.4.2), did not return to the topic of the fall of the rate of profit and did leave clues about his own doubts about such 'law.'[3] However, there is evidence that this perspective was not abandoned: Marx's own comments about the relation between volumes III and I, and, most importantly, the very essence of the manuscripts of the 1870s, should be considered.[4] In 1875, in the manuscript 'Rate of surplus-value and profit rate mathematically considered', Marx wrote:

> When considering the rate of profit—as distinct from rate of surplus-value—we proceed from a given capital, with a given composition and a given rate of valorisation. We then let it undergo through the possible series of changes that produce changes in the rate of profit, which is finally a function of different variables and we find the laws that determine the rise, fall or constancy of the rate of profit, in one word, the laws of its movement. The laws discovered this way are valid for the social capital, considered as one capital, therefore, for the rate of profit considered as a proportion between the functioning social capital and the surplus-value produced by it. (MEGA II.14, p. 128)

In other words, at the end of his trajectory, Marx had not abandoned his perspective because that would imply abandoning also the perspective of Volume I. The laws that govern the rate of profit are another way to approach the laws described in the process of accumulation in Volume I, if one describes the laws of composition of capital, competition and surplus-value, there the law of the tendency for the profit rate to fall can be found. Nevertheless, as 'laws of movement', they are not inevitable, they are never a part of a theory of the breakdown of capitalism. The law describes a process of capitalist society, which, as Marx was aware, is contradictory, and should hence include the countertendencies as a key element.

*2.5 Marx's insights, countertendencies and reciprocal effects*

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[3] Without access to the MEGA2 edition, Grolll and Orzech (1989) also advance this argument, based on a note Marx wrote on his copy of *Capital* vol. 1, published in the third German edition.

[4] In fact, on April 30th 1868, Marx wrote to Engels about 'one of the greatest triumphs over the *pons asini* of all previous economics', referring to his analysis about the tendency of profit rate to fall. (MEW, v. 32, p. 73).

Marx's manuscripts of Volume III present another very important insight for the understanding of capitalist dynamics—the role of extra surplus value (1867, p. 433-437) or surplus profit (1894, p. 279) and the profits capitalists gain by using the most modern production techniques. This decisive element of Marx's elaboration on competition was well understood by Schumpeter (1911), who developed a complete theory of capitalist dynamics based on a straight relationship between innovation and profits.

The struggle between the factors that push the rate of profit down and the counter-acting factors that push it up suggests that the rate of profit is a resultant; a 'synthesis of multiple determinants', a variable with multiple causes—a multidimensional variable. Being the result of those multiple causes, the profit rate impacts the dynamics of the whole system, in its ups and downs. Probably a variable that may be under the 'reciprocal effects' dynamics: both cause and effect—*Wechselwirkung*.

Does such interplay, those reciprocal effects between the tendency and the countertendencies to the fall of the rate of profit, lead to a dynamic beyond Newtonian physics? What are the methodological implications of the theoretical break (rupture) that Marx establishes with Ricardo's approach and his gravitation analogies? Is the world of Newtonian physics enough to deal with Marx's insights? The next section turns to this discussion.

## 3.  Notes on methodology: Complexity and non-linearity

Beyond Newtonian physics there is the world of complexity. Complexity has broader impact, as a special issue of *Science* (2 April 1999) illustrates. Goldenfeld and Kadanoff (1999 p. 88-89) review 'the development of complexity in physics', with references to pioneering researchers in this new field: 'long ago, Katchlalsky and Prigogine described the formation of complex structures in non-equilibrium systems' (p. 88). Another important reference regarding the origins of complexity is Lorenz (1972). The works from Katchlalsky and Prigogine are from 1967 and 1977. Goldenfeld and Kadanoff also mention Mandelbrot (whose pioneering book is from 1977) and Philips W. Anderson (a paper from 1972). Interestingly, P. W. Anderson, a Nobel laureate in Physics in 1977, in the last paragraph of

his paper, offers two examples from economics: one of them is Marx, who 'said that quantitative differences become qualitative ones' (Anderson 1972, p. 396).

It is suggestive that in a pioneering paper on complexity there is a reference to Marx or, to be more precise, to Marx's Hegelian roots. This reference furthers our suspicion that the tools developed by the physics of complexity may be useful to deal with some of Marx's insights.

Evolutionary economists have followed these developments with great attention (Arthur 2013). This journal has done the same, with an article on 'the complexity era in economics' (Holt *et al.* 2011). Richard Nelson highlights that '[r]ecent developments in understanding the mathematics of nonlinear dynamic systems, and recognition that many physical systems display properties that such models can explain and illuminate, is yet another stimulus to evolutionary theorizing in economics' (1995, p. 52). Goldenfeld and Kadanoff (1999, p. 87) suspect that there might be differences between complexity in the physical world and 'in biological or economic situations'.

## 3.1 The economy and a non-equilibrium system

An equilibrium system (i.e. a system in thermodynamic equilibrium) is the subject of a line of inquiry of statistical mechanics that deals with systems isolated from the rest of the universe. This isolation may be achieved by improvements (thicker walls, adequate material etc.) until the environment has a negligible effect on the system.

This line of statistical mechanics also deals with systems in contact with a large reservoir with constant temperature, so that there is no energy unbalance between them, and/or with constant chemical potential, so that no chemical (number of particles) unbalance obtains between them. If the system reaches such a state, it is in thermodynamic equilibrium. This is an equilibrium system.

Having defined a system in (thermodynamic) equilibrium, a non-equilibrium system is anything else. Therefore, a system is non-equilibrium when it is neither isolated from the rest of the universe nor in contact with a constant temperature and/or bath of constant chemical potential. A system that is under transient effects before reaching equilibrium is also non-equilibrium. External fields (e.g. electric or magnetic) or perturbations can also induce a non-equilibrium state, as the system is not isolated.

How might those concepts be applied to an economic system, since it is not strictly a thermodynamic system as understood in physics? In general, the economic system is formed by a large number of individuals, firms, banks, institutions, and countries. Those components interact with each other through their consumer preferences, production of goods, trade rules, sales and credit operations, monetary wage relationships, monetary flows, knowledge flows etc.

Individuals enter and leave the system, determining different levels of demand for products and different levels of employment. Firms are created and may survive or die, producing different levels of activity. New technologies lead to new firms, new products and lower production costs. Taking individuals, firms, banks and institutions as the 'particles' forming this system, the economic system can be interpreted as a non-fixed-particles thermodynamic system.

Furthermore, state intervention leads to different rules for the domestic and international production and trade of goods. Wars may lead to different rules of marketing and supply of goods. This dynamic of definition of rules can be compared to the influence of external fields in a thermodynamic system. Therefore, using these analogies, the economic system can be seen as a non-equilibrium system.

### 3.2 Complexity and fractals

Although there is no generally agreed, concise definition of a complex system, available definitions converge on some key points. Centrally, a complex system is formed by a large number of interacting components and exhibits different organisations at different scales of observation, which lead to different behaviours at different scales (Goldenfeld and Kadanoff 1999; Anderson 1972)

Bringing the economic system into the frame again, the smallest possible level of disaggregation are the individuals that interact with other individuals (and other components of the system). Additionally, some groups of individuals are organised, along specific rules, creating firms and other organisations. These firms, which are more than an aggregation of individuals (i.e., they are a scale of their own and have got emergent properties), become complex systems in themselves and interact with others firms and individuals through rules different than those presiding over the interaction between individuals. The growth of firms

and resulting market structures are defined by complex interactions between production costs, the reinvestment of profits and competition strategies. These complex processes lead (at this level or scale) to the logic of the market and of capital accumulation. Other forms of organisation take place with the formation of governments, institutions and universities. These in turn interact with individuals, firms and between themselves, through specific rules that differ from those of other patterns of interaction. The result (at this higher scale) is new institutions, such as laws, labour qualification, job training, public finance, the stock market, (national) innovation systems etc.[5]

As the elements of this scale are organised, countries emerge at a higher level and interact with each other and with institutions (of lower levels) through particular rules. This leads to the logic of exchange, international trade, and global innovation systems (the international flow of knowledge). There are also intermediate, overlapping, non-hierarchical and entangled scales, such as local clusters, sectoral innovation systems, global cities and so on. The descriptions that integrate 'the twin difficulties of scale and complexity' (Anderson 1972, p. 393) show how a capitalist economy fits so well in a description of complexity. As Goldenfeld and Kadanoff (1999 p. 88) posit, 'complex systems form structures and these structures vary widely in size and duration. Their probability distributions are rarely normal, so that exceptional events are not that rare'.

How do those methodological remarks help our investigation? These different organisations at myriad levels produce distinct behaviours and features at each scale, all of which influence or contribute to the dynamics of the system. The rate of profit is determined with the contribution of all those organisations, at different scales (of different orders of magnitude): as a 'synthesis of multiple determinants', the complex operation and interactions between the factors that lead to a falling rate of profit and their counteracting ones are represented in the movements of this rate.

Do these movements of the rate of profit behave as an evolutionary system? The decomposition of the frequencies that comprise the behaviour of a large variety of evolutionary systems at low frequencies exhibits a power law $1/f$ relation (where $f$ equals

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[5] Mandelbrot and Hudson (2004) evaluate stock market as a complex system. This interpretation may be an example of how constitutive institutions of a modern capitalist economy can be analyzed as complex systems.

frequency). This $1/f$ behaviour has been observed in a wide range of systems, such as condensed matter system, river discharge, DNA base sequence structure, cellular automata, traffic flow, financial markets and other complex systems with self-organising elements (Gontis *et al.* 2004; Gilden *et al.* 2005; Wong *et al.* 2003; Kaulakys *et al.* 1998; Yamamoto *et al.* 1995; Maxim *et al.* 2005).

It is common for parts of such systems to be fractals. F. D. Dyson, quoted by Mandelbrot, explains that

> Fractal is a word invented by Mandelbrot to bring together under one heading a large class of objects that have [played] ... an historical role ... in the development of pure mathematics. A great revolution of ideas separates the classical mathematics of the 19th century from the modern mathematics of the 20th. Classical mathematics had its roots in the regular geometric structures of Euclid and the continuously evolving dynamics of Newton. Modern mathematics began with Cantor's set theory and Peano's space-filling curve. Historically, the revolution was forced by the discovery of mathematical structures that did not fit the patterns of Euclid and Newton. These new structures were regarded ... as 'pathological,' ... as a 'gallery of monsters,' kin to the cubist painting and atonal music that were upsetting established standards of taste in the arts at about the same time (Mandelbrot 1977, p. 3).

Fractals and their statistics exhibit scaling (i.e. their behaviour is the same in any scale of measurement). The universality[6] of power law behaviour suggests that it does not arise as a consequence of particular forms of interaction, but that it is rather a characteristic signature of complexity and self-organisation (due the contribution of different scales of organisation to the global behaviour of the system).

There is an important difference here, vis-à-vis classical mathematics. Classical mathematics is used to describe the linear system wherein its response is proportional to the perturbations implemented on it. Thereby, the dynamic of this kind of systems is, in general, smooth and well behaved. However, several natural systems with non-well behaved properties were discovered which dynamics were described by fractals and using the tools of

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[6] This means a large group of systems display similar characteristics of interest (e.g. the relation between the frequency of events and their magnitude) regardless of the mechanisms that generate them.

non-linear mathematics. One kind of these systems is the complex systems that also show a behaviour expressed by fractals due the non-linearity of the interactions among its several components and the contribution of different scales of the system.

There are multiple self-organised systems. One of the most popular is the sand pile model (Maslov *et al.* 1999). This nonlinear model was introduced to explain 1/*f* noise as a result of self-organised criticality.[7] The great interest of this line of research regards the complex behaviour that mimics a noise with fractal characteristics. In this model, however, noise originates from nonlinear deterministic interactions.

It is also possible to define a stochastic model exhibiting fractal statistics and 1/*f* noise. Explaining the evolution of complexity into a chaotic regime, stochastic processes may be used to describe phenomena that occur as random sequences of events, exhibiting scaling of several statistics (Thurner 1997). A considerable part of real stochastic sequences of events in physics, biomedicine, geophysics and economics are fractal (Kaulakys *et al.* 1998; Kaulakys *et al.* 1999; Gontis 2001; 2002).

This short methodological note puts forward two questions for our investigations: (i) has the rate of profit empirically verifiable fractal properties; and, (ii) if yes, can we formalise its behaviour in a simple set of equations to model its long-term dynamics? The next two sections address these questions.


## 4.   Two centuries of capitalism: Countertendencies, data and a test


*4.1 Countertendencies in the history of capitalism*
The history of capitalism in the last 150 years show its flexibility, it shows that limits may be overcome and that capitalist dynamics include a strong capacity to introduce changes in the system as a whole. Therefore, metamorphoses of capitalism are part of these dynamics, and today we are in a position to see that they should be taken into account (Furtado 2002). The considerable literature on the best periodization of capitalism (Albritton *et al.* 2001) discusses the role of crises as moments of transition between phases—they have heretofore

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[7] Self-organized criticality suggests that certain systems *inherently* tend towards the critical state wherein fractal characteristics and 1/*f* noise are observed.

been associated with the reshaping of capitalism, not with its collapse (Duménil and Lévy, 2002).

Three approaches can be presented as an attempt to understand this complex system called capitalism (Marques 2014). First, technology is a driving force of change. This feature is illustrated by the approach focused on the technological dimension of capitalist dynamics: the long waves of the capitalist development research agenda (Freeman and Louçã 2001). Second, the leading centres of capital accumulation have changed over time. Those geographical changes are grasped by an approach that incorporates geopolitical struggles, deeply connected to the nature of world money: the systemic cycles of accumulation agenda (Arrighi 1994). Third, the nature of the locus of capital accumulation—the firm—has been transformed over time, with a systematic expansion towards new sectors and new regions. This feature is identified by the approach focused on the internationalisation of capital (Dunning and Lundan 2008).

What might connect those different and certainly complementary approaches?: the movements of the profit rate.

Technological innovation pushes the profit rate upwards as it opens new sectors for capital accumulation, improves existing ones and, at a microeconomic level, preserves the profit rate of leading firms in oligopolistic structures. Geopolitical changes, transitions of hegemony, mean the flow of capital towards new regions where profits are higher. The internationalization of capital is a combination of the first two dimensions, as leading firms, supported by strong innovative capabilities, are able to expand production and innovation towards new geographic regions and sectors. Multinational firms can be seen as machines to explore and transform the international division of labour in their favour, to increase profits globally—multinationality is a new source of profits. And, as they do this, they change the face of global capitalism; they are agents of the metamorphoses of capitalism.

This investigation has in Grossmann (1929) a relevant contribution, which may be read as a book on the *countertendencies* to the fall of the rate of profit. This might seem a paradox, given that the collapse of capitalism is in the book's title. Nevertheless, Grossmann actually

devotes 132 pages to the factors that contribute to the fall of the rate of profit and 166 pages to the countertendencies.[8]

The creation of new countertendencies is very clear after the crisis of 1929: the long list of Grossmann's countertendencies need to be even further extended, with a new role of the state as a strong countertendency to the fall of the rate of profit. This was left for Poulantzas: 'state interventions' should be investigated as 'countertendencies' to the tendential fall of the rate of profit (1978, p. 199). This allows for combining changes to the state (quantitative and qualitative) and metamorphoses of capitalism.

What did we see in the last 150 years of capitalism? First, technological revolutions created new sectors (chemistry, electricity, combustion engines, electronics, computers, health-related technologies, microelectronics, internet) and transformed health and information into commodities—and, therefore, into new spaces for capital accumulation. Second, there has been growth in the role of the state in capitalist economies (both quantitatively and qualitatively). Third, there has been exponential growth of share capital. Fourth, capital has experienced geographical dislocation, including a change in the leading country (and several small peaks of capital accumulation in important, but not leading, countries such as Germany, Japan, China). Fifth, there has been a tremendous geographical expansion of capital, now directed to an invented new region: the internet and the world wide web. The latter were created inside scientific institutions (CERN, Switzerland, at its core), as part of strong innovation systems,[9] and were later appropriated by capital; they constitute a strong countertendency, brought forth by non-market institutions. This list supports the

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[8] The discussion of Grossmann's elaboration is beyond the subject of this paper. Pannekoek (1934) presents a very interesting critique, focused on politics, labour organization and action, against an automatic and economically determined end to capitalism. He concludes his analysis indicating that the 'self-emancipation of the proletariat is the collapse of capitalism.' Grossmann's work has two important contributions compared to his predecessors. First, he was able to establish a more consistent relation between the different levels of abstraction in Marx's *Capital*, integrating accumulation, reproduction and the rate of profit, which represents the completeness of the system. Second, Grossman was able to point out some of the countertendencies that were only latent in Marxian times. He explored the role of capital-money and of the plethora of capitals (credit) in the recovery from a crisis (1929, p. 322ff); unproductive classes (p. 348ff); and the abolition of ground rent (p. 341ff). He was the first author to analyze Marx's theory of business cycles, as presented in the third volume of *Das Kapital* (MEGA II.15, p. 461ff), with the inclusion of the economic impact of wars as an important countertendency.

[9] WWW was proposed in 1989 by Tim Berners-Lee at CERN, Switzerland.

conjecture that capitalism has been an endless creator of countertendencies to the fall in the rate of profit.

*4.2 Rate of profit for the United States, 1869-2011*

Data on the profit rate are not easy to find, process and evaluate. There are, however, important works that organise and discuss it, pointing to complications and alternative calculations, such as Duménil and Lévy (1993), Maito (nd), Basu and Vasudevan (2013) and Jones (2016).

Maito's paper stresses the falling rate of profit in the long-term, as his graphs with global averages show (see his Figure 8). Nevertheless, there is another phenomenon visible in Maito's graphs – over time, the profit rate sequentially peaks in different key capitalist economies. Maito's (nd) Figure 2 shows how between 1860 and 1920 the United Kingdom's (UK) profit rate was higher than that for the US, and how Germany's rate was, between 1870 and 1885, higher than that of the UK. Figure 3 in turn shows how Japan's rate of profit between 1950 and 1975 was the highest of all developed countries. Finally, Maito's Figure 7 shows that the rate of profit in China between 1978 and 2007 was far above the 'world average.'

Maito's data hint at broad international movements that indicate the operation of countertendencies to the fall of the rate of profit –geographical movements of capital towards new regions. They also show that a strictly national framework is incapable of grasping all movements of the profit rate. They might display an Arrighian sequence: a juxtaposition of data for the UK, the US and China might suggest a pattern of international movements of the rate of profit. In sum: Maito's (nd) paper may be read as a snapshot of the international capital mobility in the long-run. Therefore, the conclusions of the analysis Basu and Manolakos (2013) present for the US economy between 1948 and 2007 may be put in a broader perspective: 'the rate of profit declines at a rate of approximately 0.2 percent per annum after controlling for the counter-tendencies' (p. 93)—the long-term decline of the rate of profit in the US is confronted by rises elsewhere.

Given this general background, this section focuses on the data for the US (Duménil and Lévy 2015). The US reached the summit of the technological application of science, and of financial organisation, together with the largest public sector (in absolute terms) of the

capitalist world. Therefore, it is the economy of global capitalism where the countertendencies operated more fully. However much an analysis of the US cannot be equated to an analysis of global capitalism, it is the necessary first step in doing so. This is the economy that, during the 20th century, had the greatest impact on the world economy; therefore, understanding the dynamics of the US profit rate, with their irreducible historical dimension, is an important contribution to comprehending the tendencies of capitalism and the profit rate.

Figure 1 shows the rate of profit for the USA between 1869 and 2011.[10]
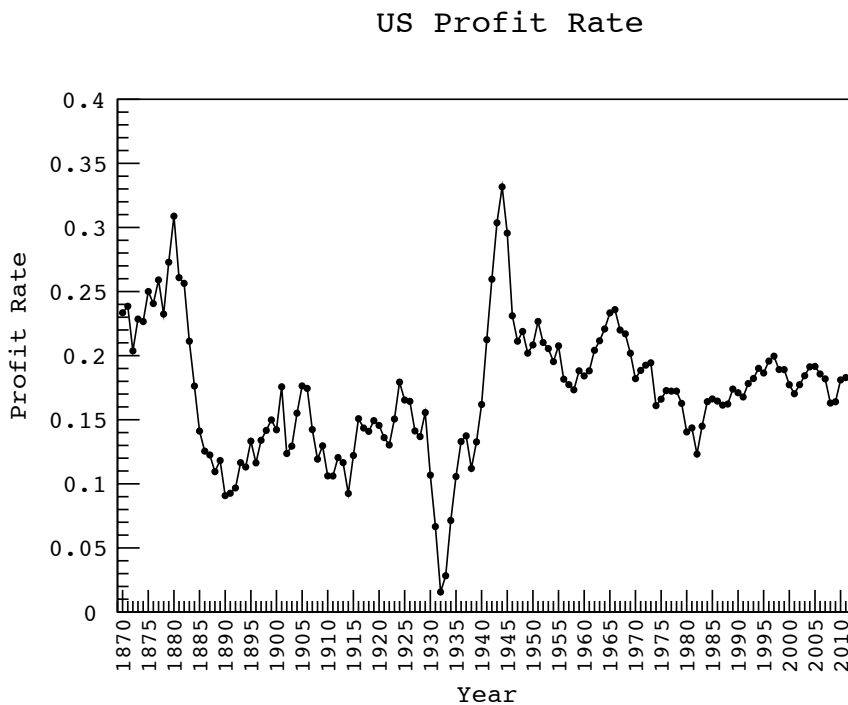
## US Profit Rate



Figure 1. Rate of profit in the United States, 1869 to 2011.

Source: Duménil and Levy (2015).

During this period capital accumulation expanded, as measured by a substantive increase in the GDP of the US: from US$ 98 billion in 1870, it reached US$ 7,394 billion in 1998—

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[10] Duménil and Lévy (2015) define the rate of profit as follows: Profit Rate = (NDP-W*L)/KN; where, NDP is Net Domestic Product (current US$); W is Hourly Wage (current US$), L is Number of Hours Worked, and KN is Net Stock of Fixed Capital (current US$).

an important change in the scale (size) of that profit-producing economy (in 1990 international US$, according to Maddison 2001, p. 184).[11] The ups and downs of the rate of profit are, of course, related to broad institutional changes that took place in the history of the US, involving both technological revolutions, the New Deal reforms, new roles in the world economy, World Wars, the growth of state intervention etc.

*4.3 Fractal properties of the rate of profit in the United States*

Given our conjecture—capitalism as a complex system, where the rate of profit is a key variable—this section processes those data with a decomposition of the cyclical movements of the rate of profit for the US.

The turbulent (in the economic sense) behaviour of the rate of profit in Figure 1 looks like a 'fractal curve.' We employ a commonly used tool (mathematical function) to analyse this kind of curve: the Fourrier transform (FT). The FT decomposes a function into its constituent frequencies. The transformation writes the original function as an infinite sum of other periodic functions ($\cos(...) + i \operatorname{sen}(...)$), each with a different period/frequency, until it sweeps all possible periods/frequencies. This kind of decomposition might have implications for debates on long wave research, as a possible quantitative methodology to measure their duration and periodicity.

Figure 2 shows the result of applying the FT to the data on the profit rate of the US.[12] The horizontal axis is the frequency of the periodic function and the vertical axis is the coefficient that multiplies this function, which means the weight of the frequency in the original function.

The resulting 'power law regression' ($y=0.0004*x^\wedge-1.0001$) in Figure 2 confirms that the profit rate curve (Figure 1) is a fractal, and the exponent near -1 (-1.0001) indicates the behaviour of a complex system (see section five for a more detailed discussion). In brief, the key implication of having obtained a $1/f$ distribution is that it provides us with the information necessary to characterize the processes and systems that generated this phenomenon. As this kind of distribution is a fingerprint of complex systems in self-organising states, we can infer

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[11] Those data have implication for later evaluation. First, 'more is different' (Anderson, 1972) and size matters—different scales might be related to properties linked to fractal phenomena. Second, because although the rate of profit may be lower, the mass of profits could be larger.
[12] A Fast Fourrier Transform (FFT) is applied here.

some features of the studied process, such as having long-range and high inter-temporal correlations. On the other hand, if we had obtained a Gaussian distribution, we could infer, due to the central limit theorem, that the processes generating such distributions are random, with short-term and low inter-temporal correlations.
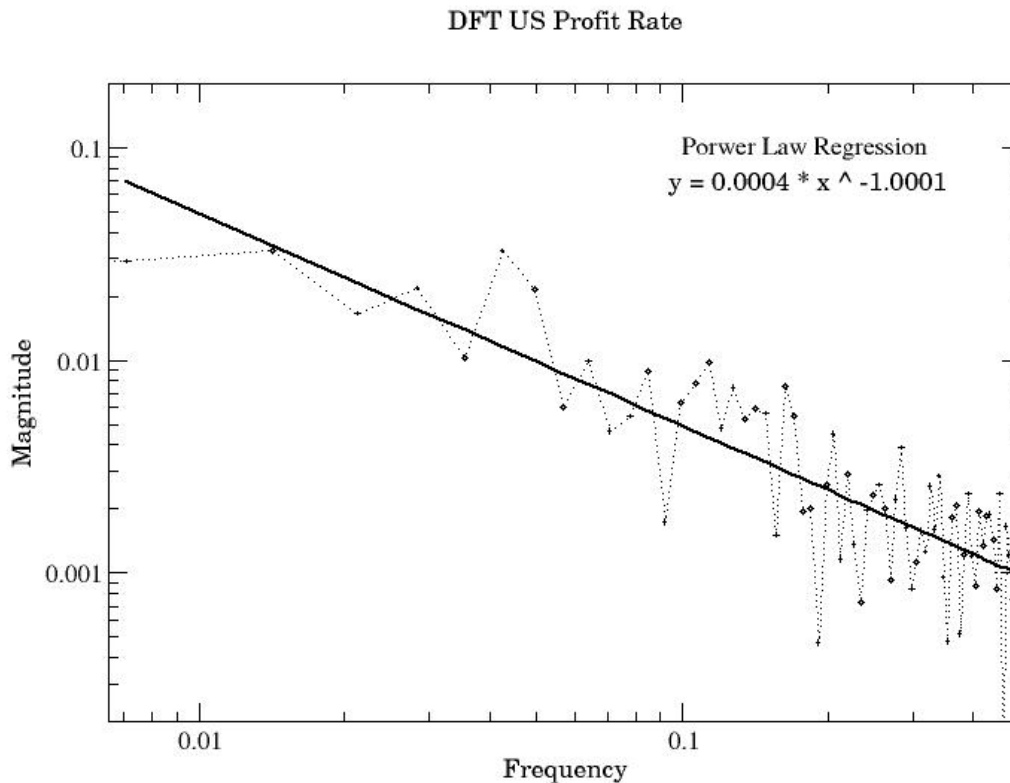


Figure 2. US profit rate data: Magnitude and frequency of the FT decomposition
Source: Authors' elaboration.

Figure 3 reorganises the FT results (Figure 2), showing at the horizontal axis the period (the inverse of frequency, $T=1/f$ where $T$ is the period and $f$ is the frequency). This facilitates the visualization of those cycles with a greater weight in the behaviour of the profit rate.

Figure 3 shows that, in the decomposition of the data for the US profit rate, the most important cycles are the 23-, the 20-, the 70- and the 35-year long ones.[13] The decomposition also shows diverse and overlapping cycles of different temporalities, revealing the combination and superposition of different dynamics. Again, an identification of properties related to complex systems and fractal properties.

Mandel (1981) and Shaikh (1992) discuss long waves focusing on the rate of profit. On the one hand, Figure 3 shows a 47-year cycle, very close to the Kondratiev cycle—approximately 50 years long, according to Schumpeter (1939). On the other hand, there are four other more important ones: the 23-, the 20- cycle, the 35-, and the 70-year ones. This evaluation is not the subject of this article, but as a by-product of our current investigation this analysis might contribute to discussions on this subject—a topic for further research.
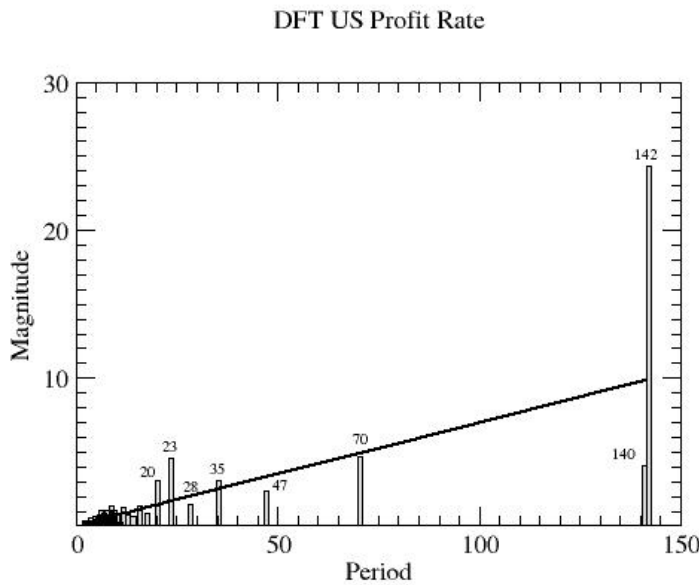
DFT US Profit Rate

Figure 3. USA profit rate data: Magnitude and period of the FT decomposition
Source: Authors' elaboration.

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[13] The highest period calculated by the Fast Fourier Transform is the sample size, which in our case is 142. Therefore it corresponds to a frequency of 0, i.e., just a shift in the y-axis of the original curve. It appears with high weight in our FFT result just because the profit rate curve has an average different from zero. Hence, we have analyzed only the periods below this maximum period.

In sum, the empirical analysis of the US rate of profit suggests features typical of complex systems, with fractal properties.

## 5. Marx's insights translated into a system of equations

From this point of view, and in light of the behaviour obtained from the empirical analysis, we propose a model based on a stochastic process, representing the struggle between the tendency and the countertendencies to the fall of the rate of profit. This model is built translating insights from Marx's chapters 13, 14 and 15 of Volume III into simple equations, through five steps. After the elaboration of this model we run a simulation test and analyse the results to test its ability to replicate the real long-term dynamics of capitalist economies.

*5.1 Five steps for a system of equations*

*Step 1 – Factors that pull the rate of profit down*

The first step is the translation into an equation of factors that pull the profit rate down—a formalization of Marx's insights in MEGA II.4.2 (p. 285-301), which Engels edited as chapter 13 of *Das Kapital* Volume III.

The basic factor for Marx regards the consequences of the growing organic composition of capital on the profit rate. Over time, investment in constant capital grows more intensively than that in variable capital, and so—supposing a constant surplus rate—the rate of profit tends to fall. The reason that pushes the organic composition of capital upwards is largely derived from the technological dynamic of the system, as capitalists search for extra-surplus or surplus profit 'for those producing under the best conditions in any particular sphere of production' (Marx 1894, p. 300). As an unintended effect, what for an individual capitalist is a way to get surplus profits, for the whole capitalist class pulls the rate of profit down. This long-term process of substituting machines for labour—mechanisation and/or automation— is one of the most important 'natural trajectories' indicated by neo-Schumpeterian research at the end of the 20th Century (Klevorick *et al.* 1995, p. 200).

Other factors may pull the rate of profit down in Marx's scheme. One is the decrease of the rate of exploitation, as organised labour and democratic institutions may reduce the

surplus rate through better wages, better working conditions and a smaller working day. Other factors might be taxes paid by firms as deductions from their profits.

Those factors may also be illustrated by earlier elaborations. Adam Smith (1776, I. 9.1), for example, expounded on the effects that capital accumulation has on competition and the consequent pressure for lower prices, which would affect profits. Authors writing after Marx are also relevant, such as the Schumpeterian view of intercapitalist competition through imitation of earlier innovations. According to this view, it would erode the temporary monopoly of innovation, on which profit is grounded.

Thus, how can we write this first step?

Formally:

$$D(t) = \alpha_D * \xi$$

where $D(t)$ is the intensity of the factors that pull the profit rate down at time ($t$) (the tendency of the rate of profit to fall); $\xi$ is a random number uniformly distributed between 0 and 1 (generated as many times as used in the equations—i.e., the several terms are not equal). The term $\alpha_D * \xi$ varies between 0 and $\alpha_D$. That is, $\alpha_D$ is their maximum. This term ($\alpha_D * \xi$) does not explicitly depend upon either the tendency or the countertendency for the rate of profit to fall. They are just random terms resulting from all the actions of the system's components and their interactions. Those interactions might result in a tendency for the profit rate to fall.


*Step 2 – Factors that push the rate of profit up*

The second step is the translation into an equation of factors that push the profit rate up - a formalization of Marx's insights in MEGA II.4.2 (p. 301-309), which Engels edited as chapter 14 of *Das Kapital* Volume III.

Marx's list of counteracting factors involves the six topics presented earlier in section two: 'more intense exploitation of labour', 'reductions of wages below their value', 'cheapening of elements of constant capital', 'the relative surplus population', 'foreign trade' and 'share capital'. Two of those counteracting factors are directly related to technological changes: first, the cheapening of elements of constant capital, centrally caused by productivity hikes in capital goods-producing sectors; and, second, the creation of 'new branches of production', which would begin with a lower organic composition of capital and 'only gradually pass through the same trajectory as other branches' (1894, p. 344). This trajectory of new sectors

might be found in the literature on industry life cycles (Klepper 1996), which investigated sectors such as automobiles in the early 20th Century.

Previously, Marx (1857-1858, p. 751) mentioned the contribution of 'lowering of taxes', 'reduction in the ground rent'.

Earlier than Marx, the classical economists had pointed to movements that could provide escape from the fall of the rate of profit: Adam Smith pointed to the escape towards new branches of business and new regions (1776, I.9.12), Ricardo indicated the role of machinery (1821, 6.29), and Mill (1848, IV.4.19 and IV.4.25) the role of improvements in production and emigration of capital. Schumpeter (1911) is an illustration of post-Marx authors. He pointed to a key variable that pushes the rate of profit up: innovation, with his typology of five forms of innovation.

As discussed above, the history of capitalism has shown a capacity to create new countertendencies that renew the system and provide flexibility in the long-term. Poulantzas (1978) indicates that the modern state's function as a countertendency was one of its main traits, and this may be an illustration of the broad changes throughout the 20th Century.

Formally:[14]

$$I(t) = \alpha_I * \xi$$

where $I(t)$ is the intensity of the factors that push the rate of profit up, at that same moment $t$ (the countertendency); $\xi$ is a random number uniformly distributed between 0 and 1 (generated as many times as used in the equations—i.e., the several terms are not equal). The term $\alpha_I * \xi$ varies between 0 and $\alpha_{D,I}$. That is, $\alpha_I$ is their maximum. This term ($\alpha_I * \xi$) does not explicitly depend upon either the tendency or the countertendency for the rate of profit to fall. They are just random terms resulting from all the actions of the system's components and their interactions. Those interactions might result in a tendency for the profit rate to fall.

*Step 3 – Simultaneous and contradictory operation*

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[14] This model will simulate the impact of factors that lead to an increase or a decrease in the rate of profit upon the tendency or countertendencies discussed above. Those factors may affect the tendency or counter-tendencies in different ways. Those impacts may even be non-deterministic, or dependent on other exogenous factors. Thereby, we are simulating their effects in a stochastic way. So the alpha term ($\alpha$) is the maximum intensity of the increase or decrease in those factors, and the multiplicative stochastic epson term ($\xi$) makes the intensity vary randomly between zero and its maximum.

The third step is the translation into two equations of Marx's understanding that the factors that pull the rate of profit down and the factors that push the rate of profit up 'cannot just be considered as existing quietly side by side, which is how Ricardo treats them'. This third step translates the simultaneous and contradictory operation of those two movements—a formalisation of Marx's insights in the *Manuscripts 1863-1865* (MEGA II.4.2, p. 309-340), which were edited by Engels as chapter 15 of *Das Kapital* Volume III.

Here is the most original contribution of Marx to this subject. Marx suggests and develops a way to integrate, in a contradictory way, the behaviour of the tendencies and the countertendencies for the rate of profit to fall: there would be a specific mechanism that triggers an increase in the upward trend of profit rate when there is an increase in the forces that pull the rate of profit down. Conversely, when the forces that push the rate of profit up increase, this mechanism triggers counteracting forces that reduce the rate of profit.

This mechanism is described and illustrated by Marx in five paragraphs (MEGA II.4.2, p. 323), which Engels did not re-write (see Marx, 1894, p. 357).[15] After criticizing Ricardo, he argues that 'the contending agencies function simultaneously in opposition to one another', and then presents three ways in which those contending agencies function 'simultaneously' (*Gleichzeitig mit*...). He concludes that they may 'exhibit themselves' sometimes 'side by side, spatially, at other times one after another, temporally'. This simultaneity is explicitly highlighted, as an articulation between the tendency and countertendency: 'Simultaneously with the fall in the profit rate, the mass of capital grows, and this is associated with a devaluation of the existing capital, which puts a stop to this fall and gives an accelerating impulse to the accumulation of capital value' (Marx 1894, p. 357; MEGA II.4.2, lines 15-18, p. 323).[16]

To translate this simultaneous and contradictory operation of tendency and countertendencies, we need terms that integrate the equations presented in steps 1 and 2.

Formally:

$\beta_D * I(t-1) * \xi$ and $\beta_I * D(t-1) * \xi$

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[15] Engels only included Marx's second paragraph (MEGA II.4.2, line 10, p. 323) in the first of those five paragraphs in the English edition (1894, p. 357). Therefore, there were 6 paragraphs in Marx's manuscripts and 5 paragraphs in Engels's edition.

[16] Once again, the same approach would reappear in the 1875 manuscript (MEGA II.14).

The adverb (simultaneously) is translated to the system of equations including those two terms, ($\beta_D*I(t-1)*\xi$) in equation 1 and its analogue in equation 2. Therefore we have completed our two first equations:

(1) $D(t) = \alpha_D * \xi + \beta_D * I(t-1) * \xi$;

(2) $I(t) = \alpha_I * \xi + \beta_I * D(t-1) * \xi$;

The term ($\beta_D*I(t-1)*\xi$) in equation 1 and its analogue in equation 2 integrate, through a crossing, the behaviour of the tendencies and the countertendencies for the rate of profit to fall. This crossing means that an increase in the forces that pull the rate of profit down triggers an increase in the upward trend of profit rate. Conversely, when the forces that push the rate of profit up increase, this triggers counteracting forces that reduce the rate of profit. This crossover behaviour, called 'coupling' in Physics, does not allow the collapse of the system (the rate of profit going to zero or infinity).

*Step 4 – Movements of the rate of profit (RoP)*

Finally, we need an equation that expresses the change in the profit rate ($RoP(t)$ - $RoP(t-1)$) as the difference between the intensity of the factors that push the rate of profit up ($I(t)$) and the counteracting factors that pull it down ($D(t)$).

Formally:

$RoP(t)$ - $RoP(t-1) = I(t)$ - $D(t)$

*Step 5 – The system of equations*

Now we have our three-equation system, which summarises Marx's insights on the behaviour of the rate of profit and that can feed a simulation model.

Formally:

(1) $D(t) = \alpha_D * \xi + \beta_D * I(t-1) * \xi$;

(2) $I(t) = \alpha_I * \xi + \beta_I * D(t-1) * \xi$;

(3) $RoP(t)$ - $RoP(t-1) = I(t)$ - $D(t)$

*4.2 Towards a non-linear world*

We have highlighted in our literature review and in Step 3 above that Marx's specific contribution was the simultaneous and contradictory operation of the tendencies and the countertendencies of the rate of profit to fall. This insight of the simultaneous operation of tendencies and countertendencies was translated by the terms suggested in Step 3: the terms that make the 'coupling' of the two equations. This 'coupling' is what transforms this system of equations into a non-linear system, it is this 'coupling' that generates non-linear behaviour.

Without this 'coupling', that is, without the insights from *Das Kapital*'s chapter 15, the equations would be only those in Steps 1 and 2. They would lead to the sort of behaviour that classical economists such as Ricardo predicted: a falling rate of profit, comprising a system for which gravitation analogies are sufficient. With the simultaneous and contradictory operation of tendencies and countertendencies, with the 'coupling' suggested in Step 3, this insight from Marx leads us to the world of complexity.

*4.3 Results from a simulation*

The system of equations shown in Step 5 summarises the model derived from Marx's insights. Figure 4 shows the profile of the movements in the rate of profit generated by a simulation of this model, starting at t=0 with a rate of profit equal to 0.7, parameters $\alpha D = \alpha I = \beta D = \beta I = 0.2$ and running 300,000 iterations. We also run simulations varying each parameter ($\alpha_D$, $\alpha_I$, $\beta_D$ and $\beta_I$) from 0.01 to 1.00, with 0.01 steps. The model does not collapse, indicating its robustness. This is centrally due to the coupling between the tendency of the profit rate to fall and its countertendencies.
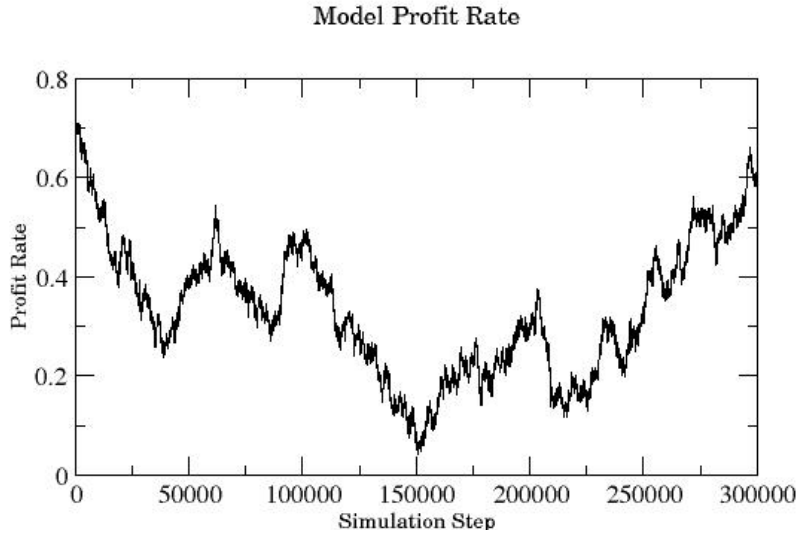
Model Profit Rate



Figure 4. Model 1: Movements of the rate of profit in a simulation with 300,000 iterations

Source: authors' elaboration

The profile of the movements of the profit rate once again look like a fractal, with widely varying peaks replicated in different scales of the graph. As with the profit rate of the US (Figure 1), we also apply a FT to analyse the data our simulation generated (Figure 4). The FT results are presented in Figure 5.[17]

The power law regression ($y=5.3462e-05*x^{-1.0003}$) presents an exponent near -1 (-1.0003), similar to the case of our empirical test with data for the USA (Figure 2). The exponent of the power law, frequently called the signature of the relation, is a particularly important result—roughly speaking, it describes the temporal structure of the phenomenon. It identifies, in mathematical terms, the relationship between the frequency of the signals that comprise the result and their intensity. In this case, the intensity is inversely related to the frequency, meaning short-term cycles have a smaller impact on the profit rate. Furthermore, the exponent close to -1 is characteristic of processes with fractal characteristics, a feature observed in the data for the US.

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[17] Although Figure 1 presents data for 150 years of history of US capitalism and Figure 4 presents a simulation with 300,000 iterations, the self-similarity as a property of fractals is what allows us to compare those two results. Remember that, as written in section II, fractals and their statistics exhibit scaling—their behavior is the same in any scale of measurement. This is a property called self-similarity. This property defines that a 'wide range of scales' can be 'decomposed in self-similar' segments—in Mandelbrot's example there is a reference to 'a wide range of scales turbulence... decomposable into self-similar eddies' (Mandelbrot, 1977, pp. 18-19)

The importance of this power law with -1 exponent is that correspond to the $1/f$ behaviour discussed in section (IV.1). The universality of this result suggests that it does not arise as consequence of a particular interaction, but it is a characteristic signature of complexity and self-organisation. Therefore, it indicates, empirically, that capital is at a self-organised state, without inbuilt tendencies to collapse or converge to a fixed profit rate. In other words, this result confirms that the profile of the profit rate movements in our simulation is also a fractal, indicating a complex system behaviour.
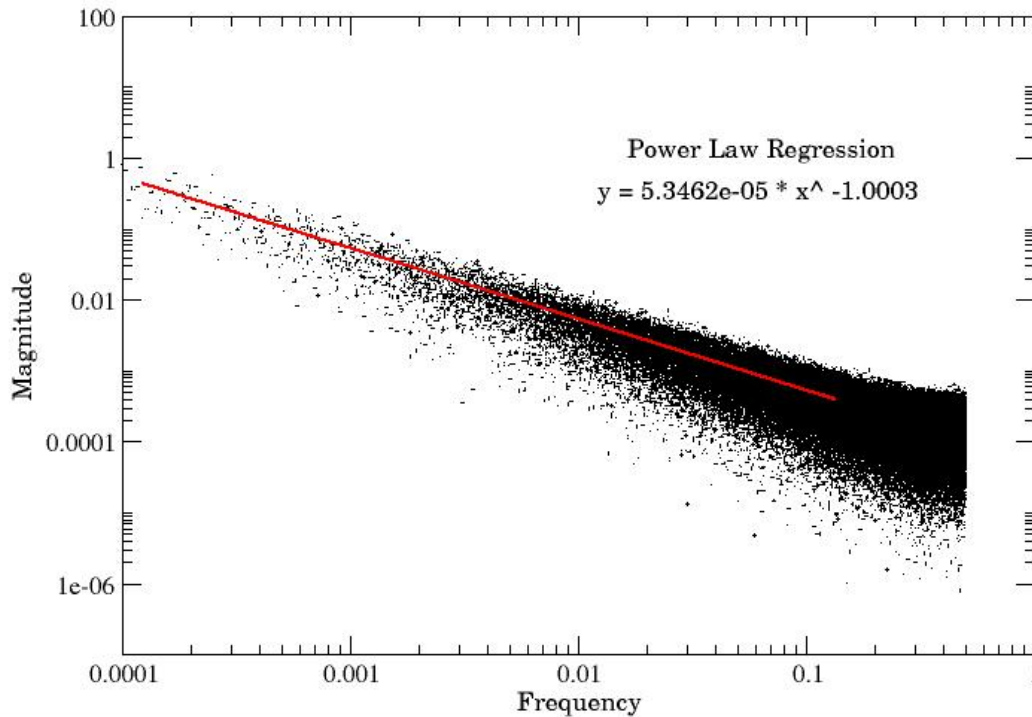


Figure 5. Model 1 data: Magnitude and frequency of the FT decomposition

Source: Authors' elaboration

Figures 4, 5 and our interpretation of their results suggest that the simulation model proposed in this section replicates key features of the data presented above for the US.

Namely, the long-run dynamics of the profit rate display features related to complexity, self-organisation and out-of-equilibrium behaviour. Capital is fractal.

## 6. Preliminary conclusions

An empirical analysis of data for the US between 1869 and 2011 has uncovered clues (physically speaking: signatures) of an out-of-equilibrium, complex system with self-organising properties. Those clues (signatures) were revealed by applying a Fourier transform and identifying that the behaviour of the profit rate in the US can be described by a 'power law'.

These findings underpin the suggestion of a three-equation simulation model based on two very simple rules, inspired by an interpretation of Marx's insights. The FT analysis of the results of this simulation model also showed a 'power law' behaviour. Importantly, both 'power law regressions' (Figures 1 and 4) delivered very similar exponents, near -1, which suggests that the simulation model replicates the real long-run dynamics and the temporal structure of the profit rate in the US. This should not be taken in the sense of the model being able to predict the evolution of the profit rate, or that it is a precisely calibrated model for such rate in the US. The result rather indicates that this approach to modelling captures essential characteristics of the system, and thus some of its fundamental dynamic properties—namely, out-of-equilibrium behaviour, complexity, self-organisation and fractal properties.

Those results stimulate two broader reflections. First, it supports this article's conjecture that Marx's insights—about the behaviour of the profit rate as a result of simultaneous and contradictory interactions between a tendency and a countertendency to its fall—can help understand the long-term dynamics of capitalism. The latter manuscripts about rate of profit and rate of surplus-value (MEGA II.14) and the manuscript of 1863-1865 (MEGA II.4.2) differ in one important aspect: in 1875, before stating the 'law', Marx had to describe the complete forces acting on the profit rate, that is to say, tendencies and countertendencies. For that purpose, although he had the theory and a lot of data, the 'method of exposition' was not ready. He could not simply repeat what was stated and maintained during the many editions of volume one. On the other hand, he cared little about deadlines or the mere publishing of

his ideas, leaving for posterity the difficult task of developing his theory on this particular subject.

Second, this real capitalist dynamics shows fractal properties. It is revealing that an investigation of such a synthetic variable (rate of profit) may unveil features related to complexity, self-organisation and out-of-equilibrium behaviour; in short, an identification of capitalist economies as complex systems.

An agenda for further research might have at least five related lines:

(i) a discussion of the theoretical implications of empirical findings of this article (the nature of Marx's insights, the need for a dialogue with other approaches, a critical evaluation of long waves of capitalist development, the role of crises and their aftermath in reshaping capitalism);

(ii) a search for other databases and an effort to systematize the available data, including other key countries in the empirical analysis of the profit rate;

(iii) it seems that the Fourier transform is a technique useful to the investigation on long waves of capitalist development, therefore data may be organised and analysed using this technique;

(iv) a fine-tuning of the suggested simulation model, to include other factors in the contradictory interaction between the tendencies and countertendencies of the rate of profit to fall, advancing, when feasible, towards a more global simulation model; and

(v) an investigation using available data to track the ups and downs of profits before and after the last crisis (2007-2008), with a focus on inter-sectoral and inter-firm differentiation—an investigation of the turbulent underpinnings of the average rate of profit.

Today we know that Marx's elaboration on the rate of profit is incomplete and unfinished. However, this article suggests that there are new reasons to reconsider those discussions on the rate of profit. The movements of the rate of profit summarise key issues of capitalist economies. The use of modern tools from non-linear mathematics and the physics of complexity show that Marx's insights were ahead of his time. With the advantage of MEGA II and new knowledge about Marx, after 150 years of the history of capitalism and data about its behaviour, and with the tools of complexity, we may return to Marx's elaboration on the rate of profit and see how insightful they were and still are.

**Acknowledgements**

**Disclosure statement**

No potential conflict of interest was reported by the authors.

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