UNIVERSITY OF CAMBRIDGE
Faculty of Education

ISSN 2043-8338

Journal of Trainee Teacher Education Research

A critical analysis of the use of 'Sumaze!' within a secondary school mathematics classroom

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Abstract

This case study examines the usefulness of the Sumaze! game for teaching mathematics in secondary school. The game was recently released by Sigma jointly with MEI in December 2015. The study consisted of two lessons which were taught to a year 9 class as part of their usual mathematics lessons. The first was taught in exam conditions and students were given questions to answer on paper. For the second lesson students had the choice of working in small groups and had no questions to answer on paper. Selected students were then interviewed and their knowledge of the topics that they had explored was assessed via a set of questions. Findings suggest that students found the game engaging and were interested in playing the game again. However, there was little evidence to suggest that they learnt mathematics from it in either of the formats that it was presented to them.

https://doi.org/10.17863/CAM.86185

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Introduction

In recent times, numerous apps have been released which claim to be useful for the learning, and occasionally teaching, of mathematics. Many claim to aid in developing a very different kind of mathematics education in which well-designed digital technologies use the strengths and curiosities of students to fuel sophisticated mathematical thinking. There is nothing new about such claims in general, however many apps now make use of one's sense of touch which appears to be a much more direct interaction with the mathematics itself. One such app of this kind is entitled 'Sumaze!' (see below).

In this study I critically examined Sumaze!, with the aim of exploring it's potential use within a modern-day mathematics classroom. I did so by planning and teaching a pair of lessons, which were centred on Sumaze!, to a mixed ability group of year 9 students in a UK secondary school as part of their usual mathematics lessons. I taught the lessons as a trainee teacher during my second professional placement for my PGCE.

The first lesson was taught in strict exam conditions and students were given questions to answer on paper, for which they knew they would be scored. This was done as a means of assessing student attainment. For the second lesson students had the choice of working in small groups and had no questions to answer on paper. In each lesson, students were given a questionnaire to complete and student progress at different points of time was recorded by my mentor. Correlation between student progress after 30 minutes and test scores from the first lesson was then calculated. Six students, two high prior attaining, two middle prior attaining and two low prior attaining students, were then interviewed. Their knowledge of the topics that they had explored via Sumaze! was then assessed via a set of questions. Findings suggest that students found the game engaging and were interested in playing the game again. However, there was little evidence to suggest that they learnt mathematics from it in either of the formats that it was presented to them.

In the following sections I explain what Sumaze! is, how it works and possible research questions, before moving into a review of the literature, after which my research questions are reduced and refined. I then clarify the methodology used and give explanations for all decisions that were made as well as outlining ethical considerations before presenting my findings. I will then discuss my findings and conclude by answering my research questions, and making considerations for future research.

Sumaze

What is Sumaze!?

Sumaze! is a free mathematics based puzzle app which was recently released (December 2015) by the body 'Mathematics in Education and Industry' (MEI) who are a UK based curriculum development organisation, jointly with Sigma, a network which aims to support students in their understanding of mathematics and statistics. The game can be downloaded for free in the App store for iPhone and iPad and in the Google play store for Android, it is also available for free online play. The developers themselves claim Sumaze! to be "a place where mathematics is learnt, problem-solving skills are developed and fun is had" (MEI, 2016).

How does Sumaze work?

The game comprises of one hundred levels called "rooms" split into seven sections, each with fourteen to sixteen rooms, which are entitled: "Arithmetic"; "Negatives"; "Powers"; "Inequalities & Modulus"; "Logarithms"; "Numbers" and "Fermat's Rooms". The online version splits "Inequalities & Modulus" into two separate sections and has a section entitled "Complex numbers", resulting in nine sections altogether. In this study I use the words 'room' and 'level' interchangeably.

The player operates a single block and the aim in each room is to navigate the room and the various mathematical operations to reach the "Goal" (see Figure 1). The only player controls are up, down, left and right which a player can operate using a keyboard or by swiping in the desired direction on their phone. Each level can consist of any combination of red, blue and green blocks which the player's block can pass through, all which function differently. Red blocks perform operations such as “add 2”, “subtract 1”, “multiply by 3” etc and can only be used once before they disappear. Blue blocks perform similar operations but can be used repeatedly and never disappear. Green blocks essentially act as barriers; they only allow the player’s block to pass through when it matches the condition set on the green block. For example, if a green block displays “= 6” and the player’s block has a value of five, then the player’s block will not be able to pass through.

The game does not allow the player to perform moves which would cause the player’s block to become outside of the range of integers from -999 to 1000.

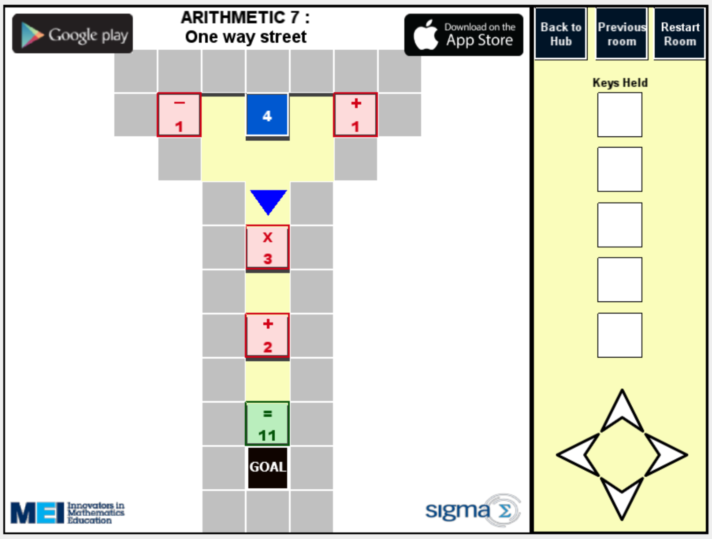

Figure 1: In this room the player’s block starts with a value of 4 and must become 11 to pass through to the goal

Game interface

There is some difference in interface between the phone app versions and the online version. On the phone app a player must complete sections in the order stated above. The next section does not become available for play until the current section is completed, whereas on the online version a player can work through the nine sections in any order they wish. Both the online version and the phone version feature buttons to restart the level and to return to the main menu where all sections are listed. The online version also has a button to return to the previous room.

In each room the title of the room is written at the top of the screen, e.g. “Powers 1”, and beneath this there is some text which is usually either a pun (“log jam” in logarithms 10), information which is useful for solving the puzzle ("32 = 2 x 2 x 2 x 2 x 2" in powers 1), or some indication of the mathematical topic to which the puzzle relates ("binary finery" in powers 10).

Unlike many traditional games, but similar to modern games, Sumaze! does not feature any initial instructions. Instead textual information appears on either side of each puzzle as the player progresses through the game and the first few levels act as instructions.

Questions

My mentor and I considered the potential of using Sumaze! within a classroom. Through conversation we devised the following seven questions, which were then condensed to three research questions following the literature review.

1. Can Sumaze! give students an initial insight into mathematical ideas that they have not yet come across and therefore assist them in developing a conceptual understanding of the mathematics behind the sections they play?
2. Can students develop fluency in several mathematical topics via the sections of Sumaze! which relate to mathematics that they have previously come across?
3. Can playing the game, and persevering through levels, help develop students' problem solving skills?
4. Can Sumaze! be integrated with ease into the school curriculum?
5. Will students find the game to be engaging and want to play the game outside of lessons?
6. Will there be any technical issues when using the game within a classroom? How can these be avoided?
7. What is the best way to present the game within a lesson and organise the classroom?

Literature review

In this section I review recent literature relating to technology and embodiment in mathematics education. The referenced literature is almost exclusively taken from academic papers which have been published in peer-reviewed education journals, with the exception of two books by notable authors in the field. The literature is not exclusively based on British research and this arguably provides some contextual limitation to my discussion. These potential limitations arise because there is a perceived shortcoming of using research which was not conducted in the UK and attempting to apply corollaries of the conclusions to schools and students in the UK. Of course, in the social situation of a classroom there could be great risk in extrapolating internationally. However, in essence all of the research I have used concerns itself with individuals independently engaging in digital technology without reference to the social context in which a learner finds themselves, hence I believe I have largely avoided the potential hazards of using international research.

By examining the literature, it is apparent that over the past few decades there has been a heightened interest in how to theorise the learning of mathematics where digital technologies are involved. In the previous century researchers were interested in the results of comparative studies, whereas in recent times, as Sinclair writes, the emphasis has been on “trying to understand how digital technologies themselves are implicated in learning and what they change about both the mathematics and how it is learned.” (Sinclair, 2014, p.171).

Within the literature there is a strong sense that new digital technologies, in contrast with the norm of paper-and-pencil technology, offer new opportunities to break away from the symbolic, language-based forms of mathematical expression and communication. Sinclair, a Canadian mathematics education researcher focusing on embodied cognition, argues that the personalisation, and to some extent anthropomorphisation, these technologies offer is essential to the learning potential they possess, she suggests that “in these environments driven by the hand or body, the human is constantly reinscribing herself into the idealized, abstract mathematics” (Sinclair, 2014, p.168). This seems an overly idealised notion but the view is supported by Lakoff , a cognitive linguist, and Núñez, a psychologist, who argue in their embodied cognition manifesto that all aspects of cognition, including high level mathematical thinking, are shaped by our everyday sensorimotor experiences (Lakoff & Núñez, 2000). Of course these arguments lend themselves to the common Frege (German Mathematical Philosopher) type criticisms. Firstly, they fail to make sensible adjustments for dealing with large numbers. It can be argued that one understands that 1 + 1 is 2 by seeing two individual objects placed together, however one can understand that 99 + 1 is 100 without seeing 99 objects placed next to another single object. Next, they make no mention of how one can come to cope with abstractions which have no physical interpretation. An individual cannot have sensorimotor experiences with abstractions such as the infinite or empty set. Moreover, mathematical ideas such as the 'unit' have a significantly greater depth than that which can be interpreted physically. However, despite the overarching faults these arguments contain, the substantive idea of embodiment appears useful for accounting for the learning potential of games such as Sumaze!.

Theorising embodiment

The literature sheds light on the difficulties faced when attempting to theorise embodiment. The conflicting views of mathematical subjectivity set Kant (German Philosopher) inspired theories against Hume (Scottish Philosopher) inspired theories. Theories inspired by the Kantian view argue that "cognitive faculties synthesise sense perception" (de Freitas & Sinclair, 2011, p.135), whereas Humean inspired theories argue that "conceptual categories are constituted through perceptual routine habits and material interactions" (ibid.). Indeed, the Humean tradition is an empirical one, whereas Kantian views assume that our experiences of the world are "structured through internal categories" (ibid.). Although the Kantian view seems more legitimate for reasons given above, researchers of mathematics education largely position themselves within the Humean empiricist tradition. Hence education researchers almost always approach the question of subjectivity by closely examining specific concrete human experiences. This convention is open to great criticism as it appears to be the result of the ease with which empirical evidence can be gathered rather than any well-posed argument.

Within this empiricist tradition, 'thinking' and any other related cognitive constructs are always external. Indeed, Nemirovsky, a British Education researcher focusing on embodied cognition, and Ferrara, an Italian Mathematics Education researcher focusing on technology and movement, argue that "thinking is not a process that takes place 'behind' or 'underneath' bodily activity, but is the body itself" (Nemirovsky & Ferrara, 2009, p.159). However, this theory of subjectivity assumes an intellectualist mind and has the burden of answering the question of how internal mental representations can refer to, or relate to, anything that is not a mental representation. Indeed, the view of Nemirovsky and Ferrara is in contention with the view of Roth who offers alternative views on subjectivity by arguing that touch is prior to intention and subjective mental representations. Roth, a Canadian researcher with broad interest, states that "in Kant's constructivist approach, the knowing subject and the object known are but two abstractions, and a real positive connection between the two does not exist. The separation between inside and outside, the mind and the body, is inherent in the intellectualist approach whatever the particular brand" (Roth, 2010, p.9). Roth uses the example of a student playing with a cube, whereby the movement of the student's hands emerge without intention or any obvious governing principles. Convincingly, Roth uses this example to argue that it is within the hand that memories of the prior encounters with cubes are immanent, going on to imply that the world emerges through touch and the coordination of movement between the eyes and hands. It is true that the movement of the hands is an embodied activity that is prior to all verbal framing, however my major concern with this theory of embodiment is that it attempts to *locate* knowledge within the individual's body, thus not sufficiently addressing the entirety of the collective social body. The early work of French philosopher Deleuze and Guattari (French reaffirm my concerns by asserting the social body to be connected and constituted through a rhizomatic lattice of material and social interaction (Deleuze & Guattari, 1987), whereby a rhizome has no centre or primary root, but instead grows and disperses itself through multiple entry and exit points. However, despite my concerns of Roth's argument I struggle to find fault when he asserts that "the next time the movement is executed, the renewed effort will be less, and the motor that has enacted the movement cannot but recognize the difference as its own will" (Roth, 2010, p.13). It is certainly true that muscle memory can make physical tasks easier to carry out when repeated over time, but whether this is true when digital technology is concerned is yet to be determined.

Instrumental genesis and situated abstraction

One of the major issues that early researchers in this field faced revolved around the fact that before students could utilise digital technology for themselves, they would have to initially learn how to use the tool or play the game itself. Some even went as far as claiming that "learning to use such a digital technology isn't just about learning to use the tool, but also about learning mathematics as well" (Sinclair, 2014, p.171). In the context of games such as Sumaze! this assertion seems utopian and limited at best because the straight forward interface of some games ensures that learning to play the game requires very little thought. Nonetheless, this led to the development of the theory of instrumental genesis which asserted that all tools had either pragmatic or epistemic roles (Trouche & Drijvers, 2014). This binary classification seems an over simplified model since there are many examples of software tools, for example the angle bisector tool in Geogebra, which give good practical representations of mathematical ideas whilst also revealing good theoretical ideas about the mathematics to which they relate.

As a result of the setbacks in the theory outlined above, the previously suggested theory of situated abstraction was revived and bought back into the foreground of research. The theory of situated abstraction, as given by Hoyles, and Noss (Professors of Mathematics Education at the institute of education in London), attempted to investigate how the mathematical ideas learned through technology differ from the standard mathematical understanding developed with traditional tools. Hoyles and Noss claimed the strength of situated abstraction was that it "recognises and legitimates mathematical expression even when it is remote from (or not represented by) standard mathematics" (Hoyles & Noss, 2008, p.92). Sinclair is a proponent of situated abstraction and makes clear the benefits it brought to our understanding of how students learn using technology, she explains that "It built on the cognitivist idea of scaffolding and abstraction, recognizing the situatedness of learning, while also trying to account for the problematic issue of 'transfer' from the computer-based situation to the official mathematics" (Sinclair, 2014, p.171). For me the main strength of this theory comes from the subtlety in the notion of abstraction. Far from the Aristotelian (Aristotle – Ancient Greek Philosopher) sense of abstraction, which involves a departure from context, situated abstraction regards a sense of abstraction *within,* which focuses on "how learners construct mathematical ideas by drawing on the webbing of the particular setting which, in turn, shapes the way the ideas are expressed" (Hoyles & Noss, 2008, p.122). Putting this into the context of technology in the mathematics classroom, in the absence of recognisable scaffolding, some well-designed digital technologies can transcend the need for support.

In a chapter based on the use of theoretical perspectives in digital technology based research, Drijvers et al. (2009) suggest that there are two distinct causal classifications in digital technology based research. For them approaches could either be strictly in the direction of 'mathematics-to-technology' or 'technology-to-mathematics'. Hence they argued that since situated abstraction "begins with existing frames within mathematics education research and adapts them to digital technology environments" (p.94) it is distinctively a 'mathematics-to-technology' approach. They contrast this with the 'technology-to-mathematics' approach for which "constructs are conceptualised specifically for use in certain digital environments and then adapted to the particular mathematical situation" (p.94). Personally I think this distinction fails to take into account the inextricability of mathematics and technology as driving forces of one another. Sinclair also disagrees with the distinction and argues that the view arises from a lack of comprehension of the vast diversity that forms of technology can take, indeed she gives the example of the "frequently unnoticed paper, pencil and blackboard" (Sinclair, 2014, p.171).

School mathematics

Throughout the literature it is expressed that one of the main problems faced when using digital technology in mathematics lessons, is the divide between the discourse of school mathematics and technology-based mathematics. For example, Hoyles directly emphasises this discourse when discussing the potential issues of implementing the Logo software in schools (Hoyles, 1993). However, rather than regard this as a sign that digital technology had no place in the classroom, researchers such as Sinclair identified the need to "examine the passage, bridge, or gulf" (Sinclair, 2014, p.171) between school mathematics and digital technology. However, researchers seem overly optimistic when they begin their research with "the hypothesis that the digital technology did do something mathematically relevant and powerful" (ibid.) and suggest that it simply requires a re-tooling of constructs such as scaffolding and abstraction. Based on my experience, I would argue that the greatest limitation involved in the scaling up of digital technology to the classroom is the pedagogical way the technology is used, specifically the means by which the teacher plans for students to interact with the technology.

Research questions

By reviewing the literature, the themes associated with games such as Sumaze! which are of interest to me have become clear. As a result, my initial questions (see Introduction) can be refined to suitable research questions as follows:

Research question 1 (RQ1): Can Sumaze! be used as a preliminary learning experience in secondary mathematics lessons to provide students with a conceptual understanding of a topic they have not yet come across?

Research question 2 (RQ2): What is the best way to present and carry out lessons involving Sumaze!?

Research question 3 (RQ3): Will students find Sumaze! interesting and will this result in students wanting to play the game in their free time?

Methodology

For this study, I carried out two lessons incorporating Sumaze! alongside my mentor (an experienced mathematics teacher) with a mixed ability year nine group which I regularly teach at my placement school.

Why did I choose a case study?

The aim of this study was to critically analyse the usefulness of Sumaze! in the mathematics classroom. Case study research is appropriate for this type of study as it 'entails the detailed and intensive analysis of a single case' (Bryman, 2008, p.52), and allows me to go into great depth to discover complex insights which may have been missed in more general research. It is this 'understanding complexity in particular contexts' (Simons, 1996, p.225) which will help me provide the detailed answers required by my research questions. In this instance the unit of analysis must be the whole class as it is the practicality for an entire classroom, rather than for an individual, which I am interested in.

Why did I choose this group?

In light of RQ1, I chose this group because I was aware that the students had not yet come into contact with the laws of indices and one of the early sections in Sumaze! is entitled 'powers', meaning that I could use the game as a preliminary learning experience.

The group comprises of eight girls and twelve boys with current attainment levels ranging from 3a to 6b (See Table 1). I chose to work with a mixed ability group so that my results are as general as possible, and, as such, reveal things about a wide spectrum of students rather than students of a particular attainment level.

Student	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
Attainment	M	M	H	L	M	M	H	H	M	M	H	H	L	L	L	M	H	M	L	L
Current level	5b	4a	5a	4a	5b	4c	6c	5b	5a	6c	6b	5a	3a	3a	4a	4a	6b	5a	4c	3a

Table 1: Students' current working levels based on national level descriptors

Highlighted students were chosen for interview. L, M or H denotes whether the student is regarded to be a low, middle or high attaining student in mathematics by the school. The national level descriptors mentioned were judged by my mentor using data from recent assessments and were referenced using 'The National Curriculum - Level descriptors for subjects' document. The letter indicates whether the teacher feels the student is close to the level above (a), close to the level below (c) or consistently within that level (b).

For RQ2 I wanted to gain some insight into the best way for a classroom teacher to present the game to students. Considering that multiple teachers often struggle to manage the behaviour of this group and keep students engaged, it seemed a perfect class with which to attempt this research. Essentially there were two ideas I wanted to explore:

1. Through which medium is best to allow students to experience the game, i.e. through a phone, tablet, laptop or computer?
2. Which classroom environment is best for students to make progress through the game? Strict silent exam conditions or group work with discussion?

Since this group was mixed, it would be of interest to see which students would carry on playing the game after the study had ended and this would aid me in providing a detailed answer for RQ3.

Why did I use the online version?

The game was originally designed for touch screen use as a downloadable phone/tablet app, however the game has now been released online for use on laptops and computers. As can be seen from the literature, using the game on a touch screen leads to an increased sense of embodiment which is argued to be useful for generating conceptual understanding**.** However, I opted to use the online version with the class for multiple reasons.

1. Firstly, the online version provides greater detail when an operation is not allowed. I considered this detailed information to be necessary to ensure that misconceptions about possible mathematical operations were not adopted by the students.
2. Secondly, the online version allows sections to be completed in any order. This was useful for me when planning the lessons because it meant I could ensure that students were only engaging in sections of the game which were useful for answering my research questions.

3. Thirdly, practical issues within the classroom such as not all students having access to a smart phone, and the use of phones in lessons being forbidden by school policy, meant it was more appropriate to use laptops and computers.

As a result, I opted to use laptops for the first lesson and utilise the school's computer suite for the second lesson.

How was the data gathered?

Table 2 below displays the ways data was gathered, the benefits and drawbacks of these methods and steps which were taken to minimise drawbacks (Cohen, Manion, & Morrison, 2003).

Method	Benefits	Drawbacks	Actions taken
Lesson observations	• Based on actions of students rather than words • Can identify occurrences in terms of environment • Participants with weak verbal skills can be observed	• Difficult to observe all students at once • Subjective interpretations of conversations • Reactive effects of students being aware they are being observed	• My mentor and I circulated the room repeatedly • My mentor and I discussed all conversations of interest after the lesson • Lesson taught as a usual lesson
Lesson tests	• Numerical data can be easily analysed and compared • Non-subjective	• Tests alone do not reveal all relationships • Students can cheat • May create anxiety	• Tests use alongside other methods to develop an argument • Test sat in exam conditions
Questionnaires	• Views of entire class taken into consideration • Results easily quantified • Can be used to measure change • Requires little time	• Respondent may be forgetful or not thinking with the full context of the situation • Subjective interpretation of answers • Inadequate to understand changes in emotion	• Students asked to take a moment to consider the lesson as a whole • Interpretations checked and agreed by two people • Followed up with semi-structured interviews
Semi-structured interviews	• Interviewer able to direct conversation to ensure all major thematic ideas are covered • Able to generate rich data where students can discuss things in their own terms	• Time consuming • Difficult to compare answers • Cause and effect cannot be inferred • Investigator effects may introduce bias	• Not all students interviewed • Students responses were not compared but rather collated • Cause and effect were not inferred from the interviews

Table 2: Benefits, drawbacks and actions taken regarding research methods

Lesson observations

I decided to carry out two lessons for two reasons. Firstly, so that students could learn how the game works by completing the arithmetic section (a topic which they are expected to have a good understanding of) in the first lesson before moving onto the powers section in the second lesson (a topic which none of them were expected to have an understanding of). Secondly, I wanted to experiment with classroom climate and whether or not this affected students' feelings towards the game.

In the first lesson students were seated in a seating plan, this made sense as part of the environment of strict exam conditions that I wished to create. During the lesson my mentor circulated the room and wrote down which level of the section students were on after 5 minutes, 15 minutes and 30 minutes. The intervals were spaced this way for two reasons. Firstly, so that any initial technology issues could be identified and remedied early, and secondly, so that students could gain increasing independence with less opportunity for teacher input as time progressed. This led to some issues in the accuracy of this data. Since each of the three circulations took approximately two minutes each to complete, it is feasible that students who were observed last in the list had more time than is stated. To mitigate this risk as best as we could, my mentor reversed the order he travelled in for the second circulation and started in the middle for the third circulation. For the second lesson students were allowed to sit wherever they wished, this made sense as part of the relaxed environment desired for the second lesson. Once all students were seated we wrote down where the students were so that we could be time effective when making observations. My mentor and I also made notes of specific comments and conversations we heard throughout both lessons.

Lesson tests

In the first lesson students were told that they must complete a test in 40 minutes (Appendix 1). For each level in the Arithmetic section there was a question, sometimes split into multiple parts, which aimed to either:

1. Probe students' understanding of how the game works (Questions 1, 2, 4, 5, 6A, 6B, 8A)
2. Encourage students to recognise the mathematics they were doing (Questions 1, 3, 7A)
3. Scaffold the students' work so that they may progress with greater ease (Questions 8B, 9A, 9B, 10, 11, 14A)

4. Sway students into interpreting their methods mathematically and take the underlying mathematics further (Questions 7B, 7C, 9C, 12A, 12B, 13A, 13B, 14B)

The worksheet came with strict instruction to not progress through a level without answering the question and commanded students to write down their working, including any ideas they had. Students were also told that they would not gain any marks for progressing through levels without writing an answer to the question on the sheet.

Carrying out this test was essential to create the environment necessary to answer RQ2. For the second lesson there was no worksheet in order to maintain a relaxed atmosphere.

Lesson questionnaires

At the end of each lesson students were given a questionnaire to complete.

The first questionnaire asked the following questions:

1. "What did you like and dislike about playing Sumaze! today?" – This was asked to gauge the students' general feelings towards the game after using it in an exam style lesson.
2. "Did you have any difficulties getting the game to work? If so, what were the issues?" – This was asked so that I could identify any technical or interface based issues when using the game on laptops within a classroom.
3. "What do you think about using technology in maths lessons?" – This was done to ensure that the opinions of all students were taken into account when considering the overriding major thematic idea of this study.
4. "Any other comments?" – A space for students to share any relevant views they had that might not have been addressed in any of the other questions.

The second questionnaire asked the following questions:

1. "What did you like and dislike about playing Sumaze! today?" – This was done so that I could compare results from asking the question in the last lesson with the students' feelings at the end of this lesson with a calmer atmosphere.
2. "Did you have any difficulties getting the game to work today? If so, what were the issues?" – This was asked to assess whether there were more or less technical issues playing the

game on a computer as opposed to playing the game on a laptop in the last lesson. Also, to see if students maintained any difficulties understanding the game interface.

3. "Do you think this game is useful for learning? Why?" – This was asked to see whether or not students believed that their education benefited from playing the game and so that any claims could be compared with the reality of what they learnt.
4. "Would you carry on playing this game?" – This was useful for answering RQ3.
5. "Any other comments?" – Another space for students to share any relevant views they had that might not have been addressed in any of the other questions.

The data from these two questionnaires was essential as it provided information on all of the students' feelings towards the game immediately after playing it, rather than allowing a break before interviewing a select group of students.

Post lesson semi-structured interviews

After both lessons had been carried out I opted to interview three pairs of students. I selected a pair of low attaining girls (S & T), a pair of high attaining boys (G & Q) and a pair of middle attaining boys (F & P). These pupils were selected because they are representative of the unit sample, both in terms of their attainment and their behaviour in lessons. Their current working levels are displayed in table 1 above. Each interview lasted approximately fifteen minutes, was held at the school at lunchtime and was recorded.

Each interview began with some questions relating to the game in general to ensure that there would be no technical issues. Then students were given some time to play a section of the game. The low and middle attaining students were asked to play the powers section again whereas the high attaining students were asked to choose a section that they had not heard of before because they were already aware of the laws of indices from work outside of school.

Whilst the students were playing the game I observed and created questions based on levels which they had completed successfully. I then asked the students these questions, writing each students' input in a different coloured pen. After this I probed the students on their views of technology in mathematics lessons and whether or not they would carry on playing Sumaze!.

How was the data analysed?

The lesson observations were processed using Microsoft Excel and included progress that students made throughout the lesson. The lesson worksheets were marked using a mark scheme that I created (Appendix 1). Student progress after 5, 15 and 30 minutes was averaged, removing anomalous results from the calculation, and rounded to the nearest whole number. Graphs were then produced which display student progress in the Arithmetic and Powers sections. I then calculated the Pearson Product Moment Correlation Coefficient between student progress after 30 minutes and student test scores. This was done to see if there was any indication that doing well in the test correlated with doing well in the game.

For each questionnaire I recorded each student's response to each individual question as either positive, negative or neutral. I then totalled these and produced graphs to display the results of the entire class.

Ethical considerations

Before conducting this study, I ensured that all of the planned elements of the research met all of the guidelines published by the British Educational Research Association (BERA) (2011) as well as the Code of Conduct published by the Faculty of Education, University of Cambridge. However, in order to carry out this research I required the permission of all relevant parties. Hence, I discussed the study with my subject lecturer, school mentor and professional tutor, all of whom signed my completed ethics form from the faculty. All of the participating student's parents were informed that the study would take place and signed permission was obtained. Further to this, a copy of the assignment was left with the school was future reference.

The major ethical considerations I took into account are as follows:

1. Anonymity - All students and the school involved in the study have been anonymised.
2. Informed consent - Students were aware of the purpose of the lessons and permission was gained before recording interviews.
3. Working with young people - Efforts were made to ensure that my research could not cause distress or discomfort.

4. Practitioner research - I taught these lessons as the student's normal classroom teacher and only gave them honest advice as usual throughout the study.

Findings

In this section I present my findings. If something is particularly useful for answering one of the research questions then it is written in brackets at the end of the statement.

Lesson 1

The classroom atmosphere in the first lesson was largely negative. Students were extremely anxious for two reasons. Firstly, they felt as though they were being assessed and this led to comments such as: "Are these results really important?" and "You didn't tell us we had a test". Secondly, the students were apprehensive about using the laptops in a mathematics lesson because they had not done so before. Attempts to reassure the students that they were not being formally assessed by an external body were futile because of the seating plan, the presence of an answer sheet and having to work independently in silence. Comments throughout the lesson made it apparent that the most disliked element of the lesson was having to complete an answer sheet whilst playing the game. (RQ2)

Table 3 shows information about which level of the Arithmetic section each student reached at various points throughout the lesson. The data shows that 7 students completed the section in the allocated 40 minutes, 2 of which completed the section in just 22 minutes. The students which finished the section in the time given were a mixture of middle and high attaining students.

Student	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	Average:
Attainment	M	M	H	L	M	M	H	H	M	M	H	H	L	L	L	M	H	M	L	L	
5 mins	5	5	6	6	6	7	7	7	6	0	5	7	2	7	4	7	9	9	3	5	6
15 mins	9	8	9	9	9	8	11	11	8	8	10	9	7	8	8	10	12	12	8	8	9
30 mins	13	13	12	12	13	13	14	14	12	12	12	14	11	11	11	11	15	15	9	9	12
Finish (mins)					36		32	33			35	35					22	22			
Test score	6	11	14	12	14	4	13	17	11	15	13	14	5	4	8	16	15	15	5	6	11

Table 3: Students' progress in the Arithmetic section and their test scores

Note that Level 15 implies that the student has finished. A blank entry implies that the student did not finish in the allocated 40 minutes. L, M or H denotes whether the student is regarded to be a low, middle or high attaining student in mathematics by the school. Students which were selected for interview are highlighted.

The product moment correlation coefficient between students' progress after 30 minutes and their test scores is 0.5733. This moderate positive correlation implies that students which made good progress in the game generally had high test scores. However, Table 3 shows that the highest test scores were not strictly recorded by students who finished the section in the allocated 40 minutes.

Figure 2 shows the progress that students made in the Arithmetic section of the game over the course of 30 minutes.

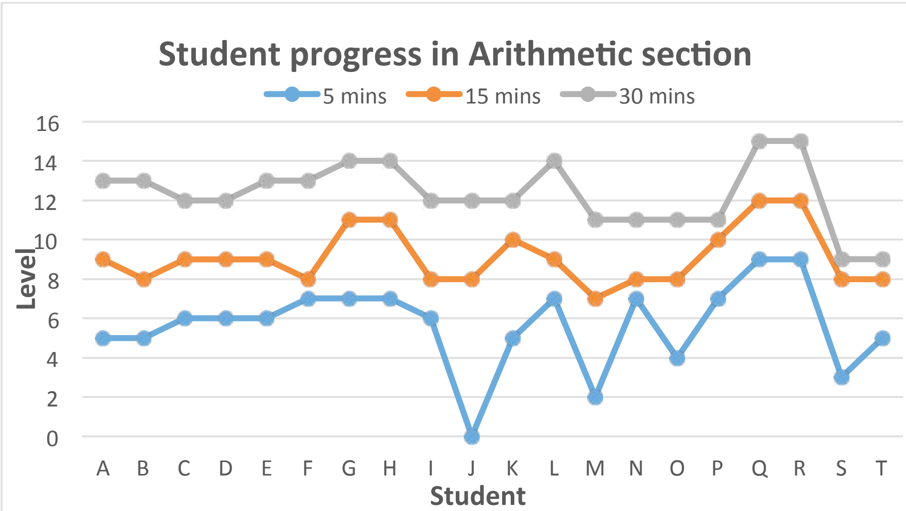

Figure 2: Students' progress in the Arithmetic section

Specific observations:

1. All students, regardless of attainment, were able to reach at least level 9 in this section after 30 minutes, on average students reached level 12 after 30 minutes.
2. The rate of progress over time decreased. After the first 5 minutes students had reached an average of level 6. In the next 10 minutes students made an average of 3 levels of progress, and in the following 15 minutes they made an average of 3 levels of progress in this longer time period. This is most likely to do with increasing difficulty rather than apathy because it

was observed that students were engaged throughout the lesson and became increasingly fixated on progressing through the levels.

3. Although student J had a technical error which lead to no progress in the first 5 minutes, they were still able to catch up and make good progress in the game overall. Based on information about the student being a middle attaining student and observations in the lesson, It is more likely that this is the result of the early levels being very straight forward rather than any trait possessed by the student.

Figure 3 shows the results of the first questionnaire (see questions listed on page 467 earlier) which was given to students at the end of the first lesson.

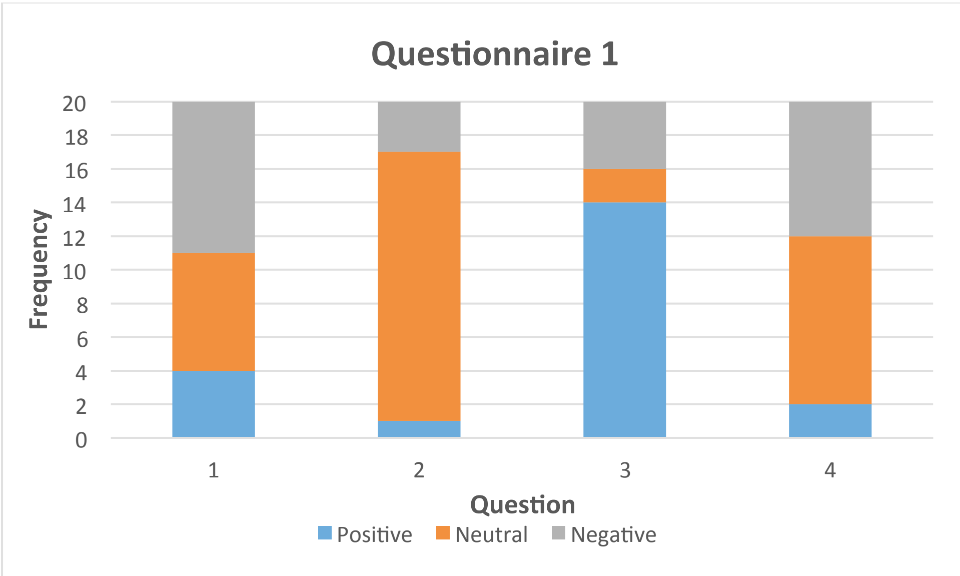

Figure 3: Students' responses to questionnaire 1

For question 1, almost all students described the game as either difficult or challenging. However, some students interpreted this difficulty positively, stating that they "enjoyed the challenge of the game", whereas other students had negative interpretations, stating that the game caused them "too much stress". This could be related to the negative atmosphere of the lesson, but it seems more likely that this is largely linked to the mind-set of the individual student and the way they perceive challenge. In fact, multiple students wrote that they disliked the game because it made them think. Many students disliked getting things wrong continuously and resultantly gave up quickly. One student even informed me that she had given up on a level because she had been working on it for "5 whole minutes!" and had made no progress. (RQ3)

The responses to Question 2 reveal that three students found the game difficult to access and use. The difficulties faced by two of these students were not related to the game design in any way, instead they were to do with internet connection and laptop failure. The third student has difficulty seeing certain colours and so could not distinguish between the different colours of the blocks. This problem was remedied by changing the colour settings on the laptop.(RQ2)

For question 3 the majority of students stated that they believed using technology was useful in maths lessons. However, the reasons given were almost exclusively either "because using technology is fun" or "it's better than having to write", rather than anything to do with understanding of mathematical topics. Nevertheless, it is clear that these students found the use of technology to be engaging. (RQ3)

Question 4 led to no comments of interest. Students simply reaffirmed what they had previously stated.

Lesson 2

This lesson had a significantly more relaxed atmosphere. Students could choose where they sat, they could work collaboratively to solve levels and, most importantly, there was no answer sheet to complete. The absence of the answer sheet seemed to be the change which students favoured most and prevented negative attitudes towards the game being formed. This led to a much more positive working environment where students appeared happier, less competitive and more engaged. (RQ2)

Table 4 shows information about which level of the Powers section each student was on at various points throughout the lesson. The data shows that just one student managed to complete the section in the given time, but almost all students had made it to level 14 after the first 30 minutes.

Student	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	Average:
Attainment	M	M	H	L	M	M	H	H	M	M	H	H	L	L	L	M	H	M	L	L	
5 mins	5	11	8	11	1	10	10	11	5	7	9	6	6	5	5	10	9	10	7	7	8
15 mins	12	14	13	13	12	13	13	14	13	12	13	12	13	12	13	13	13	13	9	8	12
30 mins	14	14	15	14	14	14	14	14	14	14	14	14	14	14	14	14	14	14	13	14	14
Finish (mins)			30																		

Table 4: Students' progress in the Powers section

As before Level 15 implies that the student has finished. A blank entry implies that the student did not finish in the allocated 40 minutes. L, M or H denotes whether the student is regarded to be a low, middle or high attaining student by the school. Students which were selected for interview are highlighted.

Figure 4 shows progress that students made in the Powers section of the game over the course of 30 minutes. The result for student E after 5 minutes is due to a technical error. Similar to student J in the previous lesson, they were still able to make good progress in the game.

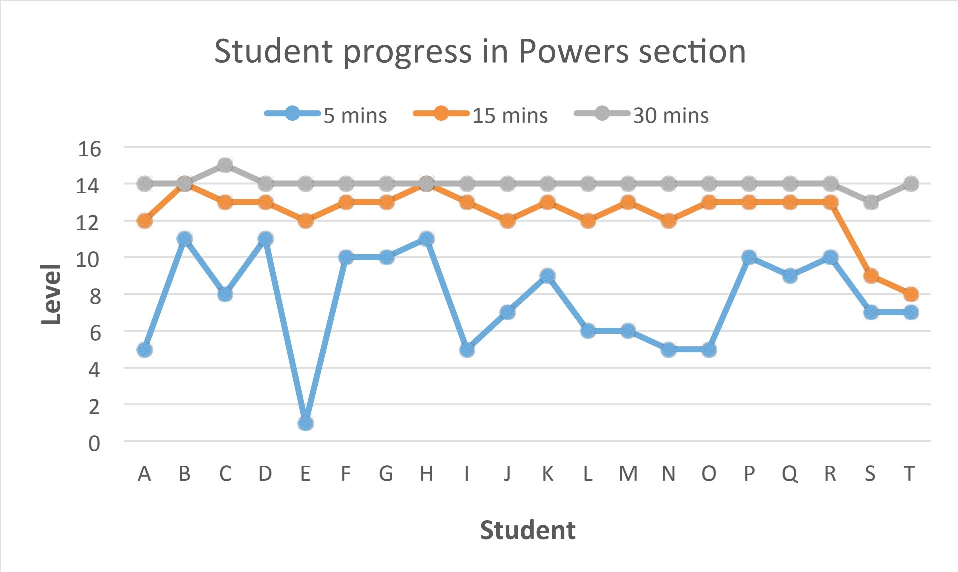

Figure 4: Students' progress in the Powers section

Specific observations:

1. All students were able to reach at least level 13 in this section after 30 minutes, on average students reached level 14 after 30 minutes, which is greater than the progress made in the Arithmetic section in the same time period, despite this section being more difficult.
2. Similar to the Arithmetic section, the rate of progress over time decreased. After the first 5 minutes students had reached an average of level 8. In the next 10 minutes students made an average of 4 levels of progress, and in the following 15 minutes they made an average of 2 levels of progress, with many being stuck on the last level.

Observations imply that the reasons why students surprisingly appeared to make greater progress in this more difficult section of the game are threefold. Firstly, the students were already used to the game and how it worked. Secondly, students were allowed to work collaboratively and this meant that once they all became stuck the entire class seemed to progress at the same rate by helping one another. Lastly, the absence of the answer sheet meant that students could focus more time on progressing through the levels and less time on writing things down.

Figure 5 shows the results of the second questionnaire (see questions listed on page 467 earlier) which was given to students at the end of the second lesson.

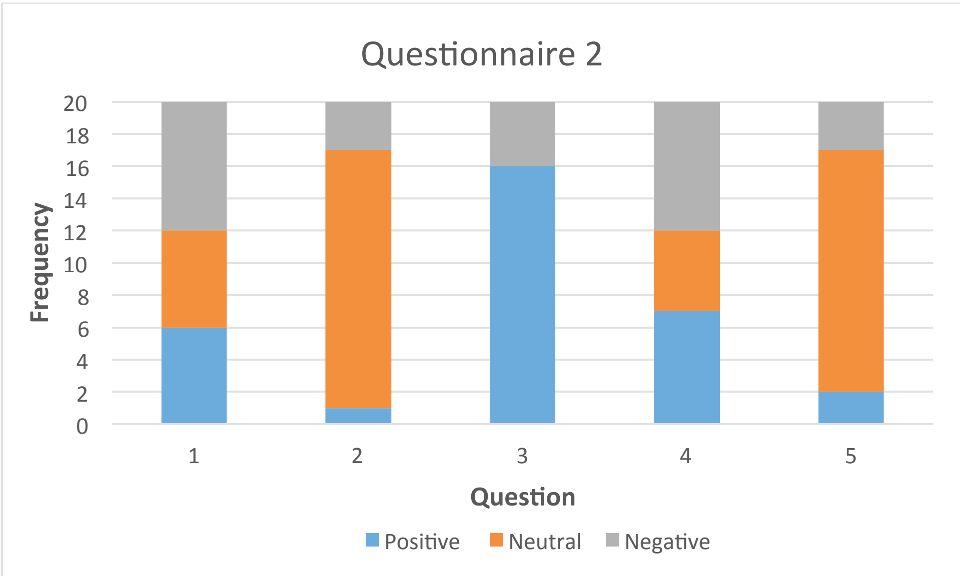

Figure 5: Students' responses to questionnaire 2

The responses for question 1 show that a greater proportion of students had a positive or neutral comment about the game after the second lesson than after the first lesson. This can be accounted for by the atmosphere of the lesson. (RQ2 & RQ3)

The responses for question 2 reaffirm that students have very few difficulties getting the game to work or using the game interface. Any problems involved the hardware they were using and not the game at all. (RQ2)

Question 3 shows that the majority of the students, 80% in fact, felt that the game is useful for learning. The negative responses comprised of students who found the game to be "frustrating" or "annoying" because it provided them with constant challenge with which they were not accustomed. (RQ3)

Question 4 shows that approximately one third of students would definitely play the game again, and one quarter said they were undecided. The students that were undecided said they would enjoy playing the game in school but would not do so in their free time. (RQ3)

Question 5 led to no significant comments being made that have not already been discussed.

Interviews

All six interviewed students were able to demonstrate their ability to access and use the game with ease. The students considered the interface to be natural and straight forward to use. Students G, Q, S and T preferred working together, whereas students F and P preferred working independently, claiming this would result in less distraction. (RQ2)

Table 5 below lists the drawbacks and benefits of Sumaze! stated by the students in interview and whether or not they would carry on playing the game. (RQ3)

Students	Sumaze! drawbacks	Sumaze! benefits	Continue playing
G & Q (High attaining)		• Avoids writing and hence quicker • Easier to remember things • Greater independence, pleasure of doing things for yourself	• Yes, for test revision • Possibly for initial learning
F & P (Middle attaining)		• Provides a good challenge • Variety of topics covered • A range of core skills covered • Greater accuracy than a teacher • Instant results, no time spent waiting for marking	• Yes, already downloaded and enjoying the game
S & T (Low attaining)	Cannot skip levels	• Able to work at own pace • Enjoyable	• Yes, already downloaded and play at home

Table 5: Students' responses to semi-structured interview questions

First interview

Students G and Q were asked to play the logarithms section. Since the mathematics was unknown to them, the students approached the game in a largely experimental way. For most levels the students simply guessed and used trial and error to find solutions. They admitted to one another that they were not aware of how things worked and many solutions were "accidents". Although the students were clearly using their problem solving skills, any hints of mathematical thinking were not specific to logarithms but instead related to the size of a solution and other mathematical operations such as squaring. The students' work largely involved spotting patterns rather than thinking things through. (RQ1)

Although the students were able to reach level 11 of this section in under 10 minutes, upon further questioning the students could not demonstrate any knowledge of logarithms. Even with numerous attempts at scaffolding and referring back to the game, they struggled with the basic definition, simple logarithm calculations and could not identify related topics. Playing the game gave the students a false sense of understanding, when discussing the benefits of the game one student remarked "I didn't even know what a logarithm was ten minutes ago, but I do now". (RQ1)

Second interview

Students F and P were asked to play the powers section, after 5 minutes they were able to reach levels 13 and 11 respectively. Whilst they played the game there was a sense of competition between them, they constantly looked at each other's screens to see who had made more progress. Upon later questioning the students were not initially able to read and solve questions such as "Compute 3^4", "simplify 2^3 x 2^5" and "simplify $(2^3)^2$". With interviewer assistance and reference to particular levels of the section, the students were able to understand and compute 3^4, but they could not make recognisable progress in the other questions. (RQ1)

Third interview

Students S and T were also asked to play the powers section. After 5 minutes they were able to reach level 5 by working together. These students were apprehensive to try things and demonstrated a fear of failure. As a result, the pair spent long periods of time not attempting things that could have been correct. Upon further questioning they were not able to answer any of the above questions regardless of any reasonable assistance given. (RQ1)

Discussion

Two related themes arose from my findings. In this section, both of these themes are related to the existing research and the implications that they might have for future research are considered.

Student interpretation

Observing students playing the game proved very insightful, for as Sinclair (2014) states, "When students express certain commands or make certain constructions or drag in certain ways, the teacher or researcher cannot help but feel as if she is learning much about how the student thinks." (p.172). As such, my findings suggest that the main issue surrounding the use of Sumaze! as a tool for education is the way in which the students approach the game. Rather than interpreting the mathematical features of the game and using their knowledge and understanding to progress, the students I observed largely played the game experimentally and viewed each operation merely as a structure, much in the same way they would for any game. This would account for why students often could not explain how certain solutions worked and why students could not link the game to any new mathematical ideas. In some sense Roth's view that touch is prior to intention and subjective mental representations is supported by my findings, specifically by the fact that students were able to memorise the move sequence required to pass a particular level, and then repeat this. This is exemplified by the faster progress students made in interview when playing the same section they had played in the lesson.

Game design

My results show that the game is entertaining and accessible for all of students involved in this study. All students thought the game was useful for education and almost none experienced difficulties with the Sumaze! interface.

The scaffolding of the game is subtle. The initial levels act as 'instructions' for the game and, alongside the textboxes, gradually reveal information which attempts to support students' understanding of the game's operations (Figure 6). Relating this to the theory of instrumental genesis, the setback of students having to learn how to use a technology before they can use it is minimised by the game's design. However, although the game uses text boxes to promote mathematical thinking, observations reveal that most students involved in this study did not read the information given. This poses the risk of the students developing misconceptions. Many of the plausible misconceptions relate to possible operations, for example when attempting $\log_2 9$ an explanatory message is displayed including the text "this would not be the case in real life" (Figure 7). This then led to a student stating that "$\log_2 0$ must be a decimal because it does not work", following their attempts to perform $\log_2 0$ which did not result in a textbox appearing. I would suggest that some text is required to explain situations of this kind. However, the issue of students not reading the text given remains. One possible solution could be to pause the game for a few seconds each time a text box arises. Although, I would predict that forcing students to pause and read information would be frustrating and reduce the entertainment value of the game, nonetheless there is scope for further research into how this issue can be overcome.

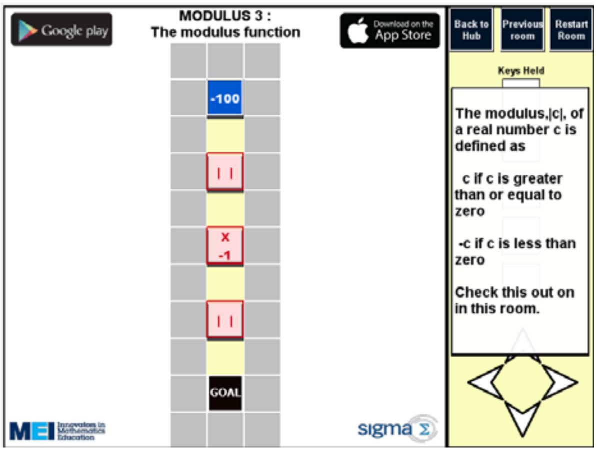

Figure 6: Sumaze explanation of the modulus function in Modulus 3

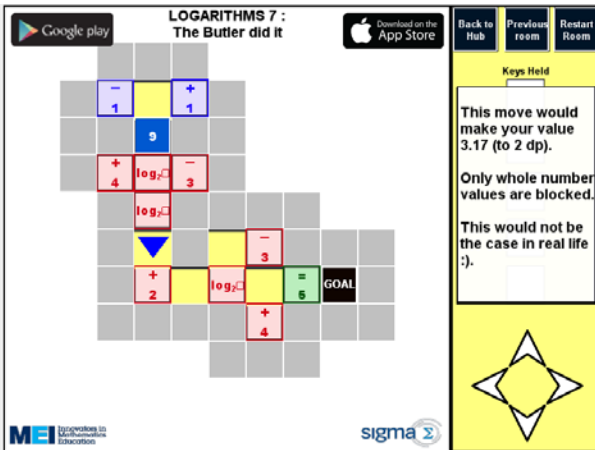

Figure 7: Sumaze explanation of $\log_2 9$

Future work

Despite the issues discussed above, this study has shown that Sumaze! can be engaging and enjoyable for students. It is an entirely original concept and it is able to provide sufficient scaffolding without spending time explaining game functions.

Taking this further, I believe there are more ways the Sumaze! game could be used in the classroom which are yet to be explored. I would suggest that a creative element, similar to that of Geogebra, where students and teachers can produce their own levels could prove useful for learning. If a teacher could produce a level which exemplified an example of a calculation or concept they wanted to show their students, then this could conceivably have enormous learning potential. Hence, I would suggest further research into the uses of Sumaze! within the classroom.

Conclusion

As a result of this research I am able to provide the following answers to my research questions and consider implications of my work for practitioners.

Research Question 1

The evidence suggests that Sumaze! cannot be used as a preliminary learning experience in a secondary mathematics classroom. Students were unable to independently demonstrate any knowledge of the topics which relate to sections of the game they had played. However, it is still unknown whether or not Sumaze! can be integrated into a sequence of lessons, not necessarily at the start.

Research Question 2

My findings suggest that presenting Sumaze! with a relaxed atmosphere where students are free to work collaboratively works better than presenting Sumaze! to a class in exam conditions. It is best to avoid any element of formal testing and having to write things down. However, it is beneficial for students to be prompted to write down their ideas when trying to solve levels.

Research Question 3

The majority of students felt the game was useful for learning and almost all students were engaged in the game during lessons. Hence there is evidence to suggest that students find Sumaze! interesting. A significant number of students said they would play the game again and many had already downloaded the game. This result was promisingly attained from a mixture of different ability students. However, some of the students said they would only play the game if allowed to do so in school time.

Despite the answers gained from this very small study, there are still many unanswered questions regarding the use of Sumaze! within the classroom (see Introduction). As apps of this kind became more popular the temptation for teachers to use them in the classroom is likely to grow (Drigas & Pappas, 2015). However, as this study shows, it is necessary to be cautious when attempting to use an app for teaching in the classroom as good results from an app do not necessarily imply good understanding of a topic. As such, a key consideration of practitioners should be the pedagogical value of such apps as opposed to keeping up with popular trends.

Acknowledgement

I would like to extend special thanks to MEI & Sigma for allowing me to use Sumaze! with my students and for their support during the period I was undertaking this project. I would also like to make a special mention in this respect to Richard Lissman, MEI online resource coordinator, for additional permission to incorporate the images of screenshots within this paper.

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Appendix 1

Worksheet & Mark Scheme

Sumaze! - Arithmetic section

Time allowed: 40 mins

Calculators not allowed

Full Name:

Age:

Date:

School:

Teacher:

For each level answer the corresponding question(s):

(i.e answer question 1 for level 1 and question 2 for level 2 etc)

DO NOT PROGRESS THROUGH A LEVEL WITHOUT ANSWERING THE QUESTION(S)

SHOW YOUR WORKING – WRITE DOWN ANY IDEAS YOU HAVE

1) Write down the calculation that you did.

2) Write down what the green blocks do.

3) What two answers can you get by using both blocks?

4) Which is the maximum value allowed in the game?

5) What sort of numbers are not allowed? What else?

6) A) What is the difference between blue blocks and red blocks?

   B) Who does "/" mean?

7) A) Write down the calculation you did. (Hint: Work backwards.)

   B) How could you solve this by working backwards?

Turn over for question 7C

   C) How could you solve this using algebra?

8) A) What do the keys do?

   B) Write down all of the answers you get as you go.

9) A) Write down all of the answers you get as you go.

   B) How many possible answers are there?

   C) How do you know if you have found every possibility or not?

10) Write down each chain of answers you get as you do the question.
    Eg 10 → 5 → 7 → 14 → 28. Does this help?

11) What are the next three terms in the sequence 1, 2, 4, 8, 16?

12) A) Why are 18, 20 and 22 not possible? (Hint: Think about odd and even numbers.)

    B) What does this level tell you about **ALL** odd numbers?

13) A) What does this question have to do with multiples of 5?

    B) What would be the answer if the minus 1's were plus 1's?

14) A) Write down all of the answers you get as you go.

    B) What does this question have to do with factors of 100?

Sumaze! – Arithmetic section answers and mark scheme

1 mark per section of question. Questions with multiple sections are worth multiple marks. E.g. each part of question 9 is worth a mark so question 9 is worth 3 marks altogether. 23 marks available.

1) $1 + 2 - 1 \times 3 = 6$ (Accept separate steps)

2) Act as conditions. I.e. your block must match the condition written on the green block to pass through. OR It doesn't let you pass if you don't have 6.(Or anything similar)

3) 5 & 6 (Must include both)

4) 1000

5) Fractions, decimals, negatives (Any two)

6) A) Blue blocks can be used repeatedly, red blocks cannot. (Or anything similar)

   B) Divide (Division symbol acceptable)

7) A) $4 - 1 \times 3 + 2 = 11$ (Accept separate steps)

   B) $(11 - 2) \div 3 + 1 = 4$ (Accept separate steps)

   C) $x \times 3 + 2 = 11$ (Accept any equivalent)

8) A) Key unlock boxes to allow you to reach the goal. (Or anything similar)

   B) Any 3 possible values OR routes to attain 5 and 2.

9) A) Mark for writing down any 3 possible values.

   B) 11 (All integers from 1 to 12 except 11)

   C) By trying every possible combination of the 4 boxes. (Or anything similar)

10) Any feasible chain of at least 3 numbers.

11) 32, 64, 128

12) A) 18, 20 and 22 are all even. You need an odd number.

   B) Multiplying any number by 2 and adding 1 gives you an odd number. (Or anything similar)

13) A) The solution is a multiple of 5 with a either 1 or 2 subtracted.

   OR The solution is 28 because 28 is 30 minus 2 and 30 is a multiple of 5.

   B) Any of the other 3 possible answers. (16 or 22 or 11 or any combination)

14) A) Any list of at least 3 possible answers.

   B) You can only multiply by 2 or add 2 so you know you must either reach half of 100 or a number close to half of 100. (Or anything sensible)