Abstract: Measurements of 3D turbulent flame surface area and burned gas were carried out for spherically expanding flames in different methane-air and hydrogen-air mixtures using a high frequency swinging laser sheet technique based on Mie scattering. The corresponding turbulent burning velocities were measured simultaneously using the rate of pressure rise, at turbulence rms velocities between 0.3 and 2.0 m/s. The ratio of turbulent burning velocity enhancement to flame surface area enhancement was measured as a function of turbulence rms velocity. For the methane-air flames, the turbulent burning velocity enhancement is close to that of the flame surface area enhancement. For the hydrogen-air flames, the former can exceed the latter by a factor of up to 6 at the largest values of turbulence rms velocity tested. The large discrepancy suggests that in the case of hydrogen-air flames, the measured rate of burning per unit flame area is significantly enhanced by the turbulence. For the reconstructed, the corresponding are also discussed.
Ms. Ref. No.: CNF-D-21-00235R1
Title: Three dimensional measurements of surface areas and burning velocities of turbulent spherical flames
Combustion and Flame

Dear Dr. Ahmed,

The reviewers have commented on your above paper. Rev 1 still indicates that additional information is required to compare 2D and 3D data. I assume that you can suitably address the reviewers' comments (included below) and I invite you to revise your manuscript. Please carefully address the issues raised in the comments.

Please submit the following after revision, within 30 days from the receipt of this notice:

a) a revised version of your manuscript,

b) a second version of your revised manuscript with the changes you have introduced highlighted so that it is easy to see your modifications,

c) a separate document, outlining each change made (point by point) as raised in the reviewer comments and providing a suitable rebuttal to each reviewer comment not addressed,

d) any revised supplemental material if appropriate.

To submit your revision, please do the following:

1. Go to: https://www.editorialmanager.com/cnf/
2. Enter your login details
3. Click [Author Login]
   This takes you to the Author Main Menu.
4. Click [Submissions Needing Revision]

I look forward to receiving your revised manuscript.

For further assistance, please visit our customer support site at http://help.elsevier.com/app/answers/list/p/7923. Here you can search for solutions on a range of topics, find answers to frequently asked questions and learn more about EM via interactive tutorials. You will also find our 24/7 support contact details should you need any further assistance from one of our customer support representatives.

Include interactive data visualizations in your publication and let your readers interact and engage more closely with your research. Follow the instructions here: https://www.elsevier.com/authors/author-services/data-visualization to find out about available data visualization options and how to include them with your article.

Yours sincerely,

Thierry Poinsot, PhD
Editor in Chief
Combustion and Flame
**Reviewers' comments:**

Reviewer #1:
Some of my comments have addressed successfully. However, I still have one comment which require additional discussions.

I may understand that it is difficult to present a detailed and extensive comparison of the 2D/3D post-processing of the volumetric data, due to the size limit imposed by the journal. However, I still recommend (as one of the other reviewers) a preliminary comparison to highlight the gain of this new technique versus the classical 2D method. The consistency of this comparison with previous similar DNS results would be also a nice opportunity to assess the quality of the measurements.

**Response:** Many Thanks for the suggestion. The Reviewer's comments are addressed, and a plot comparing the 2D, 3D and DNS work is presented on page 27 and 28.

Reviewer #3:

The authors addressed my concerns and comments in a satisfactory manner in the revised manuscript. I recommend acceptance for publication.

**Response:** We thank much for your recommendation.
Three dimensional measurements of surface areas and burning velocities of turbulent spherical flames

P. Ahmed1*, B. Thorne1, M. Lawes1, S. Hochgreb1, G.V. Nivarti2, R.S. Cant2

1School of Mechanical Engineering, University of Leeds, Leeds, United Kingdom.
2University Engineering Department, Trumpington Street, Cambridge, United Kingdom.

*Corresponding author email: ahmed.pervez@ucl.ac.uk

Abstract

Measurements of 3D turbulent flame surface area and burned gas were carried out for spherically expanding flames in different methane-air and hydrogen-air mixtures using a high frequency swinging laser sheet technique based on Mie scattering. The corresponding turbulent burning velocities were measured simultaneously using the rate of pressure rise, at turbulence rms velocities between 0.3 and 2.0 m/s. The ratio of turbulent burning velocity enhancement \( u_{tm}/u_t \) to flame surface area enhancement \( A_{3D}/a_{3D} \) was measured as a function of turbulence rms velocity. For the methane-air flames, the turbulent burning velocity enhancement is close to that of the flame surface area enhancement. For the hydrogen-air flames, the former can exceed the latter by a factor of up to 6 at the largest values of turbulence rms velocity tested. The large discrepancy suggests that in the case of hydrogen-air flames, the measured rate of burning per unit flame area is significantly enhanced by the turbulence. For the reconstructed \( A_{2D}/a_{2D} \), the corresponding \( A_{2D}/a_{2D} \) are also discussed.

Keywords: 3D, flame surface areas, turbulent spherical flames, swinging laser sheet technique, turbulent burning velocity, Damköhler’s first hypothesis.
Current Address:

P. Ahmed – Department of Mechanical Engineering, University College London, Torrington Place, London, UK.

Email: ahmed.pervez@ucl.ac.uk

B. Thorne – School of Life and Medical Sciences, College Lane, University of Hertfordshire, UK.

Email: b.thorne2@herts.ac.uk

G. V. Nivarti – School of Mathematics, University of Leeds, Leeds, UK.

Email: G.V.Nivarti@leeds.ac.uk
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{3D}$</td>
<td>total/instantaneous turbulent flame surface area from 3D volume (m$^2$)</td>
</tr>
<tr>
<td>$a_{3D}$</td>
<td>mean flame surface area based on total 3D flame volume (m$^2$)</td>
</tr>
<tr>
<td>$A_{2D}$</td>
<td>total/instantaneous turbulent flame surface area from 2D estimates (m$^2$)</td>
</tr>
<tr>
<td>$a_{2D}$</td>
<td>mean flame surface area based on 2D estimates (m$^2$)</td>
</tr>
<tr>
<td>$K_a$</td>
<td>Karlovitz number (-)</td>
</tr>
<tr>
<td>$K$</td>
<td>Karlovitz stretch factor (-)</td>
</tr>
<tr>
<td>$M_a$</td>
<td>burning velocity strain rate Markstein number (-)</td>
</tr>
<tr>
<td>$L_e$</td>
<td>Lewis number (-)</td>
</tr>
<tr>
<td>$L$</td>
<td>turbulence integral length scale (m)</td>
</tr>
<tr>
<td>$P$</td>
<td>pressure (MPa)</td>
</tr>
<tr>
<td>$Re_L$</td>
<td>turbulent Reynolds number based on turbulence integral length scale (-)</td>
</tr>
<tr>
<td>$R_0$</td>
<td>vessel volume equivalent radius (m)</td>
</tr>
<tr>
<td>$r_m$</td>
<td>mean flame radius based on total flame volume from pressure records (m)</td>
</tr>
<tr>
<td>$r_{3D}$</td>
<td>mean flame radius based on total 3D flame volume (m)</td>
</tr>
<tr>
<td>$r_{2D}$</td>
<td>mean flame radius based on total 2D estimates (m)</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature (K)</td>
</tr>
<tr>
<td>$u'$</td>
<td>turbulence rms velocity (m/s)</td>
</tr>
<tr>
<td>$t$</td>
<td>time (s)</td>
</tr>
<tr>
<td>$u_l$</td>
<td>unstretched laminar burning velocity (m/s)</td>
</tr>
<tr>
<td>$u_t$</td>
<td>turbulent burning velocity (m/s)</td>
</tr>
<tr>
<td>$u_{tm}$</td>
<td>turbulent mass burning velocity (m/s)</td>
</tr>
<tr>
<td>$\delta_l$</td>
<td>laminar flame thickness (m), $(v/u_l)$</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Taylor length scale (m)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Kolmogorov length scale (m)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>kinematic viscosity (m$^2$/s)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>equivalence ratio (-)</td>
</tr>
</tbody>
</table>
1. Introduction

The rate of turbulent burning determines the heat release rate in most combustion devices. As yet, there is no general model that can fully predict how turbulence affects the rate of burning for a given fuel-air mixture. The topic of how the burning rate of flames is affected by turbulence has been debated since Damköhler’s pioneering work [1], and has yet to be fully resolved, either with the help of experimental measurements or numerically via DNS. In spite of significant computational advancements, DNS remains limited by the maximum feasible size of the simulation domain. The corresponding turbulence integral length scale is typically limited to about 10 flame thicknesses. Previous DNS work using single-step chemistry has attributed significant deviations of the enhancement of the burning rate relative to the flame area enhancement observed for hydrogen-air flames to the increased frequency of flame self-interactions and the formation of cusps on the flame surface, where the local flame speed is enhanced disproportionately [2]. Recent DNS simulations for lean CH₄-air mixtures in turbulence conditions up to Ka around 36 (K ~ 10) show that the resulting turbulent flame speed enhancement closely tracks that of the flame surface area [3]. Simulations for lean mixtures of CH₄-air and H₂-air using detailed chemistry and at much higher Ka [4] seem to indicate that the main differences between the behaviour of the two fuel-air mixtures at low Ka arises from their different diffusive-reactive behaviour: the rate of heat release in the case of CH₄ takes place within a narrow range of temperatures within the flame, whereas in the case of H₂, thermo-diffusive instabilities which are apparent even at very low Ka, significantly enhance the reaction rate not only within the main reaction zone, but also at lower temperature. Recently, Chakraborty et al. [5] assessed the ratio $I_0$ using DNS for Bunsen burner CH₄ flames for values of Ka up to 2.4 using a single step global reaction rate chemistry, and found Damköhler’s first hypothesis valid in the flamelet regime, whereas relatively small deviations occur at higher turbulence intensities due to the effects of local strain and curvature.
The range of $K$ (and the corresponding $Ka$) investigated here is significantly lower and the ratio of integral length scales to flame thickness are significantly larger (~100) than in these DNS simulations (~4). However, the present measurements are consistent with the DNS findings that CH$_4$ reaction zones should follow flame isotherms, whereas H$_2$ reaction zones may diverge from flame isotherms.

A number of experimental measurements [6-9] have been made over the years to determine the ratio of the turbulent enhancement of the burning rate to that of that flame area as determined using an isotherm. Most of these experiments used 2D line measurements to determine flame surface areas based on surface density ratios using the approach of Shepherd et al.

Recently, measurements of Gülder [6] and Wabel et al. [7] used Bunsen burner flames to measure the ratio of the measured turbulent burning rate (assuming the entire flow of reactants is burned through the flame) to the extrapolated 2D burning rate inferred from the flame length to area ratio. These measurements suggest that a relative increase of a factor of up to 5 exists between the two at $Ka \sim 100$ for CH$_4$–air flames.

There is also significant disagreement in data obtained at nominally similar conditions [10, 11]. According to Damköhler [1], for turbulence scales larger than the laminar flame thickness, $\delta_l$, the increase in turbulent burning rate is a consequence of increased flame surface area due to flame wrinkling (first hypothesis), whereas for turbulence scales smaller than $\delta_l$, the enhancement arises due to the addition of turbulent diffusivities (second hypothesis). Direct Numerical Simulations (DNS) generally agree that, at low turbulent Reynolds numbers, the increase in turbulent burning rate is indeed proportional to flame surface area [12], whereas observations differ on the variation and magnitude of disproportionality at higher Reynolds numbers [3-5, 13-16].

For hydrogen-air mixtures, DNS in a box observed an exaggerated enhancement of the turbulent burning rate, exceeding the turbulent flame surface area enhancement by 14% on average [2]. For methane-air flames, similar single-step chemistry DNS in an inflow-outflow configuration suggests that proportionality is maintained strictly [15], whereas deviations from proportionality are observed when detailed chemistry and differential diffusion are included [3]; these deviations nevertheless
remain relatively small (up to 5%). Similar deviations observed in DNS of a range of hydrocarbon flames [17] were attributed to the added contribution of enhanced turbulent diffusivity. Recent DNS in the more realistic slot jet configuration [18] have reported similarly modest deviations for methane-air flames, increasing gradually with Reynolds number.

Whereas DNS is able to probe the relevant quantities such as the turbulent flame surface area, $A$, in 3D detail, experimental measurements of turbulent flame surface area have been largely extracted from 2D optical techniques, using the assumption that the average surface area per unit volume is equal to the average flame perimeter per unit area to determine flame surface density [11]. As a result, comparisons with DNS [19] have been limited in scope. This demonstrates the need for the development of advanced 3D measurement techniques. Wabel et al. [7] and Gülder et al. [6] made 2D flame surface density measurements in Bunsen burner flames at high turbulence intensities and found that the increase in the measured burning rate with turbulence intensity is significantly larger than that of the measured 2D flame surface area. Nivarti et al. [16] showed analytically that such exaggerations may arise from the corresponding enhancement of turbulent diffusivity in high turbulence intensities.

In order to test the analytical as well as DNS predictions, it is necessary to be able to measure the turbulent flame surface area in 3D. Few previous studies have used quasi-3D techniques for obtaining reacting scalar isosurfaces and their corresponding areas [20-22].

Recent studies [23-25] have demonstrated the feasibility and accuracy of using the volumetric laser induced fluorescence (VLIF) technique for surface reconstruction of burner stabilized 3D turbulent flames. However, this technique requires five cameras to capture CH radicals in the flame with a spatial resolution of 0.4 mm. Moreover, these previous studies did not present simultaneous measurements of the total burning rate at the same time as the flame surface area to investigate the validity of Damköhler’s hypothesis.

The present work adapts the method of Yip et al. [22] to visualise and reconstruct 3D turbulent flame surfaces and instantaneous volume using a high frequency laser [26-28]. The simultaneous
measurements of turbulent mass burning velocity $u_{tm}$ and flame surface area $A_{3D}$ made here allow a
direct determination of the ratio $I_0$ of burning rate enhancement and area enhancement:

$$I_0 = \left( \frac{u_{tm}/u_t}{A_{3D}/a_{3D}} \right)$$  \hspace{1cm} (1)

The experimental method used to determine the quantities $u_{tm}$, $u_t$, $A_{3D}$ and mean flame surface area, $a_{3D}$ are described in the following sections. The results are presented as a function of Karlovitz stretch factor $K$, defined as [29]:

$$K = \frac{1}{4} \left( \frac{u'}{u_t} \right)^2 Re_L^{1/2},$$  \hspace{1cm} (2)

where $Re_L = u'L/ν$ is the turbulent Reynolds number, calculated using the turbulence root mean square (rms) velocity $u'$, the turbulence integral length scale $L$, and the kinematic viscosity $ν$. The Karlovitz stretch factor $K$ may be interpreted as the ratio of turbulent strain rate based on the Taylor length scale, i.e. $u'/λ$, to the chemical rate given by $u_t/δ_l$. The laminar flame thickness is estimated as $δ_l = ν/u_t$ [30]. Another dimensionless factor relating the chemical time scale, $δ_l/u_t$, and strain rate, based on Kolmogorov length scales, $ν/η$, is the turbulent Karlovitz number, $Ka$ [31, 32]. The Karlovitz stretch factor $K$ is related to Karlovitz number as $K = √{15} Ka$.

Previous work by Markstein [33] and Law [34] in laminar flames, followed by the work of Bradley and Lawes in turbulent flames [35, 36], suggests that the burning velocity is affected by both hydrodynamic instabilities and sensitivity to strain. The former depends on the heat release rate and the ratio of thermal to species diffusivity, represented by the Lewis number $Le$ of the deficient species; the latter is represented by the Markstein number, $Ma$, defined as

$$Ma = -\frac{1}{δ_l} \frac{∂u_1}{∂a}$$  \hspace{1cm} (3)
where \( a \) is the strain rate. The methane-air mixtures considered here all have positive Markstein numbers and Lewis numbers close to unity. Thus, the laminar burning velocity is expected to decrease slightly with increasing strain rate. The hydrogen-air mixtures have large negative values of Ma and high diffusivities of the deficient species \( \text{H}_2 \), which suggests an increase in laminar burning velocity with increasing strain rate.

In the present work, the validity of Damköhler’s first hypothesis [37] is evaluated for spherically propagating turbulent flames over a range of Karlovitz numbers. The results show how the burning rates of the different mixtures change with turbulence intensity, while simultaneously measuring the total 3D flame surface area. The following section describes the experimental method including the technique for obtaining 3D flame surface areas and measurement of turbulent burning velocity. Results are shown in Section 3 followed by discussion of the measured ratio \( I_0 \) in Section 4.

2. Experimental method

Premixed turbulent spherical flames were ignited in a spherical combustion vessel of 380 mm internal diameter with three pairs of orthogonal windows of 150 mm diameter for optical access. One 2 kW electric heating coil provided controlled heating to the vessel. Initial mixture temperatures and static pressures were measured using a K-type thermocouple and a static pressure sensor [38]. Four fans, each driven by an 8 kW electric motor, generated near uniform isotropic turbulence in the central region of the combustion vessel. The turbulent rms velocity and integral length scale were determined in previous studies using laser Doppler anemometry [38, 39]. The integral length scale, \( L=20 \text{ mm} \), was determined to be independent of the fan speed between 1000 to 10,000 rpm, with a slight increase to 24 mm at the lower fan speed of 500 rpm. The turbulent rms velocity, \( u' \), was correlated with the fan speed \( N_f \) in rpm as:

\[
u' = 0.00119 N_f \quad (4)
\]

The Taylor length scale \( \lambda \) and Kolmogorov length scale \( \eta \) were found to vary between 1.2-2.6 mm and 0.03-0.12 mm, respectively.
2.1 Swinging laser sheet technique

The swinging laser sheet technique employs a double-cavity 532 nm Nd:YAG laser for Mie-scatter flame imaging, pulsing at a frequency of 54 kHz with a pulse energy of about 1.9 mJ. Multiple thin laser sheets are created, using a 16-faced rotating mirror at 12 Hz (Figure 1). A set of plano-concave and plano-convex lenses of diameters 50.8 mm and 50 mm respectively are used to converge the laser beam to a minimum beam thickness of 0.6 mm at the centre of the vessel, followed by a pair of cylindrical lenses to expand the laser beam into a sheet of less than 1 mm thickness. The focal lengths for the plano-concave and plano-convex lenses were 100 mm and 250 mm respectively, while the pair of plano-convex cylindrical lenses had focal lengths 38.1 mm and 25 mm. Using this optical system, a vertically expanded imaging laser sheet approximately 100 mm in height was produced across the central area of the combustion vessel. These laser sheets were made to swing at a velocity of about 54 m/s using the rotating mirror. Typically, 78 sheet images were recorded in each sweep of the combustion vessel separated by approximately 1 mm and 18 μs. The effective sweep duration, i.e. the time interval between the first and last sheet images recorded was 1.44 ms. The time interval between two successive sweeps was 5.2 ms. A high-speed Phantom VR camera, placed orthogonally to the imaging laser sheets captured 2D Mie-scattered images, using a fixed macro lens of focal length 105 mm, at every position of the laser sheet. A minimum aperture with large focal number of f/32 was used in order to generate a sufficiently large depth of field to cover the entire flame volume. The images were captured with an exposure time of 6 μs, at a resolution of 512 by 512 pixels. The chosen exposure time (6 μs) and large depth of field yielded sufficiently high contrast between the burned gas and the background. The size of each pixel was 0.196 mm, sufficiently small to resolve the Taylor length scale (which varies between 1.2-2.6 mm) but not the Kolmogorov scale (which varies between 0.03-0.12 mm). Olive oil droplets with density of 970 kg/m³ and measuring approximately 1 μm were used as seeding particles for the 2D Mie scattering images. The partial pressure of the seeding oil mixture (olive oil and air) was limited to less than 4% (180 mbar) of the total (fuel plus air) mixture at 0.5 MPa initial pressure. The oil alters the nominal equivalence ratios by between 1.1% to 3.8% over the range
of equivalence ratios studied. Experiments with up to 8% partial pressure of seeding mixture showed no significant change in the mean turbulent burning rates.

Figure 1. Schematic diagram for the 3D swinging laser sheet technique.

A limitation of the presently described technique is the finite time (1.44 ms) required to sweep through a developing flame. During this period, the mean flame radius grows approximately linearly with time, so the error in mean radius is proportional to the sweep interval. Increasing the rotating mirror speed reduces the time between subsequent sheets but also increases the physical spacing between these, whilst simultaneously reducing the time between successive sweeps. Thus, a compromise exists between spatial resolution and flame growth during imaging and also the number of sweeps which may be imaged before the flame expanded beyond the field of view. The use of a higher repetition rate laser would improve all of these aspects, while the use of a rotating mirror with extra facets would reduce the mean sweep time and increase the temporal resolution. However, there exists a minimum facet size required to give a sufficiently broad sweep through the region of interest, which limits the possible number of facets for a given mirror size. Mixtures were selected in the present work which possessed a sufficiently low laminar burning velocity. This allowed highly wrinkled, turbulent flames
to be imaged with relatively little flame growth during each sweep. The implications of these limits to
the uncertainty in measurements of flame radii are discussed in Section 3.

2.2 Flame surface area measurement

Mie-scattered binarized images in a sweep were stacked in a 3D volume matrix of 512 by 512 by 512
voxels in a sequence with respect to their spatial and temporal positions. The sweep-reconstructed
volume was approximately 100 x 100 x 10⁴ mm³. The resolution in the sweep direction was 0.74 mm.

Further details of the laser sheet geometry and synchronization control system are available in the
corresponding author’s PhD thesis [40]. Matlab scripts were used to process the 2D contour images
into 3D surfaces. The finite spacing between the successive images was filled by thickening the
images, using an interpolation function to generate a solid 3D reconstruction. This leads to a stepped
flame surface appearance, and hence the reconstructed flame was smoothed using an interpolative
algorithm developed by Taubin [41]. This algorithm retained the flame curvature while smoothing the
high frequency surface detail on the flame surface. The algorithm had the additional advantage of
incorporating an expansion term to minimise shrinkage during smoothing; without this, the flame
surface would be eroded by the smoothing process, giving rise to a reduction in flame (burned gas)
volume. Nevertheless, this procedure decreased the total flame surface area $A_{3D}$ by about 25% for the
largest and highly turbulent reconstructed flame in the present measurements compared to the
unsmoothed precursor reconstruction. It is important to note, however, that the original, highly stepped
unsmoothed flame reconstruction prior to smoothing is not a good representation of the flame with a
concomitant high surface area; instead, the smoothed surfaces more closely represent the appearance
of the actual flame imaged. Further quantitative analysis of the effect of surface smoothing and increasing
$u'$ on total surface area is available in [42].

Following interpolation, a triangulated surface mesh was generated and a smoothed flame surface was
obtained using Taubin algorithm [41]. The area of each triangle in the smoothed surface mesh was
calculated using the cross product of vectors that represent the sides of the triangle, and the triangle areas were summed to yield the total surface area $A_{3D}$ of the reconstructed flame.

The triangulated surface mesh was converted into a solid reconstruction consisting of voxels (volume pixel), discretising the triangulated surface by splitting and refining each face until the longest edge is smaller than half of a voxel [43]. The voxel is then set beneath the vertex coordinates of the original face to a value of unity. Therefore, with the volume of each voxel known, the entire volume of 3D reconstructed flame, $V_{3D}$, was obtained by summing up of all voxels.

The volume-equivalent radius is the radius of the equivalent spherical flame based on the volume of burned gas $V_{3D}$:

$$r_{3D} = \left( \frac{3V_{3D}}{4\pi} \right)^{1/3}$$

The equivalent mean flame area is then defined as $a_{3D} = 4\pi r_{3D}^2$.

Based on the assumption of isotropy [35], the total flame surface area corresponding to 2D measurements, $A_{2D}$, is estimated using the centre line sheet image in a volumetric sweep, by calculating the perimeter of the 2D flame image and then equating to that of an equivalent circle as, $P = 2\pi R_{2D}$. The total flame surface area from 2D estimates is calculated as, $A_{2D} = 4\pi R_{2D}^2$. The mean flame radius, $r_{2D}$, is calculated from the cross-sectional 2D flame area of the centreline image.

The corresponding mean flame area, $a_{2D}$ is then calculated as $a_{2D} = 4\pi r_{2D}^2$.

### 2.3 Turbulent flame speed measurement

The rise in pressure within the combustion vessel during flame propagation was measured using a Kistler 5110 piezo-electric pressure transducer (calibrated to ±0.5% of full scale at 5.0 MPa). The pressure record was used to obtain the volume-equivalent radius, $r_m$, of the flame according to [29, 35]:

$$r_m = R_0 \left\{ 1 - \left( \frac{P_o}{P} \right)^{1/\gamma} \left[ \frac{P_f - P}{P_f - P_0} \right] \right\}^{1/3}$$

(6)
where \( R_0 \) is the volume equivalent vessel radius, \( P_0 \) and \( P_f \) are the initial and peak pressures, and \( \gamma_u \) the ratio of specific heats for the unburned mixture.

The corresponding turbulent burning velocity \( u_{tm} \) was then obtained from the fractional burning rate according to [29]:

\[
u_{tm} = \left( \frac{P_0}{P} \right)^{1/\gamma_u} \left\{ 1 - \left( \frac{P}{P_0} \right)^{1/\gamma_u} \left[ \frac{P_f - P}{P_f - P_0} \right] \right\}^{-2/3} \frac{R_0}{3(P_f - P_0)} \frac{dP}{dt}
\]  

Under ideal conditions, it would be possible to calculate turbulent burning velocities either from the rate of change of the burned volume \( V_{3D} \) obtained from the high-speed flame reconstruction procedure outlined in 2.2 or from the pressure history. As discussed further in Section 3, the pressure rise is too small at the early burn times compatible with flame imaging, and, hence, cannot be used to determine \( u_{tm} \) accurately. Whereas differentiating the rate of change of the burned volume, \( V_{3D} \), might offer an alternative method, the sparse time history of the burned volume, \( V_{3D} \), means that the determination of the turbulent burning rate in this manner (with intervals of 5.2 ms between successive sweeps, corresponding to the mean sweep time), is also too inaccurate for direct comparison. In the future this could be alleviated by increasing the laser firing frequency to improve the temporal resolution.

In the present work, however, values for \( u_{tm} \) are extracted from the pressure records only at pressures higher than 0.5% of the initial pressure; under these conditions, pressure differences (rate of change of the burned volume differentiated over an interval of 0.02 ms) can be measured within 1.0% for the highest \( u' \) employed, yielding measurements of \( u_{tm} \) that are accurate to within 20%.

At these higher pressure rise conditions, however, the flame radii are too large to be observable with the 3D reconstruction technique. The remedy is to use the empirical observation that the calculated burning rate varies linearly with flame radius, and extrapolate the measurements obtained from the pressure record to the smaller radii where the 3D reconstruction technique can be employed.
The uncertainties in the final value of $u_{tm}(r_m)$ were estimated from the squared sum of uncertainties in $u_{tm}$ with the relative error of $r_m$. The latter was estimated from the uncertainty in the flame position owing to the growth of the radius during the laser sweep time of 1.44 ms. The relative uncertainties in $r_m$ varied between 17% to 5% from the highest to the lowest propagating speed. The uncertainties in $u_{tm}$ varied between 5% to 21% at the highest and lowest radii for different propagating speeds. The propagation of compound uncertainty in $u_{tm}(r_m)$, calculated as the sum of squares of the instrument and the rms fluctuations, due to extrapolation, at $r_m=30$ mm is estimated to be 20%. Details of the uncertainty in the extrapolation method are presented in the Appendix (Figures A5 – A12), and results are presented in Section 3.

Values of the unstretched laminar burning velocity, $u_l$, are taken from literature [44, 45], some of which have been confirmed in the present study: see Figure A4 in supplementary material. Details on the measurement of $u_l$ are presented in [46].

Table 1: Experimental conditions for the present study, (+) indicates the estimated value based on the given reference; (*) indicates values from the references for $u_l$ and Ma. Values of $u_l$ without an asterisk mean that measurements were made in the present study.

<table>
<thead>
<tr>
<th>Fuel</th>
<th>CASE</th>
<th>$\phi$</th>
<th>T (K)</th>
<th>P (MPa)</th>
<th>$u'$ (m/s)</th>
<th>$K$ (-)</th>
<th>$u_l$ (m/s)</th>
<th>Ma (-)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH₄</td>
<td>I</td>
<td>0.60</td>
<td>365</td>
<td>0.1</td>
<td>0.3-1.5</td>
<td>0.08-0.903</td>
<td>0.131</td>
<td>2.0*</td>
<td>[47]</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.70</td>
<td>300</td>
<td>0.1</td>
<td>0.3-1.5</td>
<td>0.027-0.30</td>
<td>0.210*</td>
<td>2.7*</td>
<td>[44]</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>1.30</td>
<td>300</td>
<td>0.1</td>
<td>0.3-2.0</td>
<td>0.046-0.79</td>
<td>0.160*</td>
<td>3.9*</td>
<td>[44]</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>1.25</td>
<td>365</td>
<td>0.5</td>
<td>0.3-2.0</td>
<td>0.019-0.33</td>
<td>0.241</td>
<td>4.8*</td>
<td>[47]</td>
</tr>
<tr>
<td>H₂</td>
<td>V</td>
<td>0.30</td>
<td>365</td>
<td>0.5</td>
<td>0.3-2.0</td>
<td>0.049-0.85</td>
<td>0.102*</td>
<td>-5.0*</td>
<td>[45]</td>
</tr>
<tr>
<td></td>
<td>VI</td>
<td>0.40</td>
<td>365</td>
<td>0.5</td>
<td>0.3-1.5</td>
<td>0.008-0.09</td>
<td>0.286*</td>
<td>-6.3*</td>
<td>[45]</td>
</tr>
</tbody>
</table>

3. Results

Figure 2(a) shows a typical raw 2D Mie-scattered centre line image in a sweep, acquired at 21.5 ms after ignition, for a CH₄/air flame at $\phi = 0.7$, $P=0.1$ MPa, $T=300$ K and at low value of turbulence, $u'=0.3$ m/s. Laser light intensity is higher in the centre of the image than at the edges due to the
Gaussian nature of the illumination. The flame was mildly wrinkled and, as a result, the flame edge was clearly defined. These raw flame images were then binarized, as shown in Figure 2(b) before using them for 3D reconstruction.

![Figure 2. (a) Raw Mie-scattered image (b) Binarized image](image)

Typical 3D reconstructed flame surfaces are shown in Figure 3, for a CH₄/air flame at $\phi = 0.7$, 300 K and 0.1 MPa, for $u'=0.3$ m/s. Each 3D reconstructed image corresponds to a single sweep of 2D image acquisition. For a given experiment, flame wrinkling develops with time as the flame interacts with the surrounding turbulence.

![Figure 3. Temporal development of a 3D reconstructed CH₄/air flame at, $\phi = 0.7$, T=300K and P= 0.1 MPa during a deflagration at $u'=0.3$ m/s.](image)

Figure 4 shows 3D flames for two different $u'$ values of 0.3 and 1.5 m/s. Flame surface area, wrinkling and burning rate all increase with increasing $u'$ as the flame becomes progressively more distorted. It is worth noting that the flame represented in Figure 4(b) propagated faster than that in Figure 4(a), and hence it developed more quickly and correspondingly fewer sweeps were recorded during the combustion event for flame reconstructions.
Figure 4. 3D reconstructed flame surfaces for CH₄/air flames at, ϕ =0.7, T=300 K, P= 0.1 MPa (a) \( u' = 0.3 \) m/s at \( t = 27.6 \) ms (b) \( u' = 1.5 \) m/s at \( t = 11.1 \) ms after ignition. Dashed circle indicates mean flame radius, \( r_{3D} \). The scales on the axes denote pixel numbers, where each pixel corresponds to 0.196 mm.

Figure 5. Mean flame radius for CH₄/air flames at, \( \phi =0.7, T=300 \) K, \( P= 0.1 \) MPa (a) \( u' = 0.3 \) m/s (b) \( u' = 1.5 \) m/s.

The mean radii, \( r_{3D} \), obtained from 3D reconstructions, for Figure 4, are presented in Figure 5, along with the mean flame radii, \( r_{2D} \), obtained from centre line 2D slices following the method outlined in Section 2.2. The 2D estimates are generally lower than the 3D measured radii regardless of operating condition. The values of the mean flame radii, \( r_m \), obtained from pressure records are also plotted in Figure 5 (cross symbols), and reassuringly overlap with those of \( r_{3D} \). Further comparison of the full 2D vs 3D statistics is beyond the scope of the present work and left for a future investigation.

Considerable variations in mean flame radius \( r_{3D} \), were observed between measurements, even under nominally identical conditions. This was also reflected in the variability of the flame shape, as shown
in Figure 6 by the images of three flames taken under nominally identical conditions at the same time 
$t = 16.3$ ms after ignition. In the present study, the variability in the mean flame radius $r_{3D}$ at a specified 
time, ranges between approximately 5% for small flame radii and up to 17% at larger flame radii.

![Figure 6. 3D reconstructed CH$_4$/air flame surfaces (P = 0.1 MPa, T = 300K, $\phi$ =0.7) at the same time 
t = 16.3 ms after ignition for three different runs at $u'$=0.3 m/s.]

![Figure 7. Temporal development of a 3D reconstructed H$_2$/air flame at, $\phi$ =0.4, T=300K and P= 0.5 
MPa during a deflagration at $u'$=0.3 m/s.]

An example of 3D reconstructed flame surfaces for a H$_2$/air flame at $\phi$ =0.4, 365 K and 0.5 MPa, for 
$u'$=0.3 m/s are shown in Figure 7. Unlike CH$_4$/air flame, shown in Figure 3, the H$_2$/air flame develops 
quickly and therefore only four sweeps could be captured. These flames exhibit considerable flame 
wrinkling and at any given time, larger diameters are achieved compared to CH$_4$/air flame indicating 
higher burning rates.
Figure 8. Effect of turbulence intensity on (a), (d) time history of mean flame radius $r_{3D}$ and (b), (e) area ratio $A_{3D}/a_{3D}$ as measured by 3D reconstruction as a function of mean radius. Symbols represent individual measurements and lines are best fit second order polynomials. Dashed red lines indicate the extrapolation of best fit curves to $r_{3D} = 30$ mm. (c), (f) Averaged pressure traces (right hand inverted scale) and corresponding $u_{tm}$ values calculated using Eq. (4). Error bars are standard
deviations from the three experiments. Dashed red lines indicate linear best fits, extrapolated to \( r_m = 30 \) mm.

Figure 8 shows the effect of turbulence rms velocity on the measured growth of the mean flame radius \( r_{3D} \) for the \( \text{CH}_4 \) II and \( \text{H}_2 \) V flames in Figure 8(a) and (d) while the corresponding flame surface area ratio \( A_{3D}/a_{3D} \) in Figure 8(b) and (e) respectively. Three experiments were conducted for each set of nominally identical conditions. However, for values of \( u' \) larger than 1.5 m/s, only two experiments were successfully reconstructed, because the flames drifted away from the optical access region. The largest flame that could be reconstructed measured \( r_{3D} = 46 \) mm for the \( \text{CH}_4 \) I case at \( u'=0.3 \) m/s. An overall root mean square variation in \( r_{3D} \) of up to 17% (for 3 experiments) was recorded for \( \text{CH}_4 \) I and \( \text{CH}_4 \) III flames. This variability is due to the fact that these flames were close to the flammability limits and is also confirmed from their susceptibility to quenching; in comparison, \( \text{H}_2 \) flames exhibited low variability in \( r_{3D} \) as these are expected to display stable burning (See Figure A14 in supplementary material for different burning regimes as taken from reference [48]). As discussed in Section 2.3, \( r_{3D} \) values measured from image data can only be extended up to about 30 mm, for the range of \( u' \) employed, due to the limitations of the imaging system, but the pressure rise at this radius was very small, leading to poor signal to noise ratio. The compromise requires evaluating both \( A_{3D}/a_{3D} \) and \( u_{tm} \) at a particular point, which is chosen as \( r_{3D} = r_m = 30 \) mm. The present work uses pressure-based values of \( u_{tm} \) at higher pressures and radii, followed by a short linear extrapolation towards 30 mm, as indicated by the dashed lines in Figure 8(c). Flames at this radius were well developed, independent of the spark effect [49] and the turbulent flow field was near isotropic and well characterized [39]. Estimates of the observed variance of three experiments are shown in Figure 8(c) using error bars; the variance ranges from 10-20\% for the lowest values, down to 5-10\% for the highest values of \( u_{tm} \). Assuming that the uncertainties associated with the sampling frequency and statistical variability are independent, we conclude that the uncertainties in the measurement of the turbulent flame speed are of the order of 6-34\% for the most favourable to the most unfavourable cases. Data for \( r_{3D}, A_{3D}/a_{3D} \) and \( u_{tm} \) for the remaining cases are provided in the supplementary material (see Figures. A1-A3). The
estimated increase in mean flame radius $r_{3D}$ for CH$_4$ flames during the effective sweep duration of 1.44 ms is between 3% and 11% for the lowest and highest turbulent velocities employed, respectively. However, for H$_2$ flames, the corresponding change in radius can reach up to 19% at the highest $u'$ due to the high burning rates of these flames. This maximum estimated flame radius change during the effective sweep time at the highest $u'$ values is close to the maximum observed variance of 17%. Faster repetition rate imaging lasers could eventually allow smaller uncertainties during a sweep.

The higher degree of wrinkling observed for H$_2$/air flames arises additionally due to the high initial pressures at which these measurements were conducted. Under such conditions, the onset of Darrieus-Landau and thermo-diffusive instabilities occurs at an early stage of flame development [50]. As a consequence, higher burning rates are achieved in H$_2$/air flames and this is in line with the higher $u_{tm}$ values obtained from pressure data.

A consistent comparison of these observed wrinkling phenomena would require CH$_4$/air and H$_2$/air flames to be compared at same initial pressures and temperatures. Figure 9 compares the 3D reconstructed CH$_4$/air flames and H$_2$/air flames at P = 0.5 MPa and T=365 K. These are compared at the same time for different $u'$ values. The 3D reconstructions obtained reveal significant small scale wrinkling for H$_2$ air flames even at low $u'$ values, which points to the role of Darrieus-Landau and thermo-diffusive instabilities, caused by small Le numbers for lean H$_2$ air flames. It is interesting to note that in the case of CH$_4$-air flames, the small scale wrinkling increases as $u'$ is increased, whereas it is apparently less distinguishable with increasing $u'$ in the case of H$_2$.

To better visualise this phenomenon, the corresponding centre line 2D binarised flame images are compared in Figure 10. As revealed by the 3D reconstructed flames, even at $u'$=0.3 m/s and at t=16.3 ms, H$_2$/air flames have higher small scale wrinkling compared to CH$_4$/air flames. In addition, the 2D images reveal that as the $u'$ increases from 0.3 to 1.0 m/s, more finger like structures and cusps appear in H$_2$/air flames. As $u'$ is increased further, these finger-like structures not only deepen in H$_2$/air flames but also in CH$_4$/air flames.
Figure 9. Comparison of 3D reconstructed CH4-air flames and H2/air flames at P = 0.5 MPa and T=365 K. (a), (d) at \( u' = 0.3 \text{ m/s} \) and \( t = 16.3 \text{ ms} \) (b), (e) at \( u' = 1.0 \text{ m/s} \) and \( t = 5.9 \text{ ms} \) (c), (f) at \( u' = 1.5 \text{ m/s} \) and \( t = 5.9 \text{ ms} \).
Figure 10. Comparison of binarised Mie-scattered centre line sheet flames for \( \text{CH}_4/\text{air} \) flames and \( \text{H}_2/\text{air} \) flames at \( P = 0.5 \text{ MPa} \) and \( T=365 \text{K} \), (a), (d) at \( \text{u}'=0.3 \text{ m/s} \) and \( t = 16.3 \text{ ms} \) (b), (e) at \( \text{u}'=1.0 \text{ m/s} \) and \( t = 5.9 \text{ ms} \) (c), (f) at \( \text{u}'=1.5 \text{ m/s} \) and \( t = 5.9 \text{ ms} \).
Figure 11. Variation of $u_{tm}/u_t$ with $K$ for the experimental conditions in Table 1. Symbols represent the average of three experiments.

Measured values of $u_{tm}/u_t$ evaluated for CH$_4$ and H$_2$ mixtures are shown in Figure 11, plotted against the Karlovitz stretch factor, $K$, at the largest resolved burned gas radius of $r_{3D} = 30$ mm. Values of $u_{tm}/u_t$ for CH$_4$ appear to increase with a small power of $K$, reaching a maximum of around 5 for $K = 1$ in the CH$_4$-II and CH$_4$-I mixtures. In comparison, $u_{tm}/u_t$ values for H$_2$ appear to be very sensitive to both composition and $K$, attaining values as high as 18 for $K = 1$. There is only sparse literature data that can be compared directly with the present conditions, particularly because the measurements were taken for mixtures far from stoichiometric. However, values of burning rates for CH$_4$ at $\phi = 1.3$ and $P = 0.5$ MPa at $u' = 2$ m/s measured by Lawes et al. [51] show $u_t$ of 0.89 m/s compared to the present measurements of 1.24 m/s. Measurements have also been made for CH$_4$ at $\phi = 0.7$ at 0.1 MPa and $u' = 0.5$ to 2 m/s in combustion vessels by Shy et al. [52] with measured $u_t$ within 20% of the current values. In the case of H$_2$, the measurements of $u_t$ from Kitagawa et al. [53] at 300 K (rather than the current 365 K) and 0.5 MPa at 1.6 m/s are about 34% higher than the current measurements.
Given the variability in these measurements and expected errors due to shot to shot experimental variations in the instantaneous flow field and burning rate, the current measurements of $u_t$ are within 10-34% of the previous measurements under similar conditions; any conclusions must therefore take these variabilities into account.

Figure 12. Effect of turbulence intensity represented through $K$ on the area ratio $A_{3D}/a_{3D}$ for the experimental conditions in Table 1.

In Figure 12, the corresponding values of measured flame surface area ratio $A_{3D}/a_{3D}$, are plotted as a function of $K$. Since only three experiments were conducted at identical conditions, significant variations might be expected. Nevertheless, clear trends emerge, and it is interesting to note that the values of $A_{3D}/a_{3D}$ appear to be reversed relative to the turbulent burning rate behaviour observed in Figure 11: large $A_{3D}/a_{3D}$ rise, comparable to the rise in turbulent burning rate in the case of CH$_4$ and, relatively small $A_{3D}/a_{3D}$ rising with a small power of $K$ in the case of H$_2$. For CH$_4$, at equivalence ratio of 0.6 and 1.30 (I and III), the sensitivity to turbulence level increases sharply for the higher levels of turbulence. For equivalence ratios 0.70 and 1.25, cases II and IV, no data is available beyond $K =$
0.3. For very lean or very rich conditions (I and IV), flames were often be convected away from the centreline. Yet for H₂, only a small rise in the turbulent flame area is measurable in either case, in spite of the large increase in turbulent flame speed measured, as shown in Figure 11.

![Figure 13](image_url)

Figure 13. Variations in the ratio $I_0$ as a function of Karlovitz stretch factor, $K$.

Finally, measurements can be obtained for the ratio $I_0$ of the turbulent enhancement in burning rate, $u_{tm}/u_l$ (Figure 11), to that of flame surface area, $A_{3D}/a_{3D}$ (Figure 12). The ratio, $I_0$, is shown as a function of $K$ in Figure 13. The measurements show significant differences for the two fuel-air mixtures. In the case of CH₄, values of $I_0$ are around 1.0±0.5, which is hardly distinguishable from unity within the current uncertainty range. In the case of H₂ mixtures, however, $I_0$ exceeds unity over a wide range of conditions by factors of up to 6. It is important to mention that the smoothing effect which reduces the total flame surface area, $A_{3D}$, by up to a maximum of 16% at $r_{3D} = 30$ mm increases the corresponding $I_0$ values (in Figure 11) by up to a maximum of 13% for CH₄ flames and 10% for H₂ flames. These differences do not alter the primary observation: $I_0$ values remain close to unity for CH₄ flames, but increase with $K$ to values much larger than unity for H₂ flames.
Figure 14. Effect of turbulence intensity represented through $K$ on the area ratio $A_{2D}/a_{2D}$ for the experimental conditions in Table 1.

An attempt was made to calculate the $A_{2D}/a_{2D}$ from the centre line flame images following the method outlined in section 2.2. Figure 14 shows the $A_{2D}/a_{2D}$ against increasing $K$ at mean radius $r_{2D} = 30$ mm. A general trend of increase in $A_{2D}/a_{2D}$ with $K$ is observed, however, unlike 3D measurements, here the values are found to reach as high as 20 at maximum. As discussed in section 3 under Figure 5, $r_{2D}$ are generally found to be lower than $r_{3D}$. Since the mean areas are proportional to the square of mean radius, the resulting $a_{2D}$ are much lower than $a_{3D}$ and therefore lead to higher values of $A_{2D}/a_{2D}$ in comparison to $A_{3D}/a_{3D}$ (Figure 12). Had these values been used to calculate the $I_0$ factor, for the same $u_{tm}/u_1$ presented in Figure 11, the resulting numbers would suggest that $I_0$ would be close to or less than unity. The reason for the large difference between the 3D and 2D is not entirely understood, but it is important to mention that the propensity of the flames to drift away from the centre of the vessel at high $u'$ increase, which means that the slice obtained by a 2D measurement technique through the centre of the developing flame may not be representative of the overall 3D structure.
5. Discussion

The present 3D measurements suggest that, for CH₄–air flames, the turbulent burning rate is roughly proportional to the flame area. There appears to be little enhancement of burning rate beyond that of flame surface area, even though the turbulence velocities are lower than in the case of the Bunsen flame measurements [6, 7].

By contrast, for the H₂ flames, even weakly wrinkled flames achieve significantly higher rates of burning than is suggested by the flame surface area measured simultaneously by the Mie-scattering technique. Additional mechanisms may be at work here, including thermo-diffusive instabilities or locally distributed reaction [3].

Due to the Darrieus-Landau and thermo-diffusive instabilities, apparent from finger like structures in Figure 9 and Figure 10, higher burned gas volumes and burning rates are achieved in H₂/air and CH₄-IV flames, as also apparent from Figure 8(f) (see also Figure A2(b) in supplementary material). High burned gas volumes result in larger mean flame areas, a₃D, that would lower A₃D/a₃D ratios, see Figure 12. Consequently, even at low K, the I₀ values deviate significantly from unity particularly so for H₂/air flames.

A preliminary comparison between the flame surface area ratios derived from the 3D and 2D methods is presented in Figure 15. Due to sparse 3D DNS data, data for only CH₄-II case are compared. It is apparent that the surface area ratios obtained from the present 3D technique are close to the DNS results while the 2D method over estimates the value by 4 times at high turbulence. Given the inherent nature of 3D movement of the flames, it is difficult to slice the flame through the centre (2D method) especially for flames at high turbulence levels (high K values) since these are displaced from the centre at an early stage. This explains the over estimation of A/a by 2D method. The present 3D technique has the advantage of capturing such flames with greater accuracy where the assumption of isotropy may not hold. More quantitative 3D data is required to confirm the differences discussed above.
An important outcome of the present work is to demonstrate that it is possible to measure 3D flame surface areas simultaneously with burning rates. These measurements have enabled the investigation of how turbulence simultaneously affects flame surface areas and propagation rates. The present measurements will serve to provide the experimental underpinnings for further modelling efforts.

5. Conclusions

The present results provide simultaneous 3D measurements of turbulent burning rates and turbulent flame surface areas in a closed combustion vessel, using pressure traces and a high frequency swinging laser sheet technique with flame area reconstruction. These measurements have allowed for the first time the determination of the ratio of turbulent burning rate enhancement to the flame surface area enhancement in the presence of turbulence.

Figure 15. Comparison of flame surface area ratios obtained from 3D and 2D methods (present work) along with 3D DNS [15].
Measurements of the enhancement ratio $I_0$ show that for both rich and lean CH$_4$-air mixtures, the enhancement ratio varies, but is not higher or lower than the inherent uncertainty in the experimental technique. In the case of H$_2$-air flames, however, the evidence is unambiguous and shows that even at low Karlovitz stretch factor $K$ (up to ~0.5), the increase in burning rate is significantly greater than the expected increase based on the measured increase in flame surface area, by a factor of up to 6.

The present measurements also show that the usual assumptions of isotropy used to reconstruct 3D from 2D flames in spherically propagating kernels are not well supported: 3D area ratios are significantly different from 2D ratios, and that may possibly be a result of movement of flame kernels. Whereas the present work shows the feasibility of high frequency 3D reconstruction of flame surfaces, further work is needed to improve the capabilities of the sweep rate, imaging resolution and field of view beyond the current state to improve the applicability of these measurements to higher flame speeds, and to allow the direct comparison of pressure measurements and volumetric methods of determining the burning rate.

ACKNOWLEDGEMENT

PA thanks University of Leeds for a Research Scholarship Award. GVN acknowledges the funding support of EPSRC grant EP/P022286/1.

REFERENCES


Three dimensional measurements of surface areas and burning velocities of turbulent spherical flames

P. Ahmed1, B. Thorne1, M. Lawes1, S. Hochgreb1, G.V. Nivarti2, R.S. Cant2

1School of Mechanical Engineering, University of Leeds, Leeds, United Kingdom.

2University Engineering Department, Trumpington Street, Cambridge, United Kingdom.

*Corresponding author email: ahmed.pervez@ucl.ac.uk

Abstract

Measurements of 3D turbulent flame surface area and burned gas were carried out for spherically expanding flames in different methane-air and hydrogen-air mixtures using a high frequency swinging laser sheet technique based on Mie scattering. The corresponding turbulent burning velocities were measured simultaneously using the rate of pressure rise, at turbulence rms velocities between 0.3 and 2.0 m/s. The ratio of turbulent burning velocity enhancement $u_{tm}/u_t$ to flame surface area enhancement $A_{3D}/a_{3D}$ was measured as a function of turbulence rms velocity. For the methane-air flames, the turbulent burning velocity enhancement is close to that of the flame surface area enhancement. For the hydrogen-air flames, the former can exceed the latter by a factor of up to 6 at the largest values of turbulence rms velocity tested. The large discrepancy suggests that in the case of hydrogen-air flames, the measured rate of burning per unit flame area is significantly enhanced by the turbulence. For the reconstructed $A_{3D}/a_{3D}$, the corresponding $A_{2D}/a_{2D}$ are also discussed.

Keywords: 3D, flame surface areas, turbulent spherical flames, swinging laser sheet technique, turbulent burning velocity, Damköhler’s first hypothesis.
Current Address:

P. Ahmed – Department of Mechanical Engineering, University College London, Torrington Place, London, UK.

Email: ahmed.pervez@ucl.ac.uk

B. Thorne – School of Life and Medical Sciences, College Lane, University of Hertfordshire, UK.

Email: b.thorne2@herts.ac.uk

G. V. Nivarti – School of Mathematics, University of Leeds, Leeds, UK.

Email: G.V.Nivarti@leeds.ac.uk
Nomenclature

\( A_{3D} \) total/instantaneous turbulent flame surface area from 3D volume (m\(^2\))

\( a_{3D} \) mean flame surface area based on total 3D flame volume (m\(^2\))

\( A_{2D} \) total/instantaneous turbulent flame surface area from 2D estimates (m\(^2\))

\( a_{2D} \) mean flame surface area based on 2D estimates (m\(^2\))

\( K_a \) Karlovitz number (-)

\( K \) Karlovitz stretch factor (-)

\( M_a \) burning velocity strain rate Markstein number (-)

\( Le \) Lewis number (-)

\( L \) turbulence integral length scale (m)

\( P \) pressure (MPa)

\( Re_L \) turbulent Reynolds number based on turbulence integral length scale (-)

\( R_0 \) vessel volume equivalent radius (m)

\( r_m \) mean flame radius based on total flame volume from pressure records (m)

\( r_{3D} \) mean flame radius based on total 3D flame volume (m)

\( r_{2D} \) mean flame radius based on total 2D estimates (m)

\( T \) temperature (K)

\( u' \) turbulence rms velocity (m/s)

\( t \) time (s)

\( u_l \) unstretched laminar burning velocity (m/s)

\( u_t \) turbulent burning velocity (m/s)

\( u_{tm} \) turbulent mass burning velocity (m/s)

\( \delta_l \) laminar flame thickness (m), \((v/u_l)\)

\( \lambda \) Taylor length scale (m)

\( \eta \) Kolmogorov length scale (m)

\( \nu \) kinematic viscosity (m\(^2\)/s)

\( \phi \) equivalence ratio (-)
1. Introduction

The rate of turbulent burning determines the heat release rate in most combustion devices. As yet, there is no general model that can fully predict how turbulence affects the rate of burning for a given fuel-air mixture. The topic of how the burning rate of flames is affected by turbulence has been debated since Damköhler's pioneering work [1], and has yet to be fully resolved, either with the help of experimental measurements or numerically via DNS. In spite of significant computational advancements, DNS remains limited by the maximum feasible size of the simulation domain. The corresponding turbulence integral length scale is typically limited to about 10 flame thicknesses. Previous DNS work using single-step chemistry has attributed significant deviations of the enhancement of the burning rate relative to the flame area enhancement observed for hydrogen-air flames to the increased frequency of flame self-interactions and the formation of cusps on the flame surface, where the local flame speed is enhanced disproportionately [2]. Recent DNS simulations for lean CH₄-air mixtures in turbulence conditions up to Ka around 36 (K ~ 10) show that the resulting turbulent flame speed enhancement closely tracks that of the flame surface area [3]. Simulations for lean mixtures of CH₄-air and H₂-air using detailed chemistry and at much higher Ka [4] seem to indicate that the main differences between the behaviour of the two fuel-air mixtures at low Ka arises from their different diffusive-reactive behaviour: the rate of heat release in the case of CH₄ takes place within a narrow range of temperatures within the flame, whereas in the case of H₂, thermo-diffusive instabilities which are apparent even at very low Ka, significantly enhance the reaction rate not only within the main reaction zone, but also at lower temperature. Recently, Chakraborty et al. [5] assessed the ratio \( I₀ \) using DNS for Bunsen burner CH₄ flames for values of Ka up to 2.4 using a single step global reaction rate chemistry, and found Damköhler’s first hypothesis valid in the flamelet regime, whereas relatively small deviations occur at higher turbulence intensities due to the effects of local strain and curvature.
The range of $K$ (and the corresponding $Ka$) investigated here is significantly lower and the ratio of integral length scales to flame thickness are significantly larger (~100) than in these DNS simulations (~4). However, the present measurements are consistent with the DNS findings that CH$_4$ reaction zones should follow flame isotherms, whereas H$_2$ reaction zones may diverge from flame isotherms.

A number of experimental measurements [6-9] have been made over the years to determine the ratio of the turbulent enhancement of the burning rate to that of that flame area as determined using an isotherm. Most of these experiments used 2D line measurements to determine flame surface areas based on surface density ratios using the approach of Shepherd et al.

Recently, measurements of Gülder [6] and Wabel et al. [7] used Bunsen burner flames to measure the ratio of the measured turbulent burning rate (assuming the entire flow of reactants is burned through the flame) to the extrapolated 2D burning rate inferred from the flame length to area ratio. These measurements suggest that a relative increase of a factor of up to 5 exists between the two at $Ka \sim 100$ for CH$_4$–air flames.

There is also significant disagreement in data obtained at nominally similar conditions [10, 11]. According to Damköhler [1], for turbulence scales larger than the laminar flame thickness, $\delta_l$, the increase in turbulent burning rate is a consequence of increased flame surface area due to flame wrinkling (first hypothesis), whereas for turbulence scales smaller than $\delta_l$, the enhancement arises due to the addition of turbulent diffusivities (second hypothesis). Direct Numerical Simulations (DNS) generally agree that, at low turbulent Reynolds numbers, the increase in turbulent burning rate is indeed proportional to flame surface area [12], whereas observations differ on the variation and magnitude of disproportionality at higher Reynolds numbers [3-5, 13-16].

For hydrogen-air mixtures, DNS in a box observed an exaggerated enhancement of the turbulent burning rate, exceeding the turbulent flame surface area enhancement by 14% on average [2]. For methane-air flames, similar single-step chemistry DNS in an inflow-outflow configuration suggests that proportionality is maintained strictly [15], whereas deviations from proportionality are observed when detailed chemistry and differential diffusion are included [3]; these deviations nevertheless
remain relatively small (up to 5%). Similar deviations observed in DNS of a range of hydrocarbon flames [17] were attributed to the added contribution of enhanced turbulent diffusivity. Recent DNS in the more realistic slot jet configuration [18] have reported similarly modest deviations for methane-air flames, increasing gradually with Reynolds number.

Whereas DNS is able to probe the relevant quantities such as the turbulent flame surface area, $A$, in 3D detail, experimental measurements of turbulent flame surface area have been largely extracted from 2D optical techniques, using the assumption that the average surface area per unit volume is equal to the average flame perimeter per unit area to determine flame surface density [11]. As a result, comparisons with DNS [19] have been limited in scope. This demonstrates the need for the development of advanced 3D measurement techniques. Wabel et al. [7] and Gülder et al. [6] made 2D flame surface density measurements in Bunsen burner flames at high turbulence intensities and found that the increase in the measured burning rate with turbulence intensity is significantly larger than that of the measured 2D flame surface area. Nivarti et al. [16] showed analytically that such exaggerations may arise from the corresponding enhancement of turbulent diffusivity in high turbulence intensities.

In order to test the analytical as well as DNS predictions, it is necessary to be able to measure the turbulent flame surface area in 3D. Few previous studies have used quasi-3D techniques for obtaining reacting scalar isosurfaces and their corresponding areas [20-22].

Recent studies [23-25] have demonstrated the feasibility and accuracy of using the volumetric laser induced fluorescence (VLIF) technique for surface reconstruction of burner stabilized 3D turbulent flames. However, this technique requires five cameras to capture CH radicals in the flame with a spatial resolution of 0.4 mm. Moreover, these previous studies did not present simultaneous measurements of the total burning rate at the same time as the flame surface area to investigate the validity of Damköhler’s hypothesis.

The present work adapts the method of Yip et al. [22] to visualise and reconstruct 3D turbulent flame surfaces and instantaneous volume using a high frequency laser [26-28]. The simultaneous
measurements of turbulent mass burning velocity $u_{tm}$ and flame surface area $A_{3D}$ made here allow a
direct determination of the ratio $I_0$ of burning rate enhancement and area enhancement:

$$I_0 = \left( \frac{u_{tm}/u_l}{A_{3D}/a_{3D}} \right)$$  \hspace{1cm} (1)

The experimental method used to determine the quantities $u_{tm}$, $u_l$, $A_{3D}$ and mean flame surface area, $a_{3D}$ are described in the following sections. The results are presented as a function of Karlovitz stretch factor $K$, defined as [29]:

$$K = \frac{1}{4} \left( \frac{u'}{u_l} \right)^2 Re_L^{1/2}$$  \hspace{1cm} (2)

where $Re_L = u'L/\nu$ is the turbulent Reynolds number, calculated using the turbulence root mean square (rms) velocity $u'$, the turbulence integral length scale $L$, and the kinematic viscosity $\nu$. The Karlovitz stretch factor $K$ may be interpreted as the ratio of turbulent strain rate based on the Taylor length scale, i.e. $u'/\lambda$, to the chemical rate given by $u_l/\delta_l$. The laminar flame thickness is estimated as $\delta_l = v/u_l$ [30]. Another dimensionless factor relating the chemical time scale, $\delta_l/u_l$, and strain rate, based on Kolmogorov length scales, $u_{\eta}/\eta$, is the turbulent Karlovitz number, $Ka$ [31, 32]. The Karlovitz stretch factor $K$ is related to Karlovitz number as $K = \sqrt{15}Ka$.

Previous work by Markstein [33] and Law [34] in laminar flames, followed by the work of Bradley and Lawes in turbulent flames [35, 36], suggests that the burning velocity is affected by both hydrodynamic instabilities and sensitivity to strain. The former depends on the heat release rate and the ratio of thermal to species diffusivity, represented by the Lewis number Le of the deficient species; the latter is represented by the Markstein number, Ma, defined as

$$Ma = -\frac{1}{\delta_l} \frac{\delta u_l}{\delta a}$$  \hspace{1cm} (3)
where \( a \) is the strain rate. The methane-air mixtures considered here all have positive Markstein numbers and Lewis numbers close to unity. Thus, the laminar burning velocity is expected to decrease slightly with increasing strain rate. The hydrogen-air mixtures have large negative values of Ma and high diffusivities of the deficient species H\(_2\), which suggests an increase in laminar burning velocity with increasing strain rate.

In the present work, the validity of Damköhler’s first hypothesis [37] is evaluated for spherically propagating turbulent flames over a range of Karlovitz numbers. The results show how the burning rates of the different mixtures change with turbulence intensity, while simultaneously measuring the total 3D flame surface area. The following section describes the experimental method including the technique for obtaining 3D flame surface areas and measurement of turbulent burning velocity. Results are shown in Section 3 followed by discussion of the measured ratio \( I_0 \) in Section 4.

2. Experimental method

Premixed turbulent spherical flames were ignited in a spherical combustion vessel of 380 mm internal diameter with three pairs of orthogonal windows of 150 mm diameter for optical access. One 2 kW electric heating coil provided controlled heating to the vessel. Initial mixture temperatures and static pressures were measured using a K-type thermocouple and a static pressure sensor [38]. Four fans, each driven by an 8 kW electric motor, generated near uniform isotropic turbulence in the central region of the combustion vessel. The turbulent rms velocity and integral length scale were determined in previous studies using laser Doppler anemometry [38, 39]. The integral length scale, \( L=20 \) mm, was determined to be independent of the fan speed between 1000 to 10,000 rpm, with a slight increase to 24 mm at the lower fan speed of 500 rpm. The turbulent rms velocity, \( u' \), was correlated with the fan speed \( N_f \) in rpm as:

\[
u' = 0.00119N_f \tag{4}\]

The Taylor length scale \( \lambda \) and Kolmogorov length scale \( \eta \) were found to vary between 1.2-2.6 mm and 0.03-0.12 mm, respectively.
2.1 Swinging laser sheet technique

The swinging laser sheet technique employs a double-cavity 532 nm Nd:YAG laser for Mie-scatter flame imaging, pulsing at a frequency of 54 kHz with a pulse energy of about 1.9 mJ. Multiple thin laser sheets are created, using a 16-faced rotating mirror at 12 Hz (Figure 1). A set of plano-concave and plano-convex lenses of diameters 50.8 mm and 50 mm respectively are used to converge the laser beam to a minimum beam thickness of 0.6 mm at the centre of the vessel, followed by a pair of cylindrical lenses to expand the laser beam into a sheet of less than 1 mm thickness. The focal lengths for the plano-concave and plano-convex lenses were 100 mm and 250 mm respectively, while the pair of plano-convex cylindrical lenses had focal lengths 38.1 mm and 25 mm. Using this optical system, a vertically expanded imaging laser sheet approximately 100 mm in height was produced across the central area of the combustion vessel. These laser sheets were made to swing at a velocity of about 54 m/s using the rotating mirror. Typically, 78 sheet images were recorded in each sweep of the combustion vessel separated by approximately 1 mm and 18 μs. The effective sweep duration, i.e. the time interval between the first and last sheet images recorded was 1.44 ms. The time interval between two successive sweeps was 5.2 ms. A high-speed Phantom VR camera, placed orthogonally to the imaging laser sheets captured 2D Mie-scattered images, using a fixed macro lens of focal length 105 mm, at every position of the laser sheet. A minimum aperture with large focal number of f/32 was used in order to generate a sufficiently large depth of field to cover the entire flame volume. The images were captured with an exposure time of 6 μs, at a resolution of 512 by 512 pixels. The chosen exposure time (6 μs) and large depth of field yielded sufficiently high contrast between the burned gas and the background. The size of each pixel was 0.196 mm, sufficiently small to resolve the Taylor length scale (which varies between 1.2-2.6 mm) but not the Kolmogorov scale (which varies between 0.03-0.12 mm). Olive oil droplets with density of 970 kg/m³ and measuring approximately 1 μm were used as seeding particles for the 2D Mie scattering images. The partial pressure of the seeding oil mixture (olive oil and air) was limited to less than 4% (180 mbar) of the total (fuel plus air) mixture at 0.5 MPa initial pressure. The oil alters the nominal equivalence ratios by between 1.1% to 3.8% over the range.
of equivalence ratios studied. Experiments with up to 8% partial pressure of seeding mixture showed no significant change in the mean turbulent burning rates.

A limitation of the presently described technique is the finite time (1.44 ms) required to sweep through a developing flame. During this period, the mean flame radius grows approximately linearly with time, so the error in mean radius is proportional to the sweep interval. Increasing the rotating mirror speed reduces the time between subsequent sheets but also increases the physical spacing between these, whilst simultaneously reducing the time between successive sweeps. Thus, a compromise exists between spatial resolution and flame growth during imaging and also the number of sweeps which may be imaged before the flame expanded beyond the field of view. The use of a higher repetition rate laser would improve all of these aspects, while the use of a rotating mirror with extra facets would reduce the mean sweep time and increase the temporal resolution. However, there exists a minimum facet size required to give a sufficiently broad sweep through the region of interest, which limits the possible number of facets for a given mirror size. Mixtures were selected in the present work which possessed a sufficiently low laminar burning velocity. This allowed highly wrinkled, turbulent flames

Figure 1. Schematic diagram for the 3D swinging laser sheet technique.
to be imaged with relatively little flame growth during each sweep. The implications of these limits to the uncertainty in measurements of flame radii are discussed in Section 3.

2.2 Flame surface area measurement

Mie-scattered binarized images in a sweep were stacked in a 3D volume matrix of 512 by 512 by 512 voxels in a sequence with respect to their spatial and temporal positions. The sweep-reconstructed volume was approximately 100 x 100 x 104 mm$^3$. The resolution in the sweep direction was 0.74 mm. Further details of the laser sheet geometry and synchronization control system are available in the corresponding author’s PhD thesis [40]. Matlab scripts were used to process the 2D contour images into 3D surfaces. The finite spacing between the successive images was filled by thickening the images, using an interpolation function to generate a solid 3D reconstruction. This leads to a stepped flame surface appearance, and hence the reconstructed flame was smoothed using an interpolative algorithm developed by Taubin [41]. This algorithm retained the flame curvature while smoothing the high frequency surface detail on the flame surface. The algorithm had the additional advantage of incorporating an expansion term to minimise shrinkage during smoothing; without this, the flame surface would be eroded by the smoothing process, giving rise to a reduction in flame (burned gas) volume. Nevertheless, this procedure decreased the total flame surface area $A_{3D}$ by about 25% for the largest and highly turbulent reconstructed flame in the present measurements compared to the unsmoothed precursor reconstruction. It is important to note, however, that the original, highly stepped unsmoothed flame reconstruction prior to smoothing is not a good representation of the flame with a concomitant high surface area; instead, the smoothed surfaces more closely represent the appearance of the actual flame imaged. Further quantitative analysis of the effect of surface smoothing and increasing $u'$ on total surface area is available in [42].

Following interpolation, a triangulated surface mesh was generated and a smoothed flame surface was obtained using Taubin algorithm [41]. The area of each triangle in the smoothed surface mesh was
calculated using the cross product of vectors that represent the sides of the triangle, and the triangle areas were summed to yield the total surface area $A_{3D}$ of the reconstructed flame.

The triangulated surface mesh was converted into a solid reconstruction consisting of voxels (volume pixel), discretising the triangulated surface by splitting and refining each face until the longest edge is smaller than half of a voxel [43]. The voxel is then set beneath the vertex coordinates of the original face to a value of unity. Therefore, with the volume of each voxel known, the entire volume of 3D reconstructed flame, $V_{3D}$, was obtained by summing up of all voxels.

The volume-equivalent radius is the radius of the equivalent spherical flame based on the volume of burned gas $V_{3D}$:

$$r_{3D} = \left(\frac{3V_{3D}}{4\pi}\right)^{1/3} \quad (5)$$

The equivalent mean flame area is then defined as $a_{3D} = 4\pi r_{3D}^2$.

Based on the assumption of isotropy [35], the total flame surface area corresponding to 2D measurements, $A_{2D}$, is estimated using the centre line sheet image in a volumetric sweep, by calculating the perimeter of the 2D flame image and then equating to that of an equivalent circle as, $P = 2\pi R_{2D}$. The total flame surface area from 2D estimates is calculated as, $A_{2D} = 4\pi R_{2D}^2$. The mean flame radius, $r_{2D}$, is calculated from the cross-sectional 2D flame area of the centreline image.

The corresponding mean flame area, $a_{2D}$ is then calculated as $a_{2D} = 4\pi r_{2D}^2$.

### 2.3 Turbulent flame speed measurement

The rise in pressure within the combustion vessel during flame propagation was measured using a Kistler 5110 piezo-electric pressure transducer (calibrated to ±0.5% of full scale at 5.0 MPa). The pressure record was used to obtain the volume-equivalent radius, $r_m$, of the flame according to [29, 35]:

$$r_m = R_0 \left\{ 1 - \left(\frac{p_f}{p}\right)^{1/\gamma u} \left[\frac{p_f - p}{p_f - p_0}\right]\right\}^{1/3} \quad (6)$$
where $R_0$ is the volume equivalent vessel radius, $P_0$ and $P_f$ are the initial and peak pressures, and $\gamma_u$ the ratio of specific heats for the unburned mixture.

The corresponding turbulent burning velocity $u_{tm}$ was then obtained from the fractional burning rate according to [29]:

$$u_{tm} = \left(\frac{P_0}{P}\right)^{1/\gamma_u} \left\{1 - \left(\frac{P_0}{P} \right)^{1/\gamma_u} \left[\frac{P_f - P}{P_f - P_0}\right]\right\}^{-2/3} \frac{R_0}{3(P_f - P_0)} \frac{dP}{dt}$$  \hspace{1cm} (7)

Under ideal conditions, it would be possible to calculate turbulent burning velocities either from the rate of change of the burned volume $V_{3D}$ obtained from the high-speed flame reconstruction procedure outlined in 2.2 or from the pressure history. As discussed further in Section 3, the pressure rise is too small at the early burn times compatible with flame imaging, and, hence, cannot be used to determine $u_{tm}$ accurately. Whereas differentiating the rate of change of the burned volume, $V_{3D}$, might offer an alternative method, the sparse time history of the burned volume, $V_{3D}$, means that the determination of the turbulent burning rate in this manner (with intervals of 5.2 ms between successive sweeps, corresponding to the mean sweep time), is also too inaccurate for direct comparison. In the future this could be alleviated by increasing the laser firing frequency to improve the temporal resolution.

In the present work, however, values for $u_{tm}$ are extracted from the pressure records only at pressures higher than 0.5% of the initial pressure; under these conditions, pressure differences (rate of change of the burned volume differentiated over an interval of 0.02 ms) can be measured within 1.0% for the highest $u'$ employed, yielding measurements of $u_{tm}$ that are accurate to within 20%.

At these higher pressure rise conditions, however, the flame radii are too large to be observable with the 3D reconstruction technique. The remedy is to use the empirical observation that the calculated burning rate varies linearly with flame radius, and extrapolate the measurements obtained from the pressure record to the smaller radii where the 3D reconstruction technique can be employed.
The uncertainties in the final value of \( u_{tm}(r_m) \) were estimated from the squared sum of uncertainties in \( u_{tm} \) with the relative error of \( r_m \). The latter was estimated from the uncertainty in the flame position owing to the growth of the radius during the laser sweep time of 1.44 ms. The relative uncertainties in \( r_m \) varied between 17% to 5% from the highest to the lowest propagating speed. The uncertainties in \( u_{tm} \) varied between 5% to 21% at the highest and lowest radii for different propagating speeds. The propagation of compound uncertainty in \( u_{tm}(r_m) \), calculated as the sum of squares of the instrument and the rms fluctuations, due to extrapolation, at \( r_m=30 \) mm is estimated to be 20%. Details of the uncertainty in the extrapolation method are presented in the Appendix (Figures A5 – A12), and results are presented in Section 3.

Values of the unstretched laminar burning velocity, \( u_l \), are taken from literature [44, 45], some of which have been confirmed in the present study: see Figure A4 in supplementary material. Details on the measurement of \( u_l \) are presented in [46].

Table 1: Experimental conditions for the present study, (+) indicates the estimated value based on the given reference; (*) indicates values from the references for \( u_l \) and Ma. Values of \( u_l \) without an asterisk mean that measurements were made in the present study.

<table>
<thead>
<tr>
<th>Fuel</th>
<th>CASE</th>
<th>( \phi )</th>
<th>T (K)</th>
<th>P (MPa)</th>
<th>( u' ) (m/s)</th>
<th>( K ) (-)</th>
<th>( u_l ) (m/s)</th>
<th>Ma (-)</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH(_4)</td>
<td>I</td>
<td>0.60</td>
<td>365</td>
<td>0.1</td>
<td>0.3-1.5</td>
<td>0.08-0.903</td>
<td>0.131</td>
<td>2.0*</td>
<td>[47]</td>
</tr>
<tr>
<td></td>
<td>II</td>
<td>0.70</td>
<td>300</td>
<td>0.1</td>
<td>0.3-1.5</td>
<td>0.027-0.30</td>
<td>0.210*</td>
<td>2.7*</td>
<td>[44]</td>
</tr>
<tr>
<td></td>
<td>III</td>
<td>1.30</td>
<td>300</td>
<td>0.1</td>
<td>0.3-2.0</td>
<td>0.046-0.79</td>
<td>0.160*</td>
<td>3.9*</td>
<td>[44]</td>
</tr>
<tr>
<td></td>
<td>IV</td>
<td>1.25</td>
<td>365</td>
<td>0.5</td>
<td>0.3-2.0</td>
<td>0.019-0.33</td>
<td>0.241</td>
<td>4.8*</td>
<td>[47]</td>
</tr>
<tr>
<td>H(_2)</td>
<td>V</td>
<td>0.30</td>
<td>365</td>
<td>0.5</td>
<td>0.3-2.0</td>
<td>0.049-0.85</td>
<td>0.102*</td>
<td>-5.0*</td>
<td>[45]</td>
</tr>
<tr>
<td></td>
<td>VI</td>
<td>0.40</td>
<td>365</td>
<td>0.5</td>
<td>0.3-1.5</td>
<td>0.008-0.09</td>
<td>0.286*</td>
<td>-6.3*</td>
<td>[45]</td>
</tr>
</tbody>
</table>

3. Results

Figure 2(a) shows a typical raw 2D Mie-scattered centre line image in a sweep, acquired at 21.5 ms after ignition, for a CH\(_4\)/air flame at \( \phi =0.7, P=0.1 \) MPa, \( T=300 \) K and at low value of turbulence, \( u'=0.3 \) m/s. Laser light intensity is higher in the centre of the image than at the edges due to the
Gaussian nature of the illumination. The flame was mildly wrinkled and, as a result, the flame edge was clearly defined. These raw flame images were then binarized, as shown in Figure 2(b) before using them for 3D reconstruction.

Figure 2. (a) Raw Mie-scattered image (b) Binarized image

Typical 3D reconstructed flame surfaces are shown in Figure 3, for a CH₄/air flame at φ =0.7, 300 K and 0.1 MPa, for u’=0.3 m/s. Each 3D reconstructed image corresponds to a single sweep of 2D image acquisition. For a given experiment, flame wrinkling develops with time as the flame interacts with the surrounding turbulence.

Figure 3. Temporal development of a 3D reconstructed CH₄/air flame at, φ =0.7, T=300K and P= 0.1 MPa during a deflagration at u’=0.3 m/s.

Figure 4 shows 3D flames for two different u’ values of 0.3 and 1.5 m/s. Flame surface area, wrinkling and burning rate all increase with increasing u’ as the flame becomes progressively more distorted. It is worth noting that the flame represented in Figure 4(b) propagated faster than that in Figure 4(a), and hence it developed more quickly and correspondingly fewer sweeps were recorded during the combustion event for flame reconstructions.
Figure 4. 3D reconstructed flame surfaces for CH$_4$/air flames at, $\phi=0.7$, T=300 K, P=0.1 MPa (a) $u' = 0.3$ m/s at $t = 27.6$ ms (b) $u' = 1.5$ m/s at $t = 11.1$ ms after ignition. Dashed circle indicates mean flame radius, $r_{3D}$. The scales on the axes denote pixel numbers, where each pixel corresponds to 0.196 mm.

Figure 5. Mean flame radius for CH$_4$/air flames at, $\phi=0.7$, T=300 K, P=0.1 MPa (a) $u' = 0.3$ m/s (b) $u' = 1.5$ m/s.

The mean radii, $r_{3D}$, obtained from 3D reconstructions, for Figure 4, are presented in Figure 5, along with the mean flame radii, $r_{2D}$, obtained from centre line 2D slices following the method outlined in Section 2.2. The 2D estimates are generally lower than the 3D measured radii regardless of operating condition. The values of the mean flame radii, $r_m$, obtained from pressure records are also plotted in Figure 5 (cross symbols), and reassuringly overlap with those of $r_{3D}$. Further comparison of the full 2D vs 3D statistics is beyond the scope of the present work and left for a future investigation.

Considerable variations in mean flame radius $r_{3D}$, were observed between measurements, even under nominally identical conditions. This was also reflected in the variability of the flame shape, as shown
in Figure 6 by the images of three flames taken under nominally identical conditions at the same time 
\[ t = 16.3 \text{ ms after ignition} \]. In the present study, the variability in the mean flame radius \( r_{3D} \) at a specified 
time, ranges between approximately 5\% for small flame radii and up to 17\% at larger flame radii.

Figure 6. 3D reconstructed CH\(_4\)/air flame surfaces (P = 0.1 MPa, T = 300K, \( \phi = 0.7 \)) at the same time 
\[ t = 16.3 \text{ ms after ignition} \] for three different runs at \( u' = 0.3 \text{ m/s} \).

Figure 7. Temporal development of a 3D reconstructed H\(_2\)/air flame at, \( \phi = 0.4, T=300K \) and \( P= 0.5 \) 
MPa during a deflagration at \( u' = 0.3 \text{ m/s} \).

An example of 3D reconstructed flame surfaces for a H\(_2\)/air flame at \( \phi = 0.4, 365 \text{ K} \) and 0.5 MPa, for 
\( u' = 0.3 \text{ m/s} \) are shown in Figure 7. Unlike CH\(_4\)/air flame, shown in Figure 3, the H\(_2\)/air flame develops 
quickly and therefore only four sweeps could be captured. These flames exhibit considerable flame 
wrinkling and at any given time, larger diameters are achieved compared to CH\(_4\)/air flame indicating 
higher burning rates.
Figure 8. Effect of turbulence intensity on (a), (d) time history of mean flame radius $r_{3D}$ and (b), (e) area ratio $A_{3D}/a_{3D}$ as measured by 3D reconstruction as a function of mean radius. Symbols represent individual measurements and lines are best fit second order polynomials. Dashed red lines indicate the extrapolation of best fit curves to $r_{3D} = 30$ mm. (c), (f) Averaged pressure traces (right hand inverted scale) and corresponding $u_{tm}$ values calculated using Eq. (4). Error bars are standard
deviations from the three experiments. Dashed red lines indicate linear best fits, extrapolated to $r_m = 30$ mm.

Figure 8 shows the effect of turbulence rms velocity on the measured growth of the mean flame radius $r_{3D}$ for the CH$_4$ II and H$_2$ V flames in Figure 8(a) and (d) while the corresponding flame surface area ratio $A_{3D}/a_{3D}$ in Figure 8(b) and (e) respectively. Three experiments were conducted for each set of nominally identical conditions. However, for values of $u'$ larger than 1.5 m/s, only two experiments were successfully reconstructed, because the flames drifted away from the optical access region. The largest flame that could be reconstructed measured $r_{3D} = 46$ mm for the CH$_4$ I case at $u'=0.3$ m/s. An overall root mean square variation in $r_{3D}$ of up to 17% (for 3 experiments) was recorded for CH$_4$ I and CH$_4$ III flames. This variability is due to the fact that these flames were close to the flammability limits and is also confirmed from their susceptibility to quenching; in comparison, H$_2$ flames exhibited low variability in $r_{3D}$ as these are expected to display stable burning (See Figure A14 in supplementary material for different burning regimes as taken from reference [48]). As discussed in Section 2.3, $r_{3D}$ values measured from image data can only be extended up to about 30 mm, for the range of $u'$ employed, due to the limitations of the imaging system, but the pressure rise at this radius was very small, leading to poor signal to noise ratio. The compromise requires evaluating both $A_{3D}/a_{3D}$ and $u_{tm}$ at a particular point, which is chosen as $r_{3D} = r_m = 30$ mm. The present work uses pressure-based values of $u_{tm}$ at higher pressures and radii, followed by a short linear extrapolation towards 30 mm, as indicated by the dashed lines in Figure 8(c). Flames at this radius were well developed, independent of the spark effect [49] and the turbulent flow field was near isotropic and well characterized [39].

Estimates of the observed variance of three experiments are shown in Figure 8(c) using error bars; the variance ranges from 10-20% for the lowest values, down to 5-10% for the highest values of $u_{tm}$. Assuming that the uncertainties associated with the sampling frequency and statistical variability are independent, we conclude that the uncertainties in the measurement of the turbulent flame speed are of the order of 6-34% for the most favourable to the most unfavourable cases. Data for $r_{3D}$, $A_{3D}/a_{3D}$ and $u_{tm}$ for the remaining cases are provided in the supplementary material (see Figures. A1-A3). The
estimated increase in mean flame radius $r_{3D}$ for CH$_4$ flames during the effective sweep duration of
1.44 ms is between 3% and 11% for the lowest and highest turbulent velocities employed, respectively.

However, for H$_2$ flames, the corresponding change in radius can reach up to 19% at the highest $u'$ due
to the high burning rates of these flames. This maximum estimated flame radius change during the
effective sweep time at the highest $u'$ values is close to the maximum observed variance of 17%. Faster
repetition rate imaging lasers could eventually allow smaller uncertainties during a sweep.

The higher degree of wrinkling observed for H$_2$/air flames arises additionally due to the high initial
pressures at which these measurements were conducted. Under such conditions, the onset of Darrieus-
Landau and thermo-diffusive instabilities occurs at an early stage of flame development [50]. As a
consequence, higher burning rates are achieved in H$_2$/air flames and this is in line with the higher $u_{tm}$
values obtained from pressure data.

A consistent comparison of these observed wrinkling phenomena would require CH$_4$/air and H$_2$/air
flames to be compared at same initial pressures and temperatures. Figure 9 compares the 3D
reconstructed CH$_4$/air flames and H$_2$/air flames at P = 0.5 MPa and T=365 K. These are compared at
the same time for different $u'$ values. The 3D reconstructions obtained reveal significant small scale
wrinkling for H$_2$ air flames even at low $u'$ values, which points to the role of Darrieus-Landau and
thermo-diffusive instabilities, caused by small Le numbers for lean H$_2$ air flames. It is interesting to
note that in the case of CH$_4$-air flames, the small scale wrinkling increases as $u'$ is increased, whereas
it is apparently less distinguishable with increasing $u'$ in the case of H$_2$.

To better visualise this phenomenon, the corresponding centre line 2D binarised flame images are
compared in Figure 10. As revealed by the 3D reconstructed flames, even at $u'$=0.3 m/s and at $t=16.3$
ms, H$_2$/air flames have higher small scale wrinkling compared to CH$_4$/air flames. In addition, the 2D
images reveal that as the $u'$ increases from 0.3 to 1.0 m/s, more finger like structures and cusps appear
in H$_2$/air flames. As $u'$ is increased further, these finger-like structures not only deepen in H$_2$/air flames
but also in CH$_4$/air flames.
Figure 9. Comparison of 3D reconstructed CH4-air flames and H2/air flames at P = 0.5 MPa and T=365 K, (a), (d) at $u' = 0.3$ m/s and $t = 16.3$ ms (b), (e) at $u' = 1.0$ m/s and $t = 5.9$ ms (c), (f) at $u' = 1.5$ m/s and $t = 5.9$ ms
Figure 10. Comparison of binarised Mie-scattered centre line sheet flames for CH₄/air flames and H₂/air flames at P = 0.5 MPa and T=365K, (a), (d) at $u' = 0.3$ m/s and $t = 16.3$ ms (b), (e) at $u' = 1.0$ m/s and $t = 5.9$ ms (c), (f) at $u' = 1.5$ m/s and $t = 5.9$ ms.
Figure 11. Variation of $u_{tm}/u_t$ with $K$ for the experimental conditions in Table 1. Symbols represent the average of three experiments.

Measured values of $u_{tm}/u_t$ evaluated for CH$_4$ and H$_2$ mixtures are shown in Figure 11, plotted against the Karlovitz stretch factor, $K$, at the largest resolved burned gas radius of $r_{3D} = 30$ mm. Values of $u_{tm}/u_t$ for CH$_4$ appear to increase with a small power of $K$, reaching a maximum of around 5 for $K = 1$ in the CH$_4$-II and CH$_4$-I mixtures. In comparison, $u_{tm}/u_t$ values for H$_2$ appear to be very sensitive to both composition and $K$, attaining values as high as 18 for $K = 1$. There is only sparse literature data that can be compared directly with the present conditions, particularly because the measurements were taken for mixtures far from stoichiometric. However, values of burning rates for CH$_4$ at $\phi = 1.3$ and $P = 0.5$ MPa at $u' = 2$ m/s measured by Lawes et al. [51] show $u_t$ of 0.89 m/s compared to the present measurements of 1.24 m/s. Measurements have also been made for CH$_4$ at $\phi = 0.7$ at 0.1 MPa and $u' = 0.5$ to 2 m/s in combustion vessels by Shy et al. [52] with measured $u_t$ within 20% of the current values. In the case of H$_2$, the measurements of $u_t$ from Kitagawa et al. [53] at 300 K (rather than the current 365 K) and 0.5 MPa at 1.6 m/s are about 34% higher than the current measurements.
Given the variability in these measurements and expected errors due to shot to shot experimental variations in the instantaneous flow field and burning rate, the current measurements of $u_t$ are within 10-34% of the previous measurements under similar conditions; any conclusions must therefore take these variabilities into account.

![Graph](image)

**Figure 12.** Effect of turbulence intensity represented through $K$ on the area ratio $A_{3D}/a_{3D}$ for the experimental conditions in Table 1.

In Figure 12, the corresponding values of measured flame surface area ratio $A_{3D}/a_{3D}$, are plotted as a function of $K$. Since only three experiments were conducted at identical conditions, significant variations might be expected. Nevertheless, clear trends emerge, and it is interesting to note that the values of $A_{3D}/a_{3D}$ appear to be reversed relative to the turbulent burning rate behaviour observed in Figure 11: large $A_{3D}/a_{3D}$ rise, comparable to the rise in turbulent burning rate in the case of CH$_4$ and, relatively small $A_{3D}/a_{3D}$ rising with a small power of $K$ in the case of H$_2$. For CH$_4$, at equivalence ratio of 0.6 and 1.30 (I and III), the sensitivity to turbulence level increases sharply for the higher levels of turbulence. For equivalence ratios 0.70 and 1.25, cases II and IV, no data is available beyond $K =$
For very lean or very rich conditions (I and IV), flames were often convected away from the centreline. Yet for H₂, only a small rise in the turbulent flame area is measurable in either case, in spite of the large increase in turbulent flame speed measured, as shown in Figure 11.

![Diagram]

Figure 13. Variations in the ratio $I_0$ as a function of Karlovitz stretch factor, $K$.

Finally, measurements can be obtained for the ratio $I_0$ of the turbulent enhancement in burning rate, $u_{tm}/u_l$ (Figure 11), to that of flame surface area, $A_{3D}/a_{3D}$ (Figure 12). The ratio, $I_0$, is shown as a function of $K$ in Figure 13. The measurements show significant differences for the two fuel-air mixtures. In the case of CH₄, values of $I_0$ are around 1.0±0.5, which is hardly distinguishable from unity within the current uncertainty range. In the case of H₂ mixtures, however, $I_0$ exceeds unity over a wide range of conditions by factors of up to 6. It is important to mention that the smoothing effect which reduces the total flame surface area, $A_{3D}$, by up to a maximum of 16% at $r_{3D} = 30$ mm increases the corresponding $I_0$ values (in Figure 11) by up to a maximum of 13% for CH₄ flames and 10% for H₂ flames. These differences do not alter the primary observation: $I_0$ values remain close to unity for CH₄ flames, but increase with $K$ to values much larger than unity for H₂ flames.
An attempt was made to calculate the $A_{2D}/a_{2D}$ from the centre line flame images following the method outlined in section 2.2. Figure 14 shows the $A_{2D}/a_{2D}$ against increasing $K$ at mean radius $r_{2D} = 30$ mm. A general trend of increase in $A_{2D}/a_{2D}$ with $K$ is observed, however, unlike 3D measurements, here the values are found to reach as high as 20 at maximum. As discussed in section 3 under Figure 5, $r_{2D}$ are generally found to be lower than $r_{3D}$. Since the mean areas are proportional to the square of mean radius, the resulting $a_{2D}$ are much lower than $a_{3D}$ and therefore lead to higher values of $A_{2D}/a_{2D}$ in comparison to $A_{3D}/a_{3D}$ (Figure 12). Had these values been used to calculate the $I_0$ factor, for the same $u_{tm}/u_1$ presented in Figure 11, the resulting numbers would suggest that $I_0$ would be close to or less than unity. The reason for the large difference between the 3D and 2D is not entirely understood, but it is important to mention that the propensity of the flames to drift away from the centre of the vessel at high $u'$ increase, which means that the slice obtained by a 2D measurement technique through the centre of the developing flame may not be representative of the overall 3D structure.
5. Discussion

The present 3D measurements suggest that, for CH$_4$–air flames, the turbulent burning rate is roughly proportional to the flame area. There appears to be little enhancement of burning rate beyond that of flame surface area, even though the turbulence velocities are lower than in the case of the Bunsen flame measurements [6, 7].

By contrast, for the H$_2$ flames, even weakly wrinkled flames achieve significantly higher rates of burning than is suggested by the flame surface area measured simultaneously by the Mie-scattering technique. Additional mechanisms may be at work here, including thermo-diffusive instabilities or locally distributed reaction [3].

Due to the Darrieus-Landau and thermo-diffusive instabilities, apparent from finger like structures in Figure 9 and Figure 10, higher burned gas volumes and burning rates are achieved in H$_2$/air and CH$_4$-IV flames, as also apparent from Figure 8(f) (see also Figure A2(b) in supplementary material). High burned gas volumes result in larger mean flame areas, $a_{3D}$, that would lower $A_{3D}/a_{3D}$ ratios, see Figure 12. Consequently, even at low $K$, the $I_0$ values deviate significantly from unity particularly so for H$_2$/air flames.

A preliminary comparison between the flame surface area ratios derived from the 3D and 2D methods is presented in Figure 15. Due to sparse 3D DNS data, data for only CH4-II case are compared. It is apparent that the surface area ratios obtained from the present 3D technique are close to the DNS results while the 2D method over estimates the value by 4 times at high turbulence. Given the inherent nature of 3D movement of the flames, it is difficult to slice the flame through the centre (2D method) especially for flames at high turbulence levels (high K values) since these are displaced from the centre at an early stage. This explains the over estimation of $A/a$ by 2D method. The present 3D technique has the advantage of capturing such flames with greater accuracy where the assumption of isotropy may not hold. More quantitative 3D data is required to confirm the differences discussed above.
An important outcome of the present work, is to demonstrate that it is possible to measure 3D flame surface areas simultaneously with burning rates. These measurements have enabled the investigation of how turbulence simultaneously affects flame surface areas and propagation rates. The present measurements will serve to provide the experimental underpinnings for further modelling efforts.

5. Conclusions

The present results provide simultaneous 3D measurements of turbulent burning rates and turbulent flame surface areas in a closed combustion vessel, using pressure traces and a high frequency swinging laser sheet technique with flame area reconstruction. These measurements have allowed for the first time the determination of the ratio of turbulent burning rate enhancement to the flame surface area enhancement in the presence of turbulence.

Figure 15. Comparison of flame surface area ratios obtained from 3D and 2D methods (present work) along with 3D DNS [15].
Measurements of the enhancement ratio $I_0$ show that for both rich and lean CH$_4$-air mixtures, the enhancement ratio varies, but is not higher or lower than the inherent uncertainty in the experimental technique. In the case of H$_2$-air flames, however, the evidence is unambiguous and shows that even at low Karlovitz stretch factor $K$ (up to ~0.5), the increase in burning rate is significantly greater than the expected increase based on the measured increase in flame surface area, by a factor of up to 6.

The present measurements also show that the usual assumptions of isotropy used to reconstruct 3D from 2D flames in spherically propagating kernels are not well supported: 3D area ratios are significantly different from 2D ratios, and that may possibly be a result of movement of flame kernels.

Whereas the present work shows the feasibility of high frequency 3D reconstruction of flame surfaces, further work is needed to improve the capabilities of the sweep rate, imaging resolution and field of view beyond the current state to improve the applicability of these measurements to higher flame speeds, and to allow the direct comparison of pressure measurements and volumetric methods of determining the burning rate.

ACKNOWLEDGEMENT

PA thanks University of Leeds for a Research Scholarship Award. GVN acknowledges the funding support of EPSRC grant EP/P022286/1.

REFERENCES


Click here to access/download
**Supplementary Material**
Supplementary material_FINAL_REVISED.docx
Click here to access/download
**Additional Material**
Article_DATA_CnF_R1.xlsx
Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: