

Discussion of “A cantilever approach to estimate bending stiffness of buildings affected by tunnelling” by Twana Kamal Haji, Alec M. Marshall, and Walid Tizani

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**Abstract**

This discussion considers the procedure proposed by Haji, Marshall and Tizani for the assessment of the structural stiffness of frame structures subjected to tunnelling. The discussion focuses on the potential contribution of both shear and bending flexibilities to the response of frame structures to tunnelling, as well as the role of the foundation scheme on the boundary conditions at the base of the structure. The validity of applying the proposed set of equations within currently available methods of prediction of tunnelling-induced deformations, based on modification factors, is also discussed.

*Keywords:* Tunnelling, Soil-Structure Interaction, Building Response

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1 The work of [Haji et al. \(2018\)](#) is of interest to both structural and geotechnical  
2 engineers involved in tunnel-structure interaction (TSI) projects. It  
3 illustrates that the reaction response of 3D framed buildings to tunnelling-  
4 induced settlements depends on frame characteristics and configuration. Im-  
5 portantly, [Haji et al. \(2018\)](#) considers the contribution of columns to in-  
6 creasing structure stiffness, the effects of the the number of building bays  
7 and the number of building storeys, and demonstrates that upper storeys in  
8 high-rise frame building contribute only marginally to the structure response  
9 at the foundation level, which is currently neglected by stiffness assessment  
10 methods.

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11 In the following, this discussion evaluates [1] the proposed method to  
 12 estimate the structure stiffness, [2] the assumed displacement boundary con-  
 13 ditions for the frame, and [3] the possibility of integrating this method with  
 14 currently available prediction methods for tunnelling-induced deformations.

15 [1] To assess the frame stiffness of a linear elastic 3D framed structure  
 16 subjected to deformations given by a tunnelling-induced settlement trough  
 17 for an eccentric tunnel-structure configuration the following procedure was  
 18 implemented at stage 5. The structure is separated from the soil and founda-  
 19 tion. Then, the structure stiffness (i.e. reaction forces induced by nominal  
 20 displacements) is calculated imposing a mix of force (FBCs) and displacement  
 21 (DBC) boundary conditions at the frame base. To replicate the effects of  
 22 the greenfield settlement trough, vertical FBCs ( $\mathbf{P}_z$ ) and fixed vertical DBCs  
 23 ( $\mathbf{u}_z = 0$ ) are imposed at the structure base within and outside the tunnel in-  
 24 fluence zone, respectively, whereas horizontal ( $\mathbf{u}_x$ ) and rotational ( $\Phi$ ) DBCs  
 25 are fixed ( $\mathbf{u}_x = 0, \Phi = 0$ ). This approach is equivalent to defining a con-  
 26 densed stiffness matrix of the superstructure ( $\mathbf{K}_c$ ) with respect to the degrees  
 27 of freedom of the base through FEM analyses. Then, the structure response  
 28 to tunnelling is characterised by the set of FBCs  $\mathbf{P}^T = [\mathbf{P}_z \ \mathbf{P}_x \ \mathbf{M}]$  for a  
 29 given set of DBCs  $\mathbf{u}^T = [\mathbf{u}_z \ \mathbf{u}_x \ \Phi]$  (i.e.  $\mathbf{P} = \mathbf{K}_c \mathbf{u}$ ). Subsequently, a scalar  
 30 value of stiffness  $K_b$  was obtained by relating  $\mathbf{u}_x$  to  $\mathbf{P}_z$  as detailed in Equa-  
 31 tions (6) and (16). This approach allows characterising a given 3D frame  
 32 with a unique scalar value of stiffness. However, the impact of applying a set  
 33 of forces  $\mathbf{P}_z$  in the region affected by tunnelling rather than a distribution  
 34 of displacements  $\mathbf{u}_z$  equal to the greenfield settlement trough (as previously  
 35 done by [Losacco et al. \(2014\)](#)) would be of interest.

36 It is important to clarify that the parameter  $K_b$ , which was defined as  
 37 the “bending stiffness” by [Haji et al. \(2018\)](#), is a total stiffness derived from  
 38 the point load analogy given in Eq. (5). As discussed,  $K_b$  is derived from the  
 39 condensed stiffness matrix of the structure  $\mathbf{K}_c$ . In addition, if a Timoshenko  
 40 beam was used to develop the point load analogy, the total stiffness  $K_b$  would  
 41 depend on both the flexural rigidity  $EI$  and the ratio between Young’s and  
 42 shear moduli  $E/G$ , which are related to the bending- and shear-type flexibil-  
 43 ities of 3D frame structures. The terms bending- and shear-type flexibilities  
 44 describes the global deflection response of the frame within a bay as follows:  
 45 in the bending-type flexibility, the differential settlement between adjacent  
 46 columns is due to axial deformations of beams/slabs (that relates to the av-  
 47 erage curvature within a bay); in the shear-type flexibility, this differential  
 48 settlement is due to deflection of beams/slabs between columns that remain

49 vertical. Note that these two terms are not used to indicate the strains of  
50 an individual element within the 3D frame (i.e. a single columns or slab  
51 span). On the other hand, for the Euler-Bernoulli beam that is adopted  
52 to develop the point load analogy (see Equation (4)), the total stiffness is  
53 only due to the bending flexibility (i.e. deflection increase is only due to  
54 the beam curvature). Although the definition adopted by [Haji et al. \(2018\)](#)  
55 is formally correct for the adopted equivalent beam, it may be a source of  
56 misunderstanding in the context of geotechnical engineering and tunnelling.  
57 Therefore, in this discussion, the parameter  $K_b$  is referred to as the “total  
58 stiffness” to highlight that it does not distinguish between the contributions  
59 of shear and bending flexibilities.

60 In Figure (18), [Haji et al. \(2018\)](#) compared the total stiffness values  $K_b$   
61 against predictions made through the stiffness assessment method proposed  
62 by [Franzius et al. \(2006\)](#). However, the procedure of [Franzius et al. \(2006\)](#) al-  
63 lows estimating a total/equivalent flexural rigidity  $EI$  of the structure (that  
64 does not account for the shear flexibility), whereas the total stiffness  $K_b$  also  
65 accounts for the shear flexibility. Although the actual structure response to  
66 tunnelling depends on the total stiffness, it would be useful to distinguish  
67 between these two contributions to define equivalent beams/solids that are  
68 meant to represent 3D frames. In point [2], the shape of the structure set-  
69 tlement profile is further discussed.

70 [2] [Haji et al. \(2018\)](#) does not discuss the physical bases for the assumed  
71 DBCs ( $\mathbf{u}_x = 0$ ,  $\Phi = 0$ ) that, in reality, would be related to the foundation  
72 scheme. For raft or continuous strip foundations transverse to the tunnel  
73 longitudinal axis, tunnelling-induced differential horizontal movements at the  
74 structure base are minimal ([Goh and Mair, 2014](#); [Dimmock and Mair, 2008](#)),  
75 which is consistent with the DBCs adopted. For separated footing and/or  
76 strip foundations orientated along the longitudinal axis, tunnel-structure in-  
77 teraction results in differential horizontal displacements within the founda-  
78 tion ([Goh and Mair, 2014](#); [Franza and DeJong, 2017](#)); for these cases, the  
79 DBCs analysed by the authors are not representative. Therefore, the hori-  
80 zontal DBCs ( $\mathbf{u}_x$ ) considered only apply directly to raft and transverse strip  
81 foundations.

82 On the other hand, the rotational DBCs were also fixed ( $\Phi = 0$ ). Although  
83 raft foundation or separated footings may be sufficiently rigid to prevent  
84 relative rotations between the column base and the foundation, it is likely  
85 that the foundation itself rotate. For long continuous foundations (e.g. rafts  
86 or transverse strip foundations), deflections will cause associated rotations

87 that vary smoothly with the horizontal offset from the tunnel centreline.  
88 For relatively rigid separated foundations, the individual foundations may  
89 rotate quite differently from each other, and also quite differently than the  
90 local slope of the greenfield settlement profile due to interaction with the  
91 structure.

92 In general, the total structural stiffness at the ground level also depends  
93 on the foundation scheme. However, to provide upper and lower bound  
94 estimations of the impact of the foundation rotational and horizontal degrees  
95 of constraint, further research could assess  $K_b$  resulting in from four possible  
96 combinations of DBCs:  $\mathbf{u}_x$ = fixed, released;  $\Phi$ =fixed, released.

97 [3] Previous research reported the variation of the structure deformation  
98 shape with respect to the greenfield settlement trough (Farrell et al., 2014;  
99 Potts and Addenbrooke, 1997). On the other hand, in the procedure proposed  
100 by Haji et al. (2018), the length of structure affected by tunnelling (assumed  
101 to behave as a cantilever in Figure (14)) is fixed a priori and does not depend  
102 on soil-structure interaction. This assumption could lead to an erroneous  
103 estimation of the stiffness. Further research is needed to relate the deformed  
104 shape of frames and greenfield input to bending and shear flexibilities.

105 Although Haji et al. (2018) indicated that the total stiffness value can  
106 be used to inform analyses of tunnel-building interaction, it is not fully clear  
107 the envisioned application. It is important to consider the applicability of  
108 the empirical formulas proposed by Haji et al. (2018) within the modification  
109 factor framework (e.g. for computing relative structure-soil stiffness param-  
110 eters proposed by Franzius et al. (2006) and Giardina et al. (2015), which are  
111 needed to estimate deflection ratio modification factors). The design charts  
112 for modification factors were developed by modelling equivalent beam/plate  
113 structures subjected to tunnelling (which are solids with a lower height-to-  
114 length ratio compared to frames with similar  $EI$ ). These charts are based  
115 on the flexural rigidity  $EI$  of the equivalent beam/plate rather than a total  
116 stiffness value and they do not account for the characteristics of framed struc-  
117 tures (Franzius et al., 2006; Giardina et al., 2015; Potts and Addenbrooke,  
118 1997). Also for deep foundations, design envelopes suggested by Franza et al.  
119 (2017) relating relative bending stiffness parameter to deflection ratio mod-  
120 ification factors do not account for the frame characteristics. Consequently,  
121 the proposed empirical relationships could not be safely used within cur-  
122 rently available modification factor approaches. Further work is needed to  
123 implement the proposed formulas in deformation prediction methods.

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