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Isospin Analysis of Θ^+ production forbids $\gamma p \to \Theta^+ K_s$ and allows $\gamma n \to \Theta^+ K^-$

Marek Karliner [a,b*]

and

Harry J. Lipkin [b,c†]

[a] *Cavendish Laboratory*
University of Cambridge, UK
and
[b] *School of Physics and Astronomy*
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv, Israel

[c] *Department of Particle Physics*
Weizmann Institute of Science, Rehovot 76100, Israel
and
High Energy Physics Division, Argonne National Laboratory
Argonne, IL 60439-4815, USA

Abstract

The discrepancy between Θ^+ photoproduction on proton vs. deuteron can be resolved if the photon couples much more strongly to K^+K^- than to $K^o\bar{K}^o$ as indicated by the experimentally observed asymmetry between $\gamma p \to \Lambda(1520)K^+$ and $\gamma n \to \Lambda(1520)K_s$. Significant signal-background interference effects can occur in experiments like $\gamma N \to \bar{K}\Theta^+$ which search for the Θ^+ as a narrow $I = 0$ resonance in a definite final state against a non-resonant background, with an experimental resolution coarser than the expected resonance width. We show that when the signal and background have roughly the same magnitude, destructive interference can easily combine with a limited experimental resolution to completely destroy the resonance signal. Whether or not this actually occurs depends critically on the yet unknown relative phase of the $I = 0$ and $I = 1$ amplitudes. We discuss the implications for some specific experiments.

*e-mail: marek@proton.tau.ac.il
†e-mail: ftlipkin@weizmann.ac.il

1 Introduction - Effects of Possible Neutral Kaon Suppression

The recent experimental discovery [1] of an exotic 5-quark KN resonance Θ^+ with positive strangeness, a mass of ∼1540 MeV, a very small width $\lesssim$ 20 MeV and a presumed quark configuration $uudd\bar{s}$ has given rise to a number of further experiments [2] and a new interest in theoretical models [3] for exotic hadrons including models with diquark structures [4]. But the controversy between experimental evidence for and against the existence [5, 6] of the Θ^+ remains unresolved. There are also questions about isospin asymmetry [7].

1.1 Θ^+ photoproduction on proton vs neutron targets

One important issue to be explained is the difference and apparent isospin asymmetry between null Θ^+ photoproduction on protons [8] vs. clear signal on neutrons (via deuteron target), e.g. [9],[10].

A photon which turns into K^+K^- can make a $\Theta^+ K^-$ directly on a neutron, but cannot make a Θ^+ directly on a proton. A photon which turns into $K^o\bar{K}^o$ can make a $\Theta^+ \bar{K}^o$ directly on a proton, but cannot make a Θ^+ directly on a neutron. So how much of the photon appears as K^+K^- and how much as $K^o\bar{K}^o$ is an experimental question that can be clarified by measuring the neutral kaons, e.g. comparing $\gamma p \to pK^+K^-$ with the analogous reaction with neutral kaons in the final state. One can also do an analogous comparison in γd. The relevant data might in principle be available in the CLAS g11 and LEPS experiments.

We also note that the $\gamma K^o\bar{K}^o$ vertex is forbidden by an $SU(3)$ flavor selection rule and can be expected to be suppressed in comparison with the γK^+K^- vertex if $SU(3)_f$ is not badly broken.

The $\gamma K^o\bar{K}^o$ selection rule has been derived [11] as an $SU(3)_f$ rotation of the G-parity [12] forbidden $\omega \to \pi^+\pi^-$. This rotation turns isospin into the U-spin $SU(2)$ subgroup of $SU(3)_f$ (analogous to isospin, interchanging $d \leftrightarrow s$) [13]. The rotation turns $\pi^+\pi^-$ into $K^o\bar{K}^o$, while the photon is a U-spin scalar [14] particle, analogous to the isoscalar ω.

In the vector dominance picture the photon is an $SU(3)_f$ octet and a U-spin scalar combination of the ρ, ω and ϕ. In unbroken $SU(3)_f$ they are exactly degenerate and their contributions to $\gamma \to K^o\bar{K}^o$ cancel exactly. In the real world $SU(3)_f$ breaking as measured by vector meson masses is about 25%-30%, so the relative importance of the ϕ component is an open question. Further implications of this breaking are discussed below.

Another simple way to see how the $\gamma K^o\bar{K}^o$ vertex vanishes in the $SU(3)$ flavor limit is by considering what happens at the microscopic quark level. The photon creates a single quark-antiquark pair via a QED interaction. The other pair must be created via QCD by gluons. There are thus two independent amplitudes in QED-QCD for creating this pair of composite particles:

1. The photon creates a $\bar{d}d$ pair and an $\bar{s}s$ pair is added by gluon pair creation.
2. The photon creates an $\bar{s}s$ pair and a $\bar{d}d$ pair is added by gluon pair creation.

Since the d and s quarks have the same electric charge and the same couplings to gluons, these amplitudes are equal and add coherently in the $SU(3)$ flavor limit. However, if one writes down these two amplitudes carefully putting in the momenta, one can see that if the first transition creates a K^o with momentum p_1 and a $\bar{K}^o$ with momentum p_2, the second transition creates a K^o with momentum p_2 and a $\bar{K}^o$ with momentum p_1. The sum of these amplitudes therefore gives a state which is even under charge conjugation. Since the photon is odd under charge conjugation, this C-even amplitude cannot contribute to the final C-odd state and the transition vanishes.

This can be seen formally by writing down all the terms with proper phases that reflect the photon being C-odd, i.e. the matrix element for a photon creating a quark with momentum p_1 and an antiquark with momentum p_2 must be equal and opposite to the matrix element for a photon creating a quark with momentum p_2 and an antiquark with momentum p_1,

$$A\left[\gamma \to d(p_1) + \bar{d}(p_2)\right] = -A\left[\gamma \to d(p_2) + \bar{d}(p_1)\right] \tag{1}$$

The photon is flavor-blind, so the amplitude on the r.h.s. is equal, including sign, to the amplitude where s quarks are replaced by d quarks, so we have

$$A\left[\gamma \to d(p_1) + \bar{d}(p_2)\right] = -A\left[\gamma \to s(p_2) + \bar{s}(p_1)\right] \tag{2}$$

But when the primary $\bar{q}q$ pairs get "dressed" by QCD, (2) implies

$$A\left[\gamma \to K^o(p_1) + \bar{K}^o(p_2)\right] = -A\left[\gamma \to K^o(p_1) + \bar{K}^o(p_2)\right] \tag{3}$$

which means that $\gamma K^o \bar{K}^o$ vertex must vanish.

$SU(3)_f$ breaking by quark mass differences means that d and s quarks with the same momenta have different energies. Thus the cancellation is no longer exact.

The suppression of the $\gamma K^o \bar{K}^o$ vertex in the $\gamma N \to N K \bar{K}$ photoproduction reactions can be put on a firmer foundation by using the unitarity relations for these reactions:

$$\begin{aligned} \mathrm{Im}\left\langle K^+ K_s\, n\right| T \left|\gamma p\right\rangle &= \kappa \sum_i \left\langle K^+ K_s\, n\right| T^\dagger \left|i\right\rangle \left\langle i\right| T \left|\gamma p\right\rangle \\ \mathrm{Im}\left\langle K^+ K^- n\right| T \left|\gamma n\right\rangle &= \kappa \sum_i \left\langle K^+ K^- n\right| T^\dagger \left|i\right\rangle \left\langle i\right| T \left|\gamma n\right\rangle \end{aligned} \tag{4}$$

where κ is a kinematic factor and the sum is over all intermediate states $|i\rangle$. Intermediate states in the unitarity sum must be on the mass shell, and we have seen that the photon coupling to the $\gamma K^o \bar{K}^o$ vertex is forbidden by an $SU(3)$ flavor selection rule and can be expected to be suppressed in comparison with the $\gamma K^+ K^-$ vertex if $SU(3)_f$ is not badly broken. We therefore assume that the unitarity sum is dominated by the $K^+ K^- N$ intermediate state. Thus

$$\begin{aligned} \mathrm{Im}\left\langle K^+ K_s\, n\right| T \left|\gamma p\right\rangle &= \kappa \left\langle K^+ K_s\, n\right| T^\dagger \left|K^+ K^- p\right\rangle \left\langle K^+ K^- p\right| T \left|\gamma p\right\rangle \\ \mathrm{Im}\left\langle K^+ K^- n\right| T \left|\gamma n\right\rangle &= \kappa \left\langle K^+ K^- n\right| T^\dagger \left|K^+ K^- n\right\rangle \left\langle K^+ K^- n\right| T \left|\gamma n\right\rangle \end{aligned} \tag{5}$$

We now see that the transition matrix $\left\langle K^+ K_s n\right| T \left|\gamma p\right\rangle$ is proportional to the transition matrix $\left\langle K^+ K_s n\right| T^\dagger \left|K^+ K^- p\right\rangle$, where the $\Lambda(1520)$ can appear as a resonance but the Θ^+ cannot.

Conversely, the transition matrix $\langle K^+K^-n| T |\gamma n\rangle$ is proportional to the transition matrix $\langle K^+K^-n| T^\dagger |K^+K^-n\rangle$, where the $\Lambda(1520)$ cannot appear as a resonance but the Θ^+ can.

This unitarity relation holds only at low enough energies so that transitions to final states containing more particles can be neglected. The restriction to intermediate states on mass shell eliminates the need to the include diagrams with higher states found in many theoretical treatments [15, 16, 17]. Note that the necessity to consider K^* exchanges which arises in all such theoretical treatments and in all treatments well above the K^* production threshold does not arise here.

Thus the g11 reaction $\gamma p \to nK^+K_s$ proceeds via $\gamma p \to K^+K^-p \to K^+K_sn$ and there is no possibility of making the Θ^+ in any simple way in the g11 setup.

The basic physics here is that the photon couples much more strongly to charged kaons than to neutral kaons and charged kaons can make the Θ^+ simply on a neutron and not on a proton.

One test of this picture is to compare the photoproduction of isoscalar baryon resonances with positive and negative strangeness on proton and neutron targets.

$$\gamma p \to \bar{K}^o\Theta^+; \quad \gamma n \to K^-\Theta^+ \tag{6}$$

$$\gamma n \to K^o\Lambda; \quad \gamma p \to K^+\Lambda \tag{7}$$

Positive strangeness resonances like the Θ^+ will be produced on neutron targets and not on protons, while negative strangeness resonances like the $\Lambda(1520)$ will be produced on proton targets and not on neutrons.

Recent data on $\Lambda(1520)$ production confirm this picture. LEPS observes a strong asymmetry in photoproduction of $\Lambda(1520)$ on proton and neutron [10, 17]. They measured both $\gamma p \to \Lambda(1520)K^+$ and $\gamma d \to \Lambda(1520)KN$ and find that the production rate on the deuteron is almost equal to and even slightly smaller than on the proton. This implies that $\gamma n \to \Lambda(1520)K_s$ is negligible.

We therefore propose the suppression of the $\gamma K^o\bar{K}^o$ vertex as the likely explanation of non-observation of Θ^+ production on a proton target in the CLAS g11 experiment [8]. To gain confidence in this explanation it is important that other experiments confirm the relative suppression of $\gamma n \to \Lambda(1520)K$.

1.2 SU(3) breaking and the ϕ component of the photon

In the vector dominance picture with broken $SU(3)_f$ and octet-singlet mixing the ρ and ω components of the photon are still degenerate but the ϕ is now separate. In this picture for Θ^+ photoproduction the $\bar{s}$ strange antiquark is already present in the initial state in the isoscalar ϕ component. The ρ and ω components contain no strangeness and can produce the Θ^+ only via the production of an $s\bar{s}$ pair from QCD gluons. How much this extra strangeness production costs is still open. This cost does not appear in treatments using the

Kroll-Ruderman theorem [7] which involves only pions and ignores the ϕ component of the photon.

One example of an experiment that projects out the $I = 0$ state of a KN state is the photoproduction on a deuteron [10] of the final state $\Lambda(1520)K^+n$.

If the strangeness in the reaction comes from the isoscalar ϕ component of the photon, the final KN state is required by isospin invariance to be isoscalar, and the signal is observed against a purely isoscalar background. This is not true for the other K^-pK^+n final states observed in the same experiment where the K^-p is not in the $\Lambda(1520)$ and the effects of a nonresonant $I = 1$ background can give very different results.*

2 Interference between resonances and background

2.1 Specific coherence effects in some experiments

We also point out here one additional crucial factor which can explain why some experiments see the Θ^+ and others do not. All experiments search for a narrow resonance against a nonresonant background, generally with an experimental resolution much coarser than the assumed Θ^+ width. Some experiments lead to a definite final state, e.g the reactions (6).

The probability of observing the signal in such an experiment is very sensitive to the relative phase between the signal and the background. This is shown explicitly below in a toy model. Such interference effects are not expected in inclusive Θ^+ production; e.g.

$$e^+e^- \to \Theta^+ X \tag{8}$$

Here the incoherent sum over all inclusive final states X destroys all phase information.

The Θ^+ is believed to decay into a kaon and nucleon with isospin zero. But the observed decay modes K^+n and K^op are equal mixtures of states with $I = 0$ and $I = 1$ with opposite relative phase. If the nonresonant background in a given experiment is mainly from an $I = 1$ amplitude, the relative phase between signal and background amplitudes will depend upon the particular decay mode observed. The phase when the Θ^+ is detected in the K^+n decay mode will be opposite to the phase in the same experiment where the Θ^+ is detected in the K^op decay mode.

A serious isospin analysis may be necessary to understand the implications of any experiment where destructive interference between an $I = 0$ signal and $I = 1$ background can effectively destroy the signal. Simple isospin relations between similar reactions which consider only the signal and not the interference with the background can give very erroneous results. In particular one can expect apparent contradictions between negative and positive results which are connected by isospin when the interference with background is neglected. How this destruction can occur is illustrated in the toy model below.

*We recall old SLAC experiments [18, 19] which looked at photoproduction of K^+-hyperon from hydrogen and deuterium at 11 and 16 GeV. It would be interesting to re-examine these data in view of [10].

2.2 A simple model for resonance and background

Consider a simple toy model for a resonance and background and write the amplitude for the production of this resonance as

$$A = b \cdot e^{i\phi} + \frac{1}{1+ix} \tag{9}$$

where x denotes the difference between the energy and the resonance energy, the strength of the resonance amplitude is normalized to unity and b and ϕ denote the amplitude and phase of the nonresonant background. The background is assumed to be essentially constant over an energy region comparable to the width of a narrow resonance. The square of this amplitude is then

$$\begin{aligned} |A|^2 = \left| b \cdot e^{i\phi} + \frac{1}{1+ix} \right|^2 &= b^2 + 2\mathrm{Re}\left(\frac{b \cdot e^{i\phi}}{1-ix}\right) + \frac{1}{1+x^2} \\ &= b^2 + \frac{1 + 2b\cos\phi - 2bx\sin\phi}{1+x^2} \end{aligned} \tag{10}$$

For a detector which integrates the cross section over a symmetric interval from $-X$ to $+X$ the term linear in x drops out and

$$\begin{aligned} \int_{-X}^{X} |A|^2 dx &= \int_{-X}^{X} dx \left(b^2 + \frac{1+2b\cos\phi}{1+x^2}\right) = 2b^2 X + \int_{-\Theta}^{\Theta} d\theta (1 + 2b\cos\phi) \\ &= 2b^2 X + 2(1 + 2b\cos\phi)\Theta \end{aligned} \tag{11}$$

where we have set $x = \tan\theta$ and $X = \tan\Theta$.

The ratio of signal to background is then

$$\frac{\text{signal}}{\text{background}} = \frac{1 + 2b\cos\phi}{b^2} \cdot \frac{\Theta}{\tan\Theta} \tag{12}$$

We thus find that for the case where $b = 1$, i.e. the signal and background amplitudes are equal at the peak of the resonance, the ratio of integrated signal to integrated background varies from -1 to 3, depending upon the relative phase.*

Fig. 1 shows $|A|^2$ as function of x for several representative values of of the relative phase: $\phi = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ and π.

For illustration purposes we have taken $b = 1$, i.e. equal strength of the background and signal amplitudes at the peak of the resonance. To expound the effect experimental resolution being substantially coarser than the resonance width, $|A|^2$ was averaged over bins of width $\Delta x = 5$, i.e. five times wider than the resonance width. The resulting values are plotted at bin centres as red points, with additional 10% relative error bars.

*Signal strength is defined as the difference between the measured (signal+background) and background alone. Relative signal strength of -1 corresponds to a dip, cf. $\phi = \pi$ entry in Fig. 1.

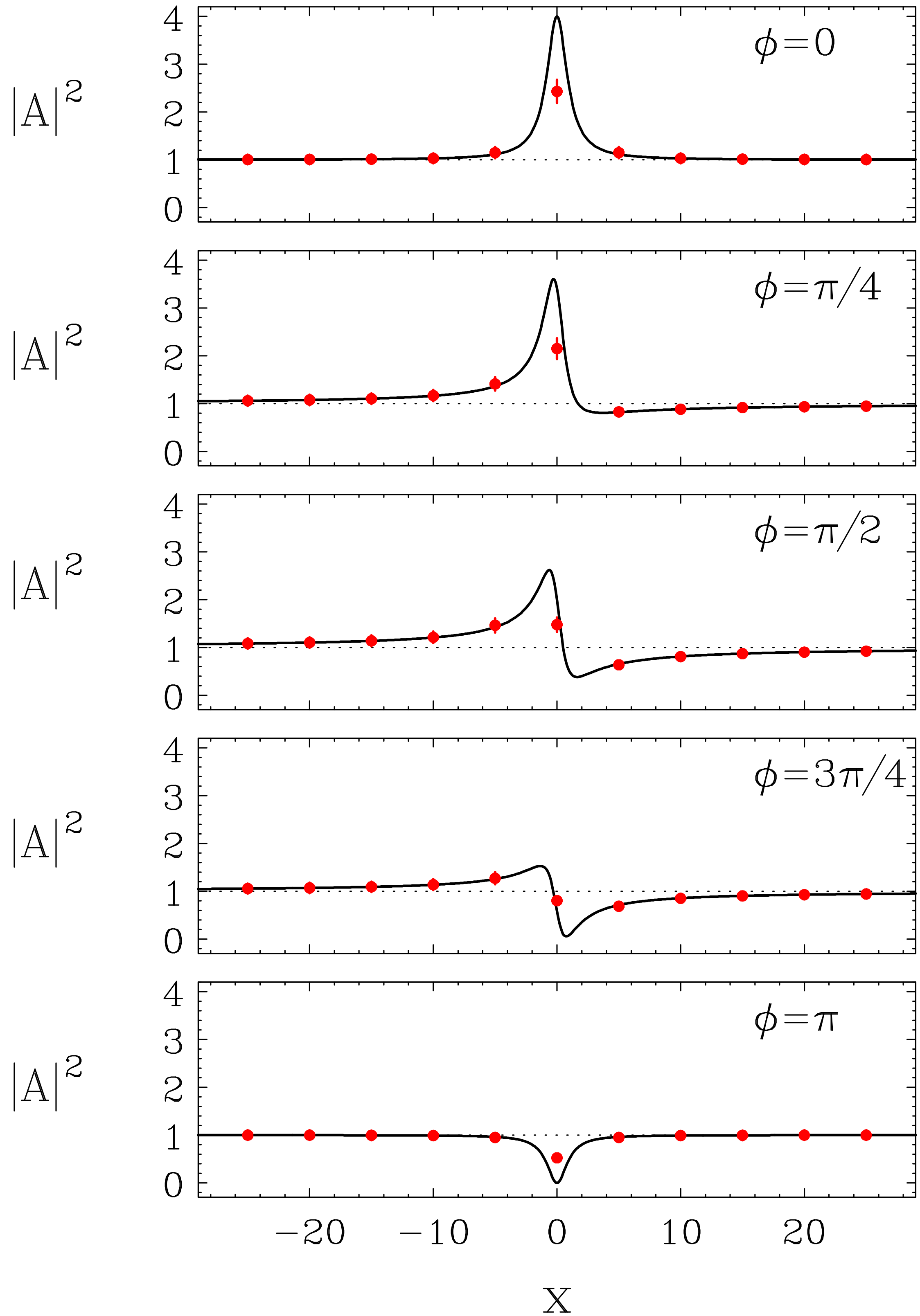

Figure 1: $|A|^2$, *eq. (10), as function of* x *for* $b = 1$ *and for different values of the relative phase,* $\phi = 0, \pi/4, \pi/2, 3\pi/4$ *and* π. *Red points denote values averaged over bins of width* $\Delta x = 5$, *assuming an additional 10% relative error.*

The interplay of the limited experimental resolution with the relative phase of signal and background can lead to rather striking results, depending on the value of ϕ. Thus e.g. for $\phi = \frac{3\pi}{4}$ the measured signal is almost completely washed out by this 'conspiracy' of the relative phase and the experimental resolution.

2.3 Mass difference in K^+n and K_sp modes

In experiments which did observe the Θ^+ there seems to be a small but systematic and non-negligible mass difference between the mass observed in the K^+n and K_sp modes [20].

It is interesting to ask if this could have something to do with isospin effects in signal-background interference. As already mentioned, K^+n and K_sp are equal mixtures of states with $I = 0$ and $I = 1$ with opposite relative phase. If the nonresonant background in a given experiment is mainly from an $I = 1$ amplitude, the relative phase between signal and background amplitudes will depend upon the particular decay mode observed. The phase when the Θ^+ is detected in the K^+n decay mode will be opposite to the phase in the same experiment where the Θ^+ is detected in the $K^o p$ decay mode.

We now consider the effect of this phase flip in our simple model of signal-background interference. A sign change of the relative phase in eq. (9) results in

$$\phi \longrightarrow \phi + \pi; \qquad A \longrightarrow \tilde{A} = b \cdot e^{i\phi+\pi} + \frac{1}{1+ix} \tag{13}$$

Since the observed signal is given by the absolute value of the amplitude, we can replace A by its complex conjugate, i.e.

$$\phi \longrightarrow \phi + \pi; \qquad |A|^2 \to |A^*|^2 = \left| b \cdot e^{i(\pi-\phi)} + \frac{1}{1-ix} \right|^2 \tag{14}$$

So if Fig. 1 were to describe the K^+n mode, the corresponding plots of $|A|^2$ for the K_sp mode can be obtained by the transformation $\phi \to \phi - \pi,\ x \to -x$, possibly resulting in the shift in the peak location.

On the other hand, backgrounds differ from experiment to experiment, so it is not at all clear why this would result in a systematic shift between K^+n and K_sp modes. It would be very interesting if our experimental colleagues could redo this analysis with a realistic background parametrization.

3 Conclusions

The theory needed to understand apparent contradictions in considering the question of why some experiments see the Θ^+ and others do not turns out to be more complicated than naively expected.

Before conclusions can be drawn from apparent violations of isospin symmetry or from negative search results using a particular final state extensive data and serious isospin analysis are needed. In particular,

1. Data from both proton and neutron targets
2. Data from both the neutral and charged kaon final states
3. Extensive isospin analyses that include the background
4. Clarification of the role of the ϕ component of the photon
5. Clarification of the degree suppression of $\gamma \rightarrow K^o \bar{K}^o$

Serious signal-background interference effects can occur in some experiments which search for the Θ^+ as a narrow $I = 0$ resonance against a nonresonant background with an experimental resolution larger than the expected resonance width. The resonance signal can be completely destroyed by destructive interference with a background having a magnitude of the same order as the signal and a destructive phase.

Experiments detect Θ^+ via the decay modes K^+n and K^op which are equal mixtures of $I = 0$ and $I = 1$ states with opposite relative phase. Unless the experiment projects out the $I = 0$ component of the signal, interference can occur between the $I = 0$ state and an $I = 1$ background. Such interference can vary greatly between final states which are related by isospin in the absence of interference.

All these considerations must be understood before reliable conclusions can be drawn. But when some experiments do not see the Θ^+, there are researchers who tend to ignore these questions and immediately conclude that the Θ^+ does not exist.

In the meantime, we note that LEPS provides fresh evidence for photoproduction of Θ^+ on deuteron [10] and CLAS is expected to release the first g10 deuteron data very soon [21]. If Θ^+ is seen in the latter as well, then understanding the seeming discrepancy between the null result in g11 proton data and the deuteron experiments will be on top of the theoretical agenda.

We think that the suppression of $\gamma K^o \bar{K}^o$ vertex and signal/background interference effects discussed here are a first promising step towards in this direction. We eagerly await additional experimental data which can help in resolving this issue.

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