Investigation of the impact of the configuration of exhaust after-treatment system for diesel engines

Chung Ting Lao\textsuperscript{a}, Jethro Akroyd\textsuperscript{a,b}, Nickolas Eaves\textsuperscript{a,d}, Alastair Smith\textsuperscript{e}, Neal Morgan\textsuperscript{e}, Daniel Nurkowski\textsuperscript{f}, Amit Bhave\textsuperscript{f}, Markus Kraft\textsuperscript{a,b,c,*}

\textsuperscript{a}Department of Chemical Engineering and Biotechnology, University of Cambridge, West Cambridge Site, Philippa Fawcett Drive, Cambridge CB3 0AS, United Kingdom
\textsuperscript{b}CARES, Cambridge Centre for Advanced Research and Education in Singapore, 1 Create Way, CREATE Tower, #05-05, Singapore, 138602
\textsuperscript{c}School of Chemical and Biomedical Engineering, Nanyang Technological University, 62 Nanyang Drive, Singapore, 637459
\textsuperscript{d}Mechanical, Automotive and Materials Engineering, University of Windsor, 401 Sunset Avenue, Windsor, ON, N9B 3P4, Canada
\textsuperscript{e}Shell Centre, York Road, London, SE1 7NA, United Kingdom
\textsuperscript{f}CMCL Innovations, Sheraton House, Cambridge, CB3 0AX, United Kingdom

\textbf{Abstract}

Exhaust After-Treatment (EAT) systems are necessary for automotive powertrains to meet stringent emission standards. Computational modelling has been applied to aid designing EAT systems. Models with global kinetic mechanisms are often used in practice, but they cannot accurately predict the behaviour of after-treatment devices under a wide range of conditions. In this study, a numerical EAT model with rigorous treatment of the catalytic chemistry is proposed to investigate the impact of the configuration of individual devices in the EAT system; one of the key design decisions. The performance of the proposed model is first critically assessed against experimental and simulation data from the literature before being applied

\*Corresponding author

Email address: mk306@cam.ac.uk (Markus Kraft)
to design a multi-device EAT system for a diesel engine. The target EAT system is composed of a diesel oxidation catalyst (DOC), an ammonia-based selective catalytic reduction (NH₃-SCR) device and a diesel particulate filter (DPF). The steady state behaviour of various EAT designs under operating conditions across the engine map are examined. The DOC-DPF-SCR layout is found to be more beneficial than the alternative DOC-SCR-DPF for the specific engine studied. Furthermore, the DPF-front system is more robust with respect to changes in emission regulations. Flux analysis is applied to study the chemical interaction in the SCR and explain the disadvantage of the SCR-front system. In addition, it is demonstrated in the study that future catalyst investigations should consider more realistic feed compositions.

**Keywords:** Exhaust After-Treatment (EAT), Diesel oxidation catalyst (DOC), Ammonia-based Selective Catalytic Reduction (NH₃-SCR), Diesel Particulate Filter (DPF), Numerical model

1. **Introduction**

Internal Combustion Engines (ICEs) are widely used to provide power for vehicles [1]. In 2018, vehicles with ICEs powered by petrol and diesel made up over 90% of the new passenger car registrations in the European Union (EU) [2], and in particular, 96% of heavy-duty engines in EU are powered by diesel [3]. However, emissions from ICE containing carbon monoxide (CO) [4], unburnt hydrocarbon (uHC) [5], nitrogen oxides (NOx) [6] and particulate matter (PM) [7] which can cause adverse impacts on human health and the environment. Correspondingly, governing bodies have set legal limits on ICE emissions that manufacturers must meet.
Since tailpipe emissions depend on the operating conditions of the ICE, the engine “test cycle” must be clearly defined such that the emission measurements are repeatable [10]. However, it has been found that emission measurements using simple test cycles such as the New European Driving Cycle (NEDC) are not representative of the actual on-road emissions [11]. And as a result, more realistic test cycles such as the Worldwide Harmonized Light Vehicles Test Cycle (WLTC) have been introduced [11]. In addition, Real Driving Emission (RDE) testing will be implemented, in which tailpipe emissions are measured when the vehicle is operating on-road [6].

Exhaust After-Treatment (EAT) is an important part of emission control of ICEs. Other emission control technologies, such as exhaust gas recirculation (EGR) and alternative fuel formulations, have also been applied by manufacturers to meet the increasingly stringent emission standards [12, 13]. In contrast to other emission control technologies, the EAT system has a smaller impact on the in-cylinder combustion. It is placed between the engine cylinders and the tailpipe in a vehicle where it reduces the regulated substances in the engine-out exhaust.

Typical EAT systems for diesel engines are composed of three individual devices [14]. Diesel EAT systems usually contain two Flow-Through Monoliths (FTMs) with a catalytic coating on the channel walls for the mitigation of gaseous pollutants. The structure of a typical FTM is shown in Fig. 1(a). The purpose of the Diesel Oxidation Catalyst (DOC) is to oxidise CO and uHC to form CO$_2$ and H$_2$O [15]. In addition to the DOC, a separate FTM is required for NOx control. The reduction of NOx to nitrogen (N$_2$) is difficult due to the high level of oxygen (O$_2$) in diesel exhaust [16]. Two technologies
are commonly applied to reduce NOx emission, namely the Lean NOx Trap (LNT) and the Selective Catalytic Reduction (SCR) device. In this work, we focus on the SCR technology since it is the preferred deNOx technology for heavy-duty applications [17, 18]. The SCR is able to reduce NOx to N\textsubscript{2} at high oxygen levels with the aid of an additional reductant. The most popular of which is ammonia (NH\textsubscript{3}) [19], stored in the form of urea solution. During operation, urea is injected into the exhaust flow and NH\textsubscript{3} is formed from the decomposition of urea [15]. SCR with alternative reductants, for example hydrocarbon [20] and CO [21], are also possible.

Whilst catalytic FTM devices are effective for controlling toxic gaseous emissions, Diesel Particulate Filters (DPFs) are required to control particulate emissions. DPFs have a Wall-Flow Monolith (WFM) structure as shown in Fig. 1(b). As opposed to having open channels like the FTM, the ends of channels in a WFM are alternately plugged. Particulates in the exhaust accumulate in the porous wall between channels when exhaust flows through the device. The backpressure in the DPF will increase during operation due to accumulation of particles in the device. This leads to worse in-cylinder combustion performance, and hence trapped particles are removed by “regeneration”, which is the combustion of the carbonaceous part of trapped particles.

Regeneration strategies can be categorised as either “active” or “passive”. During active regeneration, the temperature of the DPF is increased temporarily by the means of altering engine operation, external fuel injection [22] or microwave heating [23]. Trapped particles are then be burnt by O\textsubscript{2} at above 500°C [24]. For passive regeneration, trapped particles are oxidised by
NO$_2$, a much more powerful oxidant than O$_2$. The reaction is spontaneous at typical diesel exhaust temperatures (300°C [25]). A balance between filtration and passive regeneration may be reached without an additional heat source and hence passive regeneration is more favourable than active regeneration strategies since it has a lower associated fuel penalty [26]. However, passive regeneration may not be viable at all operating conditions and hence active regeneration remain an essential part of DPF control strategies [27].

Since more than one EAT device is required, there are several possible combinations of individual devices in the EAT system. It is important to understand the synergistic and competitive interactions between EAT devices in order to choose the optimal configuration for a certain engine. The DOC is usually the first device in the system due to its beneficial impact on downstream devices. First of all, the thermal energy released from exothermic reactions occurring inside the DOC can warm up downstream devices. This
can help reduce tailpipe emission during cold-start of engines due to catalyst inactivity at low temperature [28]. Furthermore, the DOC is able to convert NO to NO\textsubscript{2} as well as oxidising CO and uHC [29]. NO\textsubscript{2} generated from the DOC can assist passive regeneration in the DPF. The deNO\textsubscript{x} performance in the SCR can be improved if the NO\textsubscript{2}/NO\textsubscript{x} ratio is increased [30]. The DOC is also used to initiate active regeneration in the DPF, for example raising the temperature of exhaust by oxidising injected fuel [31].

On the other hand, the interactions between the DPF and the SCR are mainly competitive. If the DPF is placed in front of the SCR ("DPF-front"), the SCR will warm up more slowly during cold-start period as the DPF acts as a heat sink [32]. Passive regeneration in the DPF would consume NO\textsubscript{2} in the exhaust, preventing maximum deNO\textsubscript{x} performance in the downstream SCR. Furthermore, the downstream SCR is vulnerable to the heat generated from active regeneration in the DPF. In spite of these negative influences on SCR performance, the upstream DPF can reduce particle fouling on the wall of the downstream SCR [33]. Depending on the economic analysis, it may be preferable to have a "SCR-front" system, where the performance of the SCR is better, at the expense of suppressing passive regeneration in the downstream DPF, resulting in increased fuel consumption [14].

Given the complexity of the processes and interactions within EAT system, computational models are widely used to study and optimise the designs and control strategies of EAT systems [15, 34, 35]. The performance of the EAT system is affected by numerous phenomena, including gas flow, heat and mass transfer, chemical reaction as well as particle filtration. Furthermore, each channel in a EAT device may behave differently depending on the
inlet conditions and the relative positions of the channels inside the monolith. Depending on the purpose of the model, various simplifications of the model may be made to lower the computing cost while maintaining sufficient accuracy [36]. For example, the overall behaviour of a EAT device may be described by simulating a single representative channel with reasonable accuracy instead of simulating every channel in the monolith [37]. This is the most common approach for investigations focusing on the catalytic chemistry since the reaction kinetics is unaffected by the monolith structure [36].

One of the difficulties in EAT modelling arises from the complex reaction mechanisms of the catalyst. Simple “global kinetic” mechanisms with semi-empirical reaction rate expressions (based on work of Voltz et al. [38] for example) are often used. However they are rarely able to describe the change in catalyst behaviour under different operating conditions where the inlet composition changes significantly [39]. The global kinetic approach is also unable to describe the composition profile of species adsorbed on the catalyst which can have significant impact [40]. The capability of handling detailed microkinetic mechanisms based on elementary reactions is desirable for after-treatment models because of increasingly stringent emission testing procedures and emission standards. Furthermore, such models can allows better understanding of the intrinsic kinetics of the catalyst, which is especially useful for investigation of new catalyst formulations for the next-generation after-treatment technology.

This paper aims to propose a computational EAT model that allows a rigorous treatment for the catalytic chemistry unlike black-box models provided by most manufacturers [15]. Such a model can offer more insight into
the chemical phenomena in the EAT system, and shed light on the interaction between individual devices. This work focuses on the steady state of the after-treatment system for three main reasons. First, steady state operation provides a good indication of sufficient passive regeneration, an important phenomena for multi-device EAT system for diesel engines. Second, heavy-duty engines are subject to both transient and steady state tests. Examples from the EU regulations include the European Stationary Cycle (ESC) for heavy-duty road vehicles and the Non-Road Steady Cycle (NRSC) for non-road engines [41]. Third, EAT systems are usually designed to pass a specific emission test cycle, which may not adequately consider all operating conditions occurring under actual usage. In this work, the proposed model is applied to study the chemical interaction and performance of EAT systems at different operating points across the engine map.

The rest of this paper is structured as follows: The governing equations of the proposed model and the numerical methods used are described in Section 2. The proposed model is then applied to simulate individual EAT devices. The results are compared with experimental and simulation data from the literature in Section 3. After that, the proposed model is integrated with a virtual engine cylinder simulation software in Section 4 to investigate the advantages and disadvantages of DPF-front and SCR-front systems via a case study of a heavy-duty diesel engine. Finally, element flux analysis is applied in Section 4.2.3 to study the chemical phenomena in the EAT system.
2. Model description

This section is split into two parts. The first part describes the governing equations of the after-treatment model. The second part describes the coupling between the engine in-cylinder combustion model and the after-treatment model.

2.1. Governing equations

The model used in this work is a modified version of the WFM model described in detail by Lao et al. [42]. The key equations used in this work is summarised below.

The single channel approach is applied in this study to simulate EAT devices. It assumes that all channels in the monolith have the same behaviour. A single FTM channel is described by a two-layer one-dimensional model, where the channel and the catalytic washcoat are considered as single layers discretised in the axial direction respectively. The spatial representation of a FTM in the model is shown in Fig. 2(a). For WFM, a pair of inlet and outlet channels, as well as the porous wall in between, are modelled [42]. The channels are discretised in the axial direction. The porous wall is discretised in the through-wall direction in addition to the axial direction. The spatial representation of a WFM in the model is shown in Fig. 2(b). Ideal gas behaviour has been assumed for the gas phase mixture.

The gas flows along the channels of FTMs and WFMs are governed by the Bernoulli’s equation, whereas the gas flows in the through-wall direction across the particle cake layer and the porous wall in WFMs are governed by Darcy’s Law. The variation of gas pressure along channels and across porous
medium are described by eq. (1) and (2) respectively [42]:

\[
P_{\text{in}} + \frac{1}{2} \rho_{\text{in}} u_{\text{in}}^2 = P_{\text{out}} + \frac{1}{2} \rho_{\text{out}} u_{\text{out}}^2 + \frac{F \mu L_{\text{ch}}}{D^2} u_{\text{out}},
\]

\[
P_{\text{in}} - P_{\text{out}} = \frac{\mu v L_{\text{porous medium}}}{\kappa},
\]

where \( P \) is pressure, \( \rho \) is gas mass density, \( u \) is gas flow velocity in a channel, \( F \) is a friction factor (\( = 28.454 \) for square channel [43]), \( \mu \) is dynamic viscosity, \( L \) is the distance between volume elements, \( D \) is the side length of the cross-section of a square channel, \( v \) is the gas flow velocity across porous media and \( \kappa \) is the permeability of the porous medium. The subscripts “in” and “out” denote adjacent upstream and downstream volume elements respectively.

The model proposed by this work can describe the composition of the gas phase and the “surface” phase i.e. adsorbed species on the reactive surface of the catalyst. The composition of the gas phase is affected by the gas flow...
and chemical reactions. The governing equation of the gas phase species is shown below [42]:

\[
\frac{dn_{\gamma,i}}{dt} = \sum_{j \in S_{\text{in},i}} \frac{\dot{m}_j Y_{\gamma,\text{in},j}}{M_{\gamma}} - \sum_{j \in S_{\text{out},i}} \frac{\dot{m}_j Y_{\gamma,i}}{M_{\gamma}} + \nu_{\gamma,w} r_{w,i} V_{i} + 4 L_{\text{ch}} Sh \rho_{m,i} D_{m,\gamma,i} \Delta x_{\text{ch-wc},\gamma,i},
\]  

(3)

where \( n \) is the number of mole of a gas species, \( \dot{m} \) is the gas mass flow rate, \( Y \) is the mass fraction, \( M \) is the molar mass of a gas species, \( V \) is the volume of a volume element, \( \nu \) is the stoichiometric coefficient of a species in a reaction and \( r \) is the rate of reaction, \( \gamma \) is the index of gas phase species, \( i \) is the index of volume elements, \( j \) is the index of connection between two volume elements which allows gas flow, \( w \) is the index of reactions, \( S_{\text{in},i} \) is the set of all inflow connections to volume element \( i \), \( S_{\text{out},i} \) is the set of all outflow connections to volume element \( i \), \( W \) is the total number of reactions, \( Sh \) is the Sherwood number for convection between channel and the washcoat, \( \rho_{m,i} \) is the molar density in volume element \( i \), \( D_{m,\gamma,i} \) is the diffusivity of species \( \gamma \) in volume element \( i \) and \( \Delta x_{\text{ch-wc},\gamma,i} \) is the difference in mole fraction of species \( \gamma \) between volume element \( i \) and its corresponding channel/washcoat volume element. The Sherwood number used in this work is 3 assuming fully developed laminar flow in the channels [44]. Diffusivities of gas phase species are calculated using transport data from GRI-Mech 3.0 [45].

The composition of surface species is described by eq. (4) below:

\[
\frac{dn_{\beta,i}}{dt} = V_{i} \sum_{w=1}^{W} \nu_{\beta,w} r_{w,i},
\]  

(4)
where $\beta$ is the index of surface phase species. Adsorption and desorption of species are described as chemical reactions which is a common practice for EAT modelling [for example 46, 44]. A micro-kinetic approach is used for the treatment of the chemistry to allow the application of a detailed kinetic mechanism. The rate of a reaction is based on an Arrhenius-type equation, with capability of considering user-specified reaction order and coverage dependency of activation energy [47]. The rate of a reaction $r$ is calculated as follows:

$$r_{w,i} = \eta_{w,i} \prod_{\gamma=1}^{\Gamma} c_{\gamma,i}^{X_{\gamma,w}} \prod_{\beta=1}^{\Psi} \left( \frac{S_{i}}{V_{i} \zeta_{\beta,i}} \right)^{X_{\beta,w}},$$

(5)

$$k_{w,i} = A_w T_i^{B_w} \exp \left( -\frac{E_w}{RT_i} \right) \prod_{\beta=1}^{\Psi} \exp \left( -\frac{-\sigma_{\beta,w} \theta_{\beta,i}}{RT_i} \right),$$

(6)

where $c_{\gamma,i}$ is the molar concentration of gas species, $\Gamma$ is the total number of gas phase species, $X$ is the reaction order of a gas species, $S$ is the reactive surface area in a volume element, $\zeta$ is the surface concentration of a surface species, $\Psi$ is the total number of surface phase species, $A$ is the pre-exponential constant, $B$ is the temperature exponent and $E$ is the activation energy of a reaction. An effectiveness factor ($\eta$ in eq. (5)) is applied to describe the effect of coupled reaction-diffusion in the catalytic washcoat. The effectiveness factor is calculated as described by Mladenov et al. [48]. The coverage dependency is represented by the product of exponentials on the right hand side of eq. (6) where $\sigma$ and $\theta$ are a coverage dependent parameter and site fraction of a surface species respectively.

A sectional method is applied to numerically solve the particle size distribution in after-treatment devices [42]. The number density of trapped
particles $N_{z,i}$ of size class $z$ in the porous wall volume element $i$ is governed by the following equation:

$$\frac{dN_{z,i}}{dt} = Q_{z,i}N_{z,in} + (k_{AR}[O_2] + k_{PR}[NO_2])\left(\pi d_{p,z}^2 N_{z,i}\right), \quad (7)$$

where $Q_{z,i}$ is the filtration efficiency of particles of size class $z$, $\dot{N}_{z,in}$ is the inlet flow rate of particle number density and $d_{p,z}$ is the diameter of particle in size class $z$. $Q_{z,i}$ is calculated using the classic unit collector model [49]. It is described in detail elsewhere [42, Section 2.2]. The extended features developed by Lao et al. [42] are not included in this work since those features are applied to describe the transient behaviour of active regenerating DPFs, which is beyond the scope of this work.

The model used in this work considers passive regeneration by NO$_2$ in addition to thermal active regeneration by O$_2$. Alternative catalysed regeneration pathways due to fuel additives [50] or embedded catalysts within the DPF are not considered in this work. $k_{AR}$ and $k_{PR}$ are the rate constants for active and passive regeneration reactions. The expressions for the rate constants are as described by Kandylas and Koltsakis [29]. The following regeneration reactions are considered in the model:

$$C + \alpha_1 O_2 \rightarrow (2\alpha_1 - 1) CO_2 + (2 - 2\alpha_1) CO, \quad (8)$$

$$C + \alpha_2 NO_2 \rightarrow \alpha_2 NO + (\alpha_2 - 1) CO_2 + (2 - \alpha_2) CO, \quad (9)$$

where $\alpha_1$ and $\alpha_2$ are stoichiometric coefficients of the regeneration reactions. Different literatures [43, 49, 51, 52] suggest different values of $\alpha_1$ and
\( \alpha_2 \) and their temperature dependences. In this study, \( \alpha_1 \) and \( \alpha_2 \) take constant values of 0.8 and 1.8 respectively, within the possible range suggested in literature [29].

The thermal behaviour of the EAT system is neglected in the model used in this work. This is justified for two reasons: First, the heat of reaction in SCR is usually negligible [44]. It is found by Schejbal et al. [53] that the same applies to the DOC and passively regenerating DPF. As a result, no heat generation is expected at steady state. Second, heat loss to the environment is limited by the presence of insulation around EAT devices. Whilst heat loss to the environment can be influential to the time required for the EAT system to warm up and to eventually reach steady state, it would have negligible impact on the final steady state temperature achieved in insulated EAT devices. Therefore the heat balance equation of the EAT system is omitted in this model assuming insulations are appropriately designed to minimise heat loss of the EAT system. This is deemed acceptable under steady state operation where there is no occurrence of highly exothermic uncontrolled regeneration [54]. As a result, the temperature of the EAT system is that of the inlet flow at steady state.

2.2. Coupling with engine model

The Stochastic Reactor Model (SRM) Engine Suite [55] is used to specify the inlet boundary conditions to the EAT system. The SRM Engine Suite can predict the cylinder-out mass flow rate, gas composition and temperature of the exhaust using a Probability Density Function (PDF) based model [56]. It has been shown to be a computationally efficient tool for simulating different types of internal combustion engines including gasoline engines [59],
homogeneous charge compression ignition engines [62], natural gas engines [57] and predicting gaseous [58] and particulate emissions [66].

Whilst the mass flow rate and gas composition should remain unchanged from the cylinder to the inlet of the EAT system, there is significant heat loss when the exhaust passes from engine cylinders to the EAT system through the turbocharger and pipings. Gao et al. [68] performed an energy analysis on a heavy-duty diesel engine with turbocharger. The ratio between the enthalpy of the post-turbine exhaust and the enthalpy of the injected fuel was reported for a range of output power and engine speed [68, Fig. 9 & 15].

Assuming that the engine considered in this work exhibits the same behaviour, the enthalpy ratio can be used to estimate the temperature of the exhaust at the exit of the turbine. This assumption is justified on the basis of the similarity between both engines (8.6L, 260 kW rated power, 2100 rpm maximum speed studied by Gao et al. [68] versus 7 L, 205 kW rated power, 2200 rpm maximum speed studied here). The output power, engine speed and the amount of heat in the injected fuel are provided by the SRM Engine Suite and can be used to interpolate an enthalpy ratio from experimental data reported by Gao et al. [68]. The enthalpy of the post-turbine exhaust and the exhaust temperature at the turbine exit can then be calculated since the engine-out composition and the mass flow rate are given by the SRM Engine Suite.

In this work, the engine is assumed to be operating at steady state. Thus the boundary conditions of the EAT system does not vary with time. The modelling equations of EAT system (eq. (1)–(7)) are integrated with time until either steady state is reached or the system ceases to function correctly.
due to DPF clogging.

3. Model assessment

In this section, the proposed EAT model is applied to simulate individual EAT devices including a DOC, SCR and DPF. The results of the simulations are compared with experimental and simulation data from literature.

3.1. Diesel Oxidation Catalyst (DOC)

The main functionality of the DOC is to completely oxidise CO and uHC. It is also capable of promoting NO oxidation subject to thermodynamic constraints. The overall reactions are shown below, where uHC is represented by $C_3H_6$, a common choice in literature modelling studies [46, 48, 71]:

$$2 C_3H_6 + 9 O_2 \xrightarrow{Pt} 6 CO_2 + 6 H_2O \quad (10)$$

$$2 CO + O_2 \xrightarrow{Pt} 2 CO_2 \quad (11)$$

$$2 NO + O_2 \xrightleftharpoons{Pt} 2 NO_2 \quad (12)$$

The proposed model is applied to simulate a platinum catalyst experimentally studied by Koop and Deutschmann [46]. They developed a chemical mechanism to describe the redox reactions between platinum and exhaust gas species. They evaluated the performance of the chemical mechanism by simulating the conversion of NO of a platinum catalyst at three different temperatures and two inlet conditions, namely “lean” (12% $O_2$) and “rich”
(0.9% $O_2$). They showed good agreement between their simulation and their experimental results.

![Comparison of model prediction by the proposed model and literature model](image1)

Figure 3: Comparison of model prediction by the proposed model and literature model [46]. The outlet concentrations of NO under lean and rich conditions agreed qualitatively.

![Comparison of model prediction by the proposed model and literature results](image2)

Figure 4: Comparison of model prediction by the proposed model and literature results [46]. The proposed model is able to describe the axial NO profile along the DOC channel under lean conditions.

The performance of the proposed model is evaluated by simulating the same set of experiments using the same chemical mechanism. It should be
noted there are significant differences between the proposed model and the model used by Koop and Deutschmann [46]. The main difference between the proposed model and the literature model is the treatment for convective mass transfer phenomena. The literature model considers the catalytic channel using a two-dimensional cylindrical coordinate. As a result, radial mass transfer from the bulk gas flow to the washcoat is solved directly. In the proposed model, the convective mass transfer is modelled using a constant Sherwood number approach [44]. NO is chosen to be the rate limiting species in the effectiveness factor model following the choice made by Koop and Deutschmann [46] during the development of the mechanism. Other modelling parameters are identical to those used by Mladenov et al. [48].

The experimental measurements and simulations from the literature, as well as predicted outlet NO concentrations made by the proposed after-treatment model are shown in Fig. 3. The axial profiles of NO with “lean” inlet are shown in Fig. 4.

The differences between the model predictions by the literature model and the proposed model can be attributed to the different treatments for the convective mass transfer between the bulk gas and the catalyst. The computational cost of the proposed model would be significantly lower than the literature model due to the one-dimensional axial coordinate description of the DOC channels, and therefore the trade-off with accuracy in the prediction is deemed acceptable.

3.2. Selective Catalytic Reduction (SCR)

The purpose of a NH$_3$-SCR is to reduce NOx to N$_2$ by NH$_3$. There are three major reaction pathways, “standard-SCR” (eq. 13), “fast-SCR” (eq. 14)
and “NO$_2$-SCR” (eq. 15) [44]:

$$4 \text{NO} + 4 \text{NH}_3 + \text{O}_2 \xrightarrow{\text{cat.}} 4 \text{N}_2 + 6 \text{H}_2\text{O}$$  \hspace{1cm} (13)

$$\text{NO} + \text{NO}_2 + 2 \text{NH}_3 \xrightarrow{\text{cat.}} 2 \text{N}_2 + 3 \text{H}_2\text{O}$$  \hspace{1cm} (14)

$$6 \text{NO}_2 + 8 \text{NH}_3 \xrightarrow{\text{cat.}} 7 \text{N}_2 + 12 \text{H}_2\text{O}$$  \hspace{1cm} (15)

NH$_3$-SCR catalyst formulations are usually based on either Vanadium or Iron/Copper zeolites [72]. As zeolites have better thermal resistance than Vanadium-based catalyst, they are suitable in both SCR-front and DPF-front layouts, whereas Vanadium-based catalyst are only suitable for SCR-front layout [14]. Therefore a Copper zeolite SCR catalyst is chosen for this study.

The chemical mechanism of the Cu-ZSM-5 catalyst developed by Olsson et al. [44] is used in this work. Olsson et al. [44] calibrated the reaction mechanism against various sets of the experimental data. One of the experiments investigated the effect of changing NO-to-NO$_2$ ratio at the inlet. The inlet concentration underwent step-changes at the time specified in Table 1. The concentrations of NO, NO$_2$ and N$_2$O were measured at the outlet.

The proposed model was assessed by simulating the same experiment and comparing the model results with experimental measurements and model predictions made by Olsson et al. [44]. Olsson et al. [44] concluded that washcoat diffusion was negligible in this case and therefore the effectiveness factor model is not applied to SCR simulations. Experimental measurements and predicted concentrations of the literature model and the proposed model are shown in Fig. 5.
Table 1: Inlet conditions used in the SCR experiment [44].

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>NO (ppm)</th>
<th>NO₂ (ppm)</th>
<th>NH₃ (ppm)</th>
<th>O₂</th>
<th>H₂O</th>
<th>Ar</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>500</td>
<td>0</td>
<td>500</td>
<td>8%</td>
<td>5%</td>
<td>balance</td>
</tr>
<tr>
<td>30</td>
<td>400</td>
<td>100</td>
<td>500</td>
<td>8%</td>
<td>5%</td>
<td>balance</td>
</tr>
<tr>
<td>45</td>
<td>300</td>
<td>200</td>
<td>500</td>
<td>8%</td>
<td>5%</td>
<td>balance</td>
</tr>
<tr>
<td>60</td>
<td>250</td>
<td>250</td>
<td>500</td>
<td>8%</td>
<td>5%</td>
<td>balance</td>
</tr>
<tr>
<td>75</td>
<td>200</td>
<td>300</td>
<td>500</td>
<td>8%</td>
<td>5%</td>
<td>balance</td>
</tr>
<tr>
<td>90</td>
<td>100</td>
<td>400</td>
<td>500</td>
<td>8%</td>
<td>5%</td>
<td>balance</td>
</tr>
<tr>
<td>105</td>
<td>0</td>
<td>500</td>
<td>500</td>
<td>8%</td>
<td>5%</td>
<td>balance</td>
</tr>
</tbody>
</table>

Figure 5: Comparison of model prediction of NO, NO₂ and N₂O emission by the proposed model and literature results [44].

Qualitative agreements can be observed between the three sets of data. The differences between predictions by the literature model and the proposed model may be attributed to the different treatments of convective mass transfer. Whilst the formulations for convective mass transfer are equivalent, Olsson et al. [44] did not report the transport data used for calculations of
diffusion coefficients of gas phase species. This may lead to slightly different gas phase composition within the porous catalytic washcoat and hence small differences in predicted NOx conversions. In conclusion, the combination of the chemical mechanism and the proposed model are shown to be able to capture the general trend of the change in NOx conversion, and therefore are sufficient for investigations on the impact of device configurations on overall EAT system design.

3.3. Diesel Particulate Filter (DPF)

It is important to ensure that the proposed model can simulate both the filtration and regeneration process of the DPF with sufficient accuracy. The ability of the proposed model to simulate the filtration behaviour of DPFs is checked against results by Zhang et al. [73] who simulated a DPF that had been studied experimentally by Suresh et al. [74]. The proposed model is applied to predict the change in pressure drop of the DPF as it collects particles, as the pressure drop of the DPF is commonly used in modelling studies to characterise the filtration behaviour [75]. In this work, it is assumed that the inlet particle diameter is 100 nm which is typical for diesel engines [76]. The capability of the DPF model to describe the impact of particles has been comprehensively investigated in previous work [42].

The results of experiment and simulations are shown in Fig. 6, where we can observe excellent agreements between the literature data and predictions by the proposed model. The following parameters have been calibrated: The particle packing density = 4.0 kg/m$^3$, the percolation constant = 0.918, the porosity of the particle cake layer = 96% and the primary particle diameter of the particle cake layer = 27 nm. The first two parameters determine when
the DPF pressure drop transitions to linearly increasing with time and the latter two parameters determine the gradient of the linear phase.

Figure 6: Comparison of model prediction of DPF backpressure by the proposed model and literature results [73, 74].

The ability of the proposed model to describe DPFs under active regeneration has been shown in previous work [42]. The rate parameters for active regeneration used in this work are obtained from Bissett [77]. The ability to describe passive regeneration is checked against the work of Kim et al. [78]. They simulated the variation of the mass of trapped particles inside a DPF under three inlet conditions with various NOx-to-PM ratio. The rate parameters for the passive regeneration reactions are calibrated in this work to best reproduce the results of Kim et al. [78]. The rate of passive regeneration reaction is governed by two parameters in this work: the pre-exponential
constant and the activation energy. The value of activation energy is 95 kJ/mol [29]. However, the literature provides no consensus for the value of the pre-exponential factor and it is treated as a calibration parameter. After calibration, the value of the pre-exponential constant is $1.5 \times 10^{25}$ m/(molKs). The predicted particle mass profiles after calibration are shown in Fig. 7. It can be concluded from Fig. 7 that the proposed model is capable of describing the impact of varying NO$_2$ content on passive regeneration of DPFs.

![Figure 7: Comparison of model prediction of soot load during passive regeneration by the proposed model (lines) and the literature model (markers) [78].](image)

4. Parametric study

In this section, the proposed EAT model is used to investigate interaction between EAT devices in DPF-front and SCR-front EAT systems. The investigation is performed in the context of the design of an EAT system for
a heavy-duty diesel engine. First of all, simulations are performed to obtain the engine-out emissions. After that, the proposed EAT model is used to design the EAT system using both a DPF-front and SCR-front layout to meet the appropriate emission standards. The interaction between EAT devices are investigated by evaluation of the system performance and flux analysis.

4.1. Engine simulation

The EAT model is applied to design an EAT system for a heavy-duty diesel engine. This engine model is included in the SRM Engine Suite as “Ex.09 (6 Cylinder 7L HD Diesel LSMap TierIV)”. The engine model was calibrated for 8 operating points with varying engine speed and load. The engine-out NOx concentration and the soot emissions are shown on Fig. 8. The Brake Mean Effective Pressure (BMEP) is used as a measure for engine load.

![Figure 8: The predicted NOx and soot emissions of the engine used for the virtual design and testing of the exhaust after-treatment system.](image)

The inlet conditions of the EAT system at the 8 operating points given by the SRM Engine Suite are shown in Table 2. The exhaust temperatures at
the inlet of the after-treatment system is calculated as described in Section 2.2.

Table 2: Inlet conditions of the EAT system studied in this work.

<table>
<thead>
<tr>
<th>Operating point</th>
<th>Unit 1</th>
<th>Unit 2</th>
<th>Unit 3</th>
<th>Unit 4</th>
<th>Unit 5</th>
<th>Unit 6</th>
<th>Unit 7</th>
<th>Unit 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine speed</td>
<td>rpm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BMEP</td>
<td>bar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas mass flow</td>
<td>kg/h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gas temperature</td>
<td>°C</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Particle mass flow</td>
<td>kg/h</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass fraction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO</td>
<td>ppm</td>
<td>37</td>
<td>39</td>
<td>112</td>
<td>2080</td>
<td>4270</td>
<td>182</td>
<td>7590</td>
</tr>
<tr>
<td>CO₂</td>
<td>%</td>
<td>19.8</td>
<td>17.1</td>
<td>21.5</td>
<td>17.6</td>
<td>22.9</td>
<td>23.3</td>
<td>17.9</td>
</tr>
<tr>
<td>H₂O</td>
<td>%</td>
<td>4.5</td>
<td>4.2</td>
<td>4.4</td>
<td>5.6</td>
<td>6.0</td>
<td>5.6</td>
<td>6.1</td>
</tr>
<tr>
<td>O₂</td>
<td>%</td>
<td>11.8</td>
<td>12.4</td>
<td>12.3</td>
<td>8.9</td>
<td>8.1</td>
<td>9.1</td>
<td>7.8</td>
</tr>
<tr>
<td>N₂</td>
<td>%</td>
<td>64</td>
<td>66</td>
<td>62</td>
<td>68</td>
<td>63</td>
<td>62</td>
<td>67</td>
</tr>
<tr>
<td>NO</td>
<td>ppm</td>
<td>683</td>
<td>805</td>
<td>203</td>
<td>1320</td>
<td>504</td>
<td>677</td>
<td>1030</td>
</tr>
<tr>
<td>NO₂</td>
<td>ppm</td>
<td>335</td>
<td>359</td>
<td>380</td>
<td>191</td>
<td>193</td>
<td>249</td>
<td>145</td>
</tr>
<tr>
<td>N₂O</td>
<td>ppm</td>
<td>2.5</td>
<td>2.9</td>
<td>3.7</td>
<td>2.2</td>
<td>2.9</td>
<td>3.1</td>
<td>2.4</td>
</tr>
<tr>
<td>C₃H₆</td>
<td>ppm</td>
<td>16</td>
<td>15</td>
<td>6</td>
<td>65</td>
<td>220</td>
<td>19</td>
<td>1385</td>
</tr>
</tbody>
</table>

4.2. Interaction between after-treatment devices

In this section we examine different EAT systems designs for the 8 operating points listed in Table 2. Both DPF-front and SCR-front layouts are considered. The EAT designs are developed using emission standard IIIB and IV as objectives. The impact of the emissions standard on EAT design and performances can be extracted by comparing the systems behaviours. In
order to examine the interaction between after-treatment devices, the lengths of individual devices are varied between designs.

Since large number of design parameters are needed to define an EAT system, thorough analysis is time consuming and may distract us from the objective of this study. Some simplifications have been applied during the design process so that the investigation can focus on the chemical phenomena. For example, the outer diameter of all catalytic EAT devices are fixed to 0.2 m and that of the DPF is fixed to 0.3 m. The catalyst loadings of the SCR are chosen to be the same as Section 3. The catalyst loading of the DOC is set to 40 g/ft$^3$ to represent typical commercial DOCs [32, 46].

4.2.1. DPF-front system

![Operating point 4](image1)

Figure 9: Impact of the length of upstream DOC on downstream DPF.

Firstly the interaction between DOC and DPF is examined. This is done by comparing the behaviours of two DOC-DPF systems where the lengths of the DOCs are 0.15 m and 0.4 m respectively. Their behaviours at operating
point 4 and 5 are shown in Fig. 9. The concentration profiles of NO\textsubscript{2} are shown here since it is the key reactant for passive regeneration (eq. (9)) of the DPF. It is found that active regeneration by O\textsubscript{2} is only significant for operating point 8, where the exhaust is hot enough to overcome the activation energy of the active regeneration reaction. NO\textsubscript{2} is generated from NO by the DOC and is consumed subsequently in the DPF. It can be observed from Fig. 9 that increasing the length of the DOC leads to an increase in NO\textsubscript{2} concentration at the inlet of the DPF. At operating point 4, there is sufficient passive regeneration to achieve steady state operation with the short DOC. Steady state operation can also be achieved at operating points 1, 2, 3, 6 and 7. Increasing the length of the DOC results in a lower pressure drop across the DPF, which would have positive implications on the engine cylinder combustion efficiency. This is offset by the increased capital cost of having a longer DOC and an increase in volume of the EAT system, which can be a significant constraint.

At operating point 5, passive regeneration is insufficient to achieve steady state in the system with the short DOC. The inlet channel of the DPF is clogged by particles, resulting in rapid increase in the pressure drop after about 8 hours. Whilst increasing the length of the DOC is also unable to support sufficient regeneration within the DPF, it is capable of delaying clogging by about 1 hours. This can reduce the fuel penalty associated with active regeneration. To summarise, increasing the length of the DOC is beneficial for passive regeneration occurring in the downstream DPF regardless of whether passive regeneration is sufficient to allow continuous operation at a given operating point.
Eq. (9) shows that regeneration reactions can lead to production of CO in the exhaust. The implication of this secondary CO emission is demonstrated in Fig. 10, where the CO concentration profiles along two DOC-DPF-SCR systems at operating point 7 are shown. In this case, engine-out CO are completely removed within the DOC. Regeneration reactions inside the DPF leads to an increase of CO in the exhaust. It should be noted that the length of the DPF has no impact on the steady state gas phase composition profile. This secondary CO emission then passes through the downstream SCR catalyst unreacted.

The interaction between CO and the copper zeolite catalyst was not considered during the development of the chemical mechanism in the work of Olsson et al. [44]. It is reported by Zhang et al. [79] that copper zeolite catalyst has CO oxidation capability. If this is also possible for the Cu-ZSM-5 catalyst, then the downstream SCR catalyst can prevent CO slip and this makes the DPF-front layout more attractive. Future investigations of SCR catalysts should also consider the possibility of CO oxidation in light of the secondary CO emission.

Figure 10: Secondary CO emission post-DPF due to regeneration.
4.2.2. **SCR-front system**

Figure 11: Impact of the length of upstream DOC on downstream SCR.

After examining the interaction between the DOC and the DPF, we investigate the interaction between the DOC and the SCR in systems with the SCR-front layout. The DOC can oxidise NO to NO₂, which enables the “fast-SCR” reaction (eq. 14) to be the dominant reaction pathway in the SCR. This claim is examined at operating point 2, where the concentration profiles of NOx are shown in Fig. 11. Two DOC-SCR systems with two different lengths of DOC are shown here. The NO₂:NOx ratio increased from 0.51 to 0.76 by increasing the length of the DOC from 0.15 m to 0.4 m. It should be noted that the total amount of NOx in the exhaust remains unchanged through the DOC.

There is little benefit of having a longer DOC. The overall deNOx efficiency is slightly improved by having a longer DOC at the expense of 11% increase in NH₃ dosing. In order to further investigate the influence of the upstream DOC on the performance of the SCR, flux analysis is applied and the results are discussed in section 4.2.3.

Fig. 11 has shown that whilst having a longer DOC has little impact on the
(a) Respective performances of SCR-front systems designed to meet Stage IIIB and Stage IV standard.

(b) Evaluation of the possibility of sufficient passive regeneration with Stage IV-compliant SCR-front systems.

Figure 12: Examination of the possibility to have sufficient passive regeneration for steady state operation with SCR-front system.

overall deNOx efficiency, it increased the amount of NO$_2$ at the outlet of the SCR, which is the inlet of the DPF in a SCR-front system. This leads to more NO$_2$ at the exit of the SCR catalyst, which can aid passive regeneration in a downstream DPF. Here we examine the possibility of steady state operation for EAT systems with SCR-front layout. For a Stage IIIB compliant design, it is found that there is sufficient passive regeneration occurring in the DPF to achieve steady state at operating point 2, as shown in Fig. 12(a). In addition, this design is able to achieve steady state at operating point 3 and 7.
In order to comply with the Stage IV standard where the NOx limit decreased from 2.0 g/kWh to 0.4 g/kWh [80], the length of the SCR and the amount of NH\textsubscript{3} injection must increase. When the length of the SCR is increased from 0.32 m to 0.38 m, steady state operation is only achievable at operating point 7. At operating point 2, the inlet NO\textsubscript{2} concentration for the DPF has dropped from 120 ppm to 20 ppm, which leads to decreased regeneration activity in the DPF and ultimately DPF clogging.

Alternative designs are explored to see whether steady state operation in the DPF is possible for a Stage IV compliant SCR-front EAT system. The behaviours of those systems are shown in Fig. 12(b). Firstly the length of the DOC is increased. However, the impact of the DOC is nullified by the SCR catalyst; the inlet NO\textsubscript{2} concentration for the DPF has only increased from 20 ppm to 30 ppm despite significant increase in DOC size. Fig. 12 shows that steady state operation is not possible, even with a larger DPF, since the size of the DPF does not affect the steady state gas phase composition profile. Similar to previous findings, increasing the length of the DPF can delay clogging but does not mitigate the need for active regeneration.

4.2.3. Flux Analysis

Flux analysis is a technique to investigate reaction mechanisms [81]. It allows better understanding of the behaviour of a system by tracking how elements are “transferred” between chemical species via various reaction pathways. With flux analysis, we can identify the relative importance of the different reaction pathways.

In this case flux analysis is applied to the SCR catalyst with different upstream DOCs at operating point 2. The behaviour of the system is shown
in Fig. 11. The fluxes of nitrogen atoms are analysed since SCR is designed to reduce NOx to N\textsubscript{2}. Details of the calculations of reaction fluxes can be found in work of Nurkowski et al. [81]. The fluxes of nitrogen atoms inside the SCR at operating point 2 with upstream DOCs of lengths 0.15 m and 0.4 m is shown in Fig. 13.

![Diagram of nitrogen fluxes](image)

(a) Flux with DOC length 0.15 m.  
(b) Flux with DOC length 0.4 m.

Figure 13: Flux of Nitrogen in the SCR with upstream DOCs of lengths 0.15 m and 0.4 m at operating point 2. The fluxes of nitrogen atoms (mol/m\textsuperscript{3}s) are integrated along the entire length of the SCR.

We can observe that nitrogen atoms “flow” from NOx and NH\textsubscript{3} to N\textsubscript{2} as well as a undesirable by-product N\textsubscript{2}O. The net fluxes of nitrogen from adsorbed NH\textsubscript{3} (shown as NH\textsubscript{3}\textsuperscript{*} in Fig. 13), NO and NO\textsubscript{2} to N\textsubscript{2} are due to SCR reactions taking place. In addition, the Cu-ZSM-5 catalyst also exhibits NO oxidation activity as reflected by the nitrogen flux from NO to NO\textsubscript{2}.

We can clearly see the impact of varying the length of the upstream DOC on the SCR by comparing Fig. 13(a) and 13(b). The fluxes from
adsorbed NH₃ to N₂ for both systems are similar, indicating similar overall deNOx performances. However, the system with the longer DOC would lead to increased amount of NO₂ at the inlet of the SCR, and this leads to corresponding increase in fluxes from NO₂ to N₂ and N₂O. The direction of the net flux between NO and NO₂ is since the extent of NO oxidation is limited by equilibrium.

Further analysis was performed to study the system behaviour with a wider range of NO₂/NOx ratio. The minimum NO₂/NOx ratio (0.23) can be achieved if the SCR is the first device in the EAT system. The nitrogen fluxes are shown as stacked bars on Fig. 14. The contributions of standard SCR, fast SCR and NO₂ SCR as well as the N₂O producing side-reaction vary with the inlet NO₂/NOx ratio.

![Figure 14: The variation in deNOx efficiency of the SCR with different inlet NO₂/NOx.](image)
The fluxes of nitrogen atoms from NOx to other species are shown as stacked bars for three DOC-SCR systems. The nitrogen flux correlates to the efficiency of the SCR. The critical NO₂/NOx ratio for each NH₃ dosing are shown as crosses.
In addition to the nitrogen fluxes shown on the left vertical axis, the deNOx efficiencies under different inlet NOx ratio at four NH$_3$ dosage levels are shown as curves on the right vertical axis. The deNOx efficiencies are at their respective minimum when the inlet contains 100% NO$_2$. By reducing the amount of NO$_2$ at the inlet, the SCR becomes more efficient at fixed NH$_3$ dosage. The critical NO$_2$/NOx ratio for deNOx efficiency is marked on the four efficiency curves.

It is observed that the total nitrogen flux from NOx correlates with the deNOx efficiency. The decrease in efficiency at high NO$_2$ content can be explained by the individual flux contributions. The contributions of the fast SCR and the standard SCR to NOx removal are lowest when the NO$_2$/NOx $= 0.76$ at the SCR inlet. Furthermore, the flux of N$_2$O production is the highest, which is undesirable since N$_2$O is a strong greenhouse gas [72].

The results of flux analysis can shed light on decisions needed when designing EAT systems. In this case, increasing the length of the DOC to 0.4 m has little benefit on the downstream SCR since the increased activity of fast SCR reaction is counter-balanced by the decrease in standard SCR activity. Whilst these conclusions are limited to this particular catalyst, flux analysis have been shown to be useful for other EAT design problems.

4.2.4. Comparison between DPF-front and SCR-front layout

It is apparent that the DPF-front system is superior with respect to the SCR-front system in terms of regeneration strategy. The DPF-front system is able to achieve steady state with sufficient passive regeneration activities for 6 out of 8 operating points investigated. On the other hand the Stage IIIB compliant SCR-front system can only reach steady state at 3 oper-
ating points. Passive regeneration activities can reduce the fuel penalties associated with active regeneration. When emission standards become more stringent, passive regeneration in the SCR-front system is further limited as shown in Fig. 12. This implies an increase in the need for active regeneration. Furthermore, the CO generated from DPF regeneration may be treated by the downstream SCR catalyst in a DPF-front system, since the copper zeolite catalyst may be able to oxidise CO. This remains to be investigated.

Whilst more NO\textsubscript{2} is available to the SCR in SCR-front systems, the overall deNO\textsubscript{x} efficiency is not improved. Flux analysis in Section 4.2.3 has shown that the benefit of increased fast SCR activity due to higher NO\textsubscript{2} content is cancelled by a decrease in standard SCR activity.

In conclusion, the DPF-front layout has been shown to have better steady state performance compared to the SCR-front layout in this case study. The benefit of higher passive regeneration activities is clearly shown, whereas an increase of the fast SCR activity has no significant impact on the overall deNO\textsubscript{x} performance of the EAT system under steady state operation.

5. Conclusions

A single channel EAT model with rigorous treatment for the catalytic chemistry has been proposed in this study. The performances of the proposed model for simulating a DOC, SCR and DPF are compared with experimental and simulation data from the literature. The proposed model has been shown to be able to describe the behaviours of relevant after-treatment devices with sufficient accuracy given the simplified treatment for mass transfer phenomena. The proposed model is then applied to design after-treatment
systems for a heavy duty diesel engine simulated using virtual engine software. The steady state behaviours of designs with different configurations, emission standards requirements and lengths of individual devices are investigated.

The DOC is found to promote passive regeneration in a downstream DPF which may be able to be exploited to reduce the fuel penalty associated with active regeneration frequency and backpressure. However, it is not always possible to achieve steady state operation of the passive regeneration by increasing the size of the DOC. Furthermore, increased regeneration activity could lead to significant secondary CO emissions post-DPF which cannot be treated by the DOC. In the DPF-front configuration, the CO generated during passive regeneration was found to pass through the modelled SCR without reacting. The SCR mechanism adopted in this study does not consider interaction between CO and the SCR catalyst. This should be considered in future studies on SCR chemistry. This study has not found the reportedly positive impact of an upstream DOC on the SCR catalyst [30]. Further flux analysis suggests that the particular SCR catalyst studied is insensitive to the NO$_2$ portion at its inlet, for the range of possible inlet species concentrations provided by the DOCs considered in this design study.

It was found that as more stringent emission standards are adopted, the possibility of passive regeneration is reduced for SCR-front EAT systems. The NOx emission limits constrains the sizing of the SCR catalyst, which is the only device with net NOx conversion along after-treatment devices included in this study. In addition, it is possible that the SCR catalyst exhibits CO oxidation activity and therefore the SCR catalyst can help to
mitigate secondary CO emissions due to DPF regeneration in a DPF-front system. To summarise, the DPF-front design is superior to the SCR-front design for this case study, and it is expected to become more popular in the future.

This study has focused on the steady state performance and chemical interaction of the EAT systems. Future studies on chemical interaction of multi-device EAT systems should be extended to consider transient and thermal behaviour of EAT systems under more complicated operating conditions including cold-start behaviour and active regeneration. The numerical model should be further developed to be able to simulate more advanced, integrated EAT devices such as catalysed particulate filters and dual-layer catalysts.

Supplementary material

Additional data related to this publication is available at the University of Cambridge data repository (https://doi.org/10.17863/CAM.40283).

Acknowledgements

The authors would like to thank Cambridge Centre for Advanced Research and Education in Singapore (CARES), Engineering and Physical Sciences Research Council (EPSRC) and Royal Dutch Shell for their support. This project has received funding from the European Union’s Horizon 2020 Research and Innovation Programme under grant agreement no. 724145.

Nomenclature
<table>
<thead>
<tr>
<th>Roman symbols</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Pre-exponential constant</td>
<td>(mol, cm, s)</td>
</tr>
<tr>
<td>$B$</td>
<td>Temperature exponent</td>
<td>-</td>
</tr>
<tr>
<td>$c$</td>
<td>Molar concentration of gas species</td>
<td>mol/m$^3$</td>
</tr>
<tr>
<td>$D$</td>
<td>Side length of square channel cross-section</td>
<td>m</td>
</tr>
<tr>
<td>$D_m$</td>
<td>Molar diffusivity</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>$d_p$</td>
<td>Diameter of particle</td>
<td>m</td>
</tr>
<tr>
<td>$E$</td>
<td>Activation energy</td>
<td>kJ/mol</td>
</tr>
<tr>
<td>$F$</td>
<td>Friction factor</td>
<td>-</td>
</tr>
<tr>
<td>$J$</td>
<td>Total number of connections</td>
<td>-</td>
</tr>
<tr>
<td>$k$</td>
<td>Reaction rate constant</td>
<td>(mol, cm, s)</td>
</tr>
<tr>
<td>$L$</td>
<td>Length between centres of adjacent volume elements</td>
<td>m</td>
</tr>
<tr>
<td>$M$</td>
<td>Molar mass</td>
<td>kg/mol</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Gas mass flow rate</td>
<td>kg/s</td>
</tr>
<tr>
<td>$N$</td>
<td>Particle number density</td>
<td>1/m$^3$</td>
</tr>
<tr>
<td>$\dot{N}$</td>
<td>Flow rate of particle number density</td>
<td>1/m$^3$/s</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of mole</td>
<td>mol</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure</td>
<td>Pa</td>
</tr>
<tr>
<td>$Q$</td>
<td>Filtration efficiency</td>
<td>-</td>
</tr>
<tr>
<td>Roman symbols</td>
<td>Meaning</td>
<td>Unit</td>
</tr>
<tr>
<td>---------------</td>
<td>--------------------------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>$R$</td>
<td>Universal gas constant</td>
<td>kJ/molK</td>
</tr>
<tr>
<td>$r$</td>
<td>Rate of reaction</td>
<td>mol/m$^3$s</td>
</tr>
<tr>
<td>$S$</td>
<td>Reactive surface area</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$Sh$</td>
<td>Sherwood number</td>
<td>-</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature</td>
<td>K</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>$u$</td>
<td>Channel gas flow velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$W$</td>
<td>Total number of reactions</td>
<td>-</td>
</tr>
<tr>
<td>$v$</td>
<td>Through-wall gas flow velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>$X$</td>
<td>Reaction order</td>
<td>-</td>
</tr>
<tr>
<td>$x$</td>
<td>Mole fraction</td>
<td>-</td>
</tr>
<tr>
<td>$Y$</td>
<td>Mass fraction of gas species</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Greek symbols</th>
<th>Meaning</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Stoichiometric constant for regeneration</td>
<td>-</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Total number of gas phase species</td>
<td>-</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Surface concentration</td>
<td>mol/m$^2$</td>
</tr>
<tr>
<td>Greek symbols</td>
<td>Meaning</td>
<td>Unit</td>
</tr>
<tr>
<td>---------------</td>
<td>---------------------------------------</td>
<td>-------</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Effectiveness factor</td>
<td>-</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Site fraction</td>
<td>-</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Permeability</td>
<td>$m^2$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Gas viscosity</td>
<td>Pa.s</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Stoichiometric coefficient</td>
<td>-</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Dynamic density</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Molar density</td>
<td>mol/m$^3$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Coverage dependent parameter</td>
<td>-</td>
</tr>
<tr>
<td>$\Psi$</td>
<td>Total number of surface species</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i$</td>
<td>Volume element index</td>
</tr>
<tr>
<td>$w$</td>
<td>Reaction index</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Gas species index</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Surface species index</td>
</tr>
<tr>
<td>$z$</td>
<td>Particle size class index</td>
</tr>
<tr>
<td>in</td>
<td>Inlet</td>
</tr>
<tr>
<td>out</td>
<td>Outlet</td>
</tr>
<tr>
<td>ch</td>
<td>channel</td>
</tr>
<tr>
<td>AR</td>
<td>Active regeneration</td>
</tr>
<tr>
<td>Subscript</td>
<td>Meaning</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>PR</td>
<td>Passive regeneration</td>
</tr>
</tbody>
</table>


[18] P. Forzatti, L. Lietti, I. Nova, E. Tronconi, Diesel NOx aftertreatment catalytic technologies: Analogies in LNT and SCR catalytic chemistry,


L. Wei, F. Yan, J. Hu, G. Xi, B. Liu, J. Zeng, NOx conversion efficiency optimization based on NSGA-II and state-feedback nonlinear


[46] J. Koop, O. Deutschmann, Detailed surface reaction mechanism for Pt-


[52] A. G. Konstandopoulos, M. Kostoglou, Reciprocating flow regeneration


