Risky Choice and Type-Uncertainty in "Deal or No Deal?"

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Abstract

This paper uses data from the popular television game-show, "Deal or No Deal?", to analyse the way individuals make choices under risk. In a unique approach to the problem, I present a formal game-theoretical model of the show in which both the contestant and the banker are modelled as strategic players. I use standard techniques to form hypotheses of how rational expected utility-maximisers would behave as players in the game and I test these hypotheses with the relevant choice data. The main result is that an increasing offer function is the result of optimal behaviour when the banker is uncertain about the contestant's risk attitudes. This result provides a theoretical foundation to the empirical model of the banker that pervades the literature. Estimates of the coefficient of relative risk aversion are consistent with estimates from other studies and estimates of the discernment parameter suggest contestants have difficulty making choices.

Keywords: Choice under Risk, Expected Utility, Asymmetric Information, Risk-Aversion
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1　Introduction

"Deal or No Deal?" is a popular television game-show in which contestants make choices between risky prospects in the hope of winning up to £250,000[1]. Unlike most game-shows, the contestant's winnings are not determined by his competence at some skill or his ability to answer a set of testing general knowledge questions; "Deal or No Deal?" is a game of pure risk and the size of the contestant's winnings, which are potentially life-changing, is ultimately down to luck. For this reason, it makes for a very interesting case study of how individuals make decisions under risk.

I explain the rules of the game in detail in Section 2 but I give the basic idea of what happens here. A contestant chooses a box which contains one of 22 known sums of money. The sum contained in the box is unknown but we gain information about its value through a process of elimination. In each round the contestant selects a certain number of the other 21 boxes to be opened and the contents of these boxes are revealed to give information about what is not in the contestant's box. At the end of each round the "banker" makes the contestant an offer based on the sums that remain in play. If the contestant accepts the offer he walks away with this sure sum and if he rejects the offer he moves to the next round where he opens more boxes and faces another offer from the banker. The game goes on like this until the contestant either accepts an offer or rejects all of the banker's offers and takes home the sum in his own box.

In this paper I analyse the game-show in a game-theoretical framework. Unlike previous studies, I model both the contestant and the banker as strategic players in a dynamic game and consider equilibrium strategy profiles for both players. Given evidence of heterogeneity in individuals' risk preferences I assert that, even with a fairly extensive screening process, the banker may have noisy beliefs about the contestant's preferences and I use this position to derive the main result of this paper. Namely, I show that in the presence of uncertainty about the contestant's preferences, when both players are optimising, the banker's offer becomes relatively more generous from round to round. This gives a theoretical basis to empirical models of the banker which make such an assumption. Furthermore it shows that one does not need a complex representation of the banker's preferences to explain an increasing offer function but that it can be explained by optimising behaviour in the presence of type uncertainty.

The increasing offer function allows the banker to update his beliefs about the contestant's risk preferences step by step and this helps the banker to minimise the expected payout over

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[1] "Deal or No Deal?" was first aired in Britain in October 2005 but it had previously been shown in similar forms in the Netherlands, Australia and Italy and it is now a global franchise with shows in 49 countries around the world. This paper focuses on the British version of the show.

the rest of the game. Understanding that this is the case, contestants are able to appreciate the value of staying in the game, being grouped with lower risk types and receiving a relatively higher offer in the next round. Contestants therefore need to be paid more to reveal their type and leave the game and offers are overall higher than they would be under symmetric information.

I estimate contestants' preferences using an empirical model which can be interpreted as an approximation of the game-theoretical model. I use data from 129 episodes of "Deal or No Deal?" aired in great Britain between October 2005 and April 2006 to estimate a random effects logit model in which the probability that the contestant plays deal depends on how close the offer is to his threshold value and how "discerning" the contestant is. The estimate of the coefficient of relative risk aversion under the assumption of no wealth integration is approximately 0.6 and there is evidence of heterogeneity in preferences. The estimate of the discernment parameter suggests contestants have difficulty making choices, even when the offers are substantially different from the contestant's threshold value. This conclusion is strengthened by evidence from the bounds approach to the estimation of the contestants' preferences, which shows that at least 1 in 5 contestants make inconsistent choices during the course of their game.

While it is noteworthy that the banker's preferences need not be any more complex than the basic specification to produce an interesting offer function, it is unlikely that the offer is not informed by other considerations. I discuss what these considerations may be and how they may affect the offer. For example, low offers in the opening rounds ensure the game lasts a few rounds, which is a necessary condition for an entertaining show. The possibility that contestants' choices are characterised by non-EUT theories is mentioned but I do not offer any formal analysis on this subject.

1.1   Related Literature

Game-shows can be useful case-studies for empirical estimates of risk attitudes because they generate data on individuals' choices in high-stakes gambles without incurring large costs to the experimenter. Gertner (1993) was the first to analyse choice under risk in a game-show setting, using "Card Sharks" as the subject. Other studies that followed include Metrick (1995) ("Jeopardy!"); Fullenkamp et al (2003) ("Hoosier Millionaire"); Bennett and Hickman (1993), Berk et al (1996) and Tenorio and Cason (2002) ("The Price is Right"); and Beetsma and Schotman (2001) ("Lingo").

Within the class of game-shows "Deal or No Deal?" stands out as a particularly attractive source of data for the estimation of risk attitudes. There are a numerous possible gambles

and the prizes range from as little as 1p to as much as £250,000. Furthermore, the game is not burdened by complications such as the estimation of subjective beliefs. It is a game of pure risk – at each decision node the set of possible prizes and the associated probabilities are public information – and there is no requirement to perform well at a task or answer any general knowledge questions. "Deal or No Deal?" therefore represents a new and rich source of data for the estimation of individuals' risk attitudes.

The appealing format of the game-show has provoked interest among decision theorists, resulting in the recent growth of a body of academic literature, most notably Bombardini and Trebbi (2005), Post et al. (2006), De Roos and Sarifidis (2006) and Mulino et al (2006). These studies generally set out to test Expected Utility Theory (EUT) and find estimates of individual risk coefficients, but some also look at alternative theories of choice and make comparisons with the EUT approach. For example, Post et al (2006) and Mulino et al (2006) find evidence that contestants behave in a manner consistent with a frame-dependent theory of choice, such as Prospect Theory, and De Roos and Sarifidis (2006) estimate a rank-dependent utility (RDU) model and find that it considerably outperforms EUT.

Other studies look at specific aspects of the game to test theories of choice. For example, Blavatskyy and Pogrebna (2006) use data from the final two rounds of the Italian version of the show when the contestant has either a 20% chance or a 80% chance of winning a large prize and find evidence that contestants are not less risk-averse when facing large improbable gains. In another paper, Blavatskyy and Pogrebna (2006b) focus on the choice of a box swap and find that contestants generally do not display loss aversion in their decisions.

Estimates of the coefficient of relative risk aversion vary according to the choice of wealth parameter. De Roos and Sarifidis (2006) estimate it in the range of 0.45-0.66 when there is no initial wealth, and 1.8-3.2 when initial wealth (approximated by the average annual labour income) is included. Bombardini and Trebbi (2005) use data from the Italian version of the show and estimate the coefficient of constant relative risk aversion for the sample at 1.09, using annual labour income as the measure of wealth (the estimate drops to 0.5 if one assumes zero wealth). Thus they find that logarithmic utility is a suitable representation of contestants' preferences. They also find that contestants are approximately risk-neutral at small stakes, which, according to Rabin's calibration theorem (Rabin, 2000), is necessary for EUT to produce reasonable levels of risk-aversion at high stakes. These estimates compare well with estimates from other game-show studies (e.g. Beetsma and Schotman (2001), Fullenkamp et al (2003)) but are smaller than estimates from the broader field, which generally use finance data, (e.g. Blake (1996), Aït-Sahalia and Lo (2000) and Sydnor (2005)), although there are some exceptions (e.g. Chetty (2006), which uses evidence of labour elasticities).

A major theme to come out of the literature is that there appears to be significant het-

erogeneity in individuals' risk attitudes. Post et al (2006, p17) find that "the degree of risk aversion differs widely across the contestants", De Roos and Sarifidis (2006, p20) observe "significant variation in the degree of risk aversion" and Bombardini and Trebbi (2005, p23) find "substantial dispersion in risk preferences". Mulino et al (2006) also find evidence of considerable heterogeneity and single out age and gender as statistically significant determinants of the individual's risk aversion. Such findings of heterogeneous risk attitudes are consistent with findings from other studies from the broader literature (e.g. Barsky et al (1997), Jianakoplos and Bernasek (1998) and Kliger and Levy (2002)).

If contestants are to have varying degrees of risk aversion then uncertainty about the contestant's type on the banker's part becomes a possibility. Moreover, even with a rigorous screening process, the banker would find it difficult to develop pinpoint beliefs about an individual's preferences. Thus such type uncertainty is likely. The assumption that the banker is uncertain about the contestant's preferences forms the basis of the formal analysis in this paper and it is crucial in the derivation of the main result, which is that the banker's equilibrium offer becomes relatively more generous with each round.

It has been suggested that EUT may not be an appropriate framework to model individual choices in the setting of this game-show (see above). I do not test alternative theories of choice but offer an improvement to the EUT approach. I model the problem as a game in which both the contestant and the banker are strategic players and in doing so I derive stronger theoretical results to test with the data. EUT offers itself well to be handled in this way, but the same cannot be said for alternative theories, such as Prospect Theory. In particular, Prospect Theory does not specify how to deal with multi-stage games and it does not provide a formal theory of how individuals edit the prospects or determine their reference points. As such the formal modelling in this paper is restricted to the EUT framework.

The rest of the paper is structured as follows. In Section 2 I explain the rules of the game and in Section 3 I introduce the game-show data. In Section 4 I present an empirical model of the banker and find estimates of the contestants' preferences and the banker's offer function. I then turn my attention to the construction of a complete model of the game in Section 5, where I present the game of incomplete information and derive results for the contestant and the banker under the assumption of optimising behaviour. I discuss possible extensions to the Banker's preferences in Section 6 before closing with some conclusions in Section 7.

Table 1: The 22 hidden prizes

1p	£1,000
10p	£3,000
50p	£5,000
£1	£10,000
£5	£15,000
£10	£20,000
£50	£35,000
£100	£50,000
£250	£75,000
£500	£100,000
£750	£250,000

2 The Rules of the Game

The rules of the game are straightforward. There are 22 individuals, whom I call the candidates, and 22 sealed boxes. The contents of each box are unknown to all parties involved in the game[2], but it is known that each box contains one of the values from Table 1. All values are included and none are repeated.

Before the show starts, each candidate chooses a box. Once this is done one of the candidates is randomly chosen to compete as the contestant. The other 21 candidates stay on the show, waiting to be called upon to reveal the contents of their box.

The game starts with round 1, and the contestant is asked to select 5 boxes from the 21 held by the non-competing candidates. Once the contestant has made his selection, the contents of the 5 selected boxes are revealed to all parties. It is then known that the sum in the contestant's box is not one of these 5 sums. Round 1 ends with a cash offer from the banker. If the contestant accepts the offer, the game ends and the contestant leaves the show with the sum offered by the banker. If the contestant rejects the offer, the game proceeds to round 2.

Round 2 has broadly the same format as round 1, only this time the contestant is asked to select 3 boxes from the remaining 16 held by the non-competing candidates. The contents of these boxes are revealed and, once again, the round ends with an offer from the banker. Again, if the contestant accepts the offer, the game ends and the contestant leaves the show with the sum offered by the banker. If the contestant rejects the offer, the game proceeds to round 3.

The game continues in such a way for 6 rounds, with the contestant selecting boxes and the banker making offers at the end of each round. In each round the contestant selects

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[2]Only the independent adjudicator knows the location of each sum.

Table 2: A summary of the game-show format

Round	No. of boxes remaining at offer (including own)	Offer accepted	Offer rejected
1	17	Game ends	Go to round 2
2	14	Game ends	Go to round 3
3	11	Game ends	Go to round 4
4	8	Game ends	Go to round 5
5	5	Game ends	Go to round 6
6	2	Game ends	Open your box

Table 3: Mean offers and deal frequencies

| Round	Mean Offer	Standard Deviation	# Deals
1	£3,488.26	£2,639.20	0
2	£5,926.09	£4,806.40	0
3	£8,436.57	£6,587.73	4
4	£12,316.24	£9,279.86	32
5	£17,308.68	£14,135.12	48
6	£15,127.43	£27,149.41	22

boxes from the non-competing candidates and the contents of his own box remain unknown until the game is over. At the end of the final round, when just 2 boxes remain, if the contestant rejects the banker's offer, he is instructed to open his box to reveal his prize. Table 2 describes the complete format of the game.

3 The Data

The data set comprises the relevant information (the lotteries, the banker's offer and the contestants' choices) for each round of 129 episodes of the game-show aired in Great Britain. Some useful summary statistics of the banker's offers and the contestants' decisions are given in Table 3.

Table 3 shows that the banker's offers are generally higher in the later rounds and that there is a high variance in the offers, particularly in round 6. Furthermore, the ratio of the offer to the expected value of the remaining prizes increases in each round (See Figure 1). The contestant's decisions appear to be consistent with an improving offer function, with virtually all deals being made in the second half of the game (rounds 4-6). In the next section I perform a more rigorous analysis of the contestants' choices but by glancing over the data it appears the contestants show some level of rationality. For example, Figure 1 suggests contestants reject the low offers and accept the high offers.

Figure 1: The mean ratio of the offer to the expected value of the remaining prizes
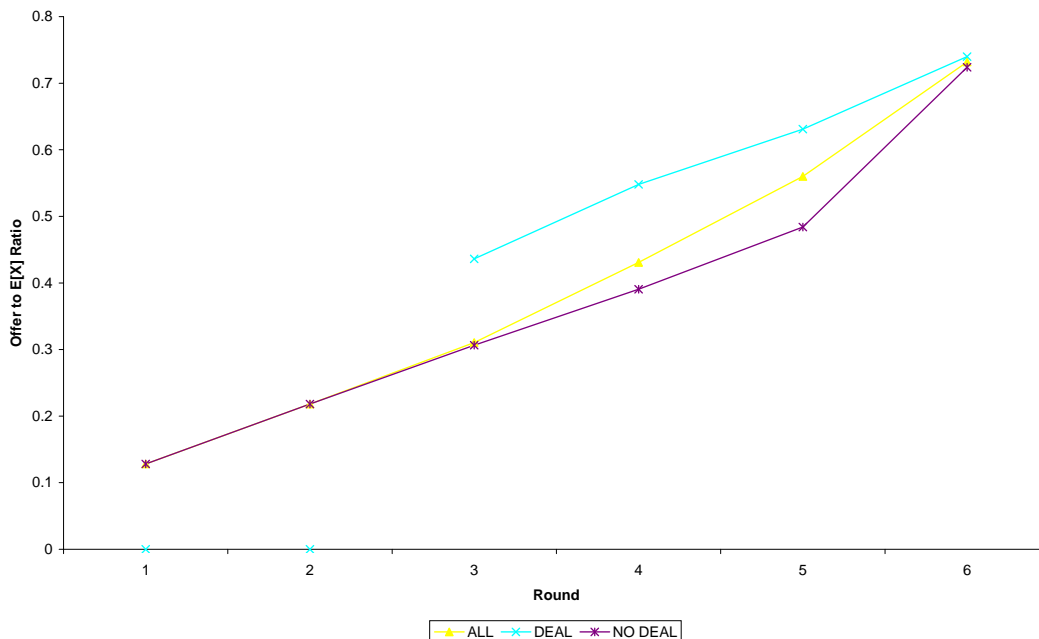
Contestant earnings are generally very high but there is also a very high variance. The mean winnings in the sample is £17,843 and the standard deviation is £17,357. Contestants should therefore understand the significance of their decisions. While there are very large amounts of money to be won there is also the chance that they walk away with as little as a penny (this happened twice in the sample). This setting makes the game ideal for modelling, because one would expect the contestants to think carefully about their choices and, as a result, better reveal their true preferences.

4   The Partial Approach - Deal or No Deal?

Existing studies generally treat the game-show as case study of how individuals make choices under risk, taking the banker's offer as given and ignoring any strategic interaction between the contestant and the banker. The banker's offer function is modelled empirically while the contestants' choices are examined under the assumption that they are optimising and from this the authors draw inference about the contestant's risk attitudes. This is a partial analysis of the game because it considers the optimising behaviour of only one of the players. In Section 5 I propose a complete strategic model of the game, in which I determine both the banker's optimal offer function, given a set of preferences, and the contestant's best response,

but first I perform a partial analysis similar to that found in the majority of the literature.

I first give details of some notation. Before the game begins the contestant chooses a prize, b, from the initial set of prizes, X. The value of b is unknown to all parties but it is known that it is uniformly distributed over X. The different rounds are indexed $t = 1, 2, ..., 6$. In round t, the set of prizes remaining when the banker makes the offer is denoted by X_t and the banker's offer is Y_t. The number of elements in X_t is smaller for larger values of t (see Table 2) but $b \in X_t$ for all values of t. I define the *straight lottery* in round t, L_t, as the simple lottery of choosing one of the remaining prizes with equal probability. That is, $L_t = \left(X_t^1, \frac{1}{K_t}; X_t^2, \frac{1}{K_t}; ...; X_t^{K_t}, \frac{1}{K_t} \right)$, where X_t^k is the kth element of X_t and K_t is the total number of elements in X_t. I distinguish this lottery from the *alternative gamble* in round t, G_t, which is defined as the lottery the contestant enters by playing No Deal in round t. These lotteries are not necessarily the same and the relation between the two is discussed below.

4.1 The contestant's preferences

I assume the contestant is an expected utility maximiser whose preferences are represented by the CRRA utility specification: $U(x; r, w) = \frac{(x+w)^{1-r} - 1}{1-r}$, where r is the contestant's coefficient of relative risk aversion and w is the contestant's initial wealth level. The contestant's optimisation problem in round t, when faced with the offer, Y_t, is:

$$\max_{p \in [0,1]} V_t \;=\; p.U(Y_t) + (1-p).E_t\left[V_{t+1}\right], \text{ for } t = 1, 2, ..., 6 \tag{1}$$

$$\text{where } V_7 \;=\; b \tag{2}$$

where $p \in [0, 1]$ denotes the probability with which the contestant plays Deal. Equation 1 implies that the contestant's best choice in round t depends on his beliefs about how the banker will set offers in future rounds. This is because by playing No Deal the contestant not only takes one step closer to the end of the final round when he can open his box, but it also gives him the chance to decide on the round $t+1$ offer. If the contestant expects future offers to be high enough, he may value the lottery over future offers above the round t offer. This rational, forward looking play can be contrasted with that of a myopic contestant, who does not take into account the value of future offers. The myopic contestant only considers the value of the straight lottery, L_t. His round t optimisation problem when faced with an offer, Y_t, can therefore be characterised as:

$$\max_{p\in[0,1]} V_t = p.U(Y_t) + (1-p).EU(L_t), \text{ for } t = 1, 2, ..., 6 \tag{3}$$

Whether the contestant is forward-looking or myopic, he will play Deal only if the offer is at least as high as the certainty equivalent of the alternative gamble ($p = 1$ only if $Y_t \geq CE(G_t; r)$), he will play No Deal only if the offer is no higher than the certainty equivalent ($p = 0$ only if $Y_t \leq CE(G_t; r)$) and he will be indifferent between playing Deal and No Deal when the offer is equal to the certainty equivalent ($p \in [0, 1]$ only if $Y_t = CE(G_t; r)$). The crucial difference is that the alternative gamble is defined differently for the forward looking and myopic contestants. When the contestant is forward looking the alternative gamble, G_t^F, comprises the lottery of the future outcomes of the optimisation problem. These outcomes may be future deals and they may be future straight lotteries, depending on the banker's offer function. When the contestant is myopic the alternative gamble in each round is the straight lottery, $G_t^M = L_t$. As this is not the result of any future optimal behaviour, it follows that $G_t^F \succeq G_t^M$.

The implication for estimation is that for a given set of prizes, the banker's offer and the contestant's decision, the forward looking model will produce a lower estimate of r than the myopic model. Since G_t^F statewise weakly dominates G_t^M, the forward-looking contestant must be at least as risk-averse as the myopic contestant if they are both to be indifferent between playing Deal and No Deal.

4.2 The banker's offer function

In Section 5, when I model the problem as a 2-player game, the contestant's beliefs are fully rational and consistent with the banker's optimal offer in the perfect Bayesian Nash equilibrium; but in this section I assume the contestant predicts the banker's offer with a simple empirical model of the form:

$$Y_t = \alpha_t + \beta_t E[X_t] \tag{4}$$

Other studies model the banker in a similar way. In Post et al (2006) the banker's offer function is modelled as a proportion of the expected value of the remaining prizes. The proportion in each round is completely determined by 2 parameters: the starting proportion and the speed at which the proportion increases to 100% (the proportion is 100% in the final round because the final round in that paper is when the contestant opens his box and he wins whatever is inside). De Roos and Sarifidis (2006) regress the offer on the expected value of the remaining prizes and the standard deviation of the remaining prizes to estimate the

Table 4: The GLS estimate of the banker's offer function						
	Round					
	1	2	3	4	5	6
α	-2891	-2523	-1949	-606.5	314.5	-187.7
	(630.9)	(865.6)	(704.5)	(937.4)	(680.6)	(681.9)
β	0.249	0.326	0.399	0.444	0.566	0.771
	(0.027)	(0.039)	(0.033)	(0.042)	(0.040)	(0.066)
R^2	0.362	0.378	0.568	0.611	0.773	0.957
Observations	129	129	129	125	93	45

banker's offer function. The authors use natural logarithms of all variables to correct for the likely heteroskedasticity and estimate a different set of parameters for each round. Mulino et al (2006) take estimates of the ratio of the offer to the expected value of the remaining prizes and separate the data not only by round but also by the size of the stakes.

Table 4 gives the results of the estimation of Equation 4 with White corrected robust variance estimates. The constant term is significant at the 5% level only in rounds 1-3 and it is negative. Further, the estimate of β_t is greater the higher the value of t, suggesting the banker becomes relatively more generous in later rounds. I refer to this observation throughout the paper and, for clarity of exposition, I find it helpful to call it an increasing offer function. The explanatory power of the model is higher for later rounds and it is especially high in round 6, suggesting there may be other things affecting the banker's offer in the early stages of the game. I come back to this in Section 6 when I discuss the banker's objectives.

The results in Table 4 are used to determine the forward-looking contestant's beliefs. For each offer in the sample we can now determine the critical value of r for which the contestant would be indifferent between playing Deal and No Deal.

4.3   Analysing the contestants' choices: a bounds approach

As stated previously, the contestant will play Deal only if the offer is at least as high as the certainty equivalent of the alternative gamble, given the contestants preferences with respect to risk. This rule can be reformulated in terms of r as follows. Denote by $\widetilde{r}_t$ the value of r for which $U(Y_t; \widetilde{r}_t, w) = U(G_t; \widetilde{r}_t, w)$, then the contestant will play Deal only if $r \geq \widetilde{r}_t$[3]. The values of $\widetilde{r}_t$ can then be used in conjunction with contestant's choices (Deal or No Deal) to determine upper and lower bounds on r for each contestant. If the contestant accepts $Y = CE(G; \widetilde{r}', w)$ then it must be that $r \geq \widetilde{r}'$, and if he rejects $Y = CE(G; \widetilde{r}'', w)$ then it

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[3]For a given straight lottery, L_t, the certainty equivalent of the alternative gamble, $CE(G_t; r, w)$, is a decreasing, continuous function of r, so, by the intermediate function theorem, there is a unique value of $\widetilde{r}_t$ that solves this equation.

must be that $r \leq \widetilde{r}''$. If the contestant never plays Deal then we cannot determine a lower bound on r.

The choice information can be used to test for contestants' mistakes. For example, keeping the notation from the last paragraph, if there is a value of $\widetilde{r}''$ for which $\widetilde{r}'' < \widetilde{r}'$, then the contestant must have made inconsistent choices. This is because he accepted an offer which is relatively less attractive than one he previously turned down. If his rejection of the previous offer represents his true preferences then he should not have accepted the offer that he did accept. On the other hand, if his acceptance of the offer represents his true preferences then he should have accepted the previous offer, which he rejected. The contestant may also make mistakes that are not observable to the experimenter. For example if he rejects all offers then we cannot construct a lower bound on r and no contradiction can be found. The contestant may well be making mistake after mistake but it will not be picked up in the data. Even if the contestant does play Deal it is possible that he may have made some mistakes that do not produce the $\widetilde{r}'' < \widetilde{r}'$ contradiction. Therefore number of actual inconsistencies is at least as large as the number of observed inconsistencies.

The CRRA formulation requires a value for the initial wealth value, w, to evaluate the individual's utility level. This is not a straightforward choice. Estimates of the coefficient of relative risk aversion are sensitive to the assumption about the contestant's wealth and different studies use different values. For example, De Roos and Sarifidis (2006) estimate their model using first zero wealth and then a value equal to the average Australian annual labour income. Post et al (2006) also consider the zero wealth scenario but use an estimate of life-time wealth for the alternative, which amounts to approximately 10 times the median household income. Bombardini and Trebbi (2005) use all three of these measures (zero wealth, annual labour income and life-time wealth measured as 10 times the annual labour income) but concentrate on the annual labour income case to be consistent with previous research[4].

The choice of wealth parameter essentially depends on the assumption one makes about the way the contestant brackets his choices. If the contestant considers his decisions in the context of their effect on life-time wealth, as a fully rational forward looking agent would, then the appropriate measure is life-time wealth. A problem with this approach is that individuals do not appear to bracket their choices so broadly. Read et al (1999) investigate how individuals bracket their choices and find evidence of narrow bracketing. They cite the example of telephone wiring insurance given in Ciccheti and Dubin (1994, p173). In their data set people pay 45 cents a month to insure themselves against an expected loss of 26 cents a month. Were individuals to consider this in the context of their life-time consumption

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[4]The authors cite Gertner (1993) and Cohen and Einav (2005) on this point.

Table 5: The results of the bounds estimate of r

Scenario	Bound	Mean	st. dev	Min	Max	Inconsistencies
Myopic,	Upper	0.467	0.145	0	0.774	26 (24.53%)
$w = 0$	Lower	0.369	0.195	−0.213	1.080	
Myopic,	Upper	0.639	0.245	0	1.849	31 (29.25%)
$w = 2000$	Lower	0.491	0.281	−0.251	1.522	
Dynamic,	Upper	1.084	0.827	0	3.790	20 (18.87%)
$w = 0$	Lower	0.546	0.498	−0.213	3.264	
Dynamic,	Upper	1.191	0.897	0.000	4.303	22 (20.75%)
$w = 2000$	Lower	0.635	0.521	−0.251	3.484	

this would be an extremely high level of risk-aversion. The fact that individuals purchase insurance at this rate seems to imply that these consumers bracket their choices in a much narrower time-frame.

Ideally the wealth parameter would be estimated but I do not do this here. I consider 2 scenarios: in the first scenario I set $w = 0$ and in the second I set $w = £2000^5$. Thus I consider the alternative behavioural assumptions that individuals 1) do not integrate wealth into the decision making process; and 2) bracket their choices in a monthly time-period.

Table 5 presents summary statistics on the upper and lower bounds on r under both the myopic and the dynamic contestant frameworks. It also details how many contestants behaved inconsistently.

As expected, the estimates of r are higher when the contestant is forward looking and higher when he integrates his wealth into the decision-making. Lying in the range of $0.4 − 1.2$, these estimates are comparable to the results of similar studies (see Section 1). The variance in the bounds is also relatively high, indicating there is some degree of heterogeneity in contestant's risk attitudes. The number of inconsistencies in each scenario suggests the contestant makes mistakes with a very high frequency. It also suggests the contestant's behaviour is best characterised by the dynamic framework and that he does not integrate wealth into his decision-making, as this combination gives the lowest number of inconsistencies.

4.4 A logit model of the contestant's behaviour

I estimate a logit model of the contestants' behaviour to obtain point estimates of the average level of risk aversion. I assume contestants sometimes make mistakes when choosing between Deal and No Deal and that the contestant's propensity to make a mistake depends on how

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[5]This is an estimate of the monthly earnings for the average contestant, based on the ONS estimate of the average household income in 2003-04.

close the offer is to the contestant's certainty equivalent[6]. In particular I estimate a logit model in which contestant i plays Deal in round t with the following probability:

$$prob(Deal_{i,t}) = \frac{e^{\lambda(r_i - \widetilde{r}_t)}}{1 + e^{\lambda(r_i - \widetilde{r}_t)}} \qquad (5)$$

This expression says that the closer to r_i is the implied level of risk aversion, $\widetilde{r}_t$, the more likely is the contestant to play Deal. An advantage of using the implied values of r to analyse contestants' choices rather than actual consumption values is that there are no limits on the values r can take. The same is not true for consumption values, which are constrained to be non-negative.

λ measures the contestant's ability to rationally decide between Deal and No Deal. I call this the coefficient of discernment. If $\lambda > 0$, the contestant shows some understanding of when he should play Deal or No Deal and the higher is the value of λ, the better is the contestant at deciding what to do. As $\lambda \to \infty$ the contestant is able to decide perfectly rationally - he will make the right decision with certainty no matter how close the offer is to the certainty equivalent. If $\lambda = 0$, the contestant makes his choices completely randomly and will play either Deal or No Deal with probability $\frac{1}{2}$ regardless of the lottery or the offer. If $\lambda < 0$, the contestant has erroneous beliefs about when he should play Deal. As $|\lambda|$ gets bigger he acts on these erroneous beliefs more confidently, making wrong decisions more often, and as $|\lambda| \to \infty$ he makes the wrong decision every time is made an offer[7].

Allowing for heterogeneous risk attitudes (i.e. $r_i = \overline{r} + v_i$, where $E[v_i] = 0$ and $var[v_i] = \sigma_v^2$), the appropriate procedure for the estimation of Equation 5 is a random effects logit model. The results are presented in Table 6.

Because of problems with the data the random effects logit model cannot be estimated for each scenario using all the data[8]. So in order to obtain results for all scenarios, I estimate the random effects logit model using data only from rounds 5 and 6. This makes use of choice data from 93 out of the original 129 contestants.

As expected, the estimates of r in Table 6 are higher when the contestant is dynamic or integrates wealth into the decision problem. Under the assumption that the contestant does not integrate wealth into the decision problem, the empirical model (with a forward

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[6]De Roos and Sarifidis (p16) and Mulino et al (p22) consider similar ways to estimate the contestant's preferences using the idea that a contestant's decision is a noisy representation of his true preferences (see Hey and Orme (1994)). This form of estimation is usually performed with utilities but I use the coefficient of relative risk aversion as the numeraire. This is not a large departure from the norm but it improves the tractability of this model.

[7]Clearly, when the offer is equal to the certainty equivalent there is no "wrong decision", so this is the only exception.

[8]The estimation procedure could not find a concave region when the estimation is performed with the whole data set for the myopic individual.

Table 6: The results of the logit model

	$\overline{r}$	σ_v^2	λ
Myopic, $w = 0$	0.484 (0.187)	0.477 (0.614)	3.954 (1.060)
Myopic, $w = 2000$	0.766 (0.214)	0 (0.387)	3.096 (0.616)
Dynamic, $w = 0$	0.636 (0.280)	1.226 (0.386)	3.615 (0.862)
Dynamic, $w = 2000$	0.872 (0.242)	0 (0.572)	2.542 (0.502)

looking contestant) produces a mean estimate of r in the region of 0.6 and a high degree of heterogeneity. This compares closely to the estimates from other studies of Deal or No Deal under the same assumption of zero wealth (e.g. Bombardini and Trebbi (2005), 0.5; De Roos and Sarifidis (2006), 0.66; and Post et al (2006), 1.15). It also compares well with estimates from studies of other game-shows (e.g. Beetsma and Schotman (2001), 0.42).

Using the average monthly income as the wealth parameter gives an estimate in the region of 0.9 but no significant heterogeneity in risk attitudes. No other studies use this wealth level, but, as one would expect, this estimate is higher than under the assumption of zero wealth and lower than estimates from other studies that assume a wealth parameter of annual or life-time wealth.

The coefficient of discernment (measured by λ) appears to be relatively low and even large deviations from the contestant's critical value yield fairly uncertain actions. For example, suppose that in the final round the two remaining prizes are £1 and £1000. The estimate of r in Table 6 indicates that the critical value for the forward looking contestant who does not integrate wealth into his decision problem values is £184.43. If the banker were to offer this sum, the contestant would play deal with a 50% probability. But if the banker were to offer £260.84, an increase of over 80%, the probability that the contestant would accept is only 61%. This suggests that contestants' choices are fairly insensitive to large deviations from the critical value.

5   The Game of Incomplete Information

Until now the banker has played a passive role in the modelling, with any strategic interaction with the contestant being ignored. In this section I present a model of the game-show that treats both the contestant and the banker as strategic players. I first set up the problem as a formal game and then look at the properties of the equilibrium solution. I show that one of these properties is an increasing offer function and thus provide a theoretical basis to the empirical model.

5.1 Strategies

The banker's strategy space in each (realised) round is the positive real line. In round t he makes an offer to the contestant, Y_t, where $Y_t \in \mathbb{R}++$. The contestant's strategy space in each round is the set of possible responses to the bankers offer. In each round he can play either Deal (D) or No Deal (ND).

5.2 Payoffs

The contestant's preferences were discussed in Section 4. I assume the contestant is forward looking so his optimisation problem is described by Equation 1. The banker is risk-neutral and cares only about his losses. In each round he uses all information available to him to minimise the expected payout over the rest of the game. This is by no means the most likely representation of the banker's preferences but it is the base case and I will show that it is enough to produce an increasing offer function in equilibrium. I discuss alternative dimensions to the banker's preferences and the effect these additions may have on the equilibrium offers in Section 6

5.3 Information

The structure of the game and the banker's preferences are common knowledge but there is an informational asymmetry with respect to the contestant's preferences. Both players know that the contestant is a risk-averse utility-maximiser whose preferences are represented by the CRRA specification, but only the contestant knows the value of r. Although the banker does not know the precise value of r for each contestant, he has a set of *a priori* beliefs about the distribution of r. These beliefs are characterised by the cumulative distribution function, $F(r)$, which is is continuous and everywhere differentiable, and the corresponding probability density function, $f(r)$.

5.4 Solving the game

The game is solved by backward induction and the relevant solution concept is the perfect Bayesian Nash equilibrium (PBNE). It specifies a strategy profile and a set of beliefs such that the strategies are sequentially rational given the set of beliefs and the beliefs are consistent, wherever possible, given the strategy profile.

    The contestant's best response to the banker's offer was derived in Section 4 so I now focus on the banker's best offer. Just as the contestant's best response depends on the expectations of the banker's future offers, so does the banker's best offer depend on the

expectations of the contestant's reply. For ease of computation I assume that the contestant always plays Deal when he is indifferent between Deal and No Deal[9]. The contestant's decision rule can therefore be characterised as follows: Accept Y_t (play Deal in round t) if and only if $Y_t \geq CE(G_t; r)$.

After each round the banker learns something about the contestant's type from the contestant's decision to play Deal or No Deal and he uses Bayes Rule to rationally update his beliefs for the next round. Since the game ends after the contestant has played Deal, the information is only useful when the contestant has played No Deal. $CE(G_t; r)$ is decreasing in r so the banker knows that for a type r' and offer Y', such that r' would accept Y', it must be that any type r'', such that $r'' \geq r'$, would also accept Y'. So when the banker updates his beliefs, he truncates the top end of the belief set, leaving the bottom end intact. Again, denote by r_t the type that is indifferent between playing Deal and No Deal when offered Y_t in round t and call this type the marginal type. Then the banker's updated beliefs in round t are that r is drawn from the truncated distribution over $[-\infty, r_{t-1}]$ characterised by $f_t(r) = f(r \mid r < r_{t-1})$ [10]. By setting no lower bound on the value of r I allow for the possibility that contestants are risk-loving. Although it is conventionally assumed that individuals are risk-averse, there is no theoretical reason why this has to be the case. Indeed it has long been established that individuals often show risk-loving behaviour when they gamble (see, for example, Friedman and Savage, 1948, pp285-86).

We can also use the above definition of the marginal type to refine the decision rule for type r: Accept $Y_t(r_t)$ (play Deal in round t) if and only if $r \geq r_t$[11].

The banker's problem is to choose the offer that minimises the expected pay-out over the rest of the game, but it is useful to think of the problem as choosing which type to make indifferent between playing Deal or No Deal. Once this choice has been made, the offer is simply the certainty equivalent pertaining to the alternative gamble, G_t, for that type. The banker's round t optimisation problem can therefore be written as:

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[9]The contestant is indifferent between Deal and No Deal when the offer is equal to the certainty equivalent, but if the offer is above the threshold by only a small amount, δ, the contestant will accept. Since the banker's strategy space is continuous, δ can be infinitesimally small and the contestant will still accept. Therefore the assumption that the contestant accepts when he is indifferent is not particularly contentious.

[10]Note that $f_1(r) = f(r)$.

[11]This is analagous to the decision rule in Section 4.

$$\min_{r_t \in [-\infty, r_{t-1}]} \Pi_t = E_t\left[\Pi_{t+1}\right] \times F_t(r_t) + Y_t(r_t) \times (1 - F_t(r_t)) \qquad (6)$$

$$\text{for } t = 1, 2, ..., 6$$

$$\text{where } \Pi_7 = b$$

$$\text{s.t. } Y_t(r_t) = CE(G_t; r_t) \qquad (7)$$

$\Pi_t(r_t)$ is flat for values of $r_t \geq r_{t-1}$, since $prob(Deal) = 0$ and $\Pi_t = E_t\left[\Pi_{t+1} \mid r_{t-1}\right]$ in this range. When the banker sets the offer so that $prob(Deal) = 0$, I assume he sets the highest offer that achieves this. This is consistent with the idea that contestants should face non-trivial decisions and that offers should be perceived to be reasonable (see Section 6). Therefore I assume that if the banker wants there to be a deal with zero probability he sets $Y_t(r_t) = Y_t(r_{t-1})$ and so the domain of the round t optimisation problem is $[-\infty, r_{t-1}]$.

In round 6 the alternative gamble is the straight lottery over the 2 remaining boxes ($G_6 = L_6$). The banker's offer is therefore the certainty equivalent of the straight lottery for the marginal type and the marginal type is chosen so as to solve the following problem:

$$\min_{r_6} \Pi_6 = E_6[b] \times F_6(r_6) + CE(L_6; r_6) \times (1 - F_6(r_6)) \qquad (8)$$

Proposition 1 *The solution to the round 6 optimisation problem is always such that $r_6 \geq 0$ if there is to be a non-zero probability of a deal in round 6.*

Proof. *The first order condition is:*

$$f_6(r_6) \times (E_6[b] - CE(L_6; r_6)) + (1 - F_6(r_6)) \times \frac{d}{dr_6} CE(L_6; r_6) = 0 \qquad (9)$$

We also know that $\frac{d}{dr_6} CE(L_6; r_6) < 0$ and $CE(L_6; 0) = E_6[b]$. Now suppose that $r_6 < 0$. Then Equation 9 implies that $(1 - F_6(r_6)) < 0$. This is impossible, so unless $f_6(r_6) = (1 - F_6(r_6)) = 0$ it must be that $r_6 \geq 0$. ∎

Proposition 2 *If $r_5 > 0$, the solution to the round 6 optimisation problem is such that $0 < r_6 < r_5$.*

Proof. Suppose $r_6 = r_5$. Then $E_6[b] = CE(L_6; r_6) = CE(L_6; r_5)$ which implies that $r_5 = 0$. But $r_5 > 0$ so it cannot be that $r_6 = r_5$. Thus as long as $r_5 > 0$, $r_6 < r_5$. Now, we have already ruled out the possibility that $r_6 < 0$ but suppose that $r_6 = 0$. Then $(1 - F_6(r_6)) = 0$. But $r_5 > r_6$ so $F_6(r_6) < 0$. Thus $r_6 > 0$ and so $0 < r_6 < r_5$. ∎

So in round 6, the banker will always update his beliefs if they include any risk-averse types. If there are no risk averse types the banker cannot benefit from paying the contestant

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to bear the risk because the price is too high. At that price the banker prefers the contestant to take the gamble and open his own box. The lowest type the banker is willing to insure is $r = 0$. If the starting beliefs are that all types are risk-loving (i.e. $r_5 < 0$), then the banker's offer is such that there is a zero probability of a deal (i.e. $r_6 = r_5$).

In round 5 the offer is set so that types $r > r_5$ play Deal (this follows from the definition of r_5 and the refined contestant decision rule). We know that types $r > r_5$ would also play Deal in round 6 were they given the choice, since $r_6 \leq r_5 < r$, so the alternative gamble for the types around the round 5 marginal type is the lottery over round 6 offers, given the choice of r_5. Therefore the round 5 offer that implements the round 6 belief set is the certainty equivalent of the lottery over round 6 offers for type r_5.

The banker chooses r_5 to solve the following problem:

$$\min_{r_5} \Pi_5 = E_5 [\Pi_6] \times F_5(r) + CE(G_5; r_5) \times (1 - F_5(r)) \tag{10}$$

Proposition 3 If $r_4 > 0$, the solution to the round 5 optimisation problem is such that $0 < r_5 < r_4$.

Proof. Suppose $r_5 \leq 0$. In round 6 the offer will be set so that there is a zero probability of a deal. This is because $CE(L_6; r) \geq E_6[b]$ for $r \leq r_5$. This implies that $E_5[\Pi_6] = E_5[b]$. It also implies that the alternative gamble for the marginal type in round 5 is the straight lottery, L_5, because the marginal type would play No Deal in round 6, and since $r_5 \leq 0$, $Y_5 = CE(L_5; r_5) \geq E_5[b]$. Therefore when $r_5 \leq 0$, $\Pi_5(r_5) \geq E_5[b]$. But $\Pi_5(r_4) = E_5[b]$ and $\Pi_5(r_4 + \delta) < E_5[b]$ for small negative values of δ so $r_5 \leq 0$ cannot be a solution to the optimisation problem. Therefore $r_5 > 0$.

To show that $r_5 < r_4$ we need to show that $CE(G_5; r_4) < E_5[\Pi_6]$. If $CE(G_5; r_4) < E_5[\Pi_6]$ then $r_5 = r_4$ cannot be a solution to Equation 10 because $F_5(r_4) = 1$. Now suppose $r_5 = r_4$. Then in round 6 the offer will be at least as high as the certainty equivalent for type r_4, $Y_6 \geq CE(L_6; r_4)$. This implies that $\Pi_6 \geq CE(L_6; r_4)$ since $CE(L_6; r_4) < E_6[b]$. Taking expectations from round 5, it follows that $E_5[Y_6] \leq E_5[\Pi_6]$. Type r_4 will accept the offer in round 6 so G_5 is the lottery over round 6 offers. Since $r_4 > 0$, it must be that $E_5[Y_6] > CE(G_5; r_4)$, and so it must be that $E_5[\Pi_6] > CE(G_5; r_4)$. Therefore $r_5 = r_4$ cannot be a solution to Equation 10. Therefore r_5 must be such that $0 < r_5 < r_4$. ∎

In rounds 1-4 the optimal offer is determined in a similar way to round 5. In round t the alternative gamble for the marginal type, r_t, is the lottery over the round $t + 1$ offers, given the choice of r_t, and the offer is chosen so as to solve Equation (6). In each round the banker chooses a belief set for the next round and sets the offer accordingly. If the contestant plays No Deal then the game proceeds to the next round and some more prizes

Figure 2: The equilibrium strategies for all contestant types
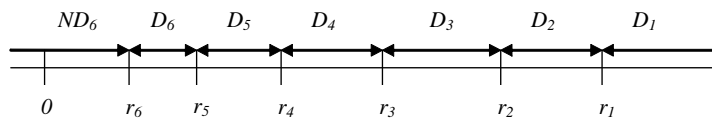
are eliminated. With the belief set he inherited from the previous round and the new set of prizes, the banker again chooses a belief set for the next round and sets the offer accordingly. The game continues until the contestant plays Deal or reaches the end of round 6.

The PBNE concept requires that the strategies be sequentially rational given the belief system and that the belief system be consistent with the strategy profile wherever possible. This means that in round t if the banker wants to implement a belief, r_{t+1}, then his round t offer has to reflect both future optimal offers and future optimal contestant decision rules in rounds $t+1$ and beyond. The possible round $t+1$ offers therefore contain information about optimal behaviour along all possible paths all the way to the end of the game.

The proposition that $0 < r_t < r_{t-1}$ if $r_{t-1} > 0$ can be proved for rounds 1-4 in a similar way as for round 5 (see the Appendix). This means that unless the *a priori* distribution of types, F, specifies that all contestants are risk-loving with unit probability, then $r_t > 0$ for $t = 1, 2, ...6$. If all types in the support of F for which $f(r) > 0$ are risk loving then the banker may set $r_t < 0$ but only on the condition that $F_t(r_t) = 1$ and $f_t(r_t) = 0$ (i.e. only if the contestant will definitely not accept the offer). Such games are not interesting because there will never be any deals so from here on I make the assumption that $F(0) < 1$ (i.e. the banker's *a priori* beliefs are such that the contestant is risk averse with a positive probability).

Under this assumption, the banker will set the offers so that in each round there is a positive probability of a deal. The banker only benefits from paying the contestant to leave the game if the contestant is risk-averse so all offers are targeted at risk-averse types.

Figure 2 illustrates the equilibrium strategy for all types of contestant. The scale represents the distribution of types and the location of the marginal types, r_t, for each round, t and above the scale are the equilibrium strategies for each type. The diagram specifies what each possible type would do For example, if r is such that $r_4 < r < r_3$, he will play No Deal in rounds 1-3 and then play Deal in round 4. The locations of the marginal types are not fixed. They are derived from the banker's optimisation problem so they depend on the prizes that are eliminated in that round and the locations of the marginal types from the previous rounds.

The contestant has a unique equilibrium strategy for a given sequence of prize eliminations

and this strategy depends on the contestant's type. The least risk averse contestants $(r < r_6)$ will never play Deal in equilibrium because the banker's offers are always too low, given their preferences. The most risk averse contestants $(r_1 < r)$, on the other hand, will always play Deal in the 1st round. Were they to participate in the later rounds they would also play Deal, but they are not prepared to take the risk of playing No Deal in the 1st round to win themselves the possibility of a better offer in future rounds. Between these ranges are types who are willing to gamble to some extent but who will eventually accept an offer.

5.5   The significance of the informational asymmetry

The informational asymmetry gives the contestant an advantage over the banker relative to the complete information scenario. Under complete information the contestant is no better off if he accepts the banker's offer than if he were playing the straight lottery (this is because $U(Y_t) = U(L_t)$) but in the game of incomplete information, with the exception of the marginal type in round 6, any type that is willing to accept an offer is strictly better off than if he were playing the straight lottery. The surplus is the result of 2 effects.

First, since the round t offer is set so as to make type r_t indifferent between Deal and No Deal, types $r_t < r < r_{t-1}$ are being paid more than they need to be to leave the game. When type $r > r_t$ plays Deal in round t he earns rent $\Gamma_t(r)$ for being pooled with lower types.

$$\Gamma_t(r) = CE(L_t, r_t) - CE(L_t, r) \tag{11}$$

Second, even the marginal types in rounds 1-5 are better off than in the game of complete information. To see why, consider the following. If a contestant accepts a deal then he must value the sure sum of money at least as highly as the lottery over future offers. That is, if type r accepts Y_t for $t = 1, 2, ..., 5$ then it must be that $U(Y_t; r) \geq E_t[U(Y_{t+\tau})]$ for all $\tau = 1, 2, ... (6-t)$. Now we already know that $0 < r_t < r_{t-1}$ if $r_{t-1} > 0$, so it follows that $Y_6 = CE(L_6; r_6) > CE(L_6; r_t)$, and so $U(Y_6; r_t) > E_6[U(b; r_t)]$. This implies that $E_t[U(Y_6; r_t)] > E_t[U(b; r_t)]$, so in round t the contestant strictly prefers the lottery over round 6 offers to L_t. But $U(Y_t; r) \geq E_t[U(Y_{t+\tau})]$ for all $\tau = 1, 2, ..., 6-t$, so it must be that $Y_t > CE(L_t, r_t)$. Therefore the marginal type in rounds 1-5 of the game of incomplete information earns more than the contestant in the game of complete information. The difference is the rent the marginal type earns because of the informational asymmetry, Δ_t.

$$\Delta_t = Y_t - CE(L_t, r_t) \tag{12}$$

While $\Delta_t > 0$ for $t = 1, 2, ..., 5$, the marginal type in round 6 earns no informational rent $(\Delta_6 = 0)$. This is because there are no future offers to consider and so there is no incentive

for the contestant to "act less risk-averse". The alternative gamble is the straight lottery, $G_6 = L_6$, and the marginal type, r_6, is indifferent between accepting Y_6 and entering the straight lottery.

The total rent earned by type r who plays Deal in round t is:

$$RENT = \Delta_t + \Gamma_t(r) \tag{13}$$

As noted, rent accrues only to types $r > r_6$. Types $r < r_6$ receive zero rent because they do not play Deal and as such cannot benefit from the premia on the offers (the offers are still too low for these types).

Like the forward-looking contestant in Section 4, the contestant realises that there is value to staying in the game. He knows that the banker will update his beliefs in the next round and set a relatively higher offer than in the current round. For some types this is enough to play No Deal even though the offer is above the certainty equivalent of the straight lottery. By playing No Deal in the current round they pool themselves with lower types and receive a relatively high offer in the next round. Unlike the types that play Deal, these types are prepared to bear the risk of the elimination of high value prizes to make themselves indistinguishable from the lower types. For this reason the banker has to pay the marginal type more than the certainty equivalent of the straight lottery to make him leave the game.

For the contestant to make gains the banker has to make losses above what he pays out in the game of complete information. The expected excess payment the banker expects to make in round t, given r_{t-1} is:

$$E_t\left[P_t\right] = \int\limits_{r_t}^{r_{t-1}} \left(\Delta_t + \Gamma_t\left(r\right)\right) f_t\left(r\right) dr \tag{14}$$

Since $\Delta_t + \Gamma_t(r) \geq 0$ for all values r and $\Delta_t + \Gamma_t(r) > 0$ for some values r, the expected excess payment is strictly positive in each round.

6 The Banker

Up to this point I have considered a simple representation of the banker's preferences and shown that when the banker is uncertain about the contestant's type the equilibrium offer becomes relatively more generous with each round. In this section I look at what other factors may motivate the banker and what effects they may have on the offer.

It is the author's understanding that the banker is an artificial personality thought up

by the producers of the show to give a voice to the offer decision, and as such his preferences represent the preferences of the producers. The question then is: what are the producers' objectives? The simple answer offered here is ratings and cost minimisation: the producers try to make the show as entertaining as possible to achieve the best possible ratings while keeping the costs down where possible. From a starting point of cost-minimisation the banker can alter the offer to increase or decrease the likelihood of a deal, depending on how exciting the game is shaping up to be. If the game contains exciting features and the producers want the contestant to continue then the offer might be set slightly lower so as to decrease the likelihood of a deal. On the other hand, if the game does not look interesting then the banker may increase the offer slightly to increase the chance of a deal. So when is the game exciting?

First of all, the game is only exciting as long as the contestant has a chance of winning a large sum of money and it is more exciting the riskier the outcomes (i.e. when there is both the possibility of large gains and the possibility of only small gains). When outcomes are risky the contestant's choices have greater significance. Thus the producers generally prefer games with high expected earnings coupled with high variance.

The style of the individual game also depends on the order of prize elimination. 2 common styles that make for good entertainment are the *comeback* and the *crash*. A comeback is what happens when a contestant experiences early losses (i.e. many of the high value prizes are eliminated) but returns to receive high offers at the end of the game; and a crash is what happens when a contestant makes early gains (i.e. few of the high value prizes are eliminated) but subsequently loses a strong position with a sequence of high value prize eliminations. When the banker spots the possibility of a comeback he may decide to keep offers relatively low, encouraging the contestant to build some hope. In this case lower offers magnify the perception of a comeback. When the banker spots the possibility of a crash the effect is not so clear. On the one hand he wants the contestant to continue (lower offers achieve this), but on the other hand he wants the crash to appear as big as possible (higher offers achieve this).

In general, the game is most exciting in rounds 4, 5 and 6. This is when the decision problem becomes relatively easy to understand and it is when the style of the game has been formed (e.g. it may be a comeback, a crash or a high variance game). The contestant has also usually managed to build a rapport with the audience by this stage. The banker may therefore be willing to give up cost-minisiation in the early rounds to see the contestant progress to the latter stages.

Finally, the essence of the game is the contestant's decision problem when he is offered a deal. If this is a trivial problem then the game cannot be exciting. The offers should therefore

be somewhere in the range where a reasonable person may have difficulty deciding what to do. This is for the most part ensured by the banker's preference for cost-minimisation, but should the banker wish all types to play No Deal, then this condition rules out the possibility of offers such as £0[12].

6.1   Framing effects

On a theoretical level, the game is fairly straightforward for the contestant. The only meaningful choice he has to make is whether to play Deal or No Deal at the end of each round. But the simplicity of the game is misrepresented by the host's theatrical performances. The host encourages contestants and viewers to believe there is much more to the game than there is and he puts emphasis on irrelevant aspects of the game. Here I look at the way the host frames the problem so as to get a better understanding of what the producers are trying to achieve. From this I hope to draw some conclusions about the banker's objectives.

The host gives the impression that the contestant's objective should be to "beat the banker". I take the meaning of this to be either:

- accepting an offer which is both higher than any other offer and higher than the sum hidden inside one's own box; or

- going all the way and finding that the sum inside one's own box is higher than any previous offer.

There are a couple of things to mention here. First, framing the problem in this way indicates that there is a complex strategic relationship between the contestant and the banker, that winning large sums can be achieved by skill and not chance. This notion is further strengthened by the banker's many comments about how well he thinks the contestant is playing. Second, it encourages ex post evaluations of the game – the correctness of a decision cannot be determined without knowledge of what is contained in the contestant's box, which is only attained at the very end of the game. This is not in the spirit of EUT, which uses an ex ante method of evaluation. Indeed, to an expected utility maximiser, these ex post evaluations are meaningless. The implication may be that one should apply significance to alternative theories of choice, such as regret theory (Bell (1982), Loomes & Sugden (1982)) and disappointment theory (Loomes and Sugden (1986)).

The host also uses certain adjectives to describe contestant's decisions. Decisions to play No Deal are usually framed positively with words such as "bravery" and "courage". The

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[12]This thinking is applied in Section 5

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banker apparently has "respect" for some players and he is apparently able to recognise an "intelligent" player. On the other hand, decisions to play Deal are often met with disapproval. There is clearly pressure on the contestant to play No Deal and the possibility that it may take some courage to play Deal is generally ignored. In the ex post evaluation, terms such as "catastrophe" and "biggest mistake of your life" are not uncommon, regardless of whether ex ante the contestant made the right decision.

These are some observations that give clues about how the banker may approach the game strategically. In Section 5 I used the most basic characterisation of the banker's preferences to model the game - expected cost minimisation. While this may not be the most precise description of the banker's preferences it forms the base case for the model to which future formulations can be compared and I showed that when there is type-uncertainty this base case assumption alone is enough to produce an increasing offer function.

7   Conclusions

In this paper I characterise the contestant and the banker as strategic players in a dynamic game and show that if there is noise in the banker's beliefs about the contestant's preferences then a preference for cost-minimisation alone is enough to produce an increasing offer function. I use this result to give theoretical backing to an empirical model of the banker which specifies a round t offer that is a linear function of the expected value of the remaining set of prizes in round t.

The empirical model of the banker with a forward looking contestant provides an approximation of the players' behaviour in equilibrium, so estimates of the coefficient of relative risk aversion based on this model can be regarded as approximations of the results of the fully specified game. Estimating the actual values is much more demanding because it involves solving out the entire game and finding the range of values that best fits the data.

Under the assumption that the contestant does not integrate wealth into the decision problem, the empirical model produces a mean estimate of r in the region of 0.6 with a high degree of heterogeneity. This is consistent with estimates from the "Deal or No Deal?" and other game-show literature. Using the average monthly income as the wealth parameter obtains an estimate in the region of 0.9 but no significant heterogeneity in risk attitudes.

It should be noted that these estimates of the coefficient of relative risk aversion are based on the choices of individuals who may not be representative of the general public. It is the opinion of this author that the screening process favours the less risk-averse individuals (a contestant who rejects relatively high offers is generally more entertaining than a contestant who accepts relatively low offers) so the estimates should be regarded as downwardly biased

estimates of the population's risk attitudes.

This claim seems plausible when one compares the estimates of the coefficient of relative risk aversion to figures from the broader literature, although estimates vary considerably. Some studies produce values far higher than the estimates discussed here (e.g. Mehra and Prescott (1985), Blake (1996), Aït-Sahalia and Lo (2000) and Sydnor (2005)), while a few produce values in the same range (e.g. Szpiro (1986) and Chetty (2006)). In general, estimates that use finance data tend to be much higher, while Chetty (2006), which uses evidence of labour supply elasticities, is notable because it produces an estimate roughly equal to unity.

The estimates of the coefficient of discernment are relatively low suggesting that contestants have a hard time choosing whether or not to accept the banker's offer, even when the offer is not close to the contestant's threshold value. In addition, the bounds approach shows that a large proportion of contestants make inconsistent choices. These results can be interpreted in one of two ways. On the one hand they can be taken to indicate that contestants suffer from cognitive limitations. On the other, they can be taken to indicate that EUT under the CRRA specification is not an appropriate framework with which to model individual choice.

The game presented in this paper is the base-case scenario in which the banker is characterised as an expected cost-minimiser. The banker's preferences are likely to be more sophisticated and some possible modifications are discussed in Section 6. Modelling these improvements more formally would be a useful topic of future research. Another useful topic would be the estimation of the wealth parameter in the CRRA utility function, which would shed light on how broadly individuals bracket their choices. Moreover, work that focuses on finding a way to solve for the whole game of incomplete information would provide the next step towards estimating a model in which the banker's beliefs are parameterised and estimated alongside the contestant's preferences.

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A   Appendix

The proof from Section 5:

Proposition 4 *If $r_{t-1} > 0$, the solution to the round t optimisation problem is such that $0 < r_t < r_{t-1}$.*

I split the proposition into 2 parts, Proposition 5 and Proposition 6. The proofs of these propositions together form the proof for Proposition 4.

Proposition 5 *If $r_{t-1} > 0$, the solution to the round t optimisation problem is such that $r_t < r_{t-1}$.*

Proof. In order to prove Proposition 5 we need to show that:

$$E_t\left[\Pi_{t+1}\right] > CE\left(G_t; r_{t-1}\right) \qquad\qquad \text{(Condition 1)}$$

If this holds then $r_t = r_{t-1}$ cannot be a solution to Equation 6 because $\Pi_t\left(r_t\right) < \Pi_t\left(r_{t-1} + \delta\right)$ for small negative values of δ. Therefore it must be that $r_t < r_{t-1}$.

I use induction to prove Condition 1 for all values of t. I first show that it holds for $t = 6$ and then show that if it holds for $t = T$ then it must also hold for $t = T - 1$.

The proof for $t = 6$ can be found in Proposition 2.

Now suppose Condition 1 holds for $t = T$. That is $E_T\left[\Pi_{T+1}\right] > CE\left(G_T; r_{T-1}\right)$. In round T the offer the banker sets when he wants to ensure a zero probability of a deal is $CE\left(G_T; r_{T-1}\right)$ and it is the lowest round T offer in the feasible set, given r_{T-1} and X_T (see Section 5). Since $CE\left(G_T; r_{T-1}\right) < E_T\left[\Pi_{T+1}\right]$ Equation 6 implies that $\Pi_T \geq CE\left(G_T; r_{T-1}\right)$.

Taking expectations from round $T-1$, it follows that $E_{T-1}[\Pi_T] \geq E_{T-1}[CE(G_T; r_{T-1})]$. But $r_{T-1} > 0$, so $E_{T-1}[CE(G_T; r_{T-1})] > CE(G_{T-1}; r_{T-1})$. Therefore

$$E_{T-1}[\Pi_T] > CE(G_{T-1}; r_{T-1}) \geq CE(G_{T-1}; r_{T-2})$$

This gives the following result:

$$E_t[\Pi_{T+1}] > CE(G_T; r_{T-1}) \implies E_{T-1}[\Pi_T] > CE(G_{T-1}; r_{T-2})$$

That is, if Condition 1 holds for $t = T$ then it must also hold for $t = T-1$. But if Condition 1 holds for $t = T-1$ then it must also hold for $t = T-2$. So if Condition 1 holds for $t = T$ it holds for $t = 1, 2, .., T$.

But we know that Condition 1 holds for $t = 6$ so it must hold for $t = 1, 2, .., 6$. Thus if $r_{t-1} > 0$, the solution to the round t optimisation problem is such that $r_t < r_{t-1}$ for all rounds, $t = 1, 2, .., 6$. ■

Proposition 6 *If $r_{t-1} > 0$, the solution to the round t optimisation problem is such that $r_t > 0$.*

Proof. Suppose $r_t \leq 0$. Since $G_t \succeq L_t$ it must be that: $Y_t = CE(G_t; r_t) \geq CE(L_t; r_t) \geq E_t[b]$. Only types $r < r_t$ will reject this offer. The round t value function must then be such that $\Pi_t \geq E_t[b]$.

Now, from Propostion 2 we know that $0 < r_6 < r_5$ and $\Pi_6 < E_6[b]$ when $r_5 > 0$ and that $r_6 \leq 0$ and $\Pi_6 = E_6[b]$ when $r_5 \leq 0$. Since $Y_5(r_5) < E_5[b]$ and $E_5[\Pi_6] < E_5[b]$ for $r_5 > 0$ it must be that $\Pi_5(r_5) < E_5[b]$ for $r_5 > 0$. Therefore $r_5 \leq 0$ cannot solve Equation 10 as it gives $\Pi_5 \geq E_5[b]$ and so it must be that $r_5 > 0$. Using the same argument in round 4, $E_4[\Pi_5] < E_4[b]$ for $r_4 \leq 0$ and so the solution to the round 4 optimisation problem must be such that $r_4 > 0$. This argument can be extended back to round 1 to prove the general result for all rounds, $t = 1, 2, ..., T$. ■