

Advanced Computational Holography for Perceptual Enhancement and Application in HUDs



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Declaration

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Abstract

This thesis explores computational holography with a focus on perceptual contrast enhancement, aiming to enable more practical and feasible holographic displays for both 2D and 3D applications. It begins with an overview of holographic displays and their advantages. Based on the core components of a holographic display system, including the laser source, spatial light modulator (SLM), and computer-generated hologram (CGH) algorithms, the challenges and limitations of conventional hologram reconstruction for real-world applications are identified. These limitations motivate the exploration of drawbacks and the development of corresponding solutions.

This thesis focuses on three aspects of the system: software, hardware, and applications. First, we propose a state-of-the-art CGH algorithm designed to enhance perceptual contrast and extend the dynamic range of reconstructed holographic images while maintaining overall luminance. The proposed scheme incorporates a non-linear merit function into the classical Gerchberg–Saxton (GS) framework, where energy redistribution is coupled with exponent-based intensity weighting and a dynamic range shift that selectively suppresses noise in perceptually sensitive areas. As a result, the method achieves superior contrast ratios and produces sharper and more vivid reconstructed holographic images with faster convergence. The optical reconstruction results indicate a 6.25% improvement in peak signal-to-noise ratio (PSNR) and a 67.85% improvement in dynamic range index (DRI).

Second, recognizing that the inevitable imperfections of SLM hardware constrain the optimal system performance, we introduce a subpixel-level optimization method that accounts for practical issues such as fringing effects. The core advantage lies in simulating subpixel behaviour through a frequency-domain shift principle without altering the computational grid size, which markedly improves efficiency compared with conventional direct expansion, reducing the computational time by 39% for a 3×3 configuration and 59% for a 5×5 subpixel lattice. Furthermore, the zero-order contribution is analysed and effectively suppressed. In parallel, the kernel shape is investigated to capture the physical characteristics of liquid crystals in LCoS devices, where anisotropy arises from the rubbed alignment layer. The analysis reveals that the effective spread of fringing field effect follows a Laplacian-like profile, with energy preferentially extending along the rubbing axis and exhibiting a positive shift due to the pretilt angle. Experimental validation demonstrates that the proposed framework enables a more

accurate simulation of realistic scenarios and achieves enhanced visual fidelity with improved diffraction efficiency in zero-order-free holographic displays.

Finally, the thesis presents a 3D head-up display (HUD) as a representative application of holographic display technology aimed at enhancing driving safety and user experience. An LCOS-based holographic engine, integrated with a relay lens system and windshield combiner, is implemented with design parameters optimized to achieve a wide field of view, a practical eyebox, and multi-depth representation. This configuration offers true phase-based depth cues that enable accurate spatial registration of virtual signs within the ambient environment, reduce visual strain, and improve driver immersion. Experimental validation, supported by enhanced CGH algorithms, demonstrates multi-layer depth reproduction with high contrast and diffraction efficiency. Furthermore, the pronounced impact of speckle noise on HUD clarity is mitigated through a time-multiplexing strategy, while zero-order diffraction is effectively suppressed. Collectively, these advances establish a viable pathway toward next-generation holographic HUDs for automotive displays.

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Nomenclature

ASM	Angular Spectrum Method
AWGS	Adaptive Weighted Gerchberg–Saxton
CGH	Computer-Generated Hologram
CSF	Contrast Sensitivity Function
DH	Digital Holography
DFT	Discrete Fourier Transform
DMD	Digital Micromirror Devices
DOE	Diffractive Optical Elements
DRI	Dynamic Range Index
FFT	Fast Fourier Transform
FWHM	Full Width At Half Maximum
GA	Genetic Algorithm
GS	Gerchberg–Saxton
HUD	Head-Up Display
LED	Light-Emitting Diode
LSI	Linear Shift-invariant
LCOS	Liquid Crystal On Silicon
MSE	Mean Squared Error
NED	Near-Eye Displays
POH	Phase-Only Holograms
PSF	Point-Spread Function

PSNR	Peak Signal-To-Noise Ratio
SG	Stochastic Gradient Descent
SLED	Superluminescent LEDs
SLM	Spatial Light Modulator
SSIM	Structural Similarity Index
VAC	Accommodation–Vergence Conflicts
VID	Virtual Image Distance
WSG	Weighted Constraint-Based Gerchberg–Saxton

List of Publications

J. Liu, M. Pivnenko, and D. Chu, "Contrast Enhancement via Residual Energy Adjustment and Effect of Non-linear Exponent in Gerchberg-Saxton Algorithm," in *Optica Digital Holography and Three-Dimensional Imaging 2024 (DH) (2024)*, Paper M1B.5 (Optica Publishing Group, 2024), p. M1B.5.

J. Liu, M. Pivnenko, X. Sui, and D. Chu, "Adaptive contrast enhancement and effect of non-linear exponential merit function on computer-generated holograms," *Appl. Opt.*, AO 64, B72–B81 (2025).

D. Chu, **J. Liu**, M. Pivnenko, P. Gao, X. Sui, and Y. Deng, "Two applications of computer generated holograms," *Computational Optical Measurement and its Education (COME 2025)*, Shanghai, China (2025).

Chapter 1 Introduction

This chapter provides an overview of the research background, together with the motivations and objectives that drive this work. It begins by outlining the evolution of holography from its origins to modern computer-generated holography, highlighting both its advantages and inherent limitations. The discussion then turns to the objectives of this thesis, which are framed around three essential components of a holographic system: coherent illumination, computational hologram synthesis, and optical wavefront modulation. The contributions of this work are introduced in this context, enhancing these elements collectively to provide more feasible solutions and to bridge the gap between theory and practice. Finally, the chapter concludes with a summary of the thesis structure, presenting a roadmap of the subsequent chapters and clarifying how the research progresses from theoretical foundations to algorithmic strategies, device-level considerations, and system-level demonstrations.

1.1 Background and Motivation

Holographic displays have recently gained significant attention due to their ability to generate realistic 3D light-wave fields. Since Gabor invented holography in 1948 as a solution for wavefront recording in electron microscopy, the field has undergone transformative development [1]. The advent of laser technology in the 1960s enabled practical optical holography through the interferential recording of object wavefronts and their diffractive reconstruction [2]. With the rapid advancement of digital devices, both the recording and reconstruction processes have transitioned into computational frameworks, significantly enhancing flexibility and precision. By digitally simulating the diffraction and interference patterns of light waves based on a target object, a computer-generated hologram (CGH) can be created and displayed on a spatial light modulator (SLM) for reconstruction. SLMs offer precise pixel-level control of light waves, enabling high-resolution and dynamic modulation of

holographic images. Approaches to CGH can be broadly categorized according to the modulation scheme supported by the underlying SLMs. Digital micromirror devices (DMDs) provide amplitude modulation, where light intensity is directly adjusted to control image brightness, though this approach is inherently limited in depth representation and energy efficiency [3,4]. In contrast, liquid crystal on silicon (LCoS) devices enable phase modulation, which preserves the wavefront phase while reshaping the optical field, thereby offering superior depth control and higher energy efficiency [5,6]. To overcome the brightness loss associated with phase-only modulation, hybrid complex-amplitude strategies combine amplitude and phase control, achieving improved diffraction efficiency [7,8]. However, this enhancement comes with the trade-off of increased optical complexity and greater bandwidth demands [9]. Together, these modulation strategies underline the strengths of holography in wavefront shaping and image fidelity, distinguishing it from stereoscopic displays that induce visual discomfort and fatigue through accommodation–vergence conflicts (VAC). Such characteristics render holography highly attractive for three-dimensional visualization in AR, VR, and other immersive technologies [10–14]. Beyond imaging, CGHs encoded onto diffractive optical elements (DOEs) [15] or metasurfaces [16] allow precise reconstruction of spatial light fields, enabling advanced applications including optical trapping [17], optical encryption [18], optical computing [19] and holographic lithography [20].

In hologram computation, the single Fourier transform is frequently adopted as a convenient strategy to simulate Fraunhofer diffraction, which corresponds to image formation at infinity. This approach enables generic holographic reconstructions and has been adapted to applications requiring precise image placement and high spatial resolution [21]. Nevertheless, single-step diffraction calculations provide only approximate solutions and fail to fully capture the complexity of wavefront propagation. To overcome these limitations, iterative refinement strategies such as the Gerchberg–Saxton (GS) algorithm are widely employed, as they effectively suppress artifacts and enhance reconstruction accuracy. Recent advances in GS-based frameworks have further extended their capability to generate images across multiple depths without additional optical elements, rendering them particularly advantageous for three-dimensional holography where depth cues are synthesized via the Fresnel transform [22–25]. However, the Fresnel formulation introduces additional propagation parameters and

necessitates more elaborate numerical modelling, which substantially increases computational demands. Alternatively, the angular spectrum method provides accurate numerical propagation over arbitrary distances, especially attractive in lensless configurations [26–28].

Despite its widespread adoption for phase retrieval, the GS algorithm still requires extensive iterations and remains prone to convergence toward local minima, which may result in suboptimal holographic reconstructions [29–31]. Recent progress in optimization algorithms, particularly those leveraging non-convex optimization strategies, such as gradient descent and its variants, have markedly improved both the efficiency and accuracy of CGH [32,33]. By exploiting gradient information to accelerate convergence and minimize error metrics, these approaches enable more accurate phase retrieval and higher-quality holographic reconstructions through the incorporation of additional constraints and refined initialization schemes [34]. Through various optimization strategies, complex-amplitude holograms encoded via double-phase methods have been demonstrated to achieve high-fidelity reconstructions [35–37]. Complex-amplitude modulation offers increased design freedom by simultaneously controlling intensity and phase distributions, although its implementation requires spatial filtering or precise alignment of modulation elements, which complicates experimental realization. In the meantime, deep learning–based approaches have emerged as a powerful alternative for accelerating computation and improving reconstruction quality. For example, autoencoder architectures incorporating Fresnel diffraction into the decoding stage have been shown to directly synthesize phase-only holograms [25], while gaze-contingent optimization combined with neural networks has achieved real-time improvements in holographic image quality [27]. Moreover, a camera-in-the-loop (CITL) optimization framework was proposed, which directly incorporates the physical display into the training loop to overcome model mismatch and achieve unprecedented image fidelity, albeit with limited generalizability since any change in system parameters requires retraining [26].

In the practical reconstruction of phase-only holograms, achieving high-fidelity reproduction remains challenging due to strong susceptibility to speckle noise. Speckle artifacts obscure fine structures and lower image contrast, with natural scenes containing smooth gradients and low-frequency components being particularly affected. To address this, both digital and optical de-speckling strategies have been explored. In the digital domain, zero-padding is widely

employed to expand the computational grid, thereby increasing spatial sampling frequency, reducing aliasing, and improving the effective resolution of the reconstructed image [38]. However, the introduction of zero-valued boundaries complicates phase retrieval, slowing convergence and degrading accuracy in iterative algorithms. To overcome this limitation, non-zero padding between 0 and 1 has been shown to provide smoother boundary conditions, mitigating phase discontinuities and reducing artifacts. This approach yields more uniform intensity distributions and improved image quality [39]. Experimentally, speckle suppression can be achieved by inserting a rotating diffuser, which dynamically alters the phase of the wavefront and averages interference patterns across time and frequency. While this method produces smoother reconstructions, excessive averaging may reduce contrast and blur fine details. Therefore, the design of advanced de-speckling strategies that jointly suppress speckle while preserving image sharpness and contrast is essential for enhancing the visual quality and practical performance of holographic displays.

Owing to its capability to accurately tailor optical wavefronts and achieve high-fidelity three-dimensional reconstructions, CGHs have become an emerging focus of both academia and industry, driving innovation across optics, information processing, and next-generation imaging systems. Nevertheless, several critical challenges remain before CGHs can be widely adopted in practical applications. The diffraction-based computation of holograms is inherently an ill-posed inverse problem, where a hologram that simultaneously satisfies physical constraints and hardware limitations is not guaranteed to exist. While optimization algorithms have markedly improved CGH performance in areas like noise reduction and computational speed, a significant gap persists between the theoretical target and the practically achievable image quality fundamentally limited by the underlying physical properties, such as source coherence and modulator imperfections. This discrepancy highlights the need for physics-oriented approaches that bridge the gap between simulation and real-world implementation, and the primary motivation of this thesis is to bridge this gap, offering high-fidelity holographic displays essential for next-generation immersive technologies.

1.2 Objectives and Contributions

A modern holographic system can be conceptually divided into three tightly interconnected components. First, the laser serves as the coherent light source that illuminates the display device. Second, the computation stage acts as the control unit of the system, employing advanced algorithms to generate the hologram pattern from a target object. Third, in the display and modulation stage, the calculated pattern is encoded onto a SLM, which modulates the incident coherent beam through diffraction to reconstruct the final wavefront.

At the algorithmic level, although state-of-the-art methods are capable of producing near-optimal holograms with low root-mean-square error in simulation, reconstructed images in practice still suffer from reduced contrast due to the coherent nature of laser illumination. While conventional speckle-smoothing techniques can suppress noise, they do so at the expense of further contrast degradation, underscoring the importance of developing strategies that improve contrast without compromising image fidelity. At the hardware level, unavoidable device imperfections, such as limited modulation accuracy, fringing fields, and phase quantization, introduce discrepancies that are often neglected in idealized CGH models, thereby preventing accurate reproduction and limiting efficiency in real-world systems. At the application level, exploiting the full capability of holography requires system designs that move beyond static demonstrations toward flexible, adaptive platforms capable of meeting real-world demands. Holographic displays are envisioned for a wide range of applications, where the ability to deliver wide field of view, accurate depth cues, and robust image quality is critical. Addressing these challenges is therefore central to advancing CGHs from theoretical development to practical, user-ready holographic displays. In line with this perspective, the present work is organized around these three aspects.

First, we propose a software-based strategy to enhance perceptual contrast in holographic reconstructions by introducing a non-linear merit function. Second, we explicitly account for device-level imperfections in the SLM, enabling more realistic and robust hologram synthesis under practical conditions, together with an approach to effectively suppress zero-order artifacts. Third, we demonstrate a representative application by designing and implementing a three-dimensional head-up display (HUD) architecture that integrates the advanced CGH

algorithms and further reduces speckle noise through temporal averaging thereby validating the effectiveness of our method in real-world conditions. This joint perspective realizes comprehensive optimization across the entire system covering algorithmic, hardware, and application levels, and paves the way for more practical, perceptually optimized holographic displays.

1.3 Thesis Structure

This thesis introduces three complementary strategies to enhance holographic systems from algorithmic, hardware, and application perspectives. It is presented in six chapters, organised as follows.

Chapter 2 establishes the theoretical foundations of computer-generated holography. It outlines the basic principles of holographic imaging through three key modules and introduces scalar diffraction theory with its historical development. Different propagation models are then discussed with respect to their applicability, and the chapter concludes with an overview of traditional and state-of-the-art algorithms, considered in terms of algorithm constraints, optimization frameworks, and initialization conditions.

Chapter 3, a non-linear merit function is introduced into the GS framework as an algorithmic enhancement to CGH. By combining intensity reweighting with adaptive dynamic range shifts, the method improves contrast, suppresses noise, and accelerates convergence while maintaining luminance. Experimental validation confirms consistent improvements in reconstruction quality, with verification in both 2D Fourier-based and 3D holography using Fresnel and angular spectrum propagation.

The fourth chapter addresses device-level limitations in holographic imaging by focusing on the fringing-field effect in liquid-crystal-based SLMs. Analytical models and compensation strategies are introduced to capture subpixel distortions, with an asymmetric kernel proposed to more accurately represent pixel coupling. Building on this, a frequency-domain shift method is developed to realize efficient and faster subpixel phase compensation while preserving the native hologram grid and ensuring energy consistency. The framework achieves higher

Structural Similarity Index (SSIM) and Dynamic Range Index (DRI) values, effectively suppresses the zero-order component, and establishes a device-aware modelling approach that links numerical reconstruction to practical hardware behaviour.

The fifth chapter, it translates the enhanced CGH algorithms into a practical demonstration through the construction of a HUD prototype. An optical architecture was developed that integrates the holographic engine with a relay lens system and windshield combiner, achieving conformal augmented reality visualization with adjustable reconstruction depths across multiple planes. This system meets key automotive HUD requirements such as FOV, eyebox, and variable image distance, while addressing critical bottlenecks including zero-order diffraction and speckle noise. The results validate the feasibility of multi-layer holographic HUDs and establish a pathway from algorithmic innovation to real-world automotive applications.

Finally, the final chapter concludes the thesis by summarizing the key findings across algorithmic, device-level, and system-level investigations. It highlights how the proposed methods bridge the gap between numerical models and practical implementations, and outlines potential directions for future work

Chapter 2 Literature Review

This chapter presents the theoretical foundations of CGH and introduces the essential background that underpins the developments in the subsequent chapters. The discussion begins with the basic principles of holographic imaging, structured around three fundamental modules: coherent illumination, algorithmic synthesis of CGH, and optical wavefront modulation modules using SLM. This section provides a detailed analysis of each module, outlining their theoretical bases, design spaces, and practical implications, while also noting the inherent limitations that motivate further developments. Particular attention is then given to the algorithmic synthesis process. In this context, scalar diffraction theory is introduced as a simplified yet effective framework for describing optical wave propagation and for illustrating how a 2D aperture gives rise to complex-amplitude distributions at the reconstruction plane. The discussion begins with its historical development, followed by different propagation theories, including Fourier-based Fresnel diffraction, Fraunhofer diffraction, and the angular spectrum method, with emphasis on their respective applicability in terms of reconstruction dimensionality, propagation distance, and modelling accuracy. The chapter concludes with an overview of traditional and modern algorithms, ranging from the GS method and its variants to optimization-based and learning-driven approaches, analysed in terms of algorithm constraints, optimization frameworks, and initialization conditions.

2.1 Fundamentals of Computational Holography

Holography is an interference-based method that enables both the recording and reconstruction of a complex optical wavefront. In classical holography, the entire process is carried out physically, as illustrated in Figure 2.1. A coherent light beam is divided into two paths: a reference beam and an object beam. The object beam illuminates the object, and the light diffracted by the object carries both amplitude and phase information and interferes with the reference beam at the photographic plate. This interference pattern, known as a hologram, preserving the full complex field of the object wavefront, allowing the original wavefront to be

reconstructed when re-illuminated with the reference wave as shown in Figure 2.2. Unlike conventional photography, which records only the intensity distribution of light and thereby discards phase information, holography captures the complete optical wavefront and enables the reconstruction of three-dimensional information about the object. Moreover, the principle of holography extends beyond visible light and can be applied to electron waves, X-rays, microwaves, acoustic waves, and so on, provided the source exhibits sufficient coherence [40].

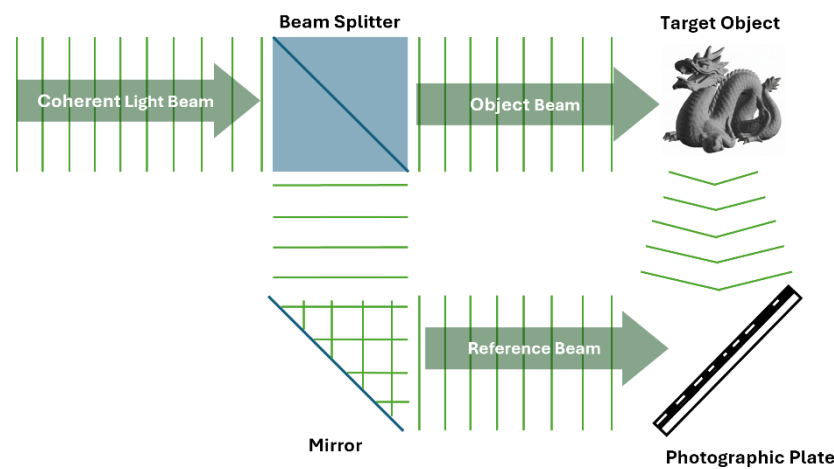


Figure 2.1 Hologram recording process.

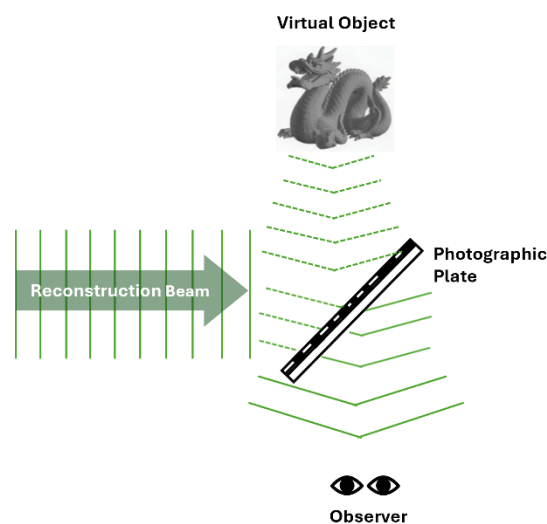


Figure 2.2 Hologram reconstruction process.

However, this traditional approach is subject to several technical limitations, including high sensitivity to mechanical vibrations and the need for chemical development, which are common

challenges in interferometric techniques. To address these limitations, modern alternatives such as Digital Holography (DH) and CGH have been developed, as both the recording and reconstruction processes in holography can be implemented digitally. In DH, the recording process is performed optically using an electronic image sensor, such as a CCD or CMOS camera to replace the traditional photographic plate. The reconstruction process is performed computationally by numerically simulating the diffraction of the reference wave through the recorded digital hologram. This approach has become feasible due to the development of high-resolution imaging sensors and the availability of enhanced computational resources [41,42].

In contrast, CGH reverses the process. The hologram is synthesized entirely through numerical simulation, without the need for a real object or physical recording. A computer calculates the interference pattern that would result from a mathematically defined virtual object. The digitally generated hologram can then be displayed on a SLM and illuminated with a coherent laser to reconstruct the corresponding optical wavefront in the physical domain.

This process is typically implemented through the system architecture illustrated in Figure 2.3, which can be decomposed into three fundamental modules: coherent illumination, CGH synthesis algorithm, and optical wavefront modulation and reconstruction. Each module introduces specific design considerations that directly influence the fidelity, stability, and flexibility of the holographic display system. The following paragraphs provide an overview of these components, establishing the basis for the subsequent in-depth analysis.

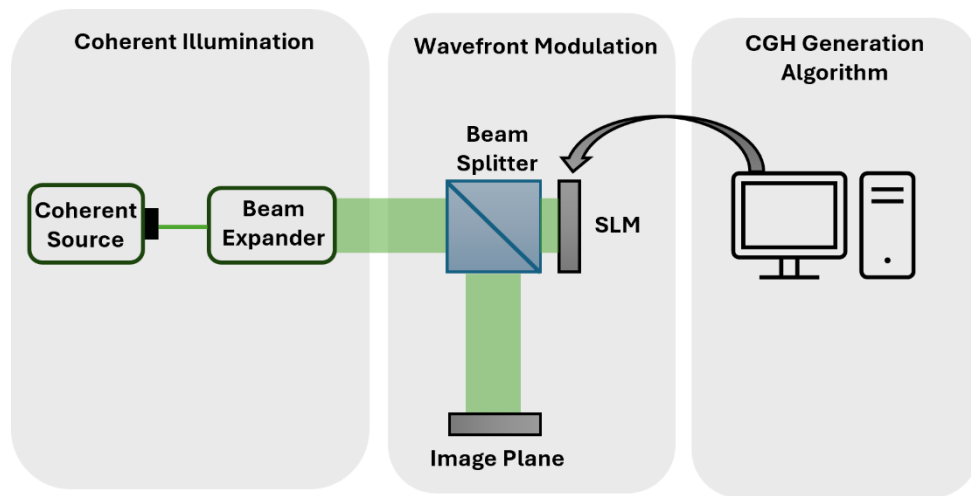


Figure 2.3 Schematic of a Computer-Generated Holography System Architecture.

In the first module, a coherent light source is expanded to provide uniform illumination across the SLM. The selection of the source is pivotal to system performance, as different candidates such as lasers, light-emitting diodes (LEDs), and superluminescent LEDs (sLEDs) exhibit distinct coherence properties that directly affect image quality. Lasers are commonly adopted in holographic displays owing to their high spatial and temporal coherence. However, the high degree of coherence also gives rise to pronounced speckle noise in the reconstructed images. Speckle manifests as a granular pattern, originating from interference among higher-order oscillations of the diffraction envelope between adjacent spatial frequency pixels. Specifically, the overall intensity profile in the replay plane is determined by the hologram pixel, while the contribution of each spatial frequency pixel follows a two-dimensional sinc function. The overlap between adjacent sinc oscillations produces strong constructive and destructive interference, resulting in speckle. The low temporal coherence of LEDs can effectively suppress speckle. However, their inherently low spatial coherence necessitates additional spatial filtering, such as pinholes, to recover image sharpness, which comes at the expense of significantly reduced optical efficiency. In principle, holographic displays assume an ideally coherent source, but practical emitters inevitably deviate from this ideal. Imperfect coherence influences holographic reconstruction in two distinct ways. Reduced temporal coherence modifies the system's impulse response, imposing an envelope on the reconstructed image and thereby lowering contrast. Temporal coherence can be quantified by the coherence length, which can be expressed as:

$$L_c = t_c c = \sqrt{\frac{2 \ln(2)}{\pi n}} \frac{\lambda^2}{\Delta \lambda} \quad (2.1)$$

where c is the speed of light, n is the refractive index of the medium, λ is the central wavelength ranging from 400 to 700nm for visible display applications, and $\Delta \lambda$ represents the spectral bandwidth, where the temporal contrast scales with the coherence length and decreases with increasing spectral bandwidth. While reduced spatial coherence removes high-frequency components, leading to a loss of sharpness. Experimental evidence shows that image sharpness is linearly proportional to spatial coherence, whereas speckle contrast is linearly proportional to temporal coherence. Hence, the optimal compromise for holographic displays is to employ a light source with high spatial coherence to preserve fine details, together with a moderately low temporal coherence to suppress speckle, enabling high-quality reconstructions with both sharp edges and reduced artifacts [43,44].

The second module involves the algorithmic synthesis of the CGH. Originating from a virtual input object, the central objective is to computationally generate a holographic interference pattern that faithfully represents the desired complex wavefront under system constraints. A comprehensive discussion of the various CGH algorithms, including iterative Fourier-based, Fresnel-based, and angular spectrum will be presented in the subsequent section.

The third module encompasses both the physical modulation of the wavefront and the subsequent optical reconstruction process. In this stage, the precomputed hologram is uploaded to the SLM, which modulates the incident beam according to the encoded phase or amplitude distribution. Different modulation schemes are implemented using different types of SLMs. Amplitude-based modulators, such as DMDs, provide binary control of light intensity, whereas phase-only devices like LCoS panels deliver high-resolution modulation with multi-bit quantization. For holographic displays, phase modulation is generally preferred, as it enables higher diffraction efficiency, superior image fidelity, and continuous phase control across the aperture. Despite the slower refresh rates typically observed in phase-only LCoS devices, this approach avoids the ghosting artifacts inherent to binary amplitude modulation and thus offers a more robust solution for high-quality holographic reconstruction. An LCoS device comprises a CMOS silicon backplane with integrated drivers, an aluminium reflective layer, and a nematic

liquid crystal layer sandwiched between alignment layers, which are in turn covered by transparent electrodes deposited on a glass substrate as demonstrated in Figure 2.4. When an external electric field is applied, the nematic liquid crystal molecules reorient. For linearly polarized light aligned with the LC's director, this reorientation modulates the effective refractive index while maintaining the polarization state. This voltage-controlled index variation results in a modulation of the phase retardation. Through the integration of liquid crystal layers with CMOS backplanes, LCoS technology combines fine electrical control with efficient optical modulation.

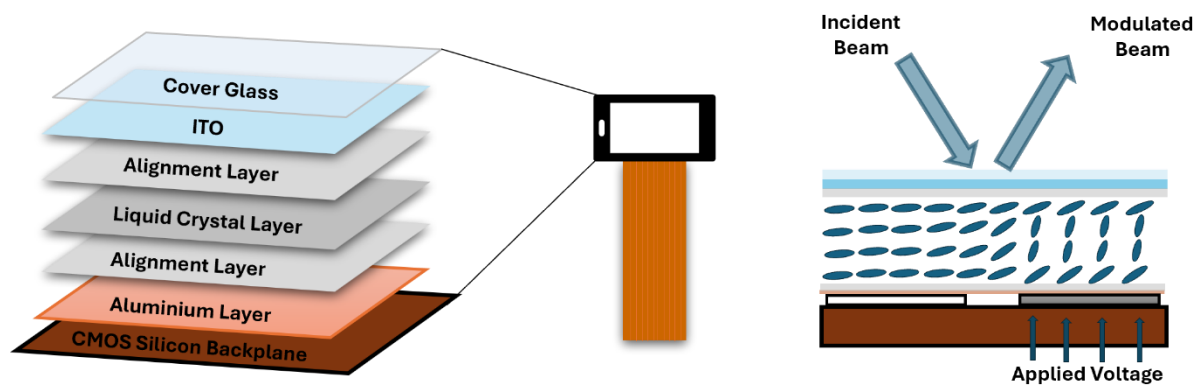


Figure 2.4 Schematic of a reflective phase-only liquid crystal on silicon (LCoS) spatial light modulator and its phase modulation mechanism using a planar aligned liquid crystal layer, with fringing field effects occurring between adjacent pixels.

Despite their advantages, phase-only LCoS devices face practical limitations. Achieving a full 2π phase retardation requires a liquid crystal layer twice as thick as that of amplitude-modulated devices, which not only slows the switching speed by up to a factor of four but also exacerbates the fringing effect [5]. The fringing effect, also known as fringing-field broadening or pixel crosstalk, arises when the electric field of one pixel extends into adjacent regions, together with the finite elastic response of the LC material, disturbing neighbouring liquid crystal molecules, as illustrated in Figure 2.4 for a planar aligned LC configuration. When a rubbed polyimide alignment layer is employed, it enforces unidirectional in-plane anchoring with a small pretilt angle, thereby defining a preferred director orientation. In the presence of strong fringing fields, the lateral components of the electric field can further induce unwanted twist of the liquid crystal directors near pixel boundaries. Such out-of-plane reorientation can

also induce changes in the polarization state of the optical beam, thereby exacerbating phase non-uniformity. Moreover, the competition between multiple possible out-of-plane director reorientation pathways can further result in slower switching dynamics [45].

These interaction lowers diffraction efficiency, reduces spatial resolution, and ultimately degrades image quality. In addition, limited modulation efficiency allows unmodulated light to leak through, giving rise to a strong zero-order component. These imperfections necessitate compensation strategies and system-level optimization to achieve high-quality holographic reconstruction.

2.2 Algorithmic Synthesis of CGHs

2.2.1 Scalar diffraction theory

In holography, the calculation and reconstruction of holograms fundamentally rely on the wave nature of light, particularly the diffraction and interference properties. Maxwell's equations describe the propagation of light as an electromagnetic wave. Within this context, diffraction theory plays a central role, explaining how light emerging from a two-dimensional aperture evolves in both amplitude and phase to form complex-amplitude distributions at the replay plane.

The development of diffraction theory began with Christian Huygens in 1678, who proposed that every point on a wavefront acts as a source of secondary spherical waves, with the new wavefront given by their envelope. In 1804, Thomas Young reinforced the wave theory of light by introducing the concept of interference. These ideas were unified by Augustin-Jean Fresnel in 1818. By introducing certain assumptions regarding the amplitudes and phases of Huygens' secondary sources and by considering the mutual interference of the resulting wavelets, successfully predicting diffraction patterns with remarkable accuracy. Later, James Clerk Maxwell identified light as an electromagnetic wave, giving the theory deeper physical significance. From Maxwell's equations, if a wave propagates in a dielectric medium that is linear, isotropic, homogeneous, and nondispersive, all components of the electric and magnetic

fields behave identically, it is possible to summarize the behaviour fully through this scalar formulation:

$$\nabla^2 u(P, t) - \frac{n^2}{c^2} \frac{\partial^2 u(P, t)}{\partial t^2} = 0 \quad (2.2)$$

where $u(P, t)$ represents any scalar field component with spatial position P and time t , n represents the refractive index of the medium, c is the speed of propagation in vacuum. For a monochromatic scalar field, it can be written as

$$u(P, t) = \Re\{U(P)e^{-j2\pi vt}\} \quad (2.3)$$

Where v is the optical frequency, and the complex function is:

$$U(P) = A(P)e^{-i\phi(P)} \quad (2.4)$$

Where $A(P)$ is the amplitude and $\phi(P)$ denotes phase at position P respectively.

Gustav Kirchhoff in 1882 placed the Huygens–Fresnel principle on a mathematical foundation by formulating boundary conditions for light propagation around obstacles, though his assumptions were later shown to be inconsistent. Sommerfeld refined the theory by applying Green's functions, leading to what is now called the Rayleigh–Sommerfeld diffraction theory.

Despite their approximations, most notably treating light as a scalar and neglecting the vectorial nature of electromagnetic fields, these formulations agree well with experiment as long as the aperture is large compared with the wavelength and observations are not made too close to the aperture. However, it must be emphasized that scalar diffraction theory breaks down in scenarios such as high-resolution gratings, sub-wavelength apertures, or nanoscale structures, where the vectorial character of light becomes essential for accurate modelling [46].

2.2.2 Fourier-Based Fresnel and Fraunhofer Diffraction

The Huygens-Fresnel principle can be mathematically expressed as follows:

$$U(P_1) = \frac{1}{j\lambda} \iint_{\Sigma} U(P_0) \frac{e^{ikr}}{r} \cos \theta \, ds \quad (2.5)$$

Where the P_0 denotes a spherical wavelet in the diffracting aperture plane illuminated by a monochromatic wave with wavelength λ as shown in Figure 2.5. While P_1 represents a point in the resultant wavefield with an angle θ and at distance r from P_0 . In addition, k is the wavenumber, defined as $2\pi/\lambda$.

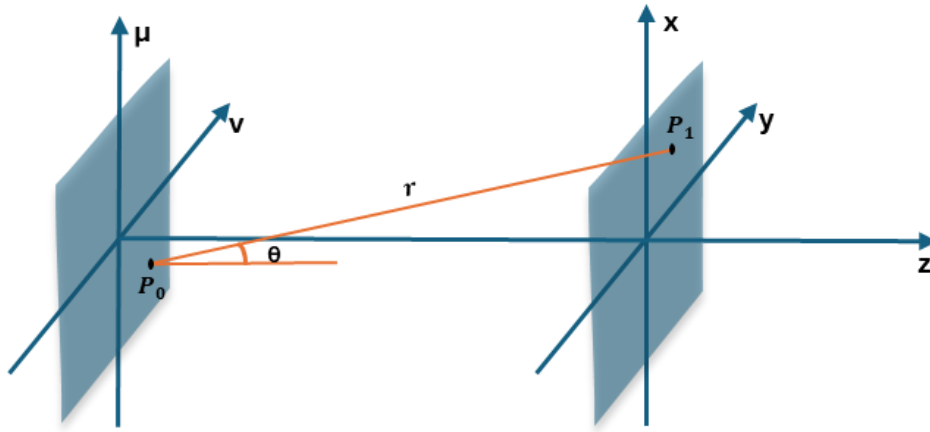


Figure 2.5 Schematic diagram of the Huygens-Fresnel diffraction geometry.

This equation can first be simplified by expressing the obliquity factor as $\cos \theta = \frac{z}{r}$ where z denotes the axial distance between the aperture and the replay plane. To further simplify the formulation, the distance r from a point $P_0(\mu, \nu, 0)$ to a point $P_1(x, y, z)$ can be rewritten in rectangular coordinates as:

$$r = \sqrt{z^2 + (x - \mu)^2 + (y - \nu)^2} \quad (2.6)$$

Assuming $(x - \mu)^2 + (y - \nu)^2 \ll z^2$, a binomial expansion can be applied to approximate r and simplify the resulting integral expression as follows:

$$U(x, y) = \frac{e^{ikz}}{j\lambda z} \iint_{-\infty}^{\infty} U(\mu, \nu) e^{i\frac{k}{2z}[(x-\mu)^2 + (y-\nu)^2]} d\mu d\nu \quad (2.7)$$

This expression can be rearranged to explicitly involve a Fourier transform, allowing for more efficient numerical implementation via Fast Fourier Transform (FFT). By factorizing out the term $e^{i\frac{k}{2z}(x^2 + y^2)}$:

$$U(x, y) = \frac{e^{ikz}}{j\lambda z} e^{i\frac{k}{2z}(x^2+y^2)} \iint_{-\infty}^{\infty} \left\{ U(\mu, v) e^{i\frac{k}{2z}[\mu^2+v^2]} \right\} e^{-i\frac{k}{z}(x\mu+yv)} d\mu dv \quad (2.8)$$

$$U(x, y) = \frac{e^{ikz}}{j\lambda z} e^{i\frac{k}{2z}(x^2+y^2)} \mathcal{F} \left\{ \left\{ U(\mu, v) e^{i\frac{k}{2z}[\mu^2+v^2]} \right\} \right\} \quad (2.9)$$

From the above equations, it is evident that the integral in this expression can be identified as the two-dimensional Fourier transform of the complex field just beyond the aperture with an additional quadratic phase exponential. This formulation provides an efficient and physically insightful representation of Fresnel diffraction. Its accuracy, however, relies on the approximation made in the expansion of the propagation distance r , where only the first-order term is retained. The approximation remains valid as long as the neglected higher-order terms contribute a phase shift much smaller than one radian. A sufficient condition to satisfy this requirement is given by Equation (2.10) [46].

$$z^3 \gg \frac{\pi}{4\lambda} [(x - \mu)^2 + (y - v)^2]_{max}^2 \quad (2.10)$$

Building upon this, it is useful to introduce a more stringent approximation that further simplifies the analysis when applicable. Depending on the propagation distance, the diffraction field can be categorized into three distinct regions: the near-field region, the Fresnel region, and the Fraunhofer region (or far-field), as illustrated in Figure 2.6 with a rectangular aperture, where the field distribution gradually evolves with distance and eventually converges to a symmetric sinc-shaped diffraction pattern. The Rayleigh–Sommerfeld region refers to the near-field zone immediately adjacent to the aperture, neither paraxial nor far-field approximations are valid and the full Rayleigh–Sommerfeld diffraction integral must be used to accurately describe the wave propagation. As the propagation distance increases, the spherical wavefronts can be approximated by parabolic wavefronts, defining the Fresnel region. When the observation distance from the aperture becomes sufficiently large, the Fraunhofer approximation holds according to:

$$z \gg \frac{k(\mu^2 + v^2)_{max}}{2} \quad (2.11)$$

In this case, the quadratic phase variation across the aperture is negligible, allowing the exponential term under the integral to be approximated as unity. Consequently, the Fraunhofer field distribution corresponds to direct Fourier transform of the aperture function with respect to Equation (2.12) and (2.13), where the computational complexity is minimized and the physical interpretation becomes more straightforward. A practically important implication is that a converging lens can naturally perform this Fourier transform. In a coherent optical system, this property provides the basis for implementing Fourier-based CGH, where the lens directly carries out the Fourier operation required for reconstruction. Thus, the complex amplitude distribution observed in the focal plane of the lens is equivalent to the Fraunhofer diffraction pattern of the aperture, even though the observation plane is located at the focal length rather than at the large distance typically required for Fraunhofer diffraction.

$$U(x, y) = \frac{e^{ikz}}{j\lambda z} e^{i\frac{k}{2z}(x^2+y^2)} \iint_{-\infty}^{\infty} \{U(\mu, v)\} e^{-i\frac{k}{z}(x\mu+yv)} d\mu dv \quad (2.12)$$

$$U(x, y) = \frac{e^{ikz}}{j\lambda z} e^{i\frac{k}{2z}(x^2+y^2)} \mathcal{F}\{\{U(\mu, v)\}\} \quad (2.13)$$

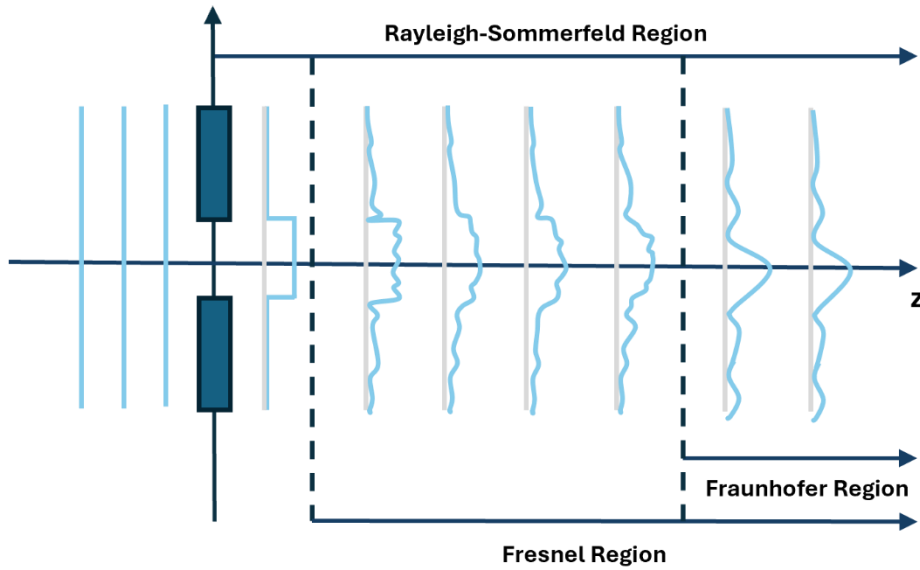


Figure 2.6 The diagram of light propagation through a rectangular aperture illustrating the near-field, Fresnel, and Fraunhofer diffraction regions.

In practice, SLMs cannot provide continuous modulation, since both the aperture function and the hologram plane are inherently pixelated and quantized. This discretization not only limits the achievable resolution but also introduces errors such as aliasing and quantization noise, which ultimately degrade the fidelity of the reconstructed wavefront. The numerical propagation from the sampled aperture to the hologram plane is therefore reformulated as a discrete Fourier transform (DFT).

2.2.3 Angular Spectrum Method

The angular spectrum method (ASM) models free-space propagation without approximating the propagation distance, using a formulation derived directly from the Rayleigh–Sommerfeld diffraction integral. This makes it well suited to lensless holographic near-eye display [47–49].

In the AS method the initial light field and its propagation are handled in the spatial-frequency domain. The basic principle is that if the complex amplitude distribution of a monochromatic wave is expressed in terms of its Fourier transform on a given plane, the resulting spatial-frequency components can be interpreted as plane waves propagating in different directions. By accounting for the phase shifts accumulated during propagation, the field on any plane

parallel to the original can be reconstructed through the coherent superposition of these directional plane waves. The complex amplitude distribution at z can be expressed as an inverse Fourier transform of its spectrum:

$$U(x, y, z) = \iint_{-\infty}^{\infty} A(f_x, f_y, z) e^{i2\pi(f_x x + f_y y)} df_x df_y \quad (2.14)$$

Where $A(f_x, f_y)$ denotes the angular spectrum of the field $U(x, y)$, obtained by decomposing the original wavefront into its spatial-frequency components. The exponential term $e^{i2\pi(f_x x + f_y y)}$ corresponds to a plane-wave basis function with transverse spatial frequencies f_x and f_y in x and y direction respectively. The integral expresses the coherent superposition of all these plane-wave components. Equivalently, $U(x, y)$ is decomposed into plane-wave components parameterized by the direction cosines:

$$\alpha = \lambda f_x, \quad \beta = \lambda f_y, \quad \gamma = \sqrt{1 - (\alpha)^2 - (\beta)^2} \quad (2.15)$$

Thus, the angular spectrum of the replay field is described by:

$$A(f_x, f_y, z) = \iint_{-\infty}^{\infty} U(x, y, z) e^{-i2\pi(f_x x + f_y y)} dx dy \quad (2.16)$$

According to Equation (2.14) and the Helmholtz equation for monochromatic fields in free space, $\nabla^2 U + k^2 U = 0$, each angular-spectrum component evolves independently along z . The angular spectrum of the replay field can be rewritten as:

$$A(f_x, f_y, z) = A(f_x, f_y, 0) e^{i \frac{2\pi}{\lambda} \sqrt{1 - (\alpha)^2 - (\beta)^2} z} \quad (2.17)$$

Moreover, from above equation, the replay field's spectrum equals the hologram spectrum multiplied by the phase transfer function as:

$$A(f_x, f_y, z) = A(f_x, f_y, 0) H(f_x, f_y, z) \quad (2.18)$$

Where

$$H(f_x, f_y, z) = e^{i \frac{2\pi}{\lambda} \sqrt{1 - (\alpha)^2 - (\beta)^2} z}$$

Conventional single-FFT Fourier-transform propagation is attractive for its simplicity and speed, yet they suffer a key drawback: output sampling pitch and effective window scale are proportional to the propagation distance z . For large z , the fixed number of samples spans a much larger field of view, coarsening the grid and washing out fine detail. For small z the Fresnel kernel's quadratic phase oscillates rapidly, the Nyquist condition is violated and high-frequency content aliases into the baseband amplifying numerical error. This limits flexibility across a wide range of distances. By contrast, the ASM overcomes this by treating the free-space propagation as a frequency-domain multiplication: take a forward FFT of the input field, multiply by a spectral transfer function, then apply an inverse FFT. With standard zero-padding to suppress circular wrap-around and appropriate band-limiting [50], the sampling pitch and window can be held independent of z , so a single grid covers a wide range of propagation distances and remains numerically stable even in the near field [51]. In concise form,

$$U(x, y, z) = \mathcal{F}^{-1}\{\mathcal{F}\{U(x, y, 0)\} \cdot H(f_x, f_y, z)\} \quad (2.19)$$

Where $\mathcal{F}^{-1}$ and $\mathcal{F}$ denote the inverse Fourier transform and the Fourier transform respectively.

Realistic 3D display is a major goal in next-generation imaging. As a promising candidate, CGH enables true 3D reconstruction and has attracted growing research interest. In CGH, a 3D object is typically decomposed into numerous basic primitives, and the hologram is synthesized by superimposing the fringe patterns of all these primitives on the hologram plane. This process, however, involves an enormous computational load, especially when dealing with complex 3D scenes or high-resolution holograms, making real-time dynamic holographic 3D display a formidable challenge. Over the past decades, a variety of approaches have been developed to enhance the performance of CGH. Depending on the type of computational primitive used, these algorithms are typically categorized into point-based methods, polygon-based methods, and layer-based methods. Point-based methods treat a 3D object as a cloud of point sources, with each point emitting a spherical wave. Although simple and intuitive, point-based methods suffer from high memory usage and computational load due to the massive number of object points. Polygon-based methods represent the 3D object as a mesh of tilted polygons and compute the diffraction from each polygonal facet. While this approach reduces the number of primitives compared to point-based methods, it involves more complex operations such as

coordinate transformations and interpolation, which remain computationally expensive. Among the three categories of CGH algorithms, the layer-based method offers a desirable trade-off between computational efficiency and reconstruction fidelity [13]. It decomposes the 3D object into multiple 2D slices parallel to the hologram plane, with each slice treated as an independent computational unit. The wave propagation from each slice can be efficiently computed using Fresnel or angular spectrum methods, depending on the propagation distance and system constraints. The final hologram is then obtained by superimposing the sub-holograms generated from all slices. Apart from Fourier-based approaches, both Fresnel and angular spectrum methods are well suited for accurate 3D hologram generation. Due to its reduced complexity and suitability for parallel processing, the layer-based method is adopted as the core computational framework throughout this work.

2.3 Overview of Traditional and State-of-the-Art Methods

Conventional hologram synthesis relying on single-diffraction computation provides only an approximate solution, inherently constrained by numerical implementation limitations. The process is an ill-posed inverse problem and guided by constraints derived from physical principles and hardware limitations. Unlike conventional phase retrieval in imaging, there is no guarantee that a hologram exists which simultaneously satisfies all constraints and reproduces the target intensity pattern. To overcome these limitations, various non-convex optimization algorithms have been proposed to seek optimal hologram solutions. These methods typically operate along three major dimensions: different constraints, optimization schemes, and initialization approaches.

2.3.1 Algorithm Constraints

To begin with, the optimization of CGH must account for several fundamental constraints that guide the inverse problem towards physically and computationally viable solutions. Failure to accurately impose the appropriate constraints within the optimization framework may lead to inconsistencies between the simulated reconstruction and the physical optical outcome, undermining the fidelity of the holographic display. The first constraint enforces the pre-

defined intensity distribution of the target object, which serves as a foundational requirement to ensure faithful optical reconstruction. The second constraint addresses the finite spatial-frequency bandwidth inherent in the propagation process. Specifically, the hologram and object wave are each confined to limited bandwidths, thereby reflecting the spectral bounds imposed by diffraction physics. The third constraint concerns the spatial extent of the hologram, which is necessarily finite in practical implementations. This limitation is typically handled through zero-padding techniques, and the optimization is carried out over a hologram of fixed size. The fourth constraint is particularly relevant for phase-only holograms (POHs), in which the hologram intensity must remain constant throughout the optimization process. This condition ensures compatibility with phase-only spatial light modulators, which are capable of modulating only the phase component while maintaining a uniform amplitude [34,52].

2.3.2 Optimization Frameworks

Having established the relevant constraints, the next focus lies in the optimization frameworks used to compute the desired hologram. Among the widely adopted frameworks, alternating projection methods serve as a fundamental approach in CGH optimization. These methods operate by iteratively projecting the solution onto different constraint sets, and by alternating between projections, the algorithm gradually refines the hologram and guiding it toward a feasible solution that satisfies all constraint. A seminal example of this approach is the Gerchberg–Saxton (GS) algorithm [29], introduced in 1972, which represents one of the earliest implementations of alternating projections that iteratively recovers the phase of a complex wavefront from intensity measurements in both the image and diffraction planes. As illustrated in Figure 2.7, the algorithm starts with a random initial guess of the phase distribution, combined with the target amplitude in the image plane, to construct an initial complex field. This field is propagated to the diffraction plane via a forward Fourier Transform, where the resulting phase distribution is retained while the amplitude is replaced by the measured diffraction-plane amplitude. The updated complex field is then propagated back to the image plane through an inverse Fourier Transform, and the amplitude is again substituted with the target image amplitude, preserving the updated phase. By repeatedly enforcing the correct amplitude in each domain while allowing the phase to evolve freely, the discrepancy

between the estimated and target amplitudes in both domains is systematically reduced over successive iterations. The iterative procedure of the GS algorithm can be mathematically expressed by the following equations:

$$G_k(\mu, \nu) = |G_k(\mu, \nu)| e^{i\phi_k(\mu, \nu)} = \mathcal{F}\{g_k(x, y)\} \quad (2.20)$$

$$G'_k(\mu, \nu) = |F(\mu, \nu)| e^{i\phi_k(\mu, \nu)} \quad (2.21)$$

$$g'_k(x, y) = |g'_k(x, y)| e^{i\theta'_k(x, y)} = \mathcal{F}^{-1}\{G'_k(\mu, \nu)\} \quad (2.22)$$

$$g_{k+1}(x, y) = |f(x, y)| e^{i\theta_{k+1}(x, y)} = |f(x, y)| e^{i\theta'_k(x, y)} \quad (2.23)$$

Here, $g_k(x, y)$ denotes the complex wavefront at the image plane in the k -th iteration, with amplitude $|f(x, y)|$ enforced as the target intensity and $\theta'_k(x, y)$ as its evolving phase. Its Fourier Transform yields $G_k(\mu, \nu)$, the wavefront in the diffraction plane, represented by its amplitude $|G_k(\mu, \nu)|$ and phase $\phi_k(\mu, \nu)$. The amplitude is then substituted with the diffraction amplitude $|F(\mu, \nu)|$, forming $G'_k(\mu, \nu)$, while retaining the computed phase. Then the field $g'_k(x, y)$ in the image plane with updated phase $\theta'_k(x, y)$ is produced by Inverse Fourier Transform. Finally, the amplitude is again replaced by the measured image amplitude $|f(x, y)|$, while keeping the phase unchanged, resulting in $g_{k+1}(x, y)$, which is utilized as the input for the next iteration. Through this alternating enforcement of amplitude constraints in both domains, the algorithm progressively converges to a phase-only solution that is consistent with the measured intensity distributions.

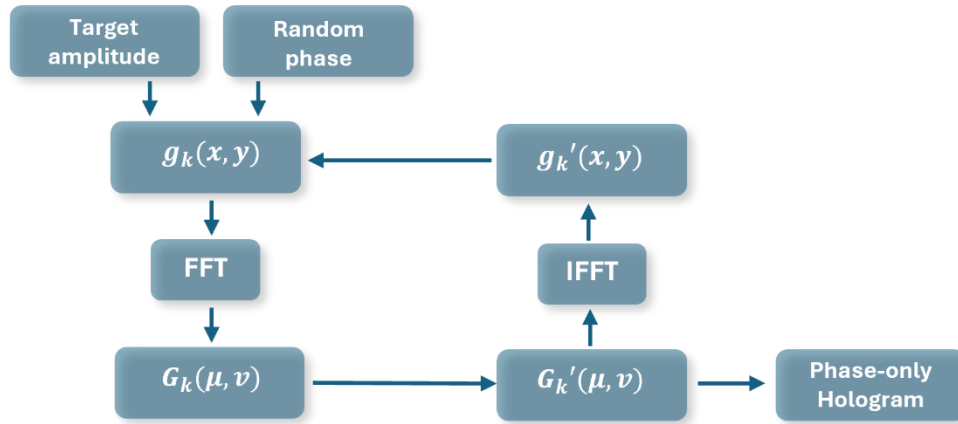


Figure 2.7 The schematic block diagram of the Gerchberg–Saxton algorithm.

The convergence of the algorithm can be monitored by computing the squared error [53]. In the Fourier domain the squared error can be expressed as:

$$E_f^2 = N^{-2} \sum |G_k(\mu, \nu) - G'_k(\mu, \nu)|^2 \quad (2.24)$$

Which can be equivalently shown as:

$$E_f^2 = N^{-2} \sum [|G_k(\mu, \nu)| - |F(\mu, \nu)|]^2 \quad (2.25)$$

According to the Parseval's theorem [54], which states the total energy of a signal is preserved under a Fourier transform, the squared error in the time domain is:

$$E_f^2 = \sum [|g_k(x, y)| - |g'_k(x, y)|]^2 \quad (2.26)$$

In the image plane, the next iteration $g_{k+1}(x, y)$ is obtained by projecting the intermediate field $g'_k(x, y)$ onto an amplitude constraint set, replacing its amplitude with $|f(x, y)|$ while retaining the phase. This operation makes $g_{k+1}(x, y)$ the closest point to $g'_k(x, y)$ within the constraint set, the updated field therefore necessarily satisfies

$$|g_{k+1}(x, y) - g'_k(x, y)| \leq |g_k(x, y) - g'_k(x, y)| \quad (2.27)$$

Therefore,

$$E_o^2 = \sum |g_{k+1}(x, y) - g'_k(x, y)|^2 \leq E_f^2 \quad (2.28)$$

Similarly, by Parseval's theorem

$$E_o^2 = \sum |g_{k+1}(x, y) - g'_k(x, y)|^2 = N^{-2} \sum |G_{k+1}(\mu, \nu) - G'_k(\mu, \nu)|^2 \quad (2.29)$$

$$|G_{k+1}(\mu, \nu) - G'_{k+1}(\mu, \nu)| \leq |G_{k+1}(\mu, \nu) - G'_k(\mu, \nu)| \quad (2.30)$$

$$E_{f,k+1}^2 \leq E_o^2 \leq E_{f,k}^2 \quad (2.31)$$

Thus, the above provides a mathematical proof of the convergence of the GS algorithm, showing that the squared error monotonically decreases or remains constant with each iteration, reducing the discrepancy between the estimated and target amplitudes, thereby ensuring convergence to a feasible solution consistent with the imposed constraints.

Nevertheless, the GS algorithm in practice often suffers from stagnation. The error drops rapidly during the initial iterations but then slows dramatically, leading to long plateaus. This phenomenon occurs because the algorithm is mathematically equivalent to a steepest-descent method with a fixed step size, which is prone to becoming trapped in shallow local minima. Over the years, numerous studies have refined the GS algorithm to overcome its limitations, focusing on accelerating convergence and suppressing noise. In the following, we highlight a few of the most representative developments. The first is a weighted constraint-based GS algorithm (WSG) [55]. In conventional GS-type methods, the optimization space is tightly restricted by uniformly enforcing the amplitude constraint across the entire image plane. While this ensures monotonic convergence, it also narrows the feasible solution set and often leads to stagnation, limiting the algorithm's ability to further reduce error or suppress noise. The WSG overcomes this issue by applying different constraint strategies in the signal and non-signal regions. The signal region corresponds to the target image area, while the noise region denotes the complementary zero-padding area. In the signal region, the amplitude is directly constrained to the target distribution using an adaptive weighting strategy, ensuring fidelity to

the desired reconstruction. In contrast, the non-signal region is not rigidly enforced but is instead regulated indirectly through total energy conservation at the hologram plane. This relaxation enlarges the feasible solution space by allowing flexibility in the non-signal region, which introduces additional degrees of freedom for the algorithm to adjust both phase and noise components. As a result, the iterative process is less prone to stagnation, converges more rapidly, and produces higher-quality hologram reconstructions. The adaptive parameters for enforced amplitude constraint that changes as the iteration progresses in the image plane is:

$$|g_{con}(x, y)| = \begin{cases} |g_k(x, y)| \left(\frac{|f(x, y)|}{|g'_k(x, y)| + \varepsilon} \right)^{\beta_k} & (x, y) \in \text{signal region}; \\ |g'_k(x, y)| & (x, y) \in \text{noise region}; \end{cases} \quad (2.32)$$

$$g_{k+1}(x, y) = |g_{con}(x, y)| \frac{g'_k(x, y)}{|g'_k(x, y)|} \quad (2.33)$$

Where the parameter β_k serves as an adaptive weighting factor that regulates the strength of the amplitude update in the signal region, initialized with a small value and updated as $\sqrt{\beta_{k-1}}$, so that it increases toward unity. By dynamically adjusting it during the iteration process, the algorithm balances convergence speed and stability. When $\beta_k \ll 1$, the update behaves similarly to GS, yielding an effective and stable early-stage convergence. As β_k approaches 1, it accelerates the convergence of GS iterations by amplifying the correction factor based on the ratio between the target and reconstructed amplitudes. Moreover, a small positive constant should be added to the denominator to avoid division by zero in the amplitude constraint, which does not affect numerical accuracy while ensuring computational stability. Furthermore, to suppress noise in the non-signal region, it is essential to impose an energy conservation constraint in the hologram plane, such that the total energy of the hologram plane equals that of the image plane. During the iteration process, the amplitude of the incident plane wave in the hologram plane is replaced by $\sqrt{\frac{E_{target}}{H}}$, which is derived as follows. The target image energy E_{target} is:

$$E_{target} = \iint |f(x, y)|^2 dx dy \quad (2.34)$$

Assume a uniform incident wave on the hologram plane with constant amplitude over hologram area H , the total energy is then defined as the sum of the squared amplitudes of the finite discrete function:

$$E_{holo} = \iint |G'(\mu, \nu)|^2 d\mu d\nu = |G'(\mu, \nu)|^2 \cdot H \quad (2.35)$$

From the Parseval's Energy Conservation theorem,

$$|G'(\mu, \nu)| = \sqrt{\frac{E_{holo}}{H}} = \sqrt{\frac{E_{target}}{H}} \quad (2.36)$$

In this way, the total energy is indirectly constrained through the hologram plane, as the iteration proceeds, energy is increasingly concentrated in the signal region, resulting in effective suppression of noise in the non-signal region.

Nevertheless, a common drawback of WGS is its susceptibility to numerical instability. When the reconstructed intensity becomes very small, ratio-based feedback may be excessively amplified, resulting in feedback failure and eventual divergence. To overcome this limitation, an adaptive weighted Gerchberg–Saxton (AWGS) algorithm was proposed based on the framework, which dynamically regulates the feedback mechanism by replacing the conventional ratio-based update with an exponential weighting function [38], as defined in Equation (2.37). This formulation avoids division by small values, guarantees stable feedback, and ensures convergence as the feedback value naturally approaches unity when the reconstructed amplitude matches the target, noting that the work presented in this thesis is built upon this framework.

$$|g_{con}(x, y)| = \begin{cases} |g_k(x, y)| e^{|f(x, y)| - |g'_k(x, y)|} & (x, y) \in \text{signal region}; \\ |g'_k(x, y)| & (x, y) \in \text{noise region}; \end{cases} \quad (2.37)$$

Another prominent category is gradient-based optimization, which can be further divided into first-order and second-order variants. For POHs, the problem is typically formulated as minimizing a loss function that measures the discrepancy between the reconstructed and target

intensities. Due to the non-convex nature of this relation, stochastic gradient descent (SGD) and its variants are widely employed. The Adam algorithm, one of the most widely used SGD-based methods, has been broadly adopted in hologram synthesis. To minimize the expected value of the loss function $\mathcal{L}_p$, Adam updates the exponential moving averages of the gradient $\nabla_{\phi}\mathcal{L}_p$ and its square $(\nabla_{\phi}\mathcal{L}_p)^2$, referred to as the first- and second-order moments, respectively. Since the optimization relies only on first-order gradients of the objective, it achieves high computational efficiency and can be implemented with relatively low memory requirements [23,26,56].

Despite these advantages, first-order methods inherently rely on gradient information alone, which often limits reconstruction accuracy. To address this, the quasi-Newton method has emerged as an important optimization framework for hologram synthesis in recent years. As a second-order approach, it minimizes the loss function by approximating the Hessian or its inverse, thereby enabling a more reliable search direction than first-order methods. A notable example is the NOVO-CGH framework, which integrates quasi-Newton optimization into computer-generated holography and has demonstrated superior reconstruction accuracy for volumetric holograms [32]. In practice, limited-memory BFGS (L-BFGS) is the predominant implementation, striking a balance between accuracy and computational feasibility [57]. Overall, quasi-Newton methods generally achieve higher reconstruction fidelity and robustness, but their substantial computational and memory demands remain major obstacles to widespread deployment. In addition to these mainstream techniques, other frameworks such as iterative shrinkage-thresholding algorithms [58] and simulated annealing [59] have also been applied to CGH. However, their usage remains relatively limited in comparison to the more established approaches.

Work has also been carried out by combining CGH with neural networks trained on ground-truth hologram datasets. Despite the rapid progress of such approaches in recent years, they still require a pre-optimization step using alternating projections as a common strategy for supervised learning [60–63]. Beyond these methods, purely learning-based frameworks have emerged, where hologram generation is directly cast as a data-driven problem. Leveraging large-scale datasets and end-to-end training, these approaches bypass explicit physical modelling and learn to approximate the hologram-to-image mapping, thereby accelerating

computation and enhancing reconstruction quality [64]. However, deep learning-based methods also face significant challenges, including high computational cost, long training times, and limited generalizability, as models often need to be retrained or fine-tuned when the dataset or optical setup changes.

In this work, we adopt the GS algorithm rather than a gradient-descent-based approach, as it offers a straightforward implementation with low computational overhead, which makes it particularly suitable for large-scale hologram generation. Unlike generic gradient descent, which requires careful step-size selection and typically converges slowly in high-dimensional phase retrieval problems, GS exploits the Fourier transform structure of holography to achieve rapid initial convergence with minimal parameter tuning. Although gradient-descent-based methods can, in principle, provide more flexible optimisation, their higher computational cost and sensitivity to hyperparameter choice make them less practical for real-time applications [65]. The GS algorithm thus provides a good balance between computational efficiency and reconstruction quality and serves as an appropriate foundation for further refinement and compensation strategies, GS particularly advantageous for real-time applications,

2.3.3 Initialization Conditions

Furthermore, the final reconstruction quality heavily depends on the initialization of the object phase. The optimization framework chosen determines the search trajectory within the feasible solution space, thereby affecting computational time, memory requirements, and convergence behavior. Among these factors, initialization plays a critical role, as the choice of the initial phase condition directly influences the convergence point and the quality of the reconstructed hologram. Different initialization strategies reflecting different physical assumptions of the object wave.

Random phase initialization is widely adopted as it mimics the scattering behaviour of real-world rough surfaces [66]. This enables reconstructions to exhibit realistic defocus blur with gradual spatial variation, while generating object waves that more accurately reflect the underlying physical principles, an essential property for faithfully reproducing three-

dimensional scenes. From this perspective, random or partially random distributions generally conform more closely to the phases of waves modulated by real-world objects. However, a major drawback of introducing random phase into optimization is the degradation of optical reconstructions due to speckle noise [67]. Speckle in CGH has been shown to originate from phase singularities, appearing as optical vortices embedded in scattered wavefronts. At these locations, the amplitude collapses to zero and the phase becomes undefined, thereby establishing a direct connection between speckle generation and singular phase phenomena [68,69].

Speckle noise can be partially mitigated by adopting a constant phase initialization [24]. Physically, this corresponds to the phase distribution of extremely smooth surfaces in nature, which produce minimal scattering and thus inherently generate fewer speckles even under coherent illumination. However, the absence of scattering also means that the object wave loses the propagating characteristics of a scattered wave, resulting in reconstructions that fail to convey a faithful 3D appearance of the object. Furthermore, quadratic phase initialization enables faithful, speckle-free planar reconstructions with higher diffraction efficiency [70,71]. However, its associated intensity distribution fails to preserve feature size during propagation and often introduces ring artifacts, thereby confining its applicability to 2D holographic projection and beam shaping. To achieve high image quality while retaining the capability of reconstructing 3D scenes, this thesis therefore adopts the conventional random phase initialization as an indispensable choice.

Chapter 3 Adaptive Contrast Enhancement

Methods

3.1 Motivation

In CGH, insufficient image quality has long hindered its widespread adoption despite its unique capability to deliver true 3D reconstructions. Achieving high visual fidelity remains challenging due to background noise that obscures fine details and reduces contrast, particularly in natural scenes dominated by low-frequency components and smooth gradients, cross talk, and uniformity. The problem is fundamentally rooted in the use of highly coherent laser illumination and the physical limitations of SLMs. The highly coherent light source introduces coherent noise, where finite aperture and restricted diffraction angle of the SLM cause each voxel reconstructed as an extended spot rather than an ideal point, resulting in interference between neighbouring voxels and the emergence of pronounced speckle noise [72]. Conventional hologram generation techniques struggle to simultaneously suppress noise and preserve structural details, often yielding reconstructions of limited visual clarity. Most speckle-reduction methods, while effective in attenuating noise, inevitably degrade contrast, which undermines perceptual image quality [73]. Moreover, phase quantization [74] and non-ideal modulation of the SLM such as dead space and crosstalk further reduce ability to deliver high-frequency components and lower diffraction efficiency [75,76], resulting in blurred fine details and compromised image contrast in the reconstructed holograms. These limitations underscore the importance of developing strategies that enhance contrast without sacrificing reconstruction fidelity, thereby addressing one of the central obstacles to practical, high-quality holographic imaging.

3.1.1 Basic Principles

Conventional hologram generation techniques face inherent trade-offs between noise suppression and the preservation of structural fidelity, frequently producing reconstructions of limited perceptual clarity. To address this limitation, an adaptive merit function is introduced, designed to redistribute energy during the iterative process in a manner that enhances contrast. By modulating its response across different regions of the image, the function prioritizes perceptually significant features while attenuating noise contributions. The mathematical formulation of this strategy is expressed in Equation (3.1), which defines how the compensated image is constructed and incorporated into the optimization to achieve contrast enhancement.

$$I_{\text{compensated}}(i, j) = \sqrt[\alpha]{I_t(i, j)^\alpha - \beta(\bar{I})^\alpha} \quad (3.1)$$

where $I(i, j)$ denotes the pixel intensity at a given spatial location, α is a non-linear exponent that regulates intensity weighting, thereby enabling adaptive redistribution of the intensity distribution across the hologram. I_t and $\bar{I}$ represent the target intensity and the global mean intensity, respectively. The parameter β introduces a dynamic range shift that controls the degree of adjustment relative to the mean intensity.

The first central component of the proposed framework is the dynamic range shift, defined with respect to the global mean intensity, which serves to suppress interference artifacts that compromise the fidelity of holographic reconstructions. A major challenge in CGH arises from low-frequency crosstalk and coherence-induced interference between neighbouring pixels. Even minor intensity fluctuations across adjacent pixels can trigger strong local interference, manifesting as speckle noise and low-frequency distortions. These artifacts typically appear as blurring that masks structural details and markedly degrades image contrast, with natural images being particularly vulnerable due to their dominance of low-frequency content characterized by smooth gradients and subtle transitions. By introducing the scaling factor β , contributions from low-intensity regions are selectively attenuated, which suppresses coherent noise arising from weak-signal interference. Since such speckle components often spread into the low-frequency spectrum, their suppression also alleviates low-frequency distortions and

improves overall image clarity. This selective suppression minimizes noise and improves image clarity without compromising the overall visual quality.

Upon simplification, the equation can be expressed as follows:

$$I_{compensated}^\alpha(i, j) = I_t(i, j)^\alpha - \beta(\bar{I})^\alpha \quad (3.2)$$

$$(I_t(i, j) + \Delta I_{compensated}(i, j))^\alpha = I_t(i, j)^\alpha - \beta(\bar{I})^\alpha \quad (3.3)$$

$$I_t(i, j)^\alpha + \alpha I_t(i, j)^{\alpha-1} \Delta I_{compensated}(i, j) = I_t(i, j)^\alpha - \beta(\bar{I})^\alpha \quad (3.4)$$

$$\Delta I_{compensated}(i, j) \cong -\frac{\beta(\bar{I})^\alpha}{\alpha} \frac{1}{I_t(i, j)^{\alpha-1}} \quad (3.5)$$

where ΔI denotes the intensity difference between the refined and target images. The formulation reveals that pixels with higher brightness exhibit smaller deviations, whereas darker pixels yield larger differences. This behaviour closely resembles the characteristics of a gamma function, which captures the nonlinear sensitivity of human vision, enhancing the visibility of subtle variations in darker regions while attenuating their perceptual impact in brighter regions [77].

Moreover, the exponent parameter α applies a nonlinear transformation to the target image intensity values. Under noisy conditions, the optimal value of α is typically greater than one, which accentuates contrast by amplifying higher intensities and suppressing lower ones, thereby strengthening the separation between bright and dark regions. This mechanism enhances structural sharpness while reducing noise contributions, particularly in low-intensity areas, and thus improves overall perceptual quality. The behavior is consistent with the characteristics of human visual perception, according to Weber's law, the just noticeable difference in intensity is proportional to the background level, implying higher sensitivity to variations in darker regions than in brighter regions [78]. Similarly, the contrast sensitivity function (CSF) indicates that the human visual system is most responsive to low- and mid-spatial frequency variations, rendering noise in low-intensity regions especially conspicuous and disruptive. Informed by these perceptual principles, the proposed algorithm selectively suppresses low-frequency noise in low-intensity regions, ensuring that reconstruction quality is preserved in the perceptual domain where the human eye is most sensitive.

Building on these principles, a range of iterative phase-retrieval techniques have been developed to improve holographic reconstruction quality. The conventional GS algorithm remains one of the most widely adopted, reconstructing a target image by initializing with a random phase and iteratively updating it under amplitude constraints, as schematically illustrated in Figure 3.1. To address the limitations of GS, including susceptibility to local minima and inadequate noise suppression, the WGS algorithm introduces differential weighting between signal and noise regions, thereby emphasizing the reconstruction of salient features while attenuating noise. However, the ratio-based weighting in WGS can cause numerical instability in low-intensity regions. To overcome this, the AWGS method replaces ratio-based feedback with exponential feedback, which enhances numerical stability and mitigates the risk of divergence.

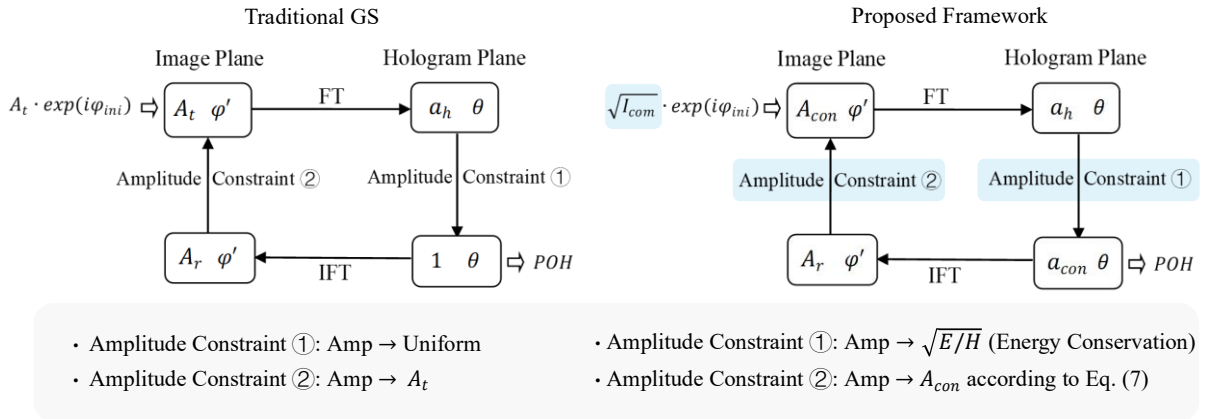


Figure 3.1 Comparison of traditional GS Algorithm and proposed framework with enhanced adaptive amplitude constraints. The blue highlighted blocks indicate the key differences, where adaptive amplitude constraints incorporating energy conservation and a perceptually based merit function are introduced. (Symbols: A_t and A_r - Target and Reconstructed Image Amplitude, θ and φ' - Hologram and Image Plane Phase, a_h and a_{con} - Hologram and Constrained Amplitude, A_{con} - Constrained Amplitude in image plane).

Building on the AWGS framework, the proposed enhancements focus on refining amplitude constraints across different planes, as depicted in the right-hand side of Figure 3.1. In the hologram plane, Amplitude Constraint ① replaces the uniform amplitude of the conventional GS algorithm with an energy-conservation criterion, ensuring that the total energy in the image plane matches that of the hologram plane. This adjustment effectively suppresses noise in non-

signal regions. Formally, the amplitude constraint ① is defined as the square root of the target image intensity summed over the hologram area. In the image plane, Amplitude Constraint ② introduces adaptive constraints according to Equation (3.7), where the compensated term derived from the merit function is embedded into the feedback loop to enable nonlinear, perceptually driven energy redistribution. This integration facilitates dynamic modulation of amplitude values during iterative phase retrieval, thereby improving convergence stability and enhancing reconstruction fidelity. The feedback function (w_{pro}) and the image-plane amplitude constraint (A_{con}) are defined as:

$$w_{pro} = e^{A_{com} - A_r} \quad (3.6)$$

$$A_{con} = \begin{cases} A_{com} \cdot w_{pro} & \text{signal region;} \\ A_r & \text{noise region;} \end{cases} \quad (3.7)$$

Where A_{com} denotes the compensated image amplitude derived from the proposed merit function, defined as $A_{com} = \sqrt{I_{com}}$. A_r denotes the reconstructed amplitude obtained at the current iteration in the image plane. To further refine the constraint, non-zero padding with values typically ranging between 0 and 1 is introduced. This strategy prevents phase values from collapsing to zero, thereby reducing boundary artifacts and discontinuities in Fourier transforms, and ultimately enhancing the fidelity of the reconstructed hologram.

3.1.2 Simulation Result in Fourier 2D Holography

This section presents simulation results of the proposed algorithm, with emphasis on the influence of the nonlinear parameter α and the dynamic range adjustment parameter β on perceptual image quality. The analysis highlights how variations in these parameters affect reconstruction performance and demonstrates their effectiveness in producing visually enhanced outputs. To approximate practical conditions, the simulations were performed under added noise, thereby emulating real-world interference and speckle, which represent major sources of quality degradation in holographic imaging.

Evaluating the quality of holographic reconstructions is essential for assessing the performance of computational holography methods. Commonly employed objective metrics include peak signal-to-noise ratio (PSNR) [79], structural similarity index (SSIM) [80], and mean squared

error (MSE) [79]. These measures provide quantitative benchmarks for comparing reconstructed holograms against their target images, capturing aspects of fidelity, structural preservation, and perceptual quality, and thereby facilitating a systematic evaluation of algorithmic effectiveness.

PSNR is one of the most widely used objective metrics for evaluating reconstruction quality. It quantifies the ratio between the maximum possible signal power and the power of noise that degrades the fidelity of the reconstructed image. PSNR is expressed in decibels (dB), with higher values generally indicating reconstructions of superior quality and lower distortion. It is formally defined as:

$$PSNR(x, y) = 10 \log \frac{255^2}{MSE(x, y)} \quad (3.8)$$

Where 255 denotes the maximum pixel value and MSE represents the mean squared error between the reference image and the reconstructed result, defined as:

$$MSE = \frac{1}{N} \sum_{i=1}^N (I_t(i) - I_{reconstruction}(i))^2 \quad (3.9)$$

Although PSNR is effective for evaluating overall reconstruction fidelity, it does not adequately capture perceptual quality, as it assigns equal weight to all pixel errors without accounting for spatial variations in human visual sensitivity. To overcome these limitations, the SSIM was introduced as a perceptual metric that incorporates luminance, contrast, and structural information to quantify similarity between a reference and a reconstructed image. Unlike PSNR, SSIM assesses localized patterns of pixel intensities, thereby providing a measure of quality that is more consistent with human visual perception. It is defined as:

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \quad (3.10)$$

μ_x and μ_y denote the local means of the reference and reconstructed images, σ_x and σ_y represent their corresponding standard deviations, and σ_{xy} is the cross-covariance between them. C_1 and C_2 are small positive constants introduced to stabilize the denominator and avoid

division by values close to zero. SSIM values range between -1 and 1 , with values closer to 1 indicating higher structural similarity and perceptual fidelity between the two images.

Different image quality metrics exhibit varying sensitivity to noise introduced in regions of different intensity levels. To illustrate this behaviour, identical Gaussian noise was applied to both high-intensity and low-intensity regions of a test image, as shown in Figure 3.2.

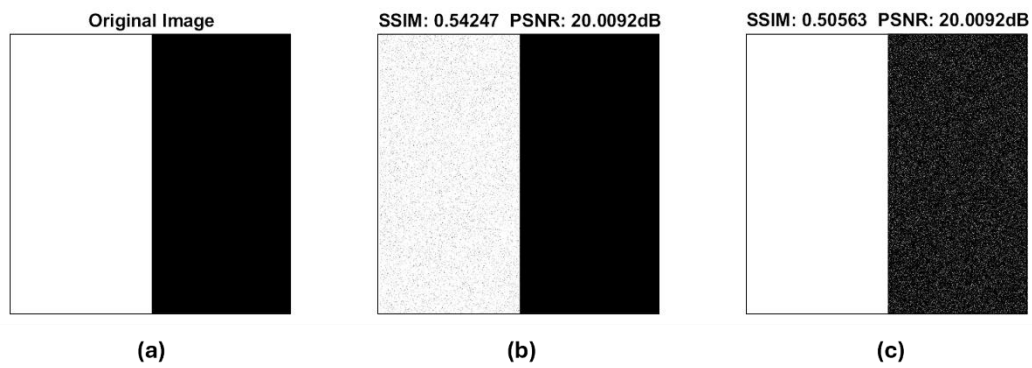


Figure 3.2 Comparison of image quality metrics (SSIM and PSNR) under identical noise conditions for bright and dark regions. (a) Original image without noise, (b) image with noise applied to bright regions, and (c) image with noise applied to dark regions.

The simulation results indicate that both cases yield identical PSNR values, reflecting the fact that PSNR is determined purely by absolute pixel-wise error and remains insensitive to baseline intensity differences. By contrast, SSIM produces markedly different outcomes: images with noise applied to bright regions achieve higher SSIM values, whereas those with noise added to dark regions show substantially lower scores. This discrepancy arises because SSIM incorporates local luminance and contrast, making it responsive to intensity-dependent variations. In bright regions, the same level of noise constitutes only a small relative perturbation against a high luminance background, resulting in minimal structural distortion. In dark regions, however, identical noise levels impose a proportionally larger disturbance, leading to a more pronounced degradation of structural similarity. These observations suggest that SSIM is a more reliable metric for holographic reconstructions, as it better reflects perceptual contrast differences and provides a more faithful assessment of visual quality.

Following the demonstration of SSIM's superiority over PSNR in capturing perceptual differences, simulations were conducted to investigate the influence of parameter

configurations. In the linear case ($\alpha = 1$), the parameter β functions as a global linear shift of pixel intensities, serving to adjust the intensity baseline and thereby suppress interference artifacts, particularly in low-luminance regions where such artifacts are most pronounced. As shown in Figure 3.3, two representative β values within the range of 0–1 were evaluated. The results indicate that introducing an optimal β correction yields a clear enhancement in image quality, reflected by a moderate increase in SSIM values for both test images, signifying improved structural similarity and reduced noise. However, the selection of β requires careful balancing: insufficient correction fails to suppress noise or enhance contrast, whereas excessive correction leads to overcompensation, resulting in an overly compressed luminance baseline and degraded visual fidelity through the loss of fine details.

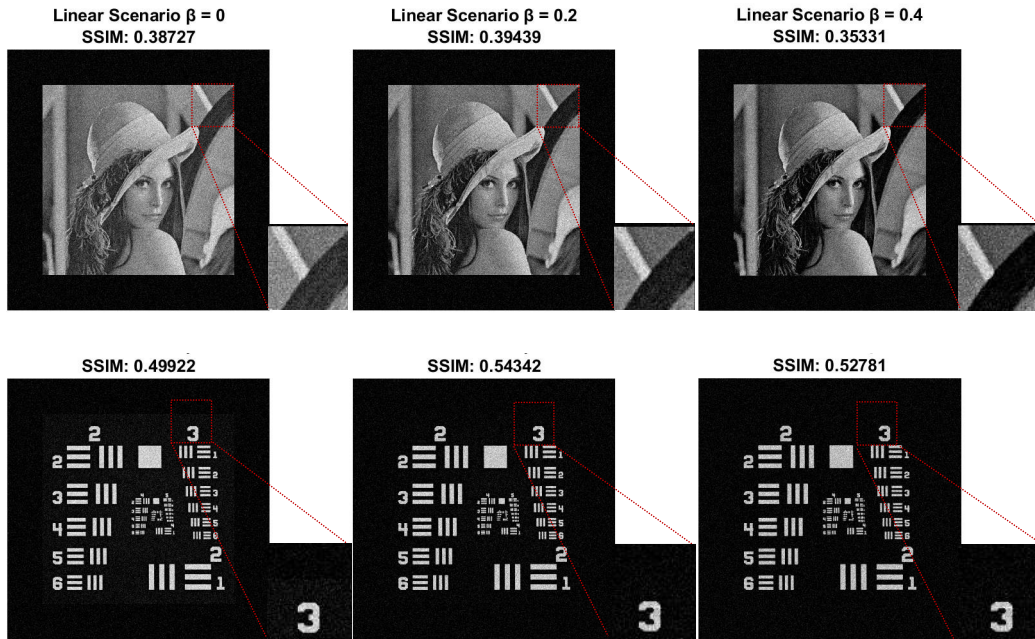


Figure 3.3 Simulation results of β correction in the linear scenario for the (a)-(c) Lena image and (d)-(f) Binary USAF resolution chart. (a), (d) No correction $\beta = 0$; (b), (e) $\beta = 0.2$; and (c), (f) $\beta = 0.4$. The magnified regions correspond to the areas enclosed by red rectangles, providing a detailed visualization of the correction effects.

Introducing the nonlinear parameter α mitigates the luminance reduction induced by the uniform shift parameter β by adaptively amplifying high-intensity regions, thereby preventing luminance distortion. This targeted adjustment provides finer control over reconstruction quality, preserving contrast and structural details, and ultimately overcoming the limitations of linear correction to achieve higher visual fidelity. The subsequent simulations present the

performance of the proposed method with optimized parameters, benchmarked against conventional AWGS and GS algorithms.

Parameter optimization for α and β was performed by employing a genetic algorithm (GA) [81], owing to its capability to efficiently explore complex, nonlinear parameter spaces, where conventional gradient-based approaches are often hindered by local minima. GA operates through evolutionary-inspired processes, including selection, crossover, and mutation, which collectively enable the exploration of a broad solution space and the refinement of candidate solutions. In the present framework, GA serves to optimize the parameters α (within the range 0.5–2) and β (within the range 0–0.5), thereby enhancing the performance of the proposed method. The algorithm was initialized with a population size of 50, generated randomly within the specified parameter ranges. The fitness of each candidate solution was evaluated using the SSIM metric, ensuring that optimized parameters directly correspond to improvements in holographic image quality. Selection was performed using the roulette-wheel strategy, which balances the prioritization of high-fitness individuals with the preservation of population diversity. Crossover was applied with a probability of 80%, recombining parameters from two parent solutions to produce offspring and thereby explore new regions of the solution space. Mutation further introduced controlled perturbations to the offspring parameters, ensuring genetic diversity and reducing the likelihood of premature convergence to suboptimal solutions. Looking forward, the framework could be extended by incorporating more advanced parameter-tuning strategies, such as learning-based optimization, which hold potential to further improve efficiency and reconstruction fidelity.

With regards to the results shown in Figure 3.4, the image reconstructed using the conventional GS algorithm exhibits pronounced noise and yields the lowest SSIM among the three methods. The AWGS algorithm achieves moderate improvement by reducing some noise and marginally enhancing structural similarity, yet the reconstructed image still contains visible artifacts. In contrast, the proposed method attains the highest SSIM, demonstrating superior noise suppression and better preservation of fine details. These results confirm its effectiveness in producing higher-quality reconstructions under challenging conditions with added noise and interference. Further comparisons are provided in Figure 3.5, which illustrates convergence behavior based on RMSE. The proposed method consistently outperforms both GS and AWGS

in terms of convergence speed and final reconstruction accuracy. In addition, Figure 3.6 presents SSIM variation plots, offering insights into the impact of parameter changes on different image types and the robustness of the proposed approach. For the Lena image (left), a wide region of stable SSIM values is observed, indicating strong tolerance to parameter variations. In contrast, the binary USAF resolution chart (right) exhibits a narrower optimal range for α and β , reflecting greater sensitivity to parameter adjustments. This analysis suggests that binary images with higher standard deviation require more precise parameter tuning, whereas natural images such as Lena demonstrate greater adaptability to parameter variations.

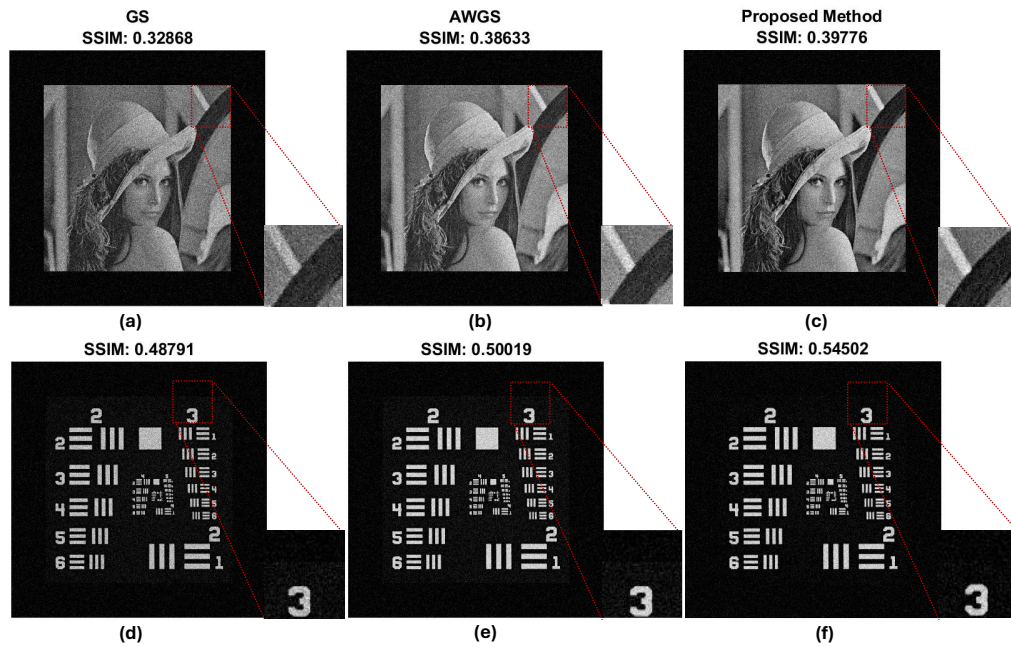


Figure 3.4 Simulation results for the (a)-(c) Lena image and (d)-(f) Binary USAF resolution chart, comparing the performance of three methods: (a), (d) GS; (b), (e) AWGS; and (c), (f) the Proposed Method. For Lena image, parameters are set to $\alpha=1.34$ and $\beta = 0.15$, while for Binary chart, $\alpha=1.14$ and $\beta = 0.14$. The magnified regions correspond to the areas enclosed by red rectangles, providing a detailed visualization of the correction effects.

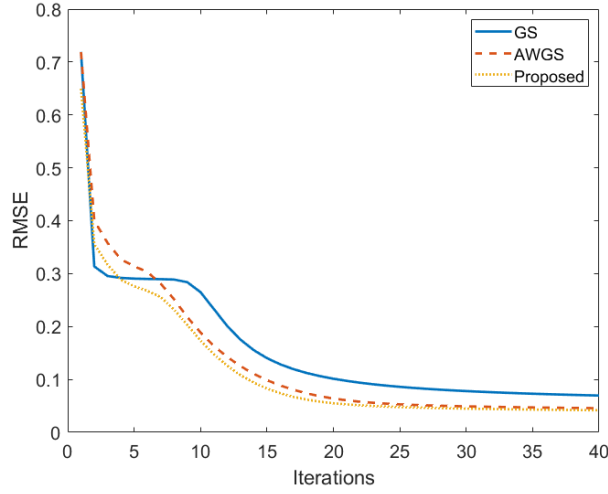


Figure 3.5 Convergence Analysis for GS, AWGS, and the Proposed Method.

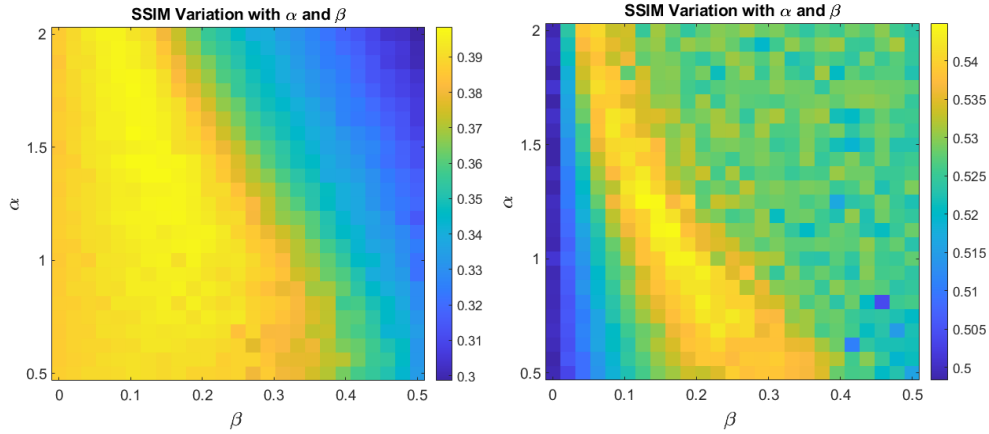


Figure 3.6 Plot of SSIM variation with α and β for (left) the Lena image and (right) the Binary USAF resolution chart.

3.2 Optical Calibration and Experimental Verification

3.2.1 Experimental Result in Fourier 2D Holography

Experiments were carried out to assess the performance of the proposed algorithm, with the optical setup illustrated in Figure 3.7. A laser operating at a wavelength of 520nm (Thorlabs, PL201) was used as the illumination source. A 2K reflective LCoS SLM, whose technical specifications are summarized in Table 3-1, was employed and calibrated using an optimized driving scheme with phase linearization. The LCoS device was fabricated in-house using

nematic liquid crystal material (MDA_98-1602, Merck) with a clearing temperature of 109 °C. The surface flatness of the LCoS was specified to be at least $\lambda/10$. The collimated laser beam was spatially filtered through a 4-f system incorporating a pinhole and subsequently directed onto the LCoS via a beam splitter. The pinhole (P20HK, Thorlabs) was with diameter 20 μm , calculated using $D = \lambda f / r$, where λ is the wavelength, f is the focal length, and r denotes the beam radius. The pinhole was used as a spatial filter to suppress unwanted side lobes of the input Gaussian beam, thereby improving its spatial coherence. Following phase modulation by the LCoS, the beam was relayed through a second 4-f system and projected onto the imaging plane, where a CCD camera recorded the reconstructions for further analysis.

Table 3-1 Specifications of the LCoS Spatial Light Modulator.

Resolution	1920 × 1080
Pixel Pitch	4.3 μm
Diagonal Size	0.94 cm
LC Alignment Mode	Horizontal
Maximum Operation Frequency	480Hz
Gray Scale	256 levels (8-bit)
Rubbing Direction	Long-axis direction of the LC cell
Fill Factor	92%
Type of LC	Nematic LC (MDA_98-1602)

Optimal utilization of the LCoS device requires careful calibration of both the driving voltage sequence and the linearization of its phase response. The process begins with determining the appropriate voltage operating window, which directly governs the achievable phase modulation depth and thus the fidelity of holographic reconstructions. In the calibration arrangement shown in Figure 3.8 (a), a polarizer (P1) is placed at 45° relative to the alignment axis of the LCoS. The reflected beam then passes through a second polarizer (P2) oriented orthogonally to P1, creating a cross-polarizer configuration. In this configuration, the phase

modulation induced by the applied voltage is translated into detectable intensity changes, which are monitored with a photodetector. A sequence of uniform grayscale inputs, ranging from 0 to 255, are displayed on the LCoS to generate the full intensity response. From these measurements, the phase modulation curve is extracted using Equation (3.11). To exploit the device's full capability, the driving conditions are chosen such that the phase span covers the complete 2π range, as depicted in Figure 3.8(b) and (c). Once the operating window is defined, a linearization step is introduced to compensate for the inherent nonlinearity of the SLM phase response. As shown in Figure 3.9, the measured phase response is mapped to an ideal linear slope to construct a lookup table, ensuring that the input grayscale levels correspond proportionally to the desired phase shifts. This correction removes the intrinsic nonlinearity of the voltage–phase relationship, as shown in Figure 3.8(d) and (e), thereby stabilizing the device characteristics and ensuring reliable, high-quality holographic performance [82].

$$\delta = 2 \sin^{-1} \sqrt{I_{norm}} \quad (3.11)$$

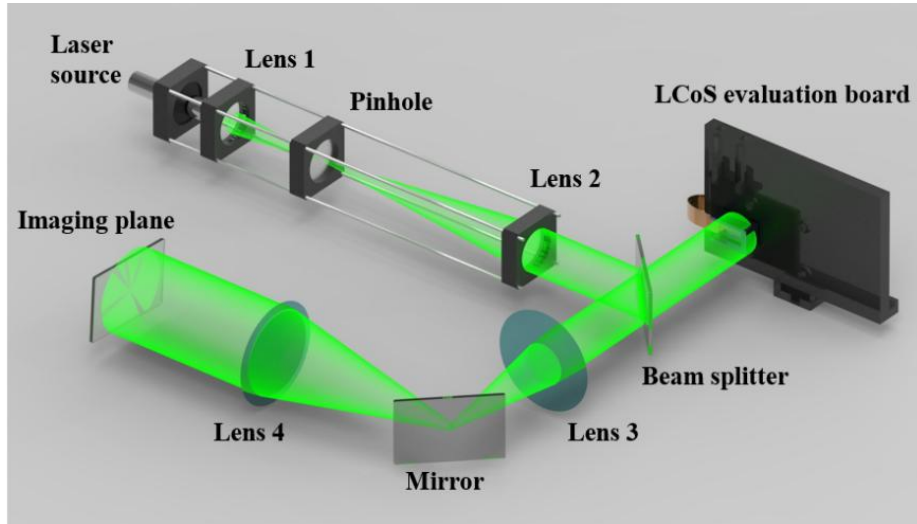


Figure 3.7 Schematic of the Experimental Setup.

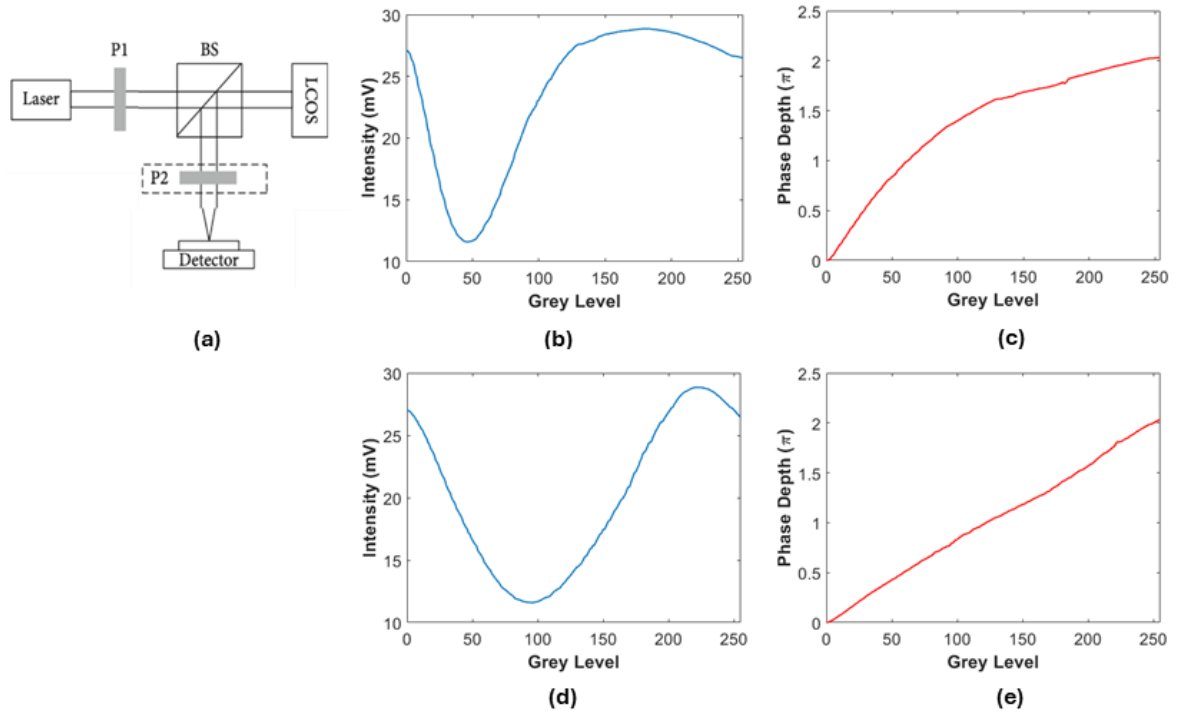


Figure 3.8 (a) Calibration of LCoS with Cross-Polarization Configuration. (b) Intensity Response Curve vs. Grey Level (0.9 - 4.5 V Window). (c) Phase Depth Curve Corresponding to (b). (d) Linearized Intensity Response Curve. (e) Linearized Phase Depth Curve.

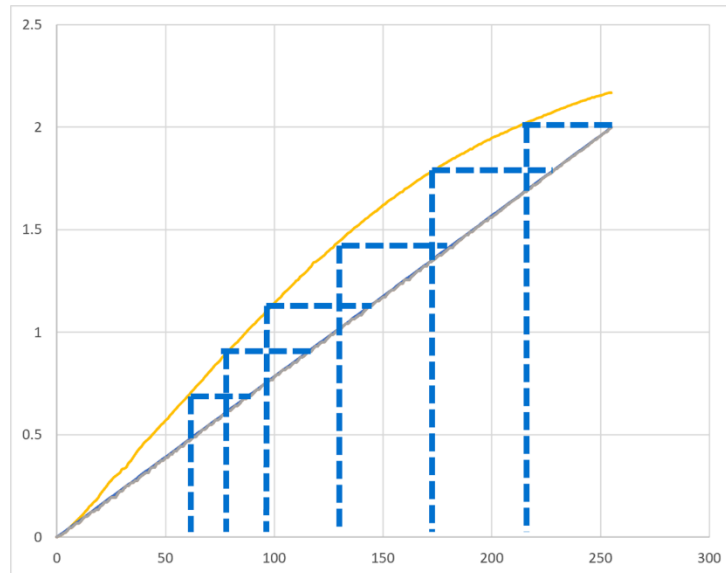


Figure 3.9 Phase Linearization Curve of the SLM. The horizontal axis denotes grayscale levels, and the vertical axis denotes phase shift. The yellow curve represents the measured nonlinear phase response, whereas the grey curve shows the optimized linearized response.

The following figures present both simulation and experimental results, highlighting the effect of varying the β parameter under a linear scenario. Consistent with the simulations, the experiments confirm that increasing β suppresses noise in low-intensity regions, producing reconstructions with improved clarity and contrast. Nevertheless, excessive β leads to overcompensation, attenuating low-intensity features and thereby degrading overall image quality. These observations suggest that employing a small β value in conjunction with the nonlinear exponent α provides an optimal balance between noise suppression and contrast enhancement. All experimental images were captured using a fixed camera exposure setting, which was determined once and kept unchanged throughout the entire evaluation. As shown in Figure 3.10, the binary resolution chart exhibits a marked reduction in background noise and a significant decrease in speckle-like patterns. For the two natural images, the influence of β is more subtle, as β increases, additional fine details become discernible, leading to sharper and more visually distinct reconstructions. It should also be noted that the reconstructed images were deliberately shifted to mitigate the influence of the zero-order component, thereby enabling a more accurate evaluation of reconstruction quality.

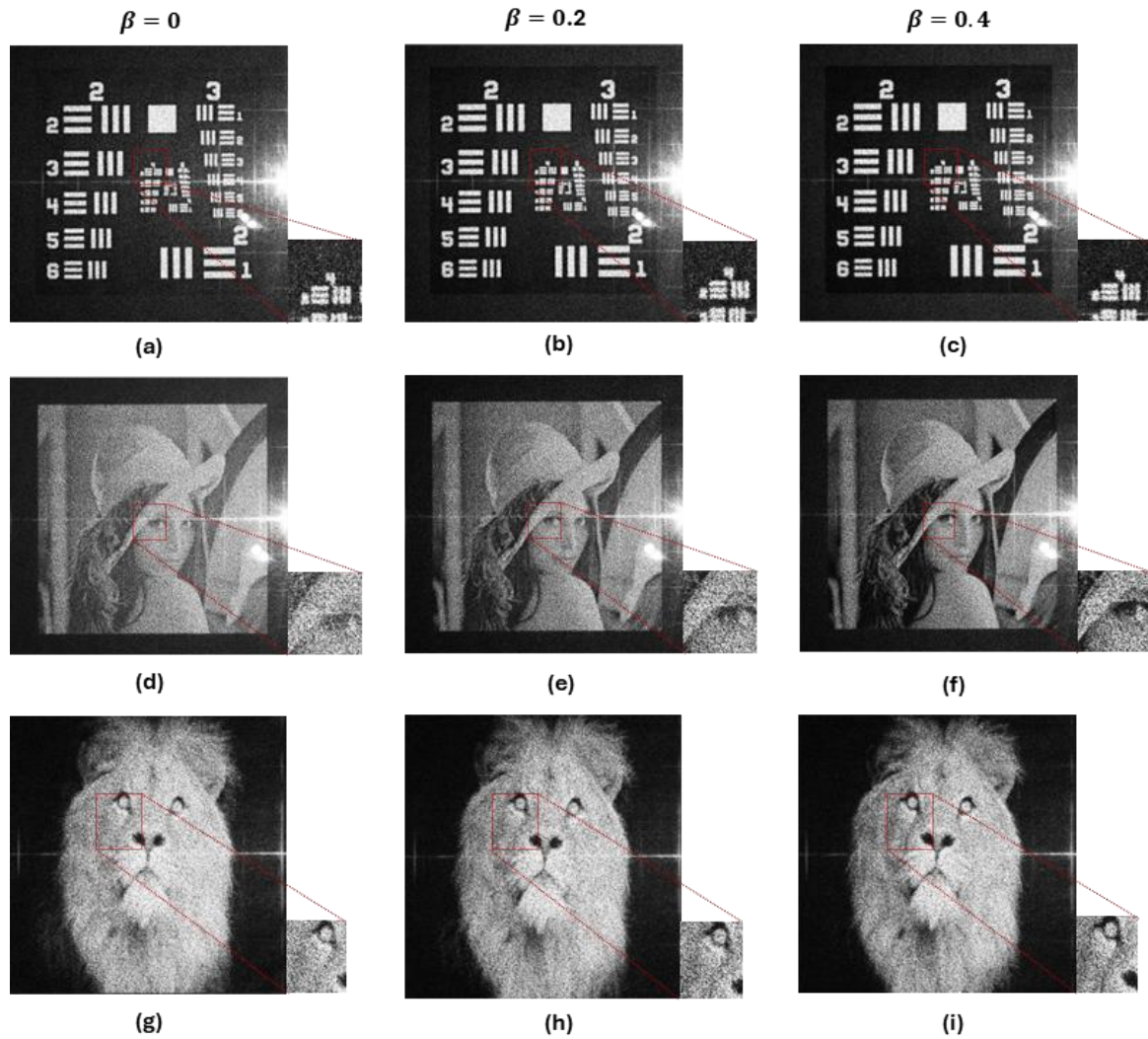


Figure 3.10 Experimental results of reconstructed images with varying β parameters in a linear scenario using three test images: (a)-(c) Binary USAF Resolution Chart, (d)-(f) Lena, and (g)-(i) Lion. Subfigures (a), (d), (g) illustrate the results for $\beta = 0$, which is equivalent to AWGS; (b), (e), (h) show the results for $\beta = 0.2$; and (c), (f), (i) correspond to $\beta = 0.4$. The zoomed-in regions, marked by red squares, provide a closer examination of key areas to facilitate detailed comparison of structural fidelity and visual quality.

Further experiments were performed to assess the proposed algorithm under the optimal α and β parameters obtained through GA optimization, with results benchmarked against the conventional GS and AWGS methods. The optimal parameter values were found to be highly dependent on the characteristics of the target images, including image size, padding value, padding dimensions, and the noise level of the optical setup. For example, for the Lena image with a resolution of 540×540 pixels and padding dimensions of 100×100 pixels, using a non-zero padding value of 0.02 and a noise level of 5%, the optimal parameters were

determined as $\alpha = 1.34$ and $\beta = 0.15$, respectively. As shown in Figure 3.11, the proposed method consistently outperforms both GS and AWGS across all three evaluation metrics for each of the test images. In addition to conventional PSNR and SSIM, the dynamic range index (DRI) was introduced to provide a more comprehensive assessment of the effective dynamic range in reconstructed images. The DRI is defined according to Equation (3.12):

$$DRI = \frac{I_{90} - I_{10}}{I_{10}} \quad (3.12)$$

I_{90} and I_{10} denote the 90th and 10th percentile intensity values, respectively, representing the ratio between the upper and lower intensity levels while avoiding the influence of extreme values. Unlike traditional contrast ratio metrics, which are highly sensitive to outliers, DRI characterizes the central portion of the intensity distribution and is therefore more robust in the presence of speckle noise, a common artifact in laser-based holographic imaging. By mitigating the impact of extreme variations and localized artifacts, DRI provides a more reliable and perceptually meaningful assessment of reconstruction quality.

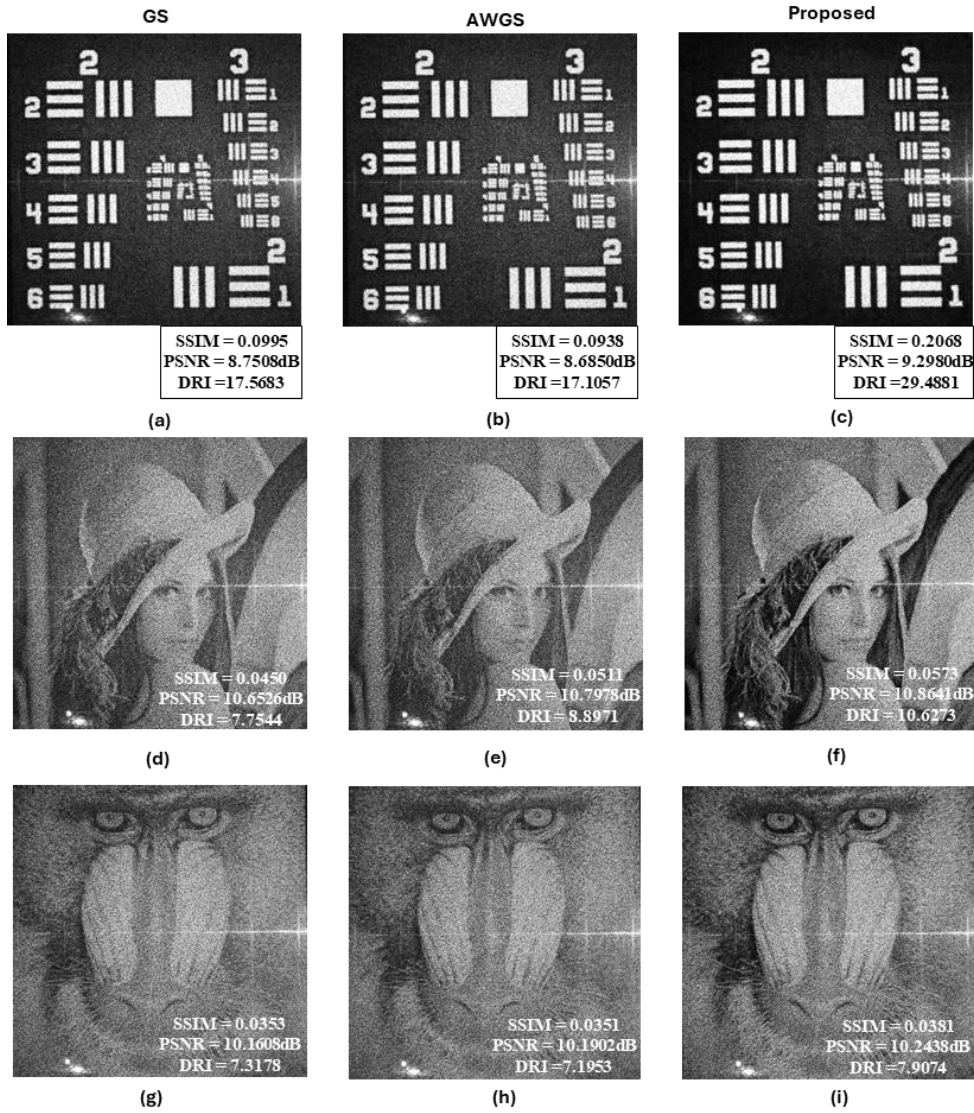


Figure 3.11 Experimental results comparing reconstructed images obtained across three test images: (a)-(c) Binary USAF Resolution Chart, (d)-(f) Lena, and (g)-(i) Mandrill using (a), (d), (g) GS; (b), (e), (h) AWGS; and (c), (f), (i) the Proposed Method. The evaluation is conducted based on three image quality metrics:

SSIM, PSNR, and Dynamic Range Index (DRI)

As illustrated in Figure 3.11, the proposed method yields marked improvements over both the GS and AWGS algorithms, producing reconstructions with sharper edges and enhanced contrast. Across all evaluated scenarios, the proposed approach consistently attains higher PSNR values, confirming its robustness in suppressing noise while preserving overall image fidelity. The corresponding SSIM results further demonstrate that the method not only reduces noise but also maintains structural coherence under adverse conditions. Notably, for the Binary

Resolution Chart, the SSIM increases substantially from 0.0995 with GS and 0.0938 with AWGS to 0.2068 with the proposed method, underscoring its effectiveness in preserving structural integrity and delivering perceptually superior reconstructions.

The DRI introduced to evaluate effective contrast while excluding extreme outliers, also exhibits notable improvement with the proposed method, particularly for the Binary Resolution Chart. This increase in DRI indicates that the proposed approach enhances contrast and reveals finer structural details more effectively than conventional methods. Higher DRI values correspond to improved separation between bright and dark regions, thereby yielding clearer reconstructions with reduced background noise. It is further observed that AWGS does not consistently outperform the traditional GS algorithm. Although AWGS frequently achieves higher PSNR values owing to its ability to suppress noise, this improvement is occasionally accompanied by a reduction in contrast, which in turn lowers SSIM in certain scenarios. These results suggest that while AWGS can mitigate noise, it may simultaneously compromise structural fidelity in the reconstructed images.

3.2.2 Further Verification in Fresnel and ASM 3D Holography

Having validated the effectiveness of the proposed method in two-dimensional holographic reconstructions, we extend the evaluation to three-dimensional holography, which represents the ultimate goal of CGH due to its capability to faithfully reproduce 3D volumetric scenes. Achieving high-quality 3D reconstructions is particularly significant, as it requires accurate preservation of depth cues and structural details across multiple planes, where conventional methods often suffer from severe background noise and loss of contrast. For this purpose, we conducted experiments using both layer-based Fresnel propagation and the ASM for multi-plane hologram generation.

The Fresnel approximation simplifies diffraction modelling by replacing the exact spherical-wave phase factor with a quadratic expansion, effectively treating the propagation as a paraxial process. In this framework, the field is expressed as a convolution integral with a quadratic phase kernel, which enables efficient Fourier-based computation and provides simple scaling properties. Physically, this corresponds to assuming that the propagation angles remain small

and that the dominant contribution to the diffracted field arises from near-axis components. Owing to its reduced computational complexity, the Fresnel approximation is particularly suited for real-time three-dimensional holographic applications.

However, it becomes unreliable at very short propagation distances where the predicted near-field oscillations deviate qualitatively from the exact solution, and it accumulates phase errors at large transverse displacements, exhibiting significant inaccuracies in the far field when the Fresnel number departs from unity. By contrast, the angular spectrum method is based on an exact plane-wave decomposition of the aperture field. The field is represented as a continuous superposition of spatial frequency components, each propagating with its precise longitudinal wavenumber determined by the Helmholtz equation. This representation rigorously accounts for both near- and far-field behaviour, including wide-angle and evanescent contributions, without invoking a paraxial assumption. The angular spectrum method therefore provides a formally exact solution within scalar diffraction theory and is well suited for accurate modelling when the Fresnel approximation becomes unreliable, such as in ultra-short propagation distances, high-numerical-aperture or wide-angle systems, high-resolution microscopy, and metasurface device simulations, albeit at the expense of higher computational cost [83–86].

As shown in Figure 3.12 and Figure 3.13, the proposed method generally outperforms traditional approaches, achieving higher SSIM and PSNR values across the evaluated reconstruction depths. The improvements are observed in terms of contrast enhancement and the recovery of fine structural details, which are less apparent in baseline methods. These results indicate that the proposed framework is robust and suitable for extension from 2D to 3D holographic imaging. The observed consistency further suggests that the proposed approach captures a capability with potential relevance for holographic imaging.

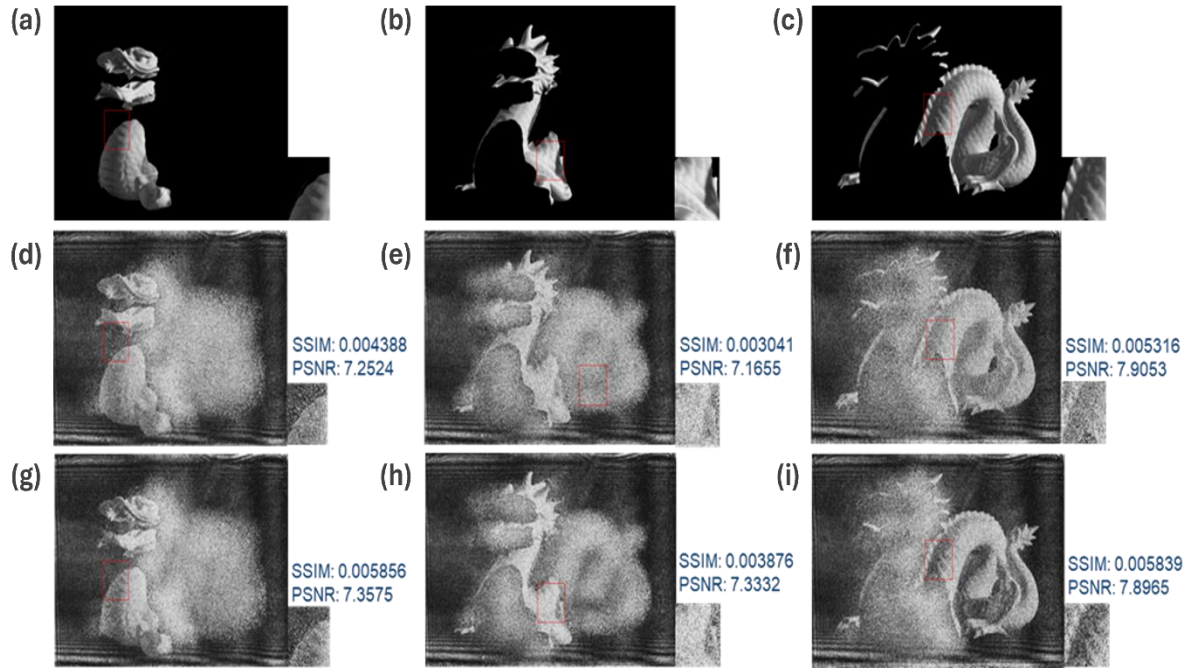


Figure 3.12 3D holographic reconstructions using ASM. (a–c) Ground-truth depth layers of the 3D object. (d–f) Reconstructions obtained with the conventional method at three depth planes. (g–i) Reconstructions obtained with the proposed method at the corresponding planes. Red boxes indicate enlarged regions that highlight detailed structural features.

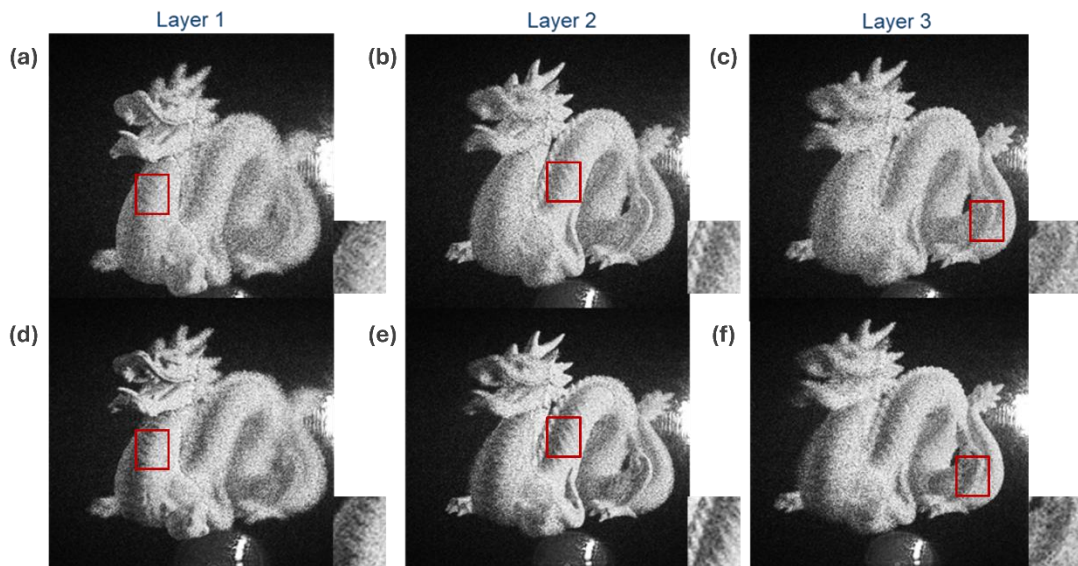


Figure 3.13 3D holographic reconstructions using Fresnel diffraction. (a–c) Reconstructions of three depth layers obtained with the conventional method. (d–f) Reconstructions of the corresponding depth layers obtained

with the proposed method. Red boxes indicate enlarged regions that highlight fine structural details and contrast improvements.

3.3 Discussion and Conclusion

This work demonstrates that embedding non-linear intensity weighting and adaptive dynamic range adjustments into the merit function of computer-generated hologram algorithms yields marked improvements in image quality. The proposed scheme enhances contrast and sharpness in reconstructed holograms without sacrificing brightness in high-intensity regions, while effectively suppressing speckle-like noise by amplifying details in darker areas. Owing to its adaptive design, the merit function responds dynamically to varying noise levels, striking a balance between contrast enhancement and noise reduction across both 2D and 3D reconstructions, reflecting a fundamental capability applicable to any propagation scheme. Experiments reveal that the β parameter is central to this trade-off, where larger values accentuate fine details in high spatial-frequency components but risk excessive compensation, underscoring the importance of optimal parameter selection through GA-based optimization. In addition, the non-linear merit function accelerates convergence, lowering computational cost while preserving fidelity. Comparative evaluations in terms of SSIM and PSNR confirm the superiority of this approach across diverse noise conditions. Beyond the reported results, the study highlights the broader potential of non-linear merit functions for holographic display algorithms, with possible extensions to multi-depth reconstruction and frequency-domain adaptive processing. In the future, more advanced parameter-tuning methods, such as learning-based optimization, could be explored to further improve robustness, efficiency, and scalability. The work presented in this chapter has been published as [87].

Chapter 4 Subpixel Phase Compensation

4.1 Motivation

In holographic imaging, particularly in POH, liquid crystal-based SLMs are widely employed owing to their fine pixel pitch and ability to provide continuous phase modulation. Their low driving voltage and cost-effective fabrication further make them highly attractive for a variety of applications. Nevertheless, these devices inherently suffer from pixel crosstalk, also known as the fringing-field effect, arising from the interaction between nonuniform electric fields across the LC layer and the elastic coupling of neighbouring LC molecules under varying applied voltages. As a consequence, electric fields from adjacent pixels overlap and distort the intended phase response. In holographic displays, this distortion smooths the phase profile between neighbouring pixels, significantly reducing diffraction efficiency from the ideal case. The smoothing effect not only introduces additional noise and degrades image quality with reduced contrast but also leads to pronounced zero-order intensity arising from unmodulated light. Unlike phase-only LC devices, amplitude-modulation approaches such as DMDs are not subject to fringing-field-induced phase errors, since they consist of micromirror arrays whose elements can be individually rotated. Most existing CGH algorithms are developed under the assumption of an idealized system with uniform and isolated pixel responses, whereas in practice each pixel is not independent [88]. Despite substantial efforts target numerical-error reduction and faster convergence, neglecting device physics produces a mismatch between simulated reconstructions and experimental results. To fully leverage liquid crystal SLMs in high-performance applications such as optical trapping or imaging, it is essential to employ approximate analytical models that accurately characterizing fringing-field broadening, together with compensation algorithms that address subpixel-level distortions. An example of the fringing-field effect in holography is shown in Figure 4.1. The figure shows the effect of phase blurring with resolution loss. It is evident that even a slight amount of crosstalk-induced blur in the hologram plane leads to a pronounced degradation in the reconstructed image, causing the energy loss and reduced contrast and diffraction efficiency. This highlights the

importance of explicitly modelling and compensating for pixel-level coupling when designing high-fidelity holographic displays.

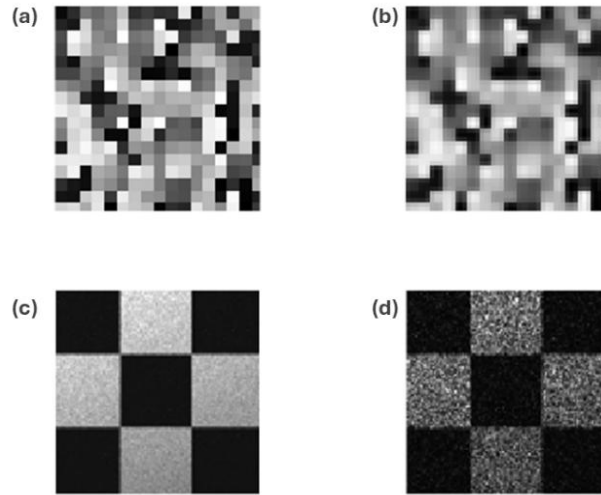


Figure 4.1 Simulation of crosstalk-induced degradation in phase-only holography. Cropped region of (a) original hologram, (b) hologram with fringing-field effect, (c) simulated reconstruction from (a), and (d) simulated reconstruction from (b).

Many studies have addressed imperfections in the phase profile of liquid-crystal devices. [89] develops an approximate analytical model for fringing effect for blazed gratings, introducing a broadening kernel that links the phase-profile broadening in LC beam-steering devices to their physical parameters, and demonstrates a linear dependence on the LC cell thickness, while [90] presents a simple method to identify fringing in SLMs by analysing the diffraction efficiency of binary gratings at different gray levels. The effect is modelled through a broadening kernel expressed as a generalized Gaussian function. The key advantage is that this approach avoids reliance on liquid-crystal parameters or fabrication details, making it easy to implement and directly applicable for evaluating and compensating commercial devices. [65] proposed a method to reduce pixel crosstalk in liquid-crystal SLMs by modifying a weighted GS algorithm, where a point-spread function (PSF) describing the crosstalk is convolved with the hologram during optimisation. This approach effectively improved the uniformity of diffraction spot intensities and enabled applications requiring precise spot control, such as holographic optical tweezers. Work has also been done to account for pixel crosstalk in the generation of arbitrary light potentials for cold-atom experiments [91]. In that study, patterns

such as Gaussian rings for atomtronics and Gaussian spot arrays for tweezer arrays were demonstrated. Pixel crosstalk was modelled by subpixel up-scaling each SLM pixel to a 3×3 grid and convolving the phase with a fitted kernel within an optimisation framework, which was further refined by camera feedback to achieve accurate and efficient holographic reconstruction.

Existing studies on pixel-crosstalk modelling have mainly addressed blazed gratings or spot arrays, which are relatively simple and computationally efficient to generate. However, such approaches have not yet been extended to holographic imaging for general image reconstruction, leaving a research gap. Holograms of complex images, such as natural images with abundant high-frequency components including edges, textures, and fine details, are particularly susceptible to SLM limitations such as finite pixel size and fringing-field-induced crosstalk. Their higher total variation further demands rapid and precise LC director reorientation, making crosstalk effects far more severe than in simple periodic patterns. Moreover, conventional subpixel up-sampling approaches for modelling crosstalk become computationally expensive and significantly slower when applied to such images. Developing efficient strategies to model and compensate crosstalk in this setting is therefore essential for advancing holographic imaging performance.

4.2 Proposed Method

We propose a novel approach to account for the fringing-field effect by incorporating it directly into iterative AWGS frameworks for complex natural images. By introducing a physics-based pre-compensation step, holograms can be optimised for real-world device behaviour, resulting in higher diffraction efficiency and reduced noise. In addition, we have successfully implemented a subpixel operation that preserves the original hologram size without requiring up-sampling. Compared with conventional up-sampling techniques, this strategy significantly lowers computational cost while maintaining accurate subpixel modelling.

4.2.1 Kernel of LSI System

In scalar diffraction theory, SLMs used in holographic displays can be regarded as linear and shift-invariant (LSI) systems, where the addressed pattern is mapped to the optical output through an effective system kernel. Such systems are fully characterised by their response to a point input, commonly represented by the Dirac delta function. In the ideal case, the impulse response reduces to a delta function, such that the kernel is unity in the frequency domain. However, in practical holographic displays, the situation deviates significantly from this idealisation. Pixel-level non-idealities such as fringing-field crosstalk broaden the effective impulse response beyond a single delta peak, leading to lateral spreading of the phase modulation into neighbouring regions. In this context, the degradation in image sharpness can be attributed to the effective PSF of the device, which characterises the extent of pixel-level spreading, and has been observed to be approximately Gaussian in profile [92,93].

Different mathematical models of the kernel have been proposed to quantify the physical limitations imposed by the fringing-field effect, including symmetric and asymmetric forms. A symmetric broadening kernel proposed in [90] is given by

$$H(p) = e^{-\left(\frac{|p|}{\sigma}\right)^m} \quad (4.1)$$

Where p denotes the spatial frequency, σ is a parameter controlling the effective width, and m as the shape parameter with $m = 2$ yields a Gaussian kernel. [94] describes the broadening kernel as an asymmetric biexponential function:

$$h(x) = \begin{cases} \frac{1}{\sigma} e^{\frac{x}{\alpha\sigma}}, & x < 0 \\ \frac{1}{\sigma} e^{-\frac{x}{(1-\alpha)\sigma}}, & x \geq 0 \end{cases} \quad (4.2)$$

Where the degree of asymmetry is controlled by α and effective width of this kernel denoted as σ . This work also shows the linear relationship between the effective width of this kernel and the thickness of the liquid crystal cell. This means that as the LC cell gets thicker, the blurring caused by the fringing fields becomes more severe.

In this work, we adopt a separable asymmetric Generalized-Gaussian kernel to represent the fringing-field response, which is defined as

$$H[i, j] = \frac{e^{(-\frac{1}{2}|\frac{x_i}{\sigma_x}|^m + |\frac{y_j}{\sigma_y}|^m)}}{\sum \sum e^{(-\frac{1}{2}|\frac{x_i}{\sigma_x}|^m + |\frac{y_j}{\sigma_y}|^m)}} \quad (4.3)$$

As in Equation (4.3), i and j are the pixel indices for the kernel, with a normalization step to make the operation energy-preserving. The scale parameters σ_x and σ_y set the effective width along the two axes and are obtained from the full width at half maximum (FWHM) according to Equation (4.16) [95]. The shape parameter m controls the roll-off, allowing more flexible axis-wise control of sharpness and tail decay, where $m = 2$ recovers a Gaussian profile, $m > 2$ yields a flatter-top and steeper-edge super-Gaussian, and $m < 2$ sharpens the central peak and thickens the tails, approaching a Laplacian distribution [96] as illustrated in Figure 4.2.

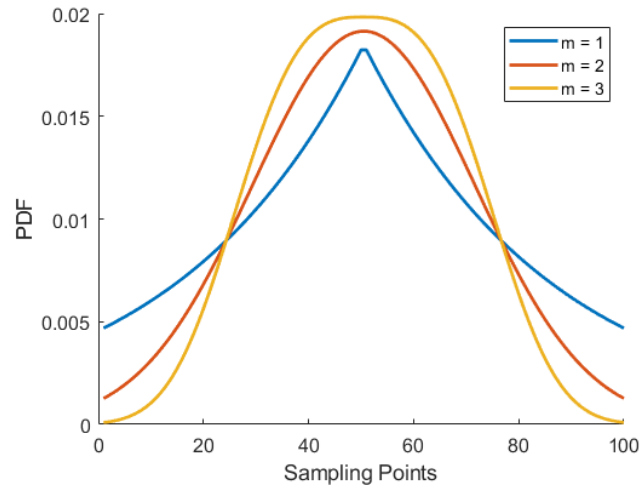


Figure 4.2 The profiles of generalized-Gaussian kernels for varying shape parameter m with total energy normalized.

Because liquid crystals exhibit intrinsic anisotropy, their electro-optic response differs along different spatial directions. In particular, the phase modulation induced by an electric field is not identical in the x and y directions, leading to an inherently asymmetric PSF. This directional asymmetry originates from the molecular structure of the LC medium, as well as the influence of surface alignment. When a rubbed polyimide alignment layer is used, it imposes

unidirectional anchoring with a slight pretilt, defining a preferred in-plane orientation and makes the director more susceptible to reorientation along the rubbing axis. Under fringe-field driving, the lateral electric field couples to this anisotropy, resulting in orientation-selective phase modulation. Specifically, fringing fields tend to leak preferentially along the rubbing axis, making crosstalk more pronounced in that direction and yielding a directionally biased phase spread [97–99]. To reflect this physical behaviour, the pixel-crosstalk kernel is modelled as asymmetric, normalised to unit energy and used as a device-aware constraint in the iterative optimisation. In our case, the fringing-field kernel was parameterized using FWHM values to represent different levels of pixel crosstalk. A smaller FWHM corresponds to a modulation response with minimal leakage into neighbouring regions. As the FWHM increases, the kernel broadens and the lateral spreading becomes more pronounced, indicating stronger crosstalk and enhanced coupling with adjacent pixels. To realize the asymmetric properties, the original symmetric spread parameter σ_x or σ_y is replaced by two distinct parameters,

$$\sigma_x^+ = \sigma_x \cdot scale^+ \quad \sigma_x^- = \sigma_x \cdot scale^- \quad (4.4)$$

$$\sigma_y^+ = \sigma_y \cdot scale^+ \quad \sigma_y^- = \sigma_y \cdot scale^- \quad (4.5)$$

where σ_x^+ and σ_x^- control the broadening on the positive and negative half-axes, respectively. This modification enables the kernel to exhibit asymmetric spreading in either the x or y direction, with Figure 4.3 showing a representative example, and in which different shape parameters illustrate the profile evolution.

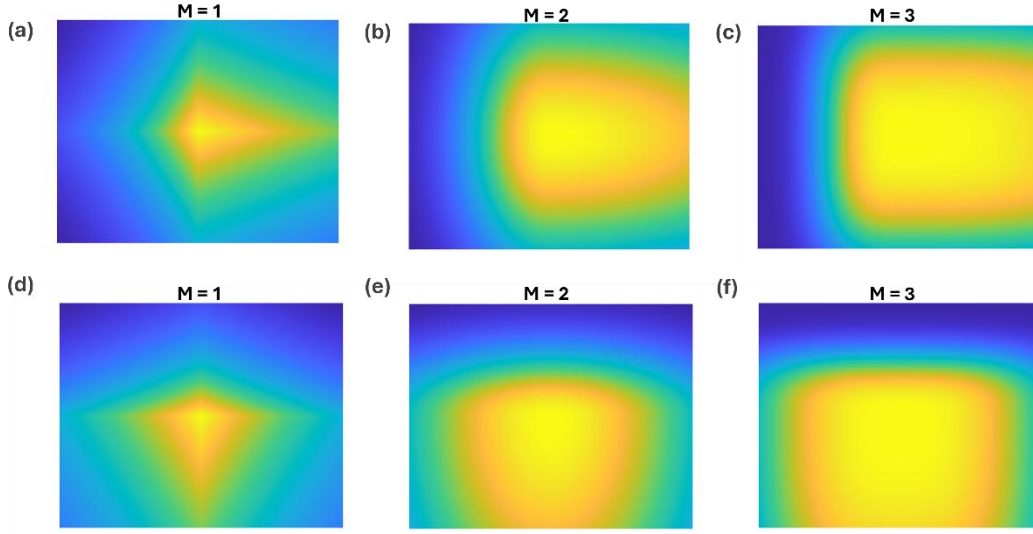


Figure 4.3 Asymmetric kernel profiles introduced by replacing the symmetric spread parameter with σ_x^+ and σ_x^- . (a–c) show asymmetry along the x-axis and (d–f) along the y-axis, with shape parameters $m=1,2,3$ from left to right.

4.2.2 Direct Expansion

The commonly used approach is the direct expansion of the hologram during iterative optimization. Studies of fringing-field effects have demonstrated that pixel crosstalk acts as a spatial low-pass filter on the phase retardation relative to the ideal response of an SLM. This behaviour can be described as the convolution of the ideal phase profile with a PSF intrinsic to the SLM. Once the PSF is characterized, the influence of pixel crosstalk can be both predicted and compensated through hologram optimization process. Building on the GS framework, direct expansion simulates pixel crosstalk by embedding a PSF-based convolution on an oversampled grid at each iteration. Starting from the target complex field as shown in Figure 4.4, the hologram plane is obtained via an FFT and is then upsampled to a larger grid, typically by a factor of 3 along both axes, to account for subpixel structure within a single pixel. On this high-resolution grid, a predefined crosstalk kernel is applied by convolution, after which an inverse FFT is performed to produce the oversampled image plane field. On this high-resolution grid, a predefined crosstalk kernel is convolved with the enlarged hologram, after which an inverse FFT is performed to produce the oversampled image plane field. The central target block is then cropped back to the original size, and the rest remains the same as in

AWGS. The loop repeats until convergence. Compared with the original $M \times N$ framework, the $3\times$ direct-expansion approach increases space complexity (working memory) by 9x and raises the computational time complexity from $O(2MN\log_2(MN))$ to $O(MN\log_2(MN) + 9MN\log_2(9MN))$.

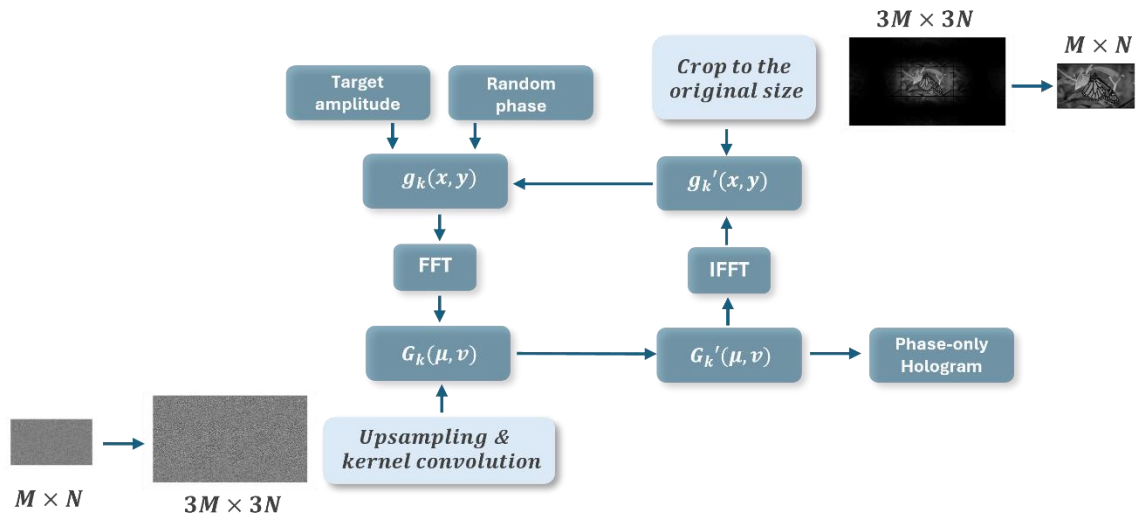


Figure 4.4 The direct-expansion subpixel phase compensation framework in AWGS for pixel-crosstalk modelling.

4.2.3 Frequency Domain Shift

In CGH computation, discretisation is inevitable, as SLMs are inherently pixelated and electrically addressed on a fixed grid. As a result, the optical wavefront must be sampled and processed on a finite, equidistant grid rather than a continuous field. Under the discrete Fourier transform (DFT) model:

$$U(x, y) = \sum_{\mu=-\frac{M}{2}+1}^{\frac{M}{2}} \sum_{v=-\frac{N}{2}+1}^{\frac{N}{2}} U(\mu, v) e^{-i2\pi(\frac{x\mu}{M} + \frac{yv}{N})} \quad (4.6)$$

Where each pixel is treated as a point sample, and the complex amplitude is evaluated at the grid nodes. This discretisation inevitably introduces modelling errors and makes subpixel simulation particularly difficult to capture [100]. A common strategy is to upsample the grid,

which improves accuracy but significantly increases computational and memory cost. To resolve this trade-off, we develop an efficient method that leverages the frequency-domain shift theorem to model sub-pixel contributions without enlarging the array, thereby preserving computational efficiency while maintaining high reconstruction fidelity.

The detailed framework of the proposed algorithm is presented below in **Algorithm 1**. The algorithm accounts for subpixel contributions analytically by (i) tiling the Fourier plane phase onto a subcell lattice, for example a 3×3 grid, (ii) smoothing the tiled phase with a fringing kernel, in our case a generalised Gaussian, and (iii) realising subpixel displacements through phase shifts in the frequency domain. The procedure runs within an iterative loop with an adaptive weight following the AWGS scheme, where the alternating optimisation updates amplitude under a device aware model that combines the ideal central pixel response with subpixel coupling, thereby steering the phase estimate toward the hardware-consistent solution.

The proposed method is fundamentally built upon the frequency domain shift theorem, under which a finite shift in the temporal or spatial domain is equivalently realised as multiplication by a linear phase factor in the Fourier domain [101]. The shift operator is realised as a complex exponential as defined in:

$$p(x - \Delta_x, y - \Delta_y) \leftrightarrow e^{-j2\pi(f_x\Delta_x + f_y\Delta_y)}P(f_x, f_y) \quad (4.7)$$

Where $P(f_x, f_y)$ is the Fourier transform of $p(x, y)$ which makes the signal processing not restricted to integer multiples of the sampling period in the time domain and therefore enables subpixel displacements on the sampling grid without interpolation. In two-dimensional hologram synthesis the same principle is applied along x and y , where fractional pixel offsets are encoded as separable linear phase terms in the spatial frequency domain, so the subpixel effect is captured analytically within the iterative optimisation. In other words, in the fringing-effect simulation, the contributions in the time domain from neighbouring subpixels must be coherently evaluated at the reference location of the central pixel rather than at their own physical coordinates. Under the assumption of shift invariance, each neighbouring subpixel would naturally produce a shifted version according to the kernel, thereby placing its response at the displaced subpixel position. To realign these contributions for coherent accumulation at the central point, a compensating linear phase ramp is applied in the frequency domain. This

effectively maps all subpixel responses back to the central location. As a result, the reconstructed field incorporates the coherent interference of subpixel contributions at a common reference, while still preserving the phase relationships imposed by their respective offsets.

The algorithm begins with the frequency-domain shift setup, where the SLM aperture pitch and pixel dimensions are used to define the total aperture lengths L_x and L_y . The corresponding frequency spacings δ_{f_x} and δ_{f_y} are then obtained, and the centred frequency grids f_x and f_y are constructed accordingly, as given in Equation (4.8)-(4.12). Following that, each SLM pixel is partitioned into a 3×3 lattice, the integer index vectors d_x and d_y enumerate the nine subcells along the two axes as in Equation (4.14). Figure 4.5 illustrates the structure of a pixelated SLM and the corresponding subpixel schematic. By multiplying these index vectors by the fractional pixel pitch, the actual subpixel displacements $shift_x$ and $shift_y$ are obtained, as given in Equation (4.15). In effect, these vectors specify the locations of the nine sub apertures within each pixel. Fringing-field crosstalk is represented by a compact, separable 2-D kernel H_{2D} with a generalised Gaussian envelope as mentioned in the early section. Anisotropy is accommodated by allowing different scale parameters along x and y and by orienting the kernel with the rubbing direction. In the following, the iterative loop starts with a complex field consists by the target amplitude and random phase, which is subsequently transformed to the frequency domain. The phase of this spectrum is extracted as $\angle(A_2)$, and to enable subpixel processing, this phase map is expanded onto a subpixel grid by a Kronecker product as Equation (4.24), which replicates each pixel phase into a 3×3 stencil, yielding a $3N \times 3M$ array that indexes the nine subpixel channels per pixel. Followed by the convolution of the predefined kernel, the blurred field $\psi_{fringing}$ is demultiplexed into nine interleaved sublattices by modulo-3 strided sampling along both axes. Each subpixel channel $S_{r,c}$, where r and c represent the row index and column indices, is an $N \times M$ map that contains all the samples and are spatially aligned with the original pixel grid. Conceptually, this operation assigns to each subpixel location the effective phase it acquires after fringing-field mixing.

The central subpixel spectrum is first generated and scaled to the target energy according to its corresponding kernel weight, then transformed back to the spatial domain via an inverse FFT.

The spectra for the eight neighbouring subpixels are obtained in the same manner, with an additional linear phase ramp in the frequency domain to realise the spatial shift associated with each neighbour as demonstrated in Equation (4.29). After inverse transformation, all nine spatial fields are coherently summed to yield the reconstructed complex field, effectively simulating the interference among subpixels through coherent superposition. The magnitude of this field provides the updated amplitude for adaptive weighting in the subsequent iteration. The phase is updated only from the central subpixel channel, ensuring that the iterative optimisation remains anchored to the unshifted reference.

As before, the AWGS adjusts the working amplitude so that the reconstruction approaches the target intensity within the signal region while suppressing artifacts in the padded area. The amplitude is updated multiplicatively through an exponential mapping of the discrepancy, which improves stability and avoids negative or overly aggressive corrections, which enables fast and stable convergence, preventing the stagnation issues. After iterations, the algorithm outputs the wrapped phase of the central sub-pixel channel as the final hologram in Equation (4.36). Because sub-pixel translations are executed exactly in the frequency domain and crosstalk is enforced by a physically interpretable kernel, the method captures sub-pixel contributions without increasing the hologram sampling grid. This provides a more effective representation of device-level behaviours compared with conventional upsampling-based methods.

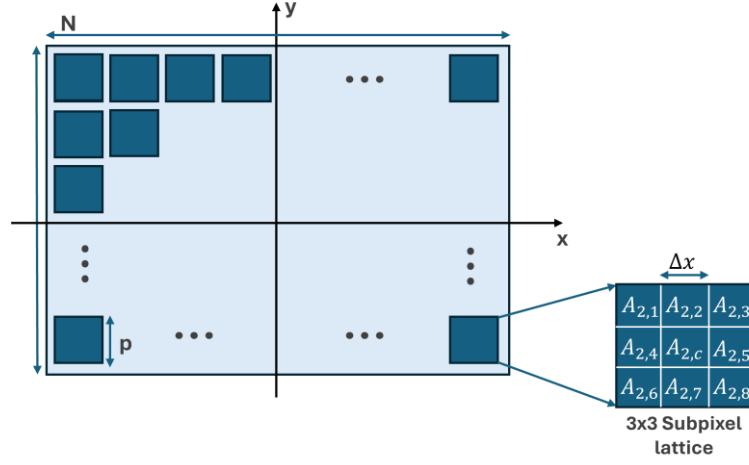


Figure 4.5 The structure of a pixelated SLM and subpixel schematic.

Algorithm 1: 3x3 Sub-pixel Phase Compensation AlgorithmInput: Padded Target image I_{target} SLM dimensions (N, M) Pixel pitch p Total target energy E_{target} Sub-division factor $s=3$ Iterations N_{iter} Output: Hologram ϕ_{final}

Frequency-Domain Shift Setup:

$$L_x = N \cdot p, \quad L_y = M \cdot p \quad (4.8)$$

$$\delta_{f_x} = \frac{1}{L_x}, \quad \delta_{f_y} = \frac{1}{L_y} \quad (4.9)$$

$$f_x = \left[-\frac{N}{2} : \frac{N}{2} - 1 \right] \cdot \delta_{f_x}, \quad f_y = \left[-\frac{M}{2} : \frac{M}{2} - 1 \right] \cdot \delta_{f_y} \quad (4.10)$$

$$F_x(i, j) = f_x(j), \forall i \in \{1, 2, \dots, M\}, j \in \{1, 2, \dots, N\} \quad (4.11)$$

$$F_y(i, j) = f_y(i), \forall i \in \{1, 2, \dots, M\}, j \in \{1, 2, \dots, N\} \quad (4.12)$$

Sub-pixel configuration (3×3 subdivision):

$$\Delta x = \frac{p}{3}, \quad \Delta y = \frac{p}{3} \quad (4.13)$$

$$d_x = [1, 0, -1, 1, -1, 1, 0, -1], d_y = [1, 1, 1, 0, 0, -1, -1, -1] \quad (4.14)$$

$$shift_x = dx \cdot \Delta x, \quad shift_y = dy \cdot \Delta y \quad (4.15)$$

Fringing effect kernel:

$$\sigma_x = \frac{FWHM_x}{2(2\ln 2)^{1/m}}, \quad \sigma_y = \frac{FWHM_y}{2(2\ln 2)^{1/m}} \quad (4.16)$$

$$\text{As } s = 3: \quad h_x^{(0)} = [-1, 0, 1], \quad h_y^{(0)} = [-1, 0, 1] \quad (4.17)$$

$$h_x = \exp\left(-\frac{1}{2}\left|\frac{h_x^{(0)}}{\sigma_x}\right|^m\right), \quad h_x = \frac{h_x}{\Sigma h_x} \quad (4.18)$$

$$h_y = \exp\left(-\frac{1}{2}\left|\frac{h_y^{(0)}}{\sigma_y}\right|^m\right), \quad h_y = \frac{h_y}{\Sigma h_y} \quad (4.19)$$

$$H_{2D} = h_y^T h_x, \quad H_{2D} = \frac{H_{2D}}{\Sigma_{i,j} H_{2D}(i,j)} \quad (4.20)$$

Initialization:

$$amp = \sqrt{I_{target}} = amp_t, \quad \phi = e^{j \cdot rand(0, 2\pi)} \quad (4.21)$$

Iterative optimization:

for $i = 1$ to N_{iter} do

$$A_1 = amp \cdot \phi \quad (4.22)$$

$$A_2 = \mathcal{F}(A_1) \quad (4.23)$$

$$\psi = \angle(A_2) \otimes J_3, \text{ where } J_3 = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad (4.24)$$

$$\psi_{fringing} = \psi * H_{2D} \quad (4.25)$$

$$S_{r,c} = \psi_{fringing}(i,j): i \equiv r \pmod{3}, j \equiv c \pmod{3} \quad \forall r, c \in \{1, 2, 3\} \quad (4.26)$$

$$A_{2,c} = \sqrt{\frac{E_{target} \cdot H_{2D}(2,2)}{N \times M}} \cdot e^{j\angle(A_2)} \quad (4.27)$$

$$A_{3,c} = \mathcal{F}^{-1}[A_{2,c}] \quad (4.28)$$

for $k = 1$ to 8 do

$$A_{2,k} = \sqrt{\frac{E_{target} \cdot H_{2D}(r_k, c_k)}{N \times M}} \cdot e^{jS_{r,c}} \cdot e^{-j2\pi(shift_x(k)F_x + shift_y(k)F_y)} \quad (4.29)$$

$$A_{3,k} = \mathcal{F}^{-1}[A_{2,k}] \quad (4.30)$$

$$A_{total} = A_{3,c} + \sum_{k=1}^8 A_{3,k} \quad (4.31)$$

$$A_{recon} = |A_{total}| \quad (4.32)$$

$$\phi = e^{i\angle(A_{3,c})} \quad (4.33)$$

$$Compensation = amp_t - A_{recon} \quad (4.34)$$

$$amp = \begin{cases} amp_t \cdot e^{Compensation} & \text{if } \in \Omega_{signal} \\ w \cdot A_{recon} & \text{if } \in \Omega_{padding} \end{cases} \quad (4.35)$$

$$\phi_{final} = \angle(A_{2,c}) \bmod 2\pi \quad (4.36)$$

Return ϕ_{final}

4.2.4 Preliminary Experimental Results

The experiment was conducted with the same optical setup shown in Chapter 3 (Figure 3.7), and Figure 4.6 below summarizes the preliminary experimental results obtained with the proposed device-aware subpixel framework. Compared with the conventional AWGS result computed on the native grid without subpixel modelling, as shown in panel (a), the inclusion of subpixel effects yields visibly sharper textures and enhanced contrast, indicating an increase in diffraction efficiency. Panels (b) to (e) illustrate the performance of both the proposed frequency-domain shift-based compensation method and the direct expansion scheme under different subpixel footprints, where the point-spread kernel is defined with a uniform FWHM of 1.5 in both x and y directions and a shape parameter $m = 2$ corresponding to a Gaussian profile. The difference is further highlighted in panel (f), where enlarged regions allow a direct visual comparison between panels (a) and (e). At a comparable level of visual enhancement, the subpixel model achieves a computational time reduction of **39.23%** for a 3×3 footprint and **59.07%** for a 5×5 footprint for a hologram with size 1080×1920 and 200 iterations, primarily due to its advantage in preserving the original reconstruction scale.

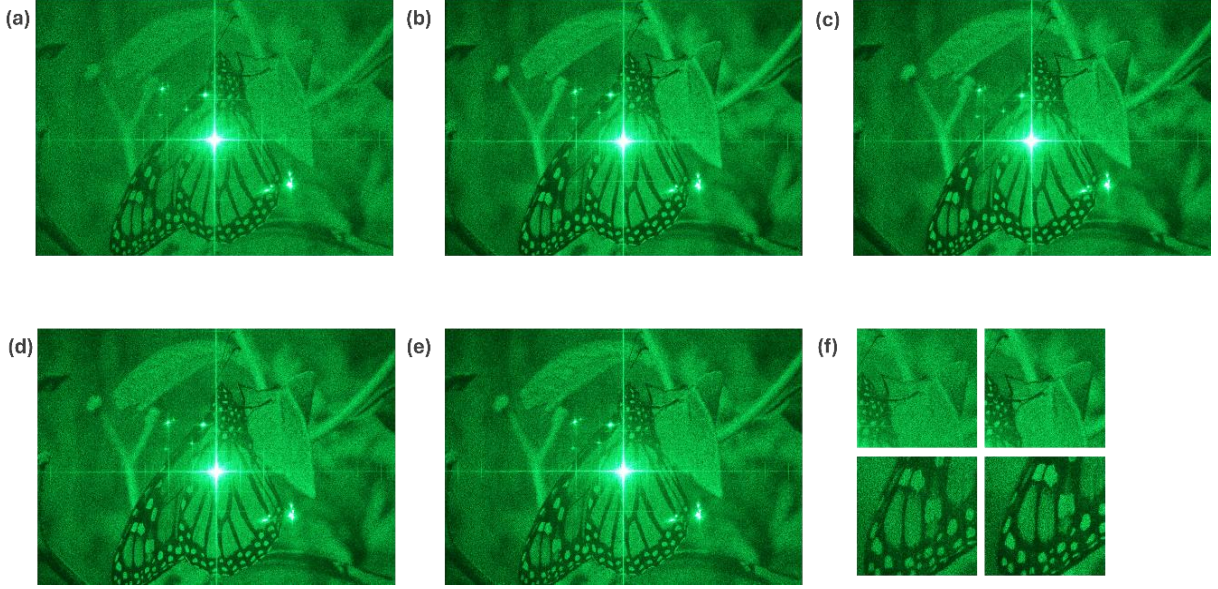


Figure 4.6 The preliminary experimental results. (a) AWGS without subpixel enhancement; (b) 3×3 subpixel; (c) 5×5 subpixel; (d) 3×3 expansion; (e) 5×5 expansion; (f) enlarged crops from (a) and (e).

However, as shown in the figure, a pronounced zero-order effect remains visible and severely degrades the image quality. This was initially mitigated by introducing a rectangular aperture placed immediately after the SLM, which spatially filters out the specular reflection from the peripheral regions of the LCOS panel, thereby effectively suppressing a significant portion of the residual zero-order component. The Figure 4.7 below presents the results obtained under this configuration together with the corresponding quantitative metrics. Grayscale presentation is adopted to remove chromatic influence and highlight luminance contrast, to which the human eye is more sensitive. Panels (a), (c), (e), and (g) correspond to an idealized scenario that without compensation of fringing-field modulation, whereas panels (b), (d), (f), and (h) show reconstructions obtained with the fringing-effect kernel explicitly incorporated. The inclusion of fringing-effect modulation produces a visible improvement in image quality, with enhanced contrast and clearer structural features. For natural images, high-frequency details are better preserved, and the quantitative evaluation further confirms this enhancement, as both PSNR and SSIM values show consistent improvement. Nevertheless, the reconstructions still exhibit a noticeable zero-order component, which continues to degrade image fidelity. The following section therefore focuses on addressing the zero-order issue, where a strategy is implemented that manages to achieve a completely zero-order-free holographic reconstruction.

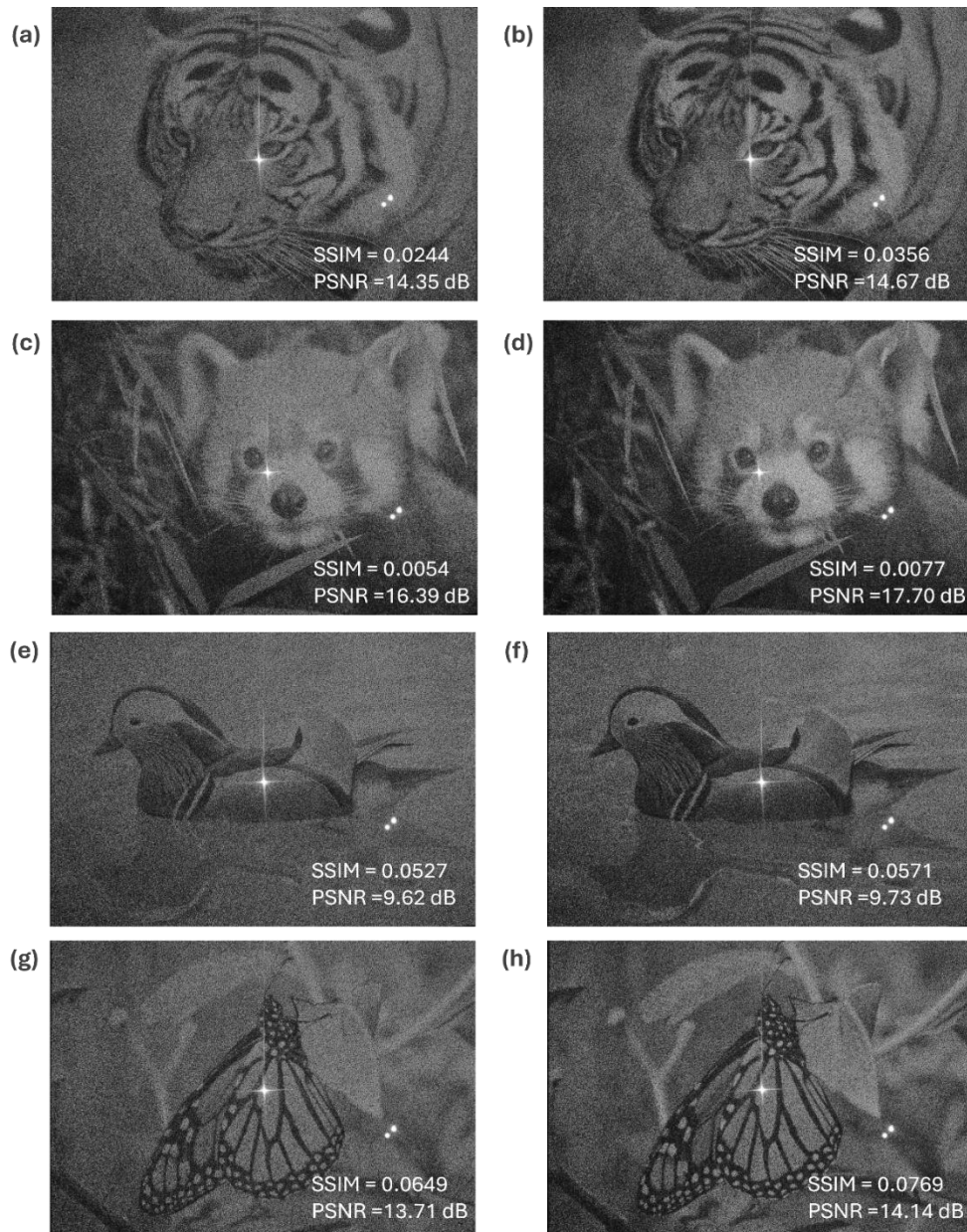


Figure 4.7 The reconstructed 4 sets of natural images [102] using a rectangular aperture for zero-order reduction. Panels (a), (c), (e), and (g) correspond to results without fringing-field modulation, while panels (b), (d), (f), and (h) incorporate the fringing-field kernel with even distribution FWHM = 1.5. All reconstructions are shown in grayscale for objective comparison, together with the corresponding PSNR and SSIM metrics.

4.2.5 Zero-order Suppression

A long-standing challenge in holographic displays is the presence of severe zero-order noise, whose high intensity degrades the image quality due to the pixelated structure of phase-only

SLMs. This noise arises from multiple factors, including the limited fill factor due to dead space, surface nonuniformities, unmodulated window reflections inherent to LCOS devices, and phase quantization. Considerable efforts have been made to suppress it both optically and digitally [103,104]. Conventional strategies to suppress the zero-order beam often rely on shifting the reconstruction pattern away from the optical axis, as demonstrated by the experimental results presented in Chapter 3. However, this approach reduces the usable aperture and lowers diffraction efficiency, while the increased off-axis angle introduces comatic aberrations. An alternative strategy achieves axial rather than transverse separation, by encoding a quadratic phase term into the hologram, thereby displacing different diffraction orders along the axis. Nevertheless, the defocused zero-order component persists as background noise [105]. A related approach is to numerically design a CGH that encodes both the target phase and an additional phase-shifted corrective term, so that the corrective beam destructively interferes with the zero-order at the optical axis. This cancellation, however, reduces diffraction efficiency since part of the available energy is redirected for suppression [106,107]. Another common approach employs a 4f system, built from two identical positive lenses and a physical block located at the Fourier plane [108]. For applications such as holographic optical traps, an effective approach is to illuminate the DOE with a converging rather than a collimated beam, thereby shifting the undiffracted zero order out of the sample plane before the objective. Residual on-axis light can be further attenuated by placing a small beam block at the intermediate focal plane. This configuration effectively suppresses the central spot and reduces ghost traps, but it comes at the cost of a modest loss of usable power, and the remaining on-axis light may still impose unwanted radiation pressure on particles near the centre of the field of view [109]. While none of these methods completely eliminates residual artifacts, a valuable insight that can be taken from them is the strategy of shifting the undiffracted zero-order component out of the image plane and blocking it at an intermediate focal plane. This effectively removes the central spot and allows only the desired diffracted light to continue propagating toward the image formation, thereby improving overall contrast and fidelity. Therefore, a hybrid strategy is employed, as shown in Figure 4.8, in which a converging lens is physically inserted into the optical path for zero-order suppression, while a quadratic phase factor of the form $e^{-i\frac{k}{2f}(x^2+y^2)}$ is numerically encoded into the hologram to emulate the action of a thin lens with focal length f . This virtual lens effectively controls the

propagation distance of the reconstructed field, thereby shifting the focal plane without the need for additional optical elements. Consequently, the unmodulated zero-order component, corresponding to the specular reflection of the SLM, is unaffected by the digital phase modulation. Placing a small beam block with a width of approximately 1 mm at the focal plane effectively removes the zero-order light, while allowing the desired reconstructed wavefront to pass through and continue to propagate toward the reconstruction plane. It is worth mention that the final reconstruction distance should be analytically correctly using the effective imaging geometry:

$$\frac{1}{f} = \frac{1}{Z_1} + \frac{1}{Z_2} \quad (4.37)$$

Where f is the focal length of the physical lens, Z_1 is the focal lens of the encoded lens, and Z_2 is the effective imaging distance.

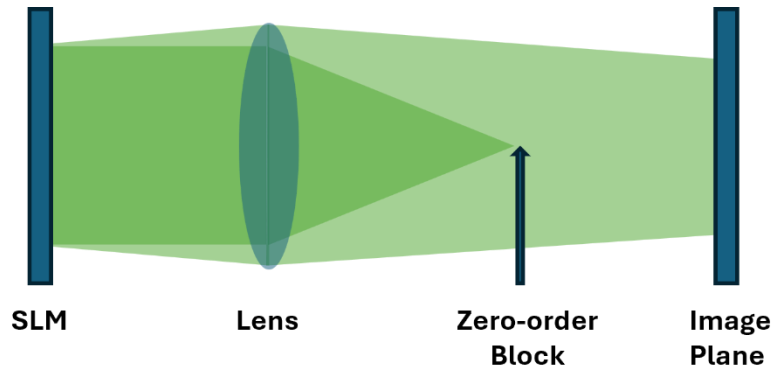


Figure 4.8 The optical configuration for zero-order suppression using a single converging lens and a focal plane obstruction.

4.2.6 Experimental Result with Zero-order Reduction

This section presents the experimental results with zero-order reduction, where performance is assessed under different kernel parameters. An anisotropic distribution is first investigated by varying the FWHM values in both the x and y directions on the USAF resolution chart with a 3×3 subpixel lattice. As shown in Figure 4.9, all reconstructions based on the subpixel phase compensation framework exhibit improved performance relative to the conventional method, represented by the black horizontal reference plane in each plot. Increasing the kernel width

along the x direction, which corresponds to enhanced energy spreading in the rubbing direction, produces the most favourable results, indicating that the fringing effect is more pronounced along this axis. The optimal configuration is obtained when the kernel spans 3 sampling points in the x direction and 0.5 sampling points in the y direction, yielding the best overall performance across all three evaluation metrics, namely SSIM, PSNR, and DRI. The detailed reconstructed images in Figure 4.10 present a comparison between the traditional AWGS method and the proposed kernel with different parameter settings. The results show a clear improvement in diffraction efficiency, while energy spreading along the rubbing direction yields a slight enhancement in contrast and a reduction of background noise.

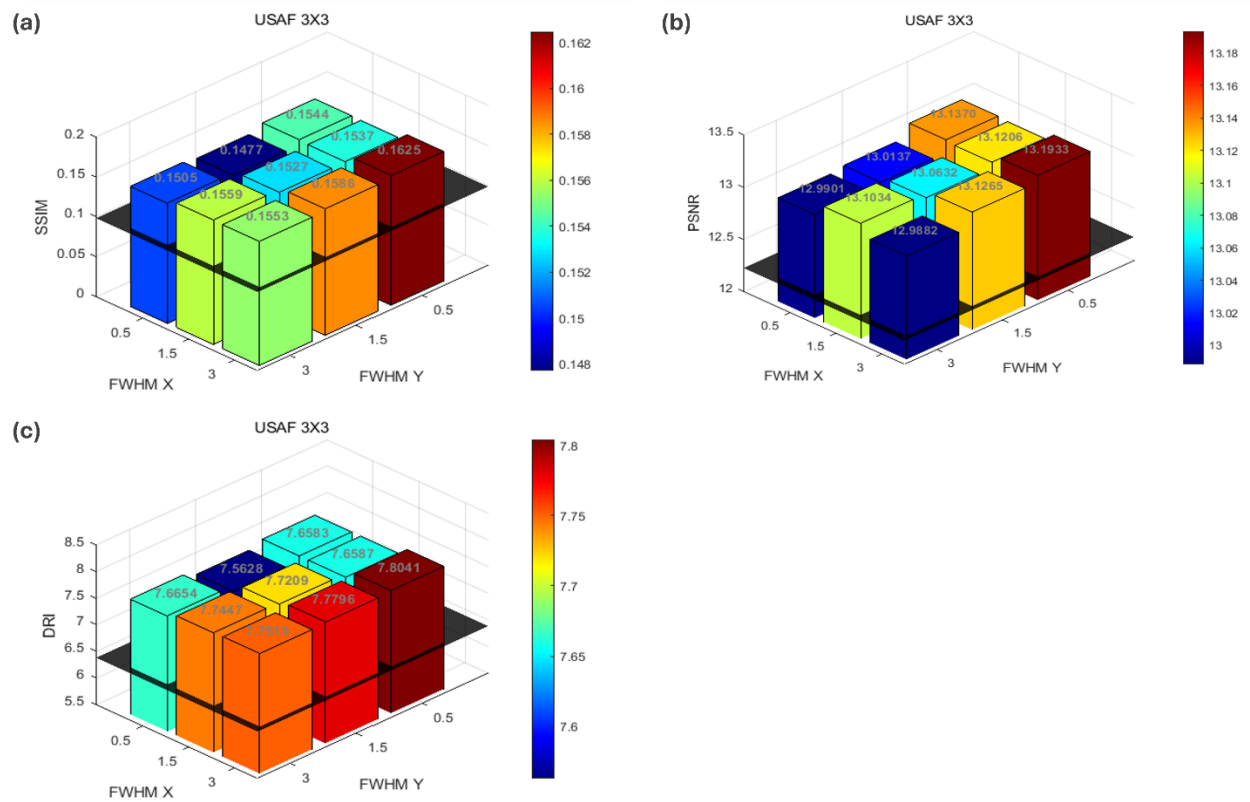


Figure 4.9 Performance evaluation of zero-order reduction on the USAF resolution chart with a 3×3 subpixel subdivision, under different FWHM values in the x and y directions: (a) SSIM, (b) PSNR, and (c) DRI.

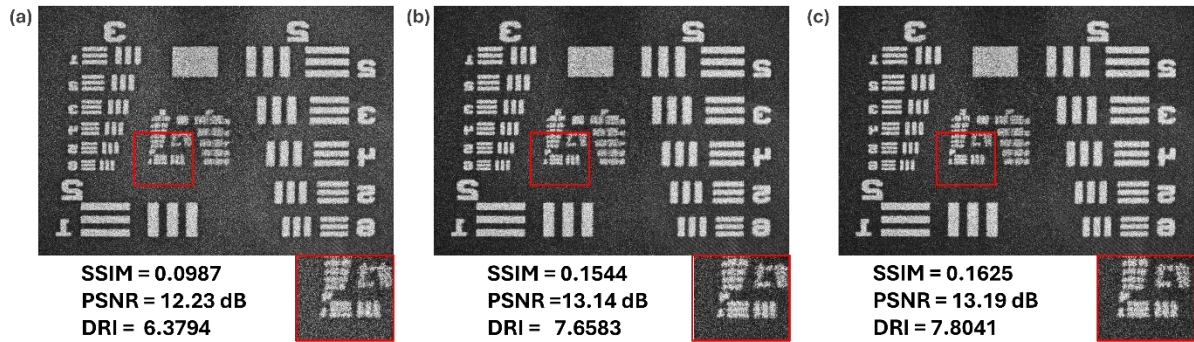


Figure 4.10 Reconstructed USAF resolution chart images with subpixel phase compensation: (a) AWGS method, (b) proposed kernel with $FWHM_x = 0.5$ and $FWHM_y = 0.5$, and (c) proposed kernel with $FWHM_x = 3$ and $FWHM_y = 0.5$. Enlarged regions are highlighted in red for comparison.

For more complex natural images with higher total variation in the hologram, the reconstruction task becomes more demanding due to the increased presence of high-frequency components and sharper intensity variations. The overall fringing effect becomes more pronounced. In this case, a broader kernel with larger FWHM is required so that more modulation energy is spread into neighbouring subpixels. Moreover, as shown in Figure 4.11, the results further demonstrate that the x-shift configuration outperforms both the even and y-shift cases. This improvement is achieved by maintaining a symmetric kernel along the y-axis while introducing an asymmetric shift along the x-axis, defined in Equation (4.4) with $scale^+ = 2$ and $scale^- = 0.5$. Such an asymmetric adjustment redistributes the energy preferentially in the positive x direction, thereby enhancing structural fidelity and reducing background artifacts. The improvement is reflected in higher SSIM and DRI values compared with the symmetric kernel, which is consistent with the physical properties of liquid crystals in LCOS devices, where the pretilt angle induced by the rubbing direction leads to anisotropic electro-optic response. Figure 4.12 further illustrates this effect with reconstructions of a natural image. Compared with the baseline AWGS method, the proposed kernel improves texture clarity and reduces background noise. The x-shift configuration provides a slight additional improvement, consistent with the quality metrics.

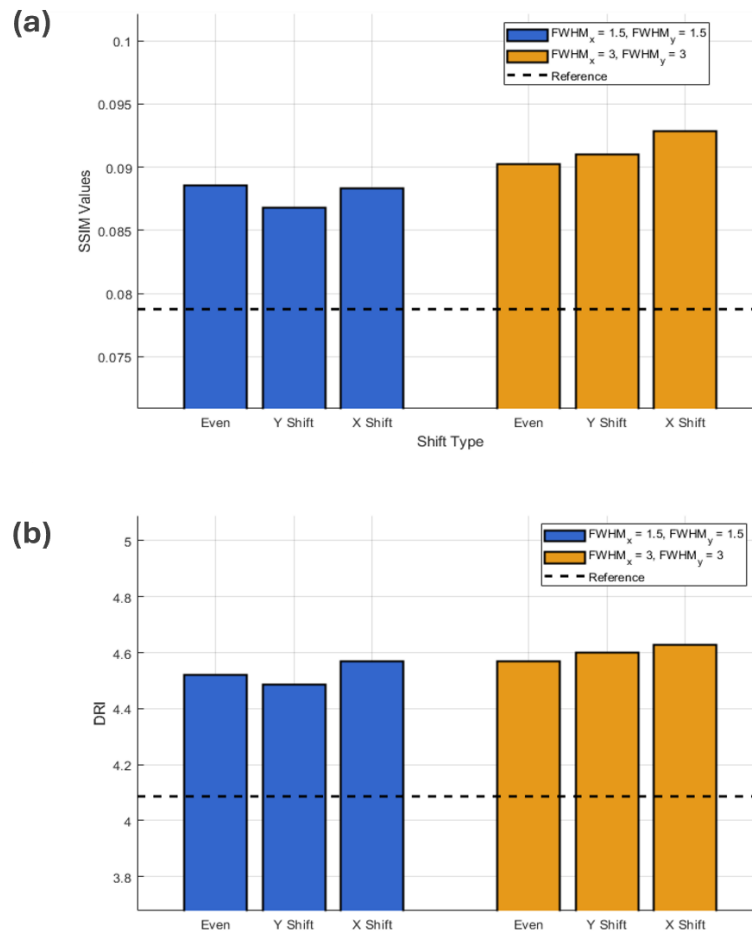


Figure 4.11 Comparison of symmetric and asymmetric kernel distributions on reconstructed image quality. SSIM (a) and DRI (b) are evaluated on natural images with a 3×3 subpixel subdivision. The dashed black line indicates the baseline AWGS result, while bars show the performance of even, y-shift, and x-shift kernels under different FWHM settings.

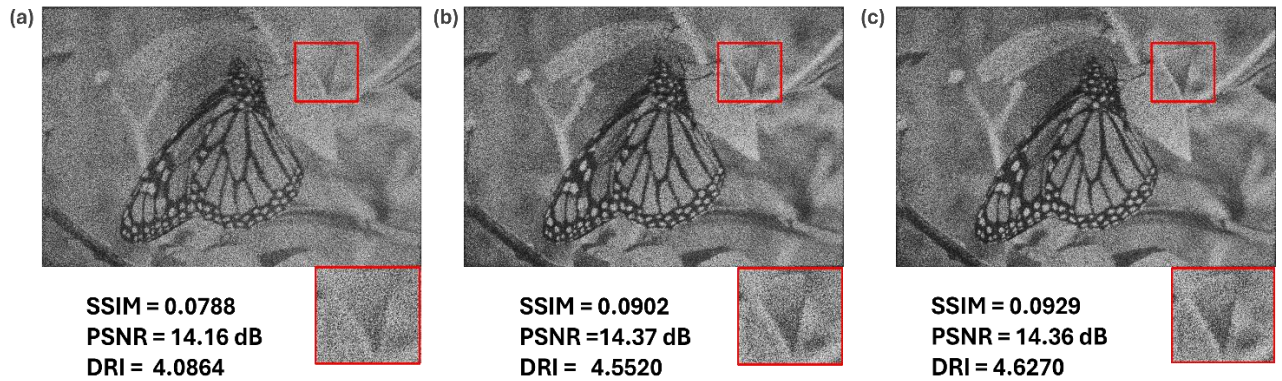


Figure 4.12 Comparison of asymmetric effects on reconstructed natural images: (a) AWGS method, (b) symmetric distribution with FWHM = 3, and (c) x-shifted kernel with FWHM = 3. Enlarged regions are highlighted in red for clarity.

Experiments were also conducted using a 5×5 subpixel configuration. The results indicate that, in general, the shape parameter $m = 1$ provides the best performance, as it allows more energy to be distributed into the surrounding regions, resembling a Laplacian distribution, as shown in Figure 4.13 (a). However, when compared with the 3×3 subdivision, the 5×5 configuration does not necessarily yield superior results. This is because that the energy partitioning and quantization noise also become critical in the 5×5 subdivision. Since the modulation energy must be distributed across 25 subcomponents and normalized according to the kernel weights, the amplitude of each component is reduced, making the reconstruction more susceptible to phase quantization and numerical noise. As a result, diffraction efficiency and background contrast may degrade rather than improve. In practice, refining the lattice from 3×3 to 5×5 often leads to redundant sampling with an increase in the computational load.

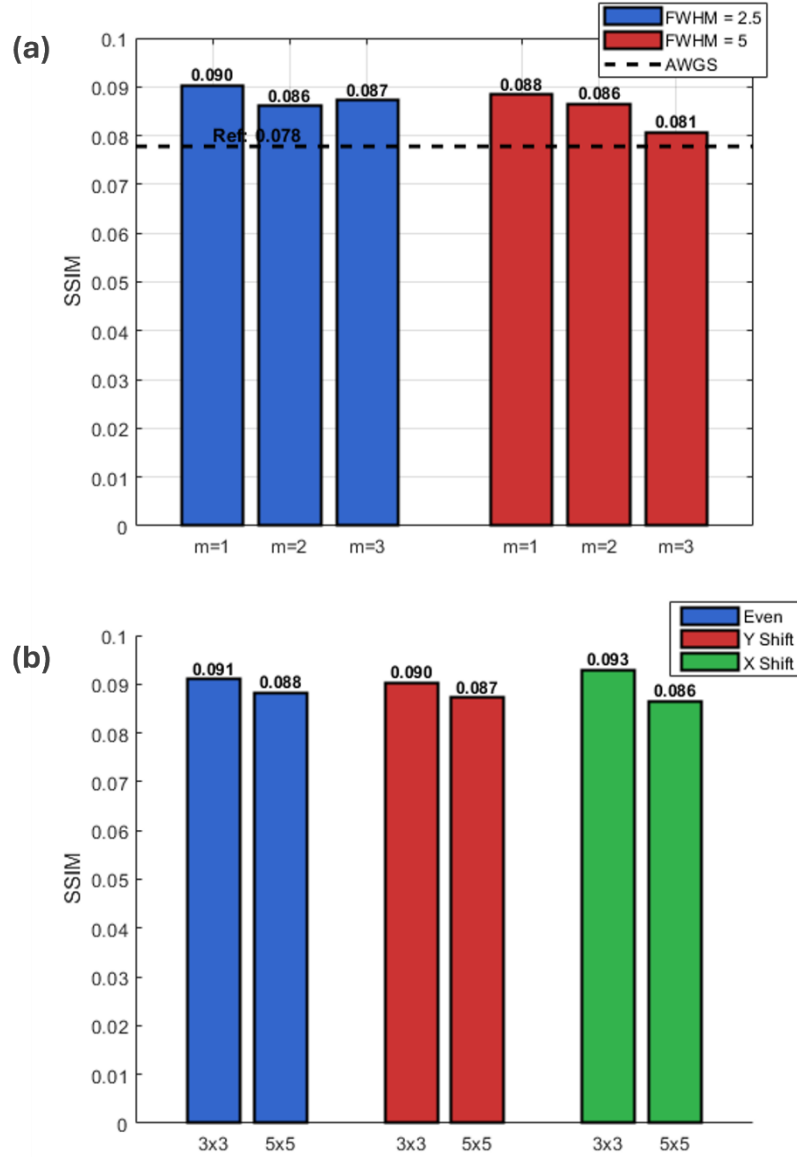


Figure 4.13 SSIM evaluation on reconstructed natural images. (a) Comparison of different shape parameters ($m=1,2,3$) for $\text{FWHM} = 2.5$ and $\text{FWHM} = 5$ with a 5×5 subpixel configuration. (b) Comparison between 3×3 ($\text{FWHM} = 3$) and 5×5 ($\text{FWHM} = 5$) subpixel configurations under even, y-shift, and x-shift kernel distributions.

4.3 Discussion and Conclusion

In conclusion, this chapter has addressed device-level limitations in holographic imaging, focusing on the fringing-field effect in liquid-crystal-based SLMs that distorts the intended phase response and reduces diffraction efficiency, contrast, and overall image quality.

Analytical models and compensation strategies were developed to capture subpixel-level distortions, providing a framework for improving fidelity in practical holographic displays. Several kernel models were evaluated, and an asymmetric kernel was proposed to more accurately represent the fringing-field effect. The asymmetric kernel achieved higher SSIM and DRI values than its symmetric counterpart, consistent with the anisotropic electro-optic response of liquid-crystal-on-silicon devices induced by the rubbing direction. The results further showed that the shape parameter $m = 1$ generally yielded the best performance by redistributing more energy into neighbouring regions in a manner resembling a Laplacian distribution. In addition, traditional direct-expansion method is discussed, in conjunction with a frequency-domain shift approach that integrates a separable fringing-field kernel, achieving efficient and significantly faster subpixel phase compensation as preserving the native hologram grid, while enforcing energy consistency, and mitigating pixel crosstalk. In this formulation, fractional offsets are encoded as independent linear phase terms in the frequency domain, ensuring accurate representation of subpixel shifts. Within the iterative AWGS loop, the amplitude is updated under a device-aware model that combines the ideal central pixel response with subpixel coupling, and this constraint steers the phase estimate toward the true device response while maintaining numerical stability and computational efficiency. It is worth mentioning that a finer 5×5 subdivision did not necessarily improve performance over the 3×3 case, as the energy distributed across much more subcomponents that reduces the amplitude of each individual, making reconstructions more susceptible to phase quantization and numerical noise. At the same time, the proposed method effectively suppressed the zero-order component without degrading overall image quality, thereby addressing one of the key challenges in practical holographic reconstruction.

Overall, the proposed approach contributes to bridging the gap between ideal numerical design and practical hardware implementation. Future work could incorporate additional device-related effects, such as non-uniformity of the SLM, dead space between pixels, and other fabrication-induced imperfections. Moreover, current models of liquid-crystal crosstalk remain approximations. A rigorous treatment of strong fringing-field effects, particularly in the case of rapid spatial variations of the LC director orientation, would require solving the light propagation problem through the full Maxwell equations [89]. With the continual

miniaturization of spatial light modulators, the reduction in pixel pitch is expected to exacerbate fringing-field effects, making accurate modelling and compensation strategies increasingly critical. At the same time, the recent development of subwavelength metasurfaces suggests further opportunities. Metasurface-assisted liquid-crystal SLMs have been shown to achieve high reflectance and polarization-independent operation, thereby extending the functional capability of conventional LCoS [110,111]. In this context, the proposed framework not only remains directly applicable to LC–metasurface systems but also establishes a basis for more advanced device-aware modelling, enabling the inclusion of additional nonidealities and paving the way for the next generation of hybrid holographic platforms.

Chapter 5 Application for 3D Head-Up Displays

In recent years, holographic display technology has progressed from an academic concept into a commercially deployable solution with transformative potential across multiple sectors. Unlike conventional stereoscopic approaches, which rely on binocular disparity and often cause vergence–accommodation conflict leading to visual fatigue and eyestrain, holography reconstructs the complete light field of a scene and thereby provides natural depth perception without physiological strain. This fundamental advantage has accelerated its adoption in applications where immersive visualization is critical. Holographic systems are now being deployed across diverse industries. In medical imaging, they enable volumetric visualization to support diagnostics and surgical planning [44,112]. In marketing and advertising, holography allows interactive product demonstrations that enhance consumer engagement [113]. The consumer electronics sector is also actively integrating holographic modules into AR/VR head-mounted devices, where compact form factors and visual comfort remain essential [114,115]. Within the automotive sector, HUDs have been proposed as a safer alternative to conventional head-down display panels, as they reduce the need for drivers to shift their gaze away from the road. Road traffic accidents cause over 1.25 million deaths and up to 50 million injuries each year, with driver distraction being a major factor [116]. By projecting navigation cues and safety warnings directly onto the windshield, HUDs help maintain continuous visual attention and situational awareness, thereby minimizing driver distraction and reducing the likelihood of accidents. Major automotive manufacturers, including BMW [117], Mercedes-Benz [118], and Jaguar Land Rover [119], have consequently invested substantially in HUD development. BMW offered HUDs in its 5 and 7 series, projecting speed, navigation, and warnings onto the windshield within the driver's field of view [120]. Jaguar Land Rover presents a head-up display with a dynamic depth-variable viewing effect enabled by an electrically tunable liquid lens [121]. In this chapter, we focus on the application of CGH to 3D HUDs. We begin with an overview of the system architecture, followed by the optical design of an LCOS-based setup with key parameters. We then present experimental demonstrations based on the proposed optical architecture, combined with

enhanced CGH algorithms, enables multi-layer depth reproduction while simultaneously addressing critical bottlenecks such as zero-order diffraction and speckle noise, paving the way toward practical holographic HUDs for next-generation automotive displays.

5.1 Overview of System Architecture

HUDs are devices that project virtual images into the driver's forward field of view, a technology that originated in military aviation before being adapted for automobiles. Early systems were dashboard-mounted combiner HUDs, which displayed information on a small transparent screen above the dashboard and offered only limited field of view and fixed image positions. In recent years, commercialized windshield-projected HUDs have become mainstream [122], with the windshield serving as the optical combiner and various optical architectures enabling wider fields of view and improved alignment with the driver's forward vision. Building on this basis, AR HUDs represent the next stage of development, extending the display with a variable virtual image distance (VID) and spatially registering virtual cues with real-world objects. Unlike traditional HUDs, which project content onto a single fixed plane typically a few meters in front of the driver, AR HUDs allow information to be superimposed directly onto the ambient environment, therefore avoid the mismatch between the accommodation distance of the displayed content and real-world objects, thereby reducing visual strain and providing greater contextual relevance by overlaying navigation markers and warnings onto the road scene [123,124]. A typical schematic of such an AR-HUD configuration is illustrated in Figure 5.1.

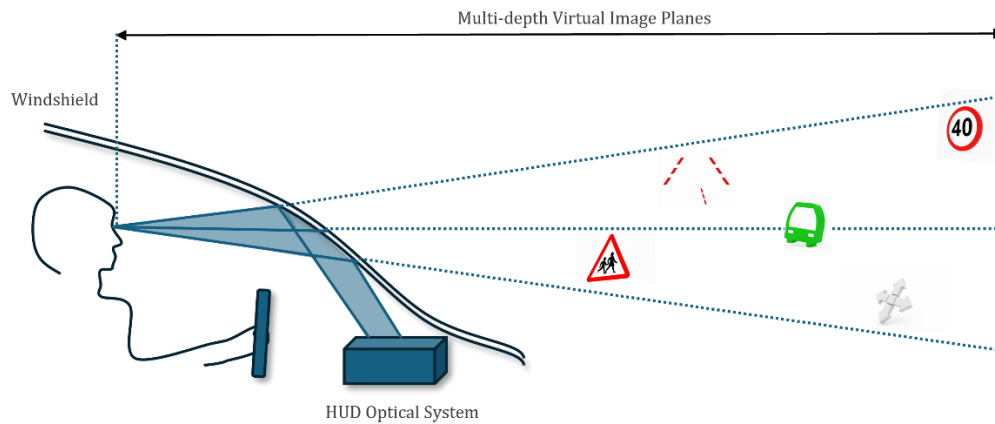


Figure 5.1 Schematic illustration of an AR-HUD optical system. The HUD optical engine projects content through the windshield to form multi-depth virtual image planes, including navigation guidance, speed limits, hazard warnings, and other contextual cues.

Several architectures have been investigated for realizing AR HUDs, each offering distinct advantages and challenges. Waveguide-based AR HUDs leverage diffractive or holographic optical elements to couple light into and out of a waveguide, enabling pupil expansion and compact form factors [125–127]. Binocular parallax-based AR HUDs employ dual light engines and focal paths to generate virtual images at different depths, thereby alleviating the accommodation–vergence conflict and enhancing visual comfort [128,129]. Holographic-based AR HUDs integrate CGH with SLM to achieve precise wavefront shaping, enabling multifocal projection with true phase depth, high spatial resolution, and flexible rendering of three-dimensional content. In addition, compact and lightweight architectures are essential for in-vehicle integration. Holographic systems, relying only on a coherent light source and an SLM, offer a promising route to such miniaturization compared with conventional bulky lens-based designs. Among existing approaches, holographic CGH shows the greatest potential for next-generation AR HUDs, as it provides full wavefront control and represents the most versatile path toward immersive, high-fidelity augmented reality displays.

The optical hardware of a HUD comprises an image generation unit and an optical projection system. The image generation unit employs projectors such as LCD, DLP, or LCOS to synthesize the source image, whereas the projection optics relay it beyond the windshield as a virtual image, incorporating functions of such as magnification and compensation for windshield-induced aberrations. The performance of a HUD optical system is typically

characterized by metrics such as FOV, VID, eyebox size, system form factor, image contrast, the depth of the image plane and so on. These parameters are inherently coupled and optimizing one often entails trade-offs with others.

The FOV denotes the angular extent over which the virtual image can be observed, with typical AR-HUDs offering a horizontal FOV of about 10° [130]. The eye box indicates the area where the user can see the entire FOV clearly without any distortion or blurry effect of the image. A large eyebox ensures comfort and usability. Conventional optical architectures face an inherent trade-off in which expanding the eyebox reduces the FOV as the product of eye box and FOV is proportional to a finite space-bandwidth product of the device. A large eyebox is particularly critical for HUD applications, compared with near-eye displays (NEDs), as it must accommodate the considerable variation in driver eye positions arising from different body heights, an eyebox greater than $90 \times 60 \text{ mm}^2$ is conventionally adopted [130,131]. The finite size and limited maximum diffraction angle of holographic optical elements constrains the eyebox, which becomes narrower as the viewing distance increases. Although enlarging the SLM size or reduce the pixel pitch can alleviate this issue, such approaches lead to challenges including increased form factor, higher fabrication complexity, and degraded optical uniformity. Consequently, a central research direction in holographic AR HUDs is to achieve a wide eyebox and support multiplane imaging without altering the device dimensions. Pupil-tracking techniques are widely investigated to simultaneously enlarge the FOV and eyebox and can be divided into active and passive schemes. Active approaches employ tracking feedback with mechanical adjustment, where exit-pupil replication generates multiple eyebox positions via spatial multiplexing and exit-pupil shifting relocates the eyebox using pupil-shifting HOEs or light-source arrays. However, these methods are constrained by slow response and bulky hardware [132]. Passive approaches, in contrast, replicate the eyebox through time multiplexing without moving parts [133]. For HUDs, diffuser-assisted lens imaging is considered more suitable [12,134], as it effectively enlarges the eyebox through directional scattering. Since the diffraction angle of LCOS devices is inherently small, directly enlarging the eyebox leads to significant design challenges. In our case, based on the equation $\varphi = \sin^{-1}\left(\frac{\lambda}{2p}\right)$ with 520 nm and 4.3 μm pixel pitch, the maximum diffraction angle is about 3.467° . By contrast, incorporating a diffuser enables eyebox expansion without relying on

temporal or spatial multiplexing, where the diffusing angle and optical path length jointly determine the final eyebox width while maintaining road visibility.

Another critical parameter is the VID, defined as the separation between the observer's eye and the perceived virtual image, where a value exceeding 10 m is generally recommended for HUD applications. Ghost images are particularly noticeable when the VID is small, whereas at larger virtual image distances the effect becomes negligible. The combiner allows the driver to perceive both the virtual image and the real-world scene simultaneously, but it also introduces ghost images. This occurs because incident beams reflect from both the first and second surfaces of the windshield at oblique angles, producing secondary rays that do not fully coincide. To mitigate this issue, a polyvinyl butyral (PVB) interlayer is typically laminated within the windshield to suppress double reflections.

Beyond these system-level requirements, further advances are needed in eliminating zero-order leakage, suppressing speckle noise, supporting high-contrast dynamic content, and enabling multifocal projection to ultimately realize real-time, high-fidelity holographic rendering pipelines [135–137].

5.2 Optical Design of LCOS-Based System and Key Parameters

This section introduces the designed optical architecture that fulfils the fundamental requirements of a holographic AR-HUD. The system employs a LCoS spatial light modulator with performance characteristics consistent with those described in Chapter 2. A phase-only CGH is generated based on the algorithms established in the previous chapters, where a layer-based Fresnel hologram is implemented to reconstruct five discrete virtual planes, as illustrated in Figure 5.1. The reconstructed image is first projected onto a diffuser, which is subsequently enlarged and positioned to meet the requirements of eyebox size and field of view. The detailed parameters and calculation procedure are provided in the following discussion and in the Appendix. By dynamically adjusting the position of the diffuser, the VID can be adjusted. The diffuser surface hosts the real image as the object plane, and the perceived image shifts with its position relative to the lens focal length: inside the focal length a magnified virtual image appears nearby, at the focal plane the image moves to infinity, and outside the focal length a

real image forms at the conjugate plane, which may obstruct the driver's view [138]. The scattered light from the diffuser is subsequently collected by a Fresnel lens and redirected toward the windshield, which serves as the combiner. In this configuration, a real reconstructed image is formed at the diffuser plane and optically enlarged by the Fresnel element to appear as a virtual image superimposed on the external road scene, the overall projection optics and variable virtual image arrangement are depicted in Figure 5.2. To reduce the form factor and cost associated with conventional refractive optics, the design adopts a Fresnel lens in place of a traditional spherical lens. Unlike conventional lenses, which are thick and heavy when a large aperture and short focal length are required, a Fresnel lens provides the same optical power in a compact form factor by segmenting the curvature into concentric zones. As illustrated in Figure 5.3, both the spherical and Fresnel lens share the same focal length, yet the Fresnel configuration achieves this with a significantly reduced thickness, approximately one-fifth that of the conventional counterpart, while maintaining a large aperture and low weight.

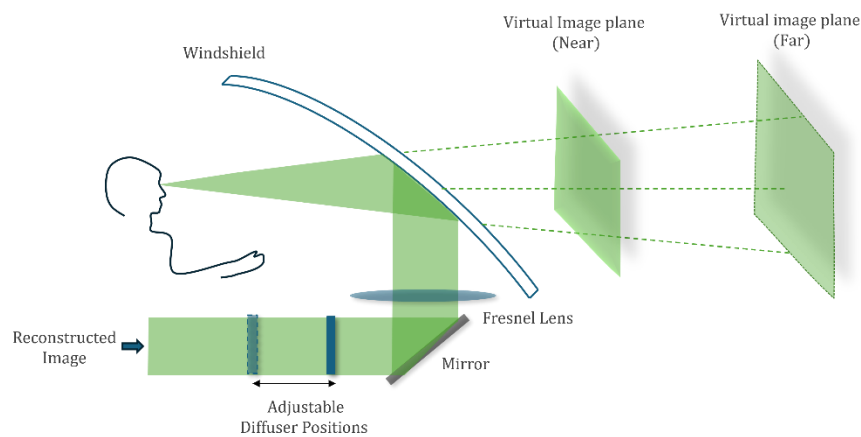


Figure 5.2 Optical layout of the projection relay in the holographic AR-HUD. The diagram highlights the reconstructed image plane, adjustable diffuser positions, Fresnel lens, and resulting virtual image planes at different depths.

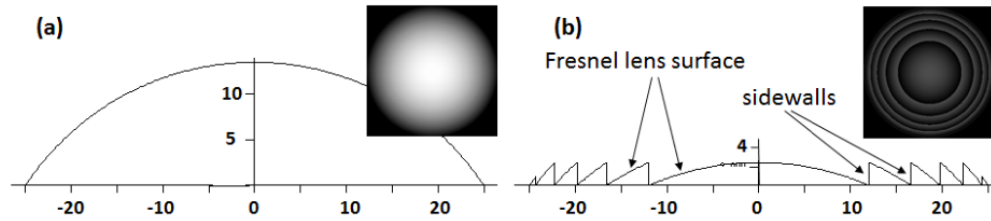


Figure 5.3 Cross-sections and top views of (a) a spherical lens (b) a Fresnel lens [139].

The optical layout of the relay lens system is illustrated in Figure 5.4, which enlarges the holographic reconstruction from the diffuser plane and projects it onto the windshield to form a virtual image at the desired viewing distance. The virtual image plane denotes the perceived image location, which is typically placed several meters away from the windshield. The diffuser plane corresponds to the position where a diffuser is inserted to expand the angular spread of light and thereby define the field of view. In the holographic HUD architecture, the diffuser primarily serves to define and optimise the eyebox. Acting as an intermediate projection screen, it receives the reconstructed real image and scatters the transmitted light into a controlled angular cone. Without a diffuser, the projected rays would propagate in narrow angular directions, and the image could only be perceived when the eye is precisely aligned with this restricted cone. While conventional diffusers such as ground glass provide only limited efficiency and often suffer from non-uniform luminance and reduced resolution, holographic diffusers are specifically engineered to overcome these drawbacks. When a diffuser is placed at the intermediate image plane, it not only expands the angular distribution of light to form the eyebox but also reduces distortion aberration, while delivering uniform luminance across the viewing area, preserving high image resolution, and maintaining contrast even under strong ambient illumination [140–142]. In this way, the diffuser ensures stable visibility within the eyebox while providing the light-emitting plane that is relayed by the Fresnel lens and windshield combiner to form the final virtual image at a distance. Its effective aperture size is denoted by W_d with diffusion angle denote as ω_d . The lens plane represents the physical position of the final Fresnel lens, which collects the diverging rays from the diffuser plane and relays them toward the eye. Downstream, the pupil plane, defines where the

lens forms the exit pupil that coincides with the observer's eyebox. This plane determines the spatial region within which the eye can be positioned while still perceiving the full image.

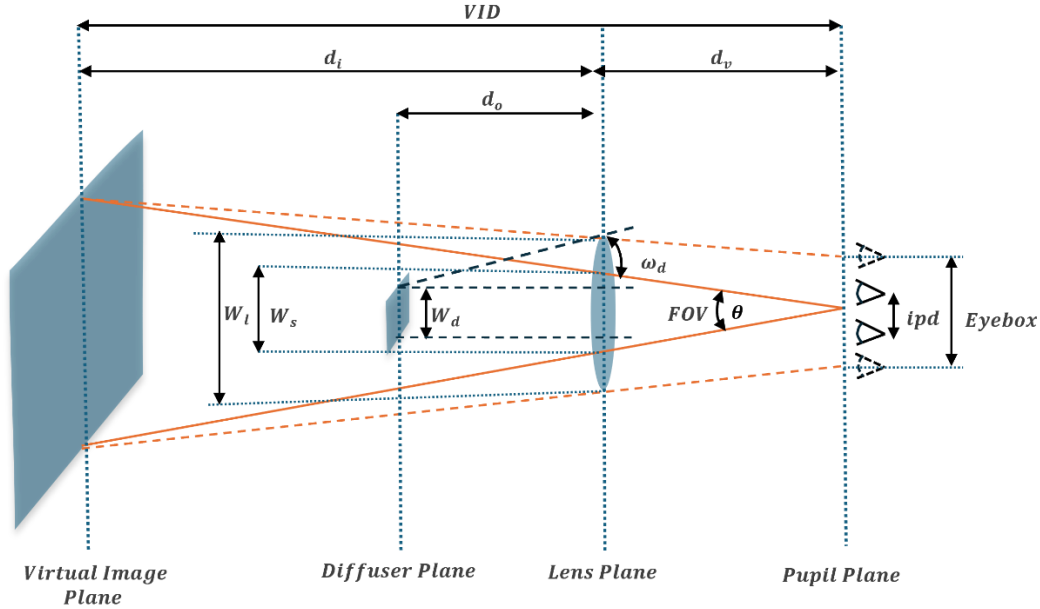


Figure 5.4 Illustration of the designed key optical parameters of relay lens system in the holographic AR-HUD system.

The distances are marked as d_i , d_o and d_v , which represent the distance between the virtual image plane and the lens, between the diffuser and the lens, and between the lens and the pupil plane, respectively. These quantities are related by the thin-lens imaging condition: $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$. The VID specifies the separation between the eyebox and the virtual image plane. The distance d_v corresponds to the optical path from the lens to the eyebox, which is also called eye relief. This value varies across vehicle models due to differences in windshield curvature and the geometric separation between the driver's eye and the windshield. The lens aperture W_l is constrained by the required eyebox size, while the FOV is expressed by the angular θ subtended at the eyebox, as given by Equation (5.2).

The system specifications of the designed holographic AR-HUD are summarized in Table 5-1. The shaded rows in blue correspond to the primary design objectives, namely the target FOV, the eyebox size, and the capability of providing multiple VIDs. These parameters represent the essential requirements for achieving an immersive and comfortable user experience. The

shaded rows in grey represent fixed constraints, where d_v corresponds to the eye relief, defined as the distance from the driver's eye to the windshield, and the focal length f of the Fresnel lens.

Table 5-1 The system specifications of the designed holographic AR-HUD with key parameters.

Specification of Holographic HUD System	
FOV	13° x 5 °
Eyebox	130mm x 50mm
Multiple-VID	3m-15m
Eye Relief (d_v)	800mm
Focal Length of Fresnel Lens (f)	400mm

Based on the specifications in Table 5-1, the magnification factor is first determined according to Equation (5.1), where d_i is obtained by subtracting the eye relief from the desired VID, ranging from 3 m to 15 m in this design. Subsequently, the required diffuser dimension is calculated using Equation (5.2), which incorporates the target FOV, the selected VID, and the corresponding magnification factor. For a single eye, the minimum lens aperture W_s is defined by the eye relief and the horizontal field of view θ , as expressed in Equation (5.3), ensuring that the entire field of view subtended at the eye can be transmitted through the projection optics. When binocular viewing is considered, the aperture must be further enlarged to accommodate the interpupillary distance (IPD) or the designated eyebox size, resulting in the lens width requirement W_l as formulated in Equation (5.4). Finally, an additional key parameter is the diffusion angle of the diffuser, which must be sufficiently large to cover the full clear aperture of the projection lens, as determined by Equation (5.5).

$$M = \frac{f}{f - d_o} = \frac{f - d_i}{f} \quad (5.1)$$

$$FOV = \tan\left(\frac{\theta}{2}\right) = \frac{MW_d}{2 \times VID} \quad (5.2)$$

$$W_s = 2d_v \cdot \tan \frac{\theta}{2} \quad (5.3)$$

$$W_l = W_s + \frac{VID - d_v}{VID} \cdot Eyebbox \quad (5.4)$$

$$\omega_d = \tan \left(\frac{W_l - W_d}{2} \div d_o \right) \quad (5.5)$$

The calculated optical parameters derived from Equations (5.1)-(5.5) are summarized in Table 5-2, while the detailed calculation procedure is provided in the Appendix.

Table 5-2 Calculated optical parameters of the designed holographic AR-HUD system

Specification of Holographic HUD System			
VID (3m)		VID (15m)	
d_o	338.46 mm	d_o	389.04mm
M	6.5	M	36.5
$W_d(H) \times W_d(V)$	105.17mm $\times$ 40.30 mm	$W_d(H) \times W_d(V)$	93.65mm $\times$ 35.89 mm
$W_l(H) \times W_l(V)$	277.63mm $\times$ 106.52mm	$W_l(H) \times W_l(V)$	305.36mm $\times$ 117.19mm
$\max(\omega_d(H), \omega_d(V))$	14.3°	$\max(\omega_d(H), \omega_d(V))$	15.22°

In order to realize a variable VID from 3 m to 15 m, the adjustment is achieved simply by varying the object distance d_o within the projection optics. As summarized in Table 5-2, d_o needs to be shifted from 338.46 mm to 389.04 mm, corresponding to a displacement of 50.6 mm. This minor mechanical adjustment enables a wide tuning range of virtual image depth, allowing the system to flexibly provide multiple VIDs. Therefore, in the subsequent design, a layer-based Fresnel hologram is employed, where adjacent layers are separated by an interval of 10 mm to realize depth-segmented virtual planes. This was further verified through Zemax simulations, which yielded consistent results with the analytical calculations, as illustrated in Figure 5.5.

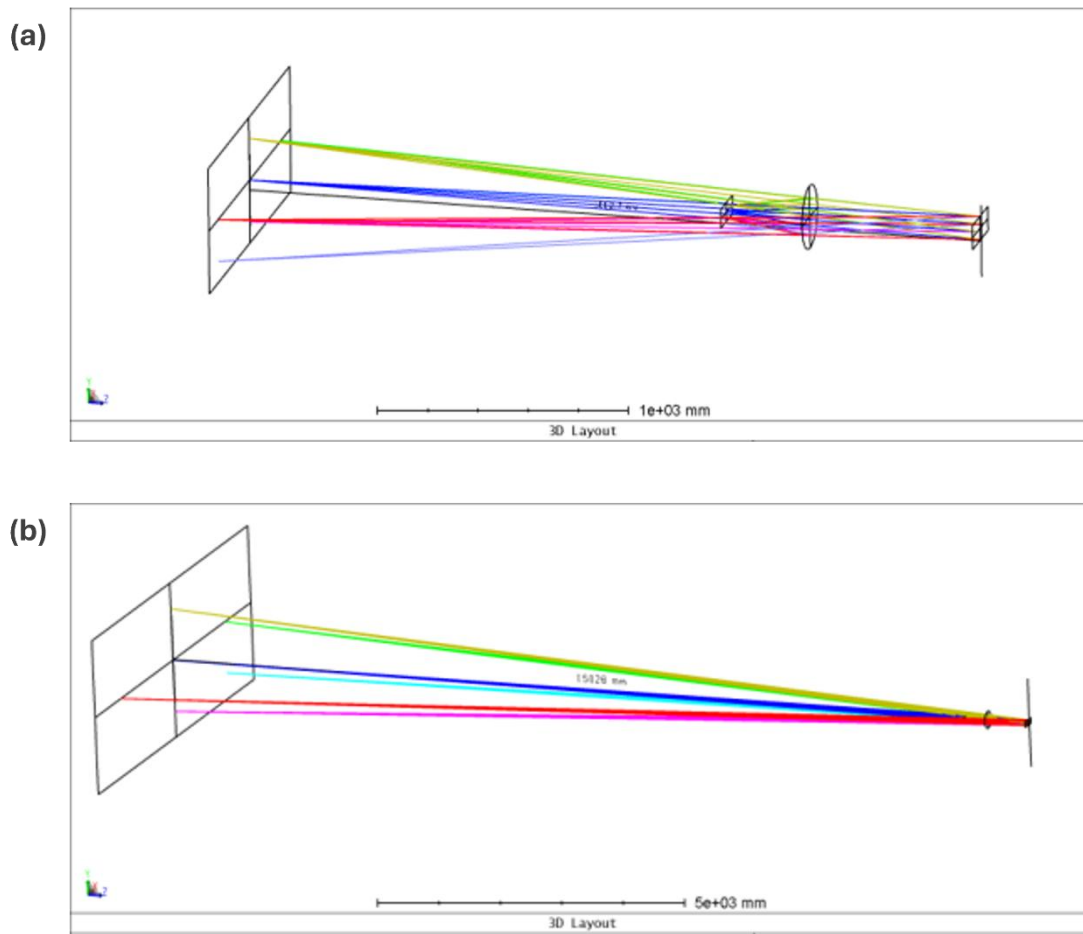


Figure 5.5 Zemax ray-tracing verification of the lens relay system designed holographic AR-HUD at Different Virtual Image Distances: (a) virtual image distance of 3 m and (b) virtual image distance of 15 m.

5.3 Experimental Results

In the previous experiments, it was observed that the reconstructed image quality was primarily degraded by speckle noise and zero-order diffraction. These effects become more pronounced under the long-distance projection and high magnification conditions of the HUD system. Although the insertion of a diffuser helped reduce the speckle noise by introducing spatial and angular diffusion, the residual speckle remained clearly visible and continued to limit the perceived image quality. Therefore, effective strategies for zero-order suppression and speckle noise reduction are essential and have been applied in this work. The specific methods are discussed in detail in this section.

5.3.1 Speckle Noise Reduction

In holographic reconstruction, the use of coherent laser illumination inevitably produces speckle noise, which manifests as a high-frequency random pattern of bright and dark grains originating from interference among scattered wavefronts. This granular noise severely degrades the image quality by reducing contrast and masking fine details. The root cause lies in the high temporal and spatial coherence of lasers: long coherence length enables random path differences to interfere, while high spatial coherence ensures that these interferences extend across the entire field of view. Several approaches have been developed to mitigate speckle. One widely used method is to lower the temporal coherence by employing partially coherent light sources such as LEDs and sLEDs, which shorten the coherence length and thereby reduce speckle contrast. Another effective strategy is to decorrelate speckle patterns temporally or spatially and average them out. For instance, introducing a rotating diffuser in the illumination path continuously changes the phase of the scattered light, generating uncorrelated speckle realizations that are time-averaged by the human eye or camera. Although these techniques reduce speckle, they often introduce trade-offs such as reduced brightness, diminished spatial coherence, or additional optical loss, which must be carefully balanced against image sharpness requirements [44,143–145].

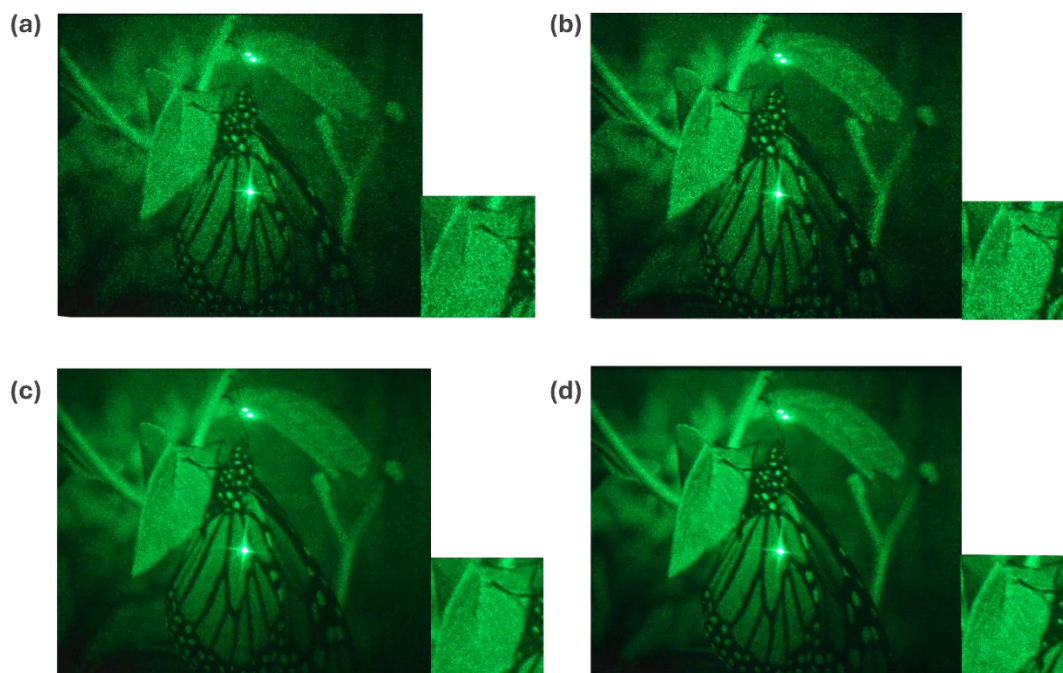


Figure 5.6 A comparison of reconstructed images under different conditions: (a) original result with subpixel phase compensation, (b) original result with subpixel phase compensation and contrast enhancement, (c) speckle-reduced version of (a), and (d) speckle-reduced version of (b). The cropped insets show enlarged regions for visualizing fine details.

To mitigate speckle noise, a temporal averaging strategy was employed. The principle of this method is that by rapidly displaying multiple, independent reconstructions of the same target image, the uncorrelated, high-frequency speckle patterns average out over the integration time of a detector, such as a camera or the human eye. Meanwhile, the desired image signal, which remains constant, is effectively enhanced. For the implementation, fifteen unique holograms of the target image were generated. Each hologram was calculated by modulating the target image with a different, randomly generated initial phase mask. These holograms were then sequentially displayed on the SLM at a refresh rate of 60 Hz. The resulting time-averaged reconstruction effectively suppresses speckle artifacts, leading to a significant improvement in image quality and a reduction in speckle contrast. While standard temporal multiplexing is effective at reducing speckle noise, its averaging nature can inherently degrade image contrast. The results in our case as shown in Figure 5.6 demonstrate that applying speckle reduction in combination with a subpixel phase compensation approach and a contrast enhancement algorithm yields reconstructions with improved visual quality. Compared with the baseline in

(a), panel (b) exhibits a clear increase in contrast, while panels (c) and (d) show a pronounced suppression of speckle, leading to smoother textures and higher image fidelity. In particular, (d) achieves both reduced speckle and enhanced contrast, offering the best overall reconstruction quality. Figure 5.7 also illustrates enhanced speckle suppression using a zero-order removal method, evaluated with a USAF resolution chart. Panels (a) and (b) suffer from severe speckle noise, which obscures fine details and reduces the clarity of the reconstructed image. In contrast, panel (c) after speckle reduction exhibits a more uniform appearance with most of the speckle removed, revealing clearer central structure lines and achieving higher effective resolution that is not affected by noise as confirmed using the USAF resolution chart. With the proposed speckle noise reduction method, the system was able to clearly resolve Group 4 elements, whereas the original result was limited to Group 3, corresponding to an improvement in resolvable spatial frequency from ≈ 11.3 to ≈ 22.6 lp/mm, nearly a two-fold increase in lp/mm. Overall, the resulting holographic reconstruction benefits from a smooth, speckle-free appearance with high diffraction efficiency, without compromising the image contrast. It should be noted, however, that this improvement comes at the expense of a reduced effective frame rate when displaying dynamic holograms.

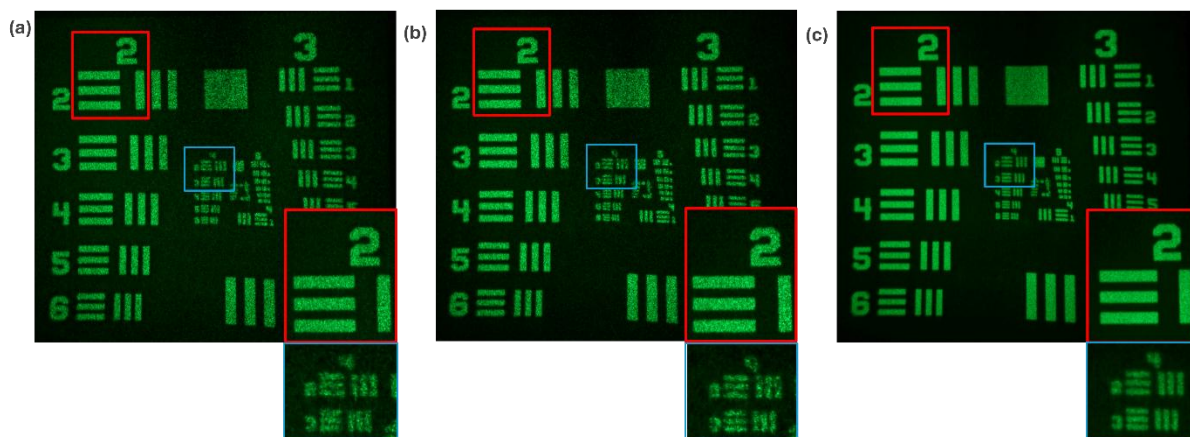


Figure 5.7 Reconstructed USAF target with zero-order reduction method under different conditions: (a) original reconstruction, (b) reconstruction enhanced with the subpixel compensation algorithm, and (c) reconstruction with speckle noise reduction. The red boxes indicate cropped and enlarged regions for visual comparison.

5.3.2 Five-Layer Fresnel Hologram Validation

The subsequent results present the five-layer hologram reconstructions obtained by varying the propagation distance. The layer-based hologram clearly shows that different features come into focus at different depths, confirming the depth-selective reconstruction capability of the method. Figure 5.8 presents the reconstructions formed as real images on the diffuser plane, where panels (a, c, e, g, i) correspond to results before speckle noise reduction and panels (b, d, f, h, j) show the results after reduction. A clear improvement is observed, as the speckle noise is effectively suppressed and the fine details appear sharper and more uniform. In contrast, Figure 5.9 shows the captured virtual images from the windshield. Although the diffuser reduces part of the speckle noise, the residual noise remains noticeable as an inherent property of the laser, further highlighting the importance of effective speckle suppression to enhance the overall image quality.

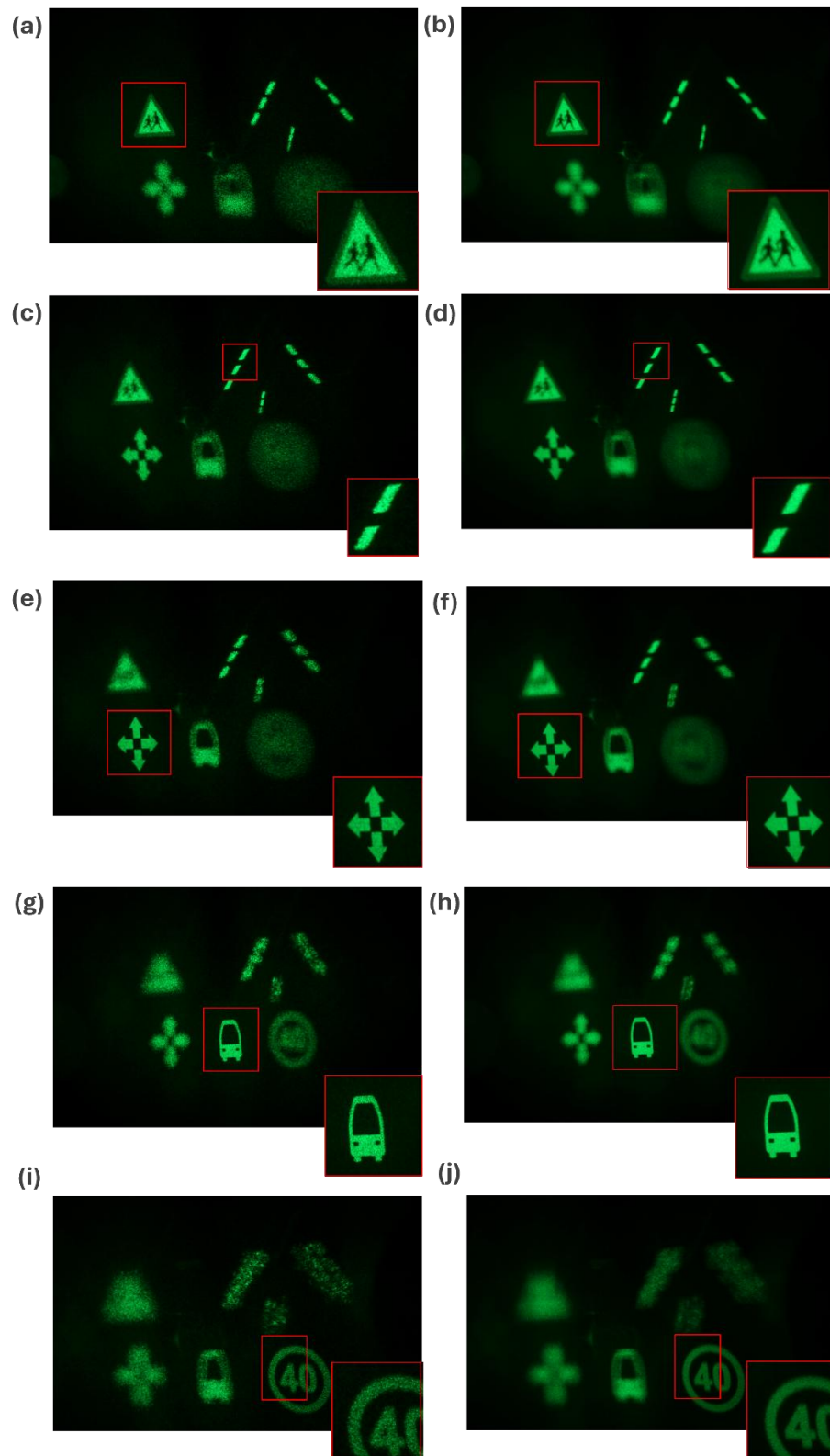


Figure 5.8 Reconstructed images captured on the diffuser plane from a five-layer Fresnel hologram, showing different symbols reconstructed at different depths. (a), (c), (e), (g), (i) correspond to results before speckle

suppression, while (b), (d), (f), (h),(j) show the results after speckle suppression. The red boxes indicate cropped and enlarged regions for visual comparison.

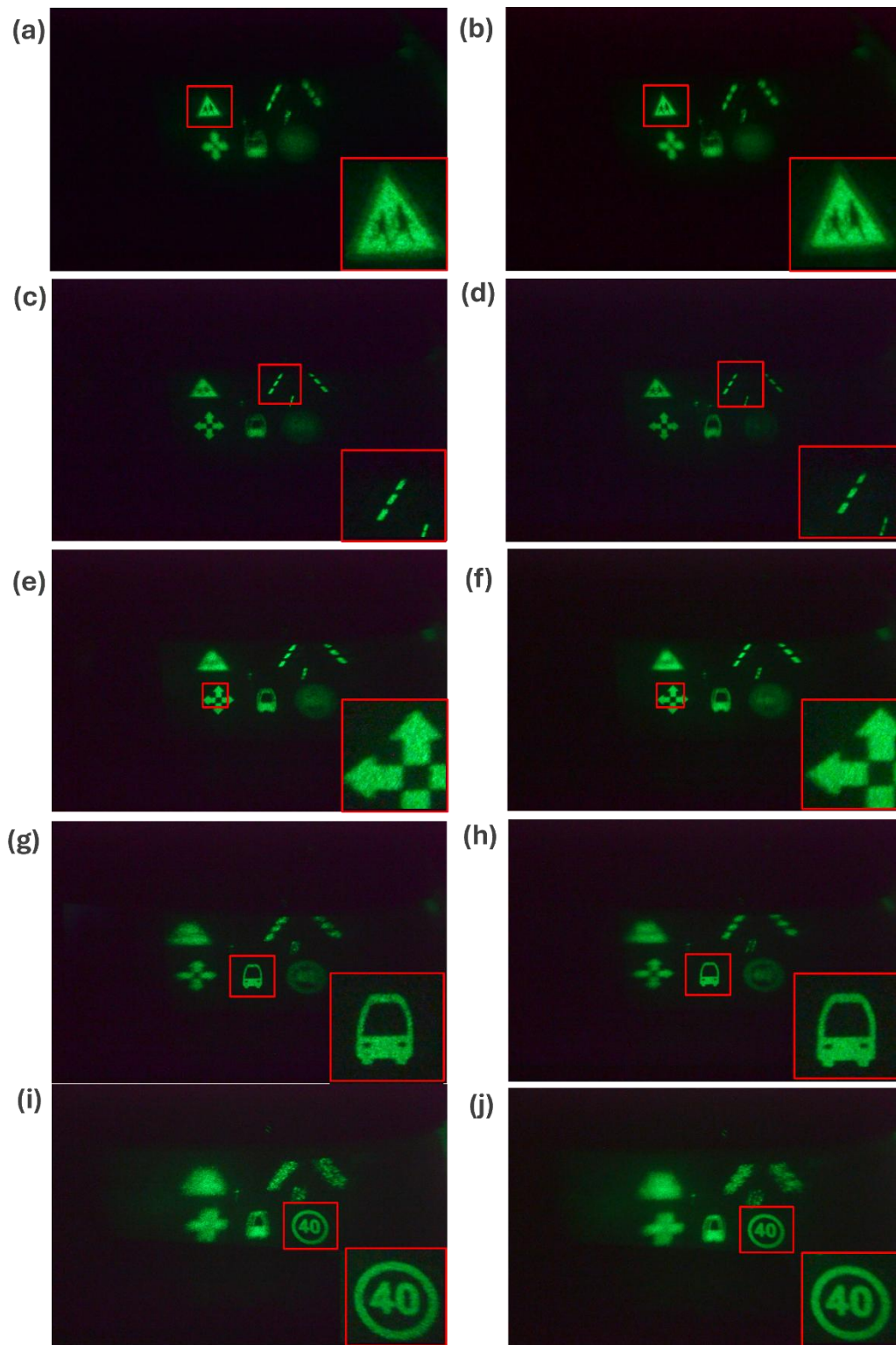


Figure 5.9 Reconstructed images captured on the windshield plane of the car, corresponding to the diffuser-plane results shown in Figure 5.8. (a), (c), (e), (g), (i) correspond to reconstructions before speckle suppression, while panels (b), (d), (f), (h), (j) show the results after speckle suppression. The red boxes indicate cropped and enlarged regions for visual comparison.

5.4 Discussion and Conclusion

Building on the enhancement of CGH algorithms that explicitly account for contrast and subpixel effects, we extended the methodology to a practical implementation by constructing a holographic-based HUD prototype. HUDs represent one of the most promising yet continually evolving application domains for holography, driven by both commercial interest and ergonomic requirements. Based on the key design parameters of an automotive HUD including FOV, eyebox and adjustable VID, we developed an optical architecture that integrates the holographic engine with the relay lens system and a windshield combiner, thereby realizing an augmented reality configuration that achieves conformal registration with the external environment while incorporating depth cues, where the use of a movable diffuser within the optical architecture enables dynamic adjustment of the reconstruction depth across five planes, demonstrating the feasibility of multi-layer holographic HUDs. This approach is consistent with studies showing that dynamic depth modulation enhances visual salience and significantly improves driver performance [146], and provides drivers with a comfortable visualization without inducing visual strain or motion sickness. In addition, the proposed system addresses two of the most critical bottlenecks that have constrained laser-based holographic HUDs from entering commercial deployment, namely zero-order diffraction and speckle noise. By overcoming these limitations within a practical optical platform, the work highlights a viable pathway from algorithmic advances to real-world applications, and points toward the broader adoption of holographic HUDs as next-generation automotive display technologies.

Several directions for future development remain. First, the reliance on temporal averaging to suppress speckle underscores the need for high-refresh-rate SLMs, since current frame rates constrain the achievable averaging capability. Second, the safety aspects of laser-based HUDs require careful trade-offs between optical power, display brightness, and eye safety standards.

Finally, as the windshield in practical vehicles is inherently curved, system-level distortions can be rigorously modelled and pre-compensated using Zernike polynomial fitting [147].

Chapter 6 Conclusion And Future Work

6.1 Conclusions

This thesis has been driven by the need to bridge the gap between numerical simulation and real-world implementation, with the overarching aim of realizing perceptually optimized and high-fidelity holographic displays for next-generation immersive systems. The principal contributions are framed around a systematic reinforcement of the holographic pipeline, achieved through three distinct yet complementary directions: the development of optimized computational algorithms, the incorporation of device-aware modelling to capture hardware nonidealities, and the demonstration of a practical application platform.

The first contribution introduces a non-linear merit function into the iterative GS framework to elevate reconstruction quality. By adaptively redistributing energy through intensity reweighting and dynamic range modulation, the algorithm increases contrast, suppresses speckle-like noise, and accelerates convergence while maintaining luminance. Consistent gains in fidelity were observed, validated across both 2D reconstructions and 3D holography using different propagation models.

The second contribution establishes a device-aware modelling scheme that explicitly incorporates pixel-level crosstalk in liquid-crystal SLMs. By formulating asymmetric kernels and introducing a frequency-domain shift strategy, the framework delivers accurate and computationally efficient subpixel phase compensation while preserving energy balance. The study also explored the physical properties of the fringing-field kernel and implemented a zero-order suppression strategy through the combination of a converging lens and focal-plane obstruction. Together, these advances address one of the key bottlenecks hindering the practical deployment of high-performance holographic displays.

The third contribution lies in the conception and realization of a holographic head-up display prototype that integrates algorithmic enhancements and device-aware compensation within a

system-level framework. The relay lens assembly and windshield combiner were tailored to satisfy critical automotive requirements such as field of view, eyebox, and tunable virtual image distance, while the inclusion of a movable diffuser facilitated multi-plane reconstructions. The resulting architecture validated the feasibility of layered holographic HUDs, effectively suppressing speckle artifacts based on temporal-averaging, and underscored the potential of holography as a practical solution for next-generation automotive display systems.

6.2 Future Works

Looking ahead, a number of important research avenues remain to be explored, and these can be considered in relation to the three main contributions of this thesis. Firstly, in algorithmic enhancement, more advanced parameter-tuning strategies, such as learning-based optimization, could be investigated to improve robustness, efficiency, and scalability. This may include preparing datasets that capture different levels of image complexity and analysing how perceptually sensitive parameters within the nonlinear merit function respond under varying conditions.

Secondly, for device-aware modelling, future work could extend the framework to capture additional hardware nonidealities such as pixel non-uniformity and inter-pixel dead space, or adopt rigorous Maxwell-based treatments to describe strongly varying liquid-crystal director fields. In addition, since the fringing-field effect also depends on parameters such as liquid-crystal layer thickness, these factors could be incorporated into the analytical kernel to enable more accurate characterisation of asymmetric leakage and to further refine compensation strategies.

For the HUD application, several promising directions remain for future exploration. First, the development of full-color holographic HUDs will require algorithms that explicitly address chromatic dispersion, as different wavelengths undergo distinct propagation behaviours, leading to misregistration in RGB reconstructions. Second, an unexplored direction lies in replacing multi-layer holographic reconstructions with a sloped configuration [148,149], which can be regarded as a volume diffuser that continuously introduces depth variation

without mechanical adjustment. While slope-based holography has so far been applied primarily in optical trapping with point sources, extending this concept to image reconstruction presents new challenges, since the inclined geometry is expected to introduce distortions within the hologram plane. Third, the FOV remains a critical factor in creating an immersive HUD experience. Recent advances in ultrahigh-resolution metasurfaces with subnanometer nanorod features [150] highlight a pathway to expand FOV and realize truly photorealistic three-dimensional holographic displays.

Beyond holographic imaging, the high-contrast, noise-suppressed, and zero-order-free reconstructions achieved through the proposed algorithms may also benefit holographic lithography. In such systems, a phase-only LCOS SLM operates as a programmable mask, where computer-generated holograms define the interference fields that expose the photoresist. Accurate control of these interference distributions is critical for fabricating complex three-dimensional micro- and nanostructures. However, the optical reconstructions are often degraded by blur, artifacts, and reduced contrast, partly due to device imperfections and light scattering [151]. The methods developed in this work, including contrast enhancement and subpixel phase compensation, provide a promising pathway to improve exposure fidelity and mitigate such artifacts, thereby enabling more reliable and scalable maskless lithography processes.

Recent demonstrations using incoherent light from smartphone screens have shown the feasibility of compact, low-cost 3D holography free from speckle and zeroth-order noise [152]. Unlike conventional coherent CGH, this approach employs incoherent CGH by treating the smartphone screen as an intensity-modulating first layer and the SLM as a phase-modulating second layer, forming a cascaded holographic system. However, the optical reconstructions still suffer from blur and low contrast. Building on this direction, the methods developed in this thesis could be incorporated to further enhance reconstruction fidelity and contrast, thereby extending the applicability of holographic techniques to emerging mobile platforms.

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Appendix A

A.1 Derivation of Key Optical Parameters in HUD Systems

1. Objective:

- VID = 3000mm
- Eye box = 130mm x 50mm
- FOV = 13° x 5°

Given:

- f = 400mm
- $d_v = 800\text{mm}$
- $d_i = -2200\text{mm}$

Calculation Steps:

- $d_o = \frac{f d_i}{d_i - f} = 338.46 \text{ mm}$
- $M = \frac{f}{f - d_o} = \frac{f - d_i}{f} = 6.5$
- $\text{FOV} = \tan\left(\frac{\theta}{2}\right) = \frac{M W_d}{2 \times \text{VID}}$
 $W_d(H) = \frac{\tan(6.5) \times 2 \times 3000}{6.5} = 105.17 \text{ mm}$
 $W_d(V) = \frac{\tan(2.5) \times 2 \times 3000}{6.5} = 40.30 \text{ mm}$
- $W_1 = 2 \times d_v \times \tan\left(\frac{\theta}{2}\right) + \frac{\text{VID} - d_v}{\text{VID}} \times \text{Eyebox}$
 $W_1(H) = 2 \times 800 \times \tan(6.5) + \frac{2200}{3000} \times 130 = 277.63\text{mm}$
 $W_1(V) = 2 \times 800 \times \tan(2.5) + \frac{2200}{3000} \times 50 = 106.52\text{mm}$
- $\omega_d = \tan\left(\frac{W_1 - W_d}{2} \div d_o\right)$
 $\omega_d(H) = \tan\left(\frac{277.63 - 105.17}{2} \div 338.46\right) = 14.3^\circ$
 $\omega_d(V) = \tan\left(\frac{106.52 - 40.30}{2} \div 338.46\right) = 5.59^\circ$

2. Objective:

- VID = 15000mm
- Eye box = 130mm x 50mm
- FOV = 13° x 5°

Given:

- $f = 400\text{mm}$
- $d_v = 800\text{mm}$
- $d_i = -14200\text{mm}$

Calculation Steps:

- $d_o = \frac{f d_i}{d_i - f} = 389.04\text{mm}$
- $\Delta t = 389.04 - 338.46 = 50.58\text{mm}$
- $M = \frac{f}{f - d_o} = \frac{f - d_i}{f} = 36.5$
- $\text{FOV} = \tan\left(\frac{\theta}{2}\right) = \frac{MW_d}{2 \times \text{VID}}$

$$W_d(H) = \frac{\tan(6.5) \times 2 \times 15000}{36.5} = 93.65 \text{ mm}$$

$$W_d(V) = \frac{\tan(2.5) \times 2 \times 15000}{36.5} = 35.89 \text{ mm}$$

- $W_l = 2 \times d_v \times \tan\left(\frac{\theta}{2}\right) + \frac{\text{VID} - d_v}{\text{VID}} \times \text{Eyebox}$

$$W_l(H) = 2 \times 800 \times \tan(6.5) + \frac{14200}{15000} \times 130 = 305.36\text{mm}$$

$$W_l(V) = 2 \times 800 \times \tan(2.5) + \frac{14200}{15000} \times 50 = 117.19\text{mm}$$

- $\omega_d = \tan\left(\frac{w_3 - h}{2} \div d_o\right)$

$$\omega_d(H) = \tan\left(\frac{305.36 - 93.65}{2} \div 389.04\right) = 15.22^\circ$$

$$\omega_d(V) = \tan\left(\frac{117.19 - 35.89}{2} \div 389.04\right) = 5.97^\circ$$

3. Lens parameters and image magnification for real image on diffuser

$$f_1 = 200\text{mm}, f_2 = 75\text{mm}$$

$$\text{LCOS}(H) = 1920 \times 4.3\mu\text{m} = 8.256 \text{ mm}$$

$$\text{LCOS}(V) = 1080 \times 4.3\mu\text{m} = 4.644\text{mm}$$

$$M = \frac{\text{image size}}{\text{object size}} = -\frac{s_i}{s_o}$$

1) Real Image, $f_2 < s_o < 2f_2$, VID =3000mm

$$M = -\frac{105.17mm}{8.256mm} = -12.74$$

$$M = -\frac{f_2}{s_o - f_2} \quad s_o = f_2 - \frac{f_2}{M}$$

$$s_o = 80.89mm$$

$$s_i = 1030.5mm$$

2) Real Image, $f_2 < s_o < 2f_2$, VID =15000mm

$$M = -\frac{93.65mm}{8.256mm} = -11.34$$

$$M = -\frac{f_2}{s_o - f_2} \quad s_o = f_2 - \frac{f_2}{M}$$

$$s_o = 81.61mm$$

$$s_i = 925.5mm$$

