

Research Article

Pilot-Aided Sequential Monte Carlo Estimation of Phase Distortions and Transmitted Symbols in Multicarrier Systems

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We address the challenging problem of the joint estimation of transmitted symbols and phase distortions in standardized multicarrier systems, including pilot or virtual subcarriers. These subcarriers create time correlation on the useful transmitted OFDM signal that we propose to take into account by an autoregressive model. Because the phase distortions are nonlinear, we set the joint estimation algorithm on the framework of the Sequential Monte Carlo methods. Simulation results are provided in terms of bit error rate (BER) and mean square error (MSE); they highlight the efficiency and the robustness of the estimator.

1. Introduction

Regarding the “Digital Subscriber lines” (DSLs), the “Digital Audio Broadcast” (DAB), the IEEE 802.11 “Wireless Local Area Network” (WLAN), or the IEEE 802.16, most of the recent communications systems are based on orthogonal multicarrier technologies. Unfortunately, such technologies are sensitive to phase noise (PHN) and carrier frequency offset (CFO) coming from the defaults of the oscillators. These phase distortions destroy the orthogonality between subcarriers and lead after Fourier Transform to both a common rotation of all the symbols and intercarrier interferences. In this paper, we propose to take into account the time evolution of the OFDM symbols for the joint estimation of the transmitted symbols, the phase noise and the frequency offset. The Sequential Monte-Carlo-based algorithm we have proposed in [1] for this joint estimation achieves unequaled performance when dealing with orthogonal multicarrier systems without virtual and pilot subcarriers. But, in multicarrier standards, virtual and pilot subcarriers are used, taking up spectral resources and leading time correlation of the useful transmitted OFDM signal. This correlation has not been considered in the algorithm developed in [1],

making it suboptimal in practical cases. As a consequence, we propose here to take into account the correlation of the unknown OFDM signal by an autoregressive model (AR) and to deduce a new dynamic state space model on which the SMC estimation algorithm is built. The rest of the paper is organized as follows. Section 2 is devoted to the formulation of the problem. In Section 3, the estimation of the AR parameters is given followed by the SMC estimator of the a posteriori distribution of the unknown OFDM symbols, the PHN and the CFO. Section 4 presents the performance of our algorithm and compares it to a perfect Common Phase Error (CPE) correction scheme which corresponds to the ideal case of [2]. Section 5 concludes this paper.

In this paper, N , N_{cp} , and T denote, respectively, the number of subcarriers, the cyclic prefix length, and the useful OFDM symbol duration. Let $\mathcal{N}(x; \mu, \Sigma)$ and $\mathcal{N}_c(x; \mu, \Sigma)$ represent, respectively, the real and circularly symmetric complex Gaussian random vectors with mean μ , and let covariance matrices Σ . \mathbf{I}_n and $\mathbf{0}_{n \times m}$ be, respectively, the $n \times n$ identity matrix and the $n \times m$ matrix of zeros. Finally, lower case bold letters are used for column vectors and capital bold letters are used for matrices; $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ denote, respectively, conjugate, transpose, and Hermitian transpose.

2. Problem Formulation

2.1. Equations of the Phase Distortions and of the Received Signal. First, input i.i.d. bits are encoded into M-QAM symbols $d_{n,i}$, where i denotes the i th subcarrier and n denotes the n th OFDM symbol. In standard systems, some of the subcarriers located at the edges of the OFDM block are not modulated. These N_g subcarriers are referred to as virtual subcarriers (VSCs) and belong to the set Ω_g . Moreover, a set of P pilot subcarriers are used. They are located on Ω_p , with $\{\Omega_p \cap \Omega_g\} = \emptyset$ and $\Omega_p \cup \Omega_g = \Omega$.

After the Inverse Discrete Fourier Transform (IDFT), the samples of the transmitted signal can be written, for $l = 0, \dots, N-1$, as follows:

$$s_{n,l} = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} d_{n,i} e^{j2\pi il/N}. \quad (1)$$

Before transmission, a cyclic prefix (CP) of length N_{cp} is introduced to remove intersymbol interference (ISI). By convention, negative time index t is used for the prefix cyclic as follows:

$$s_{n,t} = \begin{cases} s_{n,N+t} & \text{if } -N_{cp} \leq t \leq -1 \\ s_{n-1,t+N+N_{cp}} & \text{if } t < -N_{cp}, \end{cases} \quad (2)$$

where $s_{n,t}$ can be decomposed into a known data signal $k_{n,t}$ and an unknown data signal $u_{n,t}$ so that

$$s_{n,t} = k_{n,t} + u_{n,t} \quad (3)$$

with

$$k_{n,t} = \frac{1}{\sqrt{N}} \sum_{i \in \Omega} d_{n,i} e^{j2\pi it/N}, \quad (4)$$

$$u_{n,t} = \frac{1}{\sqrt{N}} \sum_{\substack{i=0 \\ i \notin \Omega}}^{N-1} d_{n,i} e^{j2\pi it/N}. \quad (5)$$

The resulting signal is transmitted in a time varying frequency selective channel $h(t, \tau)$ which is assumed both to be invariant over one OFDM symbol and to be characterized by L_p independent propagation paths. Imperfect oscillators at the transmitter and at the receiver introduce a phase noise and a carrier frequency offset Δf so that, at the sampling rate N/T of the receiver, the discrete form of the carrier delivered by the noisy oscillator on the t th sample of the n th OFDM symbol is

$$p_{n,t} = \exp(j\phi_{n,t}) \quad \forall t = 0, \dots, N + N_{cp} - 1, \quad (6)$$

where $\phi_{n,t}$ is the phase distortion which includes the Brownian phase noise and the CFO

$$\phi_{n,t} = \begin{cases} \nu_{n,0}, \\ \phi_{n,t-1} + \frac{2\pi\epsilon}{N} + \nu_{n,t} & \forall t = 1, \dots, N + N_{cp} - 1 \end{cases} \quad (7)$$

with ϵ being the normalized CFO. ϵ is a noninformative variable which is uniformly distributed on the support $[(\Delta f T)_{\min}, (\Delta f T)_{\max}]$. $\nu_{n,t}$ is a white Gaussian noise with variance $\sigma_\nu^2 = 2\pi\beta(T/N)$ where β is the bandwidth of the Brownian phase noise normalized with respect to the OFDM symbol rate $1/T$, namely, the parameter βT .

Under matrix notation, the received signal $r_{n,t}$ corrupted by phase noise and frequency offset can be written before removal of the cyclic prefix as follows:

$$r_{n,t} = e^{j\phi_{n,t}} \mathbf{h}_n^T (\mathbf{k}_{n,t} + \mathbf{u}_{n,t}) + w_{n,t}, \quad (8)$$

where

$$\begin{aligned} \mathbf{u}_{n,t} &= [u_{n,t-N_{cp}} \ \cdots \ u_{n,t-N_{cp}-L_p+1} \ \mathbf{0}_{1 \times (N+N_{cp}-t-1)}]^T, \\ \mathbf{k}_{n,t} &= [k_{n,t-N_{cp}} \ \cdots \ k_{n,t-N_{cp}-L_p+1} \ \mathbf{0}_{1 \times (N+N_{cp}-t-1)}]^T, \\ \mathbf{h}_n &= [h_{n,0} \ \cdots \ h_{n,L_p-1} \ \mathbf{0}_{1 \times (N+N_{cp}-1)}]^T. \end{aligned} \quad (9)$$

At the receiver, the cyclic prefix is first discarded, then, assuming $N_{cp} \geq L_p$ and perfect timing synchronization, a Discrete Fourier Transform is performed on the N remaining samples of the received signal. The useful signal is then corrupted by two types of distortions: a multiplicative one which is common to all subcarriers and known as Common Phase Error (CPE) and an additive intercarrier interference (ICI). The larger the phase noise rate βT is, the worse the SINR (signal to noise plus interference ratio) is. Furthermore, on a same bandwidth, a larger number of subcarriers lead to worse system performance due to the shorter subcarrier spacing for a same two-sided 3 dB bandwidth of the phase noise [3].

2.2. Autoregressive Modeling of the Unknown Data Signal Process. Due to the independence of the process between two different OFDM symbols, the process at time t can be modeled via the time domain recursion by the complex AR model of order $(t-1)$ (i.e., AR($t-1$))

$$u_{n,t} = - \sum_{i=1}^{t-1} a_{t,i} u_{n,t-i} + b_{n,t}, \quad (10)$$

where $b_{n,t}$ is a circular white Gaussian noise. The AR model parameters consist of the filter coefficients $\{a_{t,1}, a_{t,2}, \dots, a_{t,t-1}\}$ and the driving noise variance $\sigma_{b_{n,t}}^2$.

They are obtained by solving both the noise variance equation

$$\sigma_{b_{n,t}}^2 = C_{uu}(0) + \mathbf{a}_t^T \mathbf{x}_{n,t} \quad (11)$$

and the Yule-Walker equation

$$C_{n,t} \mathbf{a}_t = -\mathbf{x}_{n,t} \quad (12)$$

with

$$\begin{aligned} \mathbf{a}_t &= [a_{t,1} \ a_{t,2} \ \cdots \ a_{t,t-1}]^T, \\ \mathbf{x}_{n,t} &= [C_{uu}(1) \ C_{uu}(2) \ \cdots \ C_{uu}(t-1)]^T, \\ C_{n,t} &= E[\mathbf{u}_{n,t} \mathbf{u}_{n,t}^H]. \end{aligned} \quad (13)$$

The correlation function $C_{uu}(t-l)$ is obtained by using the cyclostationarity with period N of the process $u_{n,t}$ as follows:

$$C_{uu}(t-l) = \begin{cases} \frac{N-P-N_g}{N} & \text{if } t=l, \\ -\frac{1}{N} \sum_{i \in \Omega} e^{j2\pi i(t-l)/N} & \text{if } t \neq l. \end{cases} \quad (14)$$

This correlation function clearly shows that, for $(t-l) < N$, $u_{n,t}$ is colored. Moreover, in (5), we can denote that the term $u_{n,t}$ is the sum of $(N-P-N_g)$ independent and unit power random variables. Based on the central limit theorem, a rigorous proof given in [4] establishes that the complex envelope of a bandlimited uncoded OFDM signal converges to a Gaussian random process. Consequently, $u_{n,t}$ is modeled by a Gaussian autoregressive process. Using (10), the state equation of the vector $\mathbf{u}_{n,t}$ can finally be written in the matrix form as

$$\mathbf{u}_{n,t} = \mathbf{A}_t \mathbf{u}_{n,t-1} + \mathbf{b}_{n,t}, \quad (15)$$

where the transition matrix \mathbf{A}_t is defined as:

$$\mathbf{A}_t = \begin{bmatrix} & \xi_t^T & \\ \mathbf{I}_{(N+N_{cp}+L-2)} & \mathbf{0}_{(N+N_{cp}+L-2) \times 1} \end{bmatrix} \quad (16)$$

with, according to the cyclostationarity of $u_{n,t}$ and (10),

$$\xi_t = \begin{cases} \begin{bmatrix} \mathbf{a}_t & \mathbf{0}_{1 \times (N+N_{cp}+L-t)} \end{bmatrix}^T & \text{if } 0 \leq t \leq N-1, \\ \begin{bmatrix} \mathbf{0}_{1 \times (N-1)} & \mathbf{1} & \mathbf{0}_{1 \times (N_{cp}+L-1)} \end{bmatrix}^T & \text{if } N \leq t \leq N+N_{cp}-1 \end{cases} \quad (17)$$

and $\mathbf{b}_{n,t}$ is a $(N+N_{cp}+L-1)$ -by-1 zero mean Gaussian noise vector with the covariance matrix

$$E[\mathbf{b}_{n,t} \mathbf{b}_{n,t}^H] = \begin{bmatrix} \sigma_{b_{n,t}}^2 & 0 \\ & \vdots \\ 0 & \cdots & 0 \end{bmatrix}, \quad (18)$$

where, using (11),

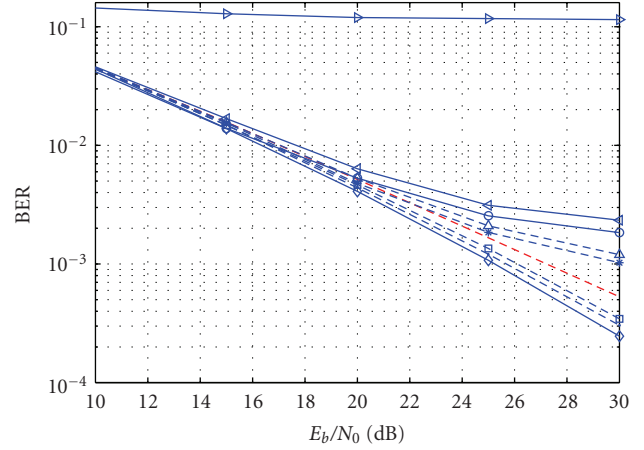
$$\sigma_{b_{n,t}}^2 = \begin{cases} C_{uu}(0) + \sum_{i=1}^{k-1} a_{t,i} C_{uu}(-i) & \text{if } 0 \leq t \leq N-1, \\ 0 & \text{if } N \leq t \leq N+N_{cp}-1. \end{cases} \quad (19)$$

2.3. Dynamic State Space Model. By using the dynamic evolution of the PHN (7) and the proposed AR modeling of the unknown OFDM signal (15), we obtain the following DSS model:

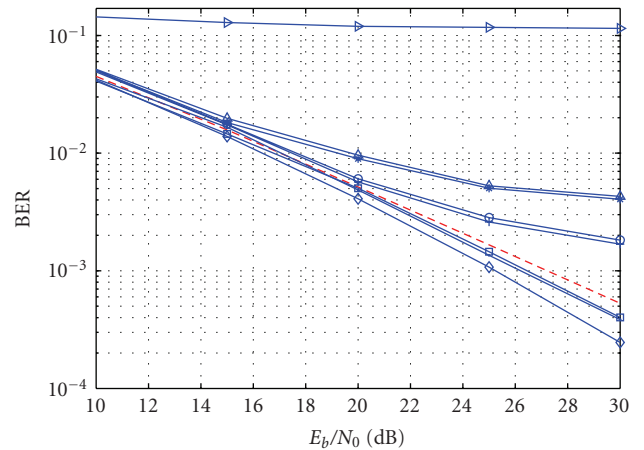
$$\phi_{n,t} = \begin{cases} \nu_{n,0} \\ \phi_{n,t-1} + \frac{2\pi\epsilon}{N} + \nu_{n,t} \quad \forall t = 1, \dots, N+N_{cp}-1, \end{cases}$$

$$\mathbf{u}_{n,t} = \mathbf{A}_t \mathbf{u}_{n,t-1} + \mathbf{b}_{n,t},$$

$$r_{n,t} = e^{j\phi_{n,t}} \mathbf{h}_n^T(\mathbf{k}_{n,t} + \mathbf{u}_{n,t}) + w_{n,t}. \quad (20)$$



(a) With AR model



(b) Without AR model

FIGURE 1: BER performance of the proposed JSCPE-MPF versus E_b/N_0 for different PHN rates βT and numbers of null subcarriers ($P=4, \epsilon=0.4$).

In order to jointly estimate $\phi_{n,t}$, ϵ , and $\mathbf{u}_{n,t}$, we need the joint posterior probability density function (p.d.f.) $p(\phi_{n,t}, \epsilon, \mathbf{u}_{n,t} | r_{n,t})$. Unfortunately, this p.d.f. is analytically intractable, so we propose to numerically approximate $p(\phi_{n,0:t}, \epsilon, \mathbf{u}_{n,0:t} | r_{n,0:t})$ via the SMC methodology [5].

3. SMC Method for Joint OFDM Signal, CFO, and PHN Estimation

The dynamic state space model (20) depends not only on the hidden state process $\phi_{n,t}$, $\mathbf{u}_{n,t}$ and the hidden variable ϵ but also on the parameters of the AR process which have to be evaluated before the estimation of the processes of interest. The first subsection is devoted to the identification of the unknown OFDM symbol by the AR process. Then the SMC-based joint estimation of the OFDM signal, the CFO, and the PHN is deduced.

3.1. AR Parameters of the Unknown OFDM Signal. The correlation of the unknown OFDM signal $u_{n,t}$ can easily be calculated from (14) so that the Yule-Walker equation (12) can be solved efficiently by the Levinson-Durbin recursion. However, as the process $u_{n,t}$ consists of a sum of $(N - P - N_g)$ sinusoids, a necessary condition is that the order of the AR process satisfies $t - 1 \leq (N - P - N_g - 1)$ [6]. With this assumption, the inverse matrix $\mathbf{C}_{n,t}^{-1}$ exists, and the Yule-Walker equation has a unique solution: $\mathbf{a}_t = -\mathbf{C}_{n,t}^{-1}\mathbf{x}_{n,t}$. Nevertheless, as shown in [7], adding a small value τ_0 to its principle diagonal enables the stability and the accuracy of larger-order AR models. This strategy is equivalent to the introduction of white noise of variance τ_0 to the original process. The addition of this spectral bias removes the bandlimitation of the original spectrum and creates a nondeterministic or regular process that in some sense closely approximates the original process. Consequently using this approach, the order of the AR process can be chosen up to $N - 1$ (the largest possible order in our context). The choice of τ_0 thus represents a good tradeoff between the improvement of the AR modeling of $u_{n,t}$ and the bias introduced in the zeroth autocorrelation lag (i.e., in the signal power).

3.2. Joint Estimation of the Phase Distortions and of the Transmitted Symbols. We notice that the DSS model (20) is similar to the one obtained when the multicarrier system has no virtual or pilot subcarriers. Only the matrices \mathbf{A}_t (16) and $\sigma_{b_{n,t}}^2$ (11) differ from the DSS model used in [1] to build the joint SMC estimation without pilot and virtual subcarriers. As a consequence, we do not have details on the JSCPE-MPF (joint signal, CFO, and PHN estimation using marginalized particle filter) algorithm in this paper and suggest that the reader refer to [1].

4. Results

With regard to the system parameters, 16-QAM modulation is assumed and we have chosen $N = 64$ subcarriers with a cyclic prefix of length $N_{cp} = 8$. A Rayleigh frequency selective channel with $L = 4$ paths and a uniform power delay profile, perfectly known by the receiver, has been generated for each multicarrier symbol. In the following simulations, the CFO term ϵ is generated from a uniform distribution in $[-0.4; 0.4]$ for each multicarrier symbol which severely degrades the received signal.

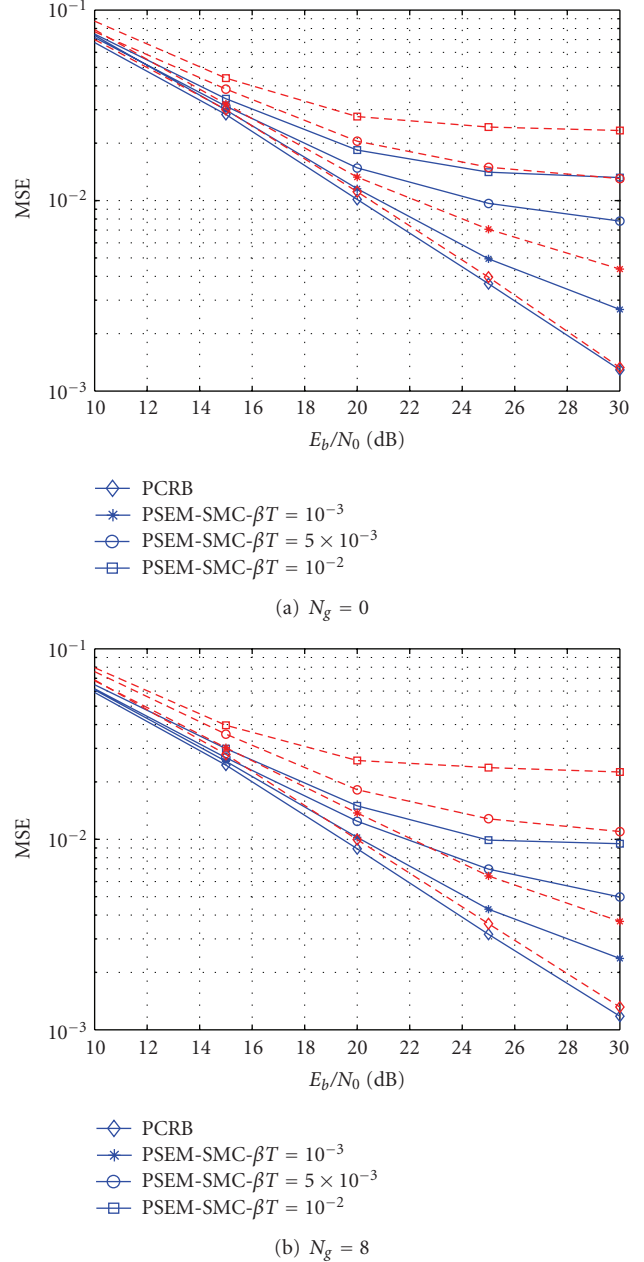


FIGURE 2: MSE of the multicarrier signal estimate using the proposed AR model (solid lines) and without the AR model (dashed lines) versus E_b/N_0 for different PHN rates βT ($P = 4$, $\epsilon = 0.4$).

Figure 1 shows the BER performance of the proposed algorithm for different PHN rates and also different numbers of null subcarriers with and without the AR modeling of the multicarrier signal. The BER performance of a multicarrier system in the absence of phase distortions, using the classical frequency domain MMSE equalizer is also depicted and denoted below by MMSE-FEQ. From this figure, it can be firstly denoted that the perfect CPE correction, which corresponds to the ideal case of [2], achieves unsatisfactory BER performance due to a large ICI induced by the severe phase impairments. Then, we can remark that the proposed

scheme exhibits very good results. Performance of our algorithm increases obviously with the number of null subcarriers, especially for high level of phase distortions. In fact, the proposed AR model of the multicarrier signal gives more prior information if the number of pilots or null subcarriers increases and thus improves the robustness of the proposed estimator. For $\beta T = 10^{-2}$ and $E_b/N_0 = 30$ dB, BER performance of the JSCPE-MPF is, respectively, 4×10^{-3} and 1.8×10^{-3} without and with the proposed AR model. In order to illustrate the gain obtained by the use of the proposed signal model, the MSE of the multicarrier signal obtained using the JSCPE-MPF with or without our AR modeling is shown in Figure 2. The performance of the proposed estimator is compared to the Posterior Cramér-Rao bound (PCRB) of a multicarrier system without phase distortions derived in [1]. The performance gap with and without the AR model really becomes significant when the number of null subcarrier increases. All these results clearly highlight the benefit from the proposed AR model which makes the SMC estimator more robust to severe phase distortions in practical system configurations.

5. Conclusion

In this paper, we address the difficult problem of data detection in pilot-aided multicarrier systems that suffer from the presence of phase noise and carrier frequency offset. The originality of this work consists in an autoregressive modeling of the OFDM signal from which we have deduced an SMC method for time domain processing of the nonlinear received signal. Numerical simulations show that even with significant PHN rates, the JSCPE-MPF achieves good performance in terms of both the phase distortion estimation and BER performance; moreover, it offers a significant performance gain in comparison to existing methods. Thus the JSCPE-MPF algorithm with AR modeling can be efficiently used with the channel estimator proposed in [8] for the design of a complete multicarrier receiver in wireline and wireless communication systems.

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