# (Mis)Allocation Effects of an Overpaid Public Sector*

Tiago Cavalcanti
University of Cambridge, CEPR and
Sao Paulo School of Economics - FGV
tvdvc2@cam.ac.uk

Marcelo Rodrigues dos Santos
Insper
marcelors2@insper.edu.br

**Abstract**

There is a large body of evidence showing that for many countries the structure of wages and pensions and the labor law legislation are different for public and private employees. Such differences affect the occupational choice of agents and might generate some type of misallocation. We develop a life-cycle model with endogenous occupational choice and heterogeneous agents to study the implications of an overpaid public sector. The model is estimated to be consistent with micro and macro evidence for Brazil, a country with a high public sector earnings premium. Our counterfactual exercises demonstrate that public-private earnings premium can generate important allocation effects and sizeable productivity losses. For instance, a reform that would decrease the public-private wage premium from its benchmark value of 19% to 15% and would align the pension of public sector workers with the one in place for private sector workers could increase aggregate output by 11.2% in the long-run without any decrease in the supply of public infrastructure. We provide a decomposition of the aggregate effect into changes in factors accumulation and changes in TFP and implement a welfare distributive analysis.

**Keywords:** Public-Private earnings premium; misallocation; and development.
*J.E.L. codes: E6; H3; J2; O1.*

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*"Programmers and media industry workers had the highest percentage of self-identified **diaosi**, but only fewer than 10 percent of civil servants self-identified as **diaosi**."* Tea Leaf Nation, June 2013.[1]

*"There are many ways of striking it rich in Brazil, but one strategy may come as a particular surprise in today's economic climate: securing a government job."* The New York Times, February 2013.[2]

*"Shaadi.com reported a 30 per cent shift away from IT grooms towards other industries, particularly civil servants and managers at state-owned companies who have higher job security and recently were awarded a large pay rise by the government... Top of the scale are civil servants, those from the ultra-elite Indian Administrative Service and Indian Police Service."* Financial Times, April 2009.[3]

# 1 Introduction

A career in the civil service can be an attractive profession for many individuals. Such a career might be associated with good pay, job stability, and in supplying essential goods and services, such as the rule of law and public infrastructure, for the functioning of any society with direct influence not only in economic outcomes but also in the quality of life of individuals. In many countries a large proportion of the labor force works in the public sector and a large share of output is produced by the government. In an economy in which the public sector is productive and factor inputs are paid according to their marginal productivity in both the public and the private sectors, the presence of a government does not necessarily generate any allocation problems. A larger government can on the contrary increase total factor productivity (TFP) if the provision of public infrastructure is below the optimal level. In many countries the structure of wages and pensions and the labor law legislation are completely different for public and private employees.[4] In general, the

[1] *Diaosi* is a Chinese expression to describe a poor, young and unattractive person in China who cannot afford to buy a house and is unlikely to marry. The Economist (2014) has also featured a piece on the topic showing that Chinese workers who least identified themselves as *Dioasi* were civil servants.

[2] This piece from the The New York Times (2013) shows anecdotal evidence on how several public employees have exceeded constitutional limits to their pay, which is roughly US$ 13,000.00. For instance, a clerk at a court in Brazil's capital, Brasília, was making more than the chief justice of the nation's Supreme Court. Other examples are of an auditor in Minas Gerais state who earned US$81,000 in one month and a librarian who got US$24,000 per month.

[3] In this report, the Financial Times (2009) shows that in the marriage market in India, public servant grooms are the most demanded, and this is explained by their relatively high income and job stability.

[4] Section 2 summarizes the empirical evidence on the public-private institutional and earnings differences.

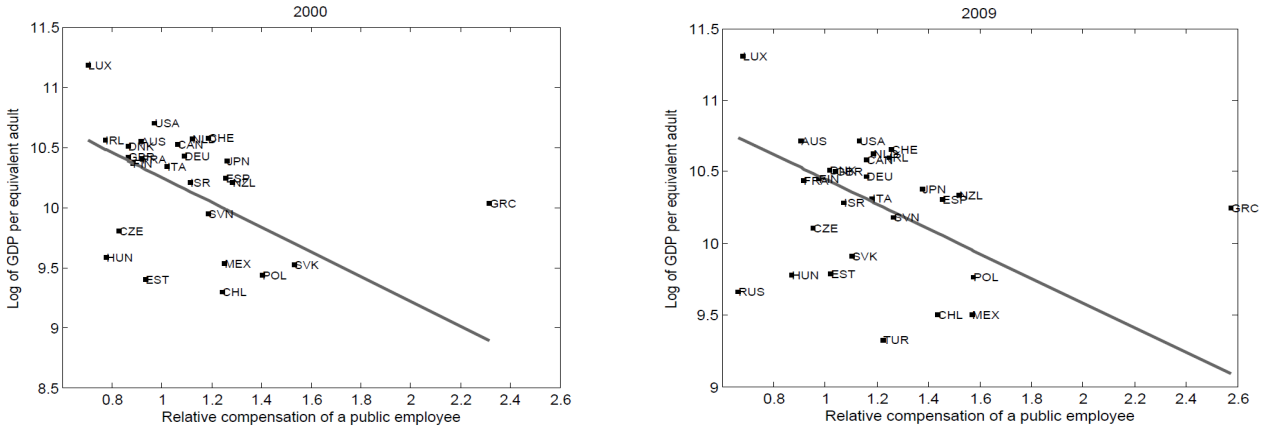


Figure 1: **Relative compensation of a public employee and GDP per adult.** *Left graph:* Data for 2000. *Right graph:* Data for 2009. *Source:* Government at a glance and Penn World Tables. The relative compensation of a public employee equals the total compensation of general government employees as a share of GDP divided by employment in general government and public corporations as a share of the labor force.

evidence for some countries suggests that even controlling for observable characteristics, public employees are on average better paid, have a more protected job and higher pension than private workers. This paper investigates whether or not differences in labor compensation (e.g. wages and pensions) and labor legislation (e.g. job security) between private and public workers affect individuals' occupational choice, investment and aggregate productivity of an economy.

Figure 1 (see also Table 6 in Appendix A with regression results) presents the relationship between compensation of a public employee relative to the income per worker of a country versus the logarithm of the level of output per adult for selected countries in 2000 (left graph) and 2009 (right graph). The best linear fit is shown by a solid line in each graph. Although the correlation in this figure does not imply a causal effect, we can observe that there is a negative relationship between the relative compensation of a public employee and the level of output per equivalent adult in a sample of basically OECD countries.[5] In addition, there is no correlation between the level of output per adult and the share of the labor force in the public sector and total compensation of public employees as a share of GDP.[6] The only negative correlation is on the relative compensation of a public employee and the output per adult person.

Differences in earnings, compensation and labor legislation between public and private workers affect the occupational decision of agents. The public sector might attract highly

[5]The pooled regression with year fixed effects yields stronger and more precise correlation. See Table 6 in Appendix A.

[6]See the last column of Table 6 in Appendix A.

productive agents looking for a more stable and higher paid job, creating a public sector job queue and crowding out private sector employment and entrepreneurship. This can lead to a negative effect on productivity and on the investment decision of agents. Therefore, countries might be able to increase their productivity in the short-run and long-run (if human capital and financial assets accumulation depend on occupational choices) by decreasing the public earnings premium and by reforming their labor legislation.

In order to guide our assessment, we construct an equilibrium model with endogenous occupational choice. Firstly, we fix ideas in Section 3 by presenting the static version of the model and deriving some analytical results. Although the static model is a useful device to understand the mechanisms of how the public sector earnings premium affects productivity, it is not an appropriate framework for quantitative analysis since investment decisions can depend on occupational choice and vice-versa. Then in Section 4 we consider a standard life-cycle model in which agents choose in each point in time whether to work in the public or private sectors or to be an entrepreneur. When born, agents draw from an invariant distribution two types of abilities: an ability to run a business and a labor market productivity. These two abilities evolve over time depending on the agents' occupational choice in a learning-by-doing and on-the-job training manner. In the same spirit of Mortensen and Pissarides (1994), there is a searching cost to apply to a public sector job and the probability of finding such a job is endogenous, depending on the public sector vacancies and number of agents searching for such a job.

At any point in time, there are two types of misallocation. Firstly, the existence of a public earnings premium influences the occupational decision of agents, generating misallocation in the extensive margin. The main idea is that such premium might attract some relative high productive entrepreneurs who would not apply for a job in the public sector if a public premium did not exist. In addition, when the public sector is large, then there will be a wedge between the earnings and the marginal productivity of public sector workers causing misallocation in the intensive margin. Finally, in the long-run, the economy might also be affected since investment decisions depend on the occupation of agents.

We calibrate and estimate the model to be consistent with key micro and macro statistics of the Brazilian economy; Brazil is an interesting case since it has a large public-private earnings premium (see Section 2). Then we perform counterfactual exercises by changing the wage premium in the public sector and by reforming the social security system such that pensions of civil servants are similar to pensions of private workers in the country.[7]

[7] We also investigate aggregate effects of changes in the stability of public sector jobs.

We keep the share of public employees constant by increasing the probability of a successful public job application. We show that a decrease in the public wage premium from 19 percent to 15 percent can produce a sizeable positive effect on long-run aggregate output (4.7 percent increase) without any significant decrease in public infrastructure. We show that factors reallocation accounts for about 19 percent of the aggregate change in output. The rest is due to changes in the accumulation of factors of production.[8] Pension reforms can also have similar aggregate effects, but such reforms change more the incentive of agents to invest in financial assets and most of the impact on output is due to changes in factors accumulation. Changes in the legislation law which decrease job stability in the public sector produce also important quantitative effects on aggregate output and productivity. We also provide a distributive welfare analysis. Welfare is measured by the ex-ante expected (with respect to idiosyncratic shocks) lifetime utility of a newborn in a stationary equilibrium. We show that all individuals would in the long-run be better-off with a policy reform that would reduce the public sector earnings premium but welfare gains are larger at the lower tail of the ability distribution. Poor individuals benefit less from the generosity of the public sector compensation since there is a lump-sum cost to apply for such a job. Although we use Brazil as a benchmark, we believe that our model could be easily applied to many different countries.

There are a number of theoretical reasons explaining why earnings differentials between the private and public sector exist. For instance, it might be the case that the bargaining power of public and private workers are different resulting in a wage differential between the two sectors (cf., Oswald, Grout, and Ulph, 1984; Holmlund, 1993; Fernández-de-Córdoba, Pérez, and Torres, 2012). The difference in bargaining power by public and private workers might be explained by historical reasons, or by the nature of economic activities in the public sector. In fact, in most countries unionization is larger in the public sector than in the private sector (cf., Gregory and Borland, 1999). Wang (2018) uses a political economy argument to show how a strong elite running the government can transfer rents to public sector workers to buy their support. It is not our goal to investigate why such earnings differentials exist and why some legislations protect more public than private employees. We take these wedges in earnings and benefits from the data and calculate their effects on investment and the allocation of talents in the economy. Note that

[8]The intuition of many readers when thinking about the effect of the wage premium on efficiency is probably that a reduction in the premium should unambiguously improve efficiency. However, this not be the case since our model contains several deviations from a frictionless setup such as public goods, the public sector wage premium, barriers to inter-sectoral labor mobility and we know that in a setting with multiple distortions, removing only one distortion may in fact lead to a worse outcome.

we abstract from any potential benefits of a public sector earnings premium. For instance, according to the efficiency wage theory, a higher wage may help to prevent civil servants from shirking, striking or becoming corrupt. This is why in the quantitative exercises we do not eliminate completely the wage premium and focus the analysis on the case in which the wage premium is reduced from 19 percent to 15 percent.[9]

This article is related to a broad literature which investigates the underlying causes of low economic development and productivity in some countries. The existing literature suggests that both factor accumulation and TFP play key roles in explaining income levels across countries.[10] Human capital and physical capital accumulation depend on the return of such investments, which in turn depend on incentives and government policies of a society. TFP, on the other hand, might vary for two main reasons: First, because countries can either use different technologies in the production of goods and services (cf., Parente and Prescott, 2000) or use similar technologies differently (cf., Acemoglu and Zilibotti, 2001); and secondly because countries do not allocate inputs efficiently. In this second case, factor inputs are misallocated and factor reallocation from less to more productive establishments or jobs could potentially increase output. Hsieh and Klenow (2009), in an influential study, show that there is greater dispersion of productivity in India and China than in the United States and factors reallocation could increase TFP in China by 30-50% and in India by 40-60%. Their goal is to measure the size of misallocation, but they, however, do not explain why it exists. See also Restuccia and Rogerson (2008) who show the potential effects of a misallocation in a growth model with heterogeneous firms.

Economists have studied different causes of why inputs of production are not allocated in the most productive manner such that the marginal productivity of input factors are equalized across different firms. Some of the theories to explain such misallocation are based on credit market imperfections and frictions,[11] taxes and regulations,[12] among others.[13] Restuccia and Rogerson (2013) provide an overview of this literature. Our article

[9] In fact, we find that the value of the public wage premium that locally maximizes aggregate output is nearly 15%, where "locally" refers to a situation where only the wage premium is changed.

[10] For a survey of this literature see Caselli (2005) and Hsieh and Klenow (2010). Caselli (2005) convincingly argues that observed differences in the factors employed in production cannot explain most of the cross-country variation in income. He also suggests that disaggregating the government sector out of the data may potentially reduce the unexplained component of GDP.

[11] See Amaral and Quintin (2010), Antunes, Cavalcanti, and Villamil (2008), Buera and Shin (2013) and Midrigan and Xu (2014). Using data from Brazil Allub and Erosa (2016) studiy the distributional effects of financial frictions and show that employers might have a vested interested in an underdeveloped credit market.

[12] See, for instance, Antunes and Cavalcanti (2007) and Guner, Ventura, and Yi (2008).

[13] Caselli and Gennaioli (2013) study the misallocation effects of dynastic family business. Their main idea

differs from this existing literature in an important way since (to our knowledge) none of the articles study the misallocation effects of the public-private earnings and institutional gaps.[14]

Related to our work is the article by Hsieh, Hurst, Jones, and Klenow (2013), who use a Roy model to investigate the aggregate productivity gains in the United States that can be attributed to decreases in labor market racial and gender discrimination. They show that 15 to 20 percent of growth in output per worker in the United States from 1960 to 2008 can be explained by the decrease in the racial and gender wage gaps. The focus and question of our work are different from theirs.[15] Song, Storesletten, and Zilibotti (2011) build a model of the Chinese economic transition in which state-owned firms have better access to outside finance than private firms, even though the latter are more productive. Chinese growth is therefore in part explained by factors reallocation from state-owned to private firms and by the reduction in capital misallocation.[16] In our model, the inefficiency is generated by a large public sector and the public earnings premium. Therefore, our results are complementary to theirs. Gomes and Kuehn (2017) investigate how differences in educational endowments and public employment account for differences in the average firm size and productivity between the United States and Mexico. Although related to our paper, the question we study is different to theirs as well as the modelling approach. Close to our ideas is a contribution by Jaimovich and Rud (2014) who study the effects of an oversized and inefficient public sector on economic performance through an endogenous occupational choice model. Their analysis is qualitative while ours is quantitative. They show that when the public sector attracts bureaucrats with a low degree of public

is that if the offspring of the family firm lacks managerial talent, then dynastic management is a failure of meritocracy that reduces a firm's TFP. David, Hopenhayn, and Venkateswaran (2014) study the role of information in misallocation.

[14] Hörner, Ngai, and Olivetti (2007) study the role of state enterprise in the rise of European unemployment since the late 1970s. Quadrini and Trigari (2007) investigate the role of the public sector on the volatility of employment and output in a matching model of the labor market. Gomes (2015) studies the role of public sector wage policies in shaping unemployment levels and business cycle fluctuations.

[15] Glomm, Jung, and Tran (2009) assess the effects of the Brazilian public sector pensions on capital accumulation and income. We differ from them in several dimensions. First, we focus not only on pensions, but also consider the effects of the public-private wage gap and other institutional differences between the two sectors. We have endogenous occupational choice and therefore we are able to capture not only the effects of the public-private earnings gap on investment but also on extensive occupational margin. Reis and Zilberman (2014) investigate the role of insurance in public employment. They show that if the share of public employment decreases, overall welfare might decrease due to general equilibrium effects, but there are welfare losses coming from a lower degree of insurance.

[16] Wang (2018) studies the Chinese transition using a political-economy framework in which the elite provides a high enough income for state sector workers to buy their support. In transition he investigates how resources are transferred from the state to the private sector.

service motivation, they will use their position to rent seek by employing an excessive number of unskilled workers and leading to an equilibrium with relatively high unskilled wages, which decreases profits and entrepreneurship. Given the focus of their question and in order to have an analytical solution, they have to simplify their analysis by for instance considering a static model and assuming exogenous levels of skills (skilled and unskilled). Our goal is to assess quantitatively the implications of earnings differentials in public and private sectors and therefore we assume endogenous skill formation and a dynamic environment.

The remainder of the paper is organized as follows. The facts are presented in Section 2. Section 4 presents the model economy. Section 5 calibrates and estimates model parameters and provides the quantitative analysis to measure the aggregate effects of public-private earnings and the institutional gap. Section 6 contains concluding remarks.

## 2 Facts

In this section we document the main facts which motivate this study and provide empirical support to some of our modelling strategies. They are described below.

**Fact 1: Public-private wage gap.** In most countries there exists a public-private wage gap. This is not a recent phenomenon. Piketty (2014) shows that in France during the time of Napoleon to World War I there was a small number of very well paid civil servants earning 50-100 times the average wage in the period, such that they could afford to live with "*dignity and elegance as the wealth heirs*". The empirical evidence suggests that in some countries a large fraction of this wage gap is not explained by differences in observed characteristics, such as education and experience. Depalo, Giordano, and Papapetrou (2015) studied the public-private wage gap for ten European countries. For some countries such as Portugal and Spain the raw average wage gap between public and private employees is 43 and 36 percent, respectively. In Portugal about half of this gap is not explained by differences in observable characteristics, while in Spain the unexplained public-private wage gap is about 2/3 of the total raw wage gap. A similar pattern is observed in Turkey (cf., Tansel, 2005), Britain (cf., Postel-Vinay and Turon, 2007), among others. Table 8 in Appendix A shows that estimates of the public wage premium in Brazil varies from 19 to 25 percent depending on the specification and this premium does not seem to be driven by unobserved variables. Most studies show that the public-private wage gap decreases for the upper tail of the conditional wage distribution and there is less dispersion in in-

come inequality in the public sector. We show that the dispersion of wages in the public sector in Brazil is similar to the level observed for private sector workers (see Table 7 in Appendix A) and the public sector wage premium is quite flat for different quintiles of the conditional wage distribution (see Figure 7 in Appendix A).

**Fact 2: Public-private pension gap.** Not only does there exist a public-private wage gap but in some cases the pension system is also different between private sector and public sector workers. As described in a report from The Economist[17] on July $27^{th}$ 2013, in America most public-sector workers can expect a pension linked to their final salary. This is not a common practice in the private sector in which only 20 percent of private-sector workers benefit from such a scheme. As The Economist points out when analysing the pension system in the United States, in general in America"*the typical public-sector worker gets a pretty good deal by private-sector standards*".[18] A similar pattern is also observed in Britain where pensions in the public sector are based on the final-salary (*defined-benefit* plan) and the pension of private sector workers rely on *defined-contribution* schemes, which is based on how much the employee and employer contributed and on the return of pension funds.[19] Brazil also has a very unequal pension system, which is divided into two main schemes: a general regime for private sector workers and a special regime for civil servants. The scheme available to private sector workers consists of a mandatory publicly managed transfer system which covers all private workers up to a ceiling of approximately US$ 1,800. Based on microdata from the private sector social security system, Afonso (2016) reports that the average replacement rate under this regime is 0.82. In the public sector, workers can retire with full salary if they are 65 years old and contributed to the pension system for 35 years. The replacement rate for civil servants is equal to one, which is consistent with the fact that they receive their last salary as benefits.[20]

**Fact 3: Public-private institutional gap.** The OECD (2011) report shows that many countries have labor legislation which translates into more secure jobs in the public than in the private sector. According to Piketty (2014) civil servants in the Great Depression were im-

[17] See The Economist, July $27^{th}$, *Who Pays the Bill?*

[18] Beshears, Choi, Laibson, and Madrian (2011) describe the pension system of the states and the largest cities and counties in the US. Although, they report substantial heterogeneity across jurisdictions, they show that unlike in the private sector, defined-benefit pensions are still the norm in the public sector.

[19] Article by Queisser, Whitehouse, and Whiteford (2008) describes different features of the pension system in OECD countries.

[20] The government has changed this for the new public employees. Those who now get a public sector job will have to contribute more in order to receive the full pay under retirement. Current public sector workers still face the previous pension scheme.

mune from the risk of unemployment and some enjoyed an increase in their real wages. Clark and Postel-Vinay (2009) construct indicators of the perception of job security for 12 European countries and find that after controlling for selection into jobs, workers feel most secure in permanent public sector jobs, which are perceived to be by and large insulated from labor market fluctuations.[21] In Brazil workers in the public sector are guaranteed life tenure after a three-year probation period and since there are no performance evaluation mechanisms in the public sector, rarely a public employee is not awarded tenure.

The economic environment to discipline our analysis is described below.

# 3 A Stylized Model

We first present a simplified version of the model to provide key intuition of our results. The full model is introduced in Section 4. We consider a static occupational model without capital so that we can easily aggregate individuals as in Lucas (1978). The economy is inhabited by a continuum of agents of measure one who live for one period. Each individual is endowed with efficiency units of labor $h_w$, and with entrepreneurial ability $h_e$, which corresponds to her capacity to employ labor, $n$, in order to produce a single consumption good, $y$. Productivity levels $h_w$ and $h_e$ follow continuous cumulative probability distributions $F_w(h_w)$ and $F_e(h_e)$, respectively. Therefore, agents are heterogeneous in the pair $(h_w, h_e)$. The production technology is represented by

$$y = G^{\chi} h_e^{1-v} n^{v(1-\varphi)},\ v, \varphi \in (0,1),\ \chi \geq 0, \tag{1}$$

where $v$ is the span-of-control parameter, and $1-\varphi$ determines the importance of labor in production. $G$ corresponds to public goods and services, such as toll free roads and the rule of law, which are made available to all firms at a zero cost. Entrepreneurs can operate only one establishment. Let $w$ be the wage rate. The problem of an entrepreneur with managerial ability $h_e$ is to choose labor, $n$, to maximize:

$$\pi(h_e; w) = \max_{n \geq 0} G^{\chi} h_e^{1-v} n^{v(1-\varphi)} - wn. \tag{2}$$

[21] Using a large-scale reform which decreased job stability in state-owned enterprises (SOEs) but not for government employees in China in the late 1990s, He, Huang, Liu, and Zhu (2018) show significant evidence of precautionary saving stemming from sudden increases in unemployment risk for SOE workers relative to that for government employees.

This problem gives labor demand and output for each entrepreneur:

$$n(h_e;w) = \left(\frac{v(1-\varphi)}{w}G^{\chi}\right)^{\frac{1}{1-v(1-\varphi)}} h_e^{\frac{1-v}{1-v(1-\varphi)}} \text{ and } y(h_e;w) = G^{\chi}h_e^{1-v}n(h_e;w)^{v(1-\varphi)}. \quad (3)$$

The profit function of each entrepreneur is given by:

$$\pi(h_e;w) = (1-v(1-\varphi))y(h_e;w). \quad (4)$$

The public good, $G$, is produced by the government using labor, $N_g$, such that:[22]

$$G = A_g N_g^{1-\alpha},\ \alpha \in (0,1), \text{ and } A_g > 0. \quad (5)$$

Individuals choose a career in order to maximize income. An individual receives gross income $\pi(h_e;w)$ if she becomes an entrepreneur, and $wh_w$ if she becomes a worker. They can also be a civil servant. We assume that the public sector wage rate $w_g$ and the size of the public sector are exogenously determined. We also assume that there is a wage premium to work in the public sector, such that civil servants with labor productivity $h_w$ receive $(1+\zeta)wh_w$, where $\zeta \geq 0$. The public sector is financed by a consumption tax, $\tau_c$.

Let $c$ be consumption of an individual whose preferences are represented by a function $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ with $\gamma > 0$. Individuals maximize utility subject to the constraint that $(1+\tau_c)c \leq \tilde{y}$, where $\tilde{y}$ corresponds to the income of each household, which depends on the agent career choice, and is characterized by the following Lemma.

**Lemma 1.** *For each $w > 0$, $\zeta \geq 0$, and $h_w > 0$, there exists an entrepreneurial ability $\bar{h}_e(h_w, w;\zeta)$, given by*

$$\bar{h}_e(h_w,w;\zeta) = \left[\frac{(1+\zeta)^{1-v(1-\varphi)}wh_w^{1-v(1-\varphi)}}{(v(1-\varphi))^{v(1-\varphi)}(1-v(1-\varphi))^{1-v(1-\varphi)}}\frac{1}{G^{\chi}}\right]^{\frac{1}{1-v}} > 0, \quad (6)$$

*such that for all $h_e \geq \bar{h}_e(h_w,w;\zeta)$, then $\pi(h_e;w) \geq (1+\zeta)wh_w$. In addition for each $w > 0$ and*

[22] We could have assumed that the government produces also the consumption good with a different technology from the one used by entrepreneurs, such as in Song, Storesletten, and Zilibotti (2011). This would generate stronger effects of an overpaid public sector on misallocation since the private and the public producing goods are perfect substitutes and the government is not needed in the economy.

*$h_w > 0$, there exists an entrepreneurial ability function $\bar{h}_w(h_w, w)$*

$$\bar{h}_w(h_w, w) = \frac{\bar{h}_e(h_w, w; \zeta)}{(1+\zeta)^{\frac{1-v(1-\varphi)}{1-v}}} \leq \bar{h}_e(h_w, w; \zeta), \tag{7}$$

*which is independent of $\zeta$, and for each $h_e \leq \bar{h}_w(h_w, w)$, then $\pi(h_e; w) \leq w h_w$.*

Therefore, any individual with a pair $(h_w, h_e)$ lying above the line $\bar{h}_e(h_w, w; \zeta)$ will choose to be an entrepreneur, while any individual with a vector $(h_w, h_e)$ lying below the line $\bar{h}_w(h_w, w)$ will choose not to be an entrepreneur. Notice that if the public sector premium is positive then every individual with a productivity vector lying below $\bar{h}_e(h_w, w; \zeta)$ would like to be a civil servant. Clearly, not all agents below this line could work in the public sector, otherwise there will be no production in the private sector and hence no resources to finance the public sector. Before we determine who will work in the public sector, it is important to understand the efficient allocation of this economy.

**Proposition 1.** *Suppose that the size of the government is determined by a benevolent social planner, then it is not efficient to pay a public sector wage premium and $G = A_g N_g^{SP}$ with*

$$N_g^{SP} = \frac{\chi(1-\alpha)}{\chi(1-\alpha) + v(1-\varphi)} \int_0^{\infty} \int_0^{\bar{h}_e^{SP}(h_w)} h_w dF_e(\bar{h}_e^{SP}(h_w)) dF_w(h_w),$$

*where $\bar{h}_e^{SP}(h_w)$ corresponds to the threshold entrepreneurial productivity function determined by the Social Planner.*

*Proof.* See Appendix B. *Q.E.D.*

The efficient allocation is therefore the one in which the marginal productivity of labor is equalized in the public and private sectors. This is not necessarily the case when the size of the government is exogenously determined.

**Proposition 2.** *Suppose that the size of the government is determined exogenously such that $G = A_g \phi_g^{1-\alpha}$ with $\phi_g > 0$. If $\phi_g \neq N_g^{SP}$, then there will be misallocation of labor in the intensive margin.*

*Proof.* See Appendix B. *Q.E.D.*

Whenever the size of the the public sector is different from the optimal level, then there will be misallocation of labor in the intensive margin and this is irrespective of whether or not a public sector wage premium exists. Were the size of the government to be too

large, then the marginal productivity of labor would be higher in the private sector than in the public sector, and moving labor from the public to the private sector would increase overall productivity.

The public sector wage premium can also generate misallocation, independently of the size of the government. In order to observe this, consider an economy with a public sector wage premium $\zeta > 0$ and with an exogenously determined public sector. There are several possible cases to determine who will be civil servants. Figure 2 presents two cases. The area above line $\bar{h}_e(h_w, w; \zeta)$ is represented by the dark grey shaded area in Figure 2. Individuals with a vector $(h_e, h_w)$ lying above $\bar{h}_e(h_w, w; \zeta)$ become entrepreneurs. Individuals whose productivity pair $(h_e, h_w)$ lies below line $\bar{h}_e(h_w, w; \zeta)$ would like to become civil servants. The left graph of Figure 2 shows the case in which selection in public sector jobs depends positively on both labor and entrepreneurial productivity, while the graph on the right presents the case in which selection for public employees is based on the labor productivity. Observe that some individuals with productivity below $\bar{h}_e(h_w, w; \zeta)$ but above $\bar{h}_w(h_w, w)$ might become entrepreneurs due to the fact that there is a limited number of jobs in the public sector. In both graphs, given the wage rate, it is possible that total efficiency might increase in this economy if an individual lying in point $A$ could become an entrepreneur while an individual lying in point $B$ could become a civil servant. The reason is that the productivity of labor of these two individuals is similar, but the entrepreneurial productivity of (*civil servant*) $A$ is substantially greater than the entrepreneurial productivity of (*entrepreneur*) $B$. Then switching occupations of agents $A$ and $B$ would not change (substantially) the production of public infrastructure but would have strong effects on entrepreneurial production increasing aggregate efficiency. This is summarized in the next proposition.

**Proposition 3.** *Suppose that there is a wage premium $\zeta > 0$ to work in the public sector, and let selection in the public sector be determined by the labor productivity $h_w$. Assuming that the marginal productivity of labor is the same in the public and private sectors, then there will be misallocation of labor in the extensive margin.*

*Proof.* See Appendix B. *Q.E.D.*

Consider the right graph of Figure 2. In equilibrium the following result can be demonstrated.

**Proposition 4.** *Let civil servants be those agents with a pair $(h_w, h_e)$ such that $h_w \geq \bar{h}_g(w; \zeta)$ and $h_e \leq \bar{h}_e(h_w, w; \zeta)$. Let the size of the public sector be exogenously determined. Then:*

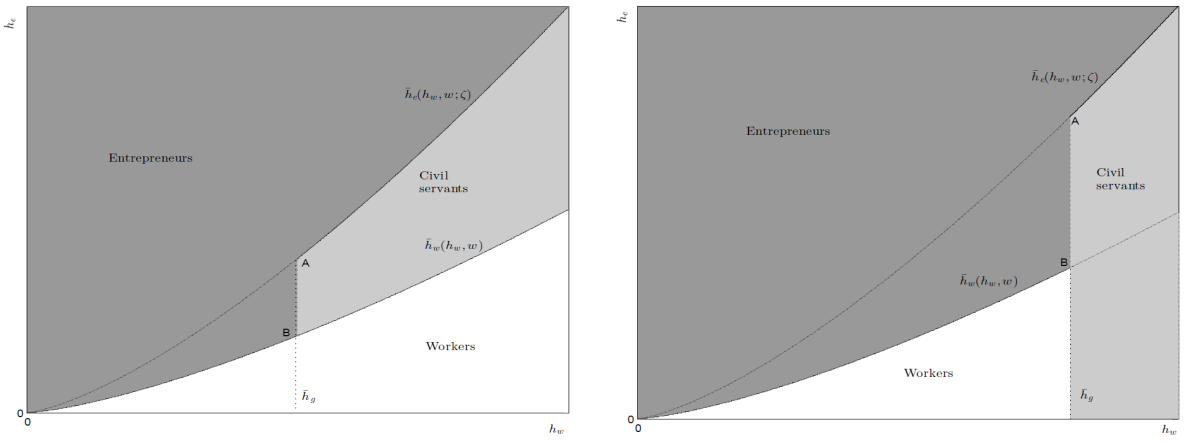


Figure 2: **Occupational choice.** *Left graph:* Selection in the public sector depends positively on $h_w$ and $h_e$. *Right graph:* Selection in the public sector depends (positively) only on $h_w$.

*i. There exists a wage rate $w(\zeta)$, which clears the labor market.*

*ii. A sufficient condition for uniqueness is that the public sector wage premium $\zeta$ is sufficiently small. In addition, for $\zeta$ sufficiently small, $w(\zeta)$ is decreasing in $\zeta$.*

*Proof.* See Appendix B. *Q.E.D.*

An equilibrium always exists and the condition for uniqueness is a sufficient requirement. Even when this condition is not satisfied the equilibrium might be unique. In general, it is not possible to analytically derive how the equilibrium wage rate varies with the public sector wage premium. However, when $\zeta$ is sufficiently small, then the wage rate decreases with a higher public sector wage premium. The reason is that a higher wage premium $\zeta$ increases the threshold productivity function $\bar{h}_e(h_w, w; \zeta)$, which decreases the demand for labor for a given wage rate. Notice that $\bar{h}_w(h_w, w)$ is independent of the public sector wage premium $\zeta$, while $\bar{h}_g(w; \zeta)$ is increasing with $\zeta$. Therefore, when the public sector wage premium increases, then the labor supply in the private sector rises for a given wage rate, and the wage rate decreases.

Aggregate output for this economy is given by:

$$Y(w, \zeta) = \int_0^{\infty} \int_{\bar{h}_e(h_w, w; \zeta)}^{\infty} y(h_e, w) dF_e(h_e) dF_w(h_w) + \int_0^{\bar{h}_g(w; \zeta)} \int_{\bar{h}_w(h_w, w)}^{\bar{h}_e(h_w, w; \zeta)} y(h_e, w) dF_e(h_e) dF_w(h_w). \quad (8)$$

The first term on the right-hand-side of Equation (8) corresponds to the aggregate output of all entrepreneurs with a vector $(h_w, h_e)$ lying above the threshold $\bar{h}_e(h_w, w; \zeta)$. The second term represents the aggregate output of those agents who would prefer to work in the public sector but given that they cannot find a job in this sector, they then choose to

be entrepreneurs rather than workers in the private sector. Without a public sector wage premium this second term would not exist. There are two effects on productivity of an increase in the public sector wage premium when the government keeps the size of the public sector constant, and selection of workers into this sector is based on labor productivity. The first one is a *selection effect*. For a given wage rate, an increase in the public sector wage premium attracts relatively productive entrepreneurs to public jobs, and therefore total output decreases. The second one is a *general equilibrium effect*, through a change in the mass of workers, entrepreneurs and the wage rate. This is clear when $\zeta$ is sufficiently small. See Equation (35) in Appendix B.

The simple framework developed above focuses on a static version of the model in which selection in the public sector is exogenously determined. The static nature of the framework prevents us from studying the allocation effects of a public pension gap, and the implications of public sector reforms on capital accumulation. Below we describe the full version of our theoretical framework that we use for quantitative analysis.

# 4 The Environment

**Demography, preferences and career choices.** Time is discrete and the economy has an infinite horizon. The economy is populated by a continuum of individuals of mass one who may live at most $T$ periods. In each time period, a new generation is born with probability one. The age profile of the population is given by $\{\mu_t\}_{t=1}^{T}$, where $\mu_t$ is the share of age-$t$ agents in the economy and satisfies $\sum_{t=1}^{T} \mu_t = 1$, such that the implicit survival rate is $\vartheta_t = \frac{\mu_t}{\mu_{t-1}}$. This survival probability implies that a fraction of the population leaves accidental bequests, which, for simplicity, are assumed to be distributed to all surviving individuals in a lump-sum basis. In order to simplify notation we omit the time subscript.

Individuals derive utility from consumption, $c_t$. Preferences over random paths for consumption over the life cycle are represented by:

$$E_{t=1}\left[\sum_{t=1}^{T}\beta^{t-1}\prod_{j=1}^{t}\vartheta_j u(c_t)\right], \text{ where } u(c_t) = \frac{{c_t}^{1-\gamma}}{1-\gamma}, \tag{9}$$

where $\beta$ is the subjective discount factor, $E_{t=1}$ is the expectations operator conditional on information at birth, and $\gamma > 0$ denotes the relative risk aversion coefficient.

In each period, agents decide whether to run an entrepreneurial activity, $e$, or to supply

their time endowment to the labor market. In the latter case, they can choose to work either in the private sector, $wk$, or as a civil servant, $cs$. At a certain age $T_r$, agents retire and receive social security payments. The social security regime in the public sector is different from the one in the private sector. We capture this feature by allowing the replacement rate to differ under the two regimes. Let $d_{m,t}$ for $m \in \{cs, wk, e, rg, rp\}$ denote an individual's occupation, where $d_{rg,t}$ denotes and individual who retired from the public sector and $d_{rp,t}$ denotes an individual who retired from the private sector. These occupational statuses are mutually exclusive, which means that, for instance, if $d_{cs,t} = 1$ then $d_{m,t} = 0$ for all $m \neq cs$.

**Private sector technology.** There are two sectors of production. Following Quadrini (2000), the first sector (*Noncorporate Sector*) is characterized by small units of production (small firms), where each business activity is related to one specific manager. Households engage in entrepreneurial activities in this sector. The second (*Corporate sector*) one is dominated by large impersonal units of production (large firms). The main feature that differentiates a small business from a big corporation is the uninsurable entrepreneurial risk. The *Corporate sector* does not face the same risks of the sector intensive in management skills and the basic model intuition holds without this sector. Its presence is important only for the quantitative analysis since in the data not all production is generated by business activities associated to one household.[23]

**Noncorporate sector.** Total production in the noncorporate sector is generated by the aggregation of all production technologies run by households engaging in entrepreneurial activities. Each technology comprises a manager with entrepreneurial ability, $h_e$, which corresponds to her capacity to employ labor, $n$, and capital, $k$, to produce a single (private) consumption good, $y_{h_e}$. Shortly we will describe how $h_e$ evolves over time. The entrepreneurial production technology is represented by

$$y_{h_e} = zA_p G^{\chi} h_e^{1-v} f(k,n)^v = zG^{\chi} h_e^{1-v} (k^{\varphi} n^{1-\varphi})^v, \; A_p > 0, \; v, \; \varphi \in (0,1), \; \chi \geq 0. \tag{10}$$

Variable $z$ is a random shock. We assume a very parsimonious representation for $z$, such that in every period with probability $\Delta$, the same technology is available for the entrepreneur and $z = 1$. However, with probability $1 - \Delta$, $z = 0$, there is no production and the individual becomes a worker. Production takes place after the realization of $z$. This captures uncertainty related to the entrepreneurial activity.

[23] In addition, the presence of the corporate sector allows us to describe the production side of the economy by a constant returns to scale technology, which makes the computation of the equilibrium much easier.

Managers can operate only one establishment. Let $w$ denotes the wage rate. The problem of an entrepreneur aged $t$ with managerial ability $h_{e,t}$ is to choose capital stock, $k_t$, and labor, $n_t$, to maximize:[24]

$$\pi_t(h_{e,t}) = \max_{k_t, n_t \geq 0} A_p G^{\chi} h_{e,t}^{1-v} f(k_t, n_t)^v - w n_t - (r+\delta) k_t, \tag{11}$$

where $r$ is the interest rate and $\delta$ is the depreciation of the physical capital stock.

**Corporate sector.** Firms in the corporate sector produce the consumption good through a standard constant returns to scale production function:

$$Y_c = G^{\chi} K_c^{\alpha} N_c^{1-\alpha}, \ \alpha \in (0,1). \tag{12}$$

They take prices as given and choose factors of production to maximize profits. The first-order conditions of a representative corporate firm are given by:

$$r = \alpha G^{\chi} \left( \frac{K_c}{N_c} \right)^{\alpha - 1} - \delta, \tag{13}$$

$$w = (1-\alpha) G^{\chi} \left( \frac{K_c}{N_c} \right)^{\alpha}. \tag{14}$$

**Government sector.** We assume that the public good is produced by the government. The public good, $G$, is produced using efficient labor units $N_g$ and capital $K_g$ according to the following technology:

$$G = A_g K_g^{\alpha} N_g^{1-\alpha}, \ A_g > 0. \tag{15}$$

This is an aggregation of labor and capital to produce public goods and services such as paved roads and the rule of law. It is important to highlight that $K_g$ in our model is capital, such as machines and equipments, employed in the public sector to produce public infrastructure. Capital employed in the public sector evolves according to the following law of motion:

$$K_{g,t+1} = I_g + (1-\delta_g) K_{g,t}, \tag{16}$$

where $I_g$ is financed through taxes.

There are two different social security regimes: A scheme for private workers and entrepreneurs and a scheme for public workers. The replacement rate for civil servants is

[24] Variable $z$ is realized before production takes place. Therefore, it does not appear in the static problem of the entrepreneur.

different from the one faced by private sector workers. Consequently, in the model we have two types of retirees: individuals who retired from the public sector and individuals who retired from the private sector.

In addition, we assume that the government carries out an exogenous flow of expenditure, $C_g$, which includes other parts of government consumption such as military expenditure that is deemed to be unproductive in our model. $C_g$ is just useful to allow the model to match the actual aggregate share of government spending in the economy and it is kept constant in the quantitative exercises. In order to finance its expenditures, the government levies proportional taxes on consumption, $\tau_c$, on labor income and profits, $\tau$, and on capital income, $\tau_k$.

**Human capital accumulation.** Individuals are heterogeneous with respect to their entrepreneurial ability, $h_e$, which is the capacity of agents to employ capital and labor more or less productively. Households are also heterogeneous with respect to their efficiency units of labor, $h_w$. We assume that the initial distribution of $h_e$ follows a log-normal distribution with location parameter $\mu_e$ and scale parameter $\sigma_e$; while the initial distribution of $h_w$ follows a log-normal distribution with location parameter $\mu_w$ and scale parameter $\sigma_w$. Individuals can enhance their future skills by investing in human capital accumulation.[25] The law of motion for $h_w$ and $h_e$ are given by:

$$h'_w = \xi_{w,m}(h_w x_w)^{\psi_{w,m}} + (1-\delta_h)h_w,\ m \in \{wk, cs\}, \tag{17}$$

$$h'_e = \xi_e(h_e x_e)^{\psi_e} + (1-\delta_h)h_e, \tag{18}$$

where $\delta_h$ is the depreciation rate and $(x_w, x_e)$ denote investments in working and entrepreneurial ability, respectively. We allow the returns to investment on the job training to be different between workers and civil servants. To simplify the model, workers are not allowed to invest in their entrepreneurial ability and entrepreneurs are not allowed to invest in their ability as workers.

**Budget constraints.** In each period of life, and conditional on the career choice, individuals make decisions about asset accumulation and investments in human capital. Individuals' labor productivity in the private sector is determined by an age-efficiency index given by $h_{w,t}exp(s_t)$, where $s_t$ is a random component that evolves according to an AR(1)

[25] We add endogenous human capital formation because it can amplify the labor distortions caused by an overpaid public sector and thus could be quantitatively important. In the quantitative analysis we will assess the important of this channel in driving our results.

process given by $s_t = \rho s_{t-1} + \varepsilon_t$ with innovations $\varepsilon_t \sim N(0, \sigma^2_\varepsilon)$. Analogously, the evolution of labor productivity in the public sector is represented by $h_{w,t}exp(s_{g,t})$, where $s_{g,t}$ is an AR(1) process given by $s_{g,t} = \rho_g s_{g,t-1} + \varepsilon_{g,t}$ with innovations $\varepsilon_{g,t} \sim N(0, \sigma^2_{\varepsilon_g})$. This is to be consistent with the fact that labor legislation regarding, for instance, firing might be different for civil servants when compared to private workers (Fact 3). This might also be consistent with differences in job characteristics of the two sectors.[26]

We can write an individual's earnings (before taxes) in occupation $m \in \{cs, wk, e\}$ as:

$$\tilde{y}_{m,t} = \begin{cases} (1+\zeta) w h_{w,t} \exp(s_{g,t})(1 - x_{w,t}), & \text{if civil servant } (m = cs); \\ w h_{w,t} \exp(s_{p,t})(1 - x_{w,t}), & \text{if private sector worker } (m = wk); \\ \pi_t (h_{e,t}(1 - x_{e,t})), & \text{if entrepreneur } (m = e). \end{cases}$$

Parameter $\zeta$ corresponds to the wage premium that public sector workers receive relative to their counterparts in the private sector. Individuals can resort to self-insurance to protect themselves against the uncertainty of labor income by trading an asset, $a_t$, which takes the form of capital. Agents are not allowed to have a negative net wealth at any age, so that $a_{t+1} \geq 0$. Furthermore, given that there is no altruistic bequest motive and death is certain at the age $T+1$, agents in the end of their life consume all their available resources, that is, $a_{T+1} = 0$. At period $t$ the budget constraint of an active individual is given by:

$$(1+\tau_c) c_{m,t} = [1 + (1-\tau_k) r]\, a_{m,t} + (1-\tau)\tilde{y}_{m,t} - I_{A,t}\theta - a'_m + tr, \tag{19}$$

for $m \in \{cs, wk, e\}$. Variable $tr$ corresponds to lump-sum transfers due to accidental bequests.

Agents in the private sector who want to work in the public sector must apply for a public job. There is a lump-sum cost $\theta > 0$ of searching for a public job and once individuals incur this cost, there is a probability $\bar{q}$ of getting a public sector job in the next period.[27] $I_{A,t}$ is an indicator function that takes value 1 if they choose to apply for a public job and 0 otherwise. Therefore, related to the matching model literature (e.g., Mortensen and Pissarides, 1994), $\theta$ is a search cost and $\bar{q}$ is the probability of finding a job offer, which

[26] We do not model unemployment explicitly. In fact, better job security and low risk of unemployment should be one of the main reasons that agents search for a job in the public sector. However, we believe that this is captured by differences in the labor income processes between workers in the private sector and in the public sector.

[27] An interpretation for this competition for public jobs is that individuals who would like to work in the public sector must take open exams and only those who obtain the best marks in these exams become eligible to fill a pre-determined number of job positions.

depends on the number of vacancies. We could condition the entry to a public sector job by the individual human capital level and ability shock. We use the above approach to simplify the analysis. Also, we view the public sector as a continuum of different jobs which would require different levels of human capital. The fixed lump-sum cost to apply for a public job implies that in equilibrium the decision to apply for a public job will be correlated with an individual's human capital, shocks and assets.

At age $T_r$, agents retire and start collecting social security payments at an exogenously specified replacement rate of the last period earnings. In line with Fact 2 above, there are two main differences in the calculation of retirement benefits in each sector. First, the replacement rate, $\eta_m$, in the public sector is higher than in the private sector. Second, benefits in the private sector are capped by a limit denoted by $\bar{b}$, while there is no benefit cap in the public sector. Thus, the budget constraint for retirees can be written as follows:

$$(1+\tau_c)c_{m,t} = [1+(1-\tau_k)r]\, a_{m,t} + b_{m,t} - a'_m + tr, \tag{20}$$

where $b_{m,t}$ denotes the benefits and is given by:

$$b_{m,t} = \begin{cases} \eta_{rg}\tilde{y}_{cs,T_r-1}, & \text{if retired in the public sector;} \\ \eta_{rp}\min\{\tilde{y}_{\tilde{m},T_r-1}, \bar{b}\},\ \tilde{m} \in \{wk, e\}, & \text{if retired in the private sector.} \end{cases}$$

**Recursive formulation of individuals' problems.** Let $V_{m,t}(\omega_t)$ denote the value function of an individual aged $t$ in the occupation $m$, where $\omega_t = (a_t, h_{w,t}, h_{e,t}, s_t, z_t)$ is the individual state space. In addition, considering that agents die at age $T$ and that there is no altruistic link across generations, we have $V_{m,T+1}(\omega_{T+1}) = 0$. Thus, the choice problem of individuals aged $t$ who work in the private sector can be recursively represented as follows:[28]

$$V_{w,t}(\omega) = \underset{a'_w \geq 0, x_w \geq 0, I_{A,t} \in \{0,1\}}{Max} : u(c_w) + \beta\vartheta_{t+1}\Big[I_{A,t}\bar{q}E_{s'}\max\left\{V_{cs,t+1}(\omega'), V_{w,t+1}(\omega'), V_{e,t+1}(\omega')\right\} + [1-I_{A,t}\bar{q}]\, E_{s'}\max\left\{V_{w,t+1}(\omega'), V_{e,t+1}(\omega')\right\}\Big], \tag{21}$$

subject to (19), where $\omega' = (a', h'_w, h'_e, s', z' = 1)$. Analogously, the recursive problem of

[28] In order to simplify the notation, we have suppressed the subscript for age from both the state and control variables.

individuals who are entrepreneurs can be represented by:

$$V_{e,t}(\omega) = \underset{a'_e \geq 0, x_e \geq 0, I_{A,t} \in \{0,1\}}{Max} : u(c_e) + \beta \vartheta_{t+1} \Big[ I_{A,t} \bar{q} E_{s'} E_{z'} \max \left\{ V_{cs,t+1}(\omega'), V_{w,t+1}(\omega'), V_{e,t+1}(\omega') \right\} + [1 - I_{A,t} \bar{q}] E_{s'} E_{z'} \max \left\{ V_{w,t+1}(\omega'), V_{e,t+1}(\omega') \right\} \Big], \quad (22)$$

subject to (19), where $\omega' = (a', h'_w, h'_e, s', z')$. Civil servants do not need to apply again for a government job in order to continue working in the public sector. As a consequence, their problem can be written as follows:

$$V_{cs,t}(\omega) = \underset{a'_{cs} \geq 0, x_{cs} \geq 0}{Max} : u(c_{cs}) + \beta \vartheta_{t+1} E_{s'} \max \left\{ V_{cs,t+1}(\omega'), V_{w,t+1}(\omega'), V_{e,t+1}(\omega') \right\}, \quad (23)$$

subject to (19), where $\omega' = (a', h'_w, h'_e, s', z' = 1)$.

Finally, since retirees only choose their next period assets, their problem is very straightforward and can be written as follows:

$$V_{m,t}(a_m) = \underset{a'_m \geq 0}{Max} : u(c_m) + \beta \vartheta_{t+1} V_{m,t+1}(a'_m), \quad (24)$$

subject to (20) for $m = rg, rp$.

**Recursive competitive equilibrium.** At each point in time, agents differ from one another with respect to age $t$ and to state $\omega_t = (a_t, h_{w,t}, h_{e,t}, s_t, z_t) \in \Omega$ . Agents of age $t$ identified by their individual states $\omega$, are distributed according to a probability measure $\lambda_t$ defined on $\Omega$, as follows. Let $(\Omega, F(\Omega), \lambda_t)$ be a space of probability, where $F(\Omega)$ is the Borel $\sigma$-algebra on $\Omega$: for each $\eta \subset F(\Omega)$, $\lambda_t(\eta)$ denotes the fraction of agents aged $t$ that are in $\eta$.

Given the asset $t$ distribution, $\lambda_t$, $Q_t(\omega, \eta)$ induces the asset $t+1$ distribution $\lambda_{t+1}$ as follows. The function $Q_t(\omega, \eta)$ determines the probability of an agent at age $t$ and state $\omega$ transiting to the set $\eta$ at age $t+1$. $Q_t(\omega, \eta)$, in turn, depends on the agents' policy functions and on the exogenous stochastic process for $z$. Now, we have all the tools to characterize the stationary recursive competitive equilibrium. Households' optimal behavior was previously described in detail above as well as the problem in the corporate sector, non-corporate sector and the government sector. It remains, therefore, to characterize the market equilibrium conditions, the aggregate law of motion, and the government budget constraint. In each period, there are two prices in this economy $(w, r)$. The equilibrium in

the labor and capital markets are defined by:

$$K_p = \sum_{t=1}^{T_R} \mu_t \int_{\Omega} d_{e,t}(\omega) k_t(\omega) d\lambda_t + K_c = \sum_{t=1}^{T} \mu_t \int_{\Omega} d_{m,t} a_{m,t}(\omega) d\lambda_t,$$

$$N_p = \sum_{t=1}^{T_R} \mu_t \int_{\Omega} d_{e,t}(\omega) n_t(\omega) d\lambda_t + N_c = \sum_{t=1}^{T_R} \mu_t \int_{\Omega} d_{w,t}(\omega) h_{w,t}(\omega) \exp(s_{p,t})(1 - x_{w,t}(\omega)) d\lambda_t,$$

$$N_g = \sum_{t=1}^{T_R} \mu_t \int_{\Omega} d_{cs,t}(\omega) h_{w,t}(\omega) \exp(s_{g,t})(1 - x_{w,t}(\omega)) d\lambda_t.$$

The consumption tax rate, $\tau_c$, is such that it balances the government's budget,

$$C_g + I_g + (1+\zeta) w N_g + B = \tau(w N_p + (1+\zeta) w N_g + \mathbf{\Pi}) + \tau_k r K_p + \tau_c C,$$

where $C$ denotes aggregate consumption, $\mathbf{\Pi}$ represents aggregate profits and $B$ denotes total benefits. The distribution of accidental bequests is given by:

$$tr = \sum_{t=1}^{T} \mu_t \int_{\Omega} (1 - \vartheta_{t+1}) d_{m,t} a'_{m,t}(\omega) d\lambda_t.$$

Finally, given the decision rules of households, $\lambda_t(\omega)$ satisfies the following law of motion:

$$\lambda_{t+1}(\eta) = \int_{\Omega} Q_t(\omega, \eta) d\lambda_t \;\; \forall \eta \subset F(\Omega).$$

# 5 Quantitative Analysis

In order to study quantitatively the effects of a generous civil servant compensation and government reforms which would change the pension and wage scheme of public employees on economic efficiency, we must assign values for model parameters. We proceed by calibrating and estimating parameters such that the model economy matches key micro and macro statistics of the Brazilian economy. Brazil is an interesting case since it has a large public-private earnings premium. The model, however, is sufficiently general to be applied to other countries, such as Spain, Portugal, India, among others. Below is the description of how we set the value of parameters.

## 5.1 Calibration and estimation

Table 1 lists the value of each parameter for the Brazilian economy and includes a comment on how each was selected.

**Model period and age distribution:** The model period is one year. We assume that individuals start their lives at the age of 25 and live until the age of 80. Therefore, the extension of their lifetimes in the model is 56 periods (T=56). The age population distribution, $\{\mu_t\}_{t=1}^{T}$, and the mortality risk are obtained from the Life Tables for the Brazilian population constructed by IBGE (National Central Statistical Agency) based on the 2010 census data.

**Utility:** There are two parameters related to preferences, $(\beta, \gamma)$. The intertemporal discount factor, $\beta$, is calibrated such that the capital-to-output ratio is about 2.60.[29] The coefficient of relative risk aversion, $\gamma$, is set at 2.0, in line with the bulk of the literature on consumption surveyed by Attanasio (1999). This value is also consistent with the literature that estimates $\gamma$ using Brazilian data, which suggests a range from 1 to 3 (see, for example, Gandelman and Hernández-Murillo (2014) and Fajardo, Ornelas, and Farias (2012)). Since this parameter is important for the question studied in the paper, we carry out sensitivity analysis on it.

**Production technologies:** In the corporate sector, we set the capital share $\alpha$ at 0.36. This number is consistent with the one reported by Gomes, Bugarin, and Ellery-Jr. (2005), when the correction suggested by Gollin (2002) and Young (1995) about the self-employed income is taken into account. We set $A_p$, $\varphi$ and $v$ so that in the entrepreneurial sector 36% is paid to remunerate capital, the share of entrepreneurs in the labor force is equal to 4.5% (see Table 7 in Appendix A) and the earnings share of the top ten percent of earners is equal to 40% (PNAD Survey). The calibrated value for these parameters were $A_p = 1.11$, $\varphi = 0.40$ and $v = 0.87$. In addition, we assume that the capital stock depreciates at a rate of 6% per year, which is consistent to the figures used in the growth and development literature (cf., Parente and Prescott, 2000). We also set $\delta_g = 0.06$. According to the Brazilian Institute of Geography and Statistics (IBGE), the ratio of public goods to output is roughly 17% - using information on production costs. Then, in order to match this ratio we set $A_g = 0.77$. To calibrate parameter $\chi$, we rely on estimates provided by Hulten (1996) who uses a cross-section of low income countries including Latin American countries and

[29] Using the Heston, Summers, and Aten (2012) Penn World Tables 7.1 and the inventory method, we find a value of 2.60 for the capital-to-output ratio in the Brazilian economy.

obtains a point estimate of 0.1 for $\chi$, which is the value we use. We set the business risk $\Delta = 0.96$ such that we match the exit rate in Brazil.

**Stochastic process on labor productivity:** The parameters that characterize the stochastic component of individuals' labor productivity are: $\rho, \sigma^2_\varepsilon, \rho_g, \sigma^2_{\varepsilon_g}$. For computational reasons, we use the algorithm described in Tauchen (1986) to approximate these stochastic processes for each sector by a first-order Markov chain with 3 points. Since there is no household panel dataset for Brazil comparable with the Panel Study of Income Dynamics (PSID) in the United States, we can not obtain direct estimates for the persistence parameters, $\rho$ and $\rho_g$. Thus, what we do is to use information on average tenure in each sector along with data on the distribution of residual wages to calibrate them. In particular, from the Mincerian regressions presented in Table 8 (columns (6) and (7)), we have that the residual variance for civil servants and private workers are nearly the same: $\sigma^2_s = \frac{\sigma^2_\varepsilon}{1-\rho^2} = 0.4279$ and $\sigma^2_{s_g} = \frac{\sigma^2_{\varepsilon_g}}{1-\rho^2_g} = 0.4290$.[30] We calibrate $\rho$ and $\rho_g$ in such a way that the average time that individuals take to change position in the grids for $s$ and $s_g$ are consistent with the average tenure in each sector, which is about 13 years for public employees and 7 years for private workers. This procedure entails that $(\rho, \sigma^2_\varepsilon) = (0.88, 0.0965)$ and $(\rho_g, \sigma^2_{\varepsilon_g}) = (0.98, 0.017)$.[31] These figures are consistent with the fact that public sector wages are more stable and more compressed than in the private sector. This implies that workers facing bad (good) shocks in the public sector are worse-off (better-off) than their comparable counterparts in the private sector.[32]

**Human capital functions:** We calibrate the parameters of the initial skill distribution of newborn agents, $\mu_w$ and $\sigma_w$ for working ability and $\mu_e$ and $\sigma_e$ for entrepreneurial ability, to match the wage distribution of workers and entrepreneurs, respectively, at age 25, which is the age individuals are born in the model. The parameters of the human capital functions are calibrated as follows. First, given that the evidence for the human capital depreciation rate ranges from 0.0016 to 0.089, with most of the estimates concentrated around 0.04 (Browning, Hansen, and Heckman, 1999), we set $\delta_h = 0.04$. As is

[30] The estimation procedure is presented in Appendix A.

[31] The associated grids for $s_g$ and $s$ are $\{-0.3752, 0, 0.3752\}$ and $\{-0.8955, 0, 0.8955\}$, respectively. In addition, the transition matrix in the public sector is $\begin{pmatrix} 0.9251 & 0.0749 & 0.0000 \\ 0.0668 & 0.8664 & 0.0668 \\ 0.0000 & 0.0749 & 0.9251 \end{pmatrix}$, while $\begin{pmatrix} 0.8729 & 0.1271 & 0.0000 \\ 0.0668 & 0.8664 & 0.0668 \\ 0.0000 & 0.1271 & 0.8729 \end{pmatrix}$ is the transition matrix in the private sector.

[32] For the sake of comparison, Kaplan (2012) estimates a similar stochastic process for labor productivity in the U.S. and finds a value of 0.94 for the persistence parameter, which is the median point of the values we found.

Table 1: **Estimation and calibration of model parameters: Brazilian economy**

| **External calibration** | | | |
|---|---|---|---|
| Parameter | Description | Values | Source |
| $\gamma$ | Risk aversion | 2 | Attanasio (1999) |
| $\chi$ | Importance of infrastructure | 0.1 | Hulten (1996) |
| $\alpha$ | Capital share - corporate sector | 0.36 | Gomes, Bugarin, and Ellery-Jr. (2005) |
| $\varphi$ | Capital share - noncorporate sector | 0.4 | Gomes, Bugarin, and Ellery-Jr. (2005) |
| $\delta = \delta_g$ | Depreciation rate | 0.06 | Growth literature |
| $\zeta$ | Public-sector wage premium | 0.19 | PNAD survey |
| $\delta_h$ | Depreciation rate, human capital | 0.04 | Browning et al (1999) |
| $\mu_w$ | Location par., initial $h_w$ | 1.15 | PNAD survey |
| $\sigma_w$ | Scale par., initial $h_w$ | 0.78 | PNAD survey |
| $\mu_e$ | Location par., initial $h_e$ | 2.06 | PNAD survey |
| $\sigma_e$ | Scale par., initial $h_e$ | 0.90 | PNAD survey |
| $\tau$ | Income tax rate | 18% | Paes and Bugarin (2006) |
| $\tau_k$ | Capital income tax rate | 15% | Paes and Bugarin (2006) |
| $\eta_{rg}$ | Replacement rate, pub. sector | 1 | Afonso (2016) |
| $\eta_{rp}$ | Replacement rate, priv. sector | 0.82 | Afonso (2016) |
| $\bar{b}$ | Ceiling for retiree income, priv. sector | $\frac{b_{-cs}^{max}}{b_{cs}^{max}} = 0.20$ | Social security legislation |
| $\rho$ | Persistence, priv. sector | 0.88 | Average tenure, priv. sector |
| $\sigma_\epsilon^2$ | Variance of innovation, priv. sector | 0.0965 | Match residual inequality |
| $\rho_g$ | Persistence, pub. sector | 0.98 | Average Tenure, pub. sector |
| $\sigma_{\epsilon_g}^2$ | Variance of innovation, pub. sector | 0.017 | Match residual inequality |
| **Internal calibration** | | | |
| Parameter | Description | Values | Target |
| $\beta$ | Discount factor | 1.003 | Capital to output ratio, 2.55 |
| $A_p$ | TFP - entrepreneurial sector | 1.11 | Share of entrepreneurs |
| $v$ | span-of-control parameter | 0.87 | Earnings share of the top 10% |
| $\xi_{cs}$ | Human capital productivity | 0.15 | Life cycle civil servants' income |
| $\xi_w$ | Human capital productivity | 0.16 | Life cycle workers' income |
| $\xi_e$ | Human capital productivity | 0.17 | Life cycle entr.' income |
| $\psi_{cs}$ | On the job training parameter | 0.70 | Life cycle civil servants' income |
| $\psi_w$ | On the job training parameter | 0.65 | Life cycle workers' income |
| $\psi_e$ | On the job training parameter | 0.90 | Life cycle entr.' income |
| $\Delta$ | Business risk | 0.04 | exit rate, IBGE |
| $\theta$ | Cost of applying for a public job | 1.54 | Flow of individuals aged 30-34 from private to public jobs |
| $1 - \bar{q}$ | Government selection criteria | 0.827 | Share of public servants |
| $\tau_c$ | Consumption tax | 30.70% | Balance gov. budget constraint |
| $A_g$ | TFP - Government sector | 0.77 | Share of public goods |
| $C_g$ | Unproductive government spending | 2% of $Y$ | Aggregate government spending |

usual in the macro-labor literature, the other parameters are then chosen in order to approximate the simulated earnings profiles to their counterparts estimated from the data.[33] This procedure is carried out for civil servants, workers and entrepreneurs and we obtain $(\psi_{cs}, \psi_w, \psi_e) = (0.70, 0.65, 0.90)$ and $(\xi_{cs}, \xi_w, \xi_e) = (0.15, 0.16, 0.17)$.[34] The resulting profiles are presented in Figure 3. In order to measure the goodness of the fit, we calculate the average (percentage) deviation, in absolute terms, between the model implied earnings profiles and the data. By this measure, on average, the model implied earnings profiles differed from the data by 2.68% in the case of civil servants, 2.23% in the case of workers and by 3.42% in the case of entrepreneurs.

As an external validation of our model, we also compare the simulated mean consumption profile with the data (see the bottom graphs of Figure 3).[35] It can be seen that the model replicates well the pattern of the life-cycle consumption for civil servants, private sector workers and entrepreneurs.

**Public sector parameters:** Based on Paes and Bugarin (2006), we assume a labor income tax rate of 18% and a capital tax of 15%. The consumption tax is determined in such a way that the government budget balances in equilibrium, which implies a tax rate equal to 30.7% in the benchmark economy. The replacement rate in the private sector, $\eta_{rp}$, is taken from Afonso (2016) who, based on microdata from the private sector social security system, provides a value of 0.82. In the public sector, the replacement rate, $\eta_{rg}$, is equal to one, which is consistent with the fact that civil servants receive their last salary as benefits. We assume that agents can get the public pension system only if they start to work in the public sector 5 years before the retirement age. We also impose a ceiling on private pensions, $\bar{b}$, such that the maximum pension in the private sector is 20% of the maximum pension in the public sector.[36] We set the public wage premium for workers to be equal to $\zeta = 0.19$, which is consistent with the estimate of the conditional public wage premium provided in regression (5), Table 8 in Appendix A. Even though 19% is the lower bound of the estimates presented in table 8, we believe that the specification (5) is the most

[33] See, for example, Huggett, Ventura, and Yaron (2006).

[34] Estimates of the elasticity parameters $\psi$ are surveyed by Browning, Hansen, and Heckman (1999). These estimates range from 0.5 to almost 1.0. Thus, we restrict our search to this interval. In the case of the scale parameters, $(\xi_{cs}, \xi_w, \xi_e)$, we consider values in the interval $[0.05, 0.20]$.

[35] The data is from the 2009 household budget survey (Pesquisa de Orçamento Familiares, POF) carried out by the Instituto Brasileiro de Geografia and Estatística (IBGE).

[36] Both the ceiling in the public sector and in the private sector are determined by the social security legislation. In the public sector, the ceiling corresponds to the salary of the Ministry of Justice of the Supreme Court.

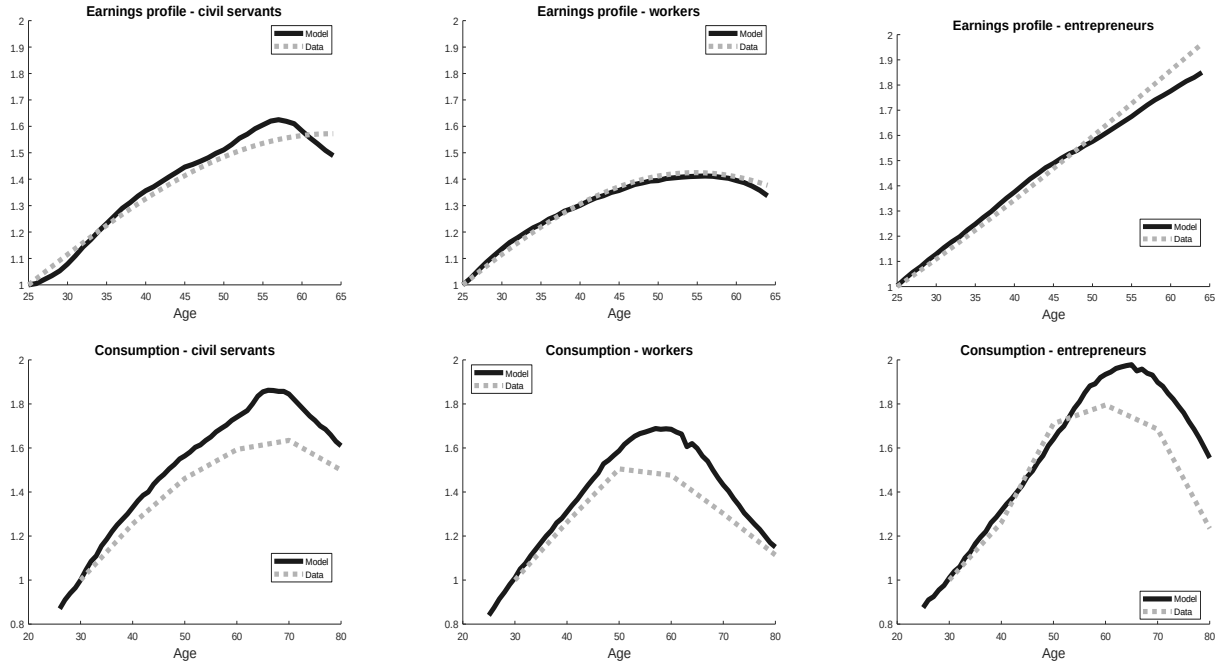


Figure 3: **Mean life-cycle earnings and consumption (model and data).** In the top panel, we show the civil servants average earnings profile (first column); the workers average earnings profile (second column); the entrepreneurs average earnings profile (third column). The average earnings at age 25 were normalized to 1. In the bottom panel, we show the mean consumption profiles. The mean consumption at age 30 were normalized to 1

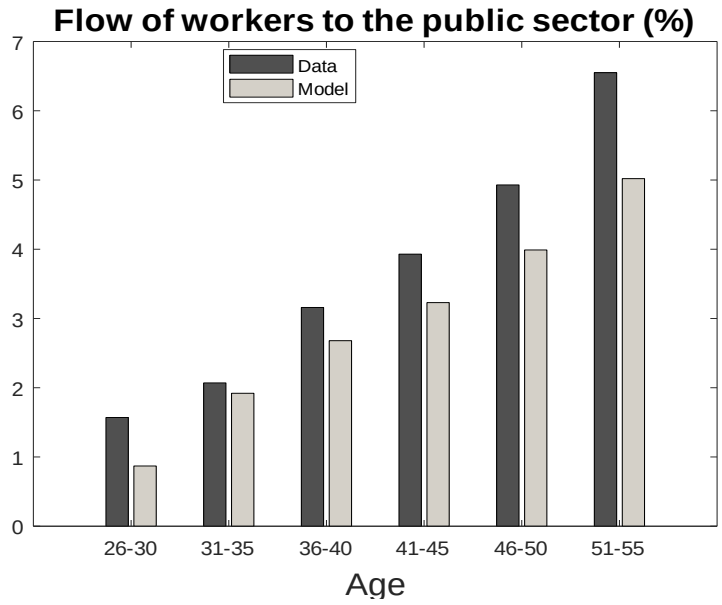


Figure 4: Flow of workers to the public sector: Model vs Data

convincing since it also controls for occupation.[37]

Two parameters remain to be set: the cost of applying for a public sector job, $\theta_t$, and the government selection criteria, $\bar{q}$. We set them to minimize the distance between the model and the data. In particular, we use as targets the public sector employment as a share of total employment and the average flow of individuals aged 31-35 going from the private sector to the public sector. This last measure is taken from the 2015 Continuous National Household Sample Survey (PNAD contínua) compiled by the Brazilian Institute of Geography and Statistics (IBGE). In Figure 4, we present the simulated and the actual flow of individuals to public jobs by age. It can be seen that this flow increases with age in the model as well as in the data. This pattern could be explained by the fact that the retirement benefits in the public sector are much more generous than in the private sector, which makes the public sector jobs more attractive as individuals approach retirement.

The model matches the Brazilian economy fairly well along a number of dimensions that were calibrated, as well as some statistics that were not calibrated, such as the income Gini index for workers and entrepreneurs. Table 2 presents some data statistics and the model counterpart. Observe that the share of public goods and the percent of civil servants in the labor force are similar in the model and in the data. The aggregate cost of job

[37] In the Companion Online Appendix we internally calibrated the model with a public sector wage premium of 25% instead of 19%. We also run counterfactual experiments using this alternative calibration. Results are in line to those presented here.

Table 2: **Basic statistics: Brazil and baseline economy.** Sources: IBGE - National accounts and PNAD.

| Variable | Brazil | Baseline model |
|---|---|---|
| Calibrated moments | | |
| % of entrepreneurs | 5.2 | 4.59 |
| % of civil servants | 10 | 10.02 |
| Share of public goods | 0.17 | 0.18 |
| Capital to output ratio | 2.55 | 2.60 |
| Income share held by highest 10% | 40% | 39% |
| Non-calibrated moments | | |
| Income Gini (all individuals) | 0.51 | 0.52 |
| Income Gini (private workers) | 0.46 | 0.46 |
| Income Gini (civil servants) | 0.48 | 0.47 |
| Income Gini (Entrepreneurs) | 0.53 | 0.54 |

applications to public jobs, which we label as the rent seeking cost, is large in the baseline economy. As can be seen in Table 3, it is roughly 3.61 percent of output. We do not have this measure for the Brazilian economy; however, in Brazil there are a large number of education institutions to prepare individuals to undertake a public exam for a public job. In our model, $\theta$ is in units of the consumption good, but in mapping this to the data we should also consider the time individuals spend in preparing for such exams and therefore their foregone income.[38]

## 5.2 Counterfactual exercises

Now, with all parameters set we can perform different policy simulations.

### 5.2.1 Reducing the public sector wage premium

Figure 5 and Table 3 display the effects of reducing the public sector wage premium on output and others relevant variables. Note that in all exercises we keep the capital in the public sector at the benchmark level. So any reallocation that we have from the public to the private sector and vice-versa is a reallocation of labor.[39] We conduct three different

[38] According to ANPAC (Brazilian National Agency for Protection and Support to Public Exams), a non-governmental institution, there are about 12 million Brazilians (5% of the population and more than 10% of the labor force) taking exams for a public job. This number also includes workers who already have a public job but are trying to upgrade to a better paid one. In our baseline economy, there are 3.38 percent of workers applying for a public job and therefore it does not seem that we are overestimating the number of applications for a public job.

[39] This assumption helps to interpret the results better. By keeping the level of capital in the public sector constant, we are able to focus on the effects of labor reallocation. In this case, for example, any change in $G$ is due to changes in the public sector employment. The results where $K_g$ is allowed to change are somewhat similar and, for the sake of space, we only report them in the companion Online Appendix.

types of experiments. In the first, we only change the public sector wage premium and allow the share of civil servants to be determined endogenously. This is the only parameter we change in the model. In this case, the reduction in $\zeta$ has an inverted U-shape effect on aggregate output.[40] To understand this result, notice that a smaller wage premium attracts fewer workers to a public sector job. However, because of higher income uncertainty and a less generous retirement pension in the private sector, individuals have more incentive to save. In addition, the consumption tax rate decreases. Hence, as one can see in Table 3, aggregate physical and human capital increase. On the other hand, the fall in public sector employment reduces the supply of public infrastructure, $G$, and therefore has a negative effect on private productivity. We find that for a reduction in $\zeta$ up to 4 percentage points from the baseline value (from 19% to 15%), the former effect dominates and output increases by as much as 3.36%. However, if the wage premium is reduced further, the later effect starts to dominate and consequently aggregate output falls. In fact, the model predicts that a public-private wage premium of $\zeta = 11\%$ would decrease output by nearly 5% in comparison to the benchmark case. Therefore, only reducing the attractiveness of the public sector does not necessarily increase aggregate output, since public infrastructure might be seriously reduced. We also document the change in TFP.[41] Notice that when we change the public sector wage premium and let the share of civil servants to adjust then most of the change in aggregate output is due to changes in the factor of production rather than changes in TFP.

In order to emphasize the role of changes in the distribution of agents, in Experiment 2 we investigate the effects of reducing the wage premium when we keep the share of civil servants unchanged. In order to maintain this share constant, for each value of $\zeta$, we adjust the cost of applying for a public sector job, $\theta$.[42] As one can see in Figure 7 and Table 3, when we reduce the wage premium from its benchmark value to 15%, for example, output still increases but by less than in Experiment 1. At the same time, this

[40] Aggregate output here is not Gross Domestic Product (GDP). GDP contains output of the public sector, which is usually evaluated at input costs. We instead focus on private output in the model.

[41] We compute the Total Factor Productivity (TFP) as it is typically done in development accounting exercises. In particular, following the procedure in Garcia-Santana and Pijoan-Mas (2014), we impose a Cobb-Douglas representative firm and use the aggregate data generated by the model to measure the increase in TFP.

[42] We think that this exercise helps us separate the effect on output due to changes in the distribution of individuals in the public sector from the one associated with changes in the share of civil servants. By keeping the share of civil servants constant, we are able to focus on the effects of labor reallocation. In this case, for example, any change in $G$ is due to changes in the distribution of individuals in the public sector. We also have results in which $G/Y$ is kept constant. They are similar to those reported here and are shown in a companion Online Appendix.

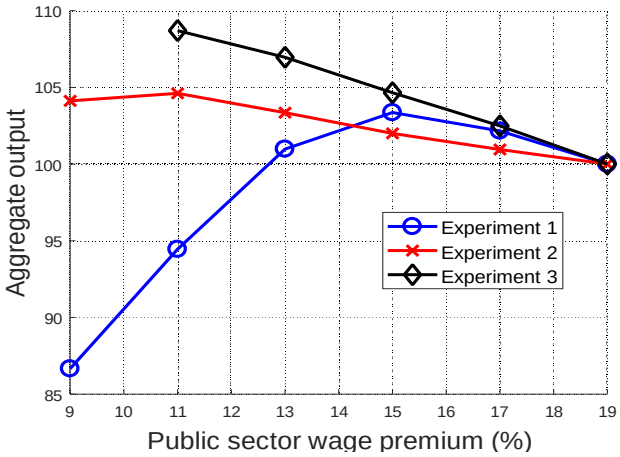


Figure 5: **Aggregate output by public sector wage premium.** In Experiment 1, we only change the public sector wage premium, $\zeta$; In Experiment 2, we change $\zeta$, but adjust $\theta$ in order to keep the share of individuals in the public sector constant; In Experiment 3, we do the same thing as in Experiment 2, but instead of adjusting $\theta$, we adjust $\bar{q}$.

reduction in $\zeta$ decreases the level of public goods, $G$, in 1.03%, even though the share of civil servants is kept constant. This reduction in $G$ is explained by the fall in the average labor productivity in the public sector due to a reallocation of workers in the economy. To understand this result, one should have in mind that the share of high ability individuals is larger in the public sector than in the private sector since ability is correlated with income and assets. Applying for a public sector job is costly and therefore households accumulate assets in order to apply for such a job. However, as the wage premium falls, the relative number of applications of individuals in lower levels of the skill distribution rises as the lump-sum cost $\theta$ decreases, reducing the average labor productivity in the public sector, but increasing it in the private sector. Moreover, it is interesting to note that if we continue reducing $\zeta$ and thus making the public sector job less attractive, output continues to go up in Experiment 2 as opposed to what happens in Experiment 1. Not only does the deadweight cost of applying for a public job decrease, but there is a reallocation of labor in the economy with a positive effect on aggregate output.[43] The increase in aggregate output is larger than the decrease in the rent seeking cost by nearly 2.3 percentage points. Observe that changes in TFP account by about 15% of the change in output when $\zeta$ decreases from the baseline value to 15%.

Experiment 3 is similar to Experiment 2, but instead of adjusting $\theta$ to keep the share of civil servants unchanged, we adjust the government selection criteria, $\bar{q}$. Related to

[43]When $\zeta = 0.13$, then $\theta = 1.04$, which is nearly 32% lower than in the benchmark. The rent seeking cost decreases in 29% . See Table 3.

Table 3: **Public sector wage premium.** In Experiment 1, we change the public sector wage premium, $\zeta$, and allow the share of civil servants to be determined endogenously; In Experiment 2, we change $\zeta$, but adjust $\theta$ in order to keep the share of individuals in the public sector constant; In Experiment 3, we do the same thing as in Experiment 2, but instead of adjusting $\theta$, we adjust $\bar{q}$.

| **Experiment 1:** % **of civil servants is endogenous** | | | | | |
|---|---|---|---|---|---|
| Variable | $\zeta = 0.19$ | $\zeta = 0.17$ | $\zeta = 0.15$ | $\zeta = 0.13$ | $\zeta = 0.11$ |
| $Y = Y_c + Y_e$ | 100 | 102.16 | 103.46 | 100.98 | 94.45 |
| $TFP$ | 100 | 100.22 | 99.81 | 99.03 | 96.02 |
| $K$ | 100 | 103.18 | 105.43 | 104.62 | 99.90 |
| $N_p$ | 100 | 104.85 | 108.29 | 110.48 | 111.49 |
| Avg. entrepreneurial ability | 100 | 100.27 | 99.36 | 98.02 | 95.61 |
| % of entrepreneurs | 4.59% | 4.67% | 4.81% | 4.98% | 5.16% |
| % of civil servants | 10.02% | 7.86% | 6.44% | 4.18% | 2.61 % |
| $G/Y$ | 18.21% | 15.41% | 13.02% | 9.47% | 5.87 % |
| $G$ | 100 | 86.44 | 73.90 | 52.49 | 30.43 |
| $\tau_c$ | 30.70% | 24.52 % | 18.78% | 11.17% | 5.10% |
| $MPL_g - MPL_p$ | -0.43 | -0.30 | -0.11 | 0.18 | 0.53 |
| $K_c/Y_c$ | 2.598 | 2.624 | 2.645 | 2.692 | 2.743 |
| $w$ | 0.799 | 0.781 | 0.767 | 0.735 | 0.683 |
| $r$ | 7.838% | 7.737% | 7.618% | 7.430% | 7.121% |
| % of applications | 3.38% | 2.87% | 2.45% | 1.68% | 1.12% |
| Rent seeking costs/Y | 3.61% | 3.12% | 2.68% | 1.93% | 1.40% |
| **Experiment 2:** % **of civil servants is constant -** $\theta$ **Adjusted** | | | | | |
| Variable | $\zeta = 0.19$ | $\zeta = 0.17$ | $\zeta = 0.15$ | $\zeta = 0.13$ | $\zeta = 0.11$ |
| $Y = Y_c + Y_e$ | 100 | 100.95 | 102.00 | 103.34 | 104.51 |
| $TFP$ | 100 | 100.13 | 100.29 | 100.30 | 100.26 |
| $K$ | 100 | 101.11 | 102.52 | 104.08 | 105.66 |
| $N_p$ | 100 | 100.99 | 102.18 | 103.35 | 104.48 |
| Avg. entrepreneurial ability | 100 | 100.52 | 100.98 | 101.06 | 100.84 |
| % of entrepreneurs | 4.59% | 4.54% | 4.49% | 4.43% | 4.38% |
| % of civil servants | 10.02% | 10.00% | 10.01% | 10.02% | 10.02% |
| $G/Y$ | 18.21% | 17.92% | 17.67% | 17.42% | 17.05% |
| $G$ | 100 | 99.39 | 98.97 | 98.85 | 97.85 |
| $\tau_c$ | 30.70% | 29.87 % | 29.43% | 28.85% | 27.52% |
| $MPL_g - MPL_p$ | -0.43 | -0.42 | -0.39 | -0.37 | -0.36 |
| $K_c/Y_c$ | 2.598 | 2.604 | 2.608 | 2.612 | 2.625 |
| $w$ | 0.799 | 0.799 | 0.800 | 0.802 | 0.801 |
| $r$ | 7.838% | 7.821% | 7.795% | 7.762% | 7.696% |
| % of applications | 3.38% | 3.44% | 3.54% | 3.64% | 3.67% |
| Rent seeking costs/Y | 3.61% | 3.35% | 3.06% | 2.56 % | 1.91% |
| $\theta$ | 1.54 | 1.41 | 1.26 | 1.04 | 0.80 |
| **Experiment 3:** % **of civil servants is constant -** $\bar{q}$ **Adjusted** | | | | | |
| Variable | $\zeta = 0.19$ | $\zeta = 0.17$ | $\zeta = 0.15$ | $\zeta = 0.13$ | $\zeta = 0.11$ |
| $Y = Y_c + Y_e$ | 100 | 102.49 | 104.65 | 106.96 | 108.70 |
| $TFP$ | 100 | 100.41 | 101.23 | 101.64 | 101.02 |
| $K$ | 100 | 103.25 | 105.24 | 109.66 | 112.89 |
| $N_p$ | 100 | 102.12 | 103.94 | 106.63 | 109.48 |
| Avg. entrepreneurial ability | 100 | 100.43 | 101.28 | 101.85 | 102.07 |
| % of entrepreneurs | 4.59% | 4.46% | 4.29% | 4.02% | 3.86% |
| % of civil servants | 10.02% | 10.02% | 10.00% | 10.03% | 10.05 % |
| $G/Y$ | 18.21% | 17.96% | 17.87% | 17.37% | 16.66% |
| $G$ | 100 | 101.13 | 102.69 | 102.05 | 99.50 |
| $\tau_c$ | 30.70% | 30.44% | 30.00% | 28.68% | 25.90% |
| $MPL_g - MPL_p$ | -0.43 | -0.42 | -0.40 | -0.39 | -0.37 |
| $K_c/Y_c$ | 2.598 | 2.617 | 2.638 | 2.658 | 2.698 |
| $w$ | 0.799 | 0.803 | 0.806 | 0.806 | 0.798 |
| $r$ | 7.838% | 7.751% | 7.660% | 7.546% | 7.363% |
| % of applications | 3.38% | 2.83% | 2.46% | 1.78% | 0.85% |
| Rent seeking costs/Y | 3.61% | 3.09% | 2.56% | 1.72% | 0.87% |
| $1 - \bar{q}$ | 0.828 | 0.793 | 0.746 | 0.629 | 0.19 |

the matching literature, this would be similar to an increase in the number of matching.[44] In this case, Figure 7 shows that as we reduce the wage premium, the aggregate output is always greater than in Experiments 1 and 2. To understand this result, note that now the share of applications falls as we reduce $\zeta$, which is due to the fact the we have kept $\theta$, the searching cost, at its benchmark value. Thus, as we increase $\bar{q}$ to maintain public sector employment, we find that not only fewer individuals are applying for public sector jobs, but also fewer ones are failing the recruitment process. The flow into public jobs increases and consequently the deadweight losses coming from applications to public jobs decrease. Note that differently from the previous experiment, the supply of public goods rises, growing 2.7% relative to the baseline when $\zeta = 0.15$ and the tax rate is roughly unchanged - see Table 3. This happens because the combination of a lower $\zeta$ with a constant $\theta$, entails the government's more efficient selection of a larger share of highly productive individuals, thereby raising labor productivity in the public sector and decreasing the deadweight losses from individuals seeking to secure a public sector job. Both human and physical capital increase substantially but TFP accounts for a large fraction of the change in output. For instance, when the public sector wage premium changes from its baseline value to 15%, then TFP accounts for about 24% of the change in aggregate output.

From these exercises we can observe that when the public sector wage premium is reduced, there are several effects on the economy (Experiment 1). First of all, there is a direct effect on taxes. The government now has to spend fewer resources to pay for public sector workers. When we adjust $\bar{q}$ to keep the share of public workers constant, then the tax rate remains roughly constant, so in this case this effect through the tax rate is not quantitatively important. Secondly, public sector jobs are less attractive so there is a lower number of applicants for these jobs. This in turn has two effects. The first is that the economy saves on application fees. When we adjust $\bar{q}$, we observe that this deadweight loss is reduced by 1.05 percentage points of output when $\zeta$ goes from its baseline value to 15%. However, aggregate output increases in 4.65%, which implies that nearly 80% of the change in output cannot be explained merely by a reduction in the application process to public sector jobs. In addition, occupational decisions change with implications not only on how factors are allocated in society but also on human capital and asset accumulation. We show that changes in allocation of factors account for about 24% of the increase in output when $\zeta$ goes from 19% to 15%. The remaining change is due to changes in the accumulation of factors of production. We provide further detailed decomposition in Table 4 below, but

[44] Our view is that changing the probability of getting a job in the public sector is a more realistic exercise, as in the matching literature, which would change endogenously.

Table 4: **Changing all factors.** The top panel displays the results for each of the following experiments: We only change the wage premium from 19% to 15% (second column); We align the parameters of the public and private social security systems (third column); We change all factors at the same time (fourth column). The bottom panel reports the importance of physical capital, human capital and occupational choices in explaining the output growth.

| **Adjusting $\bar{q}$** | | | | |
|---|---|---|---|---|
| Variable | Baseline | Wage premium | Social Security | All |
| $Y = Y_c + Y_e$ | 100 | 104.65 | 106.98 | 111.20 |
| $TFP$ | 100 | 101.23 | 100.81 | 101.57 |
| $K$ | 100 | 105.24 | 115.36 | 122.04 |
| $N_p$ | 100 | 103.94 | 100.45 | 104.03 |
| Avg. entrepreneurial ability | 100 | 101.28 | 101.67 | 102.90 |
| % of entrepreneurs | 4.59% | 4.29% | 4.50% | 4.16% |
| % of civil servants | 10.02% | 10.00% | 10.05% | 10.06% |
| $G/Y$ | 18.21% | 17.87% | 17.38% | 17.14% |
| $G$ | 100 | 102.69 | 102.13 | 104.65 |
| $\tau_c$ | 30.70% | 30.00% | 25.10% | 22.95% |
| $MPL_g - MPL_p$ | -0.43 | -0.40 | -0.42 | -0.39 |
| $K_c/Y_c$ | 2.598 | 2.638 | 2.806 | 2.882 |
| $w$ | 0.799 | 0.806 | 0.860 | 0.882 |
| $r$ | 7.838% | 7.660% | 6.826% | 6.478% |
| % of applications | 3.38% | 2.46% | 2.72% | 1.61% |
| Rent seeking costs/Y | 3.61% | 2.56% | 2.63% | 1.46% |
| $1 - \bar{q}$ | 0.828 | 0.746 | 0.756 | 0.554 |
| **Decomposing the effect on output** | | | | |
| | Baseline | Wage premium | Social Security | All |
| Full effect | 100 | 104.65 | 106.98 | 111.20 |
| Physical capital | - | 102.11 (45.4%) | 104.75 (68.0%) | 106.11 (54.6%) |
| Human capital | - | 101.63 (35.0%) | 101.64 (23.5%) | 103.40 (30.4%) |
| Occupational choices | - | 100.92 (19.8%) | 100.63 (9.02%) | 101.75 (15.7%) |

before let us learn the aggregate effects of pension reforms.

### 5.2.2 The effects of aligning the social security regimes

In Table 4, we show the effects of aligning the public and private social security regimes. In particular, we equate the retirement replacement rate in the public sector with its counterpart in the private sector and introduce the same cap on civil servants benefits that is observed for private sector workers.[45] For the sake of space, we report the results only when we change $\bar{q}$ to keep the share of civil servants constant, but in a companion Online Appendix, we also report results when $\theta$ is adjusted. For comparison, we also present results for changes in the wage premium - $\zeta$ drops from 19% to 15%, which is the value that maximizes output according to Figure 5.

As can be seen in this table, the effects of changing social security parameters in the public sector are considerable, with the output increasing by nearly 7%. In order to understand such aggregate effect, observe that by making social security less generous, gov-

[45]More specifically, we change the replacement rate in the public sector from 1 to 0.82 and we introduce a cap in the pension such that this is the same as in the private sector. The ceiling changes proportionally with changes in income levels in the economy.

ernment induces private savings and thus increases capital accumulation. The aggregate capital stock goes up by 15.36% in the long-run relative to the baseline and the capital-to-output ratio increases. With this social security reform, agents have to save more to smooth consumption over their life-cycle, since income under retirement is now lower among public sector workers. A rise in the capital stock increases the marginal productivity of labor and therefore the wage rate. Public infrastructure remains roughly constant and the tax rate on consumption decreases by 5 percentage points with a less generous pension for public sector workers. Comparing this social security reform with the change in the public sector wage premium from 19% to 15%, one can see that the mechanisms of the two policy changes are different. In the social security reform, the bulk of the effect on output is explained by a rise in the accumulation of factors of production. Change in TFP accounts for nearly 11% of the change in the accumulation of physical (mainly) and human capital. In the case in which the public sector wage premium drops from 19% to 15%, TFP accounts for about 26% of the change in output, arising from the reallocation of resources in the economy. Notice also the human capital increases by more than in the case of the social security reform. The incentive to invest in financial assets to smooth consumption is higher under the social security reform (since human capital does not rise during retirement) than in the public sector wage premium policy.

Next, we align as before the public sector system to its private counterpart and also decrease the public sector wage premium from 19% to 15%. Results are shown in the last column of Table 4. There are important aggregate positive effects on output, capital accumulation and productivity. Notice that both production in the private and public sectors increase substantially when $\bar{q}$ is adjusted, and the tax rate on consumption is severely reduced. In this case, rent seeking costs as a share of income decrease by 2.2 percentage points while aggregate output increases by 11.2%. TFP accounts for 15.7% of the change in output and the rest is due to increases in factors accumulation, given the changes in incentives to invest in both forms of capital and the lower distortions after the policy change - lower tax rate and selection in the public sector is more efficient.[46]

**Decomposition.** Changing the wage premium or the pension gap affects the aggregate

[46] Another way to decompose the effects of government reforms which decreases the public sector earnings premium on aggregate output into TFP and factors accumulation is to consider a small open economy. In a small open economy, when there is any policy change resulting in an increase in the capital stock, this leads to downward pressures in the return of capital, which consequently generates an outflow of capital searching for (international) higher rates of returns. In the companion Online Appendix, we show that changes in TFP account for roughly 20% of the change in total output when we align the public sector pension system to its private counterpart and also decrease the public sector wage premium from 19% to 15%, as in the third column of Table 4.

output through three channels: occupational choice, human capital accumulation and physical capital accumulation. Previously, through the lens of an aggregate constant returns to scale production function, we could decompose the aggregate effects into changes in factors accumulation and changes in TFP. However, this aggregation is somewhat arbitrary. In order to understand further the mechanisms generating the aggregate effects, we proceed by quantifying the importance of each of these three channels in the following way: as we run policy experiments, we maintain the individuals' choice regarding their occupation, physical capital value and human capital value, one at a time, constant at the benchmark level.[47] Therefore, the difference between the output growth in each of the counterfactual cases, and the full effect shown in the top panel of Table 4 is accounted for by the factor we kept unchanged.

As can be seen in the bottom panel of Table 4, when we keep the agents' asset holdings constant, we find that the output growth, in the case of a decrease in the public sector wage premium from 19% to 15%, is 2.54%. This is lower than the full effect of 4.65%, which implies that the increase in physical capital explains nearly 45% of the total growth in this case. Following the same procedure for the other channels, we find that the increase in human capital accumulation explains nearly 35% of the output growth, while the reminder 20% is accounted for by the changes in the occupational choices.

When we only change the social security parameters, the importance of capital accumulation is greater, accounting for 68% of the output growth. This is because of the stronger incentives to save due to the social security reform. Consequently, the importance of human capital and occupational choices are much lower, explaining 24% and 9%, respectively. The figures for the case where we implement both policy experiments at the same time are: 54.6% for physical capital, 30.4% for human capital and 15.7% for the channel acting through changes in occupational choices. Interestingly, when we use the approximated aggregate production technology to understand the role of TFP growth, we show that TFP growth accounts for approximately 14.1% of the overall effect in the case when we change both policies (last Column of Table 4).

**Welfare implications** Finally, in Figure 6, we show the welfare effects by individuals' labor and entrepreneurial abilities of reducing the public sector wage premium (from 19% to 15%) and aligning the public sector pension to the private one.[48] Welfare effects are mea-

[47] More precisely: In the bottom panel of Table 4, the row labeled as physical capital implies that agents in this counterfactual economy have the same asset value for physical capital as in the benchmark economy. For the human capital and occupational choice rows, we maintain the corresponding occupational choice and the human capital value as in the benchmark economy, respectively.

[48] We only show the case in which we adjust $\bar{q}$ to keep the share of civil servants unchanged. Results when

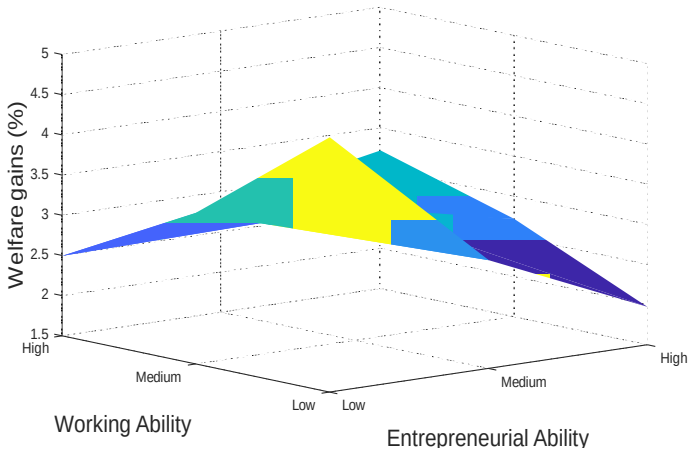


Figure 6: **Welfare.** The z axis measures the percentage change in welfare when we change the wage premium to 15% and the social security parameters, adjusting $\bar{q}$ to keep the share of civil servants unchanged.

sured by the permanent increase/decrease in consumption such that the utility of each individual in the baseline economy is similar to the economy with the government reform (i.e., pension reform and reduction in the public wage premium). Welfare is measured by the ex-ante expected (with respect to idiosyncratic shocks) lifetime utility of a newborn in a stationary equilibrium. Given that all newborns start with zero assets, welfare effects are the consumption equivalent such that the expected value function for a given individual at age 25 is the same in the baseline model and in the counterfactual economy. As can be seen in Figure 6, in the long-run, all individuals would be better-off with the aforementioned policy change. This policy reform increases the wage rate and productivity with positive effects on welfare. More importantly, the model predicts that individuals with low working and entrepreneurial abilities would be the ones who benefit the most, with their welfare increasing 4.6% in consumption-equivalent to the baseline. These individuals benefit much less from the generosity of the public sector compensation as application decisions are correlated with abilities and income. In addition, their welfare would increase by the rise in the marginal productivity of labor and lower tax on consumption.

The welfare measure report in Figure 6 compares the welfare of individuals in two different economies at the age 25 before they make their economic decisions. It shows that in the long-run all 25 year olds would be better off in an economy with a lower public earnings premium. Although this welfare figure is informative, a complete analysis of the welfare would require us to compute the transition dynamics after the reform and the welfare of all individuals during transition. Certainly the reform above would generate

we adjust $\theta$ are similar and for the sake of space we do not present them here.

some welfare losses during transition, given that individuals who are civil servants before the reform would have a major reduction in their earnings. Such individuals would have a vested interest in an overpaid public sector and could erect barriers to block reforms which would decrease the public earnings premium. A strategy to mitigate some of these welfare losses would be to implement a gradual reform, such that during transition two systems in the public sector would co-exist. An old system of earnings for existing civil servants and an alternative one for new public employees, which would be less generous. Some countries have succeeded in increasing retirement age and changing the pension system for new workers.

### 5.2.3 The effects of reducing stability

Now we introduce a probability, $1-\varrho$, that a public worker will lose her job in the public sector. Then, in the period that the worker loses her public job she will have to choose between a private job or become an entrepreneur. Once a private worker or an entrepreneur they can apply for a public job but will have to go to the whole costly application process again. The problem of the civil servant will then be described by the following value function:

$$V_{cs,t}(\omega) = \underset{a'_{cs}\geq 0, x_{cs}\geq 0}{Max} : u(c_{cs}) + \beta\vartheta_{t+1}E_{s'}\left[\varrho \max\left\{V_{cs,t+1}(\omega'), V_{w,t+1}(\omega'), V_{e,t+1}(\omega')\right\}\right.$$
$$\left. + (1-\varrho)\max\left\{V_{w,t+1}(\omega'), V_{e,t+1}(\omega')\right\}\right] \quad (25)$$

subject to (19). The rest of the model is the same. Clearly, in our benchmark case $\varrho = 1.00$. In our counterfactual exercise we decrease $\varrho$ from 1 to 0.99 such that there is a 1% probability that a public worker will lose her post in every year.[49] Table 5 contains the results for this policy experiment. We keep the share of civil servants constant by adjusting the probability of getting a public sector job, $\bar{q}$.

We can observe that output increases by more that 3% when the job destruction rate in the public sector increases from 0% to 1%. The number of applicants to public sector jobs decreases by nearly 15% and the economy reduces the inefficiency related to the public sector application process by 0.61 percentage point of output. Since the lump-sum cost $\theta$ remains unchanged, there is a larger share of highly productive individuals in the public

[49] When $\varrho = 1.00$ there is still the possibility that a public employee might choose to leave her job to become for instance an entrepreneur.

Table 5: **Decreasing job stability.** We decrease parameter $\varrho$ from 1 to 0.99. In Job Stability, we adjust $\bar{q}$ to keep the share of civil servants constant.

| Variable | Baseline | Job Stability |
|---|---|---|
| $Y = Y_c + Y_e$ | 100 | 103.42 |
| $TFP$ | 100 | 100.62 |
| $K$ | 100 | 104.64 |
| $N_p$ | 100 | 102.67 |
| % of entrepreneurs | 4.59% | 4.45% |
| % of civil servants | 10.02% | 10.05% |
| $G/Y$ | 18.21% | 18.06% |
| $G$ | 100 | 102.66 |
| $\tau_c$ | 30.70% | 29.68% |
| $MPL_g - MPL_p$ | -0.43 | -0.43 |
| $K_c/Y_c$ | 2.598 | 2.628 |
| $w$ | 0.799 | 0.806 |
| $r$ | 7.838% | 7.702% |
| % of applications | 3.38% | 2.89% |
| Rent seeking costs/Y | 3.61% | 2.99% |
| $1 - \bar{q}$ | 0.83 | 0.77 |
| **Decomposing the effect on output** | | |
| | Baseline | Job Stability |
| Full effect | 100 | 103.42 |
| Physical capital | - | 101.70 (49.8%) |
| Human capital | - | 101.14 (33.4%) |
| Occupational choices | - | 100.59 (16.9%) |

sector generating a growth in public infrastructure, which increases by 2.66% while the tax rate decreases from 30% to 29.68%. Notice that the risk of losing a job in the public sector has a strong effect on precautionary savings since physical capital increases by 4.6%, which is about twice the increase in the average human capital. Growth in TFP due to inputs reallocation accounts for 18.12% of the increase in output. When we use the decomposition introduced previously, then occupational choices account for 17% of the overall effect. Therefore, changing the attractiveness of the public sector by introducing a probability of layoff can also have sizeable output and productivity effects.

# 6 Concluding remarks

Differences in earnings and labor legislation between public and private workers might have an impact on the occupational decision of agents. The public sector might attract high productive and risk averse agents looking for a more stable and higher paid job, creating a public sector job queue and crowding out private sector employment and entrepreneurship. This can be particularly important in countries with an overpaid and large

public sector facing pressure for fiscal consolidation. We construct a life-cycle economy with endogenous occupational choice and heterogeneous agents, which is able to assess the role of public-private earnings premium on aggregate output and productivity and can be applied to different environments and countries. We quantitatively investigate the case of Brazil, which is a country with a high public wage premium and an unequal pension system for existing workers. We show that public sector reforms, which would decrease the public sector earnings premium could have important long-run aggregate effects on factor prices, productivity and output. For instance, a reform which would decrease the public-private wage premium from its benchmark value of 19% to 15% and would align the pension of public sector workers to the one in place for private sector workers could increase aggregate output by 11.2% in the long-run without any significant effect on the supply of public infrastructure. We show that factors reallocation (TFP growth) accounts for about 16% of the aggregate change in output. The rest is due to changes in the accumulation of factors of production. We show that in the long-run all individuals at age 25 (initial period of life in the model) would be better off in an economy with a lower public earnings premium. Nevertheless, during transition such reforms could generate important welfare losses for some individuals who would have a vested interest in an overpaid public sector and could erect barriers to block any reform to change the public sector earnings premium. As Olson (1982) has argued in his insightful discussion on the role of special groups in shaping policies: "The most substantial and wealthy interests are relatively better organized in the unstable society, but they often own an unrepresentative mix of a country's productive factors. They obtain policies that favor themselves and work in different ways against the interests of the larger unorganized groups in the society, thereby making the distribution of income far more unequal... " and leading to a decline in aggregate efficiency.

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# A Estimation and Regression Results

This appendix presents statistics and regression results. Table 6 presents regression results for a sample of countries in which the dependent variable is the logarithm of the GDP per equivalent adult corrected by purchasing power parity. The regressor of interest is the relative compensation of a public employee. This is constructed by dividing total compensation of general government employees as share of total labor income (i.e., $GDP$ times 2/3) by total employment in the public sector as a share of the labor force. There are data for two years: 2000 and 2009. Due to data restrictions we use employment in general

Table 6: Relative compensation of a public employee and GDP per adult. *Source:* Government at a glance and Penn World Tables.

| | Dependent variable: Log of GDP per equivalent adult | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| | 2000 | 2009 | 2009 | Pooled | Pooled | Pooled |
| Relative compensation of a | -1.55** | -0.85 | -1.29** | -1.13** | -1.40** | -2.80** |
| public employee | (-2.28) | (-1.20) | (-2.14) | (-2.26) | (-3.18) | (-2.06) |
| Total employment in the public | | | | | | -0.08 |
| sector (% of the labor force) | | | | | | (-0.98) |
| Total compensation of general government | | | | | | 0.10 |
| employees (% of GDP) | | | | | | (0.90) |
| Year | | | | Yes | Yes | Yes |
| Greece | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 26 | 28 | 26 | 54 | 52 | 52 |
| $R^2$ | 0.21 | 0.09 | 0.23 | 0.14 | 0.23 | 0.24 |

All specifications include a constant, not reported. T-Statistics are presented in parentheses, using heterosk.-consistent standard errors. * indicates significant at the 90 percent confidence level and ** a 95 percent confidence level.

government and public corporations as a share of the labor force in 2008, but we use compensation of general government employees as a share of GDP in 2009. Data come from the OECD (2011) report Government at a glance. We control for Greece fixed effects, since relative compensation of a public employee in Greece is about twice the average sample and more than 50 percent of the country with the second largest measure for this variable. There are 26 countries in the 2000 sample.[50] Column (1) uses only the 2000 sample. The 2009 sample contains the same countries as that of 2000, but has two more countries (Russia and Turkey). Columns (2) and (3) use only the 2009 sample. Column (3) corrects for any attrition problem by using the 2009 data but by considering only the countries that also appear in the 2000 sample. Notice that in all columns, except for the second column, the coefficient of the variable "relative compensation of a public employee" is negative and statistically different from zero at the 95 percent confidence level. Columns (4) to (6) use the pooled sample. In columns (5) and (6) we use only data for countries which appear in both samples. The last column add the share of employment in the public sector and the overall total compensation of public employees as a share of GDP.

This appendix also provides statistics and estimations for the Brazilian economy using microdata from the 2008 Brazilian National Household Survey - PNAD (*Pesquisa Nacional por Amostra de Domicílios*). Table 7 contains summary statistics for individuals who have

[50]The countries are: Australia, Canada, Chile, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, Ireland, Israel, Italy, Japan, Luxembourg, Mexico, Netherlands, New Zealand, Poland, Slovak Republic, Slovenia, Spain, Switzerland, the United Kingdom, and the United States.

Table 7: Summary statistics. Source: 2008 PNAD. Income per hour is constructed by dividing individuals' monthly income in their main job by the numbers of hour worked in this job: 25-65 year olds who have worked at least 20 hours per week and are not retired.

| | Fraction in the population (25-65 years) | Average income per hour (R$) | Median-mean ratio | Std dev. of income per hour | Gini of income per hour | Average tenure on the job |
|---|---|---|---|---|---|---|
| All individuals | 100% | 6.18 | 0.57 | 9.77 | 0.51 | 8.28 |
| Civil servants | 10% | 11.72 | 0.63 | 13.71 | 0.48 | 13.36 |
| Private workers | 84.8% | 5.46 | 0.64 | 7.26 | 0.45 | 6.69 |
| Entrepreneurs | 5.2% | 14.95 | 0.59 | 23.09 | 0.52 | 10.84 |
| | Average income per hour (R$) at age 25 | Std dev. of income per hour at age 25 | | | | |
| workers and civil servants | 4.27 | 3.89 | | | | |
| Entrepreneurs | 11.86 | 13.30 | | | | |

worked at least 20 hours per week, are 25-65 years old, and are not retired. We classify the occupation of individuals by their main occupation. Observe that about 10% of workers are public employees and 4.5% are entrepreneurs. The unconditional mean of income per hour is two times higher for a public employee than the mean income per hour of a private worker. Average total income for full time workers is also about twice as large for civil servants (R$ 1,959.26) as that of private employees (R$ 1,014.91). Therefore, although government jobs presumably come with a fixed workweek length, the higher wage does not seem to be driven by equalized income but, rather, by fewer hours in the public sector. Tenure on the job is about 2 times higher for public than for private employees.

Table 8 presents results for eight Mincerian wage equations in which the dependent variable is the logarithm of income per hour.[51] Columns (1)-(3) use the standard human capital variables, such as schooling and experience (represented by age and age squared) as controls. There are 16 schooling dummies, each representing the number of years of completed schooling of the individual. The dummy goes from no schooling to 15 or more years of completed schooling. We cannot differentiate if an individual has 15 or 16 years of completed schooling. However, there are only 0.4% of individuals with 15 or more years of schooling and therefore results should be robust to the case in which we could differentiate years of schooling above 15 years. In addition to these variables, in column (4) we

[51] We have also run regressions using total income for full time workers as the dependent variable. The public sector premium is almost unchanged. For instance, for the specification presented in column (3) the coefficient of the variable public is 0.21 instead of 0.25 with a t-statistic of 24.69. Results are not driven by attrition issues.

also add the variable gender and whether or not the worker has a formal job. In column (5) we add control for 13 occupations. Notice that the coefficient of the variable *civil servant* is statistically different from zero in all regressions and it ranges from 19% to 25%.[52] The potential problem of unobservable selectivity implies that our OLS regression might not be capturing the exogenous effects of public sector premium on wages. The standard approach to address this issue is to use instrumental variable (IV) techniques. However, this procedure depends on the presence of valid instruments for the indicator variable *civil servant*. Since we do not have a valid instrument in our sample and it is difficult to address this bias in non-experimental data, we use the procedure developed by Altonji, Elder, and Taber (2005) to investigate the potential size of any bias due to unobservable variables. The main hypothesis in their procedure is that selection of observable variables is the same as that of unobservable variables, such that: $\frac{Cov(\epsilon,\, civil\ servant\ (CS))}{Var(\epsilon)} = \frac{Cov(\beta X,\, civil\ servant(CS))}{Var(\beta X)}$, where $X$ is a vector of observable characteristics, and $\epsilon$ is the error term potentially correlated with *civil servants*. This is a valid procedure when the point estimates for *civil servant* are sensitive to the inclusion of additional control variables, which corresponds to our case, since when we introduce control for occupations and the formal sector dummy the estimated coefficient of the variable civil servant decreases in magnitude from 25% to 19%. The biased from OLS is $\frac{Cov(\epsilon,\, \tilde{CS})}{Var(\tilde{CS})}$, where $\tilde{CS}$ denotes the residuals from a regression of the variable *civil servant* on $X$. Although positive which is an evidence of a positive correlation of unobservable variables in the wage equation and the variable *civil servant*, the estimated bias in the two most complete specifications (columns (4) and (5)) are not statistically different from zero and it does not seem that the estimated public sector wage premium is driven by unobservable variables.

We then consider specification in column (3) of Table 6 but estimate the wage equation using quantile regression. Since quantile regression procedure produces one point estimation for each quantile, for the sake of space, we focus only on the coefficient of the indicator variable *civil servant*. Figure 7 reports the estimated coefficient of this variable for each quantile of the conditional wage distribution, as well as the 95% confidence intervals. The OLS estimate is also presented by the dotted horizontal line together with its 95 percent confidence interval. Quantile regression provides the appropriate tool to determine whether or not there is any difference in the wage premium for different quantiles of the conditional wage distribution. Observe that the coefficient of the dummy variable civil servant is positive and statistically different from zero for all quantiles. It is also quite flat

[52]The introduction of a control for rural activity does not also change the magnitude and statistical significance of the public sector premium.

Table 8: Log of income per hour. Source: 2008 PNAD.

| | (1) All indiv. (>=16 yrs) | (2) All indiv. (25-65 yrs) | (3) Workers and civil servants (25-65 yrs) | (4) Workers and civil servants (25-65 yrs) | (5) Workers and civil servants (25-65 yrs) | (6) Only workers (25-65 yrs) | (7) Only civil servants (25-65 yrs) | (8) Entrepr. (25-65 yrs) |
|---|---|---|---|---|---|---|---|---|
| Constant | -0.4075** | -0.3324** | -0.1899** | -0.2582** | 0.0318 | -0.2102** | 0.0077 | 0.6857** |
| | (-27.00) | (-10.28) | (-5.46) | (-7.66) | (-0.83) | (-5.73) | (0.07) | (3.15) |
| Schooling dummies | YES | YES | YES | YES | YES | YES | YES | YES |
| Age | 0.0439** | 0.0411** | 0.0417** | 0.0433** | 0.0426** | 0.0438** | 0.0383** | 0.0262** |
| | (57.27) | (25.49) | (24.19) | (26.01) | (25.74) | (23.92) | (7.13) | (2.60) |
| $Age^2$ | -0.0004** | -0.0004** | -0.0004** | -0.0004** | -0.0004** | -0.0004** | -0.0003** | -0.0001 |
| | (-36.59) | (-17.38) | (-17.17) | (-18.65) | (-18.46) | (-17.71) | (-3.97) | (-1.24) |
| Civil servant | 0.2379** | 0.2309** | 0.2549** | 0.2383** | 0.1875** | | | |
| | (36.91) | (32.82) | (36.11) | (34.39) | (22.89) | | | |
| Female | | | | -0.2953** | -0.2624** | | | |
| | | | | (-71.04) | (-57.14) | | | |
| Formal | | | | 0.2115** | 0.1892** | | | |
| | | | | (45.23) | (38.98) | | | |
| Occupation | NO | NO | NO | NO | YES | NO | NO | NO |
| N. of Observ. | 152,309 | 116,478 | 91,265 | 91,265 | 91,265 | 79,146 | 12,119 | 5,931 |
| Adjusted $R^2$ | 0.3649 | 0.3806 | 0.3839 | 0.4294 | 0.4393 | 0.3190 | 0.3351 | 0.2094 |
| Residual variance | 0.3976 | 0.3980 | 0.3995 | 0.3674 | 0.3890 | 0.4061 | 0.4102 | 0.4161 |

All specifications include a constant, not reported. T-Statistics are presented in parentheses, using heterosk.-consistent standard errors. * indicates significant at the 90 percent confidence level and ** a 95 percent confidence level.

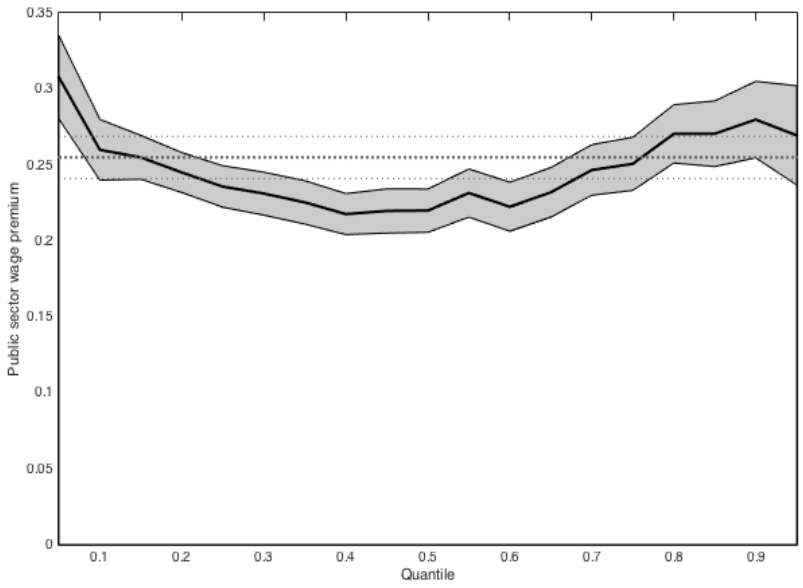


Figure 7: **Quantile regression.** Public sector wage premium for different quantile of the conditional wage distribution. *Solid line:* quantile point coefficient. *Grey area:* 95 percent confidence interval of the quantile coefficient. *Thick dotted line:* OLS point coefficient. *Thin dotted line:* 95 percent of the OLS point coefficient.

for all quantiles. It varies from 31% (low and high quantiles) to 22% (middle quantiles).

# B Proofs of the Stylized Model

**Social Planner's Problem.** Consider the model of Section 3. Abstracting from distributional issues, a benevolent planner maximizes aggregate output subject to the feasibility constraint:

$$\max_{\{\bar{h}_e(h_w),n(h_e),N_g,G\}} \int_0^\infty \int_{\bar{h}_e(h_w)}^\infty G^\chi h_e^{1-v} n(h_e)^{v(1-\varphi)} dF_e(h_e) dF_w(h_w),$$

subject to $G = A_g N_g^{1-\alpha}$ and

$$N_g + \int_0^\infty \int_{\bar{h}_e(h_w)}^\infty n(h_e) dF_e(h_e) dF_w(h_w) = \int_0^\infty \int_0^{\bar{h}_e(h_w)} h_w dF_e(h_e) dF_w(h_w).$$

The marginal productivity of labor will be equalized among all entrepreneurs, and the shadow value of a marginal worker in the public sector is the same as the shadow value of a marginal worker in the private sector. Therefore, there is no premium to work in the public sector and it does not matter how workers are allocated to work in either the private or the public sector. The equations which characterize the solution of this problem are:

$$n(h_e) = \left(\frac{h_e}{\bar{h}_e^{SP}(h_w)}\right)^{\frac{1-v}{1-v(1-\varphi)}} n(\bar{h}_e^{SP}(h_w)), \tag{26}$$

$$n(\bar{h}_e^{SP}(h_w)) = \frac{v(1-\varphi)}{1-v(1-\varphi)} h_w, \tag{27}$$

$$N_g = \frac{(1-\alpha)\chi}{(1-\alpha)\chi + v(1-\varphi)} \int_0^{\infty} \int_0^{\bar{h}_e^{SP}(h_w)} h_w dF_e(h_e) dF_w(h_w), \tag{28}$$

$$\int_0^{\infty} \int_{\bar{h}_e^{SP}(h_w)}^{\infty} \left(\frac{h_e}{\bar{h}_e^{SP}(h_w)}\right)^{\frac{1-v}{1-v(1-\varphi)}} h_w dF_e(h_e) dF_w(h_w) = \frac{1-v(1-\varphi)}{(1-\alpha)\chi + v(1-\varphi)} \int_0^{\infty} \int_0^{\bar{h}_e^{SP}(h_w)} h_w dF_e(h_e) dF_w(h_w). \tag{29}$$

Now, suppose that the size of the public sector is determined exogenously, such that $N_g = \phi_g$. If the size of the government is (not) too large, such that

$$N_g > (<) \frac{\chi(1-\alpha)}{\chi(1-\alpha) + v(1-\varphi)} \int_0^{\infty} \int_0^{\bar{h}_e^{SP}(h_w)} h_w dF_e(\bar{h}_e^{SP}(h_w)) dF_w(h_w),$$

where $\bar{h}_e^{SP}(h_w)$ corresponds to the threshold entrepreneurial productivity function determined by the Social Planner in the unconstrained problem, then the marginal productivity of labor in the government sector is smaller (larger) than the marginal productivity of labor in the private sector.

**Decentralized equilibrium.** Suppose that there exists a wage premium $\zeta > 0$ Let public employees be those individuals with a vector $(h_w, h_e)$, such that $h_w \geq h_g$ and $h_e \leq \bar{h}_e(h_w, w; \zeta)$. Firstly, just to fix ideas, assume that the marginal productivity of labor is the same in the public and private sector, then it can be shown that there will be misallocation of labor in the extensive margin. In order to see this, let individual $A$ be a civil servant, while individual $B$ is an entrepreneur. Let $wh_w^A = w\bar{h}_g(w;\zeta) + \epsilon$ and $wh_w^B = w\bar{h}_g(w;\zeta) - \epsilon$. In addition, let $\pi(h_e^A; w) = (1+\zeta)wh_w^A - \epsilon$, and $\pi(h_e^B; w) = wh_w^B + \epsilon$, for any small $\epsilon > 0$. Notice that such conditions are consistent with the occupational decision of agent $A$ and agent $B$. Since the marginal productivity of labor is the same in the public and private sectors, then there will be misallocation in the extensive margin if $\pi(h_e^A; w) - wh_w^A > \pi(h_e^B; w) - wh_w^B$. Then

$$(1+\zeta)wh_w^A - \epsilon - wh_w^A > wh_w^B + \epsilon - wh_w^B \Rightarrow \zeta wh_w^A > 2\epsilon,$$

and

$$\zeta w \bar{h}_g(w;\zeta) > (2-\zeta)\epsilon.$$

Since $\epsilon$ can be arbitrarily small, for each $\zeta > 0$ we can find an $\epsilon^{\zeta}$ in which the above condition holds and there will exist misallocation of labor in the extensive margin.

Now, Assume also that the government keeps the size of the public sector constant. The labor excess demand, $LED(w;\zeta)$, is:

$$LED(w;\zeta) = \int_0^{\infty}\int_{\bar{h}_e(h_w,w;\zeta)}^{\infty} n(h_e;w)dF_e(h_e)dF_w(h_w) + \int_0^{\bar{h}_g(w;\zeta)}\int_{\bar{h}_w(h_w,w)}^{\bar{h}_e(h_w,w;\zeta)} n(h_e;w)dF_e(h_e)dF_w(h_w) - \int_0^{\bar{h}_g(w;\zeta)}\int_0^{\bar{h}_w(h_w,w)} h_w dF_e(h_e)dF_w(h_w),$$

where the demand of each entrepreneur $n(h_e,w)$ is given by Equation (3), and threshold functions $\bar{h}_e(h_w,w;\zeta)$ and $\bar{h}_w(h_w,w)$ are defined by Equations (6) and (7), respectively. Labor in the public sector is determined exogenously, such that

$$N_g = \phi_g = \int_{\bar{h}_g(w;\zeta)}^{\infty}\int_0^{\bar{h}_e(h_w,w;\zeta)} h_w dF_e(h_e)dF_w(h_w).$$

Since the government keeps the size of the public sector constant, $\frac{\partial N_g}{\partial w} = 0$, then:

$$\frac{\partial \bar{h}_g}{\partial w}\bar{h}_g f_w(\bar{h}_g)F_e(\bar{h}_e) = \int_{\bar{h}_g}^{\infty}\frac{\partial \bar{h}_e}{\partial w}h_w f_e(\bar{h}_e)dF_w(h_w) > 0 \;\Rightarrow\; \frac{\partial \bar{h}_g}{\partial w} > 0. \tag{30}$$

In this case, we have that $\lim_{w\to 0} LED(w;\zeta) = \infty$, while $\lim_{w\to 0} LED(w;\zeta) = -\int_0^{\infty}\int_0^{\infty} h_w dF_e(h_e)dF_w(h_w) < 0$, which implies that there is a wage rate $w(\zeta)$ such that $LED(w(\zeta);\zeta) = 0$. In addition:

$$\frac{\partial LED}{\partial w} = \int_0^{\infty}\int_{\bar{h}_e}^{\infty}\frac{\partial n}{\partial w}dF_e(h_e)dF_w(h_w) + \int_0^{\bar{h}_g}\int_{\bar{h}_w}^{\bar{h}_e}\frac{\partial n}{\partial w}dF_e(h_e)dF_w(h_w) - \int_{\bar{h}_g}^{\infty}\frac{\partial \bar{h}_e}{\partial w}n(\bar{h}_e,w)f_e(\bar{h}_e)dF_w(h_w) - \int_0^{\bar{h}_g}\frac{\partial \bar{h}_w}{\partial w}n(\bar{h}_w,w)f_e(\bar{h}_w)dF_w(h_w) - \int_0^{\bar{h}_g}\frac{\partial \bar{h}_w}{\partial w}h_w f_e(\bar{h}_w)dF_w(h_w) - \frac{\partial \bar{h}_g}{\partial w}f_w(\bar{h}_g)\left(\bar{h}_g F_e(\bar{h}_w(\bar{h}_g)) - \int_{\bar{h}_w(\bar{h}_g)}^{\bar{h}_e(\bar{h}_g)} n(h_e,w)dF_e(h_e)\right). \tag{31}$$

All terms of Equation (31) are negative, except the last, which can be positive or negative. Notice that when the public sector wage premium goes to zero then $\bar{h}_e(\bar{h}_g)$ goes to $\bar{h}_w(\bar{h}_g)$, and the last term becomes negative. Therefore, there is a sufficient small public sector wage premium such that the labor excess demand will cross the zero line at only one

point. Moreover:[53]

$$\frac{\partial LED}{\partial \zeta} = -\int_{\bar{h}_g}^{\infty} \frac{\partial \bar{h}_e}{\partial \zeta} n(\bar{h}_e, w) f_e(\bar{h}_e) dF_w(h_w) - \frac{\partial \bar{h}_g}{\partial \zeta} f_w(\bar{h}_g) \left( \bar{h}_g F_e(\bar{h}_w(\bar{h}_g)) - \int_{\bar{h}_w(\bar{h}_g)}^{\bar{h}_e(\bar{h}_g)} n(h_e, w) dF_e(h_e) \right), \tag{32}$$

which is negative whenever the public sector wage premium is not too big. When the public sector wage premium is large we cannot guarantee that $\frac{\partial LED}{\partial \zeta}$ is negative. Using the implicit function theorem, we have

$$\frac{\partial w}{\partial \zeta} = -\frac{\frac{\partial LED}{\partial \zeta}}{\frac{\partial LED}{\partial w}},$$

which is negative for sufficient small public sector wage premium. Aggregate output is then given by:

$$Y(w, \zeta) = \int_0^{\infty} \int_{\bar{h}_e(h_w, w; \zeta)}^{\infty} y(h_e, w) dF_e(h_e) dF_w(h_w) + \int_0^{\bar{h}_g(w;\zeta)} \int_{\bar{h}_w(h_w, w)}^{\bar{h}_e(h_w, w; \zeta)} y(h_e, w) dF_e(h_e) dF_w(h_w). \tag{33}$$

Therefore:

$$\begin{aligned} \frac{\partial Y(w, \zeta)}{\partial \zeta} &= \int_0^{\infty} \int_{\bar{h}_e}^{\infty} \frac{\partial y}{\partial w} \frac{\partial w}{\partial \zeta} dF_e(h_e) dF_w(h_w) + \int_0^{\bar{h}_g} \int_{\bar{h}_w}^{\bar{h}_e} \frac{\partial y}{\partial w} \frac{\partial w}{\partial \zeta} dF_e(h_e) dF_w(h_w) \\ &- \int_{\bar{h}_g}^{\infty} \left( \frac{\partial \bar{h}_e}{\partial w} \frac{\partial w}{\partial \zeta} + \frac{\partial \bar{h}_e}{\partial \zeta} \right) y(\bar{h}_e, w) f_e(\bar{h}_e) dF_w(h_w) - \int_0^{\bar{h}_g} \frac{\partial \bar{h}_w}{\partial w} \frac{\partial w}{\partial \zeta} y(\bar{h}_w, w) f_e(\bar{h}_w) dF_w(h_w) \\ &+ \int_{\bar{h}_w(\bar{h}_g)}^{\bar{h}_e(\bar{h}_g)} \left( \frac{\partial \bar{h}_g}{\partial w} \frac{\partial w}{\partial \zeta} + \frac{\partial \bar{h}_g}{\partial \zeta} \right) y(\bar{h}_e(\bar{g}_g), w) f_w(\bar{h}_e(\bar{h}_g)) dF_e(h_e). \end{aligned} \tag{34}$$

In the limit we have

$$\begin{aligned} \lim_{\zeta \to 0} \frac{\partial Y(w, \zeta)}{\partial \zeta} = &\underbrace{\frac{\partial w}{\partial \zeta} \left( \int_0^{\infty} \int_{\bar{h}_e}^{\infty} \frac{\partial y}{\partial w} dF_e(h_e) dF_w(h_w) - \int_0^{\infty} \frac{\partial \bar{h}_e}{\partial w} y(\bar{h}_e, w) f_e(\bar{h}_e) dF_w(h_w) \right)}_{\text{General equilibrium effect (positive)}} \\ &\underbrace{- \int_{\bar{h}_g}^{\infty} \frac{\partial \bar{h}_e}{\partial \zeta} y(\bar{h}_e, w) f_e(\bar{h}_e) dF_w(h_w).}_{\text{Selection effect (negative)}} \end{aligned} \tag{35}$$

[53] We also have that $\frac{\partial \bar{h}_g}{\partial \zeta} \bar{h}_g f_w(\bar{h}_g) F_e(\bar{h}_e) = \int_{\bar{h}_g}^{\infty} \frac{\partial \bar{h}_e}{\partial \zeta} h_w f_e(\bar{h}_e) dF_w(h_w) > 0$.