Superradiance induced particle flow via dynamical gauge coupling

W. Zheng and N. R. Cooper

T.C.M. Group, Cavendish Laboratory, J.J. Thomson Avenue, Cambridge CB3 0HE, United Kingdom

(Dated: September 16, 2016)

We study fermions that are gauge-coupled to a cavity mode via Raman-assisted hopping in a one dimensional lattice. For an infinite lattice, we find a superradiant phase with infinitesimal pumping threshold which induces a directed particle flow. We explore the fate of this flow in a finite lattice with boundaries, studying the non-equilibrium dynamics including fluctuation effects. The short time dynamics is dominated by superradiance, while the long time behaviour is governed by cavity fluctuations. We show that the steady state in the finite lattice is not unique, and can be understood in terms of coherent bosonic excitations above a Fermi surface in *real* space.

Quantum matter interacting with gauge fields is a central topic of modern physics. In cold atom systems, although atoms are charge neutral, Abelian or non-Abelian gauge potentials can be simulated by various methods[1][2], such as rotation[3], magnetic gradients[4], Raman coupling[5][6][7], laser-assisted hopping[8][9][10][11], and lattice "shaking"[12]. However, simulation of a dynamical gauge field, possessing its own quantum dynamics, is still a great challenge[13][14].

On the other hand, subjecting quantum gases to optical cavities[15] has drawn a lot of attention in recent years. Coupling cold atoms to a quantized cavity mode can dramatically change the properties of both the atomic gas and the cavity field. For example, a Bose-Einstein condensate coupled to a cavity can undergo a quantum phase transition to a supersolid phase, with the cavity field entering a "superradiant" phase with a non-zero expectation value[16][17][18][19][20]. The Bose-Hubbard model inside a cavity exhibits a rich phase diagram, due to cavity-induced long range interactions between atoms[21][22][23]. These successful experiments have stimulated many theoretical studies[24][25][26][27][28][29][30][31][32][33][34][35][36][37][38]. Dissipation of the cavity field causes significant back action on both the dynamics[39] and the steady state distribution[40][41][42][43] of the atoms.

In this letter, we study the steady states and the non-equilibrium dynamics of fermions in a one dimensional cavity-assisted hopping lattice. The phase of the cavity mode acts on the atoms as a vector potential, which has its own quantum dynamics controlled by the atom distribution. This system differs from the models in Refs.[35][37], where the cavity-assisted hopping acts only between two legs of a ladder. Allowing hopping along an infinite lattice, we find a transition, at infinitesimal pumping threshold, to a superradiant phase in which the gauge coupling induces a directed persistent current. In a finite lattice, with open boundary conditions, we show that there can be no superradiant steady state. We study the non-equilibrium dynamics in the finite lattice, incorporating fluctuation effects beyond mean field. We find a transition from a period of coherent hopping (as for the infinite lattice) at early times, to a late-time regime
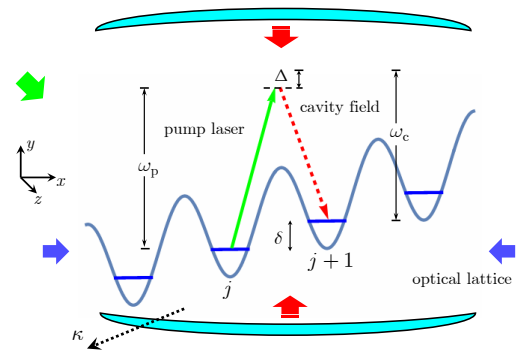

FIG. 1: The setup for cavity-assisted hopping on a lattice. A large energy offset δ prevents direct tunneling between neighbouring sites. The atoms can hop by a cavity-assisted Raman process, absorbing a pump photon at ω_p (solid green arrow) and emitting a photon at $\omega_p - \delta$ into the cavity (red dashed arrow). This emission is detuned from the cavity mode, ω_c, by $\Delta \ll \delta$. Cavity losses are described by κ.

in which dissipation dominates the transport and steady states.

Model.– We consider spinless atoms trapped by an optical lattice in a high-Q cavity, Fig.1. The optical lattice is in the x-direction, while the cavity mode is in the y-direction. The atom cloud is illuminated by a pump laser in the z-direction. We consider a strong transverse confinement to prohibit momentum transfer to the atoms, so the system is quasi one dimensional. By accelerating the optical lattice or applying a gradient magnetic field, an energy gradient can be imposed along the x-direction so that direct hopping is suppressed by a large energy offset δ between lattice sites. An atom can hop to the right by a Raman process, absorbing a pump photon (ω_p) and emitting at $\omega_p - \delta$. (We assume ω_p to be far detuned from the optical transition so the excited state population can be neglected.) We consider this emission to be enhanced by a cavity mode tuned close to this frequency, $\omega_c \simeq \omega_p - \delta$. (We assume that δ is sufficiently large that emission at $\omega_p + \delta$, corresponding to a hop to the left, is negligible.) We make a tight-binding approximation to

obtain the effective Hamiltonian ($\hbar = 1$ throughout):

$$\hat{H} = \Delta \hat{a}^\dagger \hat{a} - \sum_{j=1}^{L-1} \left(\lambda \hat{a}^\dagger \hat{c}_{j+1}^\dagger \hat{c}_j + \lambda^* \hat{a} \hat{c}_j^\dagger \hat{c}_{j+1} \right). \quad (1)$$

Here $\hat{a}$ is the field operator of the cavity photon expressed in a frame rotating at the frequency $\omega_p - \delta$ for which inter-site hopping is resonant; $\Delta \equiv \delta - \omega_p + \omega_c$ is the detuning of the cavity mode from resonance[63]; and $\hat{c}_j^{(\dagger)}$ are fermionic field operators on lattice sites j. (We shall also mention some results for hard-core bosons.) The cavity-assisted hopping $\lambda \hat{a}^\dagger$ has a phase given by the phase difference between the cavity field and the pump laser[64]. We choose to set the phase of the pump to zero, $\lambda^* = \lambda$, such that the hopping phase equals the phase of the cavity field.[65]

If the cavity were replaced by a second drive laser, at frequency $\omega_p - \delta$, such that the cavity field operator is replaced by the coherent state $\langle \hat{a} \rangle = \alpha = |\alpha| e^{i\theta}$, then the particles would experience the static Hamiltonian $\hat{H}(\alpha) = -\lambda \sum_j \left(\alpha^* \hat{c}_{j+1}^\dagger \hat{c}_j + \alpha \hat{c}_j^\dagger \hat{c}_{j+1} \right)$. The corresponding dispersion relation is

$$E_k = -2\lambda |\alpha| \cos(k + \theta) \quad (2)$$

for a particle of momentum k. Thus, the phase of the cavity field, θ, couples to the particles as a vector potential. In this driven case, the vector potential is static, set by the phase difference between the two driving lasers. Henceforth we shall treat the cavity field as dynamical, so the vector potential inherits its own quantum dynamics, linked to the distribution of particles. This differs from the cavity-assisted hopping in Refs.[34][35], where the hopping phase is fixed, and only the amplitude is dynamical.

Superradiance.– The leakage of photons from the cavity requires the full dynamics to be described by the Lindblad master equation, $\partial_t \rho = -i \left[\hat{H}, \rho \right] + \mathcal{L}[\rho]$, where ρ is the density matrix, and the Lindblad superoperator reads $\mathcal{L}[\rho] = \kappa \left(2\hat{a}\rho\hat{a}^\dagger - \hat{a}^\dagger \hat{a} \rho - \rho \hat{a}^\dagger \hat{a} \right)$. This describes a cavity photon loss rate of 2κ. The mean cavity field, $\langle \hat{a}(t) \rangle = \alpha(t)$, evolves as:

$$\partial_t \alpha = -i(\Delta - i\kappa)\alpha + i\lambda K, \quad (3)$$

where $K \equiv \langle \hat{K} \rangle$, with $\hat{K} \equiv \sum_{j=1}^{L-1} \hat{c}_{j+1}^\dagger \hat{c}_j$ the operator that couples to the cavity field (1). For steady states, $\partial_t \alpha = 0$, we obtain

$$\alpha = \frac{\lambda K}{\Delta - i\kappa}. \quad (4)$$

with Δ, κ and λ real parameters.

Consider first an infinitely long lattice. In this case, one can write $K = \sum_k e^{-ik} \langle \hat{n}_k \rangle$, where $\hat{n}_k$ counts the number of particles of momentum k. These occupations are conserved, $[\hat{n}_k, \hat{H}] = 0$, so λK in Eq. (3) can be

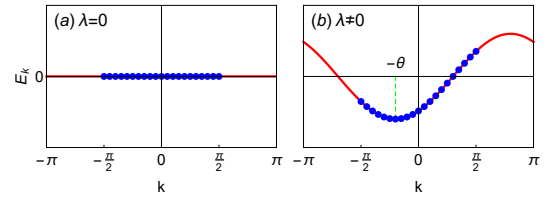

FIG. 2: Mean field band structure and momentum distribution of fermions. (a) No pumping field, $\lambda = 0$. (b) Non-zero pumping strength, $\lambda \neq 0$. Here $k = -\theta$ is the minimum of the band.

treated as an external source, determined by the initial momentum distribution. Provided the initial distribution has $|K| \neq 0$, the steady state has $|\alpha| \neq 0$, i.e. there is no threshold for superradiance. This differs from the usual Dicke-type setup[47], where superradiance appears only above a critical pumping strength.

Although the cavity field cannot change the momentum distribution of the atoms, the emergence of superradiance dramatically alters their dispersion (2). We find that superradiance leads to a directed persistent current. From (2) the particle velocity is $v_k = 2\lambda |\alpha| \sin(k + \theta)$, so the total current, $J = \sum_k v_k \langle \hat{n}_k \rangle$, may be written $J = -2\lambda \mathrm{Im}(\alpha^* K)$. Thus, there will be a non-zero net current if the phases of K and α differ. From Eq.(4), such a phase difference appears whenever there is cavity loss, $\kappa \neq 0$. For example, consider a half-filled system with $\langle \hat{n}_k \rangle = \Theta(|k| - \pi/2)$, see Fig. 2(a). One finds a real $K = L/\pi$, while the phase is $\tan \theta = \kappa/\Delta$. The minimum of the band is shifted to $k = -\theta$, see Fig. 2(b). This leads to an imbalance of left and right moving particles, resulting in a net current to the right. Thus, the dynamical vector potential self-organizes to induce a particle current. Indeed, on resonance, $\Delta = 0$, the steady state value of cavity phase θ maximizes the current ($\frac{\partial J}{\partial \theta} = 0$).

The importance of dissipation for the net current can also be seen by substituting Eq. (4) into the expression for the total current, giving $J = 2\kappa |\alpha|^2$. This has a simple interpretation. For a cavity occupation of $|\alpha|^2$, photon loss rate is $2\kappa |\alpha|^2$. To maintain the population $|\alpha|^2$, the scattering of pump photons into the cavity should compensate this loss. Each atom that scatters a photon from pump to cavity undergoes a hop by one site to the right, thus leading to a net current of $2\kappa |\alpha|^2$.

Now we switch to the finite lattice with open boundary conditions. The boundaries break translational invariance, so the cavity field can have a feedback on the distribution of the atoms. At mean field level, the equation-of-motion of the fermionic density matrix, $\rho_{ij}(t) = \left\langle \hat{c}_i^\dagger(t) \hat{c}_j(t) \right\rangle$, is

$$\partial_t \rho_{ij}(t) = -i\lambda A_{ij}(t), \quad (5)$$

where $A_{ij} = \alpha^* \rho_{i+1,j} + \alpha \rho_{i-1,j} - \alpha^* \rho_{i,j-1} - \alpha \rho_{i,j+1}$. With open boundary conditions, we can prove $\alpha^* K = \alpha K^*$
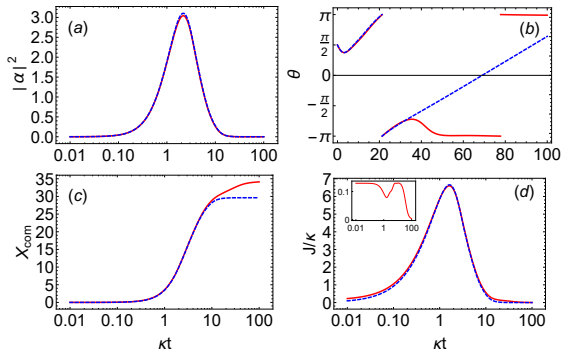
FIG. 3: The non-equilibrium dynamics of the cavity mode and the fermions. (a) The cavity field occupation $|\alpha(t)|^2$; (b) the phase of $\alpha(t)$; (c) the centre-of-mass of the fermions; (d) the total current of the fermions. Inset shows the current beyond mean field, $J^{cl} + J^{qu}$. The blue dashed lines are the mean field results with lattice length $L = 20$, particle number $N = 5$, detuning $\Delta/\kappa = 0.5$, and $\lambda/\kappa = 0.5$. The red solid lines are the results beyond mean field by solving Eq.(6) with the same parameters.

in any steady state[45]. Combining with Eq.(4), we find that the only steady state solution is $\alpha = K = 0$. The essential physics is that the boundaries preclude the steady state from carrying a net current, so the net photon scattering rate from pump mode to cavity mode must vanish.

Non-equilibrium dynamics.– If the particles start from a state with non-zero $|K|$, then, even in a finite lattice, the dynamics will first build up the cavity population $|\alpha|^2 \neq 0$. However, any eventual steady state must have $\alpha = K = 0$. To understand how the particles redistribute themselves in a finite lattice, we study the non-equilibrium dynamics. Combining Eq.(3) and Eq.(5) describes the coupled mean field dynamics of cavity field and fermions. However, Eq.(5) indicates that for $\alpha \to 0$ the fermions cannot hop. This is incorrect, since fluctuations of the cavity field will cause fermions to hop. To describe these fluctuation effects in the dynamics, we employ the Keldysh formulation of open quantum systems[48][49], and use the quasi-particle approximation to obtain[45]:

$$\partial_t \rho_{ij} = -i\lambda A_{ij}(t) + \frac{\kappa\lambda^2}{\Delta^2 + \kappa^2} B_{ij}(t), \qquad (6)$$

which supplements (5) with fluctuation corrections, $B_{ij}(t) = 2\rho_{i-1,j-1} - 2\rho_{i,j} - \sum_l (\rho_{i-1,l-1}\rho_{l,j} + \rho_{i,l}\rho_{l-1,j-1}) + \sum_l (\rho_{i+1,l+1}\rho_{l,j} + \rho_{i,l}\rho_{l+1,j+1})$. Here we ignore terms of higher order than λ^2, e.g. cavity-induced interactions between particles at order λ^4, which is valid for $\lambda \ll \kappa$. From the diagonal elements, i.e. the particle density $\rho_i = \rho_{ii}$, and the continuity equation, $\partial_t \rho_i + J_i - J_{i-1} = 0$, we derive the current J_i. This current can be separated into three parts, $J_i = J_i^{sr} + J_i^{cl} + J_i^{qu}$, where $J_i^{sr} = -\lambda \text{Im} (\alpha^* \rho_{i+1,i})$, is

the superradiant current as in mean field. The current $J_i^{cl} = \frac{2\kappa\lambda^2}{\Delta^2 + \kappa^2} (1 - \rho_{i+1}) \rho_i$ describes the semiclassical current, subject to Pauli blocking, arising from dissipative losses of the fluctuating cavity mode. This current is precisely that for classical driven-dissipative models such as the asymmetric exclusion process (ASEP)[51][52], which has interesting dynamical phase transitions sensitive to boundary conditions. The contribution $J_i^{qu} = -\frac{2\kappa\lambda^2}{\Delta^2 + \kappa^2} \sum_{l \neq i} \text{Re} (\rho_{i+1,l+1}\rho_{l,i})$, is a quantum correction to the semiclassical current induced by correlations and involving long-range coherence imposed by the fact that the cavity mode couples to all atoms. Here we see that even when the superradiance vanishes, $\alpha = 0$, the fluctuations of the cavity mode can induce a nonzero current $J_i^{cl} + J_i^{qu}$.

We have solved Eq.(6) combined with Eq.(3) numerically. Representative results are plotted in Fig.3. We choose the initial state to be the groundstate of N free fermions in a finite lattice with non-zero hopping, and the cavity mode empty. Because this initial particle state has coherence in real space, K is non-zero and, according to Eq.(3), it will first generate a superradiant state. Indeed, we find that the cavity occupation $|\alpha(t)|^2$ grows from zero, and reaches its maximum during a time interval $\tau_1 \sim \kappa^{-1}$ Fig.3(a). After that, due to the cavity loss, the superradiance decays to zero on a time scale $\tau_2 \sim \frac{\kappa\pi^2\nu(1-\nu)}{2\lambda^2\sin^2(\pi\nu)}$, where ν is the particle filling. This superradiant pulse is similar to those observed by illuminating degenerate quantum gases in free space[53][54]. Thus, the dynamics can be separated into two regimes. At short times $t \lesssim \tau_1 + \tau_2$, the particle dynamics is dominated by coherent hopping, which is assisted by the mean cavity field. In this regime, the centre-of-mass of the fermions, $X_{com} = \sum_j j \left\langle \hat{c}_j^\dagger \hat{c}_j \right\rangle$, increases quickly, see Fig.3(c). At long times, $t \gg \tau_1 + \tau_2$, the superradiance dies, and the particle dynamics is governed by the dissipative hopping. See Fig.3(c), after the collapse of superradiance, the mean field solution of $X_{com}(t)$ saturates, while the solution including fluctuations shows that $X_{com}(t)$ still grows slowly, until finally reaching a slightly larger final steady-state value.

To explore the final steady states reached long after superradiance has vanished (so $J^{sr} = 0$), in Fig.4(c,d) we plot the density distribution and the density matrix for the fermions at very late times, $\kappa t = 10^3$. Before discussing the results for fermions, it is helpful to consider the case of hard core bosons, see Fig.4(e,f). For hard core bosons, the steady-state density distribution is a step function, i.e. the rightmost N sites are fully populated, while others are empty. The single-particle density matrix shows no coherence. Since all the particles are blocked at the right side, no particle can hop to the right due to the hard core repulsion, giving zero semiclassical current, $J^{cl} = 0$. Vanishing coherence indicates the quantum correlation current J^{qu} is also zero. The situation is
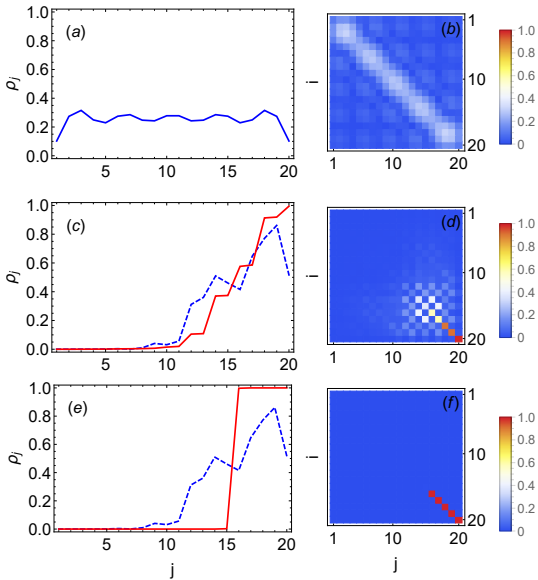
FIG. 4: (a)(b) The density distribution and the absolute value of single-particle density matrix of initial state. (c)(d) The final density distribution and density matrix of fermions (beyond mean field) after a long time evolution $\kappa t = 10^3$. (e)(f) The final density distribution and density matrix of hard core bosons starting from the same initial state. Blue dashed lines are the mean field results, while the red solid lines are the solutions beyond mean field. The lattice length $L = 20$, particle number $N = 5$, detuning $\Delta/\kappa = 0.5$, and $\lambda/\kappa = 0.5$.

different in the case of fermions. As seen in Fig.4(c,d), the density distribution is not a step function, while the density matrix retains non-zero correlations. In this case, the semiclassical current and its quantum correction do not separately vanish, instead, they cancel each other in the steady state, with $J^{\mathrm{cl}} + J^{\mathrm{qu}} = 0$. The quantum correction current and semiclassical current counteract each other in the case of fermions, while they add together in the case of bosons.

Steady states.– To understand the steady states, we adiabatically eliminate the cavity field[50], making use of the fact that superradiance is absent. We obtain the master equation for the fermion density matrix, $\partial_t \rho_{\mathrm{f}} = -i\left[\hat{H}_{\mathrm{eff}}, \rho_{\mathrm{f}}\right] + \frac{\kappa\lambda^2}{\Delta^2+\kappa^2}\left(2\hat{L}_{\mathrm{eff}}\rho_{\mathrm{f}}\hat{L}_{\mathrm{eff}}^\dagger - \hat{L}_{\mathrm{eff}}^\dagger\hat{L}_{\mathrm{eff}}\rho_{\mathrm{f}} - \rho_{\mathrm{f}}\hat{L}_{\mathrm{eff}}^\dagger\hat{L}_{\mathrm{eff}}\right)$, where $\hat{L}_{\mathrm{eff}} = \hat{K}$ is the effective Liouvillian operator, and the effective Hamiltonian reads $\hat{H}_{\mathrm{eff}} = -\frac{\kappa\lambda^2}{\Delta^2+\kappa^2}\hat{K}^\dagger\hat{K}$. Any pure state $|D\rangle$ for which $\hat{H}_{\mathrm{eff}}|D\rangle = E|D\rangle$ and $\hat{L}_{\mathrm{eff}}|D\rangle = 0$ is a steady state[55][56]. Here, these two conditions reduce to $\hat{K}|D\rangle = 0$. It can readily be verified that the step function state, $|\mathrm{step}\rangle \equiv \prod_{j=1}^N \hat{c}_{L-N+j}^\dagger |0\rangle$, in which the N particles occupy the N states furthest to the right, is a steady state. However we can also find other steady states. To construct these we define the bosonic operators $\hat{b}_s^\dagger = \sum_{j=s+1}^L \hat{c}_{j-s}^\dagger\hat{c}_j$, where

$s = 1, \cdots, L-1$. These are analogous to the bosonic operators used to solve the Tomonaga-Luttinger model, but now with real space separation s replacing momentum and with $|\mathrm{step}\rangle$ viewed as a Fermi sea in *real space* (with states occupied for $j > L - N$ and empty for $j \leq L - N$). One can verify that the state with one particle-hole excitation above this Fermi sea, created by applying one bosonic operator $\hat{b}_s^\dagger |\mathrm{step}\rangle$, is a steady state, via $\hat{K}\hat{b}_s^\dagger |\mathrm{step}\rangle = \left[\hat{K}, \hat{b}_s^\dagger\right] |\mathrm{step}\rangle = 0$ if $s \neq 1$ and $s \leq \mathrm{Min}\,(L-N, N)$. Similarly, for n_{b} such bosonic excitations the states $\prod_{\alpha=1}^{n_{\mathrm{b}}} \hat{b}_{s_\alpha}^\dagger |\mathrm{step}\rangle$, are steady states provided $s_\alpha \neq 1$, and $\sum_\alpha s_\alpha \leq \mathrm{Min}\,(L-N, N)$. Thus, we can construct a large number of steady states. For $N, (L-N) \to \infty$ (when all relevant states can be described by bosonic modes), any state that does not involve occupation of the $s = 1$ boson is a steady state. This arises because the Liouvillian operator $\hat{K}$ equals the bosonic annihilation operator $\hat{b}_1$, so dissipation can only damp the $s = 1$ collective mode. This differs from models involving coupling to a macroscopic number of dissipation channels[55][56], which lead to unique steady states. The large number of steady states for our model means that different initial conditions will lead to different final steady states.

Final remarks.– Our proposal explores one natural route to a dynamical gauge coupling in cold atom systems, using elements realized in current experiments. The dynamics of the superradiance can be observed by detecting the photons leaving the cavity, while the redistribution of the fermions could be measured by recently developed fermionic in-situ imaging in optical lattices[57][58][59][60][61][62]. Generalizing the model to higher dimensions will give similar non-equilibrium dynamics, and multiple steady states for each transverse mode.

[1] J. Dalibard, F. Gerbier, G. Juzeliūnas, and P. Öhberg, Rev. Mod. Phys. **83**, 1523 (2011).

[2] N. Goldman, G. Juzeliūnas, P. Öhberg, and I. B. Spielman, Rep. Prog. Phys. **77,** 126401 (2014).

[3] A. L. Fetter, Rev. Mod. Phys. **81**, 647 (2009).

[4] M. W. Ray, E. Ruokokoski, S. Kandel, M. Möttönen, and D. S. Hall, Nature (London) **505**, 657 (2014).

[5] Y.-J. Lin, R. L. Compton, A. R. Perry, W. D. Phillips, J. V. Porto, and I. B. Spielman, Phys. Rev. Lett. **102**, 130401 (2009).

[6] Y.-J. Lin, R. L. Compton, K. Jiménez-García, J. V. Porto, and I. B. Spielman, Nature (London) **462**, 628 (2009).

[7] Y.-J. Lin, K. Jiménez-García, and I. B. Spielman, Nature (London) **471**, 83 (2011).

[8] M. Aidelsburger, M. Atala, S. Nascimbène, S. Trotzky,

Y.-A. Chen, and I. Bloch, Phys. Rev. Lett. **107**, 255301 (2011).

[9] M. Aidelsburger, M. Atala, M. Lohse, J. T. Barreiro, B. Paredes, and I. Bloch, Phys. Rev. Lett. **111**, 185301 (2013).

[10] H. Miyake, G. A. Siviloglou, C. J. Kennedy, W. C. Burton, and W. Ketterle, Phys. Rev. Lett. **111**, 185302 (2013).

[11] M. Aidelsburger, M. Lohse, C. Schweizer, M. Atala, J. T. Barreiro, S. Nascimbène, N. R. Cooper, I. Bloch, and N. Goldman, Nature Phys. **11**, 162 (2015).

[12] J. Struck, C. Ölschläger, R. Le Targat, P. Soltan-Panahi, A. Eckardt, M. Lewenstein, P. Windpassinger, K. Sengstock, Science **333**, 996 (2011).

[13] D. Banerjee, M. Dalmonte, M. Müller, E. Rico, P. Stebler, U.-J. Wiese, and P. Zoller, Phys. Rev. Lett. **109**, 175302 (2012).

[14] D. Banerjee, M. Bögli, M. Dalmonte, E. Rico, P. Stebler, U.-J. Wiese, and P. Zoller, Phys. Rev. Lett. **110**, 125303 (2013).

[15] H. Ritsch, P. Domokos, F. Brennecke, and T. Esslinger, Rev. Mod. Phys. **85**, 553 (2013).

[16] K. Baumann, C. Guerlin, F. Brennecke, and T. Esslinger, Nature (London) **464**, 1301 (2010).

[17] R. Mottl, F. Brennecke, K. Baumann, R. Landig, T. Donner, T. Esslinger, Science **336**, 1570 (2012);

[18] R. Landig, F. Brennecke, R. Mottl, T. Donner, T. Esslinger, Nature Communications **6**, 7046 (2015).

[19] P. Domokos and H. Ritsch, Phys. Rev. Lett. **89**, 253003 (2002).

[20] D. Nagy, G. Kónya, G. Szirmai, and P. Domokos, Phys. Rev. Lett. **104**, 130401 (2010).

[21] M. R. Bakhtiari, A. Hemmerich, H. Ritsch, and M. Thorwart, Phys. Rev. Lett. **114**, 123601 (2015).

[22] J. Klinder, H. Keßler, M. R. Bakhtiari, M. Thorwart, and A. Hemmerich, Phys. Rev. Lett. **115**, 230403 (2015).

[23] R. Landig, L. Hruby, N. Dogra, M. Landini, R. Mottl, T. Donner and T. Esslinger, Nature, online 10.1038/nature17409 (2016).

[24] M. J. Bhaseen, M. Hohenadler, A. O. Silver, and B. D. Simons, Phys. Rev. Lett. **102**, 135301 (2009).

[25] S. Gopalakrishnan, B. L. Lev, and P. M. Goldbart, Phys. Rev. A **82**, 043612 (2010).

[26] M. Buchhold, P. Strack, S. Sachdev, and S. Diehl, Phys. Rev. A **87**, 063622 (2013).

[27] J. Keeling, M. J. Bhaseen, and B. D. Simons, Phys. Rev. Lett. **112**, 143002 (2014).

[28] F. Piazza, and P. Strack, Phys. Rev. Lett. **112**, 143003 (2014).

[29] Y. Chen, Z. Yu, and H. Zhai, Phys. Rev. Lett. **112**, 143004 (2014).

[30] Y. Deng, J. Cheng, H. Jing, and S. Yi, Phys. Rev. Lett. **112**, 143007 (2014).

[31] L. Dong, L. Zhou, B. Wu, B. Ramachandhran, and H. Pu, Phys. Rev. A. **89**, 011602 (2014)

[32] Y. Chen, H. Zhai, and Z. Yu, Phys. Rev. A **91**, 021602 (2015).

[33] J.-S. Pan, X.-J. Liu, W. Zhang, W. Yi, and G.-C. Guo, Phys. Rev. Lett. **115**, 045303 (2015).

[34] G. Szirmai, G. Mazzarella, and L. Salasnich, Phys. Rev. A **91**, 023601 (2015).

[35] C. Kollath, A. Sheikhan, S. Wolff, and F. Brennecke, Phys. Rev. Lett. **116**, 060401 (2016).

[36] Y. Chen, Z. Yu, and H. Zhai, Phys. Rev. A **93**, 041601 (2016).

[37] A. Sheikhan, F. Brennecke, C. Kollath, Phys. Rev. A **93**, 043609 (2016).

[38] N. Dogra, F. Brennecke, S.D. Huber, T. Donner, arXiv:1604.00865.

[39] G. Kónya, G. Szirmai, D. Nagy, and P. Domokos, Phys. Rev. A **89**, 051601(R) (2014).

[40] T. Griesser, H. Ritsch, M. Hemmerling, and G. R. M. Robb, Eur. Phys. J. D **58**, 349 (2010).

[41] W. Niedenzu, T. Griesser, and H. Ritsch, Europhys. Lett. **96**, 43001 (2011).

[42] S. Schutz, H. Habibian, and G. Morigi, Phys. Rev. A **88**, 033427 (2013).

[43] F. Piazza, and P. Strack, Phys. Rev. A **90**, 043823 (2014).

[44] D. Jaksch, and P. Zoller, New Journal of Physics **5**, 56 (2003).

[45] See supplemental material which includes Ref. [46], for (a)The fate of superradiance in a finite lattice with open boundary condition; (b)The derivation of the quantum kinetic equation for fermions.

[46] A. Altland and B. Simons, *Condensed Matter Field Theory* (Cambridge University Press, New York, 2010), 2nd ed., Chap. 11.

[47] R. H. Dicke, Phys. Rev. **93**, 99 (1954).

[48] L. M. Sieberer, M. Buchhold, S. Diehl, arXiv:1512.00637.

[49] J. Rammer, *Quantum Field Theory of Non-equilibrium States* (Cambridge University Press, New York, 2007), Chap. 7.

[50] C. Maschler, H. Ritsch, Phys. Rev. Lett. **95**, 260401 (2005).

[51] O. Golinelli and K. Mallick, J. Phys. A: Math. Gen. **39**, 12679 (2006).

[52] R. A. Blythe and M. R. Evans, J. Phys. A: Math. Theor. **40**, R333 (2007).

[53] S. Inouye, A. P. Chikkatur, D. M. Stamper-Kurn, J. Stenger, D. E. Pritchard, W. Ketterle, Science **285**, 571 (1999).

[54] P. Wang, L. Deng, E. W. Hagley, Z. Fu, S. Chai, and J Zhang, Phys. Rev. Lett. **106**, 210401 (2011).

[55] S. Diehl, A. Micheli, A. Kantian, B. Kraus, H. P. Büchler, and P. Zoller, Nature Phys. **4**, 878 (2008).

[56] S. Diehl, W. Yi, A. J. Daley, and P. Zoller, Phys. Rev. Lett. **105**, 227001 (2010).

[57] L. W. Cheuk, M. A. Nichols, M. Okan, T. Gersdorf, V. V.Ramasesh, W. S. Bakr, T. Lompe, and M. W. Zwierlein, Phys. Rev. Lett. **114**, 193001 (2015).

[58] E. Haller, J. Hudson, A. Kelly, D. A. Cotta, B. Peaudecerf, G. D. Bruce, and S. Kuhr, Nat. Phys. **11**, 738 (2015).

[59] M. F. Parsons, F. Huber, A. Mazurenko, C. S. Chiu, W. Setiawan, K. Wooley-Brown, S. Blatt, and M. Greiner, Phys. Rev. Lett. **114**, 213002 (2015).

[60] G. J. A. Edge, R. Anderson, D. Jervis, D. C. McKay, R. Day, S. Trotzky, and J. H. Thywissen, Phys. Rev. A **92**, 063406 (2015).

[61] A. Omran, M. Boll, T. A. Hilker, K. Kleinlein, G. Salomon, I. Bloch, and C. Gross, Phys. Rev. Lett. **115**, 263001 (2015).

[62] L. W. Cheuk, M. A. Nichols, K. R. Lawrence, M. Okan, H. Zhang, M. W. Zwierlein, arXiv:1604.00096v1.

[63] The cavity frequency $\omega_c = \omega_c^0 + N\epsilon_c$ includes a shift from the empty cavity ω_c^0 due to the presence of the atoms. Since N is conserved, this is a constant.

[64] The pump field oscillates as $e^{i(\omega_p t + \phi_p)}$. Due to the energy gradient, the fermion operator $\hat{c}_j$ oscillates as $e^{-ij\delta t}$. These cause the cavity to be driven at a frequency $\omega_p - \delta$. So the cavity field will oscillate as $e^{i[(\omega_p - \delta)t + \phi_c(t)]}$. In the rotating frame with this frequency, the phase of hopping is $\theta(t) = \phi_c(t) - \phi_p$.

[65] This Hamiltonian can also be realized by the cavity-modulated optical lattice, similar to [8][9][10][11]. In that case, λ has a spatially dependent phase, which can be eliminated by a local gauge transformation.