

## The narrow width of the $\Theta^+$ - a possible explanation

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### Abstract

The narrow width of the exotic narrow baryon resonance  $\Theta^+$  might be explained by mixing between the two nearly degenerate states that arise in models with two diquarks and an antiquark. The only open  $\Theta^+$  decay channel is  $KN$ . When two states both coupled to a single dominant decay mode are mixed by the loop diagram via this decay mode, diagonalization of the loop diagram decouples one mass eigenstate from this decay mode as in some treatments of the  $\rho - \pi$  decay from the mixed singlet-octet  $\omega - \phi$  system, the  $K^* - \pi$  decay of the strange axial vector mesons and the  $NK$  couplings of some baryons. This mechanism can explain the narrow width and weak coupling of  $\Theta^+ \rightarrow KN$  while allowing a relatively large production cross section from  $K^*$  exchange. Interesting tests are suggested in  $K^-p$  reactions where backward kaon production must go by exotic baryon exchange.

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The recent experimental discovery of an exotic 5-quark  $KN$  resonance [1,2],  $\Theta^+$  with  $S = +1$ , a mass of 1540 MeV, a very small width  $\lesssim 20$  MeV (possibly as little as  $1\div 2$  MeV [3]), and a presumed quark configuration  $uudd\bar{s}$  has given rise to a number of theoretical models postulating diquark or triquark correlations [4–6]. However all have difficulty in explaining the narrow width. There is also the problem of explaining comparatively large production cross sections by diagrams involving kaon exchange with an  $NK\Theta^+$  coupling constant limited by the observed decay width.

Models with two diquarks and an antiquark [4–6] have two different wave functions described by different color and spin couplings of the three constituents. Any simple model for the hyperfine interaction mixes these two states [7]. In similar situations with two-state systems coupled to a single dominant decay mode, the mixing via loop diagrams has been shown to create a decoupling of one of the eigenstates from this dominant decay mode. Some examples are  $\omega - \phi$  mixing [8], the mixing of the strange axial vector mesons [9], and the “ideal mixing” [10] decoupling the  $KN$  decay mode in p-wave decays of some negative parity strange baryons [11]. The suggestion that the narrow width of the  $\Theta^+$  might be due to a decoupling mechanism has also been made in a different context [12].

We apply this approach to the diquark pentaquark models and show that the single dominant  $KN$  decay mode is decoupled to a good approximation from one of the two diquark-triquark states. The  $KN$  decay mode includes the two decay channels  $K^+n$  and  $K^0p$  which have equal branching ratios in an isospin conserving decay of an isoscalar state. Isospin breaking is negligible. Therefore the two modes are both decoupled in the same way and are considered together in the following analysis of isoscalar  $KN$  decays.

Let  $|\Theta_1\rangle$  and  $|\Theta_2\rangle$  denote any orthonormal basis for the two diquark-diquark-antiquark states with different color and spin couplings [7].

The eigenstates of the mass matrix will have the form

$$|\Theta\rangle_S \equiv \cos \phi \cdot |\Theta_1\rangle + \sin \phi \cdot |\Theta_2\rangle \quad (1)$$

$$|\Theta\rangle_L \equiv \sin \phi \cdot |\Theta_1\rangle - \cos \phi \cdot |\Theta_2\rangle \quad (2)$$

Where  $\phi$  is a mixing angle determined by the diagonalization of the mass matrix.

Since each of these states can decay by quark rearrangement to the isoscalar  $KN$  final state, we define their decay transition matrix elements respectively as  $\langle KN|T|\Theta_1\rangle$  and  $\langle KN|T|\Theta_2\rangle$ . We then find that these two states can be mixed by a loop diagram

$$\Theta_i \rightarrow KN \rightarrow \Theta_j \quad (3)$$

The contribution of this loop diagram to the mass matrix is

$$M_{ij} = M_o \cdot \langle \Theta_i|T|KN\rangle \langle KN|T|\Theta_j\rangle \quad (4)$$

We first consider the approximation where  $|\Theta_1\rangle$  and  $|\Theta_2\rangle$  are degenerate and the mass matrix is dominated by the loop diagram contribution (4) and other contributions are neglected. The mass eigenstates (1-2) are:

$$|\Theta\rangle_S = C [\langle KN|T|\Theta_1\rangle \cdot |\Theta_1\rangle + \langle KN|T|\Theta_2\rangle \cdot |\Theta_2\rangle] \quad (5)$$

$$|\Theta\rangle_L = C [\langle KN|T|\Theta_2\rangle \cdot |\Theta_1\rangle - \langle KN|T|\Theta_1\rangle \cdot |\Theta_2\rangle] \quad (6)$$

where  $C$  is a normalization factor. Then

$$\langle KN|T|\Theta\rangle_L = \langle KN|T|\Theta_1\rangle \cdot \langle KN|T|\Theta_2\rangle - \langle KN|T|\Theta_2\rangle \cdot \langle KN|T|\Theta_1\rangle = 0 \quad (7)$$

Thus in this approximation the state  $\Theta_L$  is forbidden to decay into the  $KN$  final state, while the  $\Theta_S$  should have a normal hadronic width of about 100 MeV and probably escape observation against the continuum background.

In contrast with their couplings to the  $KN$  channel, the couplings of the states  $|\Theta_1\rangle$  and  $|\Theta_2\rangle$  to  $K^*N$  final states will not satisfy these relations and therefore both states  $|\Theta_L\rangle$  and  $|\Theta_S\rangle$  can be produced without any suppression by  $K^*$  exchange.

This model thus predicts a stronger production via  $K^*$  exchanges than would be obtained from kaon exchange with an  $NK\Theta^+$  coupling limited by the observed decay width.

An interesting further test of this model would be in the baryon-exchange  $K^-p$  reactions where the kaon is observed going backward in the center-of-mass system:

$$K^-p \rightarrow \bar{K}^0n; \quad K^-p \rightarrow \bar{K}^{*0}n; \quad K^-p \rightarrow \bar{K}^0N^{*0}; \quad K^-p \rightarrow \bar{K}^{*0}N^{*0} \quad (8)$$

where  $N^{*0}$  denotes any  $I = 1/2$  electrically neutral baryon resonance.

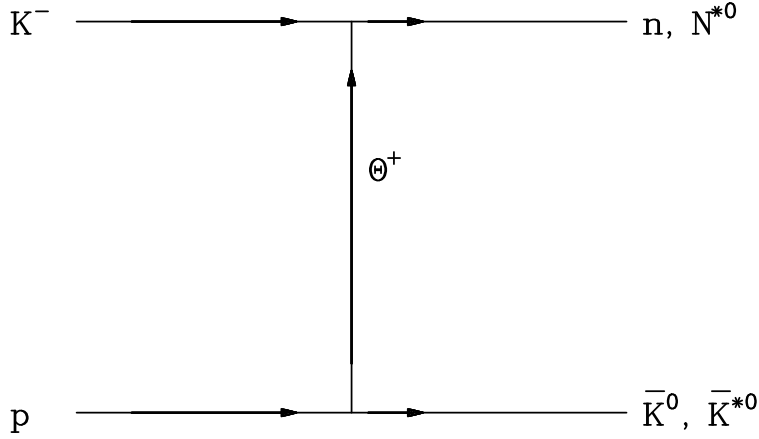


Fig. 1. Baryon-exchange diagram corresponding to the reactions (8).

These reactions shown in Fig. 1 can only proceed via the exchange of an exotic positive-strangeness baryon. But if the  $\Theta^+$  couples only weakly to  $KN$ , the  $K^-\Theta^+ \rightarrow n$  and  $p \rightarrow \bar{K}^0\Theta^+$  vertices are also weak by crossing and the  $K^-p \rightarrow \bar{K}^{*0}N^{*0}$  reaction should be much stronger than the other three which require a  $\Theta^+KN$  coupling.

The  $\Theta^+K\Delta$  coupling is forbidden by isospin if the  $\Theta^+$  is an isoscalar. Therefore the presence of the  $\Delta$  in these baryon exchange reactions is a test for the presence of exotic positive strangeness baryons with higher isospin.

A more precise calculation, not feasible at present, will consider other contributions to the mass matrix in addition to the loop diagram. However, we can show by a rough calculation

how this mechanism can explain a suppression of the width of the  $\Theta^+$  in any model where two states contribute to the decay and the other contributions to the mass matrix are not much greater than the decay width of the broad state.

In addition to the loop diagram contribution to the mass matrix, denoted by  $M_o$ , let us add a contribution which splits the masses of the two states in the absence of the loop diagram. This contribution is denoted by off-diagonal matrix elements  $M_1$  in the base where the  $|\Theta_L\rangle$  and  $|\Theta_S\rangle$  states are diagonal. The mass matrix then becomes

$$\langle\Theta_S|M|\Theta_S\rangle = M_o; \quad \langle\Theta_S|M|\Theta_L\rangle = M_1 \quad (9)$$

$$\langle\Theta_L|M|\Theta_S\rangle = M_1; \quad \langle\Theta_L|M|\Theta_L\rangle = 0 \quad (10)$$

From the trace and the determinant of this matrix the mass eigenvalues denoted by  $M_A$  and  $M_B$  are

$$M_A = M_o + \xi; \quad M_B = -\xi; \quad M_A - M_B = M_o + 2\xi \quad (11)$$

where

$$\xi \cdot (M_o + \xi) = M_1^2 \quad (12)$$

The masses are no longer degenerate in the absence of the loop diagram. The splitting without the loop diagram is obtained by setting  $M_o = 0$  in eqs. (11- 12)

$$\Delta M = 2M_1 \quad (13)$$

The eigenstates of this mass matrix differ from the loop diagram eigenstates by a mixing angle denoted by  $\alpha$

$$|\Theta_A\rangle \equiv |\Theta_S\rangle \cdot \cos \alpha + |\Theta_L\rangle \cdot \sin \alpha \quad (14)$$

$$|\Theta_B\rangle \equiv |\Theta_S\rangle \cdot \sin \alpha - |\Theta_L\rangle \cdot \cos \alpha \quad (15)$$

where

$$\tan \alpha = \frac{\xi}{M_1} = \frac{M_1}{M_o + \xi} = \frac{\Delta M}{2(M_o + \xi)} \quad (16)$$

The ratio of the  $KN$  decay widths of the two states is thus

$$\frac{|\langle KN|T|\Theta_B\rangle|^2}{|\langle KN|T|\Theta_A\rangle|^2} = \tan^2 \alpha = \frac{\xi^2}{M_1^2} = \frac{M_1^2}{(M_o + \xi)^2} = \frac{\Delta M^2}{4(M_o + \xi)^2} \leq \frac{\Delta M^2}{4M_o^2} \quad (17)$$

We thus see that the narrow state  $|\Theta_B\rangle$  is narrower than the normal broad state  $|\Theta_A\rangle$  if the mass splitting  $\Delta M$  between the two states is smaller than the loop diagram contribution  $M_o$  which gives the width of the broad state. For example for a splitting  $\Delta M = 50$  MeV and a broad width  $M_o = 100$  MeV, the width of the narrow state is suppressed by a factor 16.

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