Supplemental Material - A Comparison of Various Aggregation Functions in Multi-Criteria Decision Analysis for Drug Benefit-Risk Assessment

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1 Additional simulation results

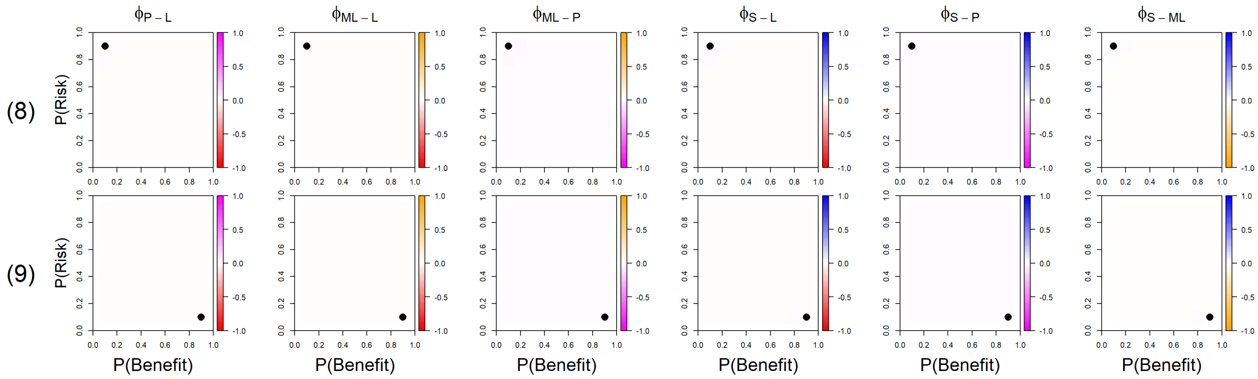

Figure S1. Results of the six pairwise comparisons of the four AM, for scenarios 8 and 9

		Linear Model	Product Model	Multi-Linear Model	Product Model
Scenario 1	≥2.5%	24/81	18/81	25/81	18/81
	≥5%	12/81	8/81	9/81	5/81
Scenario 2	≥2.5%	13/81	18/81	16/81	16/81
	≥5%	3/81	1/81	3/81	1/81
Scenario 3	≥2.5%	19/81	13/81	19/81	13/81
	≥5%	2/81	2/81	2/81	0/81
Scenario 4	≥2.5%	11/81	0/81	9/81	0/81
	≥5%	2/81	0/81	2/81	0/81
Scenario 5	≥2.5%	13/81	3/81	14/81	0/81
	≥5%	2/81	0/81	0/81	0/81
Scenario 6	≥2.5%	20/81	13/81	17/81	8/81
	≥5%	3/81	0/81	3/81	0/81
Scenario 7	≥2.5%	22/81	14/81	23/81	11/81
	≥5%	3/81	2/81	3/81	0/81
Scenario 8	≥2.5%	1/81	2/81	1/81	3/81
	≥5%	1/81	1/81	1/81	1/81
Scenario 9	≥2.5%	3/81	3/81	3/81	3/81
	≥5%	1/81	1/81	1/81	1/81
Total	≥2.5%	126/729	84/729	127/729	72/729
	≥5%	29/729	15/729	24/729	8/729

Table S1. Number of times when the difference in recommending T_1 changes by at least 2.5$\%$ or 5$\%$ between the negatively correlated criteria and the non-correlated criteria

2 Weight derivations

Product Model Firstly, note that under the product model, the resultant utility score for the two criteria example is given by:

$$z^P = u^P(\theta_{i1}, \theta_{i2}.w^P) = u_1(\theta_{i1})^{w^p} \times u_2(\theta_{i2})^{1-w^p}$$

This is then used to find the equation of the contour lines:

$$1 - u_2(\theta_{i2}) = 1 - \left(\frac{z^P}{u_1(\theta_{i1})^{w^p}}\right)^{\frac{1}{1-w^P}} \quad \text{for } u_1(\theta_{i1})^{w^p} > (z^P)^{-w^P}$$

The slope of the tangent of the contour at a given $u_1(\theta_{i1})$ for the utility score z^P is:

$$\frac{w^P}{1-w^P}\left(\frac{1}{u_1(\theta_{i1})}\right)\left(\frac{z^P}{u_1(\theta_{i1})^{w^p}}\right)^{\frac{1}{1-w^P}}$$

Now, the two equations of the slopes for the product and the linear models are set equal to each other at the point $u_{i,1}$=$u_{i,2}$=0.5. This gives the equality:

$$\frac{w^P}{1-w^P} = \frac{w^L}{1-w^L}.$$

Which further simplifies down to w^P =w^L , showing that the weight used in the linear model is also the weight used in the product model.

Multi-Linear Model Firstly, note that under the multi-linear model, the resultant utility score for the two criteria example is given by:

$$z^{ML}(\theta_{i1}, \theta_{i2}, w_1^{ML}, w_2^{ML}) := w_1^{ML}u_1(\theta_{i1}) + (w_2^{ML})u_2(\theta_{i2}) + (1 - w_1^{ML} - w_2^{ML})(u_1(\theta_{i1})u_2(\theta_{i2})).$$

This is then used to find the equation of the contour lines:

$$1 - u_2(\theta_{i2}) = 1 - \frac{z^{ML} - w_1^{ML}u_1(\theta_{i1})}{w_2^{ML} + u_1(\theta_{i1}) - w_1^{ML}u_1(\theta_{i1}) - w_2^{ML}u_1(\theta_{i1})}$$

The slope of the tangent of the contour at a given $u_1(\theta_{i1})$ for the utility score z^S is:

$$\frac{w^L}{1-w^L} = \frac{w_1^{ML}w_2^{ML} + z^{ML} - z^{ML}(w_1^{ML} + w_2^{ML})}{\left(w_2^{ML} - (u_1(\theta_{i,1}))(-1 + w_1^{ML} + w_2^{ML})\right)^2}$$

Now, we firstly set a constraint $1 - w_1^{ML} - w_2^{ML} = c$ (where c is the total weight given to the interaction terms). We then set the two equations of the slopes for the multi-linear and the linear models so they are equal to each other at the point $u_{i,1}$=$u_{i,2}$=0.5. This gives the equality:

$$w^{ML} = w^L - \frac{c}{2}.$$

SLoS Model Firstly, note that under the SLoS Model, the resultant loss score for the two criteria example is given by:

$$z^S = \left(\frac{1}{u_1(\theta_{i1})}\right)^{w^S} + \left(\frac{1}{u_2(\theta_{i2})}\right)^{1-w^S}$$

This is then used to find the equation of the contour lines:

$$1 - u_2(\theta_{i2}) = 1 - (z^S - u_1(\theta_{i1})^{-w^S})^{-\frac{1}{1-w^S}} \text{ for } u_1(\theta_{i1}) > z^{-\frac{1}{z^S}}$$

The slope of the tangent of the contour at a given $u_1(\theta_{i1})$ for the utility score z^S is:

$$\frac{w^S}{1-w^S}\left(z^S - u_1(\theta_{i1})^{-w^S}\right)^{\frac{w^s-2}{1-w^S}}\left(u_1(\theta_{i1})^{-(w^S+1)}\right)$$

Now, the two equations of the slopes for the SLoS and the linear models are set equal to each other at the point $u_{i,1}$=$u_{i,2}$=0.5. This gives the equality:

$$\frac{w^S}{1-w^S}\left(2^{2w^s-1}\right) = \frac{w^L}{1-w^L}.$$