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On a possible tetraquark cousin of the Θ^+

Marek Karliner [a,b*]
and
Harry J. Lipkin [b,c†]

[a] *Cavendish Laboratory*
Cambridge University, UK
and
[b] *School of Physics and Astronomy*
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv, Israel

[c] *Department of Particle Physics*
Weizmann Institute of Science, Rehovot 76100, Israel
and
High Energy Physics Division, Argonne National Laboratory
Argonne, IL 60439-4815, USA

Abstract

If a narrow Θ^+ pentaquark exists, it is likely that a $ud\bar{s}$–ud triquark-diquark configuration is a significant component of its wave function. If so, the mechanism responsible for the binding of a triquark and a diquark is also likely to bind the triquark to an $\bar{s}$ antiquark. We discuss the expected properties of such a $ud\bar{s}$–$\bar{s}$ tetraquark meson. In particular, we point out that for a 0^+ isoscalar $ud\bar{s}\bar{s}$ meson the lowest allowed decay mode is a four-body $KK\pi\pi$ channel with a very small phase space and a distinctive experimental signature.

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[*] e-mail: `marek@proton.tau.ac.il`

[†] e-mail: `harry.lipkin@weizmann.ac.il`

The recent experimental reports about observation [1]-[4] and non-observation [5] of pentaquarks have triggered a substantial theoretical activity. There have been suggestions as to why some experiments see the Θ^+ and others don't [6], but the experimental situation is far from clear and will likely only be resolved when the results from the new generation of CLAS experiments [7] are released.

If a narrow Θ^+ pentaquark does exist, it is likely that a $ud\bar{s}$–ud triquark-diquark configuration [8,9] is a significant component of its wave function. If so, the mechanism responsible for the binding of a triquark and a diquark is also likely to bind the triquark to an $\bar{s}$ antiquark, resulting in a manifestly exotic $ud\bar{s}$–$\bar{s}$ tetraquark meson $\mathcal{M}^+_{\bar{s}\bar{s}}$ with spin zero, strangeness $+2$ and isospin zero.

A very simple general argument spelled out in detail below shows that for any triquark-diquark model of the Θ^+, the isoscalar $S = +2$ tetraquark constructed by replacing the diquark with a strange $\bar{s}$ antiquark should be above the KK threshold by 300 MeV more than the Θ^+ is above the KN threshold.

In most theoretical analyses, the Θ^+ pentaquark is assumed to have a positive parity, corresponding to a triquark and a diquark in a P-wave. If one takes such a configuration and replaces the ud diquark by a $\bar{s}$ antiquark, the tetraquark has negative parity. It is then easy to see that such a 1^- state will have a large decay width to K^+K^0. A 0^- or 2^- state which cannot decay to K^+K^0 and must decay to $KK\pi$ still has a large phase space and is expected to have large decay width.

Here we wish to examine the experimental consequences of another possibility, namely that the $\mathcal{M}^+_{\bar{s}\bar{s}}$ tetraquark has positive parity. We also briefly discuss the constraints on the corresponding quark wave function.

A scalar-isoscalar tetraquark with strangeness $+2$ cannot decay to anything below $KK\pi\pi$ because of the selection rules that come from generalized Bose statistics for the K^+K^0 system.

The isoscalar K^+K^0 is antisymmetric in flavor and therefore must be antisymmetric in space. It therefore has odd parity and cannot couple to an even parity tetraquark. The $KK\pi$ state is excluded since any $J = 0$ state of three pseudoscalar mesons must have odd parity. A system of three 0^- states has odd intrinsic parity. If it is coupled to $J = 0$, it must have even orbital parity because there are only two independent relative orbital angular momenta and they must be equal to make $J = 0$.

Thus the lowest 0^+ state allowed for the decay of an isoscalar $(ud\bar{s}\bar{s})$ tetraquark is a $KK\pi\pi$ state. Moreover, the kaons and pions must have a rather nontrivial relative angular momentum structure. If a $KK\pi\pi$ system has isospin zero, the KK and $\pi\pi$ systems must have the same isospin. This means they must have opposite parity; if one has even L, the other has odd L. Therefore the KK and $\pi\pi$ systems must be in a relative P-wave to make a $J = 0$ state. One possible channel is a P-wave decay

$$\mathcal{M}^+_{\bar{s}\bar{s}} \;\rightarrow\; K^*(1^-)\,\kappa(0^+) \;\rightarrow\; KK\pi\pi \tag{1}$$

Thus the lowest decay mode of $\mathcal{M}^+_{\bar{s}\bar{s}}$ has a very distinctive experimental signature.

To make a rough estimate of the $\mathcal{M}^+_{\bar{s}\bar{s}}$ mass, recall that the difference $\Delta M(\Theta^+ \rightarrow K^+n)$ between the Θ^+ mass and the K^+n mass can be written as the sum of two terms:

$$\Delta M(\Theta^+ \rightarrow K^+n) = \Delta E(ud\bar{s} \rightarrow K^+d) + \Delta E(dud \rightarrow n) \tag{2}$$

where

a) $\Delta E(ud\bar{s} \to K^+ d)$ denotes the energy change due to splitting the triquark into a K^+ and a d quark and moving the d quark next to the color antitriplet diquark.

b) $\Delta E(dud \to n)$ denotes the recombination energy of the color triplet d quark with the color antitriplet diquark into a neutron.

The difference $\Delta M(ud\bar{s}\bar{s} \to K^+ K^0)$ between the $S = +2$ tetraquark and twice the kaon mass can similarly be written as the sum of two analogous terms:

$$\Delta M(ud\bar{s}\bar{s} \to K^+ K^0) = \Delta E(ud\bar{s} \to K^+ d) + \Delta E(d\bar{s} \to K^0) \tag{3}$$

where

a) $\Delta E(ud\bar{s} \to K^+ d)$ denotes the energy change due to splitting the triquark into a K^+ and a d quark and moving the d quark next to the color antitriplet diquark.

b) $\Delta E(d\bar{s} \to K^0)$ denotes the recombination energy of the color triplet d quark with the color antitriplet antiquark into a kaon.

Although there is no reliable way to estimate the first terms $\Delta E(ud\bar{s} \to K^+ d)$, it seems reasonable to assume that they are approximately equal in the two cases. It is the same splitting of the triquark which is sitting in a color antitriplet color field. The second terms have the same color-electric binding of a quark with an antitriplet. But the hyperfine energy is very different in the two cases. Combining the d quark with the spin-zero diquark to make a neutron does not change the hyperfine energy. But combining the d quark with the strange antiquark to make a kaon gains the kaon hyperfine energy which is $\frac{3}{4}$ of the K^*–K splitting, or about 300 MeV,

$$\Delta M(ud\bar{s}\bar{s} \to K^+ K^0) - \Delta M(\Theta^+ \to K^+ n) = \Delta E(d\bar{s} \to K^0) - \Delta E(dud \to n) \approx 300\,\mathrm{MeV} \tag{4}$$

This puts the $\mathcal{M}^+_{\bar{s}\bar{s}}$ above the KK threshold by about 420 MeV, which gives much more phase space for the decay than for the Θ^+. If the tetraquark has quantum numbers that forbid KK and allow $KK\pi$, this is still well above threshold. But if the Θ^+ is a triquark-diquark in an S-wave, this puts the 0^+ isoscalar $\mathcal{M}^+_{\bar{s}\bar{s}}$ above the $KK\pi\pi$ threshold by about the same amount that the Θ^+ is above the KN threshold. Moreover, the $KK\pi\pi$ system must contain at least two units of angular momentum, coupled to $J = 0$. This is likely to make $\mathcal{M}^+_{\bar{s}\bar{s}}$ very narrow. Since there has not been any search for this four-body resonance, it seems reasonable to suggest such a search.

The question of a possible 0^+ $\mathcal{M}^+_{\bar{s}\bar{s}}$ state goes back to the initial discussion [8,9] following the experimental discovery of the Θ^+: the single S-wave cluster is repelled by chromomagnetic effects. Therefore a diquark-triquark model is chosen to separate the quark pairs of the same flavor. The P-wave gives a centrifugal barrier which helps to keep them apart. But an S-wave is not ruled out with a complicated spatial configuration in a five-body wave function that keeps them apart.

An example of such complicated spatial correlations arises in the nuclear shell model, where there is known to be a strong repulsive core in the nucleon-nucleon interaction. Although the shell-model wave function has nucleons in relative S-states, the effects of this strong repulsion are removed by methods commonly used in nuclear many-body physics [10] in which the shell-model wave function is transformed to remove the repulsion. Similar arguments can be used to transform the simple S-wave diquark-triquark pentaquark wave function and the S-wave antiquark-triquark tetraquark wave function to remove the repulsion between identical quark or antiquark pairs.

At this stage we believe that there is little to be gained from doing a detailed and complicated nuclear physics calculation. But the unusual experimental signature requiring a four-body resonance is interesting because it is easy to look for and evidently hasn't been done until now [11].

The most favorable initial state for a doubly strange search might be K^+p which already has one unit of strangeness, since production of a doubly strange state from an initial state of zero strangeness requires the creation of two strange $s\bar{s}$ pairs. Examples of "factories" for inclusively producing doubly strange states are the inclusive reactions:

$$K^+ p \to \Lambda\, \mathcal{M}^+_{\bar{s}\bar{s}} + X \tag{5}$$

$$K^+ p \to \Sigma\, \mathcal{M}^+_{\bar{s}\bar{s}} + X \tag{6}$$

There is also the exclusive reaction

$$K^+ p \to \Sigma^+\, \mathcal{M}^+_{\bar{s}\bar{s}} \tag{7}$$

Since K^+ and $\mathcal{M}^+_{\bar{s}\bar{s}}$ have opposite parity, if the initial state in (7) is in an S-wave, the final state must be in a P-wave, etc.

The possibility of constructing a crypto-exotic tetraquark by replacing the diquark in a triquark-diquark model of the Θ^+ by a nonstrange antiquark has been pointed out by Jaffe [12]. The same simple general argument used above is even stronger here, since the relevant threshold is $K\pi$ and combining the d quark with a nonstrange antiquark to make a pion gains even more hyperfine energy. This gives an estimate for the cryptoexotic tetraquark mass as above the $K\pi$ threshold by 400 MeV more than the Θ^+ is above the KN threshold.

The possibility of constructing an exotic tetraquark by replacing the diquark in a triquark-diquark model of the Θ^+ by a strange antiquark has been considered by Close [13]. He does not consider the scalar tetraquark because of the repulsive short range S-wave interaction. Further detailed investigations of exotic tetraquarks by Dudek, Burns and Close [14] also do not consider the scalar tetraquark.

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