Fiscal Policy in an Unemployment Crisis

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Abstract

This paper shows that equilibrium unemployment dynamics can significantly increase the efficacy of fiscal policy. In response to a shock that brings the economy into a liquidity trap, an expansion in government spending increases output and causes a fall in the unemployment rate. Since movements in unemployment are persistent, the effects of current spending prevail into the future, leading to an enduring rise in income. As an enduring rise in income boosts private demand, an increase in government spending sets in motion a virtuous employment-spending spiral with large effects on macroeconomic aggregates.

Keywords: Fiscal multiplier, liquidity trap, zero lower bound, unemployment inertia.

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1 Introduction

The aggressive fiscal response to the financial crisis of 2008 sparked a heated debate over the merits of countercyclical government spending. Critics questioned the transmission mechanisms typically invoked to support the effect of fiscal policy, expressing concerns over their theoretical and empirical foundations. This paper offers a new perspective on these issues by exploring a channel in which equilibrium unemployment dynamics can increase the efficacy of government spending considerably.

The key mechanism underlying the main results in this paper stems from the interaction between two widely accepted premises. First, at a zero rate of nominal interest, output is largely determined by demand\[1\] If households wish to consume more, firms will also produce more. Second, the labor market is frictional. Any change in current unemployment – and thus any change in current output – will therefore partly persist into the future. The core contribution of this paper is in showing that the interaction of these two properties implies that even a temporary expansion in government expenditures can have a large and lasting impact on output.

Consider the effect of a transitory spending hike. Higher spending raises output (premise 1) and lowers the unemployment rate both in the present and in the future (premise 2). As forward looking agents desire to smooth consumption over time, a rise in future output feeds back to a rise in present private spending, and the unemployment rate falls further. This interplay between present and future economic activity triggers a virtuous employment-spending cycle which propagates the effect of demand stimulating policies. The fiscal multiplier associated with these dynamics lies typically in between one and two, with potentially large improvements in welfare.

The theoretical framework developed in this paper departs from the traditional Mortensen-Pissarides model in three important aspects. First, the elasticity of intertemporal substitution (EIS) is finite, and low. Second, the economy may fall into a liquidity trap with a zero nominal interest rate. Lastly, nominal wages are rigid.

To appreciate how these features interact, envisage a sudden temporary decline in the rate of time preference. The reduction in time preference encourages agents to save, reduces consumption demand, and puts downward pressure on the real interest rate. With the nominal interest rate at zero, a fall in the real interest rate can only materialize through a rise in expected inflation, which requires a decline in the current price level. Provided that nominal wages are downwardly rigid, falling prices raise real wages, reduce profits, discourage hiring, and provoke a decline in current economic activity. As a result, the unemployment rate increases. With persistent unemployment, however, the slump in the present also makes the future appear bleaker. Troubled by this bleak future households take further measures to smooth consumption, adding additional downward pressure on the real interest rate. But the additional desire to save only makes matters worse, and the economy is set on a downward spiral of self-reinforcing thrift.

\[1\] Here I am referring to the fact that demand tends to play a much more prominent role in a liquidity trap than otherwise.
Yet the same force that propels the economy downwards can also be turned around. An expansion in government spending increases demand and puts upward pressure on prices. The associated rise in profits spurs economic activity, encourages hiring, and the unemployment rate falls. Since a decline in unemployment is also persistent, the future appears brighter. Amidst a less troubling outlook households start to spend, and the vicious cycle is turned virtuous. In the baseline quantitative assessment of the model, an expansion in government spending gives rise to a fiscal multiplier of about 1.8.

These results relate to a long-standing literature in economics. In the context of forward looking agents, however, it has proven challenging to account for a fiscal multiplier of an empirically plausible size. In a standard flexible price, real business cycle model, for instance, a rise in government spending reduces private wealth and stimulates labor supply (e.g. Barro and King (1984) and Aiyagari et al. (1992)). Yet the same wealth effect which instills a rise in output crowds out private consumption and the multiplier must fall short of unity. Alternatively, in the new Keynesian model in which prices are rigid, the effect of fiscal policy can be considerably larger. Nevertheless, the eventual effect on economic activity depends crucially on the conduct of monetary policy, and the fiscal multiplier remains below one under a wide range of circumstances.

These circumstances do not extend to a situation of a liquidity trap. In recent seminal work, Christiano et al. (2011) and Eggertsson (2011) study the effect of fiscal policy in a standard new Keynesian model with staggered pricing. They find that a simultaneous rise in both current and expected (future) government spending can have a large effect on present output when the nominal interest rate is up against zero. The reason is that while an increase in current spending raises current inflation, an increase in expected spending raises expected inflation. Since a rise in expected inflation lowers the real interest rate, private consumption demand expands already in the present. The result is a pronounced rise in both public and private consumption, with a large associated fiscal multiplier.

However, absent a frictional labor market there are no endogenous intertemporal linkages in these models, and a purely temporary rise in government spending subdues any internal propagation. Under these conditions the positive effect on output is small and short-lived and the fiscal multiplier equals 1.

In two recent surveys Ramey (2011) and Hall (2009) conclude that the fiscal multiplier is between 0.8 and 1.5, and 0.7 and 1.0, respectively, but do not rule out values as high as 2.0. Auerbach and Gorodnichenko (2012) argue that the fiscal multiplier is around zero in expansions, but rises to two in periods of economic slack. Ramey and Zubairy (2014), however, argue that the effect of fiscal policy does not change with the state of the economy, and remains stably at around 0.8.

In the context of a flexible price model, Baxter and King (1993) show that when the rise in government spending is permanent, the long run multiplier may exceed one at the expense of dynamic efficiency. From a new Keynesian perspective, monetary policy tends to “lean against the wind”. In this context, Gali et al. (2007), Ravn et al. (2012), and Murphy (2015) show that government spending can lead to a positive response in consumption. From an empirical perspective, Fatas and Mihov (2001), Blanchard and Perotti (2002), Gali et al. (2007), Ravn et al. (2012), and Fisher and Peters (2010) find support for a positive response in consumption. But studies such as Ramey and Shapiro (1998) and Ramey (2011) do not.

In the baseline analysis, Christiano et al. (2011) find a multiplier of 3.7, and Eggertsson (2011) of 2.3. The difference can be traced back to the choice of preferences.
This mechanism exposes some important contrasts to this paper, reflecting distinct channels of policy transmissions. In both approaches, a rise in government spending increases output and makes the private sector richer through an increase in income – but also poorer through an increase in present value taxation. The net result is a wash. Then, however, the stories diverge. In this paper, the rise in public spending stimulates private consumption because the rise in income is associated with a job, and jobs last. In Christiano et al. (2011) and Eggertsson (2011), private spending takes off because the general equilibrium effect associated with a committed rise in future government spending sets off an inflationary spiral and lowers the real interest rate.

Several studies have explored the empirical support for each channel. Bachmann et al. (2015) study US households’ readiness to spend in response to changes in inflation expectations. They find that the effect is statistically insignificant outside a liquidity trap, and significant but negative inside. Dupor and Li (2015) question whether the US economy was characterized by a deflationary spiral before the passage of the American Recovery and Reinvestment Act of 2009. They argue that while measures of expected inflation had increased by a modest amount by the time of the Act’s passing, there is no evidence of a systematic link between an individual forecaster’s expectation of government spending and his or hers expectation of inflation. On the contrary, in the context of a structural VAR, Dupor and Li (2015) show that inflation systematically responds negatively to innovations in government spending – even during periods of passive monetary policy. Overall, the empirical support for the expected-inflation channel is not strong.

Evidence related to the mechanism studied in this paper is examined by Bachmann and Sims (2012). These authors assess the impact of government spending on output through a mechanism of “fundamental confidence”; a confidence measure which is strongly predictive of future, low frequency, fluctuations in output. Following the methodology developed in Auerbach and Gorodnichenko (2012), Bachmann and Sims (2012) deploy a regime-switching structural VAR and find that fiscal multipliers are small or even negative in expansions, but rise to about two in recessions. Moreover, about half of the effect of government spending on output in recessions can be attributed to the associated, causal, rise in fundamental confidence.

A multiplier of one arises once the increase in government spending is temporary. Since a temporary rise in spending fails to generate a rise in expected inflation, current private consumption demand is left unaffected.

This reasoning resembles that of DeLong and Summers (2012). In their paper, however, the fiscal multiplier is set exogenously, and cumulates because of unemployment hysteresis. In this paper, the fiscal multiplier is endogenous and large because of inertia in the labor market.

The expected-inflation channel is intimately linked to the hypothesis that negative supply shocks can have a positive effect on output. The reason is that increased production costs put upward pressure on (expected) inflation, which thereby lowers the real interest rate (e.g. Eggertsson (2012)). Wieland (2014) uses two natural experiments to test this prediction. Despite the fact that a negative supply shock indeed lowers the real interest rate, the ultimate effect on output is contractionary.

Miyamoto et al. (2015) provide some further support for this view using evidence arising from Japan’s “lost decade”. By assessing the effect of government spending on macroeconomic variables at different time horizons, they find that an increase in government spending gives rise to a fiscal multiplier in excess of one inside a liquidity trap, and causally
Monacelli et al. (2010) study the effect of government spending on factors closely related to the functioning of the labor market. Using a structural VAR they find that a rise in spending equal to one percent of GDP not only increases output by about 1.6 percent, but also raises labor market tightness by around 20 percent and employment by 1.6 percent, lowering the unemployment rate by 0.6 percentage points. Recent cross-state studies further corroborate these findings. Suárez Serrato and Wingender (2011), Chodorow-Reich et al. (2012) and Shoag (2013) assess the effect of the Recovery Act on job creation. Suárez Serrato and Wingender (2011) find that each job-year cost around $30,000 in government spending, suggesting around 3.3 job-years created per $100,000 spent. Chodorow-Reich et al. (2012) find that $100,000 in government spending generated 3.8 job-years, of which 3.2 were outside the government, health, and education sectors. Shoag (2013) finds that $100,000 in government spending added around 4.8 jobs, of which 2.5 can be attributed to a reduction in unemployment, with the addition 2.3 stemming from a rise in labor market participation. Lastly, Nakamura and Steinsson (2014) use historical data on military procurement spending and find that a rise in government expenditures equal to one percent of GDP increases the employment rate by about 1.5 percentage points.

While these results must be considered tentative they do lend support to the view taken in this paper. The transmission mechanism of fiscal policy appears to be closely intertwined with the labor market. A rise in government spending seems able to have a profound, positive effect on job creation and confidence, and to jointly raise both employment and output.

Lastly, this paper adds to a fast growing literature which explores similar demand effects shaping the dynamics of economic aggregates. In contrast to the aforementioned articles which primarily analyze these effects in a liquidity trap, Bai et al. (2012) and Michaillat and Saez (2015) study how search frictions in the goods market can give rise to similar dynamics; Benhabib et al. (2015), Angeletos and La’o (2013), Schaal and Taschereau-Dumouchel (2014), Kaplan and Menzio (2014), and Heathcote and Perri (2015) show how coordination failures can give rise to multiple equilibria and/or coordination on sentiment shocks. From a different perspective, Challe et al. (2014), Ravn and Sterk (2013) and Den Haan et al. (2015) analyze the role of precautionary saving as an important source of propagation of shocks.

decreases the unemployment rate both in the short and the long run. In contrast, during normal times the fiscal multiplier is markedly smaller, and the effect on unemployment is insignificant, irrespective of the time horizon considered. Chodorow-Reich et al. (2012) assume that jobs end immediately with the Act’s expiry, and therefore interpret these numbers as a lower bound. Using an average total compensation of $56,000, Chodorow-Reich et al. (2012) estimate the associated fiscal multiplier to be about two. Other studies such as Wilson (2012) and Feyrer and Sacerdote (2012) suggests smaller effects with a cost of around $125,000 and $107,000 per job, respectively. The differences across studies appear to lie in the precise data used, the definition of spending, and the source of exogenous variation.
2 A stylized model

This section presents a stylized version of the model to provide some key intuition. The full model is introduced in Section 3, which is self-contained. To enhance analytical tractability, I here make use of two simplifying assumptions. First, the persistence in unemployment is assumed to be exogenous. Second, because of nominal wage rigidities the model displays a disequilibrium, in which labor demand falls short of supply. Both assumptions will be relaxed in the subsequent section.

2.1 Baseline setting

In the baseline setting I describe and analyze the economy in the absence of unemployment persistence. This will provide a useful benchmark through which the the main results can be understood. Section 2.2 then explores the consequences of unemployment persistence using a minimal change in modeling assumptions.

Aggregate supply. In period $t$, a representative firm produces $y_t$ units of the output good using $n_t$ units of labor. The production function, $y_t = f(n_t)$, displays decreasing returns to scale according to

$$f(n_t) = n_t^{1+\sigma}, \quad \text{with } \sigma \in \mathbb{R}_+.$$  

(1)

The firm takes prices as given and maximizes profits

$$\max_{n_t} \left\{ p_t f(n_t) - w_t n_t \right\},$$  

(2)

where $p_t$ denotes the aggregate price level, and $w_t$ the nominal wage\footnote{Decreasing returns to scale in labor does not necessarily raise the issue of entry, nor of the number of firms operating in the market. By including another (fixed) factor of production – say land, $L$ – such that the production function is given by

$$F(n_t, L) = n_t^{1+\sigma} L^{\sigma},$$

output displays constant returns to scale. Thus, as long as firms take prices as given, all factors of production are paid their respective marginal product; profits are zero; and the number of firms is undetermined. Labor, $n_t$, then refers to workers per units of land, where the latter quantity can be normalized to one and is assumed to be owned by the households. The production function in equation (1) follows.}. Nominal wages are assumed to be rigid, such that $w_t = w$ in each period. The first order condition associated with the optimization problem is given by

$$p_t \frac{1}{1 + \sigma} n_t^{\frac{\sigma}{1+\sigma}} = w.$$  

(3)
Combining the first order condition together with the production function gives the aggregate supply relation

\[ p_t = (1 + \sigma)wy_t^\sigma. \] (4)

According to equation (4) the price level is positively related to output. The reason is that any incremental increase in output reduces the marginal product of labor, which, in the optimum, must be equal to the real wage. Since nominal wages are rigid, the price level must therefore rise for real wages to fall.12 The strength of this relationship – i.e., how changes in output map into changes in prices and vice versa – is governed by the price elasticity of aggregate supply, 1/\( \sigma \). In particular, if the price elasticity is high – i.e., if \( \sigma \) is low – a given change in output has a smaller effect on prices, which is therefore related to a “flatter” aggregate supply curve.

In the baseline setting, labor in period \( t \) is inelastically supplied at unity. Consequently, the price level that ensures full employment is given by \( p^* = (1 + \sigma)w \). As we will see, there always exists a nominal interest rate such that full employment is the equilibrium outcome, even in the case of rigid nominal wages. Under some conditions, however, the required nominal interest may be negative. Provided that negative nominal interest rates are not implementable, the price level in period \( t \) falls short of \( p^* \), and labor demand will fall short of labor supply. In this case, employment is rationed and determined by the aggregate supply relation in equation (4) only.

Labor supply beyond period \( t \) is assumed to be equal to labor demand, and is – for the time being – exogenously determined. Future output is therefore treated as a known exogenous variable.

**Aggregate demand.** Aggregate demand is determined by private consumption, \( c_t \), as well as government spending, \( g_t \). Government spending, \( g_t \), is discretionary and treated as exogenous. Since the main focus is on the aggregate effects of a temporary rise in spending, \( g_{t+s} \), for \( s \geq 1 \), is set to zero.

A representative household allocates consumption between periods \( t \) and \( t + 1 \) according to the Euler equation

\[ u'(c_t) = \beta (1 + i_{t+1}) \frac{p_t}{p_{t+1}} u'(c_{t+1}), \] (5)

where \( i_{t+1} \) denotes the nominal interest rate, and \( u(\cdot) \) the utility function. The utility function is of constant relative risk aversion such that \( u(c) = c^{1-\gamma}/(1 - \gamma) \), with the EIS given by \( 1/\gamma \). The discount factor, \( \beta \), governs the private sector’s propensity to save. I consider a negative “demand shock” a situation in which \( \beta \) temporarily rises to a higher value, \( \hat{\beta} \). To keep the analysis as simple as possible I otherwise consider a deterministic setting.

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12Conversely, any incremental increase in the price level reduces the real wage, which, in the optimum, must be equal to marginal product of labor. As a consequence, employment and output expand.
Monetary policy. The natural interest rate, $i^*_t$, is defined as the nominal rate that ensures clearing of the labor market; or, equivalently, that $p_t = p^*$. The monetary authority sets the nominal interest rate, $i_{t+1}$, equal to the natural rate provided that it is positive, and sets it to zero otherwise. Thus, the monetary policy rule is given by

$$i_{t+1} = \max\{i^*_t, 0\}. \tag{6}$$

2.1.1 Results

A competitive equilibrium is defined as a price level, $p_t$, and an interest rate, $i_{t+1}$, such that aggregate demand (for goods) is equal to aggregate supply (of goods), i.e., $y_t = c_t + g_t$. Inserting this equilibrium condition, together with the aggregate supply relation in equation (4), into the household’s Euler equation in (5) yields

$$u'(y_t) = \beta(1 + i_{t+1}) \left(\frac{y_t}{y_{t+1}}\right)^\sigma u'(y_{t+1}), \tag{7}$$

where $g_t$ is, for the time being, left out. Equation (7) fully characterizes the equilibrium, and reveals that for any value of $y_{t+1}$ there exist a (possibly negative) interest rate, $i_{t+1}$, such that $y_t = 1$, $p_t = p^*$, and the labor market in period $t$ clears. In addition, for any $y_{t+1}$ in the neighborhood of – or above – one, there exist a labor-market clearing equilibrium with a strictly positive interest rate, $i_{t+1}$. Lastly, the equilibrium interest rate is a decreasing function in the discount factor, $\beta$, and increasing in future output, $y_{t+1}$.

Define $\beta^*$ as a value of the discount factor such that

$$u'(1) = \beta^* \left(\frac{1}{y_{t+1}}\right)^\sigma u'(y_{t+1}), \tag{8}$$

and notice that $\beta^*$ is a strictly increasing function in $y_{t+1}$. Then for any shock $\hat{\beta} \leq \beta^*$ the nominal interest rate is weakly positive and satisfies equation (7) with $y_t = 1$. The fiscal multiplier is in this case zero as a rise in government spending raises the nominal (and real) interest rate sufficiently to crowd out private consumption entirely.\footnote{This follows as for any $g_t > 0$ there exist an $i_{t+1} > 0$ such that

$$u'(1 - g_t) = \hat{\beta}(1 + i_{t+1}) \left(\frac{1}{y_{t+1}}\right)^\sigma u'(y_{t+1}).$$

Appendix B analyzes the effect of fiscal policy at a positive nominal interest rate in the context of the richer framework developed in Section 3.}
For any $\hat{\beta} > \beta^*$, however, the nominal interest rate is instead zero, and period $t$ output satisfies

$$u'(y_t) = \hat{\beta} \left( \frac{y_t}{y_{t+1}} \right)^\sigma u'(y_{t+1}), \quad (9)$$

with $y_t < 1$. In this case, output falls below its full employment level since the rise in the discount factor prompts agents to consume less and to save more. With the nominal interest rate tied at zero the demand for goods falls short of supply, and the price level, $p_t$, declines in order to restore market clearing. But since nominal wages are rigid, the decline in prices raises real wages and firms employ less workers. The result is a reduction in both the demand and supply for goods, with a contraction in equilibrium output.

An important implication of equation (9) is that a lower value of $y_{t+1}$ – i.e., “bad news” of future income – exacerbates the contraction in current output. The reason is twofold. First, a perceived decline in $y_{t+1}$ is isomorphic to a perceived decline in future consumption, which reinforces the initial motive to save. Second, a decline in $y_{t+1}$ is followed by a perceived fall in the future price level, $p_{t+1}$, which leads to less expected inflation and a higher real interest rate. The higher real interest rate encourages agents to save even further. These two forces form the foundation of an important feature of the model, in which a causal link between future and current output emerges. The ensuing subsection will establish the converse relationship, which causally relates current to future output. As we will see, the resulting two-way interaction can give rise to profound consequences of even small demand shocks, and simultaneously enhance the efficacy of fiscal policy.

Under the current circumstances, Proposition 1 reveals that the fiscal multiplier is between zero and one.

**Proposition 1.** For $\hat{\beta} > \beta^*$ the fiscal multiplier is given by

$$\frac{\partial y_t}{\partial g_t} = \frac{1}{1 + \frac{\sigma}{\gamma}} < 1. \quad (10)$$

**Proof.** The Euler equation is in this case given by

$$u'(y_t - g_t) = \hat{\beta} \left( \frac{y_t}{y_{t+1}} \right)^\sigma u'(y_{t+1}). \quad (11)$$

A straightforward application of the implicit function theorem around $g_t = 0$ gives the result. □

The result in Proposition 1 emerges since a rise in government spending brings – ceteris paribus – goods demand above supply. As a consequence, the price level increases in order to restore market clearing. With higher prices real wages fall, and firms expand supply. However, since the increase in prices also reduces expected inflation, the real interest rate rises and stifles private demand. The equilibrium effect on output is therefore positive, but falls short of the increase in government spending.
itself; indeed the “consumption multiplier”, which is straightforwardly given as the fiscal multiplier net of one, is negative.

Lastly, and before I extend the model with an inertial labor market, it is useful to solve for \( y_t \) in terms of the shock \( \hat{\beta} \), and under the condition that \( y_{t+1} \) is equal to one. That is, \( \beta^* = 1 \) and for any \( \hat{\beta} > 1 \) output satisfies

\[
y_t = \hat{\beta}^{-\frac{1}{1+\sigma}} < 1.
\]  

(12)

That is, the decline in output is smaller for larger values of both \( \sigma \) and \( \gamma \). With a less elastic supply curve – i.e., a larger value of \( \sigma \) – a decline in output is associated with a larger fall in the current price level; a more pronounced rise in expected inflation; and therefore a lower real interest rate. The lower real interest rate leads to a muted decline in consumption demand and in equilibrium output. The effect of \( \gamma \) is more mechanical. At a lower value of the EIS – i.e., at a higher value of \( \gamma \) – agents are less willing to alter their consumption plans in response to intertemporal disturbances. As a consequence, a similar-sized shock to the discount factor has a smaller effect on agents’ willingness to save, and therefore also on equilibrium output.

### 2.2 An inertial labor market

To introduce the notion of labor market inertia, employment in any future period is assumed to follow the law of motion

\[
n_{t+1} = n_t^\alpha, \quad \alpha \in [0, 1).
\]  

(13)

The parameter \( \alpha \) governs the degree of persistence in the labor market. If \( \alpha \) is equal to zero, employment in period \( t+1 \) is equal to one, which is interpreted as full employment. In this case the model collapses to that of the previous subsection. If \( \alpha \) is close to one, however, employment displays hysteresis and a reduction in the present can last into the future. The law of motion in equation (13) is interpreted as a reduced form representation of the transitional dynamics of employment associated with a frictional labor market.\(^{14}\)

Raising each side of equation (13) to the power \( 1/(1+\sigma) \) reveals that

\[
y_{t+1} = y_t^\alpha.
\]  

(14)

As a consequence, inertia in employment causes inertia in output.

\(^{14}\)Sections 3 and 4 provide a rigorous treatment of this view. Other interpretations that encompass similar supply sided consequences resulting from temporary variations in employment – such as skill-loss or mismatch – appear equally viable.
Under the current conditions equation (7) is given by

\[ u'(y_t) = \hat{\beta}(1 + i_{t+1}) \left( \frac{y_t}{\gamma_t} \right)^\sigma u'(\gamma_t^{\alpha_t}), \]  

and for any \( \hat{\beta} > 1 \) the nominal interest rate is driven to zero with output equal to

\[ y_t = \hat{\beta}^{-\frac{1}{(1-\alpha)(1+\sigma)}} \leq \hat{\beta}^{-\frac{1}{\gamma+\sigma}}, \]  

where the expression on the right-hand side of the inequality sign repeats equation (12). Equation (16) captures the consequences of unemployment inertia on output succinctly. A more frictional labor market – i.e. a larger value for \( \alpha \) – causes a larger decline in output than would a more flexible labor market. The intuition is relatively straightforward. As in the baseline setting, a rise in the discount factor encourages saving and pushes the nominal interest rate to zero. Any further rise translates instead to a reduction in both the demand and supply of goods, with a contraction in equilibrium output. With persistent unemployment, however, the decline in current economic activity brings forth bad news of the future. And since bad news reinforce the saving motive, the recession becomes deeper, and a downward unemployment-saving spiral is set in motion.\(^{15}\)

So what can the government do to address this vicious cycle? As in the previous setting, a transitory expansion in government spending raises demand and increases output. With rising output the unemployment rate falls. But since a reduction in the current unemployment rate is also expected to last, the boost in the present makes the future appear less troubling, and the desire to save is weakened. With more private spending the unemployment rate takes yet another drop, the future appears even less troubling, and the vicious cycle turns virtuous. Proposition 2 formalizes this intuition.

**Proposition 2.** For \( \hat{\beta} > \beta^* \) the fiscal multiplier is given by

\[ \frac{\partial y_t}{\partial g_t} = \frac{1}{1 + \frac{\sigma}{\gamma} (1 - \alpha) - \alpha}. \]  

**Proof.** The Euler equation is in this case given by

\[ u'(y_t - g_t) = \hat{\beta} \left( \frac{y_t}{\gamma_t^{\alpha_t}} \right)^\sigma u'(\gamma_t^{\alpha_t}). \]  

A straightforward application of the implicit function theorem around \( g_t = 0 \) gives the result. \( \square \)

Comparing Propositions 1 and 2 suggests that persistence in the labor market gives rise to two effects that propagate the efficacy of fiscal policy; both of which are related to the idea that a rise in

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\(^{15}\)Page 8 in Section 2.1.1 provides a discussion of the channels through which “bad news” translate into a deeper contraction of current output.
government spending brings forth “good news” of future output. First, Proposition 1 showed that the
effect of government spending on output was dampened as the ensuing rise in the contemporaneous
price level raised the real interest rate, which lowered private consumption demand. With a frictional
labor market this channel is held back. The reason is that the associated perceived rise in future output
also leads to a perceived rise in the future price level, which counteracts – but does not overturn – the
adverse effect of current prices on the real interest rate. Second, a rise in future output is isomorphic to
a rise in future consumption, which strengthens agents’ willingness to spend already in the present, thus
raising private sector demand. Of course, these properties amplify each other, and the fiscal multiplier
ranges from that of Proposition 1 when \( \alpha = 0 \), to infinity as \( \alpha \) approaches one.

Before I turn to the full model, a few brief remarks are in order. First, and in contrast to Christiano
et al. (2011) and Eggertsson (2011), the above mechanism does not rely on the government setting
off an inflationary spiral. In fact, a rise in government spending leads to less expected inflation, not
more. Thus, the effect of government spending can be large independently of the expected-inflation
channel.

Second, since the effect of current spending lasts for several periods into the future the possibility of
a cumulative multiplier arises. While a precise expression for the cumulative multiplier is not available,
a lower bound is given by

\[
\sum_{j=0}^{T} \frac{\partial y_{t+j}}{\partial g_t} \geq \frac{1 - \alpha^{T+1}}{1 - \alpha} \frac{\partial y_t}{\partial g_t},
\]

which can be many times larger than the impact multiplier itself.

Lastly, the simple framework developed above focuses on the case in which firms hire workers in
a spot market with a constant nominal wage. As a consequence, a disequilibrium emerges in which
labor demand falls short of supply, with exogenously imposed persistent consequences. The richer
framework presented in the next section circumvents both these issues by considering a flexible-price
search-and-matching model with rigid nominal wages. The resulting frictional labor market implies
that job-finding is the outcome of a stochastic matching process with a long-lasting expected job
tenure. As a consequence, (un)employment emerges as an equilibrium outcome, with endogenously
determined persistence. The core mechanism of this paper, in which an increase in the desire to
save erodes (current and) future income which thereby reinforces the initial savings motive, remains
unchanged.

I will return to this simple model in Section 4 as it provides a useful lens through which some of

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16 Following equation (15), expected inflation is given by \( \pi_{t+1} = y_t^{-\alpha (1-\alpha)} - 1 \), which is decreasing in output.
17 This follows as

\[
\frac{\partial y_{t+j}}{\partial g_t} = \alpha^j y_t^{\alpha-1} \frac{\partial y_t}{\partial g_t} = \alpha^j y_{t+j} y_t \frac{\partial y_t}{\partial g_t} \geq \alpha^j \frac{\partial y_t}{\partial g_t}.
\]
the richer dynamics of the subsequent model can be understood.

### 3 Model

The economy is populated by a government, a large number of potential firms, and a unit measure of households. The planning horizon is infinite, and time is discrete. The output good, $y_t$, is produced in each period using labor, $n_t$, according to the production function $y_t = n_t$. The economy displays a cashless limit with the price level denoted $p_t$. The labor market is frictional in the Mortensen-Pissarides tradition. There is no physical capital in the conventional sense, but there are investments.

#### 3.1 Households

Households initiate their lives in period zero. They supply labor inelastically and the time-endowment is normalized to one. Employment is denoted by $n_t$, and the unemployment rate is therefore given by the difference in labor supplied and labor demanded, $u_t = 1 - n_t$. In a frictional labor market employment is beyond the control of the households and will, for the time being, be treated as given. The nominal wage rate in the economy is denoted by $w_t$.

In addition to labor income, the households own the firms and receive nominal firm profits, $\Pi_t$. Income, $W_t$, constitutes both total labor income, $n_t \times w_t$, and profits, $\Pi_t$. There are complete insurance markets across households, so each household earns income $W_t$ irrespective of whether she is employed or not.

A representative household enters period $t$ with nominal bonds $B_t$, and receives income $W_t$. Bonds are nominally riskless and pay net return $i_t$. Out of these resources, the household pays nominal lump-sum taxes $T_t$, and spends the remainder on consumption, $C_t$, or on purchases of new bonds, $B_{t+1}$. The sequence of budget constraints is therefore

$$B_t(1 + i_t) + W_t - T_t = C_t + B_{t+1} \quad t = 0, 1, \ldots, \quad B_0 \text{ given.} \quad (20)$$

Conditional on a process of taxes, prices, and income, $\{T_t, p_t, i_{t+1}, W_t\}_{t=0}^\infty$, the household decides on feasible consumption and saving plans $\{C_t, B_{t+1}\}_{t=0}^\infty$, to maximize her expected net present value utility

$$V(\{c_t\}_{t=0}^\infty) = E_0 \sum_{t=0}^\infty \beta^t u(c_t), \quad (21)$$

where $c_t$ refers to consumption of the output good, $p_t c_t = C_t$. As previously, the instantaneous utility function $u(\cdot)$ displays constant relative risk aversion, such that $u(c) = c^{1-\gamma}/(1 - \gamma)$. The expectations operator denotes the mathematical expectation with respect to future processes, conditional
on information available in period zero.

The first order condition associated with the problem in (21) subject to (20) is given by the consumption Euler equation

\[ u'(c_t) = \beta E_t [(1 + i_{t+1}) \frac{p_t}{p_{t+1}} u'(c_{t+1})]. \]  

(22)

3.2 Firms

A potential firm opens up a vacancy at cost \( \kappa \geq 0 \). The vacancy cost is denominated in terms of the output good. Conditional on having posted a vacancy, the firm will instantaneously meet a worker with probability \( h_t \). If not, the vacancy is void and the vacancy cost, \( \kappa \), is sunk. A successfully matched firm–worker pair becomes immediately productive and produces one unit of the output good in each period. The employment relation may last for perpetuity, but separations occur at the exogenous rate \( \delta \). The nominal dividend stream generated by a firm in period \( t \) is therefore given as \( p_t - w_t \), where the nominal wage, \( w_t \), is treated as given.

A representative entrepreneur seeks to maximize the real asset price of the firm, \( J_t \),

\[ J_t = 1 - \frac{w_t}{p_t} + \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} (1 - \delta) J_{t+1} \right]. \]  

(23)

As a consequence, a vacancy is posted in period \( t \) if and only if the expected benefits, \( h_t J_t \), (weakly) exceed the associated cost, \( \kappa \). Free entry ensures that \( \kappa = h_t J_t \).

3.3 Matching Market

The labor market is frictional. Let \( \hat{u}_t \) denote the beginning of period unemployment rate. That is, \( \hat{u}_t = u_{t-1} + \delta n_{t-1} \). Let \( v_t \) denote the measure of vacancies in period \( t \). Following the ideas underlying the search-and-matching literature (e.g. Diamond (1982); Mortensen and Pissarides (1994)), the measure of successful matches is given by

\[ M_t = M(v_t, \hat{u}_t). \]  

(24)

The function \( M(\cdot, \cdot) \) exhibits constant returns to scale, and a firm posting a vacancy will therefore find a worker with probability

\[ h_t = \frac{M_t}{v_t} = h(\theta_t), \quad \text{with} \quad \theta_t = \frac{v_t}{\hat{u}_t}. \]  

(25)

\[ ^{18} I follow Blanchard and Gali (2010) and assume that a vacancy posted in period \( t \) can be filled with a positive probability within the same period. This contrasts with, for instance, Hall (2005) in which it is assumed that a vacancy posted in period \( t \) can only be filled with a positive probability in period \( t + 1 \). \]
As usual, $\theta_t$ denotes the labor market tightness in period $t$. Analogously, a worker unemployed in the beginning of period $t$ will find a job with probability

$$f_t = \frac{M_t}{\bar{u}_t} = f(\theta_t), \quad \text{with} \quad f_t = \theta_t h_t.$$  

(26)

Lastly, the law of motion for employment is given by

$$n_t = M_t + (1 - \delta)n_{t-1}$$

$$= (1 - n_{t-1} + \delta n_{t-1})f(\theta_t) + (1 - \delta)n_{t-1}, \quad t = 0, 1, \ldots,$$

(27)

with $n_{-1}$ given.

### 3.3.1 Wage bargaining

Nominal wages are sticky and denoted $w$. In order to determine a flexible wage benchmark for monetary policy, however, the flexible real wage in period $t$ is defined and denoted $\hat{w}_t$.

Flexible wages, $\hat{w}_t$, are determined by Nash bargaining. Nash bargaining seeks to maximize the Nash product of each party’s surplus associated with a match. As there are complete insurance markets across households, however, unemployment has no financial meaning in the context of a household’s surplus, which is therefore always zero. To circumvent this problem, I will follow [Den Haan et al. (2000)](https://example.com) and [Gertler and Trigari (2009)](https://example.com), and view workers as separate risk-neutral entities which are owned and traded by households like an asset. As a result there is a price tag attached to each worker. The market price for an employed worker, $V_t$, and an unemployed worker, $U_t$, are then given by

$$V_t = \hat{w}_t + \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \left\{ [1 - \delta(1 - f_{t+1})] V_{t+1} + \delta(1 - f_{t+1}) U_{t+1} \right\} \right],$$

(28)

and

$$U_t = z + \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \left[ f_{t+1} V_{t+1} + (1 - f_{t+1}) U_{t+1} \right] \right],$$

(29)

respectively. Here $z$ denotes unemployment benefits, or the workers outside option, in terms of the output good. The cost of the unemployment insurance program is financed through lump-sum taxes levied on all households, and is therefore non-distortionary.

Flexible (real) wages are set according to,

$$\hat{w}_t = \arg\max \{ J_t^{1-\alpha} (V_t - U_t)\alpha \},$$

(30)

where, with a slight abuse of notation, $J_t$ denotes the flexible-wage asset price of a firm. The parameter
ω ∈ (0, 1) governs the worker’s relative bargaining power.

### 3.4 Monetary authority

Let $i^*_{t+1}$ denote the nominal interest rate that implements the flexible wage equilibrium. That is, $i^*_{t+1}$ is the natural rate. The monetary authority then sets the nominal interest rate according to the rule

$$i_{t+1} = \max\{i^*_{t+1}, 0\}. \quad (31)$$

Thus, whenever the zero lower bound is not binding, the price level is determined as

$$p_t = \frac{w_t}{\hat{w}_t}, \quad (32)$$

and the natural interest rate is determined by the Euler equation in (22). When the zero lower bound is binding, however, the nominal interest rate is zero and the price level is instead determined by the Euler equation.

Although the model is primarily analyzed at the zero lower bound, the choice of monetary policy is not innocuous. Future policy decisions can affect the present through expectations; either of future prices – and therefore the real interest rate – or of future consumption allocations. Thus, since future monetary policy may interact with current government spending, it is important not to conflate the direct effect of fiscal policy on the economy with its indirect effect on future monetary policy. The natural rate policy provides a useful benchmark in this regard, as fiscal policy can be understood under the assumption of an (ex post) optimally operating central bank. With this caveat in mind, Section 4.5.2 studies the effects of the monetary authority observing a Taylor rule.

### 3.5 Fiscal authority

Apart from lump-sum taxes, the government has access to two additional policy tools; government spending, $G_t$, and public debt, $D_t$. For ease of exposition, they are all denominated in nominal terms. Ricardian equivalence allows me to remain agnostic with respect to the timing of taxes. Thus, a fiscal plan is a process of taxes, spending, and debt \(\{T_t, G_t, D_t\}_{t=0}^{\infty}\), which satisfies the sequence of budget constraints

$$T_t + D_{t+1} = G_t + (1 + i_t)D_t, \quad t = 0, 1, \ldots, \quad D_0 \text{ given}, \quad (33)$$

---

19Ricardian equivalence holds since (i) households are (rationally) forward looking, (ii) taxes are non-distortionary, and (iii) households are treated symmetrically with respect to taxation.
as well as the no-Ponzi condition

\[
\lim_{n \to \infty} \frac{D_{n+1}/p_{n+1}}{\Pi_{n+1}/R_{n+1}} \leq 0,
\]

(34)

with \(R_{s+1} = (1 + i_{s+1})p_s/p_{s+1}\).

### 3.6 Equilibrium

**Definition 1.** Given a fiscal plan, a **competitive equilibrium** is a process of prices, \(\{p_t, i_{t+1}, J_t, w_t\}_{t=0}^\infty\), and quantities, \(\{C_t, \theta_t, n_t, v_t, B_t\}_{t=0}^\infty\), such that,

(i) Given prices, \(\{C_t, B_{t+1}\}_{t=0}^\infty\) solves the households’ problem.

(ii) Labor market tightness, \(\{\theta_t\}_{t=0}^\infty\), satisfies the free-entry condition, \(\kappa = h(\theta_t)J_t\).

(iii) Employment, \(\{n_t\}_{t=0}^\infty\), satisfies the law of motion

\[
n_t = [(1 - n_{t-1}) + \delta n_{t-1}]f(\theta_t) + (1 - \delta)n_{t-1}.
\]

(iv) Asset prices, \(\{J_t\}_{t=0}^\infty\), satisfy equation (23).

(v) Nominal wages, \(\{w_t\}_{t=0}^\infty\), are rigid and satisfy \(w_t = w, t = 0, 1, \ldots\)

(vi) Vacancies, \(\{v_t\}_{t=0}^\infty\), satisfy equation (25).

(vii) The nominal interest rate, \(\{i_{t+1}\}_{t=0}^\infty\), satisfies equation (31).

(viii) Bond markets clear, \(B_t = D_t\).

(ix) Goods markets clear, \(Y_t = C_t + G_t + I_t\), with \(Y_t = p_tw_t\) and \(I_t = p_tv_t\kappa\).

**Proposition 3.** There exists a unique steady state competitive equilibrium.

**Proof.** In Appendix [A]

Consolidating the private and public sectors’ budget constraints and imposing the equilibrium conditions gives

\[
W_t = C_t + G_t.
\]

(35)

Since \(W_t = n_tw + \Pi_t\), profits are given by

\[
\Pi_t = n_t(p_t - w) - \kappa v_t p_t.
\]

(36)

Profits are therefore given as nominal dividends net of investment/vacancy-posting costs.

To provide an intuitive understanding of how market clearing occurs in the model, it is instructive to compare the framework developed here with the simple model of Section 2. In line with the simple model, the nominal interest rate and the price level are determined through goods market clearing; i.e.,
by equalizing aggregate demand and supply of goods in period $t$. Excluding government spending, aggregate demand is again given by the Euler equation,

$$u'(c_t) = \beta E_t \left[ (1 + i_{t+1}) \frac{p_t}{p_{t+1}} u'(c_{t+1}) \right].$$

(AD)

Thus, as in the simple model a ceteris paribus rise in the current price level contracts consumption demand by raising the real interest rate.

Aggregate supply, however, is now provided by the asset value of the firm, the free-entry condition, and the law of motion for employment. That is,

$$J_t = 1 - \frac{w}{p_t} + \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} (1 - \delta) J_{t+1} \right],$$

$$\kappa = h(\theta_t) J_t,$$

$$n_t = (1 - n_{t-1} - \delta n_{t-1}) f(\theta_t) + (1 - \delta) n_{t-1}.$$  

(AS)

As a consequence, a ceteris paribus rise in the price level lowers the real wage and increases the asset value of the firm. The higher asset value raises labor market tightness, employment, and the supply of goods. Thus, as in the aggregate supply relation of the simple model – equation (4) – the price level is positively related to production, with the law of motion for employment in (AS) capturing the inertia previously provided by equation (13).

The market clearing mechanism of the model can then be illustrated as follows. A temporary rise in the discount factor, $\beta$, reduces consumption demand according to equation (AD). This reduction in demand does not, however, necessarily pose a problem for the economy, since the initial effect can be countervailed by a reduction in the nominal, and therefore real, interest rate. The price level would then adjust to mimic the flexible wage outcome according to equation (32), and there are no material consequences on the economy.

If the shock to the discount factor is large enough, however, the required nominal rate is negative and the economy falls into a liquidity trap. But even in a liquidity trap there is still scope to restore market clearing by lowering the real interest rate. Yet, with the nominal rate tied at zero, a lower real rate can only materialize through expected inflation, which requires a decline in the current price level. Since nominal wages are sticky the decline in the price level causes a rise in real wages, which reduces the asset value of the firm, hampers vacancy posting, and contracts supply. The combination of a decline in both supply and demand leads to a contraction in equilibrium output, with elevated unemployment.

The law of motion for employment in (AS) adds a final spin to this story. Since future employment is negatively affected by the decline in current activity, expected future consumption is perceived to fall as well. As a consequence, current demand takes another drop, the contraction in the present becomes deeper, and so on.
It may seem misplaced to put the transitional dynamics of employment at the source of this propagation. After all, the gross worker flows in the labor market are typically large enough for unemployment to be well approximated by its conditional steady state, rendering its own persistence irrelevant (e.g., Shimer (2005)). The framework considered here is no exception; conditional on equilibrium gross worker flows, the resulting unemployment rate is well approximated by its conditional steady state. However, the equilibrium worker flows are determined endogenously, and in the current framework the transitional dynamics are crucial for their determination.

To appreciate why, suppose that a decline in current employment has only a small negative effect on future employment. If the EIS were to be infinite – which is the case in most standard search-and-matching models – this would bear no further consequence on current employment, and the transitional dynamics would be irrelevant in determining the equilibrium. If the EIS is finite, however, the decline in future employment has a negative effect on current demand. With the nominal interest rate tied at zero and with sticky nominal wages, a decline in current demand causes a further contraction in the present. As a consequence, a positive feedback loop emerges of which the fixed point is the equilibrium outcome, with potentially large implications on both current and future employment. Absent the transitional dynamics, this feedback loop would come to an immediate halt, and give rise to fundamentally different results. Thus, while the transitional dynamics in this framework are fast, they are not instantaneous; and with a finite EIS, sticky nominal wages, and a zero rate of interest, this matters.

4 Numerical analysis

This section provides a numerical analysis of the above model. The objective is to understand the mechanisms involved in the richer framework of Section 3, and to gauge the effect of fiscal policy in an equilibrium setting with endogenous labor market frictions.

4.1 Calibration

The model is calibrated to target the US economy at a quarterly frequency. Thus, the discount factor, $\beta$, is set to $1.03^{1/4}$ which corresponds to a 3 percent annual real interest rate. The EIS is set to $1/2$.

The matching function is of a standard Cobb-Douglas type, and given by $M(v_t, \hat{u}_t) = \varphi v_t^\eta \hat{u}_t^{1-\eta}$. Following Petrongolo and Pissarides (2001), the elasticity of job finding with respect to labor market

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20See Appendix E.1. The conditional steady state is defined as the value to which unemployment would converge provided that the current labor flows remain constant; see equation (37) below.

21Indeed, in this hypothetical case unemployment is not an “informationally relevant state variable”, and can be ignored when determining equilibrium labor flows (Mortensen and Nagypál 2007).

22The effect of government spending outside a liquidity trap is studied in Appendix B and Appendix D studies the effect of government spending in a model with an endogenous separation rate.
tightness, $\eta$, is set to 0.5. To calibrate overall match efficiency, $\phi$, and the separation rate, $\delta$, define $u_t^*$ as the steady state unemployment rate conditional on the current job finding rate,

$$u_t^* = \frac{\delta(1 - f_t)}{\delta(1 - f_t) + f_t}. \quad (37)$$

Rearranging the law of motion for employment in equation (27) gives

$$(u_t - u_t^*) = \alpha_t (u_{t-1} - u_t^*), \quad (38)$$

with

$$\alpha_t = (1 - \delta)(1 - f_t). \quad (39)$$

Thus, the parameter $\alpha_t$ determines the speed of convergence of unemployment to its conditional steady state. Let $\phi_t$ denote the corresponding time to convergence defined as the duration required to close 90 percent of the current gap. That is,

$$\phi_t = \frac{\ln 0.1}{\ln \alpha_t}. \quad (40)$$

Barnichon and Nekarda (2012) estimate $\phi_t$ to be between 3 to 5 months depending on the amount of slack in the labor market. In the recent financial crisis, the time to convergence reached an unprecedented value of 9. I therefore calibrate the separation rate, $\delta$, and the efficiency of the matching function, $\phi$, to match a steady state unemployment rate of 5 months and a steady state unemployment rate of 6 percent. The resulting separation rate, $\delta$, is about 0.15. The associated steady state value for $\alpha_t$ is about 0.25.

Unemployment benefits are set to 0.5, which approximates a 50 percent replacement rate (Chetty, 2008). The workers’ bargaining power, $\omega$, is set to target a 3 percent profit margin of firms. As a result, $\omega$ is equal to 0.767. This is in line with Shimer’s (2005) and Kaplan and Menzio’s (2014) value of 0.72 and 0.74 respectively, but higher than both Hall’s (2005) value of 0.5 and Hagedorn and Manovskii’s (2008) of 0.052. However, it should be noted that neither the bargaining power nor the replacement rate are, in isolation, important factors for the model’s properties. Rather it is their combined effect on firms’ profit margins that matter. Here, the profit margin is 3 percent, which can be compared to 3.4 percent in Hall (2005), 2.25 percent in Hagedorn and Manovskii (2008), 2 percent in Pissarides (2009), and 0.7 percent in Shimer (2005).

Given a steady state labor market tightness normalized to one, the cost of posting a vacancy $\kappa$ is set to $h(1)J^{23}$. Real non-discretionary government spending equals 20 percent of steady state output. I

---

23Notice that it is possible to multiply the steady state value of $\theta$ with a factor of $\lambda$ and $\phi$ with a factor of $\lambda^{-\eta}$ without altering the steady state values of $f$ and $n$. The steady state value of $h$ and $\kappa$ are then scaled by $1/\lambda$. Thus, as in Shimer...
set the steady state nominal wage, \( w \), equal to the real wage of 0.97. The implied steady state price level is therefore one.

The calibrated parameter values are summarized in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
<th>Source/steady state target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>Inverse of EIS</td>
<td>2</td>
<td>Convention</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Discount factor</td>
<td>0.993</td>
<td>Annual real interest rate of 3%</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Efficiency of matching</td>
<td>0.704</td>
<td>Unemployment rate of 6%</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Separation rate</td>
<td>0.152</td>
<td>Time to convergence of 5 months</td>
</tr>
<tr>
<td>( \omega )</td>
<td>Workers bargaining power</td>
<td>0.767</td>
<td>Steady state profit margin of 3%</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Elasticity of ( f(\theta) )</td>
<td>0.5</td>
<td>[Petrongolo and Pissarides (2001)]</td>
</tr>
<tr>
<td>( \kappa )</td>
<td>Vacancy posting cost</td>
<td>0.134</td>
<td>Steady state ( \theta ) normalized to one</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>Unemployment benefits</td>
<td>0.5</td>
<td>[Chetty (2008)]</td>
</tr>
<tr>
<td>( g )</td>
<td>Steady state fiscal spending</td>
<td>0.188</td>
<td>20% of steady state output</td>
</tr>
</tbody>
</table>

Notes. This table lists the parameter values of the model. The calculations and targets are described in the main text. One period in the model corresponds to one quarter.

4.2 Experiments

The economy is in the steady state at time \( t - 1 \). In period \( t \) the discount factor, \( \beta \), unexpectedly increases to \( \hat{\beta} = \beta + 0.015 \). In each subsequent period the discount factor may revert back to its steady state with probability \( q \), but remain elevated with probability \( (1 - q) \). Once the discount factor has reverted back to its steady state value, it remains there for perpetuity. As a consequence, the economy falls into a liquidity trap in period \( t \) with expected duration of \( 1/q \) quarters.

Discretionary government spending may or may not increase in response to the above shock. If it does increase, spending remains elevated in the subsequent period with probability \( \rho \), conditional on the economy still remaining in a liquidity trap. With the complementary probability, \( (1 - \rho) \), however, spending reverts back to its steady state value and remains there.

The systems of first order conditions inside and outside a liquidity trap are linearized around the steady state, giving rise to a rational expectations regime-switching linear system (see, for instance, [Farmer et al. (2009)]). This has some important implications. The resulting linear policy functions imply that there are no interaction effects across variables. The effect of a deeper crisis, for instance, is therefore only a scaled version of the above benchmark. Similarly, conditional on the economy being in a liquidity trap, the effectiveness of fiscal policy is unrelated to both the depth of the crisis, the size of the stimulus package and the precise timing of spending. As a consequence, the economy can be fully analyzed through the impulse response functions with respect to shocks at various durations and with different spending profiles.

[Farmer et al. (2009)], the steady state value of \( \theta \) is intrinsically meaningless and can be normalized.
The computational details are described in Appendix C.

4.3 Results

This section proceeds in three parts. In Section 4.3.1, I explore the case in which the rise in the discount factor and government spending are deterministic and known to last for one period only, i.e., \( q = 1 \) and \( \rho = 0 \). The purpose of this exercise is not to quantitatively gauge the predictions of the model, but rather to gain some intuition and explore the mechanisms at a manageable level. In Section 4.3.2, I consider the effect of a stochastic duration of the shock to the discount factor under the more realistic assumption of a long lasting liquidity trap, accompanied by a committed rise in spending, i.e., \( q = 0.1 \) and \( \rho = 1 \). The third and final part will illustrate the main results for a large set of combinations of \((q, \rho)\)-values and evaluate the welfare implications.

Section 4.4 then discusses the empirical plausibility of the core mechanism, and Section 4.5 explores the sensitivity of the model with respect to both some key parameters, and the effects of an alternative monetary policy rule.

4.3.1 The deterministic case

The solid line in Figure 1 illustrates the response of the economy to a temporary one-period rise in \( \beta \). Time is given on the \( x \)-axis, and the \( y \)-axis illustrates the percent deviation of a variable from its steady state value. In response to a discount factor shock that brings the economy into a liquidity trap for one quarter, output, consumption, and investment all fall together.

The dashed line in Figure 1 illustrates the response to an identical shock, but now accompanied by a one-shot expansion in government spending of 0.15 percent of steady state output. The increase in spending staves off a 0.18 percentage point fall in output, suggesting a multiplier on impact of around 1.2. But more importantly, the effect on impact persists over time. Output in period \( t+1 \) is 0.05 percentage points higher than it would be if there was no fiscal expansion in period \( t \). And output in period \( t+3 \) is still higher than in the absence of fiscal policy. Thus, an expansion in government spending does not only boost output and employment in the present, but also in the future.

The mechanics predicating these results are as follows. An increase in government spending brings – ceteris paribus – goods demand above supply. As a consequence, the price level rises in order to restore the equilibrium. The rise in the price level, however, comes with two implications. First, it reduces consumption demand by lowering expected inflation and thereby raising the real interest rate. Second, a higher price level increases supply by lowering the real wage. Thus, to a first approximation, the rise in government spending increases output, but by less than the increase in spending itself.

However, since the impact of spending persists over time, there is an associated rise in expected future consumption. Since a perceived rise in future consumption bolsters current demand, the price

\[24\text{The causal mechanism driving these results were discussed in Section 3.6, page 17.}\]
Figure 1: Impulse responses to macroeconomic and labour market variables, $q = 1$.

Notes. The black line illustrates the effect on the economy of a one time rise in the discount factor shock of 0.015. The dashed line illustrates the equivalent impulse under the hypothesis of an accompanied rise in government spending of 0.15 percent of steady state output. Unemployment and the profit margin are in levels.
level increases further, real wages fall, and the above process repeats itself. The equilibrium outcome is given by the fixed point of this propagation mechanism, with a large effect on output, both in the present and in the future.

The intertemporal nature of the above mechanism exposes the important role played by the EIS. A perceived rise in future consumption increases, ceteris paribus, consumption demand already in the present. But while the associated rise in the current price level expands supply by lowering the real wage, it also stifles the initial desire to consume by raising the real interest rate. With a lower EIS, the former effect gains more prominence, and the propagation mechanism strengthens. As a consequence, a rise in future consumption causes a rise already in the present, with the EIS governing the strength of this channel.

How powerful is this mechanism? Figure 2 illustrates the level deviation of output from its steady state (left) and the fiscal multiplier (right). The solid line in the right graph illustrates the result on impact; i.e., the change in output in any given period divided by the change in government spending in period \( t \). The impact multiplier is about 1.2 in period \( t \), and tapers off over time. The dashed line in the same graph illustrates the cumulative response. The cumulative fiscal multiplier reaches about 1.67.

![Figure 2: Output (left) and the fiscal multiplier (right), \( \Delta g_t = 0.0015 \times y_{ss} \).](image)

Lastly, the ratio of output in period \( t + 1 \) to output in period \( t \) is about 0.3, with and without a rise in government spending. This number reflects the stable eigenvalue of the system, and corresponds to the parameter \( \alpha \) in the simple model of Section 2 and loosely to the parameter \( \alpha_t \) in the calibration of Section 4.1. Of this, 0.25 can be attributed to the transitional dynamics of employment conditional on the steady state job finding rate (see Section 4.1). The remaining 0.05 can be traced to the slightly restrained job finding rate throughout the transition.  

\[ 25 \] This can be calculated using the right graph of Figure 2. That is, \( 0.35/1.18 \approx 0.3 \).

\[ 26 \] Following the bottom left graph of Figure 1 the job finding rate is somewhat restrained during a recovery, since the intertemporal marginal rate of substitution is below one, which lowers the asset value of a firm.
Decomposing effects  Which forces underpin the dynamics of Figures 1 and 2? Combining the aggregate resource constraint with the linearized Euler equation in period $t$ gives

$$\tilde{y}_t = \tilde{g}_t + \kappa \tilde{v}_t + \frac{c_{ss}\beta}{\gamma} \tilde{\pi}_{t+1} + \beta \tilde{c}_{t+1} + \frac{c_{ss}\beta}{\gamma} (1 - \tilde{\beta}),$$  \hspace{1cm} (41)

in which $\pi_{t+1} = p_{t+1} - p_t$, and for any variable $x_t$, $\tilde{x}_t = x_t - x_{ss}$, with $x_{ss}$ referring to its steady state value. Differentiating equation (41) with respect to $g_t$ gives the decomposition

$$\frac{\partial y_t}{\partial g_t} (1.18) = 1 + \kappa \frac{\partial v_t}{\partial g_t} (0.45) + \frac{c_{ss}\beta}{\gamma} \frac{\partial \pi_{t+1}}{\partial g_t} (-0.72) + \beta \frac{\partial c_{t+1}}{\partial g_t} (0.47),$$

where the numbers in parentheses reflect those emerging from the exercise in Figures 1 and 2. Thus, the fiscal multiplier is 1.18 on impact, of which 0.45 can be attributed to investments ($\kappa v_t$), 0.47 to the rise in expected future consumption, and $-0.72$ to the decline in expected inflation.

There are two aspects of this decomposition that are worth emphasizing. First, investments are not part of demand in the conventional sense, but emerge as a byproduct from the roundabout production technology alongside an increase in either private or public consumption. For instance, if private consumption, or government spending, were to increase by $x$, output must increase by more than $x$ in order to produce enough goods to satisfy the rise in demand and to finance the associated vacancy-posting costs. Second, equation (42) reflects an accounting exercise, and does not allow for counterfactual interpretations. For instance, the fiscal multiplier absent the expected-inflation channel is not $1.18 + 0.72 = 1.9$, since the effect of expected consumption – and therefore investments – respond to such changes as well.

Thus, to provide a deeper account of the mechanisms governing the results of this paper, Figure 3 illustrates the decomposition in equation (42) under three alternative scenarios, alongside the baseline described above. In all cases the economy is exposed to the same shocks to the discount factor and to government spending.

Case 1 reports the results when future employment is unaffected by the current increase in government spending. That is, when the main channel of this paper is suppressed. This is accomplished by exposing the economy to an anticipated future job-destruction shock, such that employment beyond period $t$ is left unaffected by the rise in current government spending. The fiscal multiplier falls from 1.18 to 0.82 on impact, with no cumulative consequences. The negative effect of expected inflation

\footnote{Investment is not a large share of output in this model, which is given by $\kappa (1 - n_{ss} + \delta n_{ss})/n_{ss} \approx 2.88\%$. At the steady state, a marginal increase in output yields a marginal increase in investments according to $\frac{\partial \kappa v_t}{\partial y_t} = \frac{\kappa}{\varphi \eta} \approx 0.38$, which aligns well with the numbers provided in equation (42).}
is partially muted, and expected consumption has, by construction, no influence on the efficacy of policy. As a consequence, the main channel of this paper gives rise to a propagation mechanism that amplifies the effect of a government spending shock by a factor of about 1.45 on impact, and more than doubles it over the course of one and a half years.

In Case 2, the profit margin is lowered from 3 to 0.1 percent, which results in a sharp decline in the vacancy posting cost, \( \kappa \), and therefore in steady state investments. Three discernible differences emerge relative to the baseline: The fiscal multiplier is slightly larger and around 1.36, with a cumulative value of 1.86; the rise in investment is negligible; and the expected-inflation channel is largely irrelevant. Two reasons predicate these results. First, since investment is a very small fraction of output – around 0.1 percent – its contribution to the fiscal multiplier is therefore negligible. Second, with a low profit margin, the price elasticity of aggregate supply increases. As a consequence, even a small rise in prices has a large impact on employment, and the fall in expected inflation is muted. Since the latter effect dominates the former, the efficacy of fiscal policy is enhanced.

![Figure 3: Decomposition of the fiscal multiplier in under three alternative scenarios.](image)

**Figure 3:** Decomposition of the fiscal multiplier in under three alternative scenarios.

**Notes.** Case 1 refers to a situation in which future output is unaffected by current spending; Case 2 reduces the role of investments; and Case 3 suppresses the effect of expected inflation. Further details are provided in the main text. All decompositions follow equation (42).

Case 3 illustrates the results when both the current and future price levels are left unaffected by government spending. That is, when the expected-inflation channel is suppressed. To accomplish this, I expose the economy to a sequence of current and future (anticipated) nominal-wage shocks, such that the resulting price sequence in the presence of spending coincides with that of its absence. The result is a pronounced increase in both investments and expected consumption, and, of course, a nullified response in expected inflation. Together, these forces contribute to a large response in output, with a

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28 The simple model of Section 2 provides a remarkably accurate account of these results. Proposition 1 suggests that a value of \( \sigma = 0.42 \) gives rise to a fiscal multiplier of 0.82 (for \( \gamma = 2 \)). Using this value of \( \sigma \) together with a value of \( \alpha = 0.3 \), Proposition 2 predicts a fiscal multiplier of 1.18, with a cumulative lower bound of 1.68.

29 The rise in the price elasticity of aggregate supply is an unavoidable consequence of restraining the role of investments, and the two effects cannot be perfectly distinguished.
fiscal multiplier ranging from 4.3 on impact, to about 6.1 over the course of one and a half years.

Everything considered, the results reported in Figure 3 support the view that frictional unemployment can enhance the efficacy of fiscal policy considerably, and constitutes an independent transmission mechanism of government spending.

### 4.3.2 A stochastic crisis

The solid lines in Figure 4 illustrate the response of the economy to a liquidity trap with an expected duration of 10 quarters, i.e. $q = 0.1$. Each thin line represents a possible outcome, or sample path, with a darker shading indicating a more likely outcome. The thick solid line represents the expected outcome.

While all variables follow a similar pattern to Figure 1, there are some marked quantitative differences. In response to a shock of identical magnitude as before, the drop in macroeconomic and labor market variables are much more pronounced. Output falls by 5 percent, asset prices by 35 percent, and the unemployment rate peaks at around 11 percent. In addition, the fall in output relative to the fall in the price level is about three times greater than in the $q = 1$ case. Thus, in a prolonged liquidity trap the model displays more endogenous propagation.

Returning to the simple model of Section 2 – but extended to accommodate the uncertain structure analyzed here – the Euler equation in (15) can be rewritten as

$$u'(\hat{y}) = \beta [q \hat{y}^{\sigma(1-\alpha)}u'(\hat{y}^{\alpha}) + (1-q)u'(\hat{y})],$$

or simply

$$u'(\hat{y}) = q \beta \frac{\hat{y}^{\sigma(1-\alpha)}u'(\hat{y}^{\alpha})}{1-\beta(1-q)} \chi,$$

where $\chi$ is around 1.08. Thus there is an implicit rise in the discount factor that is seven times larger than previously, explaining a large share of the additional amplification in Figure 4.

This is not merely a technicality. In a long lasting liquidity trap, expected future consumption is depressed for two reasons. First, if the economy leaves the liquidity trap, the transition is not immediate

---

30 As output is entirely demand driven it remains constant in a liquidity trap, and the same level of output appears on both the right- and the left-hand side of the Euler equation.
Figure 4: Impulse responses to macroeconomic and labour market variables, $q = 0.1$, $\rho = 1$.

Notes. The shaded lines illustrate all possible paths back to the steady state after a demand shock absent government spending. A darker shade represents a more likely outcome. The dash-dot line and the solid black line illustrate the expected path of each variable with and without government spending. Unemployment and the profit margin are in levels. Government spending is equal to 1 percent of steady state output.
and consumption remains below its steady state value for some time. This effect was extensively analyzed in Sections 2 and 4.3.1. Second, however, if the economy remains stuck in the liquidity trap, output continues to remain depressed. The latter effect amplifies the former, leading to a pronounced drop in consumption and output.

Figure 4 echoes this intuition. With consumption levels expected to remain low in both future states, demand is further depressed and the price level drops. But as the expected future price level also remains low, the current price level must fall significantly more than previously in order to generate the same decline in the real interest rate. The result is a pronounced slump. Furthermore, as the price level is expected to remain low for an extended period of time, so are profits, and asset prices plummet. As a consequence, output falls more relative to the contemporaneous price level when compared to the case in which $q = 1$.

The dashed line in Figure 4 illustrates the expected response to the same shock but now accompanied by a committed rise in government spending equal to 1 percent of steady state output, with $\rho = 1$. In light of the previous discussion, the effects are anticipated. Government spending puts upward pressure on the price level, both in the present and in the future, raising profits, asset prices, and the job finding rate. The result is a fall in unemployment and a rise in output. In the absence of government spending, output falls by about 5.4 percent. With spending, this decline is limited to about 3.5 percent. Since the rise in government purchases is equal to 1 percent of steady state output, the fiscal multiplier associated with these dynamics is slightly below 1.9.

Figure 5: Expected inflation (left) and difference in expected inflation with and without government spending (right).

In similarity to Section 4.3.1, these results do not hinge on particularly favorable inflation dynamics. The solid black line in the left graph of Figure 5 shows the evolution of expected inflation absent fiscal interventions. The dashed line in the same graph illustrates the equivalent series in the presence of government spending. The right graph depicts their difference. Two features stand out: First,
expected inflation rises substantially on impact and swiftly reverts back to its pre-crisis trend. Second, an increase in government spending reduces the rise in expected inflation. Thus, and as in Section 2, the expected-inflation channel operates in reverse, and lessens the efficacy of policy. Both these two aspects contrast markedly to those in the new Keynesian literature, in which a liquidity trap is characterized by persistent expected deflation, that is mitigated by a rise in government spending.

The mechanism governing the above dynamics is relatively straightforward. A sudden rise in the discount factor leads to a sharp contraction in both the supply and demand for goods. Supply, however, is comprised by two components; the stock of retained employees from the previous period, and the number of new matches. Upon the impact of the shock, the stock of retained employees is relatively large, as it is set equal to its steady state value. As a consequence, the decline in the number of new matches must absorb the entire fall in output, which necessitates a pronounced decline in the immediate price level. In the subsequent periods, the stock of retained employees is lower and the needed adjustment in matches is therefore smaller, which is reflected in a less suppressed level of prices. Together, these two effects explain why the economy experiences a sharp rise in expected inflation at the onset of the crisis, with a swift reversion towards its pre-crisis trend.

The empirical relevance of these dynamics is discussed in Section 4.4.

4.3.3 Fiscal policy and welfare

To explore the effect of government spending on welfare let $c(n; g, q, \rho)$ represent the consumption policy function at state $n_t$ conditional on a liquidity trap with expected duration $1/q$, government spending $g$, and policy duration $\rho$. The associated value function is then given by the fixed point of the functional equation

$$v(n_t; g, q, \rho) = u[c(n_t; g, q, \rho)] + \hat{\beta}E[v(n_{t+1}; g, q, \rho)]$$

subject to the laws of motion for $n_t$, $\hat{\beta}$ and $g$.

Define $\hat{c}(g; q, \rho)$ as the level of consumption that leaves an agent indifferent between consuming $\hat{c}(\cdot)$ for perpetuity and experiencing a crisis with duration $1/q$ under policy $(g, \rho)$. That is,

$$\hat{c}(g; q, \rho) = [(1 - \beta)(1 - \gamma)v(g, q, \rho)]^{\frac{1}{1-\gamma}}.$$
Thus, an easily interpretable measure of welfare is given by

$$W(q, \rho) = [1 - (1 - q)\rho] \times \frac{\partial \hat{c}(g; q, \rho)}{\partial g} \times \frac{1}{1 - \beta},$$

(47)
evaluated around $g = 0$. The partial derivative in equation (47) can be interpreted as the marginal, perpetual, consumption-equivalent change in utility stemming from a rise in government spending under policy $\rho$. To convert this perpetual stream of consumption into a present value, the derivative is divided by $(1 - \beta)$. Lastly, since

$$\sum_{j=0}^{\infty} E_t[\Delta g_{t+j}] = \frac{\Delta g}{1 - (1 - q)\rho},$$

(48)
I multiply through with $[1 - (1 - q)\rho]$. As a consequence $W(q, \rho)$ captures the welfare associated with policy $\rho$ measured as the dollar gained in consumption-equivalents per dollar spent by the government.

The results below are reported with respect to the relative duration of government spending, $\zeta$, defined as the expected duration of spending relative to the expected duration of the liquidity trap, net of the first period. That is,

$$\zeta = \frac{1 - (1 - q)\rho}{q - 1}, \quad q < 1,$$

(49)
where the numerator after the first equality sign provides the expected duration of government spending (minus one), and the denominator the duration of the liquidity trap (minus one). The fiscal multiplier is defined as the quantity

$$\mu(q, \zeta) = \frac{\sum_{s=0}^{\infty} E_t[\Delta g_{t+s}]}{\sum_{s=0}^{\infty} E_t[\Delta y_{t+s}]}.$$

(50)
Although $\mu(q, \zeta)$ is often thought of as a cumulative multiplier, it should be noticed that $\mu(q, 0)$ corresponds to the fiscal multiplier on impact. Similarly, $\mu(q, 1)$ corresponds to the cumulative multiplier that arises under a committed expansion in government spending which lasts throughout the entire crisis.

The left graph in Figure 6 shows the fiscal multiplier for various combinations of the expected duration of the crisis and the relative duration of spending. The efficacy of government spending is increasing in its duration, and decreasing in the duration of the liquidity trap on most of the domain.

[33]The precise value of $\rho$ has very different implications depending on the duration of the liquidity trap, $1/q$, and is therefore difficult to interpret. The parameter $\zeta$ is more accessible, and can be thought of as the fraction of the liquidity trap’s net expected duration that the government is expected to engage in spending. For instance, a value of 0.5 implies that the government is expected to raise spending throughout half of the duration of the liquidity trap, and so on. Notice that for $q = 1$, $\zeta$ has no meaning, and is assumed to equal one.

30
Returning to the simple model in equation (43) augmented to accommodate the stochastic nature of spending yields

\[ u'(\hat{y} - g) = \hat{\beta} \{ q \sigma (1-\alpha) u'(\hat{y}^\alpha) + (1-q) |\rho| u'(\hat{y} - g) + (1-\rho) u'(y) \}, \tag{51} \]

where \( y \) denotes output in a liquidity trap absent of government spending. The fiscal multiplier is in this case given by

\[ \frac{\partial y}{\partial g} = \frac{1}{1 + \left[ \frac{\sigma}{\gamma} (1-\alpha) - \alpha \right] \times \left[ 1 - \frac{\hat{\beta}(1-q)(1-\rho)}{1-\hat{\beta}(1-q)\rho} \right]}. \tag{52} \]

Thus, the fiscal multiplier is decreasing in the expected duration of the liquidity trap and, provided that \( q < 1 \), increasing in the expected duration of spending. These features align well with Figure 6.

The right graph in Figure 6 illustrates the welfare implications at various combinations of \( q \) and \( \zeta \). A prolonged crisis accompanied by a committed rise in spending suggests that a dollar spent by the government raises welfare by the equivalent of 0.65 dollars of private consumption. For shorter durations – either of the crisis or of relative spending – the welfare effects are smaller, and turn negative at short-lived expansions in countercyclical spending. While expansionary fiscal policy may indeed be welfare-detrimental for a substantial share of the parameter space, it should be kept in mind that government purchases are here considered wasteful. As a consequence, the results in the right graph of Figure 6 should be interpreted as a lower bound.

\[ ^{34} \text{To give some quantitative merit to these ideas, consider the two cases,} \ \rho = 1 \text{ and} \ \rho = 0. \text{ In the former case, the fiscal multiplier is identical to that of Proposition 2 independently of} \ q. \text{ Setting} \ \alpha \text{ equal to 0.3, this corresponds to a cumulative multiplier of 1.68. In the latter case, the cumulative multiplier ranges from 1.43 for values of} \ q \text{ close to zero, and rises relatively linearly to 1.68 at} \ q = 1. \text{ Thus, the effect of fiscal policy spans a larger spectrum of values at lower values of} \ \rho. \text{ A similar picture emerges when instead altering} \ \rho \text{ for the cases,} \ q = 1 \text{ and} \ q = 0. \]
4.4 Discussion

A prediction of the model is that a demand induced recession is marked by a nearly constant nominal wage and a depressed price level. In light of a generally acyclical real wage, this may seem problematic. This is not, however, a model aiming to capture run-of-the-mill business cycles. Rather it is a model of rare occurrences in which a large demand shock pushes the nominal interest rates to zero and leaves conventional monetary policy without traction. Two such occurrences can be observed in the last century; the great depression of 1929-1939, and the financial crisis of 2008-2010.

The evidence regarding the great depression is strong. The price level in the United States fell by 20 percent between 1929 and 1934 while nominal wages barely budged. O’Brien (1989) describes this event as “one of the largest increases in real income of any group in the history of the country” (pp. 719-720). In a panel study containing 22 countries, Bernanke and Carey (1996) further corroborate this evidence and conclude that real wages in 1931-1934 were up 20 to 40 percent on those in 1929, and that this rise was attributable to sharp declines in the price level that were not met by a comparable fall in nominal wages.

![Graph showing employment costs, price level, and productivity](image)

Figure 7: Nominal and real employment costs, the price level, and labor productivity throughout the financial crisis.

Notes. The data are seasonally adjusted quarterly averages by the BLS and the Bureau of Economic Analysis (BEA). All series are normalized to 100 in the fourth quarter of 2007. The PCE refers to the implicit price deflator for personal expenditures from the BEA. Output per worker is defined as real output per person inflated by its own implicit price deflator and re-deflated by the PCE. The Employment Cost Index (ECI) refers to total compensation: all civilian.

Figure reveals a qualitatively similar pattern with regards to employment costs during the financial crisis of 2008. The left panel shows the evolution the employment cost index (ECI), the implicit price deflator for personal expenditures (PCE), and output per worker from 2004 to 2014. The right panel

---

35 The “group”, of course, referring to those who remained employed.

36 The ECI measures the cost of a particular job, and not compensation to workers. As the main mechanism of this
depicts the implied real employment costs. At the time Lehman Brothers fell, the price level dropped but employment costs did not. The result was a pronounced rise in real employment costs that were not justified by a comparable rise in labor productivity. These data are of course not conclusive and their precise implications are certainly open to debate.\footnote{37} But the figure does indeed lend support to the view that the financial crisis was marked by nearly constant nominal employment costs and falling prices.

Lastly, in response to a large, adverse, shock to the economy, the model predicts a sharp rise in expected inflation followed by a reversal to its pre-crisis trend within one quarter (see Figure 5 of Section 4.3.2). This is partly at odds with the data, which display only modest deviations from trend without any particular sharp movements\footnote{38}. This discrepancy is, however, largely mechanical and does not conflict with the main properties of the model. The reason is threefold. First, the initial rise in expected inflation is driven by a large decline in the contemporaneous price level. This drop occurs as the reduction in output in the first quarter is accommodated through a large decline in the number of matches.\footnote{39} As the stock of employed workers is lower in any future period, output remains suppressed even at a less severely depressed matching rate, which therefore allows for a slightly higher price level. The result is a pronounced rise in expected inflation at the onset of the crisis, with a swift reversal. If, however, the shock to the discount factor instead would gradually build up over time – or be pre-announced – this effect would be muted.\footnote{40} Second, the rise in expected inflation stabilizes the economy since it lowers the real interest rate. Thus, absent the rise in expected inflation the contraction would be deeper. Third, the effect of government spending – which is the main focus of this paper – is unrelated to the dynamics of the shock itself. In fact, \textit{any} shock that brings the economy into a liquidity trap would have identical implications on fiscal policy, irrespective of its effect on expected inflation.\footnote{41}

\footnote{37}{For instance, real wages, as measured by BLS’ hourly compensation index, did not exhibit any pronounced pattern during the recent financial crisis irrespective of the choice of deflator.}

\footnote{38}{Breakeven inflation, often defined as the difference in yield between a regular Treasury bill and a Treasury Inflation-Protected Security (TIPS), experienced a pronounced drop by the end of 2008. This drop is, however, mainly attributable to three consecutive trading days following the Federal Reserve’s announcement of the Large-Scale Asset Purchase Program, and not to inflation expectations per se \cite{Dupor and Li 2015}.}

\footnote{39}{See Section 4.3.2, page 29 for a more detailed exposition of this mechanism.}

\footnote{40}{The same logic of course applies if unemployment upon impact of the shock was higher than its steady state value.}

\footnote{41}{Consider, for instance, the simple idea of a shock that temporarily paralyzes the central bank, preventing it from altering the nominal interest rate. The effect of government spending under this hypothesis is identical to the effects explored in the paper, even in the absence of a crisis.}
4.5 Robustness

4.5.1 Parameter sensitivity

The mechanism explored in this paper is not an artefact of a unique combination of calibrated parameters. Its quantitative importance may, however, vary. Table 2 shows the fiscal multiplier for six different combinations of the time to convergence, $\phi$, and the inverse of the EIS, $\gamma$. For each different $(\phi, \gamma)$ combination, the fiscal multiplier is provided given nine combinations of $q$ and $\zeta$. Two key messages emerge from this exercise.

First, irrespective of the time to convergence; the duration of the crisis; or the duration of spending, the fiscal multiplier unambiguously decreases with the elasticity of intertemporal substitution. This aligns well with the message from the stylized model; a lower EIS implies that agents are more reluctant to observe consumption fluctuations absent large changes in the real interest rate. Consequently the propagation mechanism of the paper is amplified.

Second, a longer time to convergence can either increase or decrease the potency of government spending. This may appear contradictory to the message of the stylized framework, where a long time to convergence maps into a higher value of $\alpha$ which bolsters the efficacy of spending. The full model brings forth, however, an additional conflicting force: A shorter time to convergence is related to a higher separation rate which front-loads the valuation of profits, and therefore generate additional sensitivity of asset prices to changes in the current and near-future movements in the price level. Thus, depending on which effect dominates, a longer time to convergence can either strengthen or weaken

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Notes. The table shows the fiscal multiplier associated with a combination of the coefficient of risk aversion, $\gamma$, and the time to convergence, $\phi$. A time to convergence of $\phi = \{3, 5, 7\}$ corresponds to a separation rate of $\delta = \{0.35, 0.15, 0.09\}$, respectively. Each $3 \times 3$ cluster details the multiplier for each $(\gamma, \phi)$-pair for nine combinations of the duration of the crisis, $q$, and the duration of government spending, $\zeta$. The calculations are described in the main text.
Figure 8: Impulse responses to macroeconomic and labour market variables, $q = 0.1$, $\zeta = 1$.

Notes. The shaded lines illustrate all possible paths back to the steady state after a demand shock absent government spending. A darker shade represents a more likely outcome. The dash-dot line and the solid black line illustrate the expected path of each variable with and without government spending. Monetary policy follows a Taylor rule. Government spending is 1 percent of steady state output.
the role of fiscal policy. The latter counteracting effect tends to dominate for short demand spells, i.e. 
\( q = 1 \), short durations of spending, i.e. \( \zeta = 0 \), and when the EIS is relatively large, \( \gamma = 1 \).

### 4.5.2 Alternative monetary policy

To further explore the sensitivity of the model, I assume that the monetary authority must observe the
Taylor rule

\[
1 + i_{t+1} = \frac{1}{\hat{\beta}} \left( \frac{\beta}{\hat{\beta}} \right)^\xi \left( \frac{p_t}{p_{t-1}} \right)^{\phi_{\pi}} \left( \frac{y_t}{y_{ss}} \right)^{\phi_y},
\]

(53)

with \( \phi_{\pi} = 1.5 \) and \( \phi_y = 0.5 \). The parameter \( \xi \) is set to a sufficiently large value to ensure the economy
remains in a liquidity trap throughout the crisis. The remaining calibration follows that of Section 4.1.

Figure 8 illustrates the impulse responses of this model to a discount factor shock, \( \hat{\beta} \). The
quantitative differences between this figure and Figure 4 are small, and the qualitative differences
unnoticeable. Macroeconomic aggregates as well as labor market variables fall, however, somewhat
less in Figure 8 compared to Figure 4 as the depressed level of output leads to a boost in inflation
once the economy leaves the liquidity trap and monetary policy regains traction. The resulting higher
expected inflation lowers the expected real interest rate, and some of the adverse demand effects
associated with the shock are mitigated. Put simply, the Taylor rule is associated with some forward
guidance.

These additional effects are also operating in the presence of government spending. As a rise in
spending closes the gap in output, the accommodative nature of monetary policy is partly offset. The
fiscal multiplier is therefore slightly lower, and about 1.4.

### 5 Concluding Remarks

This paper has shown that equilibrium unemployment dynamics can significantly enhance the efficacy
of fiscal policy. With nominal interest rates stuck at zero and sticky nominal wages, output is largely
determined by demand. As a consequence, a temporary rise in government spending increases output
and lowers the unemployment rate. Since movements in unemployment are partly persistent, a
reduction in the present is also expected to last into the future. With a sufficiently low elasticity of
intertemporal substitution, forward-looking agents desire to smooth consumption over time, and a
reduction in future unemployment feeds back to a further rise in current demand. The outcome of this
two-way interaction is a large response in output – both in the present as well as in the future – with
potentially significant gains in welfare.

The mechanism explored in this paper differs substantially from that of the standard new Keynesian
framework, in which the efficacy of fiscal policy hinges crucially upon the government setting off a
spiral of expected inflation. Here, government spending is associated with less expected inflation, not more. Instead, a rise in government spending is associated with jobs of a relatively long expected tenure. And with a non-trivial expected job tenure, a newly employed agent desires to spend already in the present, setting off a virtuous employment-spending spiral. Thus, the mechanism proposed operates independently of the “expected-inflation channel”, and reflects a distinct transmission mechanism of fiscal policy.

Lastly, it should be noted that this mechanism hinges on the assumption of a frictional labor market, comprised by an otherwise homogenous workforce. If movements in unemployment were primarily driven by structural changes, such as mismatch, the effect of demand stimulating policies are likely to play a less prominent role.
A Proof of Proposition 3

In a steady state all (real) quantities are constant. The equilibrium asset price must therefore satisfy

\[
\frac{\kappa}{h(\theta)} = \frac{1 - \frac{w}{p_t} + \beta(1 - \delta)}{p_t + \beta(1 - \delta)} \frac{\kappa}{h(\theta)},
\]

where \(w\) denotes some fixed nominal wage. Hence the price level is constant \(p_t = p_{t+1} = \ldots = p\). The Euler equation reveals that \(i_{t+1} > 0\). With the monetary authority setting the interest rate equal to the natural rate, the price level must satisfy

\[
\frac{w}{p} = (1 - \omega) + \omega b + \beta(1 - \omega)(1 - \delta)f(\theta) \frac{\kappa}{h(\theta)}.
\]

If the monetary authority instead operates according to a Taylor rule, the price level is not determined without an initial condition on \(w/p_{-1}\). I will assume that \(w/p_{-1}\) satisfies equation (A.2).

Inserting equation (A.2) into equation (A.1) gives

\[
\frac{\kappa}{h(\theta)} = \omega(1 - b) + \beta(1 - \delta) \frac{\kappa}{h(\theta)} [1 - (1 - \omega)f(\theta)].
\]

Define the function \(g(\theta)\) as,

\[
g(\theta) = \frac{1 - \beta(1 - \delta)[1 - \omega f(\theta)]}{\beta(1 - \eta)(1 - b) - \frac{h(\theta)}{\kappa}},
\]

which, in equilibrium, should equal zero. Under standard conditions imposed on the matching function, \(f\) and \(h\) satisfy \(f'(\theta) > 0, h'(\theta) < 0\) as well as the limits: \(\lim_{\theta \to 0} h(\theta) = \infty, \lim_{\theta \to 0} f(\theta) = 0, \lim_{\theta \to \infty} h(\theta) = 0, \) and \(\lim_{\theta \to \infty} f(\theta) = \infty\).

As a consequence \(g(\cdot)\) satisfies \(g'(\theta) > 0, \lim_{\theta \to 0} g(\theta) = -\infty, \) and \(\lim_{\theta \to \infty} g(\theta) = \infty\). Thus there exists a unique steady state equilibrium. \(\square\)
B Government spending outside of a liquidity trap

Figure B.1: Impulse responses to a government spending shock outside the zero lower bound.

Notes. The left column illustrates the evolution of output, the real interest rate, the price level, and the job finding rate following a rise in government spending equal to 1 percent of steady state output with high, $\rho = 0.7$, and low, $\rho = 0$, persistence, and a natural rate policy. The right column illustrates the same variables under a Taylor rule.
Figure B.1 illustrates the effect of an increase in government spending when the economy is not in a liquidity trap. With the monetary authority implementing the natural rate, a rise in government spending depresses both output and the job finding rate, and raises prices as well as the real interest rate. These results contrast somewhat to those of the stylized model in Section 2 in which output is left unchanged, private consumption is reduced, and real interest rates are raised. The rise in the real interest rate is, however, immaterial in the stylized framework. This is not true in the full model. By raising the real interest rate, asset prices fall, and the economy contracts slightly (i.e. there is crowding out of investment).

![Figure B.2: The cumulative fiscal multipliers associated with Figure B.1](image)

With a Taylor rule of the type

$$1 + i_{t+1} = \frac{1}{\beta} \left( \frac{p_t}{p_{t-1}} \right)^{1.5} \left( \frac{y_t}{y_{ss}} \right)^{0.5} \quad (B.1)$$

these conclusions change. A rise in government spending increases prices much more than under the natural rate, leaving real wages severely depressed. While the real interest rate increases, the rise in profits is sufficiently large to increase the return on investment, and the unemployment rate falls. The result is, in contrast to the case with a natural rate, a boom. A Taylor rule, therefore, puts the equilibrium dynamics somewhere in between the results inside a liquidity trap and those of a natural rate policy outside a trap.

Figure B.2 shows the fiscal multipliers associated with both monetary policy rules.
C Computational Details

Outside a liquidity trap the equilibrium is characterized by the following seven equations

\[ 0 = -J_t + 1 - \frac{\omega_{ss}}{p_t} + \beta_t E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} (1 - \delta) J_{t+1} \right], \quad (C.1) \]

\[ 0 = -n_t + [(1 - n_{t-1}) + \delta n_{t-1}] f(\theta_t) + (1 - \delta) n_{t-1}, \quad (C.2) \]

\[ 0 = -w_{ss} + p_t \left\{ (1 - \omega) + \omega b + \beta_t E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} (1 - \omega) f(\theta_{t+1})(1 - \delta) J_{t+1} \right] \right\}, \quad (C.3) \]

\[ 0 = -\kappa + h(\theta_t) J_t, \quad (C.4) \]

\[ 0 = -n_t + c_t + \kappa \theta_t [(1 - n_{t-1}) + \delta n_{t-1}] + g_t, \quad (C.5) \]

\[ 0 = -g_t + g_{ss}, \quad (C.6) \]

\[ 0 = -\beta_t + \beta_{ss}, \quad (C.7) \]

where equation (C.3) ensures that the real wage coincides with the Nash bargaining outcome. With a Taylor rule equation (C.3) is replaced by equation (C.8)

\[ 0 = -\frac{u'(c_t)}{\beta_t u'(c_{t+1})} \frac{p_{t+1}}{p_t} + (1 + i_{t+1}), \quad (C.8) \]

with

\[ 1 + i_{t+1} = \frac{1}{\beta} \left( \frac{p_t}{p_{t-1}} \right)^{1.5} \left( \frac{y_t}{y_{ss}} \right)^{0.5}. \quad (C.9) \]

Inside a liquidity trap the equilibrium is instead characterized by the following seven equations

\[ 0 = -J_t + 1 - \frac{\omega_{ss}}{p_t} + \beta_t E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} (1 - \delta) J_{t+1} \right], \quad (C.10) \]

\[ 0 = -n_t + [(1 - n_{t-1}) + \delta n_{t-1}] f(\theta_t) + (1 - \delta) n_{t-1}, \quad (C.11) \]

\[ 0 = -p_t + \frac{1}{\beta_t} E_t \left[ \frac{u'(c_t)}{u'(c_{t+1})} p_{t+1} \right], \quad (C.12) \]

\[ 0 = -\kappa + h(\theta_t) J_t, \quad (C.13) \]

\[ 0 = -n_t + c_t + \kappa \theta_t [(1 - n_{t-1}) + \delta n_{t-1}] + g_t, \quad (C.14) \]

\[ 0 = -g_t + (1 - \rho) g_{ss} + \rho g_{t-1}, \quad (C.15) \]

\[ 0 = -\beta_t + \beta_{t-1}. \quad (C.16) \]

Linearizing system (C.1)-(C.7) around the steady state gives

\[ 0 = Ax_{t-1} + Bx_t + CE_t[x_{t+1}], \quad (C.17) \]
where $x_t = (n_t, J_t, p_t, \theta_t, c_t, g_t, \beta_t)'$, and $A$, $B$, and $C$ are the Jacobians of (C.1)-(C.7) with respect to $x_{t-1}$, $x_t$, and $x_{t+1}$, respectively. The linearized system in (C.17) can be solved using standard methods. Denote the policy function emerging from the solution as $F$. That is, $x_t = FX_{t-1}$.

Linearizing system (C.10)-(C.16) around the steady state gives

$$
\hat{D} = \hat{A}x_{t-1} + \hat{B}x_t + \hat{C}[(1-q)E_t[x_{t+1}] + qFx_t],
$$

or

$$
0 = \hat{A}x_{t-1} + (\hat{B} + \hat{C}qF)x_t + \hat{C}(1-q)E_t[x_{t+1}] - \hat{D}.
$$

Again, the linearized system in (C.19) can be solved using standard methods with policy function $x_t = \hat{E} + \hat{F}x_{t-1}$.

A sample path in which the economy stuck in a liquidity trap for $n$ periods occurs with probability $Pr(n) = (1-q)^{n-1}q$, and is defined as $x_{t+s+1}(n) = \hat{E} + \hat{F}x_{t+s}(n)$, for $s = 0, \ldots, n$, and $x_{t+s+1}(n) = Fx_{t+s}(n)$, for $s = n + 1, \ldots, T$.

The expected value of a variable is calculated as

$$
E_t[x_{t+s}] = \sum_{n=1}^{S} Pr(n)x_{t+s}(n) + (1-q)^s x_{t+s}(s).
$$

Notice that

$$
\sum_{n=1}^{S} Pr(n) + (1-q)^s = q \sum_{n=1}^{S} (1-q)^{n-1} + (1-q)^s
$$

$$
= q \frac{1 - (1-q)^S}{q} + (1-q)^s
$$

$$
= 1.
$$

![Figure C.1: Nominal interest rates.](image-url)
The left graph in Figure C.1 shows the desired natural and actual nominal interest rate as a function of the discount factor shock, with \( q = 1 \). The right graph shows the desired natural (dashed line) and actual (solid line) interest rate throughout an impulse response. The sample length of each impulse response is, for expositional clarity, set equal to its expected duration.

The left graph is computed using the policy function \( F \), and illustrates the desired nominal interest rate under the assumption that it is implementable both in the present and in the future. As can be seen from the graph, the natural interest rate is not implementable for shocks that brings the discount factor above 1.003.

The right graph is computed as

\[
x_t = \hat{E} + \hat{F} x_{t-1}, \tag{C.24}
\]

with

\[
\hat{E} = \{B + C[qF + (1 - q)\hat{F}]\}^{-1}[-C(1 - q)\hat{E}], \tag{C.25}
\]

\[
\hat{F} = \{B + C[qF + (1 - q)\hat{F}]\}^{-1}(-A). \tag{C.26}
\]

That is, the right graph depicts the desired nominal interest rate under the hypothesis of a one-shot deviation from the liquidity trap.

Figure C.1 shows that the economy is in a liquidity trap as long as the discount factor is elevated, but not beyond. Thus, even though the economy contains one endogenous state variable, the duration of the liquidity trap is entirely determined by the shock to the discount factor and is \textit{de facto} exogenous.

\footnote{Setting \( q \in [0, 1) \) does not result in any discernible difference and is therefore omitted.}
D Fiscal Policy in a Model with Endogenous Separations

To allow for endogenous separations I follow [Mortensen and Pissarides (1994)] and introduce match specific shocks. In particular, in each period each firm-worker pair is exposed to an independent and identically distributed (iid) productivity shock, \( x_t \). If the shock is large enough, the firm-worker pair turns sufficiently unproductive to merit an efficient separation. The introduction of idiosyncratic productivity shocks implies that wages, labor productivity, and separations become functions of one additional state variable; the value of the iid shock below which a separation is efficient, \( \tilde{x}_t \).

Let \( w(x_t) \) denote the nominal wage as a function of the iid productivity shock and let \( F(x_t) \) denote the cumulative distribution function. For any function \( y_t(x_t) \) define \( \hat{y}_t = E[y_t(x_t)|x_t \geq \tilde{x}_t] \). The job finding probability of an unemployed worker is defined as

\[
J_t = [1 - F(\tilde{x}_t)]^M_t. \tag{D.1}
\]

The asset value of a firm is then given by

\[
J_t(x_t) = x_t - \frac{w(x_t)}{p_t} + \beta E_t [u'(c_{t+1})/u'(c_t)] [1 - F(\tilde{x}_{t+1})] J_{t+1}. \tag{D.2}
\]

The analogous asset value for an employed worker is

\[
V_t(x_t) = \frac{w(x_t)}{p_t} + \beta E_t [u'(c_{t+1})/u'(c_t)] ([1 - F(\tilde{x}_{t+1})(1 - f_{t+1})] \hat{V}_{t+1} + F(\tilde{x}_{t+1})(1 - f_{t+1}) U_{t+1}) \tag{D.3}
\]

and for an unemployed worker

\[
U_t = b + \beta E_t [u'(c_{t+1})/u'(c_t)] [f_{t+1} \hat{V}_{t+1} + (1 - f_{t+1}) U_{t+1}] \tag{D.4}
\]

An employed worker’s surplus of a match is given by

\[
V_t(x_t) - U_t = \frac{w(x_t)}{p_t} - b + \beta E_t [u'(c_{t+1})/u'(c_t)] [1 - F(\tilde{x}_{t+1})](1 - f_{t+1}) (\hat{V}_{t+1} - U_{t+1}) \tag{D.5}
\]

Let \( S_t(x_t) \) denote this surplus. Total surplus is then

\[
J_t(x_t) + S_t(x_t) = x_t - b + \beta E_t [u'(c_{t+1})/u'(c_t)] ([1 - F(\tilde{x}_{t+1})](J_{t+1} + \hat{S}_{t+1}) - f_{t+1} \hat{S}_{t+1}) \tag{D.6}
\]

The cut-off value of the match specific shock below which the firm and the worker separate, \( \bar{x}_t \), is determined by

\[
J_t(\bar{x}_t) + S_t(\bar{x}_t) = 0. \tag{D.7}
\]
Figure D.1: Impulse responses to macroeconomic and labour market variables, $q = 0.1$, $\zeta = 1$.

Notes. The shaded lines illustrate all possible paths back to the steady state after a demand shock absent government spending. A darker shade represents a more likely outcome. The dash-dot line and the solid black line illustrate the expected path of each variable with and without government spending.
Or, equivalently,

$$\hat{x}_t = b - \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \left[ (1 - \delta(x_{t+1})) (\hat{J}_{t+1} + \hat{S}_{t+1}) - f_{t+1} \hat{S}_{t+1} \right] \right].$$  \hfill (D.8)

In addition to equation (D.7), the model is characterized by the following five equations.

$$f_t = \hat{x}_t - \frac{\hat{w}_t}{p_t} + \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \left[ 1 - F(\hat{x}_{t+1}) \right] f_{t+1} \right],$$  \hfill (D.9)

$$n_t = \left[ 1 - F(\hat{x}_t) \right] (M_t + n_{t-1}),$$  \hfill (D.10)

$$\kappa = \left[ 1 - F(\hat{x}_t) \right] \eta(x_{t+1}) \hat{J},$$  \hfill (D.11)

$$n_t = c_t + \kappa \theta_t \left[ (1 - n_{t-1}) + F(\hat{x}_t) n_{t-1} \right] + g_t,$$  \hfill (D.12)

$$\hat{S}_t = \frac{\hat{w}_t}{p_t} - b + \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \left[ 1 - F(\hat{x}_{t+1}) \right] (1 - f_{t+1}) \hat{S}_{t+1} \right].$$  \hfill (D.13)

I use a uniform cumulative distribution function,

$$F(x) = \frac{x-a}{b-a},$$  \hfill (D.14)

with $a$ and $b$ set such that $F(\bar{x}_{ss}) = \delta$, and $E[x|x \geq \bar{x}_{ss}] = 1$. Solving the model under the baseline calibration yields a fiscal multiplier of around 1.5. However, the model also delivers the counterfactual prediction of a sharp fall in the separation rate. To understand why, consider the equation determining total surplus and, ultimately, the cut-off, $\hat{x}_t$, (repeated for convenience)

$$J_t(x_t) + S_t(x_t) = x_t - b + \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \left[ (1 - F(\hat{x}_{t+1})) (\hat{J}_{t+1} + \hat{S}_{t+1}) - f_{t+1} \hat{S}_{t+1} \right] \right].$$  \hfill (D.15)

Since this equation is largely independent of wages, total surplus will fall only if the term $f_{t+1} \hat{S}_{t+1}$ rises. However, as $f_{t+1}$ declines, $\hat{S}_{t+1}$ must therefore rise substantially. Since $\hat{S}_{t+1}$ is a worker’s surplus of a match it depends positively on real wages. As real wages rises in the recession, the latter effect can dominate if $\hat{S}_{t+1}$ displays sufficient volatility. Repeating the expression for $\hat{S}_t$

$$\hat{S}_t = \frac{\hat{w}_t}{p_t} - b + \beta E_t \left[ \frac{u'(c_{t+1})}{u'(c_t)} \left[ 1 - F(\hat{x}_{t+1}) \right] (1 - f_{t+1}) \hat{S}_{t+1} \right].$$

Thus, $\hat{S}_t$ will display a lot of volatility if the steady state value of the term $\frac{\hat{w}_t}{p_t} - b$ is quite small. Therefore, following the ideas in [Hagedorn and Manovskii (2008)], I recalibrate the model and set $b = 0.9$ and the workers bargaining power to 0.3. This recalibration leaves steady state wages largely unchanged, but substantially increases the volatility of $\hat{S}_t$. The results can be found in Figures D.1 and D.2. The fiscal multiplier associated with these dynamics is around 2.3.
Figure D.2: Impulse responses to the separation rate and labor productivity, $q = 0.1$, $\zeta = 1$. 
E Comparison of the model with the data

Figure E.1 illustrates the (expected) impulse responses of the model and compares them with their empirical counterpart during the recent financial crisis. As in Kaplan and Menzio (2014) the empirical counterpart is simply defined as the evolution of the corresponding variables throughout the recent financial crisis. The impulse responses, as well as the data, are normalized to take on the value zero at the third quarter of 2008. The data is seasonally adjusted quarterly averages by the Bureau of Labor Statistics (BLS) and the Bureau of Economic Analysis (BEA). Output denotes real gross domestic product; consumption real personal consumption expenditures; the price level the consumer price index; and profits are given as corporate profits after tax (BEA). Profits in the model are calculated as \( 1 - \frac{w_t}{p_t} \). Labor market variables are provided by the BLS, with calculations following Shimer (2005). Asset prices are referring to the S&P 500. All variables are expressed as logarithmic deviations from a HP filter with a smoothing parameter of 1,600. Nominal wages are assumed have a price elasticity of 0.3 (Bernanke and Carey, 1996), such that \( w_t = wp_t^{0.3} \), and monetary policy follows the Taylor rule of Section 4.5.2.

There are a few differences between this figure and Figure 4. In particular, as nominal wages partly respond to movements in prices, the fall in the price level is here more pronounced. In addition, as monetary policy follows a Taylor rule, prices are more sluggish and revert back to the steady state value more slowly. This latter feature implies that macroeconomic and labor market variables display more persistence. The multiplier associated with these dynamics is 1.4.

While the model generally fits the data well, it is off in two dimensions. First, the price level drops more in the model than in the data. This follows as wages are only partly rigid while prices are fully flexible. Second, the drop in corporate profits is more pronounced in the model than in the data. This latter finding reflect the well known fact that asset prices display more volatility than what is justified by their respective dividend flows (Shiller, 1981). As a consequence, by accurately matching the behavior of dividends/asset prices, inevitably implies a failure to match the other. It is worth keeping in mind, however, that while the model has only one endogenous state variable – employment – and is exposed to only one type of shock – the discount factor – it is assessed against eight different observables.

E.1 Transitional dynamics

The grey line in the left graph of Figure E.2 illustrates the evolution of quarterly US unemployment from 1948 until 2014. The black line shows the approximate value when unemployment is set equal to its conditional steady state, \( u^* t \). To calculate the approximate value, I impute a series of the job-finding rate according to the law of motion for employment in equation (27). That is,

\[
f_t = 1 - \frac{u_t}{\delta(1 - u_{t-1}) + u_{t-1}}, \tag{E.1}
\]
Notes. The data is seasonally adjusted quarterly averages by the BLS and the BEA. Output denotes real gross domestic product; consumption real personal consumption expenditures; price level the consumer price index; and profits are given as corporate profits after tax (BEA). Labor market variables are provided by the BLS, with calculations following Shimer (2005). Asset prices are referring to the S&P 500.
with the separation rate calibrated according to Section 4.1, i.e., $\delta \approx 0.15$. I then find the approximate value of unemployment according to,

$$u^*_t = \frac{\delta (1 - f_t)}{\delta (1 - f_t) + f_t},$$

(E.2)

where equation (E.2) repeats equation (37) for convenience. As can be seen from the figure the discrepancy between actual and approximate unemployment is very small. Thus, the unemployment rate is well approximated by its conditional steady state.

The right graph of Figure E.2 illustrates the impulse response of unemployment from a temporary 30 percent decline in the job-finding rate, using the approximation in equation (E.2), as well as the true law of motion in equation (27). The approximation overstates the effect on impact, but understates the ensuing dynamics. Revisiting the results in Figure 3, page 25 provides direct evidence for what these discrepancies imply for the model. The fiscal multiplier in the baseline scenario is 1.18 on impact and cumulates to 1.67. In Case 1, in which the transitional dynamics are absent, both these numbers decline to about 0.82. As a consequence, the transitional dynamics gives rise to a propagation that amplifies a shock to unemployment by a factor of about 1.45 on impact, and more than doubles it over the course of one and a half years.
References


