Essays in Modern Macroeconomics

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Declaration

This thesis is the result of my own work and includes nothing which is the outcome of work done in collaboration except as declared in the Preface and specified in the text.

I further state that no substantial part of my thesis has already been submitted, or, is being concurrently submitted for any such degree, diploma or other qualification at the University of Cambridge or any other University or similar institution except as declared in the Preface and specified in the text.

It does not exceed the prescribed word limit for the relevant Degree Committee of 60,000 words.

Daniel Wales
October 2022

Abstract

Essays in Modern Macroeconomics; Daniel Wales

This PhD thesis consists of a short introduction followed by three papers. Each paper examines a different topic within the broad area of modern monetary and international macroeconomics.

The first paper, *Product Quality, Measured Inflation and Monetary Policy*, written in collaboration with Alex Rodnyansky and Alejandro van der Ghote, fills a gap in the New Keynesian literature, which has largely ignored product quality adjustments. This paper proposes a tractable model of a New Keynesian (NK) economy where, in addition to the standard price and quantity channels, firms are able to endogenously adjust the quality of their products in response to shocks. This new model, featuring endogenous product quality changes subject to adjustment costs, nests the canonical New Keynesian model, which is frequently used as the starting point for policy analysis by central banks. In this framework, endogenous product quality choices imply a larger slope than the traditional NK Phillips curve as, for a positive productivity shock that lowers marginal costs, quality-adjusted prices decline because firms are simultaneously able to increase the quality of their products. Allowing firms to adjust product quality also amplifies the economy's response to productivity shocks. Following a positive productivity shock the natural real interest rate decreases by more as households look to smooth a larger increase in consumption, which is boosted by a rise in both the quantity and quality of the goods they consume. As a result, monetary policy responds by altering the nominal interest rate by more for a given productivity shock. Model misspecification of imperfectly observable quality adjustments matters more for macroeconomic stabilization than the mismeasurement of those adjustments. With no misperception of product quality by the monetary authority, the principles for optimal monetary policy are, nonetheless, unchanged as the product quality extensions to the canonical NK model preserve divine coincidence.

My second PhD paper, *The Impact of Large-Scale Asset Purchases on Wealth Inequality* examines the relationship between monetary policy and household wealth inequality through changes in the size and composition of the central bank's balance sheet. I focus on the impact on household wealth inequality through the financial portfolio rebalancing channel of monetary policy transmission. I construct a theoretical model that has multiple assets (of differing liquidity), banks and heterogeneous agents, who experience idiosyncratic labor productivity shocks. This model is carefully calibrated to reproduce theoretical levels of wealth inequality which match those observed in the US Survey of Consumer Finances. I use the model to replicate the changes in the Federal Reserve's balance sheet which arose in the aftermath of the 2007/2008 financial crisis. This shows that an expansion of the central bank's balance sheet can materially alter the distribution of wealth, causing inequality to increase,

while even extreme changes in the composition of the central bank's balance sheet (for example through maturity extension) have little effect. This arises as central bank purchases of longer-term assets cause households to hold additional liquid financial wealth. Liquid financial assets are unevenly distributed in the population, and hence wealth inequality measures increase. When the model is calibrated to match the Federal Reserve's Large Scale Asset Purchases (LSAPs) from 2008 until 2014, wealth inequality increases by 3.8%, as measured by the Gini coefficient, suggesting this channel leads to a significant increase in wealth inequality.

The final PhD paper, *The Rise of Harrod-Balassa-Samuelson*, begins by documenting two stylised facts. Firstly, over the past 70 years the positive cross-country relationship between aggregate consumer prices and real output per capita has strengthened (i.e. a rise in the Harrod-Balassa-Samuelson effect), as demonstrated using data from the Penn World Tables. Secondly, border frictions have increased over the same time frame, with international borders effectively becoming wider and an increasing failure of the Law of One Price (LOOP). I construct my own dataset of city-level relative prices using national sources across five continents to document the increasing failure of the LOOP. I then use a two-country endowment model with a domestic distribution services sector to construct an equilibrium failure of the LOOP. An increase in the relative size of the distribution services sector can simultaneously explain both stylized facts, while the standard explanation (a higher share of non-traded goods) may only explain the first. Furthermore, I extend the model to include production by monopolistically competitive firms, before solving and calibrating the model to closely replicate the two stylised facts.

JEL Codes: D31, E31, E32, E52, E58, F31, F41.

To my wife, Diana.

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Chapter 1

Introduction

Modern macroeconomics has greatly evolved during the last four decades, starting with the introduction of microeconomic foundations. The ubiquitous Hicks (1937) and Hansen (1953) IS-LM framework faced difficulties in simultaneously explaining low output growth and high price inflation during the 1970s, particularly in the United States. Partly as a result, mainstream macroeconomists recognised an urgent need to reconcile their theoretical models with the latest empirical observations (Lucas and Sargent, 1979). Progress arose in several stages. One new strand of the literature incorporated optimising behaviour by firms and households. This built upon earlier microeconomic results, including the general equilibrium theories of Arrow and Debreu (1954). This approach, with fully flexible prices and continually efficient markets, became known as the Real Business Cycle (RBC) theory (Kydland and Prescott, 1982). An alternative research agenda found explanations for nominal rigidities through efficiency wages, menu costs, staggered pricing and labour market hysteresis. Mankiw and Romer (1991a,b) provide a two volume collection of the seminal works in this area which together formed the basis of New Keynesian Phillips Curve (NKPC). Other frictions, for example as a result of money creation (Kiyotaki and Moore, 1997) or imperfect competition (Hart, 1982), were also adopted into the new theories. Gradually, the two approaches were combined to form the New Keynesian (NK) model (Goodfriend and King, 1997; Yun, 1996). Subsequent extensions have produced the more complex Dynamic Stochastic General Equilibrium (DSGE) class of economic models (Christiano et al., 2005; Smets and Wouters, 2007). These DSGE models were the foremost tools for macroeconomic policy analysis during much of the last decade. Most recently, advances in computational economics have enabled researchers to analyse the impact of non-degenerate household wealth distributions in these settings (Guerrieri and Lorenzoni, 2017; Kaplan et al., 2018).

Against this historical background, each paper in this thesis addresses a distinct issue within the broad area of modern monetary and international macroeconomics. The first two papers discuss the conduct and effects of monetary policy while the final paper builds a model to explain two stylised facts within the field of international economics. Although there are many substantial differences between these papers, they share several common themes in the methods and techniques they use to answer these questions, drawing upon recent developments in the macroeconomics literature.

The first theme shared by all three papers is their use of microeconomic foundations to build macroeconomic models. In the first paper this manifests as an extension to the canonical New Keynesian (NK) model of the economy, discussed at length in Galí (2015), where forward-looking households and firms optimise their decisions, subject to a series of constraints. In

the second paper these microeconomic foundations are again present for households who make portfolio investment decisions based upon expectations for their individual future labour productivity and state of the aggregate economy. The final paper uses microeconomic foundations for households and firms across a "Home" and "Foreign" country to construct a model of the international economy, building upon a large modern international economics literature, such as the seminal textbook by Obstfeld and Rogoff (1996).

The second theme shared across these papers is their use of realistic economic "frictions" to alter established results in the literature. The first paper assumes intermediate goods firms must decide their level of product quality, which is costly to produce but raises demand. Past quality decisions determine the size of current quality adjustment costs, smoothing the desired adjustments over time. These real frictions are accompanied by a standard Rotemberg (1982) nominal pricing friction. The second paper assumes frictions within financial markets, which are modelled as incomplete. Alongside heterogeneous labour productivity, this produces an endogenous non-degenerate distribution of household wealth and hence household wealth inequality. Frictions in the final paper arise in the form of local distribution costs for internationally traded intermediate goods, which cause an endogenous failure of both Purchasing Power Parity (PPP) and the Law Of One Price (LOOP) as in Burstein et al. (2005, 2003) and Corsetti and Dedola (2005).

A final theme, explored in papers two and three, is a use of heterogeneity to understand transmission mechanisms in greater depth and provide realism to otherwise representative theoretical models. The second paper moves away from the traditional representative agent framework to model an economy comprised of heterogeneous households, with a focus on the wealth inequality resulting from heterogeneous productivity in the presence of incomplete financial markets. The final paper appeals to a fundamental change in the structure of modern economies, as it establishes that when local wholesale distribution costs represent a larger proportion of product prices, this may act to increase the cross-country wedge between the consumer price levels of traded goods. In this model, accounting for differences in local production requirements across countries, wholesale firms choose to charge heterogeneous prices, when expressed in the same currency units. This results in differential consumer retail prices being charged for the same products in the Home and Foreign country.

Chapter 2

Product Quality, Measured Inflation and Monetary Policy[1]

2.1 Introduction

A growing body of empirical work has documented that product quality adjustments play a prominent role in the overall response of firms to a wide array of economic shocks.[2] In this paper, we investigate how the addition of a product quality choice by firms into an otherwise standard New Keynesian framework influences macroeconomic outcomes and the transmission mechanism of monetary policy.

Specifically, we propose a New Keynesian model augmented by endogenous product quality choices by firms, which nests the canonical framework. This approach shows how firms may endogenously determine the optimal product quality level alongside price and production quantity. Intuitively, to determine product quality, two opposing forces are simultaneously operative. If firms choose to produce higher quality products, then they face larger marginal costs of production, reducing profits. However, increasing product quality also raises demand, and hence revenue. In an equilibrium where symmetric firms adjust the price and quality of their products sluggishly due to Rotemberg (1982) adjustment costs, we thus add one additional condition to determine product quality, thereby obtaining dynamics for the quality of output that are inherently different from the usual quantity dynamics. In this setting quality-adjusted prices typically adjust by more than in the canonical New Keynesian model.

Product quality plays a central role in many areas of economics. For instance, it explains why some firms are more successful than others; why production input intensities vary; how exporters set prices; and how households respond to business cycles.[3] These empirical results on the importance of product quality serve as motivating evidence for our analysis. Despite a rich literature and a long tradition, particularly in international trade and industrial organization, most workhorse business cycle models do not feature any quality decisions

[1] This chapter is based upon work completed in collaboration with Alex Rodnyansky and Alejandro van der Ghote.

[2] For instance, Goetz and Rodnyansky (2021) show that exchange rate devaluations can cause importers to substitute away from high-quality imported goods more quickly than low-quality items while Medina (2020) finds that an increase in trade-induced competition leads firms to upgrade the quality of their output to escape competition from low-wage countries.

[3] Hottman et al. (2016) show firm appeal (a residual partly determined by product quality) explains 76% of variation in firm sales; Jaimovich et al. (2019a) and Jaimovich et al. (2019b) find that high-quality goods are more labour-intensive, particularly for high skilled workers; Manova and Zhang (2012) propose a model with heterogeneous quality to explain six stylised facts about firm's export prices from the universe of Chinese customs data; Jaimovich et al. (2019a) document that consumers traded down in the quality of goods consumed during the Great Recession.

and assume that firms only adjust the price and quantity of their products.[4] We augment the standard New Keynesian model to understand how product quality choices affect the transmission of shocks and, in particular, we show how the transmission of monetary policy shocks depends critically upon the relative rigidity of price and quality adjustments.

It turns out that a New Keynesian model with endogenous product quality adjustments has several different implications for inflation dynamics, shock propagation, and monetary transmission. First, the quality-augmented model implies a larger slope of the traditional theoretical NK Phillips curve. This result arises because, for a given productivity shock that changes marginal costs, firms adjust their hedonic (i.e. quality-adjusted, welfare-relevant) prices by more when they are able to simultaneously adjust the quality of their products. Product quality adjustment provides an additional margin which reinforces the standard effect. Furthermore, the quality-augmented NK Phillips curve includes an additional term, which captures contemporaneous product quality choices. The slope of the Phillips curve increases in the relative ratio of the cost of price adjustment to that of quality adjustment. The lower the relative costs of quality adjustment vs. price adjustment the higher the adjustment in quality, and hence the higher the slope of the Phillips curve. That is, as quality rises relatively more quickly after a positive productivity shock, this will reduce the aggregate measures of quality-adjusted inflation more strongly.

Second, endogenous quality choices further amplify the response of the baseline economy to productivity (or cost) shocks as changes in productivity now cause firms to alter both the price and quality of their products. After an initial positive shock to productivity, current marginal costs fall. Firms then seek to re-optimize by cutting prices, to re-establish an optimal markup, and also by increasing product quality, which becomes cheaper to produce. Both of those effects lead to an increase in demand for each variety. In equilibrium this results in a larger decline in the natural real interest rate as households look to smooth a larger increase in consumption, which is boosted by a rise in both the quantity and quality of the goods they consume.

Third, for calibrated versions of the model, and assuming the monetary policy authority responds to shocks according to a standard theoretical Taylor rule (based upon quality-adjusted inflation), we show that the amplification result leads to larger changes in the nominal interest rate, for a given productivity shock. We also analyze the empirically relevant scenarios of *bad measurement*, when the national statistical authority cannot accurately measure product quality adjustments, as well as the extreme case where the central bank neglects product quality changes altogether, which we will refer to as a *bad model*. We

[4]For example Benigno and Benigno (2003), Galí and Monacelli (2005), Galí (2015), Campa and Goldberg (2005, 2008), Sutherland (2005), and Nakamura and Zerom (2010).

find that bad measurement of product quality almost does not change the monetary policy reaction, whereas, in the case of a bad model, the central bank tends to stabilize the economy substantially less than intended.

In building a theoretical model, our framework allows for an explicit analytical formulation of the difference between the true rate of price inflation and a potentially biased measure of (sticker price) inflation, which does not account for changes in product quality. Such quality-adjusted inflation *may* be captured by statistical authorities. Discussions around this topic have a long history dating back at least to Boskin et al. (1996) and, more recently, economic accounting changes in ESA (2010)[5] and the Bean Review (2016)[6], which document how statistical agencies in the United States, Euro Area, and United Kingdom, respectively, have struggled with those issues. We show how the difference between the two measures has important implications for the optimal conduct of monetary policy.

If monetary policymakers misperceive the level of the output gap or inflation (perhaps due to mismeasurement), the economy nonetheless responds in a similar fashion. However, if the monetary policy authority misperceives the true model and reacts to productivity shocks assuming product quality does not adjust, it will stabilize the economy by less than intended. The principle for optimal monetary policy is nonetheless unchanged under no misperception as "divine coincidence" also applies in this broader setting, with monetary policymakers thus seemingly able to simultaneously stabilize the output gap, sticker price inflation and product quality movements. In this setting, as product quality and sticker price inflation are stabilized, so too are hedonic measures of inflation.

2.1.1 Related Literature

Our work integrates several strands of the literature. Empirically, there are at least two channels through which product quality may matter for the macroeconomy, as a source of a product's price variation (Nakamura and Steinsson, 2008) or sales variation (Hottman et al., 2016), and as a key component in heterogeneous product replacement (or upgrading) decisions. The product life-cycle is likewise important as innovative higher-quality products replace outdated ones (Argente et al., 2020).[7]

[5] European System of Accounts 2010 (ESA 2010), paragraphs 10.13-10.18. Available at https://ec.europa.eu/eurostat/web/esa-2010.

[6] Paragraphs 3.76-3.90. Available at https://www.gov.uk/government/publications/independent-review-of-uk-economic-statistics-final-report.

[7] De Ridder (2019) also considers longer-run developments, showing that investment in corporate research and development in the United States has steadily increased by 61% as a fraction of national income over the last 30 years, reflecting the ever increasing desire on the part of firms to produce higher quality versions of goods than their counterparts.

Our paper is also related to work by Adam and Weber (2019) that studies monetary policy when firms face Calvo (1983) and Yun (1996) style stickiness in setting both quality and prices. The authors show that the optimal level of trend inflation depends on the number of firms entering the market, and their relative productivity (and hence pricing) differences to incumbents. For instance, when each cohort of new firms are more productive than the existing firms it is socially optimal for new firms to charge lower prices. In this case the optimal level of trend inflation may be negative. Schmitt-Grohé and Uribe (2012) compute a theoretical level of optimal trend inflation which varies depending on whether sticker prices or quality-adjusted prices are subject to an adjustment rigidity. As opposed to those papers, product quality adjustments are endogenous in our setting, and as such depend on monetary policy, and the focus is on business cycle fluctuations rather than on long-term trends. In contrast to much of the literature on product quality choice (including Fajgelbaum et al. (2011); Khandelwal (2010); and Jaimovich et al. (2019a)), our results do not rely upon non-homothetic utility function assumptions, although this would reinforce our main findings.

We are not the first paper to consider product quality movements in the context of business cycles, and the consequences of new products replacing existing ones as part of an ongoing process of innovation. Bils and Klenow (2001) used the U.S. Consumer Expenditure Survey to quantify the quality bias in consumer durable goods, and found that quality grew on average 3.8 percent per year in the 1980-1996 period. More recently, Garcia-Macia et al. (2019) show that most productivity growth in the U.S. is generated by incumbent firms improving their existing products rather than the creation of new varieties, and that this force is more important than creative destruction. Other work has focused on mismeasurement of growth and inflation due to product quality changes which are unaccounted for (e.g. Broda and Weinstein (2010); Nakamura and Steinsson (2012); and Aghion et al. (2019)).

Our paper connects to a rapidly growing theoretical and empirical literature on the "disappearance" of the traditional Phillips curve relationship. The lack of sensitivity of inflation to changes in employment over the recent decades, and especially during the Great Recession, has led many to believe that the Phillips curve has flattened. This expanding body of work offers a number of explanations for those patterns, such as shifting inflation expectations or globalization and intermediate input flows (e.g. Ball and Mazumder (2011, 2019); Barnichon and Mesters (2020); Blanchard et al. (2015); Coibion and Gorodnichenko (2015); Del Negro et al. (2020); Geerolf (2019); Hall (2013); IMF (2013); Stock and Watson (2019); and Rubbo (2020)), though recent work has also questioned the extent of this fall (Hazell et al., 2021). We contribute to this strand by showing that the apparent flattening of the Phillips curve does not have to imply a flaw in the New Keynesian model once it is augmented to account for

endogenous quality changes, provided the relative ease of adjusting product quality, relative to sticker prices, has risen.

Finally, we relate to studies on the role of product quality in international finance and trade. Linder (1961) first noted the role of quality as a determinant of the direction of trade, while Rodriguez (1979) developed a simple model to analyse the equivalence between tariffs and quotas in a setting with endogenous quality. Hummels and Klenow (2005) use export quantities and proxies for the number of varieties to argue that product quality differences are necessary to explain (at least part of) the observed differences in unit export values. Chen and Juvenal (2018); Levchenko et al. (2011); and Bems and Di Giovanni (2016) have found evidence that the disproportionate drop in the value of international trade after the global negative income shock in 2008 was caused by the higher quality of traded goods combined with non-homotheticity of demand. Previous work also examined the relationship between trade distances and product quality (including Alchian and Allen (1964); Hummels and Skiba (2004); and Feenstra and Romalis (2014)). Recent contributions have also shown that firms may choose to up- or downgrade their product quality in response to exchange rate movements (Bastos et al., 2018; Goetz and Rodnyansky, 2021), or because competing with inexpensive imports from low-wage countries induces companies to upgrade, as in Martin and Mejean (2014) and Medina (2020).[8]

The remainder of the paper proceeds as follows. Section 2.2 provides the model and section 2.3 discusses the implications of product quality adjustments for the slope of the Phillips curve and the business cycle fluctuations in the natural rate of interest. Section 2.4 solves a calibrated version of the model and then investigates the implications of *bad measurement* by the statistical authority and a *bad model* in being used by the central bank. Section 2.5 concludes.

2.2 Model

The model is comprised of five agent types: households, final goods producers, intermediate goods producers, the government and the central bank. In the model households are identical. Final goods producers operate under prefect competition while intermediate goods producers operate under monopolistic competition. The government fiscal authority runs a balanced budget. Throughout this model lower case variables are defined as the natural logarithm of their upper case equivalents, i.e. $x = \ln X$.

[8]Other trade shocks that can drive firms to quality upgrade include cheaper intermediate inputs (see Verhoogen (2008), Fieler et al. (2018) and Bas and Strauss-Kahn (2015)) or access to larger markets (see Bustos (2011), Lileeva and Trefler (2010), and Aw et al. (2011)).

2.2.1 Household Problem

A set of identical households seek to maximize their expected lifetime utility, given by:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{L_s^{1+\eta}}{1+\eta} \right), \tag{2.1}$$

where $0 < \beta < 1$ denotes their subjective discount factor, $\sigma > 0$ denotes their coefficient of relative risk aversion, $\eta \geq 0$ denotes their inverse Frisch elasticity of labour substitution, C_t denotes their consumption of a composite good, L_t denotes their labour supply, and subscripts indicate the time period. Households have access to one-period nominal bonds which they may use to smooth consumption over time. Thus households face a nominal budget constraint given by:

$$P_t C_t + B_{t+1} = (1 + i_t) B_t + W_t L_t + P_t T_t + P_t D_t. \tag{2.2}$$

B_{t+1} represents the bonds purchased by households in period t. These pay out the gross nominal interest rate, $1 + i_{t+1}$, in period $t + 1$. The nominal wage is given by W_t and T_t denotes the real level of lump sum transfers (or taxes when negative) given to households by the government fiscal authority. D_t is the lump sum value of real dividend income given to households by some intermediate goods firms (see subsection 2.2.3 for details on these firms). P_t is the aggregate price level.

The solution of this problem then yields the standard Euler equation, defining the relationship between marginal utilities across time periods, as well as the intratemporal trade-off between current consumption and current labour supply:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + i_t}{1 + \Pi_{t+1}} \right], \tag{2.3}$$

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\eta}, \tag{2.4}$$

where we define the aggregate gross rate of household price inflation as $1 + \Pi_t \equiv P_t / P_{t-1}$.

2.2.2 Final Goods Producers

Producers of the final composite good, Y_t, operate under perfect competition. Y_t is a Constant Elasticity of Substitution (CES) aggregate of a continuum of intermediate goods, $Y_t(j)$, with $j \in [0, 1]$:

$$Y_t = \left[\int_0^1 [\mathcal{G}(F_t(j))]^{\frac{1}{\varepsilon}} Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}j \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{2.5}$$

where $\mathcal{G}(\cdot)$ is a generalized demand shifter with the property $\mathcal{G}_F(\cdot) \equiv \partial\mathcal{G}(\cdot)/\partial F_t(j) > 0$, while its argument, $F_t \geq 0$, represents the number of "features" a given product or variety has—i.e., its quality. This formulation generalises existing specifications used in the macroeconomics and international trade literature.[9] The parameter $\varepsilon > 1$ is the elasticity of substitution across varieties, and a measure of competition in the economy. Having noted that the function $\mathcal{G}(F_t(j))$ may be idiosyncratic across the intermediate goods depending upon their level of quality, we will suppress this as $\mathcal{G}_t(j) \equiv \mathcal{G}(F_t(j))$ for notational convenience.

Using (2.5), the cost minimization problem of the final goods producers results in a demand function for each intermediate variety given by:

$$Y_t^d(j) = \mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t, \tag{2.6}$$

where $P_t(j)$ is the sticker price of variety j. In addition, the aggregate hedonic (welfare relevant, quality-adjusted) price level can be found as the minimum expenditure to purchase one unit of final output, Y_t:

$$P_t = \left[\int_0^1 \mathcal{G}_t(j)P_t(j)^{1-\varepsilon}\,\mathrm{d}j\right]^{\frac{1}{1-\varepsilon}}. \tag{2.7}$$

This shows an inverse relationship between the aggregate quality-adjusted price and the level of product quality of individual intermediate goods, as captured by $\mathcal{G}_t(j)$. Further, the final goods price level, (2.7), reduces to the standard business cycle formulation whenever for all intermediate producers $\mathcal{G}_t(j) = 1$. This outcome, with an absence of product quality effects, may be used to define a cost-of-goods index (i.e., a price level that is not adjusted for product quality). The literature refers to this as the sticker price index, P_t^S:

$$P_t^S = \left[\int_0^1 P_t(j)^{1-\varepsilon}\,\mathrm{d}j\right]^{\frac{1}{1-\varepsilon}}. \tag{2.8}$$

More formally, this sticker price index represents the minimum expenditure to purchase one unit of standard final output, Y_t^S, without extra features, so it does not account for product quality:

$$Y_t^S = \left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}\,\mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{2.9}$$

[9] Including Adam and Weber (2019) and Schmitt-Grohé and Uribe (2012) in the former, and Hallak (2006); Kugler and Verhoogen (2012); and Feenstra and Romalis (2014) in the latter.

2.2.3 Intermediate Goods Firms

A continuum of intermediate goods firms, indexed by $j \in [0, 1]$, operate under monopolistic competition. A typical intermediate firm faces a production function given by:

$$Y_t(j) = N_t(j)\mathcal{H}(A_t, F_t(j)), \tag{2.10}$$

where $N_t(j)$ is the level of labour input and the function $\mathcal{H}(\cdot)$ takes the place of the productivity of labour. This function shifts the production process depending on both the economy-wide level of productivity, A_t, and the level of quality firm j chooses to produce, $F_t(j)$. Again we take as given the arguments of this function, and suppress the notation as $\mathcal{H}_t(j) \equiv \mathcal{H}(A_t, F_t(j))$ for convenience. Throughout we will assume that $\mathcal{H}_{F,t}(j) \equiv \partial\mathcal{H}(\cdot)/\partial F_t(j) < 0$, such that producing goods with a higher quality reduces the level of labour productivity, and $\mathcal{H}_{A,t}(j) \equiv \partial\mathcal{H}(\cdot)/\partial A_t > 0$, as is standard. Given the economy-wide level of productivity and level of quality produced by firm j, real marginal costs can be written as:

$$MC_t(j) = \frac{W_t}{P_t}\frac{1}{\mathcal{H}_t(j)}. \tag{2.11}$$

This potentially differs across firms through heterogeneous quality levels.

Optimality Under Flexible Price and Quality Adjustments

Under flexible pricing, the real profit (dividend) flow for a given intermediate goods producer is given by:

$$D_t(j) = (1 + \tau_t)\frac{P_t(j)}{P_t}Y_t^d(j) - \frac{W_t}{P_t}N_t(j). \tag{2.12}$$

The variable τ_t represents a production subsidy given to intermediate goods firms by the fiscal authority. This subsidy will be set to eliminate distortions from monopoly power (see subsection 2.2.4). Using (2.10) and (2.11) to rewrite labour costs and then using the market clearing condition under monopolistic competition to equate (2.6) and (2.10), the static profit maximization problem for each variety of the intermediate goods takes the form:

$$\max_{P_t(j),F_t(j)} \left[\frac{(1+\tau_t)P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t, \tag{2.13}$$

where firms are able to choose both prices, $P_t(j)$, and quality, $F_t(j)$.

The first order condition with respect to prices, $P_t(j)$, reveals the standard condition for the optimal level of prices:

$$\frac{P_t(j)}{P_t} = \frac{\varepsilon}{\varepsilon - 1} \frac{MC_t(j)}{1 + \tau_t}, \tag{2.14}$$

which shows that in this framework, and absent production subsidies, the optimal relative price remains a constant mark-up above marginal costs. This condition holds even though marginal costs may increase when increasing the quality of goods produced. The optimal level of product quality is determined by the first order condition of the profit function (2.13) with respect to $F_t(j)$. Using (2.6) this gives:

$$\underbrace{\frac{\partial MC_t(j)}{\partial F_t(j)} Y_t(j)}_{\text{1. Marginal Cost Increase}} = \underbrace{\left[\frac{(1+\tau_t)P_t(j)}{P_t} - MC_t(j)\right]}_{\text{Markup}} \underbrace{\frac{\partial Y_t^d(j)}{\partial F_t(j)}}_{\text{2. Demand Increase}} . \tag{2.15}$$

This states that, while optimally setting prices to maintain their desired markup, according to (2.14), firms are simultaneously able to further increase profits through altering their product quality. This clearly shows the two influences:

1. Higher quality products reduce profits as they increase marginal costs.
2. Higher quality products increase profits as they increase demand.

Graphical Interpretation

This situation is shown graphically in Figure 2.1. Due to imperfect competition in the intermediate goods sector, individual firms face a downward sloping relative demand curve using (2.6) and, given both the quality of their products and aggregate prices, marginal costs (2.11) are constant in the quantity produced.

The initial position is shown in panel (a), where the production point for the monopolist of each variety arises at the intersection of marginal cost (the **black dotted** line) and marginal revenue (the **black dashed** line).[10] In this case, prices are a fixed markup over marginal costs,

[10] Marginal revenue in Figure 2.1 is the derivative of total revenue share:

$$\frac{P_t(j)Y_t^d(j)}{P_t Y_t} = \left(\frac{Y_t^d(j)}{Y_t}\right)^{\frac{\varepsilon-1}{\varepsilon}} \mathcal{G}_t(j)^{\frac{1}{\varepsilon}} \tag{2.16}$$

with respect to relative demand. So, it is equal to:

$$\left(\frac{\varepsilon - 1}{\varepsilon}\right)\left(\frac{Y_t^d(j)}{Y_t}\right)^{-\frac{1}{\varepsilon}} \mathcal{G}_t(j)^{\frac{1}{\varepsilon}}. \tag{2.17}$$

and profits are then the **black shaded** area. An increase in the quality of products supplied is then shown in panel (b) as the movement from the **black** lines toward the **blue** lines.

As quality improves, we observe two effects:

1. Marginal costs increase, as higher quality products are more costly to produce from a given level of factor inputs (the effective productivity of labour falls).

2. Higher quality increases the relative demand, and thus marginal revenue, for this variety.

The resultant profit maximization is then shown in panel (c), where profits are now represented as the **blue shaded** area between the new demand schedule and new marginal costs at the optimal prices and quantities. For reference, the initial level of profits is also shown in the diagram to highlight how the firm will use this simultaneous shift in marginal costs and demand to determine the optimal level of product quality (or product features) that maximizes profits. The firm prefers the new level of quality if the new profit area is larger than the initial one and adjusts level of product quality so that it maximises profits.

Optimality Under Price and Quality Rigidities

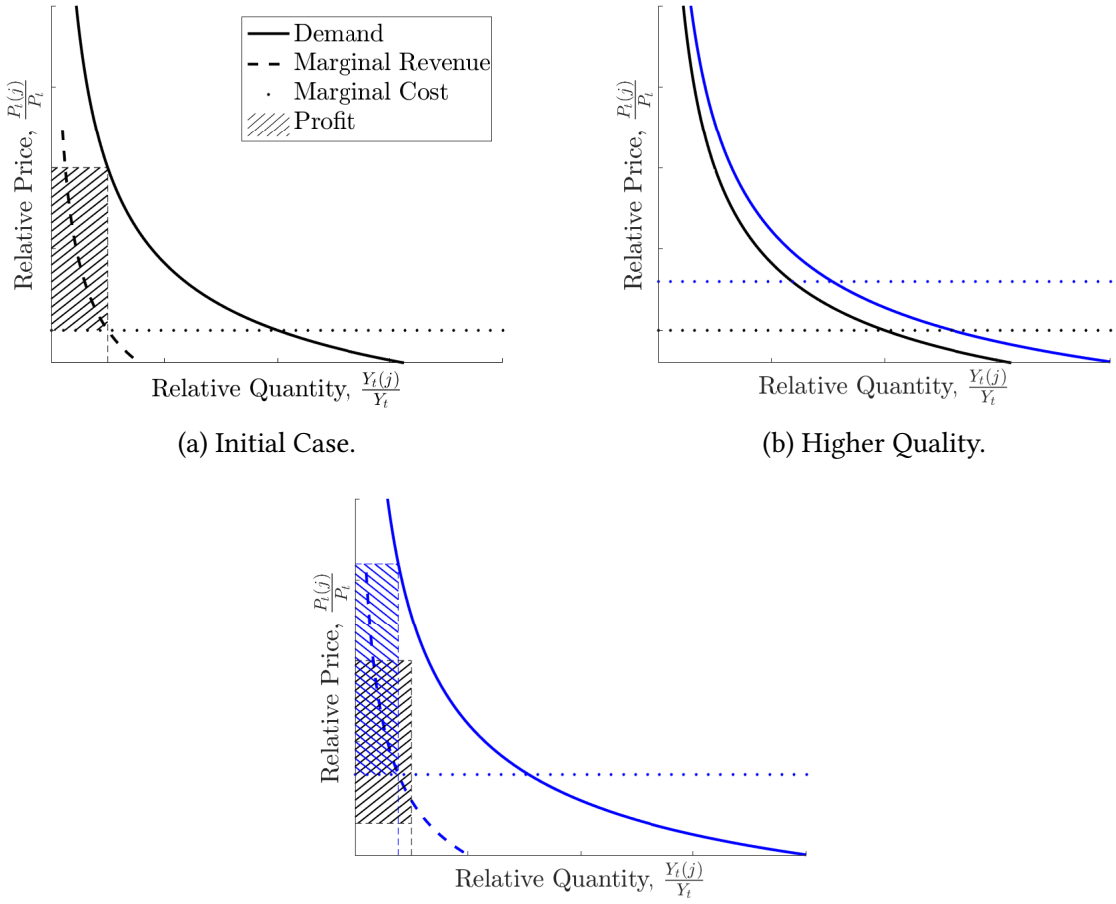
Having outlined the benchmark case for the optimality conditions for intermediate goods firms under flexible prices and quality, we now turn to optimal decisions in a New-Keynesian framework under Rotemberg (1982) adjustment costs for both sticker prices and product quality, which generate clear analytical results. The real profit flow (dividend) for a given intermediate goods producer is given by:

$$\begin{aligned} D_t(j) = (1+\tau_t)\frac{P_t(j)}{P_t}Y_t(j) - \frac{W_t}{P_t}N_t(j) \\ - \frac{\theta_P}{2}\left(\frac{P_t(j)}{(1+\bar{\Pi}^S)P_{t-1}(j)} - 1\right)^2 Y_t - \frac{\theta_F}{2}\left(\frac{F_t(j)}{(1+\bar{\Pi}^F)F_{t-1}(j)} - 1\right)^2 Y_t, \end{aligned} \tag{2.18}$$

where, again, τ_t represents a production subsidy given to intermediate goods firms by the fiscal authority. The constants $1+\bar{\Pi}^S$ and $1+\bar{\Pi}^F$ represent the gross rate of change in the deterministic steady state of sticker prices and product quality, respectively. The parameters $\theta_P > 0$ and $\theta_F > 0$ are, respectively, coefficients which calibrate the difficulty of sticker price and product quality adjustments. Although both θ_P and θ_F are assumed to be positive, $\theta_P = 0$ and $\theta_F = 0$ are considered as limiting cases.

The incorporation of adjustment costs in this fashion merits discussion. Both the convexity of adjustment costs and the separability between sticker prices and product quality require justification. Equation (2.18) extends the original motivation given by Rotemberg (1982) who

Figure 2.1 Increase in Product Quality Under Monopolistic Competition



(a) Initial Case.

(b) Higher Quality.

(c) Profit Comparison.

Notes: A representation of how increasing product quality increases both relative demand and marginal costs, and potentially increases profits. Panel (a) shows the initial situation, panel (b) shows the demand and marginal cost movements with a higher level of quality as the movement from the **black** lines toward the **blue** lines. Panel (c) shows the resultant change in profits.

claimed that such adjustment costs represent the "costs borne by firms as they render their customers unhappy with recurrent price changes." Rotemberg's insight captured a notion that customers would be more upset when nominal prices are adjusted substantially, hence justifying the convexity. Alternatively, there are real managerial and menu costs associated with decisions to adjust nominal prices. These costs are greater when deciding to adjust by larger amounts (for example requiring more management time, analysis and marketing expenses). As such, adjustment costs are increasing in the level of adjustment. Similarly, when product quality is adjusted there may be a physical cost to rearranging production structures, which is increasing in the degree of adjustment.

In part, the separability between sticker prices and product quality adjustment costs reflects the monetary illusion of sticker prices. Real world customers react to both changes in sticker prices and product quality. But some customers may react more to sticker price changes than product quality adjustments despite both influencing the hedonic price. Indeed, if sticker prices are unchanged, many customers may not even notice when product quality adjusts. We therefore assume product quality improvements incur convex adjustment costs that are isomorphic to those for sticker prices. Appendix A.1.2 details the implications of using Rotemberg (1982) adjustment costs for hedonic prices.

The inclusion of trend inflation in (2.18) enables a framework where the real resource value of adjustment costs for a given percentage change in prices, or product quality, is not eroded over time. In an economy with stable inflation and product quality growth the costs of adjusting by this trend are likely to be insubstantial. Finally, this form of adjustment costs has attractive tractability properties which enable an analytical form of the Phillips curve to be found.

Under monopolistic competition, intermediate goods firms seek to maximize the present discounted flow of profits by choosing the price, $P_t(j)$, and quality, $F_t(j)$, of goods they produce. The symmetry of the problem across all intermediate goods firms means they choose the same price, $P_t(s)$, and product quality, $F_t(j)$. In a symmetric solution the aggregate sticker price index (2.8) becomes:

$$P_t^S = P_t(j), \tag{2.19}$$

so the aggregate hedonic price level (2.7) becomes:

$$P_t = [\mathcal{G}_t]^{\frac{1}{1-\varepsilon}} P_t^S, \tag{2.20}$$

where intermediate goods firm indexation is suppressed.

The gross rate of consumer price inflation is denoted as $1 + \Pi_t \equiv P_t/P_{t-1}$, while the symmetry across intermediate goods firms permits the denotation of the gross growth rate of

aggregate product quality as $1+\Pi_t^F \equiv F_t/F_{t-1}$ and the gross rate of sticker price inflation as $1+\Pi_t^S \equiv P_t^S/P_{t-1}^S = [\mathcal{G}_t/\mathcal{G}_{t-1}]^{\frac{1}{\varepsilon-1}}(1+\Pi_t)$, where the second equality uses (2.20) and (2.19).

Appendix A.1 shows how the two first order conditions for this problem can therefore be written as:

$$\begin{aligned}(1+\tau_t)(\varepsilon-1) &= \varepsilon MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} - \theta_P\left(\frac{1+\Pi_t^S}{1+\bar{\Pi}^S}-1\right)\frac{1+\Pi_t^S}{1+\bar{\Pi}^S} \\ &\quad + \beta\theta_P\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{1+\Pi_{t+1}^S}{1+\bar{\Pi}^S}-1\right)\frac{1+\Pi_{t+1}^S}{1+\bar{\Pi}^S}\right], \qquad \text{(FOC P)}\\ \frac{(1+\tau_t)F_t\mathcal{G}_{F,t}}{\mathcal{G}_t} &= MC_tF_t\mathcal{G}_t^{-\frac{1}{\varepsilon-1}}\left(\frac{\mathcal{G}_{F,t}}{\mathcal{G}_t}-\frac{\mathcal{H}_{F,t}}{\mathcal{H}_t}\right)+\theta_F\left(\frac{1+\Pi_t^F}{1+\bar{\Pi}^F}-1\right)\frac{1+\Pi_t^F}{1+\bar{\Pi}^F} \\ &\quad - \beta\theta_F\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{1+\Pi_{t+1}^F}{1+\bar{\Pi}^F}-1\right)\frac{1+\Pi_{t+1}^F}{1+\bar{\Pi}^F}\right]. \qquad \text{(FOC F)}\end{aligned}$$

Together, the two equations above form the intermediate goods firm's First Order Conditions for sticker Prices (FOC P) and product quality (FOC F). These expressions are fundamental in determining the modified forms of the New Keynesian Phillips curves. The first states that firms optimally chose sticker prices based upon current real marginal costs and their expectations for future price levels.

It is clear that, within this framework, the (FOC P) condition, derived from the optimality conditions of individual firms, is cast in terms of sticker price inflation, rather than the rate of change of aggregate consumer prices. This arises as a result of the assumption that adjustment costs depend on the change in sticker prices. This may be seen when assuming product quality is constant, with $\mathcal{G}_t = 1$, such that $1+\Pi_t = 1+\Pi_t^S$. Using this restriction reduces (FOC P) to the standard condition with Rotemberg (1982) adjustment costs:

$$\begin{aligned}(1+\tau_t)(\varepsilon-1) &= \varepsilon MC_t - \theta_P\left(\frac{1+\Pi_t}{1+\bar{\Pi}}-1\right)\frac{1+\Pi_t}{1+\bar{\Pi}} \\ &\quad + \beta\theta_P\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{1+\Pi_{t+1}}{1+\bar{\Pi}}-1\right)\frac{1+\Pi_{t+1}}{1+\bar{\Pi}}\right]. \qquad (2.21)\end{aligned}$$

Assuming symmetry across intermediate firms, alongside (2.20) and (2.19), implies the two first order conditions of the intermediate goods firms problem under flexible sticker price and product quality adjustments, (2.14) and (2.15), become:

$$\mathcal{G}_t^{\frac{1}{\varepsilon-1}} = \frac{\varepsilon}{\varepsilon-1}\frac{MC_t}{1+\tau_t}, \qquad (2.22)$$

and:

$$-\frac{\mathcal{H}_{F,t}}{\mathcal{H}_t} MC_t \mathcal{G}_t^{\frac{1}{1-\varepsilon}} = \left[(1+\tau_t) - MC_t \mathcal{G}_t^{\frac{1}{1-\varepsilon}}\right] \frac{\mathcal{G}_{F,t}}{\mathcal{G}_t}. \tag{2.23}$$

respectively. These represent the limiting cases of (FOC P) and (FOC F). When $\theta_P = 0$ (FOC P) becomes (2.22) and when $\theta_F = 0$ (FOC F) becomes (2.23).

The second optimality condition, (FOC F), may be rewritten as a decomposition of four intuitive elements:

$$\begin{aligned}
&\underbrace{-MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} \frac{\mathcal{H}_{F,t}}{\mathcal{H}_t}}_{\text{1. Marginal Cost Increase}} \\
&+ \underbrace{\frac{\theta_F}{F_t}\left(\frac{1+\Pi_t^F}{1+\bar{\Pi}^F} - 1\right)\frac{1+\Pi_t^F}{1+\bar{\Pi}^F} - \beta\frac{\theta_F}{F_t}\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{1+\Pi_{t+1}^F}{1+\bar{\Pi}^F} - 1\right)\frac{1+\Pi_{t+1}^F}{1+\bar{\Pi}^F}\right]}_{\text{Rotemberg Effect}} \\
&= \underbrace{\left[(1+\tau_t)\mathcal{G}_t^{\frac{1}{\varepsilon-1}} - MC_t\right]}_{\text{Markup}} \quad \underbrace{\mathcal{G}_{F,t}\mathcal{G}_t^{-\frac{\varepsilon}{\varepsilon-1}}}_{\text{2. Demand Increase}}
\end{aligned} \tag{2.24}$$

In this way (FOC F) directly extends the results from subsection 2.2.3 under flexible price and quality adjustments, which are described by (2.15), to include an additional term reflecting Rotemberg adjustment costs. The equation therefore states that, as before, product quality is set optimally based upon the balance between: higher product quality increasing marginal costs, thereby reducing profits, and higher product quality increasing demand for a given variety, thereby increasing profits. In addition, with price and quantity rigidities, the optimal product quality choice of intermediate goods firms includes a new term. This accounts for the cost of current product quality adjustments on the set of future product quality choices. The expectations of intermediate goods firms are therefore captured in this additional Rotemberg effect term.

2.2.4 Government

The government fiscal authority finances the intermediate goods producer's subsidies by imposing lump-sum taxes (i.e. $T_t < 0$) on households to maintain a balanced budget:

$$0 = T_t + \tau_t \int_0^1 \frac{P_t(j)}{P_t} Y_t(j)\,\mathrm{d}j \tag{2.25}$$

We assume the aim of government fiscal policy is to offset the impact of imperfect competition in distorting pricing decisions of intermediate goods firms, in the steady state.[11] As shown in Appendix A.3.1, the steady state of this model has constant inflation, with both sticker and hedonic consumer price inflation equal to the central bank's target. In this steady state the economy-wide level of productivity is constant, by assumption, which results in the optimal level of product quality chosen by intermediate goods firms also being constant.[12]

The subsidy, τ_t, is therefore set at a constant level to deliver the efficient level of marginal costs in this steady state. In a steady state, sticker price inflation is constant, so (FOC P) implies:

$$(\varepsilon - 1)(1 + \bar{\tau}) = \varepsilon \bar{MC} \bar{\mathcal{G}}^{-\frac{1}{\varepsilon-1}}, \tag{2.26}$$

where a bar denotes the steady state level of a variable. In the efficient frictionless allocation, derived in Appendix A.2, it is shown that, in the steady state:

$$\bar{MC} = \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}}. \tag{2.27}$$

In the standard case where product quality has no impact on demand, $MC = 1$ is efficient and the real wage is equal to the marginal product of labour: $\frac{W}{P} = \mathcal{H}$. However, in the current setting, the former condition is replaced by (2.27) as product quality has additional value to households and a higher efficient level of product quality, $\bar{\mathcal{G}}$, increases marginal costs for intermediate goods firms, $\bar{MC}$.

Using (2.26) and (2.27), the production subsidy will therefore ensure the steady state production level is undistorted by monopolistic competition to deliver the frictionless first-best allocation whenever:

$$1 + \tau_t = 1 + \bar{\tau} \equiv \frac{\varepsilon}{\varepsilon - 1}. \tag{2.28}$$

2.2.5 Monetary Policy

The monetary authority sets the reference one-period nominal interest rate, which in equilibrium is perfectly arbitraged with the nominal bond yield. In the baseline scenario, the monetary authority follows a standard interest rate rule, following Taylor (1993), of the form:[13]

$$\frac{1 + i_t}{1 + \bar{i}} = \left(\frac{1 + \Pi_t}{1 + \Pi^*}\right)^{\phi_\pi} \left(\frac{Y_t}{Y_t^n}\right)^{\phi_y} e^{\zeta_t^i}, \tag{2.29}$$

[11] This formulation is consistent with much of the existing literature in this area, for further discussion see Woodford (2003), chapter 6.

[12] Appendix A.6 shows how trend productivity growth may be incorporated into the model.

[13] This closely follows the specification in Miao (2020), chapter 19. Equivalent log-linearised forms are presented in Galí (2015) and Woodford (2003).

where $1 + \bar{i}$ is the nominal interest rate in the deterministic steady state with gross inflation at the target rate; $1 + \Pi^*$ is the monetary authority's target for the aggregate gross rate of household price inflation; Y_t^n is the level of aggregate output which would arise under perfectly flexible prices (often called the natural or potential level of output), and ζ_t^i is an *iid* normally distributed random variable with mean 0 and standard deviation σ_i^2. Additionally, we assume $\phi_y > 0$ and $\phi_\pi > 1$, which ensures the Taylor principle is obeyed.

As the baseline scenario (subsection 2.4.2), we consider the case in which product quality adjustments are perfectly measurable, and the monetary policy authority responds according to (2.29). But, afterwards (subsection 2.4.3) we also consider cases in which the monetary authority cannot measure product quality adjustments properly or, even worse, neglects product quality adjustments altogether. In these cases, (2.29) no longer captures the behaviour of the monetary authority, and is replaced with alternatives. We present the (log-linearised) Taylor rules for the alternative scenarios in section 2.4, where we conduct the monetary policy analysis.

2.2.6 Equilibrium and Market Clearing Conditions

In equilibrium, one-period nominal bond holdings are zero in every period:

$$B_t = 0. \tag{2.30}$$

The labour market clearing condition may be written as:

$$L_t = \int_0^1 N_t(j)\mathrm{d}j. \tag{2.31}$$

The market clearing condition for each intermediate good is:

$$Y_t^d(j) = Y_t(j) \qquad \forall j, \tag{2.32}$$

and the aggregate level of dividends are denoted:

$$D_t = \int_0^1 D_t(j)\mathrm{d}j. \tag{2.33}$$

Using the bond, labour and intermediate goods market clearing conditions, (2.30), (2.31) and (2.32) respectively, alongside the aggregate level of dividends, (2.33), the household budget constraint, (2.2), the intermediate goods firms' profit function, (2.18), the government's budget constraint, (2.25), and the assumption of symmetry across intermediate goods firms,

the aggregate resource constraint describing equilibrium in the final goods market for this economy is given as:

$$Y_t = C_t + \frac{\theta_P}{2}\left(\frac{1+\Pi_t^S}{1+\bar{\Pi}^S} - 1\right)^2 Y_t + \frac{\theta_F}{2}\left(\frac{1+\Pi_t^F}{1+\bar{\Pi}^F} - 1\right)^2 Y_t. \tag{2.34}$$

This states that output is either consumed or spent on the wasteful adjustment costs of prices and quality. Using the production functions for the final goods producer, (2.5), and intermediate goods firms, (2.10); market clearing conditions for the labour market, (2.31), and intermediate goods markets, (2.32); and symmetry across intermediate goods firms', the aggregate production function may be found as:

$$Y_t = \left[\int_0^1 [\mathcal{G}_t(j)]^{\frac{1}{\varepsilon}} [N_t(j)\mathcal{H}_t(j)]^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}j\right]^{\frac{\varepsilon}{\varepsilon-1}} = \mathcal{G}_t^{\frac{1}{\varepsilon-1}} L_t \mathcal{H}_t. \tag{2.35}$$

Finally, the model is made stochastic with an AR(1) process for technology:

$$\ln A_t = \rho_A \ln A_{t-1} + \zeta_t^a, \tag{2.36}$$

where $0 < \rho_A < 1$ ensures stationarity and convergence to the deterministic steady state solution, $\bar{A} = 1$, and ζ_t^a is an *iid* normally distributed random variable with mean 0 and standard deviation σ_a^2.

The full set of model equations is outlined in Appendix A.3 using general functional forms for $\mathcal{G}$ and $\mathcal{H}$. This appendix also presents a method to derive the solution of this system for prices and quantities in the symmetric steady state. This method first substitutes deterministic values for each of two random variables, using their solution to identify the solution to 6 of the endogenous variables. The remaining 6 endogenous variables are then be solved using an iterative process.

2.3 Quality Adjustments, the Phillips Curve, and the Natural Rate

This section discusses the implications of product quality adjustments for the slope of the Phillips curve and business cycle fluctuations in the natural rate of interest. To keep the analysis tractable, the section assigns specific functional forms to the demand shifter, $\mathcal{G}_t(j)$, and to labour productivity, $\mathcal{H}_t(j)$. The precise steps, log linearisation and model solution method are presented in detail in Appendix A.4. In this section, hedonic consumer price inflation is used for the central bank's target, while technology and product quality are

constant in the steady state. Altering (2.36) to incorporate trend growth in technology (and hence product quality) has no impact on our conclusions, as shown in Appendix A.6.

Notation. Bars ($\bar{x}$) denote steady state levels, hats ($\hat{x}$) denote log deviations from steady state level, which are approximately equal to percentage deviations, and tildes ($\tilde{x}$) denote log differences from natural levels. Variables are written in upper case with their natural logarithm in lower case.

2.3.1 Functional Forms

The functional forms for $\mathcal{G}_t(j)$ and $\mathcal{H}_t(j)$ are specified to enable the firm block of the model to be log-linearised. The generalized demand shifter, $\mathcal{G}_t(j)$, is set as:

$$\mathcal{G}_t(j) \equiv [F_t(j)]^{\phi}, \tag{2.37}$$

with first derivative:

$$\mathcal{G}_{F,t}(j) = \phi\,[F_t(j)]^{\phi-1}. \tag{2.38}$$

The parameter $\phi > 0$ may be used to control the elasticity of individual variety demand to changes in product quality.[14] The impact of quality on the production process is set as:

$$\mathcal{H}_t(j) \equiv \left[A_t^{\frac{\nu-1}{\nu}} - \kappa[F_t(j)]^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{2.39}$$

with first derivative:

$$\mathcal{H}_{F,t}(j) = -\kappa\left[A_t^{\frac{\nu-1}{\nu}} - \kappa[F_t(j)]^{\frac{\nu-1}{\nu}}\right]^{\frac{1}{\nu-1}}[F_t(j)]^{-\frac{1}{\nu}}, \tag{2.40}$$

where $\nu < 0$ and $\kappa > 0$ are constant parameters, such that the individual firm level productivity index is a CES aggregate of economy-wide productivity and the individual firm's level of product quality produced.[15,16] The parameter ν is the elasticity of substitution between aggregate productivity and individual product quality and captures the ease, or difficulty, of producing high quality products. The parameter κ is a share parameter which captures the reduction in productivity accounted for by higher product quality. The canonical NK model will be obtained if both $\phi = 0$ and $\kappa = 0$ such that $\mathcal{G}_t(j) = 1$ and $\mathcal{H}_t(j) = A_t$.

[14] This closely follows the formulation in Feenstra and Romalis (2014); Hallak (2006); Kugler and Verhoogen (2012) and Jaimovich et al. (2019a) among others.

[15] This is again a similar formulation to Kugler and Verhoogen (2012) and Feenstra and Romalis (2014).

[16] Appendix A.7 presents an alternative framework, based upon differentiated labour inputs and the worker skill premium, which results in the same form of $\mathcal{H}_t(j)$.

In this setting, using (FOC F) alongside (2.27) and (2.28), the steady state level of product quality can be implicitly characterised by:

$$\frac{1}{\varepsilon - 1} = -\frac{\bar{\mathcal{H}}_F}{\bar{\mathcal{H}}} \frac{\bar{\mathcal{G}}}{\bar{\mathcal{G}}_F} \tag{2.41}$$

Using (2.37) to (2.40), this can be solved analytically as:

$$\bar{F} = \bar{A} \left(\frac{1}{\kappa}\right)^{\frac{\nu}{\nu-1}} \left[\frac{\phi}{\varepsilon - 1 + \phi}\right]^{\frac{\nu}{\nu-1}}. \tag{2.42}$$

This implies that, in the steady state, the endogenous level product quality is increasing in the exogenous economy-wide level of productivity. Moreover, using (2.42), the steady state values of (2.37) to (2.40) are respectively given as:

$$\bar{\mathcal{G}} = \bar{A}^{\phi} \kappa^{\frac{\phi\nu}{1-\nu}} \left[\frac{\phi}{\varepsilon - 1 + \phi}\right]^{\frac{\phi\nu}{\nu-1}}, \tag{2.43}$$

$$\bar{\mathcal{G}}_F = \phi \bar{A}^{\phi-1} \kappa^{\frac{(\phi-1)\nu}{1-\nu}} \left[\frac{\phi}{\varepsilon - 1 + \phi}\right]^{\frac{(\phi-1)\nu}{\nu-1}}, \tag{2.44}$$

$$\bar{\mathcal{H}} = \bar{A} \left[\frac{\varepsilon - 1}{\varepsilon - 1 + \phi}\right]^{\frac{\nu}{\nu-1}}, \tag{2.45}$$

$$\bar{\mathcal{H}}_F = -\kappa \left[\frac{\kappa(\varepsilon - 1)}{\phi}\right]^{\frac{1}{\nu-1}}. \tag{2.46}$$

2.3.2 Model Solution

A first-order log-linearisation of the model around the steady state with trend inflation gives rise to:

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\tilde{\pi}_{t+1}] - r_t^n - \pi^*), \tag{Dynamic IS}$$

$$\tilde{\pi}_t^S = \frac{\varepsilon(\sigma + \eta)}{\theta_P} \tilde{y}_t + \beta \mathbb{E}_t \left[\tilde{\pi}_{t+1}^S\right], \tag{Log Linearised NKPC-P}$$

$$\tilde{\pi}_t^F = -\frac{\varepsilon\phi(\sigma + \eta)}{\theta_F(\varepsilon - 1)} \tilde{y}_t - \frac{\phi(\nu - 1)(\varepsilon - 1 + \phi)}{\nu\theta_F(\varepsilon - 1)^2} \tilde{f}_t + \beta \mathbb{E}_t \left[\tilde{\pi}_{t+1}^F\right], \tag{Log Linearised NKPC-F}$$

$$i_t = \rho + \pi^* + \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t + \zeta_t^i, \tag{Taylor rule}$$

where $\rho \equiv -\ln\beta$ and **blue font** has been used to highlight differences from the canonical NK model. In addition, it yields the following block of auxiliary equations:

$$r_t^n = \rho + \sigma\psi_a \mathbb{E}_t[\Delta a_{t+1}], \tag{2.47}$$

$$\tilde{\pi}_t^F = \tilde{f}_t - \tilde{f}_{t-1}, \tag{2.48}$$

$$\tilde{\pi}_t^S = \frac{\phi}{\varepsilon - 1}\tilde{\pi}_t^F + \tilde{\pi}_t, \tag{2.49}$$

$$a_t = \rho_A a_{t-1} + \zeta_t^a, \tag{2.50}$$

where $\psi_a \equiv \frac{\eta+1}{\sigma+\eta}\frac{(\varepsilon-1)+\phi}{\varepsilon-1} > 0$. The model may be solved for the 8 endogenous variables: $\tilde{y}_t$, $\tilde{\pi}^S$, $\tilde{\pi}_t^F$, i_t, r_t^n, $\tilde{\pi}_t$, $\tilde{f}_t$ and a_t.

By inspection, this simplified model shares many characteristics with the canonical NK model. Both the dynamic IS curve and NKPC-P are unchanged, though as sticker prices are in general different to hedonic prices, the latter equation makes this distinction and holds only for "sticker" prices. The AR(1) technology process and closing the model with a Taylor rule for nominal interest rates are also standard.

In contrast to the standard setting, the equilibrium now contains an additional equation, namely the simplified form of the Log Linearised NKPC-F, which describes how the quality of goods evolves through time. As was the case prior to log-linearisation, and shown in equation (2.24), the Log Linearised NKPC-F may be rewritten to highlight the decomposition of several elements:

$$\underbrace{\frac{\phi(\sigma+\eta)}{\varepsilon-1}\tilde{y}_t + \frac{\phi(\nu-1)(\varepsilon-1+\phi)}{\nu(\varepsilon-1)^2}\tilde{f}_t}_{\text{1. Marginal Cost Increase}} + \underbrace{\theta_F\tilde{\pi}_t^F - \beta\theta_F\mathbb{E}_t\left[\tilde{\pi}_{t+1}^F\right]}_{\text{3. Rotemberg Effect}} = \underbrace{-\phi(\sigma+\eta)\tilde{y}_t}_{\text{2. Demand Increase}} . \tag{2.51}$$

This formulation demonstrates how current product quality choices depend upon three factors:

1. The response of marginal costs to a given shock, as captured by the first two terms in (2.51). Higher product quality will increase marginal costs, thereby reducing profits. The first term represents the indirect (general equilibrium) impact of higher real wages increasing marginal costs for a given level of product quality. The second term captures the direct impact of higher product quality increasing marginal costs.

2. The response of aggregate demand to a given shock, as captured by the term on the right hand side of (2.51). Higher product quality will increase demand for a given variety, thereby increasing profits. This may be seen by inspection of the demand function for each intermediate variety given in (2.6), noting the first derivative of $\mathcal{G}_t(j)$ in (2.38).

However, the simplified form of the Log Linearised NKPC-F also captures the indirect impact of changes in product quality on aggregate demand. In a symmetric solution, a general increase in product quality (with constant sticker prices) will result in a lower aggregate hedonic price level, P_t. This indirect equilibrium effect on aggregate demand may be seen through (2.20). The lower aggregate hedonic price level will increase the relative price and thus reduce demand for each individual variety at given sticker prices. As the latter (indirect) effect outweighs the former (direct) effect, the final term in (2.51) is negative whenever the output gap is positive, i.e. with $\tilde{y}_t > 0$.

3. Finally, intermediate goods firms also account for the impact of changes in their choice of product quality on their Rotemberg (1982) adjustment costs. Product quality is costly to adjust. A contemporaneous increase in product quality will increase current adjustment costs, with $\tilde{\pi}_t^F > 0$. Eventually, a current increase in product quality will also lead to an increase in future adjustment costs as product quality falls and returns to the steady state level, with $\mathbb{E}_t\left[\tilde{\pi}_{t+1}^F\right] < 0$. This second effect is discounted as it impacts future profits. The third and fourth terms in (2.51) capture the present discounted value of these adjustment costs. This effect smooths the transmission of quality adjustment through time, such that changes to product quality arise gradually as opposed to being concentrated in a single time period.

   Whenever the Rotemberg (1982) adjustment costs for product quality are turned off, with costless quality adjustment, as $\theta_F \longrightarrow 0$, the reduced form of the Log Linearised NKPC-F becomes:

$$\tilde{f}_t = -\frac{\nu\varepsilon(\sigma+\eta)\,(\varepsilon-1)}{(\nu-1)(\varepsilon-1+\phi)}\tilde{y}_t. \tag{2.52}$$

   This implies that, absent product quality adjustment costs, the product quality gap is negatively associated with the output gap.[17] A positive output gap corresponds to higher marginal costs, and intermediate goods firms therefore optimally respond by reducing product quality.

2.3.3 The Phillips Curve

In this setting, with endogenous product quality adjustment, the two decisions by intermediate goods firms (for optimal sticker price and product quality) may be summarised in a single equation. This equation characterises how the aggregate hedonic price level responds to shocks, after accounting for the changes made to product quality by intermediate goods firms.

[17] This arises as $\sigma > 0$, $\eta > 0$, $\varepsilon > 1$ and $\phi > 0$, while $\nu < 0$.

Proposition 2.3.1 (Phillips Curve). *The reduced form relationship between aggregate price inflation and the output gap, with endogenous product quality adjustment, may be represented as the following Phillips curve:*

$$\tilde{\pi}_t = \frac{\varepsilon(\sigma+\eta)}{\theta_P}\left(1 \quad + \quad \frac{\phi^2\theta_P}{\theta_F(\varepsilon-1)^2}\right)\tilde{y}_t \quad + \quad \frac{\phi^2(\nu-1)(\varepsilon-1+\phi)}{\nu\theta_F\,(\varepsilon-1)^3}\tilde{f}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}\right]. \tag{2.53}$$

Proof. Substituting the Log Linearised NKPC-P and Log Linearised NKPC-F into (2.49) and simplifying yields (2.53), where the relationship between sticker and hedonic price inflation, (2.49), has also been used one-period ahead to rewrite:

$$\mathbb{E}_t\left[\tilde{\pi}_{t+1}\right] = \mathbb{E}_t\left[\tilde{\pi}^S_{t+1} - \frac{\phi}{\varepsilon-1}\tilde{\pi}^F_{t+1}\right]. \tag{2.54}$$

□

The **blue font** in (2.53) highlights the elements that do not appear in the canonical New Keynesian framework. In this augmented framework the impact of an increase in marginal costs, either through higher real wages (via a positive output gap) or through higher product quality, cause the aggregate rate of hedonic inflation to increase.[18] Two corollaries are thus immediately apparent as a consequence of proposition 2.3.1.

Corollary 2.3.1.1 (Slope). *The product quality augmented Phillips curve in (2.53) has a larger slope than the canonical New Keynesian Phillips Curve (NKPC).*

This arises as the new component of the constant multiplying the output gap is positive, i.e. $\frac{\phi^2\theta_P}{\theta_F(\varepsilon-1)^2} > 0$. The intuition behind this result is as follows: Following a positive productivity shock, firms seek to lower sticker prices, but they also seek to improve the quality of their products since quality is now relatively cheaper. Both the lower sticker prices and higher product quality act to lower quality-adjusted prices. Thus, the change in product quality makes the slope of the Phillips curve larger than before. The magnitude of this effect depends critically upon the relative cost of price and quality adjustments. When the costs of price adjustment rise relative to those of quality adjustment, i.e. θ_P/θ_F rises, the change in product quality will be larger and the slope of the Phillips curve increases. If, over time, the relative cost of adjusting product quality compared to sticker prices has risen, i.e. θ_P/θ_F falls, a flattening of the product quality augmented Phillips curve would arise.

[18] In calculating the aggregate Phillips curve (2.53) the growth rate of product quality, $\tilde{\pi}^F_t$, is removed through focusing on hedonic price changes. However the *level* of product quality (deviation from natural level) remains a part of the equation as marginal costs depend both upon the quantity and quality of output produced.

Corollary 2.3.1.2 (Bias)**.** *Empirical reduced-form estimation of the Phillips curve must control for quality adjustments or suffer omitted variable bias.*

The second term in **blue font** in (2.53) shows how the current level of product quality matters for reduced-form Phillips curve estimation. The sign of this bias depends upon the unconditional correlation between $\tilde{f}_t$ and $\tilde{y}_t$. In the case with costless product quality adjustment this is negative according to (2.52). In model simulations (see subsection 2.4.2) this unconditional correlation is -0.71, which implies reduced-form estimates of the Phillips curve will underestimate the true value of the Phillips curve slope unless accounting for changes in product quality.[19]

2.3.4 The Natural Rate

This subsection discusses the transmission of aggregate productivity shocks in the model. As in the canonical setting, shocks to the aggregate level of productivity enter the model through changes in the natural real interest rate. However, the magnitude of this transmission is amplified in a setting with endogenous product quality adjustments.

Proposition 2.3.2 (Amplification)**.** *Product quality adjustment amplifies the response of the natural real interest rate to aggregate productivity shocks.*

Proof. The natural real interest rate evolves according to:

$$r_t^n = \rho + \sigma\psi_a \mathbb{E}_t[\Delta a_{t+1}], \tag{2.55}$$

where:

$$\psi_a \equiv \frac{\eta+1}{\sigma+\eta}\frac{(\varepsilon-1)+\phi}{\varepsilon-1}, \tag{2.56}$$

which, with $\phi > 0$, is larger than the coefficient in the canonical case, with constant product quality. Given the functional form for $\mathcal{G}_t(j)$, (2.37), the model may be collapsed to generate the canonical NK case with constant product quality as $\phi \to 0$. In this case the **blue font** terms in (2.56) normalise to 1 to give:

$$\lim_{\phi\to 0}\psi_a = \frac{\eta+1}{\sigma+\eta} = \psi_a^{\text{NK}}. \tag{2.57}$$

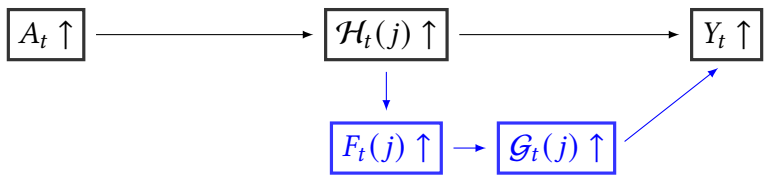
□

[19]The arguments presented here presume researchers are already able to overcome issues surrounding the inclusion of expected future inflation, $\mathbb{E}_t[\tilde{\pi}_{t+1}]$ and are able to measure the "natural" level of output and product quality. Typically, the literature uses either leads and lags of inflation (Ball and Mazumder, 2011), smoothed filters (Stock and Watson, 2019), or professional forecasts and surveys (Coibion et al., 2013) to instrument for this.

The amplification channel described in proposition 2.3.2 is illustrated in Figure 2.2. After an initial increase in aggregate productivity, current marginal costs fall and labour productivity, $\mathcal{H}_t$, rises. At a given level of labour input, intermediate goods firms increase the quantity of their production and aggregate output therefore rises. This standard transmission is shown in the **black lines** of Figure 2.2. In a setting with endogenous product quality adjustment, lower marginal costs (and a higher level of labour productivity, $\mathcal{H}_t$) cause intermediate goods firms to increase the quality of their products. A higher level of product quality increases the quality-adjusted level of aggregate output, Y_t. This amplification effect is shown in the **blue lines** of Figure 2.2. Both of these effects result in greater aggregate goods production, via a quantity (volume) and quality channel respectively. The new quality channel thereby amplifies the quantity channel.

There are two important caveats to note while interpreting this transmission and the representation in Figure 2.2. Firstly, the general equilibrium involves a more elaborate mechanism than described by the two effects in the previous paragraph. For instance, there is the standard additional effect as labour input rises after a positive productivity shock, which further increases aggregate demand. Secondly, the description in the previous paragraph, and depicted in Figure 2.2, does not show that the increase in product quality also reduces the labour productivity of individual intermediate goods firms, captured in (2.39). This mitigating effect dampens the initial mechanism, through a lower net rise in $\mathcal{H}_t$ in general equilibrium. The net impact increases aggregate output by more than in the canonical case, as captured by (2.56).

Figure 2.2 Amplification of Productivity Shocks



Note: A representation of the amplified effect (in **blue**) of a positive aggregate productivity shock on current output due to product quality adjustment.

In a model with flexible prices and product quality, there are two perspectives why these effects manifest as changes in the natural real rate of interest. The greater supply of goods in the current period resulting from the rise in productivity lowers the relative price of consumption in this period, reducing the real interest rate. As product quality improvements amplify the extent of the increase in final goods supply, so the fall in the natural rate of interest is now larger. Alternatively, from a loanable funds perspective, households now consume more in the current period, and wish to smooth their consumption over time. By

increasing their intended current savings the real interest rate falls. Again, the effect is now larger as consumption has increased in terms of both the quantity and the product quality.

2.4 Monetary Policy Analysis

This section firstly calibrates the model before presenting baseline results. It then investigates positive aspects of monetary policy and finally focuses on normative considerations..

2.4.1 Calibration

By design, our model with endogenous product quality adjustment nests the workhorse New Keynesian model. We therefore choose a number of parameters to match standard values found in the literature, for a quarterly frequency.[20] The parameters chosen for our baseline specification of the Taylor rule are also standard in the literature. (See Table 2.1 for details). Investigating alternative levels of productivity persistence leaves the qualitative results unchanged.[21] This leaves five parameters to specify: θ_P, θ_F, ϕ, ν, and κ.

Table 2.1 Baseline Parameter Calibration

Description	Parameter	Value	Target
Time-discount factor	β	0.99	Standard value
Coefficient of relative risk aversion	σ	1.00	Standard value
Frisch elasticity of labour substitution	$1/\eta$	∞	Standard value
Elasticity of sub. across intermediate varieties	ε	6.00	Standard value
Inflation target (gross)	$1+\bar{\pi}^*$	1.00	Standard value
Productivity level	$\bar{A}$	1.00	Standard value
Productivity persistence	ρ_A	0.90	Standard value
Productivity shock variance	σ_A^2	0.01^2	Standard value
Interest rate shock persistence	ρ_i	0.50	Standard value
Interest rate shock variance	σ_i^2	0.0025^2	Standard value
Taylor rule inflation coefficient	ϕ_π	1.50	Baseline
Taylor rule output gap coefficient	ϕ_y	0.50/4	Baseline
Rotemberg price adj. cost parameter	θ_P	29.41	Match Calvo-Yun with $\theta = 2/3$
Rotemberg quality adj. cost parameter	θ_F	29.41	Match price rigidities
Elasticity of demand to quality	ϕ	1.00	Baseline
Elasticity of sub. across productivity and quality	ν	-0.80	Baseline
Quality cost	κ	1/6	Baseline

[20]For example, see the calibration in Chapter 3 of Galí (2015), as shown in Table 2.1. We set σ_i^2 such that a one standard deviation shocks will generate a 25bps (100bps annualized) shock to the nominal interest rate on impact.

[21]For instance varying ρ_A between 0.70 and 0.95.

In our baseline specification, we choose the parameter controlling Rotemberg (1982) price adjustment costs, θ_P, so that the evolution of marginal costs under a standard New Keynesian model with Rotemberg (1982) pricing coincides with Calvo (1983) and Yun (1996) pricing with a probability of price adjustment of $1 - \theta = 1/3$, taken from Galí (2015). The Rotemberg and Calvo-Yun specifications align whenever:

$$\theta_P = \frac{(\varepsilon - 1)\theta}{(1 - \theta)(1 - \theta\beta)}, \tag{2.58}$$

such that $\theta_P \approx 29.41$ given our other parameter specifications.

Our baseline specification sets the Rotemberg cost of quality adjustment equal to that of prices: $\theta_P = \theta_F$. Alternative strategies to calibrate θ_F suggest lower values are plausible. Nakamura and Steinsson (2008) calculate the mean implied duration for the prices of goods contained within the U.S. consumer price index basket between 1998-2005 as 9 months (for goods excluding sales and substitutions). This would also generate a quarterly Calvo-Yun pricing parameter of $\theta = 1 - 3/9 = 2/3$. In addition, they show this falls to 7.7 months when including product substitutions (but still excluding sales). This may indicate a relative ease of adjusting quality, rather than prices with an associated quarterly Calvo-Yun parameter of $\theta = 1 - 3/7.7 \approx 0.61$, corresponding to a Rotemberg (1982) cost of quality adjustment of $\theta_F \approx 19.78$. A second method, employed by Adam and Weber (2019), incorporates an exogenous Calvo-Yun style probability of quality adjustment, which is set at 11.5% per year. This is the midpoint between the average establishment birth rate (12.4 percent) and the average establishment exit rate (10.7 percent) over the period 1977 to 2015 reported in the Business Dynamics Statistics (BDS) of the US Census Bureau and is similar to the product entry and exit rates observed in Argente et al. (2020). This would generate a quarterly Calvo parameter of $\theta = 1 - 0.115^{0.25} = 0.42$, and again a lower estimated cost of quality adjustment at $\theta_F \approx 6.20$. Reducing θ_F to allow for these considerations does not change our qualitative results.

Turning to the elasticity of demand with respect to product quality, in the international trade literature, ϕ is often used as a term *relative* to the United States. This has a different interpretation than in our model where the *absolute* level of ϕ matters. We therefore use a baseline specification with $\phi = 1$ for unitary demand as our starting point. This is supported by Khandelwal (2010) who regresses estimates of the quality of imported goods on per capita GDP from the exporting country and finds a demand elasticity of quality with $\phi = 0.8$.[22] Note

[22]He also provides a range of plausible estimates for the demand elasticity of a change in industry-level productivity on product quality that varies between 1.05 and 0.83, depending on the regression specifications.

that (2.6) and (2.37) yield:

$$\frac{\partial Y_t^d(j)}{\partial F_t(j)} \frac{F_t(j)}{Y_t^d(j)} = \frac{\mathcal{G}_{F,t}(j) F_t(j)}{\mathcal{G}_t(j)} = \phi, \tag{2.59}$$

where the final equality uses the functional forms given in (2.37) and (2.38).

Finally, we set $\nu = -0.8$ to ensure productivity and quality are weak complements, consistent with the empirical evidence in Feenstra and Romalis (2014). Plugging parameter values into (2.42) gives the steady state level of product quality as:

$$\bar{F} = (6\kappa)^{-\frac{4}{9}}. \tag{2.60}$$

$\kappa = 1/6$ is then used as a normalization to target $\bar{F} = 1$.

2.4.2 No Misspecification in Inflation Behavior

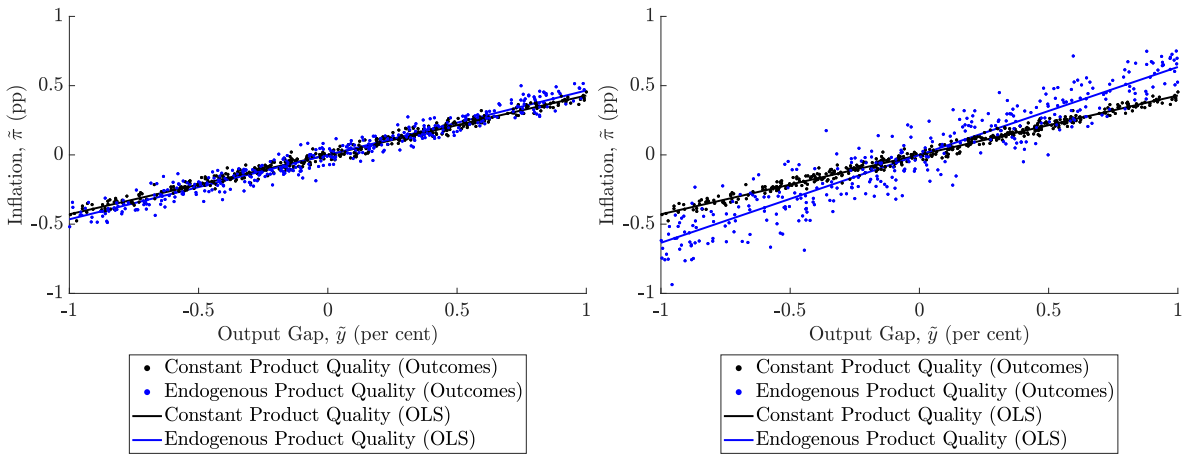
To highlight corollary 2.3.1.1, the baseline model is simulated for 500 periods and the output gap and inflation are plotted against one another. This is shown in Figure 2.3. Panel (a) shows that even in the case of unitary elasticity of demand with respect to quality, with $\phi = 1$, the observed Phillips curve relationship has a (slightly) larger slope when firms are able to adjust the quality of their products. This is demonstrated further in Panel (b) of the same diagram, which shows that as the elasticity of demand with respect to quality increases, to $\phi = 5$, the difference between the slope of the Phillips curve in the canonical NK model increases.[23] As outlined in Galí (2015), in theory, this relationship between the equilibrium outcome for output gap and inflation is generally ambiguous, as it depends critically both upon the elasticity of labour supply and the weight on output gap stabilization assigned by the central bank.

To illustrate proposition 2.3.2 for the baseline model, the Impulse Response Functions (IRFs) to a productivity and monetary policy shock are shown in Figures 2.4 and 2.5 respectively. In each figure, the standard IRFs of the canonical NK model are shown by **black solid** lines, while the IRFs allowing firms to update the quality of their products are shown in **blue dashed** lines. All models use a common Taylor rule.

The impact of an exogenous 1% increase in aggregate productivity is shown in Figure 2.4. As productivity improves firms expand their supply of goods to consumers, raising the level of current output and reducing prices. As described in subsection 2.3.4, this causes the natural

[23] In both instances the relationship also appears more diffuse when product quality is endogenous. This effect arises whenever both interest rate and economy-wide productivity shocks are turned on. With endogenous product quality interest rate shocks have a larger impact on inflation and a smaller effect on the output gap (see Figure 2.5), resulting in a relationship between these outcome variables which appears less concentrated.

Figure 2.3 Simulated Phillips Curve Relationship



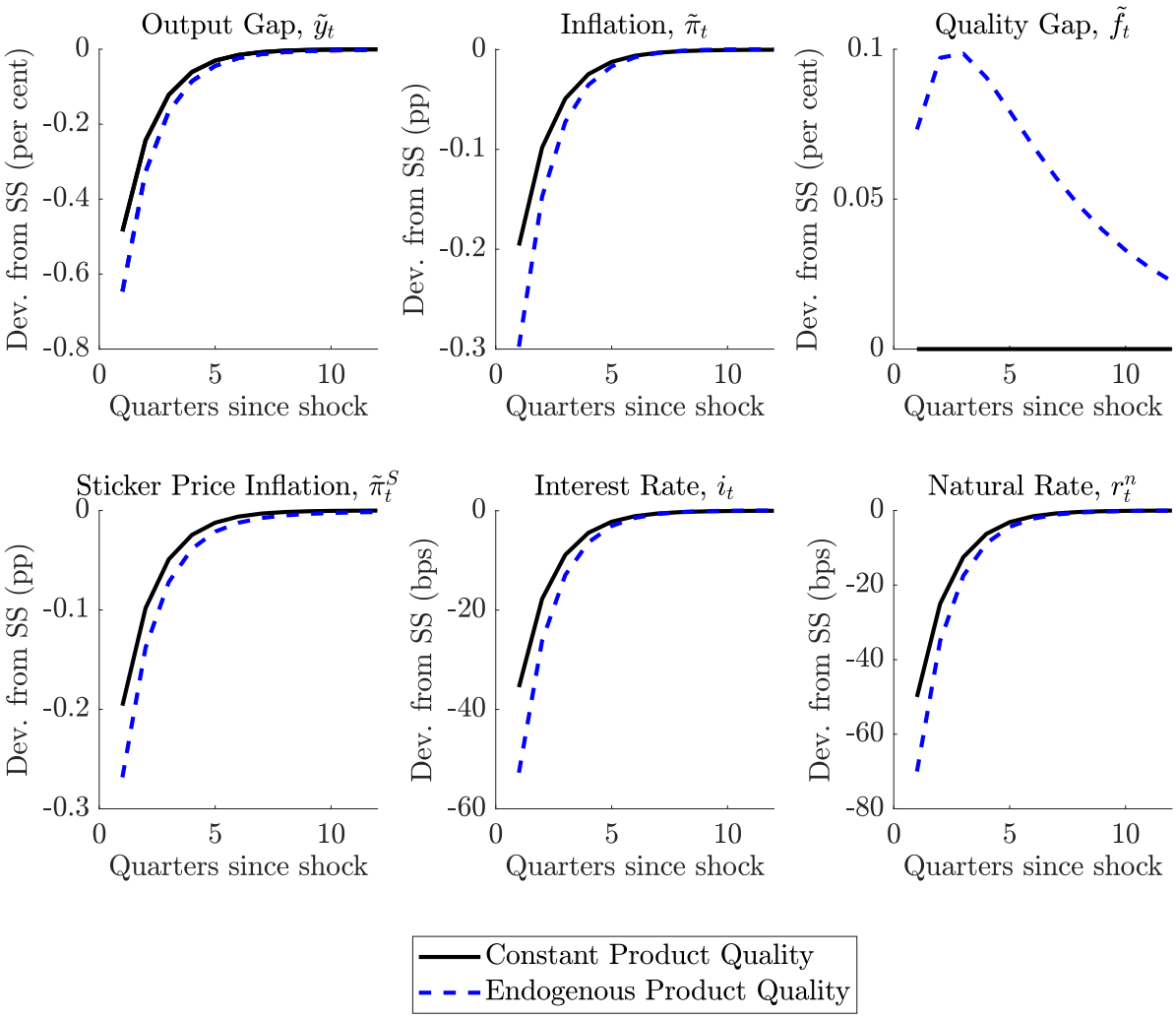
(a) Baseline, with $\phi = 1$ (b) Alternative, with $\phi = 5$

Notes: Baseline model, using parameters in Table 2.1, simulated for 500 periods. **Black dots** show the model-implied relationship between the equilibrium outcome variables: consumer price inflation and the output gap, using a model with constant product quality. **Blue dots** show the same relationship between the equilibrium level of hedonic consumer price inflation and the output gap in the baseline model with endogenous product quality. In each case solid lines show the Ordinary Least Squares (OLS) fit for the same data. Panel (b) shows an alternative simulation with a higher elasticity of quality demand, set at $\phi = 5$.

rate of interest to fall as greater supply lowers the relative price of current consumption. Persistence in the AR(1) productivity process causes this to effect to dissipate gradually over time. When sticker prices are costly to adjust, firms expand output by less than the expansion of the natural output level, causing the output gap to fall. The central bank reacts to this shock by cutting the nominal interest rate to stimulate output (hence reducing the magnitude of the output gap) and increase inflation back towards its target.

In concordance with proposition 2.3.2, the impulse responses are of a greater magnitude when firms are able to update product quality. In this case, product quality improvements amplify the response of quality-adjusted output, causing the natural real interest rate to fall by more, and they lead to a greater fall in hedonic measures of inflation, $\tilde{\pi}_t$. As a result, the central bank seeks to cut interest rates by a greater amount. Despite this, under the baseline calibration, the equilibrium path for both the output gap and inflation is still more volatile than the corresponding case with constant product quality. In this setting, the non-monotonic impulse response of product quality arises as it is costly to adjust. Therefore, to avoid large initial changes in the quality of their products after a given productivity shock, intermediate goods firms use a more gradual adjustment path. For smaller values of θ_F the impact response (period 1 impulse response) of product quality moves closer to the peak impulse response.

Figure 2.4 Impulse Response Functions to Positive Productivity Shock



Notes: Baseline calibration, using parameters in Table 2.1. Productivity, A_t, increases by 1%. **Black solid** lines show response with constant product quality. **Blue dashed** lines show response with endogenous product quality.

The impact of an exogenous increase in the nominal interest rate is shown in Figure 2.5. This monetary policy shock is equivalent to an increase in the nominal interest rate of 25 basis points (bps). Although the shock is persistent, under the baseline specification with $\rho_i = 0.5$, it fades relatively quickly throughout the first year of impact. The exogenous monetary tightening increases the real interest rate as prices only gradually adjust due to convex adjustment costs. This causes households to save more in the current time period. As there is no change in the natural real interest rate (as productivity growth is unchanged), this shock causes the current output gap (and hence inflation) to fall. The fall in the output gap and inflation causes the nominal interest rate to decline via a feedback loop in the Taylor rule, partially offsetting the initial exogenous shock.

When intermediate firms are able to adjust product quality, the impulse responses to an interest rate shock are similar. However, to deliver these endogenous response paths for output and inflation, the nominal interest rate rises by less than the case with constant product quality due to a stronger feedback channel through the Taylor rule. This arises as, with a negative output gap, product quality is relatively cheaper to produce due to lower real wages in the current period. In turn, higher product quality reduces aggregate hedonic inflation, which strengthens the offsetting feedback loop to the nominal interest rate from an exogenous shock.

2.4.3 Monetary Policy under Mismeasurement or Model Misspecification

Until now, we have implicitly assumed that monetary policymakers are able to correctly ascertain the influence that model variables have upon the economy. However, in reality, there are major empirical impediments to quality-adjusting price data at national statistical offices that limit the ability of policymakers to understand movements in hedonic terms. Ultimately these considerations can be summarized as either:

1. Mismeasurement of $\tilde{\pi}_t$, $\tilde{y}_t$, or both, which we will refer to as *bad measurement*.

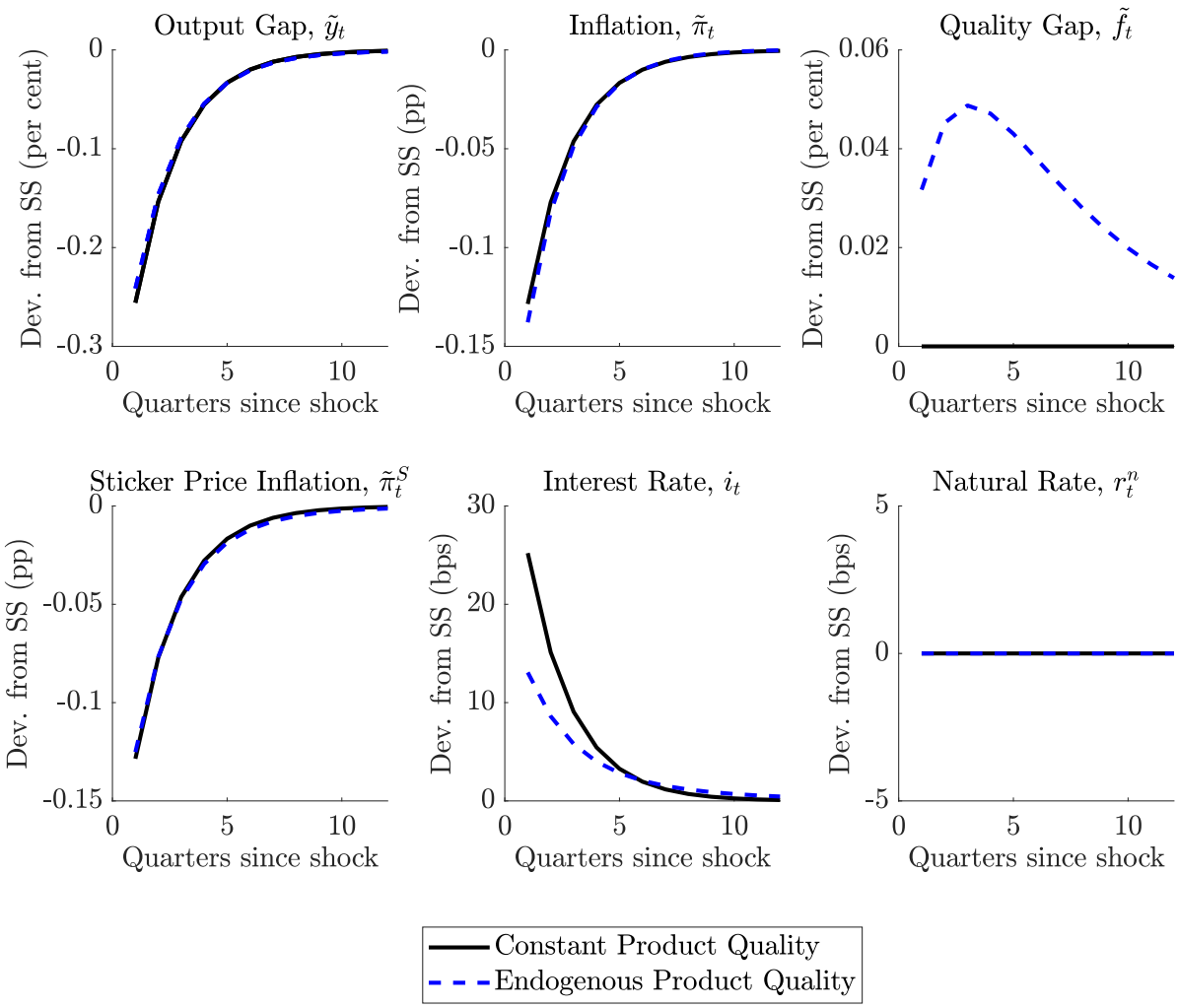
2. Model misspecification, not accounting for quality movements, which we will refer to as a *bad model*.

Each case is now considered in turn.

Bad Measurement. This may arise whenever quality-adjusted price indices are unavailable or unreliable. In such instances, monetary policymakers may replace their Taylor rule with an alternative function such as:

$$i_t = \rho + \pi^* + \phi_\pi \tilde{\pi}_t^S + \phi_y \tilde{y}_t^S + \zeta_t^i, \tag{2.61}$$

Figure 2.5 Impulse Response Functions to Contractionary Monetary Policy Shock



Notes: Baseline calibration, using parameters in Table 2.1. The nominal interest rate, i_t, increases by 25 bps. **Black solid** lines show response with constant product quality. **Blue dashed** lines show response with endogenous product quality.

where ϕ_π and ϕ_y are the same as before but now apply to the quality-unadjusted variables, $\tilde{\pi}_t^S$ and $\tilde{y}_t^S$. These terms are the potential sources of Measurement Error (ME) and shown in **red font** in (2.61). This form of the Taylor rule may be rewritten as:

$$i_t = \rho + \pi^* + \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t + \zeta_t^{\text{ME}} + \zeta_t^i, \tag{2.62}$$

where the additional measurement error term, ζ_t^{ME}, is defined as:

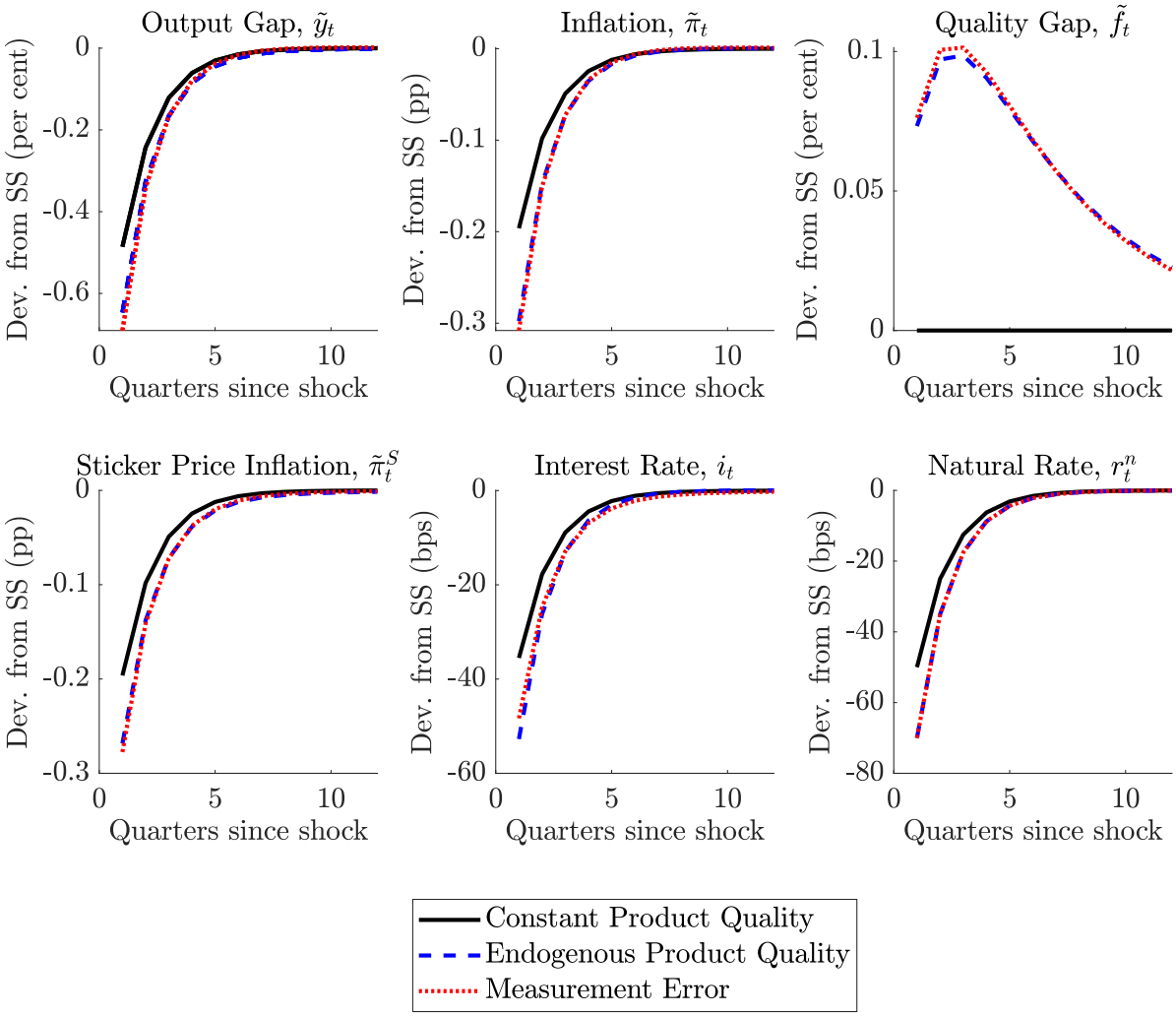
$$\zeta_t^{\text{ME}} \equiv \phi_\pi(\tilde{\pi}_t^S - \tilde{\pi}_t) + \phi_y(\tilde{y}_t^S - \tilde{y}_t). \tag{2.63}$$

It is therefore clear from (2.63) that measurement error enters the policy decision through differences between the quality adjusted and unadjusted inflation and the output gap. A model with constant product quality has no measurement error term.

Interestingly, in a model with endogenous product quality adjustment, and under the baseline calibration, there are also few implications if monetary policymakers are unable to avoid this source of misperception. Figure 2.6 shows that using quality-unadjusted variables, due to bad measurement, to replace components of the monetary policy rule has little impact on the response of the economy following productivity shocks. In particular, the **red dotted** line showing the case of mismeasurement align almost exactly, and on top of the **blue dashed** lines. This shows that when policymakers respond to quality-unadjusted variables, the economy responds in largely the same way. Alternatively put, for the baseline calibration, the combined multiplication of each source of measurement error and their Taylor rule coefficients, as in (2.63), is small. The economy therefore behaves almost identically to the economy without bad measurement.

One reason for this is that an increase in product quality will reduce quality-adjusted measures of inflation while simultaneously increasing quality-adjusted measure of the output gap. Therefore, these two effects at least partially offset each other in (2.63), resulting in a small overall source of additional error. Even when this is not the case (becasue either inflation or the output gap have been correctly measured so there is no offset; or because the relative weights on each component in the Taylor rule are extremely unequal), the effect generally remains small in magnitude provided other parameters are calibrated to reasonable values. For instance, with a lower elasticity of substitution across intermediate goods varieties, at $\varepsilon = 2$, changes in product quality have such a large impact on quality-adjusted prices that monetary policymakers who mismeasure these changes will stabilise the economy by less in the short-run, resulting in larger deviations of inflation and the output gap. But bad measurement of economic variables is generally not a big problem for monetary policymakers in this context.

Figure 2.6 Impulse Response Functions to Positive Productivity Shock Under Measurement Misperception



Notes: Baseline calibration, using parameters in Table 2.1. Productivity, A_t, increases by 1%. **Black solid** lines show response with constant product quality. **Blue dashed** lines show response with endogenous product quality. **Red dotted** lines show response with endogenous product quality, but monetary policymakers mismeasure both inflation and the output gap and respond to their quality-unadjusted measures instead, according to (2.61).

Bad Model. This case arises whenever monetary policymakers follow the canonical NK model, but in reality intermediate firms are able to update both the price and quality of their goods. In a parallel to the above case of bad measurement, in this alternative situation monetary policymakers may replace their Taylor rule with an alternative function such as:

$$i_t = \rho + \pi^* + \phi_\pi \tilde{\pi}_t^{\text{NK}} + \phi_y \tilde{y}_t^{\text{NK}} + \zeta_t^i, \tag{2.64}$$

where $\tilde{\pi}_t^{\text{NK}}$ and $\tilde{y}_t^{\text{NK}}$ respectively refer to the level of inflation and output taken from the canonical NK model which experiences identical shocks.

The relevant impulse response functions for this situation are shown in the **red dotted** lines in Figure 2.7. The response of the canonical NK model is shown in **black solid** lines while the response where product quality may vary is given in **blue dashed** lines. After a given productivity shock, monetary policymakers respond in the same way as they would in the canonical NK model. For this reason the **black solid** lines and the **red dotted** lines perfectly align for the nominal interest rate, i_t.

However, instead of stabilising the economy to the **black solid** lines, as would arise in the canonical NK model, the response of both the output gap and inflation is now even more pronounced than the case where product quality adjustments are accounted for. This arises as the underlying shock to the natural rate of interest, r_t^n, is larger than in the canonical NK model, yet monetary policymakers reduce the nominal interest rates by the same amount.[24]

For a given change to the nominal interest rate, the model misperception therefore manifests as an under-stabilisation of the economy by monetary policymakers *relative* to their anticipated paths for the output gap and inflation. The output gap and inflation both respond by more than intended to the productivity shock, as evidenced by the fact that the **red dotted** lines are below the **black solid** lines (which are the monetary policymakers *intended* paths) for $\tilde{y}_t$ and $\tilde{\pi}_t$.

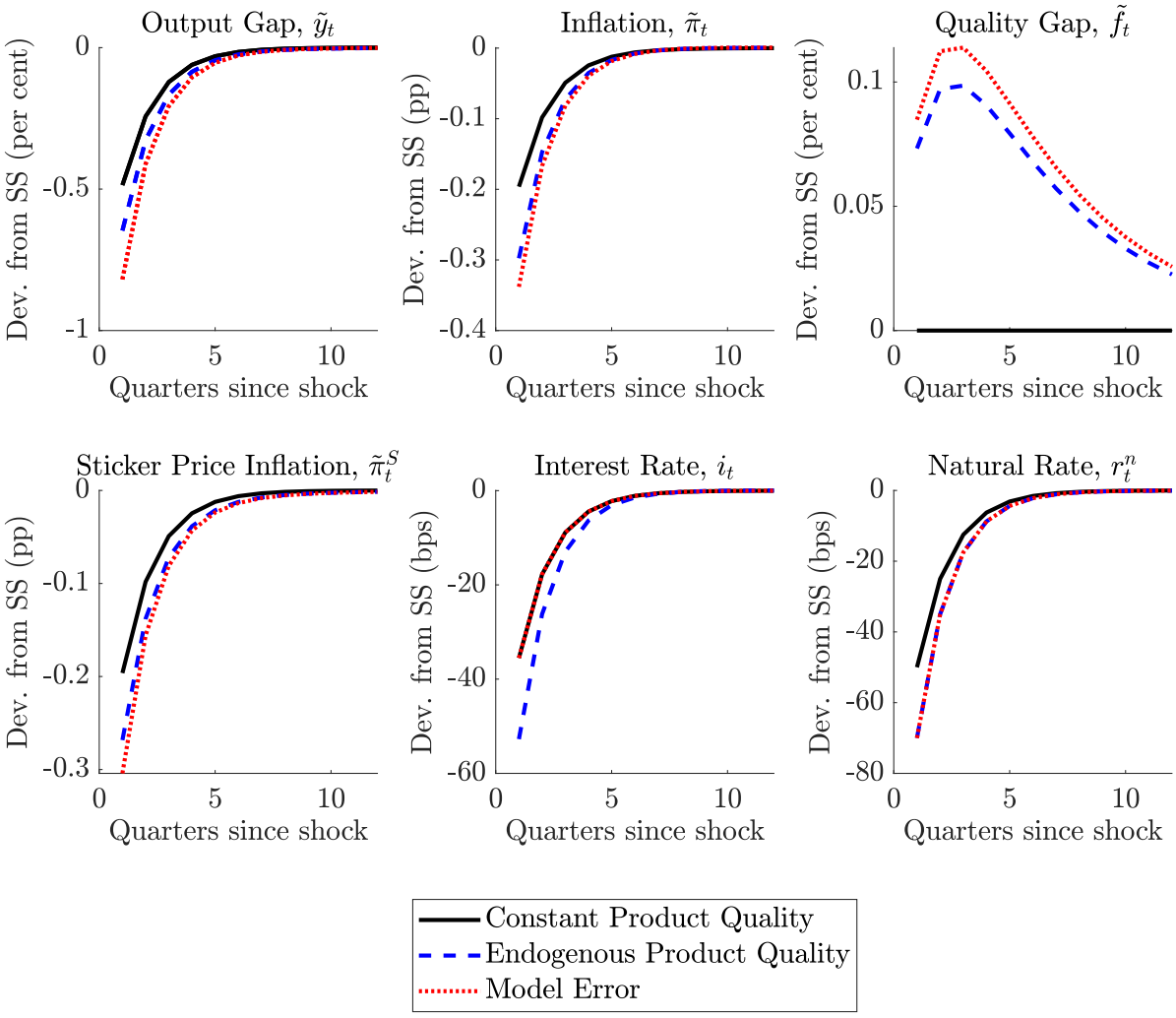
Although the use of a bad model leads to an unintended consequence of an under-stabilized economy, this analysis remains positive in nature. This statement says nothing about the welfare implications of such effects, which we turn to for our normative results in the next subsection.

2.4.4 Normative Analysis

The introduction of product quality into the otherwise standard NK model does not break the classic Blanchard and Galí (2007) divine coincidence whereby monetary policymakers

[24]Indeed, the impulse response of the natural real interest rate is identical to that of the quality augmented model, so the **blue dashed** lines and the **red dotted** lines perfectly align in Figure 2.7.

Figure 2.7 Impulse Response Functions to Positive Productivity Shock Under Model Misperception



Notes: Baseline calibration, using parameters in Table 2.1. Productivity, A_t, increases by 1%. **Black solid** lines show response with constant product quality. **Blue dashed** lines show response with endogenous product quality. **Red dotted** lines show response with endogenous product quality but monetary policymakers misperceive the underlying economic model and instead assume the economy behaves according to the canonical NK model, and respond according to (2.64).

face no trade-off between stabilizing inflation and the output gap. This arises as non-trivial real imperfections remain absent from the product quality augmented model presented above. In other words, the introduction of an endogenous product quality choice does not change the optimal equilibrium path for the real interest rate which a social planner would implement under full flexibility.[25] By inspection of the reduced-form model, one monetary policy rule to implement the social planner's optimal solution would set:

$$i_t = r_t^n + \pi^*, \tag{2.65}$$

which is identical to the desired optimal monetary policy rule in the standard NK model. It is straightforward to observe how this monetary policy rule delivers the social planner's solution, as under this nominal interest rate policy the reduced-form model simplifies to:

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] + \frac{1}{\sigma}\mathbb{E}_t[\tilde{\pi}_{t+1}], \tag{2.66}$$

$$\tilde{\pi}_t^S = \frac{\varepsilon(\sigma+\eta)}{\theta_P}\tilde{y}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^S\right], \tag{2.67}$$

$$\tilde{\pi}_t^F = -\frac{\varepsilon\phi(\sigma+\eta)}{\theta_F(\varepsilon-1)}\tilde{y}_t - \frac{\phi(\nu-1)(\varepsilon-1+\phi)}{\nu\theta_F\left(\varepsilon-1\right)^2}\tilde{f}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^F\right], \tag{2.68}$$

$$i_t = r_t^n + \pi^*, \tag{2.69}$$

$$r_t^n = \rho + \sigma\psi_a\mathbb{E}_t[\Delta a_{t+1}], \tag{2.70}$$

$$\tilde{\pi}_t^F = \tilde{f}_t - \tilde{f}_{t-1}, \tag{2.71}$$

$$\tilde{\pi}_t^S = \frac{\phi}{\varepsilon-1}\tilde{\pi}_t^F + \tilde{\pi}_t, \tag{2.72}$$

$$a_t = \rho_A a_{t-1} + \zeta_t^a. \tag{2.73}$$

The monetary policy rule, (2.65), is therefore consistent with both inflation at target and a welfare-relevant output gap of zero, as one solution to the simplified reduced model (2.66) to (2.73) is:

$$\tilde{y}_t = \tilde{\pi}_t = \tilde{\pi}_t^S = \tilde{\pi}_t^F = \tilde{f}_t = 0 \qquad \forall_t, \tag{2.74}$$

$$i_t = r_t^n + \pi^*, \tag{2.75}$$

$$r_t^n = \rho + \sigma\psi_a\mathbb{E}_t[\Delta a_{t+1}], \tag{2.76}$$

$$a_t = \rho_A a_{t-1} + \zeta_t^a. \tag{2.77}$$

[25] The level of product quality chosen by a benevolent social planner without sticker price or product quality frictions ($\theta_P = \theta_F = 0$) is derived in Appendix A.2.

This states that every endogenous variable matches the equilibrium behaviour of the frictionless first best solution.

However, this first best solution is not unique and therefore there is no guarantee that monetary policymakers implementing the monetary policy rule, (2.65), will attain the optimal social planner's solution. The problematic issues surrounding monetary policy rule indeterminacy are well documented.[26] One solution conjectures an alternative whereby monetary policy contains endogenous components following a rule of the form:

$$i_t = r_t^n + \pi^* + \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t, \tag{2.78}$$

which may restore a determinant solution to the model provided the values for ϕ_π and ϕ_y ensure policymakers respond sufficiently to any change in the natural real interest rate, r_t^n. The precise conditions upon ϕ_π and ϕ_y are outlined in Appendix A.5. For reasonable parameter values, the issue of interest rate rule determinacy is no more problematic in a model with endogenous product quality than the standard setting.

2.5 Conclusion

In this paper we fill a gap in the literature by developing a tractable model of a NK economy with endogenous adjustments in both product price and product quality. In our model, intermediate goods firms adjust the quality of their products to increase their demand but face higher marginal costs when doing so. This contemporaneous trade-off generates endogenous movements in product quality whenever economy-wide productivity shocks alter marginal costs.

To analyse the business cycle properties of this extension to the NK model, we include Rotemberg (1982) adjustment costs such that both nominal prices and product quality respond sluggishly to productivity shocks. We find that, relative to the canonical NK model with adjustment costs in product prices alone, the inclusion of product quality adjustments increases the slope of the Phillips curve. This result arises as, in the quality-augmented model, firms seek to adjust both the quality of their products and their nominal prices following a change to marginal costs. The inclusion of endogenous product quality also amplifies the economy's response to productivity shocks through greater changes in the natural real interest rate, as a new product quality channel of transmission adds to the existing quantity channel. One consequence of this extension to the canonical NK model is that, in general, monetary policymakers must respond more to a given productivity shock in order to deliver similar equilibrium paths for the endogenous variables, including the output gap and inflation.

[26] For example see Blanchard and Kahn (1980) and Bullard and Mitra (2002).

The quality-augmented model presents several challenges to monetary policymakers, who may be unable to correctly ascertain the influence of product quality upon the economy. Whenever quality-adjusted price indices are unavailable or unreliable, inflation and/or the output gap are observed with measurement errors. Under the baseline parametrisation, these issues relating to "bad measurement" remain small and do not substantially influence the path chosen by monetary policymakers for the nominal interest rate, or the other endogenous outcome variables, including the output gap and inflation. In an alternative setting when monetary policymakers follow a "bad model" due to misspecification of the quality-augmented model, the impact is more substantial with monetary policymakers under-stabilising the economy relative to their intended path. Under no misperceptions in product quality, nonetheless, the principles for optimal monetary policy are unchanged, with monetary policymakers seemingly able to jointly stabilize the output gap, inflation and product quality movements.

Chapter 3

The Impact of Large Scale Asset Purchases on Wealth Inequality

3.1 Introduction

In response to the 2007-2008 global financial crisis, monetary policymakers used a variety of novel measures in an effort to stimulate aggregate demand. However, the channels through which these more unconventional forms of monetary policy transmit to the real economy are widely debated, as are their efficacy and distributional consequences. This paper investigates the impact of central bank Large Scale Asset Purchases (LSAPs) on an *endogenous* distribution of household wealth. The primary contribution to the literature is through the presentation of a theoretical model which can account for both financial income and portfolio transmission channels. This includes a set of heterogeneous households who face idiosyncratic risks due to labour productivity shocks and incomplete financial markets which prevent an equilibrium solution with full risk-sharing. The model is calibrated to match the US household wealth distribution prior to the 2007-2008 global financial crisis, and this is used to investigate the effect of changes to the Federal Reserve's balance sheet on wealth inequality.

My main finding is that the Federal Reserve LSAP programmes contributed to an increase in US wealth inequality through financial portfolio channels, raising the Gini coefficient by 3.8%. This occurred through quantitative easing programmes as central bank purchases of longer term assets encouraged households to hold additional liquid wealth. Initially, liquid wealth is disproportionately held by wealthy households. The higher demand for shorter term liquid assets increases their price, benefiting the existing wealthy households through capital gains. As a result, multiple measures of wealth inequality increase. The Federal Reserve's maturity extension programme is found to slightly mitigate the impact of quantitative easing.

In this paper I specify monetary policy operations using a balance sheet approach. This enriches the existing contributions in this literature which typically use Taylor-type rules and focus upon conventional monetary policy changes to the real interest rate and a comparison to homogeneous agent models, (Auclert, 2019; Broer et al., 2020; Kaplan et al., 2018; Lee, 2021). Although Sterk and Tenreyro (2018) use a balance sheet approach to model central bank open market operations, I extend this analysis by allowing the central bank to purchase assets with differing liquidity. This allows changes to the central bank asset composition to be investigated alongside changes to its size.

The remainder of the paper is structured as follows. Two stylised examples are presented in Section 3.2. The existing theoretical and empirical literature is then discussed in Section 3.3. Section 3.4 presents a theoretical model with household heterogeneity which explicitly

accounts for the composition of the central bank's balance sheet and Section 3.5 discusses the model calibration. Section 3.6 presents the main results of the paper. Section 3.7 concludes.

3.2 Stylised Examples

This section presents two rudimentary methods to uncover the potential implications from asset price and quantity movements, consistent with those surrounding LSAPs.[1] Both methods provide evidence to suggest LSAPs may have increased household wealth inequality.

3.2.1 Price Movements (Constant Allocations)

Holding asset allocations constant, asset price movements have contributed toward the increase in wealth inequality between 2007 and 2019. A counterfactual analysis can be conducted to determine the plausible change in wealth inequality as a result of asset price movements alone. This assumes fixed cross-sectional asset allocations such that asset price movements are the sole contributor to changes in wealth inequality. The procedure occurs in three steps. Initially a detailed picture of household wealth portfolios across the distribution of households is obtained from the 2007 US Survey of Consumer Finances (SCF).[2] Next, price movements are assigned to each component of household wealth, following the methodology outlined in Kuhn and Ríos-Rull (2016). This matches the individual components of household wealth to real house and stock market price movements, taken from Shiller (2015).[3] At each quintile of the wealth distribution, household wealth portfolios are then re-evaluated over time to account for the observed changes in asset prices.

The results of this exercise suggest wealth inequality has increased since the great recession, as illustrated in Figure 3.1, which plots the (indexed) real value of household wealth for the 2nd and 5th quintiles of the 2007 wealth distribution.[4] During the great recession, a fall in both house and stock prices caused a fall in the value of wealth portfolios for both the 2nd and 5th quintiles. Initially, the reduction in value of wealth is of a similar magnitude for both quintiles. However, between 2009 and 2012 the slower recovery in house prices, than in the stock market, combined with a greater portfolio allocation towards housing for households in the 2nd quintile of the net wealth distribution, caused wealth valuations to diverge. A gap opens between the two valuations, indicating an increase in wealth inequality. Overall, the

[1] Although the focus of this paper is on developments in the United States similar results would be attainable for other developed countries. In particular, the UK's wealth distribution shares many similarities with the US, as discussed in Appendix B.1.2.

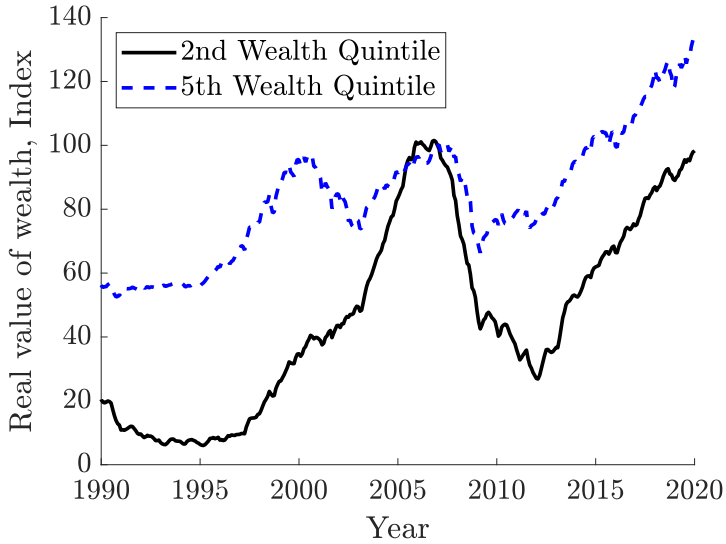
[2] See Appendix B.1.1 for further details on the US SCF.

[3] Subsequent updates for more recent data are available via Robert Shiller's website: http://www.econ.yale.edu/~shiller/data.htm.

[4] The 2nd quintile is used instead of the 1st to remove issues relating to negative wealth held by this group.

reduction in the real value of wealth between 2007 and 2012 is of a similar magnitude to the increases seen in the early 2000s, for both quintiles.

Figure 3.1 Pure Effect of Asset Prices on the Value of Net Wealth, by Quintile



Sources and Notes: US SCF (2007, see Appendix B.1.1), Shiller (2015) including subsequent updates from http://www.econ.yale.edu/~shiller/data.htm, and author's calculations. Each asset in household net wealth in the US SCF (2007) is matched to a price index from Shiller (2015) according to Kuhn and Ríos-Rull (2016). This generates an evolution for the value of wealth over time accounting only for asset price changes. For each household net wealth in January 2007 is indexed to 100, and the 5th quintile (richest households) and 2nd quintile (second least wealthy) are graphed.

Although this exercise is instructive in recovering the pure effect of asset price changes on inequality through changes to the value of wealth, it has severe limitations. The assumption of fixed asset portfolios is strong but difficult to overcome in empirical work. Moreover, as households rebalance their portfolios, the accuracy of these valuation effects reduces when moving further from the initial date. Finally, this exercise invites a false presumption that all asset price changes during this period were driven by Federal Reserve policy.[5] For these reasons, a theoretical model can help to uncover general equilibrium effects, particularly those arising from the portfolio rebalancing channel of transmission.

Nonetheless, this exercise also provides a mechanism to enable a quick calculation which suggests the pure impact of real asset price changes over this period may be responsible for a 3.1% increase in the Gini coefficient and a 14.3% increase in the cross-sectional coefficient of variation of household net wealth. These back-of-the envelope calculations are shown in Table 3.1, which highlights the impact of a change in property and stock market prices on measures of wealth inequality. In net, during the 10-year period between January 2007 and

[5]The aggregate impact of LSAPs on asset prices and economic activity is discussed in subsection 3.3.2.

January 2017, real house prices fell by 15.6% while stock prices increased by 40%. Holding asset allocations fixed at their 2007 SCF levels, these asset price changes both served to increase the Gini coefficient and the cross-sectional coefficient of variation. The increase in stock prices had a larger impact on increasing wealth inequality according to the Gini coefficient, while the fall in real house prices had a larger impact on the cross-sectional coefficient of variation. Taken together, the asset price changes, with fixed portfolios, appear to have considerably increased wealth inequality over this period.

Table 3.1 Pure Effect of Asset Price Changes on Wealth Inequality

	Data (2007 SCF)	Property	Stocks	Total
% Price Change (2007-2017)		-15.6	40.0	
Gini Coefficient	0.849	0.857	0.870	0.875
% change		0.9	2.5	3.1
Cross-sectional Coefficient of Variation	6.6	7.2	7.0	7.6
% change		8.5	5.7	14.3

Sources and Notes: US SCF (2007, see Appendix B.1.1), Shiller (2015) including subsequent updates from http://www.econ.yale.edu/~shiller/data.htm, and author's calculations. Real price changes are taken as the 10 years between January 2007 and January 2017. These are then combined with a fixed asset allocation taken from the US SCF (2007) to calculate the impact on these measures of wealth inequality.

3.2.2 Price and Quantity Movements

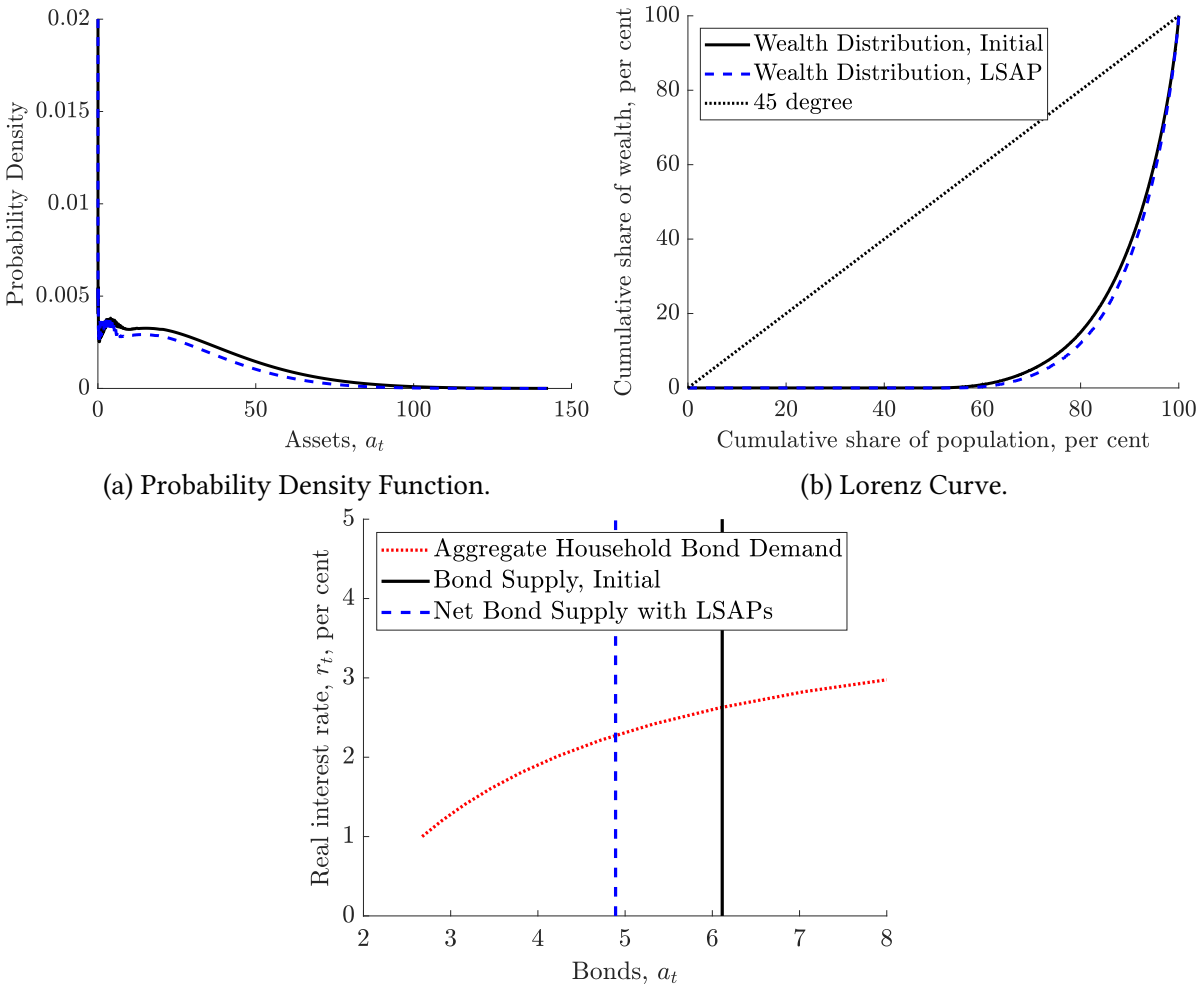
An alternative, purely theoretical, exercise also supports the conclusion from the previous section: that LSAPs may increase household wealth inequality. In this illustration a conventional Aiyagari (1994) model is calibrated to match the observed US wealth distribution from the 2007 SCF, as shown in Figure 3.2. Two steady states are compared.[6]

From this initial position the net availability of government bonds is then reduced by 20% to reflect the central bank's LSAPs. From a bond market perspective, LSAPs reduce the net supply of government bonds to the private sector. This increases the price of bonds and reduces bond yields (interest rate). In aggregate, the market clears as fewer households are willing to purchase bonds with a lower interest rate (Figure 3.2, Panel c).

The impact of this shock to effective asset supply on the distribution of household wealth is shown in Figure 3.2, Panels a and b. As fewer assets are available, the distribution of

[6]The comparison of two steady states when evaluating the impact of LSAPs is justified as these changes to the central bank balance sheet have been both large and persistent. The scale and length of time over which central banks have maintained balance sheets larger than the historical average suggest considering small, temporary, perturbations around a fixed steady state is unsuitable.

Figure 3.2 Comparative Statics of the Standard Aiyagari (1994) Model to Net Bond Supply Shock.



(a) Probability Density Function.

(b) Lorenz Curve.

(c) Bond Market Equilibrium.

Notes: This "standard" case is an Aiyagari (1994) model with one asset (government bond), calibrated to match the observed US wealth distribution from the 2007 SCF. Asset supply matches the wealth-to-income ratio in the US Survey of Consumer Finances. The LSAP scenario restricts net asset availability by 20%.

household assets shifts to the left. Asset holdings are not reduced proportionally by each household. Instead, wealthier households are more willing to save at a lower interest rates, as their precautionary motive is stronger due to greater relative income volatility and convex utility. Alternatively put, at a given interest rate, wealthier households have a higher marginal propensity to save. All else equal, bond demand from wealthier households therefore falls by less than for those further down the wealth distribution (who still have initially positive bond demand). In addition, the least wealthy households already face a binding liquidity constraint, and this change in net asset supply results in a greater concentration of households at this fixed point. Overall, wealth remains concentrated among the richest section of the population (even though these household individually each hold fewer assets).

This downwards shift in the number of assets held, combined with an increasing number of households at their borrowing constraint, shift the Lorenz curve for household wealth outwards, particularly in the middle of the distribution. Household wealth inequality as measured by the Gini coefficient increases, by 2.4%. The impact on aggregate household wealth is theoretically ambiguous, as lower bond holdings are combined with higher bond prices.

Both of these stylised examples conclude by suggesting LSAPs are likely to have increased wealth inequality after the global financial crisis. The empirical exercise highlights a role for the composition of different assets within the population, with more wealthy households benefiting during the economic recovery, while the theoretical model suggests these households are able to maintain their *relative* position as, in equilibrium, these individuals are less likely to be credit constrained following a large and persistent intervention by the central bank.

3.3 Related Literature

The related literature can be divided into three primary strands. Firstly, one section of the literature provides the essential modelling assumptions and theoretical justification required to analyse the consequences of LSAPs on wealth inequality. This includes both frictions, usually in financial markets, which are vital to allow monetary policy to impact real variables, and household heterogeneity, which is required to model wealth inequality. A second strand discusses the aggregate impact of LSAPs on asset prices, investors' asset allocations, GDP and inflation. Although precise magnitudes vary across studies and countries, a large empirical literature finds a strong positive statistical relation between LSAPs and asset prices, and an associated portfolio rebalancing effect. It also finds a significant positive impact of LSAPs on GDP growth, and inflation. Finally, the third strand considers the relationship between inequality and monetary policy through an inflation, labour and financial channel of

transmission. The majority of empirical analysis here focusses on conventional monetary policy. In the next three subsections I discuss each of these strands of the literature in turn.

3.3.1 Essential Modelling Assumptions

Two modelling assumptions are required to study the impact of monetary policy on wealth inequality. Firstly, frictions, usually in financial markets, are necessary to enable monetary policy to have real effects in the long-run.[7] Secondly, any model requires a distribution of heterogeneous agents with differing wealth levels. The model presented in section 3.4 will make use of both modelling assumptions without nominal frictions.

The first assumption required to analyse the impact of monetary policy on wealth inequality is market frictions. In his influential Jackson Hole paper Woodford (2012) highlighted the need for market frictions to ensure central bank LSAPs have a real effect on the economy, as a specific case of the general Wallace (1981) irrelevance result. The general result shows that central bank open market operations have no effect on the real economy when accompanied by fiscal transfers ensuring an invariant income distribution. A reallocation of assets between economic agents has no economic impact if assets are only valued for their payoffs and budget constraints are the sole restriction on investors' portfolio positions. Market frictions are therefore required for monetary policy to have real effects in the long-run, and the literature suggests a variety of forms these could take, each capturing a particular feature of the monetary transmission mechanism. Eggertsson and Woodford (2003) provide an alternative to market frictions through the role of expectations, while Gertler and Kiyotaki (2010) explicitly model disruptions to financial intermediation to highlight the central bank's role in providing liquidity support to the financial sector. However, when considering the impact of LSAPs, the most appropriate market friction to break the Wallace (1981) irrelevance result should focus on the portfolio rebalancing channel of monetary transmission, as this most clearly distinguishes LSAPs from other forms of monetary policy.

Two model varieties focus on the portfolio rebalancing channel of monetary policy transmission. In one approach, investors have preferred-habitat demand preferences for assets with differing term structures. Vayanos and Vila (2021) show how the preferred-habitat view of the term structure can generate asset demand which differs substantially from no-arbitrage restrictions, with the government bond term structure instead determined through an interaction between investors with preferences for bonds of specific maturities and risk averse arbitrageurs. In an alternative approach, Andrés et al. (2004) introduce a model with portfolio adjustment costs and exogenous financial market participation restrictions to model the separate impact of monetary policy on short- and long-term assets. This allows them to

[7] Nominal rigidities would be sufficient for a short-run impact.

separate the impact of open market operations on long-term assets through the expected path of short-term rates and an additional influence of monetary policy on relative prices through term premia. Chen et al. (2012) calibrate preferred-habitat, adjustment costs and exogenous financial market participation restrictions in a medium-scale DSGE model and simulate the impact of Federal Reserve LSAPs. They find little aggregate impact when matching the low degree of market segmentation observed in the data.

The second vital modelling assumption, to include a distribution of heterogeneous households with differing wealth levels, is often accompanied by an assumption of incomplete financial markets.[8] This class of models has undergone a series of transformations since first being proposed by Bewley (1980, 1983, 1986). Huggett (1993) analyses a partial equilibrium with trade in a zero-net-supply asset, while Aiyagari (1994) extends this analysis to a general equilibrium setting, for a production economy. The interaction between heterogeneous households and incomplete financial markets in these models generates an intrinsic precautionary saving motive, as households save to self-insure against possible future adverse idiosyncratic states.[9] Further work by Krusell and Smith (1998) introduces a concept of bounded rationality to permit solutions for models with additional aggregate uncertainty resulting from aggregate business cycle productivity shocks.

Comprehensively matching moments of the household wealth distribution is challenging, even in a model with incomplete financial markets and both idiosyncratic and aggregate productivity risks. Typically the strength of the precautionary saving motive declines as wealth increases, relative to labour income. This results in the poorest households holding more wealth than the data suggests, while the richest households hold less.[10] Krueger et al. (2016) note how these differences bias idiosyncratic household impulse responses towards those from representative agent models and propose increasing the persistence of earnings risk, to substantially increase wealth dispersion.[11] In most modifications to the standard model of heterogeneous agents with incomplete markets, financial markets remain limited to a single risk-free one-period bond, which is insufficient to study the full effects of Large Scale Asset Purchases, which requires multiple assets and the possibility of portfolio rebalancing.

[8] An entire *distribution* is necessary here as although a two-agent model may describe the macroeconomic effects of monetary policy the impact on wealth inequality would be trivial and potentially misleading in such a setting without accounting for the wider distribution.

[9] Greater uncertainty may heighten this motive (Bayer et al., 2019).

[10] In Carroll (1997) households stop saving once they reach a buffer-stock of capital, beyond which richer households have a negative saving rate.

[11] A similar solution is also suggested in Carroll et al. (2015) who add buffer-stock saving to the standard Krusell and Smith (1998) model, and Ravn and Sterk (2017), where search and matching frictions in the labour market generate this persistence through additional unemployment risk.

3.3.2 The Aggregate Impact of LSAPs

The empirical evidence suggests LSAPs have a significant positive impact on the price of a variety of assets. Two different empirical exercises dominate the literature. In the first, the authors construct short event windows around monetary policy announcements to capture the effect of policy announcements on asset prices. Typically, unexpected expansionary announcements reduce the yields on government bonds and mortgage-backed securities, increase equity valuations and depreciate nominal exchange rates. Although the precise effects differ depending on both the empirical methodology and specific LSAP programme, when scaled to a common size, LSAP programmes (prior to Covid-19) which purchase assets worth 10% of nominal GDP are usually found to reduce 10-year bond yields by 70-100bps in the US and UK, but by a far smaller amount in the euro area and Japan. These results are shown in Table 3.2.

The second empirical method uses dynamic term structure models to distinguish the effect of LSAP announcements on the risk premium and the expected path of future policy rates.[12] These models find a smaller, yet significant, effect of LSAPs on the risk premium component of asset prices, for example reducing 10-year government bond yields by between 40-60bps in the US. Where applicable, these results are reported in parentheses in Table 3.2.

Further empirical work suggests investors adjust their asset allocations after central bank LSAPs, rebalancing portfolios away from government bonds towards alternative asset classes. In the US, Carpenter et al. (2015) highlight the role of segmented financial markets, with heterogeneous asset sales to the Federal Reserve across multiple investor categories. They found that, during the global financial crisis, the Federal Reserve ultimately purchased US Treasury bonds from households,[13] who rebalanced their portfolios towards corporate bonds, commercial paper and municipal debt. Joyce and Tong (2012) use a counterfactual analysis to show that UK LSAP programmes induced similar portfolio rebalancing behaviour across a set of individual life insurance companies and pension funds, who reduced their government bond asset allocations in favour of corporate bonds.

While central bank LSAPs appear to substantially influence asset prices and allocations, their broader impact on economic activity and prices is less clear. The impact on aggregate measures of economic activity is critical in assessing the impact on wealth inequality, as although an initial shock to relative asset prices may be sufficient to alter wealth inequality, given heterogeneous asset positions, subsequent changes in aggregate variables may plausibly

[12] In a complementary analysis Wu and Xia (2016) use a non-linear term-structure model to translate the estimated impact of LSAPs into a "shadow" short-term nominal interest rate which would have prevailed if not for the effective lower bound.

[13] The aggregate household category in the US Flow of Funds data includes hedge funds.

Table 3.2 Impact of LSAPs on 10-year Government Bond Yields

Study	Sample	Method	Yield Reduction in bps	
US				
Krishnamurthy and Vissing-Jørgensen (2011)	2008-2009	Event study	90	
Krishnamurthy and Vissing-Jørgensen (2011)	2010-2011	Event study	90	
Gagnon et al. (2011)	2008-2009	Event study	77	(60)
Christensen and Rudebusch (2012)	2008-2009	Event study	76	(25)
D'Amico et al. (2012)	2009-2010	Weekly time-series regression	170	
D'Amico and King (2013)	2009-2010	Micro cross-section regression	144	
Bauer and Rudebusch (2014)	2008-2009	Event study	75	(48)
Swanson (2015)	2009-2015	Factor model event study		(40)
Neely (2015)	2008-2009	Event study	79	
Lloyd (2017)	2008-2012	Event study	117	(60)
UK				
Joyce et al. (2011a)	2009-2010	Event study	77	
Joyce et al. (2012)	2009-2010	Event study, time series	75	
Christensen and Rudebusch (2012)	2009-2011	Event study	34	(46)
McLaren et al. (2014)	2009, 2012	Event study	65	
Euro Area				
Altavilla et al. (2015)	2014-2015	Event study	42	
Andrade et al. (2016)	2015-2016	Event study	41	
De Santis (2016)	2004-2015	Error correction model	63	
Japan				
Lam (2011)	2008-2010	Event study	9	(4)
Fukunaga et al. (2015)	1992-2014	Cointegrating regression	13	(30)
Imakubo et al. (2015)	1992-2014	Natural rate comparison	12	

Source and Notes: Author's calculations from reported values in cited studies. Yield reduction is measured in basis points. Across studies, results are normalised to an LSAP programme with asset purchases worth 10% of nominal GDP. Term premia effects, also measured in basis points, are in parentheses.

unwind these effects. Empirically, as aggregate variables are further removed from the initial shock, stronger methodological assumptions are usually required to ascertain the broader impact of LSAPs. Nevertheless, the existing literature uses two main methods to uncover plausible effects. The first technique uses various restrictions on vector autoregressive (VAR) models to simulate counterfactual paths for GDP and inflation, or present impulse response functions (IRFs). A second popular technique is to embed central bank asset purchases into a DSGE model and simulate shocks. Both methods generally show LSAPs to have a substantial positive impact on output and inflation, as presented in Table 3.3. Overall, the various Federal Reserve LSAPs implemented in the aftermath of the global financial crisis are found to increase output and inflation, but the magnitude of these effects is unclear. The same is true in the UK, although the Bank of England's LSAP programmes are generally found to have had a smaller impact. The more recent programmes in the euro area (enacted between 2012 and 2019) are found to have effects which are smaller still.

Comprehensive reviews of the impact of the pre-pandemic central bank LSAP programmes, as well as other unconventional monetary policy measures, can be found in both Borio and Zabai (2016) and Haldane et al. (2016).

3.3.3 The Effect of Monetary Policy on Inequality

The theoretical literature identifies several potential channels of transmission for monetary policy shocks to influence the distribution of household wealth.[14] Auclert (2019) analytically decomposes the response of heterogeneous households into three broad classes (inflation, labour and financial), then showing how variation along each dimension can generate different marginal propensities to consume from a monetary policy shock. Using this framework, contributions to the literature may also be divided by their channels of transmission, with the net impact of monetary policy shocks on wealth inequality depending both on the balance between these channels of transmission and also on the extent to which the response of *income* inequality translates over time into a *wealth* effect. In response to an expansionary monetary policy shock these channels are as follows:

1. **Inflation and savings redistribution channel**. An (unanticipated) increase in inflation following an expansionary monetary policy shock erodes the real value of nominal debt contracts, thereby transferring wealth from savers to borrowers, reducing wealth inequality. The empirical relevance of this transmission channel is explored extensively by Doepke and Schneider (2006b), who find a 'moderate' inflationary surprise (a 5pp

[14] Colciago et al. (2019) present a summary of the related empirical literature concluding that the impact of unconventional monetary policy on measures of income and wealth inequality is unclear.

Table 3.3 Impact of LSAPs Aggregate Activity and Prices

Study	Sample/Simulation	Method	Peak Impact (pp) Output	Inflation
US				
Baumeister and Benati (2013)	2009	VAR & counterfactual	0.9	0.5
Gertler and Karadi (2013)	QE1	DSGE simulation	3.5	4.0
Chen et al. (2012)	QE2	DSGE simulation	0.1	0.0
Gertler and Karadi (2013)	QE2	DSGE simulation	1.0	1.4
Engen et al. (2015)	2009-2015	DSGE simulation (FRB/US)		0.5
Chung et al. (2012)	2010-2012	DSGE simulation (FRB/US)	3.0	1.0
Gambacorta et al. (2014)	2008-2011	VAR IRF	4.3	2.6
Weale and Wieladek (2016)	2009-2014	VAR IRF	4.4	4.7
UK				
Baumeister and Benati (2013)	2009	VAR & counterfactual	0.7	2.0
Joyce et al. (2011b)	2009	VAR & counterfactual	1.5	0.8
Kapetanios et al. (2012)	2009	VAR & counterfactual	1.4	1.2
Gambacorta et al. (2014)	2008-2011	VAR IRF	3.9	0.0
Bridges and Thomas (2012)	2009-2011	Various	2.0	1.0
Pesaran and Smith (2016)	2009-2011	ARDL & counterfactual	1.0	
Weale and Wieladek (2016)	2009-2014	VAR IRF	3.1	3.9
Euro Area				
Lenza et al. (2010)	2008-2010	VAR & counterfactual	2.5†	0.2
Gambacorta et al. (2014)	2008-2011	VAR IRF	1.4	0.9
Gambetti and Musso (2017)	2009-2016	VAR & counterfactual	0.2	0.4

Source and Notes: Author's calculations from reported values in cited studies. Results are not normalised, with initial shocks varying. In Kapetanios et al. (2012), Pesaran and Smith (2016) and Bridges and Thomas (2012) model averages are reported. The peak impact in Weale and Wieladek (2016) multiplies the peak impulse response by the largest policy announcement shock. Estimates in Gambacorta et al. (2014) use central country-specific impulse response estimates and observed balance sheet changes.
†The output estimate in Lenza et al. (2010) uses industrial production.

increase, sustained for 10 years) leads to a sizeable redistribution of wealth. The largest gains (from 2.6 to 5.9 percent of GDP) accrue to middle-class households under age 45, who hold substantial fixed-rate mortgage positions. In contrast, middle-class retirees (over age 65) lose most as inflation erodes the real value of savings. This inflationary surprise episode also causes a material redistribution from the household sector to the government, the primary net debtor to the household sector.[15] In a theoretical contribution, Akyol (2004) examines the welfare implications of different steady state rates of inflation, showing that in an incomplete markets setting the Friedman rule (setting the nominal interest rate to zero) is not optimal. Instead, when money is useful for consumption smoothing, the monetary authority may use a small positive level of inflation to achieve a higher level of social insurance, as a tax on households who receive large endowment shocks with seigniorage revenue evenly distributed. The benefit of this, relative to self-insurance, diminishes as inflation increases.

2. **Labour channel**. Theoretically, households should re-optimise their labour supply decisions in an attempt to mitigate the impact of a change in relative asset prices and real wages, resulting from an expansionary monetary policy shock. However, empirical estimates of this facet of household behaviour (the intertemporal substitution effect) vary substantially (Havranek et al., 2015). Although most estimates suggest expansionary monetary policy reduces inequality through labour channels, as expansionary monetary policy shocks are usually associated with higher aggregate demand and additional job creation, the empirical approach usually abstracts from decomposing the equilibrium effect into labour supply and demand channels. Instead, researchers typically combine price, quantity, supply and demand into one overall effect. For example, by combining Romer and Romer (2004) exogenous monetary policy shocks with household-level consumer expenditure data, Coibion et al. (2017) find monetary policy shocks alter overall US earnings inequality by a similar order of magnitude as their impact on GDP and inflation. A 100bps expansionary conventional monetary policy shock is found to decrease income inequality by 1.5% as measured by the Gini coefficient. Estimates for the strength of this channel are slightly larger in the UK (Evgenidis and Fasianos, 2021; Mumtaz and Theophilopoulou, 2017) and perhaps lower for the euro area (Guerello, 2017). These effects on income inequality are reported in Table 3.4.

   In an extensive summary of the differences in monetary policy transmission between representative and heterogeneous agent models, Kaplan et al. (2018) argue the endogenous response of indirect channels, including labour demand and fiscal policy, are

[15] In related work, Doepke and Schneider (2006a) also show such a zero-sum wealth redistribution may generate persistent aggregate effects.

Table 3.4 Impact of Contractionary Monetary Policy Shocks on Income Inequality

Country	Study	Inequality Measure	Response
US	Coibion et al. (2017)	Gini coefficient	1.5%
UK	Mumtaz and Theophilopoulou (2017)	Gini coefficient	3%-10%
Euro Area	Guerello (2017)	Gini coefficient	1%
Panel	Furceri et al. (2018)	Gini coefficient	2%

Source and Notes: Author's calculations from reported values of the cumulative response, scaled to a +100bps increase in the monetary policy instrument against income inequality. Studies use a similar VAR methodology with the exception of Furceri et al. (2018), who use local projection.

responsible for over 75% of the first-quarter consumption response, compared to under 5% in a typical representative agent model. They show real interest rate movements and intertemporal substitution effects primarily stem from these indirect effects. Lüetticke (2021) draws similar conclusions in a model calibrated to replicate business cycle statistics. This indirect aggregate channel of transmission is also important as central bankers have argued the positive impact of LSAPs on job creation absolves any medium-term influence of these policies on wealth inequality as low-skilled workers should benefit more from lower unemployment (Haldane, 2014; Lee, 2021).

3. **Financial channel**. This channel may be further divided into two components:

   (a) **Capital gains**. As a result of an unexpected expansionary monetary policy shock, bond prices typically rise, while yields fall. These financial asset price increases cause wealth inequality to increase through valuation effects which fall upon the richer individuals who are more exposed to financial markets. As monetary policy alters *relative* asset prices, and households have heterogeneous asset portfolios, a given change (shock) in asset prices may generate disproportionate responses to different households, with a resulting impact on wealth inequality. In a partial equilibrium setting (abstracting from portfolio rebalancing), Domanski et al. (2016) find an increase in equity valuations since the global financial crisis has benefited the wealthiest members of society, increasing wealth inequality across a range of countries. A more even distribution from house price gains only partially offsets this. Doepke et al. (2015) also conclude that the house price channel of transmission is not large enough to mitigate the impact on wealth inequality of inflationary shocks. These empirical results are not universally accepted, with O'Farrell et al. (2016) finding monetary policy shocks to be an insignificant determinant of changes in net wealth inequality.

(b) **Financial income**. Given the variance in household asset levels and portfolio composition, the capital income boost from an expansionary monetary policy shock results in heterogeneous relative income shocks. Wealthier households typically hold a larger proportion of wealth as financial assets and therefore experience a larger income rise through this channel. Broer et al. (2020) focus on this aspect of the monetary transmission mechanism using a version of the standard New Keynesian model with capitalists and workers to highlight the role of profits in generating the standard increase in output following expansionary monetary policy shocks. Coibion et al. (2017) also present factor income heterogeneity as an important transmission channel. Empirically, they find aggregate labour income responds little to monetary policy shocks, whereas financial and business income fluctuate extensively.

Overall the effect of an expansionary monetary policy shock on wealth inequality largely depends on the balance between lower inequality through labour channels and a greater level of inequality which arises as a result of financial channels. Recent reviews by the Bank of England (Bunn et al., 2018), European Central Bank (Lenza and Slacalek, 2021), Bundesbank (2016), Bank d'Italia (Casiraghi et al., 2018), and International Monetary Fund (Bonifacio et al., 2021) all conclude that the total effect of pre-pandemic LSAPs on various measures of income and wealth inequality has been small. Indeed, Lenza and Slacalek (2021) find these policies slightly reduce the income inequality Gini coefficient.

The theoretical models discussed in this section generally rely upon a monetary policy rule specifying the central bank's reaction function, without specifically modelling open market operations and balance sheet changes. This is potentially problematic when assessing the impact of LSAPs. This point is also noted by Sterk and Tenreyro (2018), a paper highly related to this one. Sterk and Tenreyro (2018) specifically focus on modelling central bank balance sheet policies, and suggest two distinct channels of monetary policy transmission towards heterogeneous households: through inflation altering the value of real wealth (channel 1, above), and through an associated change in transfers to households that occur as a result of remittance income from the central bank to the Treasury (channel 3b, above). An important contribution they make is to model different sectors of the economy, concluding that consumers substitute towards durable goods in response to expansionary monetary policy shocks. Sterk and Tenreyro (2018) introduce household heterogeneity through an overlapping generations model, which naturally focuses on monetary policy transmission across demographic groups within a life-cycle structure.

Several theoretical papers consider the impact of large-scale asset purchases on the economy, but few explicitly consider the role of central bank liquidity transformation. To

the best of my knowledge, none do this in the presence of heterogeneous agents. Sterk and Tenreyro (2018) and Harrison (2012) use a balance sheet approach without a role for central bank liquidity transformation. In Sterk and Tenreyro (2018) changes in central bank policy directly contribute to changes in the short-term, liquid, asset market, while in Harrison (2012) the central bank holds only long-term bonds. Other representative household models with explicit central bank balance sheets include Sims and Wu (2021) and Carlstrom et al. (2017) who respectively highlight the central bank's role in asset transformation and leverage reduction during LSAPs. Although the models presented in Gertler and Karadi (2011) and Lee (2021) contain more sophisticated banking sectors, central bank policy consists of a combination of control over the short-term, liquid, interest rate alongside either credit spread policy or long-term asset holdings respectively.[16] Similarly, in Chen et al. (2012) central bank policy is enacted through use of the short-term interest rate and credit spreads, while in Cui and Sterk (2021) the central bank uses a regime switching policy, and either controls the short-term interest rate or the balance sheet.

This paper contributes to the literature in two regards. Firstly, it presents a theoretical model which can account the financial transmission channels. It then calibrates the model to match recent US LSAPs to show these forms of unconventional monetary policy have increased wealth inequality through these financial channels.

3.4 Model

The model features six agent types: households, firms, banks, the government, the central bank and a mutual fund. Heterogeneous households largely follow a *Bewley-Huggett-Aiyagari* style model, extended to include a durable goods representation for each household's stock of housing wealth. There is no separation between home-owners and renters. The central bank's balance sheet is modelled in detail. This enables a theoretical definition for different LSAP programmes. Firms are modelled simply. To cleanly distinguish the positive, rather than normative, impact of central bank policies, neither the government nor central bank is engaged in optimising behaviour. For reference, a schematic diagram of the model is provided in Appendix B.2.

Notation. Upper case variables denote nominal variables while real variables are in lower case. First subscripts denote the agent; i indicates household-specific variables; g and s denote the supply of a variable from the fiscal and monetary authority respectively; b indicates a bank; and m the mutual fund. Second subscripts denote the time period. Finally, whenever

[16] Lee (2021) models quantitative easing as temporary perturbations around a steady sate level of asset holdings.

relevant, superscripts are used to denote the asset to which a net rate of return, proportion, price or demand refers.

3.4.1 Households

The household side of the model closely follows the durables goods extension to Guerrieri and Lorenzoni (2017). A continuum of heterogeneous households (with unit mass) maximise their expected lifetime utility:

$$\max_{\{c_{i,j},\ell_{i,j},d_{i,j+1},h_{i,j+1}\}_{j=t}^{\infty}} \mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} u(c_{i,j}, h_{i,j}, 1-\ell_{i,j}), \tag{3.1}$$

where $\beta \in (0,1)$ is the household discount factor and the time-separable, concave and increasing period-t felicity function, u, for household i depends on their individual level of real consumption, $c_{i,t}$, the flow utility gathered from housing which is taken to be proportional to the stock, $h_{i,t}$, and labour supply, $\ell_{i,t}$, which is normalised to be within the unit interval. As in Castañeda et al. (2003), households are assumed to care equally about their own utility and that of their descendants. Utility maximisation occurs subject to a series of period budget constraints. Stated in real terms, for household i, the period-t budget constraint is:

$$c_{i,t} + d_{i,t+1} + p_t^h h_{i,t+1} + \phi(h_{i,t+1}, h_{i,t}) = (1+r_t^d)d_{i,t} + p_t^h(1-\delta)h_{i,t} + \tau_t + \pi_{i,t} + w_t\zeta_{i,t}\ell_{i,t}. \tag{3.2}$$

The left-hand side of the budget constraint represents the household's expenditure. Each period households use available resources to purchase consumption, $c_{i,t}$, deposit real money balances with retail banks, $d_{i,t+1}$, and purchase illiquid housing stock, $h_{i,t+1}$ at its relative price, p_t^h. Finally, agents must pay a real cost, $\phi\left(h_{i,t+1}, h_{i,t}\right)$, of adjusting their stock of illiquid asset holdings from $h_{i,t}$ to $h_{i,t+1}$, where $\phi(\cdot)$ is assumed to be convex with $\phi\left((1-\delta)h_{i,t}, h_{i,t}\right) = 0$ and $\phi(\cdot) \geq 0$, such that adjustment costs are paid only when housing stock is adjusted by an amount other than its rate of deprecation, given by the parameter $\delta \in (0,1)$. The right-hand side of the budget constraint represents household income. Expenditures are financed using the gross real return from previously invested, risk-free, bank deposits, $(1+r_t^d)d_{i,t}$, housing assets which may be sold to fund purchases, $p_t^h(1-\delta)h_{i,t}$, fiscal transfers τ_t, idiosyncratic profit payments, $\pi_{i,t}$ (discussed further in section 3.4.7), and finally labour income, $w_t\zeta_{i,t}\ell_{i,t}$, which itself depends on an idiosyncratic productivity level, $\zeta_{i,t}$ and the real wage w_t. A subset of households, who will be called 'retired', receive an idiosyncratic productivity level $\zeta_{i,t} = 0$, such that their optimal decision will be to supply no labour.

Households face two further constraints on their choices: a collateral constraint for bank deposits and a liquidity constraint for housing stock:

$$(1 + r^d_{t+1})d_{i,t+1} \geq -\Phi^d p^h_{t+1}(1-\delta)h_{i,t+1}, \tag{3.3}$$

$$h_{i,t+1} \geq 0, \tag{3.4}$$

When binding, these constraints limit household portfolio choices. The first constraint, (3.3), states that household banking debt must be collateralised by those housing assets available to make repayments in future periods. Φ^d therefore represents the fraction of housing assets available as collateral to household borrowing. In this way, whenever household bank deposits are negative it may be taken to represent an adjustable-rate bank loan, renewed each period at the interest rate r^d_{t+1}. Large bank loans are therefore similar to adjustable-rate mortgages (ARMs).[17] A restriction $0 < \Phi^d < 1$ is used to guarantee households may always fully repay banking debt. The calibration of Φ^d directly controls how frequently the bank deposit collateral constraint binds. In this setting, idiosyncratic labour income risk prevents uncertain future labour being used as collateral for borrowing. The second constraint, (3.4), normalises the minimum level of permissible housing stock. Taken together, incomplete financial markets therefore introduce a precautionary saving motive for heterogeneous households who seek to self-insure against adverse labour productivity shocks.

At this stage it is useful to represent the real value of household net wealth, $x_{i,t}$, at the start of period t as:

$$x_{i,t} \equiv (1 + r^d_t)d_{i,t} + p^h_t(1-\delta)h_{i,t}. \tag{3.5}$$

So, suppressing indexation for each individual, the household's problem may be rewritten in recursive form, with the Bellman equation:

$$v(x,h,\zeta) = \max_{c,\ell,x',h'} \Big\{ u(c,h,1-\ell) + \beta\mathbb{E}[v(x',h',\zeta')] \Big\}, \tag{3.6}$$

subject to the constraints:

$$c + d' + p^h h' + \phi(h',h) = x + \tau + \pi + w\zeta\ell, \tag{3.7}$$

$$x = (1 + r^d)d + p^h(1-\delta)h, \tag{3.8}$$

$$x' \geq (1-\Phi^d)p^{h'}(1-\delta)h', \tag{3.9}$$

$$h' \geq 0, \tag{3.10}$$

[17] In this model, following Sterk and Tenreyro (2018) and Broer et al. (2020), but unlike Kaplan et al. (2018) and Lüetticke (2021), there is no exogenous household credit spread as households may borrow and save at the the same interest rate, r^d_{t+1}. Including such a credit spread will not alter the main results.

For computational purposes, it is important to note the household's state variables. For household i these consist of: total asset holdings, $x_{i,t}$, and the composition of this in the form of deposits, $d_{i,t}$, and illiquid housing assets, $h_{i,t}$; the idiosyncratic productivity level, $\zeta_{i,t}$; and the set of aggregate price and quantity variables. The cross-sectional joint distribution of asset holdings and productivity is also a state variable, and can be represented by the cumulative distribution function, $F_t(\zeta_{i,t}, d_{i,t}, h_{i,t})$. Shocks to idiosyncratic labour productivity are assumed to be persistent, and are discussed further below, in section 3.5.1.

3.4.2 Firms

Production technology is assumed to be symmetric across the consumption and housing services markets. Firms are perfectly competitive and have access to a linear production function in labour. In terms of the consumption goods price, the representative firms' problems are therefore:

$$\max_{n_t^c} z_t^c n_t^c - w_t n_t^c, \tag{3.11}$$

$$\max_{n_t^h} p_t^h z_t^h n_t^h - w_t n_t^h. \tag{3.12}$$

where z_t^c and z_t^h represent exogenous productivity levels for consumption and housing producers respectively, while n_t^c and n_t^h represent the level of labour used in the production of consumption and housing. While the term $z_t^c n_t^c$ in (3.11) is the production function used to produce the aggregate level of consumption goods and real costs from housing adjustment, $z_t^h n_t^h$ in (3.12) describes the production function used to produce the aggregate gross investment in the housing stock (equivalent to the change in aggregate housing stock, plus depreciation). This is stated explicitly in the equilibrium conditions (3.46) and (3.47) in section 3.4.8. The first order conditions for (3.11) and (3.12) determine both the real wage and relative price of housing as:

$$w_t = z_t^c, \tag{3.13}$$

$$p_t^h = \frac{z_t^c}{z_t^h}. \tag{3.14}$$

3.4.3 Government

Fiscal policy is not the focus of this paper, and a number of simplifying assumptions are made. The government focuses on debt issuance, as fiscal policy is formed without government spending, or distortionary taxation, with net transfers to households made on a

lump-sum basis.[18] The period t nominal government budget constraint can be written as:

$$A_{g,t+1} + V_t^B(B_{g,t+1} - B_{g,t}) + S_t = (1 + R_t^A)A_{g,t} + B_{g,t} + \mathcal{T}_t. \tag{3.15}$$

The fiscal authority has access to two financing options: (1) a one-period nominal coupon bond, $A_{g,t}$, with coupon rate R_t^A, and (2) an infinitely-lived (perpetual annuity) nominal consol, $B_{g,t}$, which pays one unit of currency in every time period. The current price of the one-period bond is taken as the numéraire, while the end-of-period (post-coupon) value of a consol is V_t^B. Newly issued consols, which start paying coupons in the next period, have the same market value as the post-coupon value of those issued previously, such that the value of new consol issuance is given as the second term in (3.15). The budget constraint, (3.15), states that the value of new government debt issuance and the transfer income received from the central bank, S_t (discussed in the next subsection), must equal the value of outstanding debt obligations and nominal value of fiscal transfers made to households, $\mathcal{T}_t$. This budget constraint, (3.15), can be re-written in real terms:

$$\frac{A_{g,t+1}}{P_t} + \frac{V_t^B B_{g,t+1}}{P_t} + \frac{S_t}{P_t} = \frac{(1 + R_t^A)P_{t-1}}{P_t}\frac{A_{g,t}}{P_{t-1}} + \frac{(1 + V_t^B)P_{t-1}}{P_t V_{t-1}^B}\frac{B_{g,t}V_{t-1}^B}{P_{t-1}} + \frac{\mathcal{T}_t}{P_t}, \tag{3.16}$$

$$a_{g,t+1} + b_{g,t+1} + s_t = (1 + r_t^a)a_{g,t} + (1 + r_t^b)b_{g,t} + \tau_t, \tag{3.17}$$

where P_t is the price of the aggregate consumption good[19] and (3.17) uses the following definitions of the real value and one-period real rate of return for newly issued one-period

[18] The analysis here closely follows Harrison (2012) and Sterk and Tenreyro (2018), who explore this exposition in greater detail.

[19] For the formulation discussed in section 3.5 this is $P_t = \frac{(p_t^h)^{1-\theta}}{\theta^\theta(1-\theta)^{1-\theta}}$, where θ captures the expenditure share of consumption, up to the presence of the collateral constraint.

coupon bonds and consols, real fiscal transfers and the real value of seigniorage:

$$a_{g,t+1} \equiv \frac{A_{g,t+1}}{P_t}, \tag{3.18}$$

$$1 + r_t^a \equiv \frac{(1 + R_t^A)P_{t-1}}{P_t}, \tag{3.19}$$

$$b_{g,t+1} \equiv \frac{V_t^B B_{g,t+1}}{P_t}, \tag{3.20}$$

$$1 + r_t^b \equiv \frac{(1 + V_t^B)P_{t-1}}{P_t V_{t-1}^B}, \tag{3.21}$$

$$\tau_t \equiv \frac{\mathcal{T}_t}{P_t}, \tag{3.22}$$

$$s_t \equiv \frac{S_t}{P_t}. \tag{3.23}$$

In this way, the characterisation of long-term government debt as consols ensures that the rate of return depends only on the change in their relative price over a single time period. In real terms, the government budget constraint (3.17) states that government payments on previously issued debt along with fiscal transfers to households are funded either through new government debt issuance or via remittance income from the central bank.

Implicit in this representation of central government constrained behaviour is a coordination with the central bank, such that the central government does not simply offset changes in the relative holdings of the central bank's assets by altering its debt issuance, a point particularly highlighted in Kaplan et al. (2018). This assumption is in line with actual practise, as debt management offices do not typically alter debt issuance as a result of temporary monetary policy decisions. To ensure this holds, when solving the model, the nominal supply of each asset, $A_{g,t+1}$ and $B_{g,t+1}$, will be calibrated at constant values to match observed data.

3.4.4 Central Bank

Although no frictions are assumed to exist between the central bank and Treasury, a conceptual distinction is made between the fiscal and monetary authorities. The central bank's balance sheet is now modelled explicitly. Each period the central bank issues nominal reserve balances, $M_{s,t+1}$. As the economy is cashless, reserve balances also represent the measure of base money. Against this liability, the central bank holds one-period liquid government nominal coupon bonds. It holds a proportion, $\lambda_{t+1}^a \in (0, 1)$, of the total supply. The central bank also holds a proportion, $\lambda_{t+1}^b \in (0, 1)$, of the total supply of government issued infinitely-

lived nominal consols.[20] The central bank is assumed to hold no other assets while central bank capital is assumed to be 0. Furthermore, any net income is remitted to the treasury at the start of the period while government balances are also transferred to households.[21] In nominal terms, at the start of period t the value of the central bank balance sheet can therefore be written as:

$$M_{s,t} = \lambda_t^a A_{g,t} + \lambda_t^b V_{t-1}^B B_{g,t}. \tag{3.24}$$

Holding prices and government debt issuance constant, an expansion of the monetary base is associated with an increase in either λ_{t+1}^a or λ_{t+1}^b.

The central bank earns income on assets held from the previous period, and pays a nominal rate of return, R_t^M, on central bank issued reserve balances. Each period, I assume the central bank first liquidates its entire portfolio and then purchases assets and issues nominal reserve balances for the next period.[22] As a result of this process all central bank net income (or loss) is assumed to be given directly to the fiscal authority as seigniorage payments, S_t, which can then be defined as:

$$\begin{aligned} S_t = & \underbrace{(1+R_t^A)\lambda_t^a A_{g,t}}_{\text{Gross revenue of bonds}} + \underbrace{(1+V_t^B)\lambda_t^b B_{g,t}}_{\text{Market value of consols, coupon and capital gain}} \\ & - \underbrace{(1+R_t^M - \hat{M}_{s,t+1})M_{s,t}}_{\text{Expenses due on reserve balances and balance sheet change}}, \end{aligned} \tag{3.26}$$

where $\hat{M}_{s,t+1}$ is the percentage change in nominal central bank issued reserve balances:

$$\hat{M}_{s,t+1} \equiv \frac{M_{s,t+1} - M_{s,t}}{M_{s,t}}. \tag{3.27}$$

Central bank seigniorage payments may be rewritten in real terms:

$$s_t = (r_t^a - r_t^m)\lambda_t^a a_{g,t} + (r_t^b - r_t^m)\lambda_t^b b_{g,t} - \mu_t m_{s,t}, \tag{3.28}$$

[20] It can be assumed that a practical upper bound exists for both λ_{t+1}^a and λ_{t+1}^b far below 1. Prior to 2019, the Bank of Japan were closest to reaching a practical upper limit, with an asset purchase programme twice the size of new government issuance, though still held only 40% of the total outstanding stock of Japanese Government Bonds. Since then many countries have extended their LSAP programmes holding ever-higher proportions of overall supply. $\lambda_{t+1}^a > 0$ and $\lambda_{t+1}^b > 0$ are assumed to ensure $M_{s,t+1} > 0$.

[21] As government balances begin each period at 0, and the government pays obligations on existing debt before receiving income, I assume the government may run an interest-free negative balance at the central bank, an overdraft, within each period.

[22] This assumption is made for analytical simplicity. Assuming the central bank remits its net income flow:

$$S_t = R_t^A \lambda_t^a A_{g,t} + \lambda_t^b B_{g,t} - R_t^M M_{s,t} + (M_{s,t+1} - M_{s,t}), \tag{3.25}$$

results in the same steady state as with (3.26).

where (3.24), (3.34) and the following definitions, consistent with (3.18) to (3.22), are used for real seigniorage, s_t, the real value of central bank issued reserve balances, $m_{s,t}$, and their real rate of return, r_t^m and μ_t:

$$s_t \equiv \frac{S_t}{P_t}, \tag{3.29}$$

$$m_{s,t+1} \equiv \frac{M_{s,t+1}}{P_t}, \tag{3.30}$$

$$1 + r_t^m \equiv \frac{(1 + R_t^M)P_{t-1}}{P_t}, \tag{3.31}$$

$$\mu_t \equiv \frac{\hat{M}_{s,t+1}P_{t-1}}{P_t}, \tag{3.32}$$

$$\mu_t m_{s,t} = \frac{M_{s,t+1} - M_{s,t}}{P_t}. \tag{3.33}$$

The final term in (3.28), $\mu_t m_{s,t}$, which is simplified in (3.33) captures the standard seignorage term from an expansion of base money which is paid to the Treasury. Two other sources of seigniorage revenue arise in (3.28) as a result of differences between the rate of return on central bank base money and either one-period nominal coupon bonds or government consols. For a given balance sheet size, seigniorage payments can always be made positive if the nominal rate of return on central bank issued reserve balances is sufficiently small.

Assuming a constant supply of assets, $A_{g,t}$ and $B_{g,t}$; constant asset prices; and a constant aggregate price level, three different stylised central bank policies are now considered:

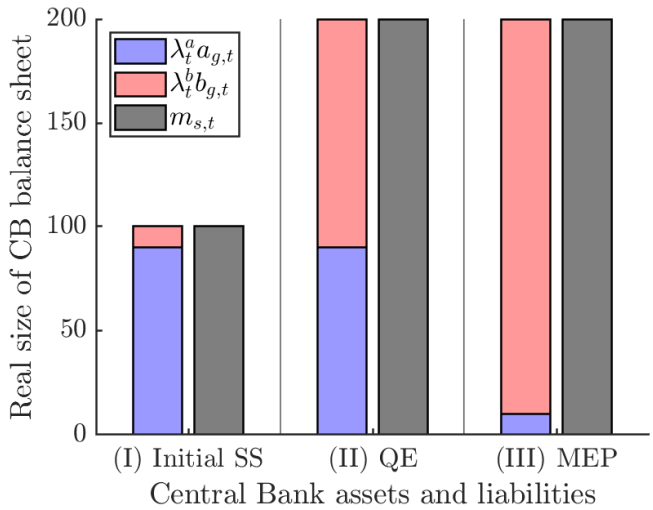
1. In an initial steady state (SS), the central bank holds a constant fraction of both one-period liquid real government bonds and illiquid assets. In addition, the central bank's supply of base money is unchanged. The real size of the balance sheet is therefore given as:

$$\underbrace{m_{s,t}}_{\text{Liabilities}} = \underbrace{\lambda_t^a a_{g,t} + \lambda_t^b b_{g,t}}_{\text{Assets}}. \tag{3.34}$$

   A stylised example of this is shown in the panel (I) of Figure 3.3, where the real size of the balance sheet in this initial steady state is normalised to 100. The contribution of illiquid assets to the central bank's balance sheet is small, with $\lambda_t^a a_{g,t} \gg \lambda_t^b b_{g,t}$. The assumptions of this policy for monetary policymakers' decision variables are shown in the first column of Table 3.5.

2. Under a Quantitative Easing programme (QE), the central bank purchases a large amount of illiquid assets, funded by creation of additional base money. Holdings of liquid assets are unchanged. An example of this can be seen by moving from the panel (I) to (II)

Figure 3.3 Central Bank LSAP Programmes



Notes: Figure shows real assets ($\lambda_t^a a_{g,t} + \lambda_t^b b_{g,t}$) and liabilities ($m_{s,t}$) of the central bank's balance sheet in three stylised scenarios. (I) In the initial steady state (SS) the central bank holds predominately liquid government assets, $\lambda_t^a a_{g,t} \gg \lambda_t^b b_{g,t}$. (II) Under a Quantitative Easing programme (QE) the central bank purchases illiquid assets, $\lambda_{t+1}^b b_{g,t+1}$, expanding the base money supply, $m_{s,t+1}$. (III) Under a Maturity Extension Programme (MEP) the central bank alters the composition of the asset side of the balance sheet, purchasing illiquid assets and selling liquid assets. Liabilities are unchanged after a MEP.

of Figure 3.3, where the real size of the balance sheet doubles, with the increase on the asset-side of the balance sheet entirely due to the increase in illiquid assets. The assumptions of this policy for monetary policymakers' decision variables are shown in the second column of Table 3.5.

3. Under a Maturity Extension Programme (MEP), the central bank restructures the asset side of its balance sheet, without increasing the overall size. The central bank's liquid asset holdings fall to offset an increase in illiquid assets (a sterilised open market operation). As a result of this programme $\lambda_{t+1}^a a_{g,t+1} \ll \lambda_{t+1}^b b_{g,t+1}$. This can be seen by moving from panel (II) to (III) in Figure 3.3. The implications of this policy for monetary policymakers' decision variables are shown in the final column of Table 3.5.

Table 3.5 Assumptions for Central Bank LSAP Programmes

Steady State	QE	MEP
$\lambda_{t+1}^a = \lambda_t^a$	$\lambda_{t+1}^a = \lambda_t^a$	$\lambda_{t+1}^a < \lambda_t^a$
$\lambda_{t+1}^b = \lambda_t^b$	$\lambda_{t+1}^b > \lambda_t^b$	$\lambda_{t+1}^b > \lambda_t^b$
$m_{s,t+1} = m_{s,t}$	$m_{s,t+1} > m_{s,t}$	$m_{s,t+1} = m_{s,t}$

3.4.5 Retail Banking Sector

The banking sector is modelled simply, with the problem stated as follows. A continuum of retail banks operate under prefect competition, taking prices as given. Following Andrés and Arce (2012) and Li (2020), all bank profits are assumed to be distributed as dividends to households each period. Additionally, there is no bank capital and therefore each retail bank has an start-of-period-t balance sheet given by:

$$a_{b,t} + m_{b,t} = d_{b,t}. \tag{3.35}$$

which states that, in real terms, retail banks hold assets consisting of liquid one-period government coupon bonds, $a_{b,t}$, and central bank issued reserve balances, $m_{b,t}$. These retail banks also hold deposits received from households, $d_{b,t}$, on their balance sheet which, given the lack of a credit spread in this model, are taken to be net of bank loans for analytical convenience.

In each period-t, retail banks interest make payments, in real terms, to their existing depositors, $r_t^d d_{b,t}$, make dividend payments to households, $\pi_{b,t}$, purchase new one-period government coupon bonds, $a_{b,t+1}$, and may increase their deposits held at the central bank, $(m_{b,t+1} - m_{b,t})$. Similarly, inflows to retail banks consist of adjustments made by households to increase their net deposit holdings, $(d_{b,t+1} - d_{b,t})$, the gross payments from existing one-period government bonds, $(1 + r_t^a)a_{b,t}$, and interest paid on money held at the central bank, $r_t^m m_{b,t}$. Assuming costless financial intermediation and no default possibility, the budget constraint for retail banks may be written as:

$$r_t^d d_{b,t} + \pi_{b,t} + a_{b,t+1} + (m_{b,t+1} - m_{b,t}) = (d_{b,t+1} - d_{b,t}) + (1 + r_t^a)a_{b,t} + r_t^m m_{b,t}. \tag{3.36}$$

Alternatively, the real value of period-t profits, the difference between income and expenditure, may be written as:

$$\pi_{b,t} = (1 + r_t^a)a_{b,t} + (1 + r_t^m)m_{b,t} - (1 + r_t^d)d_{b,t} - a_{b,t+1} - m_{b,t+1} + d_{b,t+1}. \tag{3.37}$$

Under perfect competition, each retail bank's period-t problem is to choose the amount invested in each asset, $a_{b,t+1}$ and $m_{b,t+1}$, as well as the size of the balance sheet, via its choice of $d_{b,t+1}$, in each period to maximise its franchise value (the expected present discounted value of profits):

$$\max_{\{a_{b,j}, m_{b,j}, d_{b,j}\}_{j=t+1}^{\infty}} = \mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} \pi_{b,j}. \tag{3.38}$$

subject to the balance sheet identity (3.35), where β is the same discount factor used by households. Using (3.35) and (3.37), the period-t profit function may be simplified to:

$$\pi_{b,t} = (r_t^a - r_t^d)a_{b,t} + (r_t^m - r_t^d)m_{b,t}. \tag{3.39}$$

Hence the first order conditions of (3.38) with respect to $a_{b,t+1}$ and $m_{b,t+1}$ respectively are:

$$0 = \beta\mathbb{E}_t[r_{t+1}^a - r_{t+1}^d], \tag{3.40}$$

$$0 = \beta\mathbb{E}_t[r_{t+1}^m - r_{t+1}^d], \tag{3.41}$$

as $\beta > 0$ these imply, under perfect competition, a zero interest rate margin for each asset is expected to be earned by banks and hence $\mathbb{E}_t[\pi_{b,t}] = 0$. The model contains no aggregate uncertainty and therefore expectations operators may be dropped from (3.40) and (3.41).

3.4.6 Mutual Fund

The primary role of the mutual fund is to provide a market for illiquid government assets. A representative mutual fund holds infinitely-lived (perpetual annuity) nominal government consols and pays lump-sum dividend payments to retired households. Each household owns one share of the mutual fund. This gives a claim to payments once the household retires. These shares may not be traded and are therefore not in the household budget constraint (3.2). This set-up is equivalent to a state provided pension fund.

The mutual fund has been provided an initial level of illiquid government assets, $b_{m,t}$. It uses the proceeds of this resource to make payments and must decide how many units of the long-term consol to purchase next period and how many lump-sum payments to make to retired households. Therefore, the mutual fund faces a real budget constraint given by:

$$\pi_{m,t} + b_{m,t+1} = (1 + r_t^b)b_{m,t}. \tag{3.42}$$

I assume the mutual fund smooths the flow of lump-sum dividend payments to retired households with the following objective function:

$$\max_{\{\pi_{m,j}, b_{m,j+1}\}_{j=t}^{\infty}} = \mathbb{E}_t \sum_{j=t}^{\infty} \beta^{j-t} \frac{(\pi_{m,t})^{1-\sigma} - 1}{1-\sigma}, \tag{3.43}$$

subject to (3.42), where $\sigma > 0$ represents an inverse elasticity of intertemporal substitution.[23] The first order condition of this problem is:

$$1 = \beta \mathbb{E}_t \left[\left(1 + r^b_{t+1}\right) \left(\frac{\pi_{m,t+1}}{\pi_{m,t}}\right)^{-\sigma} \right]. \tag{3.44}$$

So, the extent of payment smoothing depends critically upon the expected path of the effective interest rate for illiquid government assets, r^b_{t+1}. Again, the expectations operator may be dropped from (3.44) due to the lack of aggregate uncertainty.

3.4.7 Profit Payments

To close the model a profit payment scheme is required which maps ownership of retail banks and mutual fund equity capital to individual households. Each period, profit payments from retail banks and the mutual fund to households are distributed in the following two-part form:

$$\pi_{i,t} = \begin{cases} \pi_{b,t} + \pi_{m,t} \left(\int_{\zeta_{i,t}=0} 1 dF_t \right)^{-1} & \text{if} \quad \zeta_{i,t} = 0, \\ \pi_{b,t} & \text{otherwise.} \end{cases} \tag{3.45}$$

So, mutual fund profits are received as pension payments by households in their retirement state with $\zeta_{i,t} = 0$ (appropriately scaled for this population sub-group), while household ownership of bank capital is spread evenly throughout the population such that each household receives an equal share of profits in each period. In equilibrium households receive no income, either ex ante or ex post, from this latter source due to perfect competition in the retail banking sector.

3.4.8 Equilibrium Definition

In each time period, a recursive stationary equilibrium is defined as a set of policy functions, $c_{i,t}(x_{i,t}, h_{i,t}, \zeta_{i,t})$, $\ell_{i,t}(x_{i,t}, h_{i,t}, \zeta_{i,t})$, $x_{i,t+1}(x_{i,t}, h_{i,t}, \zeta_{i,t})$ and $h_{i,t+1}(x_{i,t}, h_{i,t}, \zeta_{i,t})$; set of household-specific quantities $\tau_{i,t}$, $d_{i,t+1}$ and $\zeta_{i,t}$; real returns r^a_t, r^m_t, r^d_t and r^b_t; aggregate prices w_t and p^h_t; and aggregate quantities, n^c_t, n^h_t, $a_{g,t}$, $a_{b,t}$, $b_{g,t}$, $m_{s,t}$, $m_{b,t}$, $d_{b,t}$, $\pi_{b,t}$, $\pi_{m,t}$ and s_t; such that:

1. Households' policy functions maximise utility (3.1) subject to (3.2), (3.3) and (3.4), given any aggregate pricing vector.
2. Aggregating across these household policy functions generates feasible allocations which simultaneously clear all seven markets at the equilibrium aggregate pricing

[23] This construction follows the money market mutual fund in Lee (2021), though in that setting the fund smooths holdings of liquid assets.

vector, such that:

$$z_t^c n_t^c = \int_i c_{i,t} + \phi(h_{i,t+1}, h_{i,t}) dF_t, \tag{3.46}$$

$$z_t^h n_t^h = \int_i h_{i,t+1} - (1-\delta) h_{i,t} dF_t, \tag{3.47}$$

$$d_{b,t} = \int_i d_{i,t+1} dF_t, \tag{3.48}$$

$$a_{g,t} = a_{b,t} + \lambda_t^a a_{g,t}, \tag{3.49}$$

$$b_{g,t} = b_{m,t} + \lambda_t^b b_{g,t}, \tag{3.50}$$

$$m_{s,t} = m_{b,t}, \tag{3.51}$$

$$n_t^c + n_t^h = \int_i \zeta_{i,t} \ell_{i,t} dF_t, \tag{3.52}$$

respectively defining equilibrium conditions in the markets for goods, (3.46), housing services, (3.47), retail deposits, (3.48), one-period government bonds, (3.49), government consols, (3.50), central bank reserves, (3.51), and labour, (3.52).

3. The distribution of productivity and asset holdings across households, $F_t(\zeta_{i,t}, d_{i,t}, h_{i,t})$, is invariant.

Equilibrium is therefore determined as the solution to the recursive system (3.6) to (3.10), alongside the two first order conditions from firms (3.13) and (3.14); the government budget constraint in real terms, (3.17); the central bank's level of seigniorage, (3.28), and balance sheet, (3.34); the retail bank's balance sheet, (3.35), profit function, (3.37), and first order conditions for asset holdings (3.40) and (3.41); the mutual fund's budget constraint, (3.42), and first order condition, (3.44); the household's profit transfer payment schedule, (3.45); and the market clearing conditions (3.46) to (3.51). In this case the labour market will clear according to (3.52) by Walras' law, as shown for a steady state in appendix B.3.3. The full model is presented and solved for an aggregate steady state in Appendix B.3.

3.5 Calibration & Computation

Before the model can be solved, a functional form of the period-t felicity function is chosen:

$$u(c_{i,t}, h_{i,t}, 1-\ell_{i,t}) = \frac{\left(c_{i,t}^{\theta} h_{i,t}^{1-\theta} + \frac{\chi \zeta_{i,t} \left(1-\ell_{i,t}\right)^{1-\eta} - 1}{1-\eta}\right)^{1-\sigma} - 1}{1-\sigma}, \tag{3.53}$$

where the coefficient of relative risk aversion, $\sigma > 0$, is chosen to ensure strict convexity of marginal utility with respect to consumption such that prudence is positive, ensuring

households display precautionary saving behaviour. The inverse of the Frisch elasticity of labor supply is η, and χ is a parameter which scales the equilibrium level of hours worked. One aim of choosing this form of the household utility function, common across much of this literature, e.g. Lüetticke (2021) and Lee (2021), is to focus analysis on financial transmission and balance sheet effects. In addition, this normalisation of the utility trade-off between consumption and leisure greatly reduces the computational burden, as households of every (positive) productivity type each supply the same number of hours worked in equilibrium according to these Greenwood et al. (1988)-style preferences. The parameter θ captures the expenditure share of consumption, up to the presence of the collateral constraint, with an elasticity of substitution between consumption and housing services which is assumed to be 1, following Ríos-Rull and Sanchez-Marcos (2008), Chambers et al. (2009), Iacoviello and Pavan (2013), Favilukis et al. (2017) and others.[24]

The functional form of the real housing adjustment costs is taken from the investment literature (Kiyotaki and West, 1996; Thomas, 2002) as:

$$\phi(h_{i,t+1}, h_{i,t}) = \frac{\phi}{2}\left(\frac{h_{i,t+1} - (1-\delta)h_{i,t}}{h_{i,t}}\right)^2 h_{i,t}, \tag{3.54}$$

where $\phi > 0$. This form ensures households do not pay adjustment costs when their housing stock only falls due to depreciation.

3.5.1 Calibration

The model calibration procedure occurs in three parts. The first sets parameters equal to those generally found in the literature, where possible. A second set of parameters are set equal to their natural empirical counterparts. A final set of parameters are calibrated targeting moments of the initial wealth distribution, closely following the procedure outlined in Castañeda et al. (2003). Table 3.6 summarises the baseline calibration, with parameters of particular interest or sensitivity to the results described in greater detail below. The model is parameterised at an annual frequency with GDP normalised to 1 in the initial steady state, with GDP-ratios equal to aggregate variables.

[24]Several studies attempt to estimate the elasticity of housing substitution with little consensus (Piazzesi and Schneider, 2016). Macroeconomic studies generally find a parameter value consistent with housing and non-housing consumption being substitutes, while studies at the household level find these distinct components of consumption to be complements. For example, Davis and Martin (2009) provide estimates using aggregate data and Li et al. (2016) using household-level data. For this reason most articles use assume an elasticity of 1 and investigate properties around this. Davis and Ortalo-Magné (2011) show the expenditure share of housing services to be stable over both time and metropolitan statistical areas in the United States.

Table 3.6 Baseline Parameter Calibration

Description	Parameter	Value	Target
Literature			
Time-discount factor	β	0.98	Standard value
Coefficient of relative risk aversion	σ	1.00	Standard value
Utility coefficient on leisure	χ	12.48	Guerrieri and Lorenzoni (2017)
Inverse Frisch elasticity	η	1.5	Guerrieri and Lorenzoni (2017)
Housing expenditure share†	$1-\theta$	0.30	Favilukis et al. (2017)
Housing deprecation rate	δ	2.5%	Favilukis et al. (2017)
Collateralisable fraction of housing stock	Φ^d	0.95	Hintermaier and Koeniger (2010)
Housing adjustment cost coefficient	ϕ	0.05	Hintermaier and Koeniger (2010)
Empirical			
Government real supply of one-period bonds	$a_{g,t}$	0.07	SIFMA issuance data
Initial real value of government consols	$b_{g,t}$	0.20	SIFMA issuance data
Aggregate productivity for consumption sector	z_t^c	1.43	$w_t(n_t^c + n_t^h) = 1$, in initial steady state
Aggregate productivity for housing services	z_t^h	1.43	Symmetry across sectors

†For households for whom the collateral constraint does not bind.

The majority of the parameters calibrated to those found in the literature are standard, with the labour parameters calibrated to match Guerrieri and Lorenzoni (2017) and housing parameters to match Favilukis et al. (2017) and Hintermaier and Koeniger (2010).

Among the most important empirically estimated variables are the aggregate levels of the exogenous short-term, $a_{s,t}$, and long-term, $b_{s,t}$, real asset supply. Data on the size of the outstanding US Treasury market are taken from the Securities Industry Financial Markets Association (SIFMA). Short-term, liquid government assets are taken to be US Treasury Bills. For the 10-year period prior to the financial crisis, between 1996 and 2006, these averaged 7% of nominal GDP. Long-term assets are taken to be Treasury Notes and Bonds (assets with an initial maturity of at least 2 years). An analogous exercise shows these long-term assets average 20% of nominal GDP in the 10 years prior to the financial crisis.

Policy variables for the central bank holdings of short-term, λ_t^a, and long-term, λ_t^b, government securities are calibrated by combining the detailed US Treasury security issuance composition data, taken from SIMFA, with Federal Reserve balance sheet data from the Federal Reserve's Flow of Funds accounts at a quarterly frequency. As the Flow of Funds data also decomposes US Treasury holdings by US Treasury Bills and other US Treasury securities, we may match Federal Reserve holdings to both short-term and long-term outstanding US Treasury securities. The Flow of Funds also details the level of each security held outside the United States. As the model presented in section 3.4 assumes a closed economy, these US

Treasury holdings are subtracted from the overall amount outstanding, leaving the calibrated values of λ_t^a and λ_t^b as the ratio of Federal Reserve holdings to the outstanding, domestically held, stocks of these short- and long-term US Treasuries. The results of this exercise are shown in Table 3.7. The impact of individual LSAP programmes may then be evaluated, with the overall impact taken as the difference between the initial (pre-QE1) steady state and the final (post-QE3) steady state.

Table 3.7 Policy Calibration

Policy	Date	λ_t^a	λ_t^b
Pre-QE1 (Initial)	Q4 2008	0.17	0.09
Post-QE2	Q2 2011	0.10	0.25†
Pre-MEP	Q3 2011	0.09	0.23†
Post-MEP	Q4 2012	0.01	0.24
Pre-QE3	Q3 2012	0.01	0.24
Post-QE3 (Final)	Q4 2014	0.01	0.31

Sources and Notes: SIMFA and US Flow of Funds. λ_t^a and λ_t^b respectively refer to the proportion of domestically held short-term and longer-term US government asset supply held by the Federal Reserve. QE denotes Quantitative Easing while MEP denotes the Maturity Extension Programme.
†The change in λ_t^b between Q2 and Q3 2011 primarily reflects long-term issuance which increased $b_{g,t}$

The three time periods shown in Table 3.7 cover several Federal Reserve LSAPs. Between Q4 2008 and Q2 2011, the Federal Reserve undertook two Quantitative Easing (QE) policies, QE1 and QE2. The combined impact of these policies was to raise the overall size of the Federal Reserve's balance sheet. The proportion of the outstanding stock of longer-term domestically held US Treasuries, λ_t^b, increased from 9% to 25%. During the same period the Federal Reserve reduced the proportion of outstanding stock of domestically held short-term US Treasuries on its balance sheet, λ_t^a, from 17% to 10%. The reduction in λ_t^a primarily reflects the natural evolution of bond holdings, as fewer short-term, liquid, bonds were renewed as they matured during the three year period of this LSAP. The Federal Reserve's Maturity Extension Programme (MEP) occurs in the middle of the period studied, between Q3 2011 and Q4 2012. During this programme the Federal Reserve reduced its holding of short-term assets, replacing them with longer-term assets. This results in a reduction in λ_t^a, while λ_t^b increases. The increase in λ_t^b is far less than the fall in λ_t^a as the outstanding long-term debt stock is higher than for short-term Treasury securities. Finally, during the third Quantitative Easing programme (QE3) the Federal Reserve dramatically increased its share of outstanding long-term Treasuries. This is reflected in the rise of λ_t^b from 24% in Q3 2012 to 31% in Q4 2014.

Income and Productivity Processes

The final empirically estimated parameters concern the idiosyncratic productivity process and have significant implications for the model solution, as individuals facing small, infrequent shocks have little incentive for precautionary saving behaviour. The calibration procedure for the income process therefore warrants special attention, as this is vital in achieving a realistic starting level for the distribution of household wealth, from which to investigate the impact of LSAPs. Key to delivering the required result is allowing households to have both the ability and desire to generate a distribution of household assets that greatly resembles the data. To this end, the calibration procedure captures two of the most important features of the income distribution:

1. Substantial cross-sectional heterogeneity. This is achieved through a persistence of different productivity levels between households.

2. A life-cycle structure of earnings, which increase rapidly towards middle age before falling back dramatically for households who retire at ages around 65.

This paper follows the literature (Shorrocks, 1976; Storesletten et al., 2004; Tauchen, 1986) in assuming household specific log-labour productivity follows a first order autoregressive process:

$$\ln \zeta_{i,t} = \rho \ln \zeta_{i,t-1} + \varepsilon_{i,t}. \tag{3.55}$$

This process is then discretised into four productivity states, $\zeta_{i,t}$, and a Markov transition matrix, $\Xi_{i,t,t+1}$, which is used to link them. Both the level of productivity in each state and the transition process are then used to target the distribution of household wealth following Castañeda et al. (2003), with the outcome parameters given in Table 3.8.[25] Households with the highest productivity represent a small proportion of the population (2.7% in equilibrium). Yet the risk faced by these households of a fall in their productivity is relatively larger than the remainder of the population, as these households have further to fall, without the possibility of a higher productivity state. This motivates substantial precautionary savings from these households, resulting in wealth inequality.

Alongside this productivity process for employed households, which satisfies the cross-sectional income heterogeneity criterion above, a life-cycle structure is also imposed. In this, households from each productivity state face a common probability of retirement, and following this death. As in Castañeda et al. (2003), these common probabilities are set to

[25] Although Shorrocks (1976) also suggests the dynamics of productivity (and income) could be matched more accurately using a second-order Markov chain, this paper follows the spirit of Heer and Maussner (2009) who argue this accuracy improvement does not justify the greater complexity.

Table 3.8 Discretised Idiosyncratic Productivity Process

Productivity State, $\zeta_{i,t}$	Transition Matrix, $\Xi_{i,t,t+1}$			
0.74	0.94	0.05	0.01	0.00
0.87	0.05	0.90	0.05	0.00
0.97	0.01	0.06	0.93	0.01
5.95	0.00	0.01	0.05	0.94

Notes: Table shows discretised version of income process given in (3.55) with four productivity states. As transition probabilities (i.e. the rows of matrix $\Xi_{i,t,t+1}$) are reported to two decimal places these may not sum to 1 due to rounding.

ensure an average working-life of 45 years and an average length of retirement of 18 years.[26] Following death, descendants enter the working population according to proportions governed by the non-stochastic steady state.

3.5.2 Computation

Determining a steady-state equilibrium for this model is computationally intensive, primarily because the numerical procedure requires a simultaneous solution for the household portfolio choice problem alongside aggregate equilibrium prices. The main elements of the algorithm are described in both Judd (1998) and Heer and Maussner (2009). The procedure involves three key stages for any given level and composition of the central bank's balance sheet.

In the first stage, aggregate prices are guessed by starting with the solution under complete financial markets. The household problem is then solved, and policy functions obtained through value function iteration. As this step is repeated multiple times for every iteration, improving the convergence speed of this step drastically reduces overall computational time. To this end, two computational tricks are used. Firstly, a policy function iteration accelerator block is added. Secondly, value function iteration uses an endogenous end-of-period grid for net wealth, given in (3.5), following the method outlined in Carroll (2006) and developed further in Hintermaier and Koeniger (2010).

The second stage requires convergence of the ergodic household distribution, F_t. When asset grids for deposits and illiquid securities are combined with the discretised idiosyncratic productivity variables, an exact probability measure can be constructed across households. When this probability density measure is combined with the household policy functions, Mat-

[26]This translates into a probability of retirment close to 2% for any given year of work and a probability of death close to a 6% for any given year of retirement.

lab's sparse matrix routine can be used to iteratively approximate the household distribution, F_t directly controlling numerical errors. Solving for the ergodic distribution is therefore direct, without simulation error.

In the final stage, a Bisection method is used to converge on the initial guess for the aggregate pricing vector, ensuring market clearing conditions are met.[27]

3.6 Results

In solving the model from section 3.4, policy functions are found for each household in the population. An example of these is shown in Figure 3.4.[28] These demonstrate how households with a higher level of net wealth both consume more (Figure 3.4, panel a) and save greater amounts into liquid financial deposits (Figure 3.4, panel b) and their illiquid housing stock (Figure 3.4, panel c). Overall households increase their net wealth holdings in each period, when working, as evidenced by the **black solid** line being above the 45 degree line in Figure 3.4, panel d. Upon retirement, policy functions shift from the **black solid** lines to the **blue dashed** ones. Retired households with the same level of net wealth as their working counterparts consume and invest less, as their pension income is below that earned when working. At low levels of net wealth, the marginal propensity to consume, both through the consumption good and via housing, increases dramatically. This is evidenced most clearly for retired households with low wealth levels by the steeper gradient in Figure 3.4, panels a and c.

The impact of LSAP programmes conducted by the Federal Reserve is now investigated in the next three subsections. The first accounts for the overall change in the wealth distribution as a result of the Federal Reserve LSAP programmes enacted between 2008 and 2014. The second subsection details the model-implied behaviour of the wealth distribution for individual LSAP policies. The final subsection compiles a series of robustness checks, altering key parameters from the theoretical model.

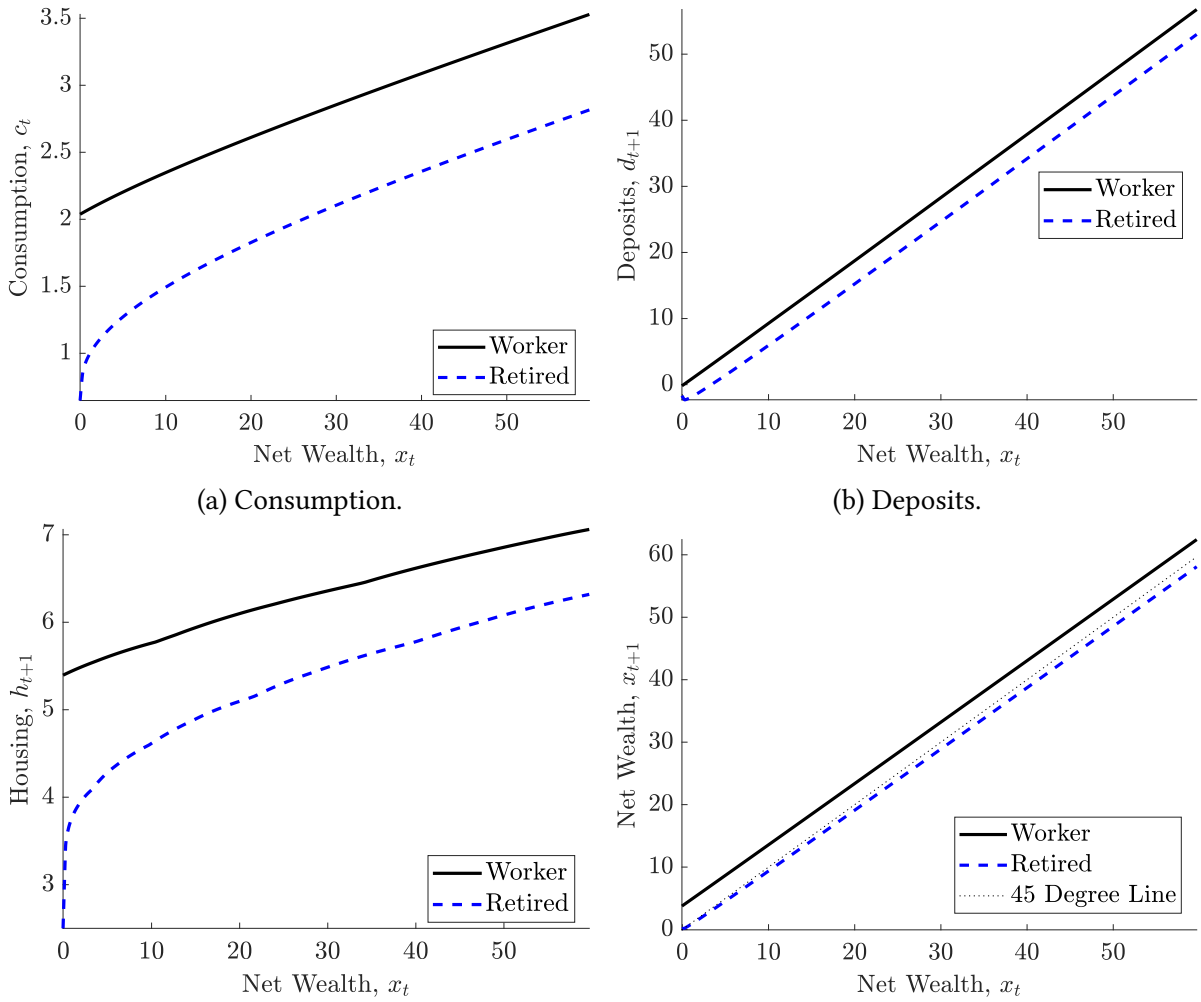
3.6.1 Overall Effect of US LSAPs

The primary result of this paper is that Federal Reserve LSAP programmes, in the aftermath of the global financial crisis, increased inequality of household net wealth, $x_{i,t}$, through financial transmission channels. Table 3.9 highlights the impact of US LSAPs on aggregate household variables by various LSAP program. The left-hand columns of Table 3.9 show

[27] Asset grids each contain 50 nodes. Value function iteration converges when the norm of the change in value function is below 1×10^{-9}. Similarly the distribution, F_t, is taken to have converged when the norm of change in the probability statistic falls below 1×10^{-9}. Bisection tolerance is set to 0.1% of supply along each dimension.

[28] A worker with the highest productivity level is chosen as, graphically, this gives the greatest difference between decisions when working and when in retirement.

Figure 3.4 Policy Functions.



(a) Consumption.

(b) Deposits.

(c) Housing.

(d) Net Wealth.

Notes: Baseline model, using calibrated parameters from Tables 3.6, 3.7 and 3.8. Equilibrium policy functions for the initial steady state prior to Federal Reserve LSAPs. Worker is of the highest productivity state while currently retired were previously workers of this same productivity state.

changes to the household balance sheet, while the right-hand columns show changes to net wealth inequality measures. The main result of the paper can therefore be seen in the right-hand columns of Table 3.9. The Gini coefficient and cross-sectional coefficient of variation both increase as a result of these asset purchase programmes. The total impact is substantial, with the Gini coefficient increasing by 3.8%, and the coefficient of variation by 2.9%.

Table 3.9 Impact of US LSAPs on Household Sector

	Aggregate Household Balance Sheet			Net Wealth Inequality Measures		
	Liquid Deposits	Illiquid Housing	Total	Gini Coefficient	Coefficient of Variation	90-20 Ratio
Pre-QE1 (Initial)	0.29	4.13	4.42	0.840	6.47	40.54
Post-QE2	0.40	4.05	4.45	0.855	6.58	41.52
Pre-MEP	0.39	4.05	4.44	0.855	6.57	41.52
Post-MEP	0.43	4.04	4.47	0.858	6.61	41.67
Pre-QE3	0.43	4.04	4.47	0.858	6.61	41.67
Post-QE3 (Final)	0.47	4.01	4.48	0.872	6.66	42.28
Total Change (%)	59.6	-2.8	1.3	3.8	2.9	4.3

Notes: Total percentage change is taken from initial to final.

The intuition for how LSAPs increase wealth inequality in this model is as follows. When enacting unanticipated quantitative easing, the central bank purchases additional long-term government bonds. Temporarily, this leads to an increase in long-term bond prices and a fall in their yield.[29] After these central bank asset purchases, the domestic household sector, via ownership of mutual funds, owns fewer long-run government bonds, consistent with the empirical evidence (Carpenter et al., 2015). The central bank thereby transforms long-term government assets into central bank base money, increasing the availability of short-term liquid assets available to banks. The higher monetary base is then passed on by banks to households, through a deposit creation and a bank lending channel. The impact upon household wealth inequality is twofold.

Firstly, as wealthier households have a larger marginal propensity to save, more of these additional bank deposits are purchased by wealthier households. This enables richer households to limit their use of the housing stock for consumption smoothing purposes. In all cases, a greater share of bank deposits, $d_{i,t}$, are held by wealthy households while less wealthy

[29] In steady state, the general equilibrium solution for the price of long-run consols is determined by the stochastic discount factor of the mutual fund, given in (3.44), which implies equilibrium interest rates are constant across steady states.

households use adjustments in their housing stock ownership, $h_{i,t}$, to absorb income shocks. However, with a greater availability of bank deposits this is exacerbated after LSAPs. Holding a greater share of wealth as liquid bank deposits ensures wealthier households face fewer real adjustment costs to rebalance their illiquid housing stocks, after income shocks. This increases wealth inequality.

After LSAPs, households hold additional bank deposits and fewer illiquid assets, which therefore places additional weight on the (less-equal) liquid asset distribution when computing overall net wealth inequality, than in the initial steady state. This represents a portfolio rebalancing effect where higher income households disproportionally move into liquid assets, increasing wealth inequality.

Secondly, as the mutual fund holds fewer assets, pension payments fall and total income inequality therefore rises. The cumulative impact of this increases wealth inequality. Additionally, knowledge of lower future pension payments encourages precautionary saving among current workers, further exacerbating the increase in wealth inequality.

In general, LSAPs have an ambiguous impact on the household net wealth distribution, which may also vary depending on the inequality measure used. As the Gini coefficient has received the most attention in existing literature, I focus analysis on this measure of net wealth inequality. In the baseline calibration the Gini coefficient increases by 3.8%. This arises as the 1.3% overall increase in the aggregate household balance sheet is disproportionally concentrated among wealthier households, who reallocate their portfolios away from housing towards holding financial assets for consumption smoothing purposes, while simultaneously holding the majority of liquid financial assets initially. This may also be inferred from the larger increase in the 90-20 net wealth ratio (ratio of wealth held by the 90th percentile of household net wealth distribution to that of the 20th percentile), which increases by 4.3% across all programmes, compared to other measure of net wealth inequality which place greater weight on observations in the middle of the net wealth distribution, such as the Gini coefficient and cross-sectional coefficient of variation, and hence increase by less overall.

One limitation of this modelling environment is evident when considering the collapse of the money multiplier which arose between 2007 and 2013 in the US, both as banks were unwilling to lend and as households and firms were reluctant to extend borrowing without repaying existing obligations (Arnold and Soederhuizen, 2018; Carpenter and Demiralp, 2012; Seghezza and Morelli, 2020). The focus throughout this paper is upon the impact of LSAPs as a policy tool, rather than a precise modelling of financial crises. This is particularly true for the model presented in section 3.4. In this setting, banks remain willing to lend while constrained households desire additional credit to smooth their consumption through time. For this reason, the model implies a counterfactual increase, by a substantial 59.6%, in the

household sector's liquid deposit holdings without a corresponding collapse in the money multiplier.

To further investigate the transmission of LSAPs in this economy we may consider how these policies impact sub-groups of the population. In particular, in this model, households may be split between workers and retirees. This decomposition is shown in Table 3.10. Working households hold over three times as many assets as retirees. In part this gap is explained by working households representing a greater share of the population, but the life cycle structure of the model also encourages households to save while young, and dissave during retirement. After the central bank implements LSAPs, working households experience a greater balance sheet expansion as their marginal propensity to save (for retirement) is greater than retirees. These households are also richer, on average, than the retiree group. However, after LSAPs both groups hold a greater level of net wealth. Net wealth among retirees is substantially more evenly distributed than workers, as measured by the Gini coefficient. This arises as, in the model, the only non-financial source of income for this group, pension transfers, is constant across all wealth levels in retirement. After the central bank implements LSAPs, net wealth inequality rises by more for retired households, as wealthy retired households increasingly rely upon (relatively small) income from liquid financial assets to support consumption during retirement, rather than pension transfers. Both the larger level of net wealth and greater extent of net wealth inequality in the working population, compared to retirees, are consistent with the 2007 SCF and the existing literature.[30]

Table 3.10 Differential Impact of US LSAPs on Workers and Retirees

	Aggregate Balance Sheet			Net Wealth Gini Coefficient		
	Workers	Retirees	Total	Workers	Retirees	Total
Pre-QE1 (Initial)	3.45	0.97	4.42	0.857	0.766	0.840
Post-QE3 (Final)	3.50	0.98	4.48	0.874	0.856	0.872
Total Change (%)	1.5	0.8	1.3	2.0	11.7	3.8

Notes: Total percentage change is taken from initial to final.

3.6.2 Effect by US LSAP programme

When the impact of specific Federal Reserve's LSAP programmes on household wealth inequality is investigated, higher net wealth inequality is found to arise predominantly through quantitative easing programmes. Increases were smaller during Federal Reserve's maturity

[30]For example as reported in Table 31 of Kuhn and Ríos-Rull (2016).

extension programme. For instance, only the third decimal place of the Gini coefficient changes during MEP, as reported in Table 3.9, demonstrating that even substantial changes in the composition have little effect on household net wealth inequality. Percentage changes in the measures of household net wealth inequality (the Gini coefficient, cross-sectional coefficient of variation and 90-20 wealth ratio) are reported in Table 3.11. Although each measure of net wealth inequality increases as a result of the Federal Reserve's MEP, as the Federal Reserve's balance sheet increased by a smaller magnitude during this programme, it's impact is correspondingly smaller, despite the compositional change in assets held by the central bank. In contrast, the QE programmes dominate the impact on wealth inequality, increasing these measures of household net wealth inequality.

Table 3.11 Change in Measures of Household Net Wealth Inequality During US LSAPs

	Gini Coefficient	Coefficient of Variation	90-20 Ratio
QE1 & QE2	1.8	1.7	2.4
MEP	0.3	0.6	0.4
QE3	1.6	0.8	1.5
Total Change	3.8	2.9	4.3

Notes: Reported as percentage changes taken from initial to final programme dates. Total represents the cumulative impact of all programmes and may not be exact sum due to rounding.

Though the change in the Gini coefficient is of a similar magnitude for both QE programmes, changes in the cross-sectional coefficient of variation and the 90-20 wealth ratio are substantially larger for the combined QE1 & QE2, than for QE3. This highlights how a broad overview of the changes in a range of measures is required when analysing the impact of LSAPs on wealth inequality, as each measure places different weights on different groups in the wealth distribution.

Similarly, the ordering of the magnitude of the change in these measures of household net wealth inequality is not constant across the LSAPs considered. For instance, although the cross-sectional coefficient of variation increases by the smallest magnitude overall, and in both QE episodes, this measure increased by more than the Gini coefficient and the 90-20 wealth ratio during MEP. This arises as, although MEP has little impact on the overall level of financial assets held by households in the economy, by selling additional liquid financial assets to households while purchasing long-term assets from the mutual fund, this programme encouraged households in the middle of the wealth distribution to hold additional assets for retirement, relatively more than QE programmes.

3.6.3 Robustness

In this subsection I briefly review some robustness checks around the key conclusions of the paper. To do this I re-estimate the model using key alternative parameter assumptions. The baseline results use parameters as described in section 3.5, while alternative scenarios each vary a single parameter of interest. The results of these exercises on the total change in household net wealth inequality are described in Table 3.12, with the baseline calibration presented in the first row. In each alternative scenario the broad conclusion of the paper remains, i.e. that Federal Reserve LSAPs increased household net wealth inequality, though the magnitude of this effect varies across scenarios and inequality measures. Two parameters are particularly important in determining the desire and ability of households to hold assets for precautionary purposes.

Table 3.12 Robustness of Impact of US LSAPs on Household Net Wealth Inequality

	Gini Coefficient	Coefficient of Variation	90-20 Ratio
Baseline	3.8	2.9	4.3
Higher coefficient of relative risk aversion: $\sigma = 1.2$	2.0	2.6	3.5
Smaller collateralisable fraction of housing stock: $\Phi^d = 0.8$	4.7	2.8	5.2
Higher housing adjustment cost coefficient: $\phi = 0.08$	2.5	3.3	7.6
Lower housing expenditure share: $1 - \theta = 0.25$	5.2	2.8	7.5

Notes: Baseline model uses calibrated parameters from Tables 3.6, 3.7 and 3.8. Table shows total percentage change taken from initial to final.

Firstly, when households are relatively more risk averse than in the baseline calibration, with $\sigma = 1.2$, the change in net wealth inequality is smaller for each measure. The choice of σ does not influence the sign of the estimated impact, though the Gini coefficient appears more responsive to this parameter than the other measures of net wealth inequality, falling from a 3.8% response to 2.0%. Under a scenario with greater risk aversion, every household desires a greater level of financial wealth to smooth consumption in the event of adverse income shocks. As a result, the initial level of net wealth inequality is lower than the baseline case, as reported by measures which place substantial weight on the middle portion of the net wealth distribution, such as the Gini coefficient and cross-sectional coefficient of variation. However, the initial 90-20 ratio is higher than the baseline, as households at the very top of the wealth distribution have a stronger prudence motive causing a greater change in precautionary savings. When households have a greater initial precautionary savings motive, changes to the central bank balance sheet have a more limited impact on wealth inequality.

The second line in Table 3.12 outlines the results from a scenario where a smaller fraction of the housing stock may be collateralised, with $\Phi^d = 0.8$. This impacts the ability of households to use housing to smooth income shocks. In this alternative scenario, ownership of each unit of durable housing stock requires a greater financial commitment from households. This tighter collateral constraint results in less housing consumption in equilibrium, and a greater reliance upon liquid financial deposits to smooth income through time. The impact of this parameter change on differing measures of net wealth inequality is non-uniform, with the Gini coefficient and 90-20 net wealth ratio responding more to LSAPs than in the baseline, whereas the cross-sectional coefficient of variation responds by less in this scenario.

Robustness checks on other key parameters also highlight how changes to the initial underlying distribution of household net wealth result in estimates of the effect of LSAPs which strengthen the response of some measures of net wealth inequality, while weakening the response of others. For instance, while the scenario increasing the housing adjustment cost coefficient, ϕ dampens the responds of the Gini coefficient and amplifies that of the cross-sectional coefficient of variation and 90-20 net wealth ratio, in the scenario with a smaller housing expenditure share, $1 - \theta$, the cross-sectional coefficient of variation and the 90-20 wealth ratio move in opposing directions, relative to the baseline.

3.7 Conclusion

This paper considers the impact of changes in the size and composition of the central bank's balance sheet on household wealth inequality through the financial portfolio rebalancing channel of monetary policy transmission. I first demonstrate how two rudimentary methods can uncover the potential implications from asset price and quantity movements, consistent with those surrounding LSAPs. These methods find wealth inequality increased by between 2.4% and 3.1%, as measured by the Gini coefficient.

The primary contribution of the paper is the formulation of a model of the economy which can distinguish between the impact of different central bank LSAPs on an endogenous distribution of household wealth. This necessitates using a balance sheet approach to model the central bank, while households may choose between investing in two assets (of differing liquidity) and experience heterogeneous, idiosyncratic shocks to their labour productivity.

Once the model is calibrated to fit US data from 2007 to 2013, it shows the contribution of Federal Reserve LSAPs to the increase in household net wealth inequality over this period, as the distribution of household wealth moved to a new steady state. The increase in household net wealth inequality, across multiple measures, arises primarily as the result of quantitative easing policies conducted by the Federal Reserve, while maturity extension had a smaller effect. Taken together, Federal Reserve LSAPs increase wealth inequality by 3.8% as measured

by the Gini coefficient, with similar increases in the cross-sectional coefficient of variation and 90-20 wealth ratio. Higher net wealth inequality arises due to the differing degrees of inequality within households' financial asset portfolios. After LSAPs are implemented by the Federal Reserve, households hold additional liquid and fewer illiquid assets, which thereby places additional weight on the (less-equal) liquid asset distribution when computing overall net wealth inequality, than in the initial steady state. This represents a portfolio rebalancing effect where higher income households disproportionally move into liquid assets, increasing wealth inequality. These estimates of the endogenous increase in measures of household net wealth inequality from the more sophisticated model are somewhat larger than those estimated by more rudimentary methods.

Further research could seek to decompose the effects of central bank LSAPs undertaken during the recent COVID-19 pandemic on wealth inequality, and contrast this experience with that of the great recession. Although likely to generate new challenges, this may mitigate one of the limitations of the modelling environment of this paper, as a precise modelling of financial crises is less relevant.

Chapter 4

The Rise of Harrod-Balassa-Samuelson

4.1 Introduction

Purchasing Power Parity (PPP) has a well established history in economics with some versions stretching back as far as the Salamanca School in 16th-century Spain. By the late 19th-century the writings of British economists Ricardo, Mill and Marshall had formalised many of these ideas. Indeed the modern formulation, usually attributed to Cassel (1916, 1918), is itself over a century old.[1] However, throughout this history of economic thought, progress has not always been straightforward.

> "[PPP] has been considered an identity, a truism, an empirical regularity or a grossly misleading simplification" (Dornbusch (1987b), p.1).

> "Unless very sophisticated indeed, PPP is a misleading, pretentious doctrine, promising what is rare in economics, detailed numerical prediction" (Samuelson (1964), p. 153).

Nonetheless, today PPP is ubiquitous as a fundamental concept and benchmark for most theoretical and empirical work in international macroeconomics. This is particularly true when analysis focusses upon secular trends or uses a flexible pricing framework. In this paper, I document, and explain, how these foundations have shifted.

In section 4.2, I present two stylised facts. Firstly, over the past 70 years the positive cross-country relationship between aggregate consumer prices and real output per capita - the so called Harrod-Balassa-Samuelson[2] (HBS) effect - has strengthened. In particular, during the 1950s an increase in relative real output per capita would not have typically been associated with an increase in relative consumer prices. In contrast the latest data suggest a strong positive association. This relationship may be captured by the slope in the cross-sectional Ordinary Least Squares (OLS) estimate, from a regression of relative consumer prices to relative real output per capita, which is typically used to highlight violations of PPP (Krugman et al., 2013; Obstfeld and Rogoff, 1996). Estimates of this slope have increased over time. This empirical result, which is robust to country composition and time horizon, may not be explained using the traditional justification of relative productivity advances in the tradable

[1] The terminology of "purchasing power parity" is introduced in Cassel (1918) to relate differences between internal and external currency valuations as a result of post WWI inflation differences. This may be understood using modern language as a statement about *relative* PPP.

[2] Harrod (1933), Balassa (1964) and Samuelson (1964).

goods sector.[3] Tradable goods productivity, although higher in level than for non-tradables, has not dominated recent productivity growth.

Secondly, the Law of One Price (LOOP) has weakened, with relative cross-country price movements increasingly explained by border effects. I reconsider the established city-level cross-sectional relative price results from Engel and Rogers (1996), augmenting their results for North-America by including European, South American, African and Australasian cities. I then show that the magnitude of cross-sectional relative price variation explained by a border effect has increased since the 1980s, when these data series start. Again, this result is robust, holding across borders on different continents, for a range of CPI items and distance measures.

In section 4.3, I present a two-country endowment model in which traded goods are required to use non-traded domestic distribution services. This results in an equilibrium failure of the LOOP, and can simultaneously explain the two stylised facts. In this model countries are endowed with a fixed level of traded and non-traded goods. Traded goods require some domestic non-traded services in order for products to reach the consumers. This drives a supply-side wedge between producer and consumer prices, breaking the LOOP. It also causes international differences in the relative price of non-traded goods. Over time, as local distribution requirements increase, real income differences are reflected more in overall consumer price level differences, through the non-traded component of the price differential in traded goods. As a result, the HBS effect strengthens. In contrast, the standard explanation of a higher share of non-traded goods, as discussed by Taylor and Taylor (2004), can only explain a strengthening HBS effect.

In section 4.4, I present a production version of this framework. This extends the endowment model, of section 4.3, by adding monopolistically competitive intermediate goods firms. When local distribution services are required to bring tradable products to market, the optimisation problem of these intermediate goods firms results in endogenous pricing-to-market. One consequence of this is to violate the LOOP whenever countries have asymmetric levels of productivity. When the share of national income spent on local distribution services increases over time, this richer model can simultaneously replicate a strengthening HBS effect and an increasing failure of the LOOP. In section 4.5 I solve and simulate the model to closely replicate the two stylised facts. Finally, section 4.6 concludes.

[3] This would lead to a constant slope estimate unless relative productivity advances in the tradable goods sector are continually increasing over time. Instead, an increasing slope may result from a Baumol and Bowen (1966) effect, the tendency for the within-country relative price of services to increase, as discussed by Froot and Rogoff (1995).

4.1.1 Related Literature

This paper is related to several strands of the literature. Many studies attempt to explain observed movements in the real exchange rate between countries, both empirically and theoretically. Over time, observed real exchange rate movements appear to temporarily fluctuate around a longer-term fundamental value. Although suggested as far back as Cassel (1916), 20th century evidence supporting the PPP hypothesis is mixed. Early empirical studies used simple regression techniques aiming to (linearly) relate the nominal exchange rate and relative consumer prices. Frenkel (1978) found a near linear relationship for the 1920s, which broke down when 1970s data are analysed, as discussed in Frenkel (1981). Later work challenged the validity of these empirical techniques due to the relatively slow response of the real exchange rate after exogenous shocks.[4] Frankel (1986, 1990, 1991) argued this could lead to biased empirical estimates due to potential nonstationarity and the presence of a unit-root, particularly at high frequencies. This problem may be exacerbated if, as found in this paper, the HBS effect is time varying. To overcome these issues, more recent investigations utilise co-integrating relationships to estimate the response of the real exchange rate to temporary exogenous shocks. Eichenbaum and Evans (1995) show contractionary monetary policy shocks appear to cause persistent appreciation of the nominal and real exchange rate, while Ravn et al. (2012) document that increased government expenditure depreciates the real exchange rate.

In this paper, I investigate the cross-country relationship between price levels and output per capita. Across countries, observed real exchange rates are higher in richer nations, as documented through numerous studies including several vintages of the Penn World Tables.[5] In early work linking real exchange rate movements to relative changes in productivity, Hsieh (1982) found that relative growth rates of labor productivity between traded and non-traded sectors can help explain deviations of exchange rates from PPP as predicted by the Harrod (1933), Balassa (1964) and Samuelson (1964) theory. Relative to this literature, I find evidence to show the HBS effect is increasing, extending the ideas in Taylor and Taylor (2004) and Bergin et al. (2006). In theory, relative productivity differences are ubiquitous as the fundamental structural source for the HBS effect, typically through unbalanced productivity differences across the traded and non-traded goods sectors. While Ghironi and Melitz (2005) provide microeconomic foundations for the HBS effect as an endogenous outcome of firm entry in the presence of heterogeneous productivity, Cardi and Restout (2015) highlight the

[4] Rogoff (1996) presents the "consensus view" of the half-life of this real exchange rate response at 3-5 years. Estimates in more recent work, for instance Steinsson (2008) and Chen and Engel (2005), align with this earlier consensus.

[5] The most recent version is 10.0 with major changes outlined in Feenstra et al. (2015).

role of imperfect labor mobility in improving the quantitative fit of relative price responses to relative productivity differentials between the tradable and non-tradable goods sectors. Bergin et al. (2006) use a model with endogenous international goods tradablity to explain an increasing HBS effect.[6] This paper connects to this literature by showing how a model with local distribution requirements may also reconcile HBS theory with data.

This paper adds to the current international macroeconomics literature on local distribution services by establishing changes in this economic sector as an important channel to help explain both the observed secular changes in the HBS effect and the increasing empirical failure of the LOOP. Local distribution services have been used to empirically justify the slow speed of PPP convergence, as this effectively increases the (relatively less price responsive) non-traded income share, and an increase in the relative productivity of distribution services can therefore lead to a real exchange rate appreciation (Burstein et al., 2003; MacDonald and Ricci, 2001). This channel is particularly evident during large real exchange rate depreciations arising after large nominal devaluations (Burstein et al., 2005) and can also explain the cyclical behaviour of the real exchange rate (Burstein et al., 2006). More recent empirical evidence also shows how the primary channel of exchange rate pass-through from exchange rate changes to consumer prices arises through indirect cost changes from imported inputs, combined with local distribution services (Goldberg and Campa, 2010). Theoretically, incorporating this sector can lead to endogenous deviations from the LOOP and incomplete exchange rate pass-through, as a wedge exists between wholesale and retail prices (Corsetti and Dedola, 2005; Corsetti et al., 2008).

This paper also contributes directly to the empirical literature on the failure of the LOOP. In its most simplistic theoretical form PPP assumes the Law of One Price holds for tradable goods. A failure of the LOOP may therefore lead to a breakdown of PPP and by extension a change in the HBS effect. In a seminal study, Engel and Rogers (1996) show, after accounting for distance, the presence of an international border increases the variance of CPI items between US and Canadian cities. This is attributed to the non-traded components in the final consumer prices of traded goods, which are generally less internationally mobile. Using more granular barcode data to compare precise goods across the same border, Broda and Weinstein (2008) suggest a substantial compositional effect is also present, with goods bundles differing more across countries than within. Nevertheless, Broda and Weinstein (2008) estimate a significant, but smaller, border effect than Engel and Rogers (1996).[7] Also using unique product codes, Gopinath et al. (2011) use variation driven by costs shocks to conclude that

[6] Other related work which use endogenous tradablity to explain real exchange rate fluctuations includes Bergin and Glick (2007, 2009) and Naknoi (2008).

[7] While Broda and Weinstein (2008) use the ACNielsen data, which largely covers grocery items, Imbs et al. (2010) find a significant border effect is also present in euro area television pricing.

national borders segment markets as retail prices respond to changes in costs in neighbouring stores within the same country but not across the border. This paper adds to this debate by showing how the border effect in final consumer prices has increased over time, across five continents.[8] Though product-level data are excellent in determining the current forces behind LOOP violations, they are less well suited to analysis of secular changes due to data limitations in the time dimension (Gopinath et al. (2011) analyse 178 weeks; Broda and Weinstein (2008) 16 quarters).[9]

The literature has established many possible reasons for the failure of the LOOP including: transaction, transport and trade costs, often introduced in "iceberg" form (Krugman, 1991; Samuelson, 1954); the existence of non-traded goods; short-run nominal price rigidity, including through local currency pricing (Gopinath and Rigobon, 2008); and a breakdown of perfect competition leading to pricing to market (Dornbusch, 1987a; Krugman, 1987). The model presented in this paper uses the latter theories to explain the long-run LOOP failure (Rogoff et al., 2019).

Although not central in this paper, related work interprets the observed relationship between consumer price levels and output per capita across countries as a result of specific factor endowments, rather than productivity differences as in the HBS theory. In this alternative explanation, due to Bhagwati (1984) and Kravis and Lipsey (1983) (BKL), a fixed capital-to-labor endowment combined with a relatively high capital intensity for tradable goods production generates high wages in countries with higher capital-to-labor ratios, who ultimately become relatively wealthy. In its simplest form the BKL hypothesis posits that capital is not freely mobile between countries. The primary differences between these competing hypotheses may also be understood as analogous to those between productivity-driven Ricardian trade models (HBS theory) and relative-factor-endowment Heckscher-Ohlin trade models (BKL theory). Empirically analysing these competing hypotheses, Bergstrand (1991) finds a role for both HBS and BKL mechanisms in explaining the relationship between international price differences, also adding non-homothetic demand structure (Linder, 1961) as a competing "demand-side" mechanism.

Finally, many surveys of this literature explain the developments in our understanding of both PPP and the LOOP in recent years. Officer (1976) surveys the early empirical literature seeking to explain PPP convergence, while Froot and Rogoff (1995); Rogoff (1996); Taylor and Taylor (2004) and Burstein and Gopinath (2014) are excellent broader summary articles on

[8] I am not aware of other work which considers the time-series properties of this effect and, while Engel and Rogers (1996) investigate the US-Canada border and Beck et al. (2009, 2016) investigate the euro area, I am not aware of other researchers using data from South America, Africa or Australia and New Zealand.

[9] Crucini and Telmer (2020) and Crucini and Landry (2019) somewhat overcome this using product-level data across several years, starting in 1990. Crucini et al. (2005) use product-level European data from 1975, but have only a single observation for each country.

these subjects. Dornbusch (1987b) provides a historical account of the development of PPP theory.

4.2 Stylised Facts

In this section I present two stylised facts. The first demonstrates how the HBS effect has strengthened over the past 70 years, while the second documents a weakening of the LOOP through stronger border effects.

Stylised Fact 1 (HBS). *The Harrod-Balassa-Samuelson effect has strengthened over the past 70 years.*

Economics textbooks frequently remind us that "*rich countries have high price levels*", and motivate this empirical deviation from absolute PPP using charts showing cross-country relative real GDP per capita and consumer price levels.[10] Usually, this is then attributed to the HBS effect. This exercise is replicated in Figure 4.1, for the year 1950 in panel (a) and the year 2004 in panel (b). These use data from the Penn World Tables version 10.0 (Feenstra et al., 2015) and are constructed relative to the United States.[11] The sample includes 54 countries with available data in every year between 1950 and 2019, and each country is represented by a **blue dot**. A **black solid** line has been included showing the fitted value from a simple Ordinary Least Squares (OLS) regression with a constant term:

$$\ln(P_{i,t}/P_{US,t}) = \alpha_t + \beta_t \ln(Y_{i,t}/Y_{US,t}) + \varepsilon_{i,t}, \tag{4.1}$$

where $P_{i,t}$ is the consumer price level and $Y_{i,t}$ is the real GDP per capita in year t. The results of this exercise are reported in Table 4.1 for the first and last years in the sample (1950 and 2019) along with results for 2004, which is the single year with the largest estimated value for $\hat{\beta}_t$.[12] Although a popular empirical exercise, a regression of this type almost certainly does not represent a causal relationship and regression estimates of β_t are therefore unlikely to be stable. In particular, the error terms, $\varepsilon_{i,t}$, are likely to be correlated with the regressors $\ln(Y_{i,t}/Y_{US,t})$ through simultaneity bias, as both $(P_{i,t}/P_{US,t})$ and $(Y_{i,t}/Y_{US,t})$ are endogenous economic variables.[13]

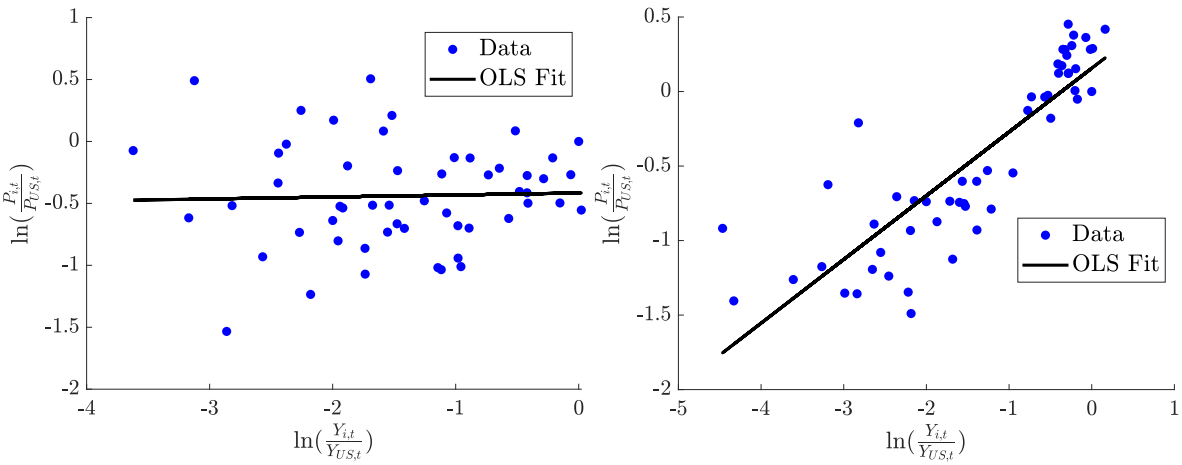
[10] E.g. Obstfeld and Rogoff (1996), Fig 4.1 p. 201; and Krugman et al. (2013), Fig 16.3 p. 402.

[11] Specifically, this analysis uses the price level of real consumption, (pl_con); output-side real GDP at chained PPPs (rgdpo); and population (pop).

[12] The smallest estimated value of $\hat{\beta}_t$ arises in 1950.

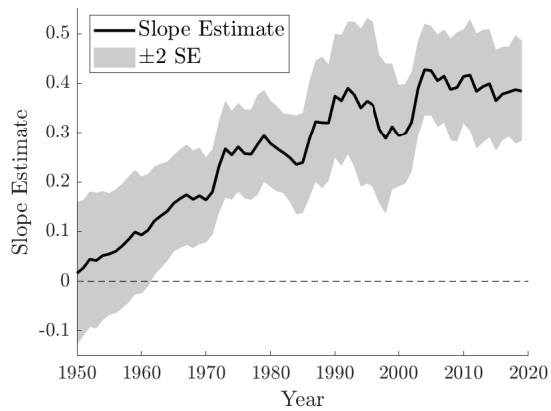
[13] Other confounding factors include, but are not limited to: omitted variables, including productivity and factor endowments; missing data, with no entries for some countries during earlier parts of the sample period; and measurement error, particularly of the real value of output.

Figure 4.1 Estimates of HBS Relationship



(a) Data from 1950.

(b) Data from 2004.



(c) Rolling HBS Estimates.

Source and Notes: Penn World Tables version 10.0 (Feenstra et al., 2015). Panels (a) and (b) show the natural logarithm of the relative consumer price level against the natural logarithm of the relative level of output per capita in 1950 and 2004, respectively. Each country is represented by a **blue dot** with the USA used as the base country. Fitted values from the OLS regression given in equation (4.1) have also been added to these panels in a **black solid** line. The **black solid** line in panel (c) shows how the annual estimate of the slope coefficient, $\hat{\beta}_t$, in the OLS regressions given by equation (4.1) has changed over time between 1950 and 2019, while the **gray** region in panel (c) forms a ±2 standard error band around this central estimate. Data underlying these charts uses a fixed sample of 54 countries for whom data are available in all years.

Table 4.1 Cross-Sectional HBS Relationship

	(1) 1950	(2) 2004	(3) 2019
$\hat{\alpha}$	-0.42***	0.16***	-0.07
	(0.09)	(0.06)	(0.05)
$\hat{\beta}$	0.02	0.43***	0.38***
	(0.07)	(0.05)	(0.05)
N	54	54	54
Adj-R^2	-0.02	0.73	0.68

Source and Notes: Penn World Tables version 10.0 (Feenstra et al., 2015). This table reports coefficients from the OLS regression given in equation (4.1), with robust standard errors reported in brackets. Coefficients which are significantly different from zero at the 1% , 5% and 10% significance level are denoted with asterisks ***, ** and * respectively. The number of observations (countries) included in the sample is also reported along with the adjusted R^2 regression statistic. Countries are only included in these regressions when they have data available for all sample years between 1950 and 2019. Sample: (1) uses cross-sectional data from 1950; (2) from 2004; and (3) from 2019.

Nevertheless, these data suggest this non-causal cross-country relationship between relative consumer price levels and relative real output per capita has been positive since 1950, and significant since the mid 1960s. Higher income countries do tend to experience higher relative consumer price levels. Furthermore, when analysing the time series dimension of this data, the strength of this cross-sectional association has been increasing over time.[14] Indeed, Figure 4.1, panel (c), shows that estimates of the slope coefficient, $\hat{\beta}_t$, from the series of annual regressions described in OLS regression equation (4.1) have increased substantially. This is shown in the **black solid** line, while the **gray region** forms a ±2 standard error band around this central estimate. At the start of the sample, in 1950, the cross-country relationship between relative consumer prices and relative real output per capita is positive but insignificant at 0.02, with price levels and output per capita seemingly unrelated to one another. But nearly 70 years later this estimate of the slope coefficient has increased to 0.38, signifying that for every 1% increase in real output per capita, relative to the United States, a country's domestic consumer price level tends to increase by 0.38%. The largest estimated value for $\hat{\beta}_t$ arises in 2004 and over the past 15 years estimates have fallen slightly raising the possibility of a structural break in this upward trend. This issue is discussed formally below.

The initial secular empirical increase in the magnitude of the HBS effect, prior to 2004, is robust to several alternative specifications. To highlight this, the exercise is replicated for several alternative cross-sectional samples. In a second stage, the time series trends of these

[14]This observation is also noted in Bergin et al. (2006) for an earlier sample.

$\hat{\beta}_t$ estimates are investigated using the regression:

$$\hat{\beta}_t = \gamma_0 + \gamma_1 \text{Year}_t + v_t. \tag{4.2}$$

The results from this exercise are shown in Table 4.2, Panels A and B. Column (1) shows the baseline OLS specification with cross-sectional regressions for a fixed sample of 54 countries which have available data in all years from 1950 until 2019, highlighting the increase in the OLS regression slope coefficient, $\hat{\beta}_t$, over this period. Other samples also display the same trend. Columns (2) and (3) partition the cross-country sample based on whether countries are judged to have a free capital account. An indicator variable is used to partition the sample drawing upon the International Monetary Fund's (IMFs) Annual Report on Exchange Arrangements and Exchange Restrictions (AREAER).[15] According to this measure, the proportion of countries judged by the IMF to have capital controls increases during the sample period, from 76% of countries in 1967 to 84% of countries in 2019.[16] The results in columns (2) and (3) of Table 4.2, panels A and B, show that the trend increase in the OLS regression slope coefficient, $\hat{\beta}_t$, applies both for the subset of countries with and without capital account controls. This result is useful in ruling out changes to international financial restrictiveness as the primary force behind these changes, though free capital account countries have tended to experience an upwards trend which is more than double that of countries who have capital account controls.

The implications from an alternative division of the cross-sectional sample according to country income levels are shown in columns (4) to (6) of Table 4.2. In recent work Hassan (2016) highlights the importance of non-linearities in this cross-sectional relationship between consumer prices and real income per capita, which turns negative for low income countries. To allow more cross-sectional variation the sample is split by income. The 54 countries are evenly divided into high, middle and low income brackets according to their level of 2019 real GDP per capita. All three groups show a significant positive time trend in their slope coefficient, which is largest for the highest and median income groups. Alternative income partitions generate analogous results.[17]

Finally, to highlight the potential role of bias resulting from an evolving sample selection, column (7) uses cross-sections from all available data. The cross-sectional sample size starts at 54 countries in 1950 and has a maximum of 182 from 2005 onwards, with the majority of

[15] Between 1999 and 2019 data are downloaded from the IMF's AREAER website indicating whether a given country has restrictions or controls on capital transactions or payments. I manually appended these data by the same indicator using the annual reports from 1967 until 1998 which are not available in the online database. Though these reports begin in 1950, indicator variables in an appendix table only appear from 1967 onwards.

[16] An alternative specification using measures from the Fernandez et al. (2016) dataset does not change the econometric results presented here.

[17] For instance a division with either 25%, 50%, 25% in high, medium and low income groups respectively or 20%, 60%, 20% in these groups.

Table 4.2 Time Trend in HBS Deviations

	(1) Baseline	(2) Free Capital Account	(3) Capital Account Controls	(4) High Income	(5) Medium Income	(6) Low Income	(7) Evolving Sample
Panel A: 1950-2019†							
$\hat{\gamma}_1$	0.56***	0.28***	0.12***	0.96***	1.13***	0.17***	0.39***
	(0.03)	(0.06)	(0.03)	(0.08)	(0.07)	(0.04)	(0.02)
T	70	53	53	70	70	70	70
Adj-R^2	0.88	0.22	0.21	0.66	0.84	0.12	0.82
Panel B: 1950-2004†							
$\hat{\gamma}_1$	0.66***	0.69***	0.16**	1.26***	1.50***	0.16*	0.54***
	(0.04)	(0.11)	(0.06)	(0.11)	(0.07)	(0.08)	(0.02)
T	55	38	38	55	55	55	55
Adj-R^2	0.88	0.53	0.13	0.70	0.92	0.06	0.93
Panel C: 2004-2019							
$\hat{\gamma}_1$	-0.28***	-0.65***	-0.16*	1.41**	-0.73***	-1.30***	-0.33**
	(0.06)	(0.18)	(0.07)	(0.58)	(0.22)	(0.16)	(0.12)
T	16	16	16	16	16	16	16
Adj-R^2	0.53	0.57	0.16	0.21	0.38	0.85	0.37

Source and Notes: Penn World Tables version 10.0 (Feenstra et al., 2015). This table reports coefficients from the OLS regression given in equation (4.2), with robust standard errors in brackets. Coefficients and their standard errors have been multiplied by 100. Coefficients which are significantly different from zero at the 1% , 5% and 10% significance level are denoted with asterisks ***, ** and * respectively. The number of observations (time periods, T) included in the sample is also reported along with the adjusted R^2 regression statistic. The Baseline regression uses a fixed sample of 54 countries for which data are available in all years from 1950 until 2019. Free Capital Account and Capital Account Controls instead restrict the sample to the list of countries to judged to have a free capital account (or capital account controls) according to the IMF AREAER. High Income, Medium Income and Low Income partition the sample into three equal sized income brackets based on 2019 income levels. Evolving Sample allows uses all countries possible in the first stage cross-sectional estimation of equation (4.1). †In regressions (2) and (3) the sample starts in 1967.

countries therefore joining the sample during the estimation period. The size of the time trend in PPP deviations using this evolving sample is similarly positive and significant as with the fixed baseline sample, though the lower value of the secular trend, given by $\hat{\gamma}_1$, shows how lower income countries entering the sample at later dates appear to be experiencing a smaller change in this relationship thereby reducing estimates of this trend. This is consistent with the difference between regressions (4) and (6). Alternatively, countries entering the cross-sectional sample may have lower values of $\hat{\beta}_t$ which would also reduce the observed trend.

The structural break in this secular relationship from 2004 onwards may be observed by contrasting results in Table 4.2 across panels. Panel A highlights results for the whole 1950-2019 sample period. Panel B focusses upon the period before 2004, and Panel C on the period after 2004. For all estimates the time trend in $\hat{\beta}_t$ is positive in the early sample. Only the trend for high income countries fails to then turn negative in the subsequent period, after 2004, highlighting that this change in trend arises most strongly in lower income countries. As these countries represent two thirds of the total cross-sectional observations, they also drive trend changes in the other measures, which pool all income groups.

An additional empirical specification is next used to provide further evidence of an increasing slope coefficient in (4.1), particularly in the early part of the sample period and for high income countries. Unlike the process described above, this is a one-stage procedure. Assuming the linear trend in $\hat{\beta}_t$ holds, (4.2) may be used alongside (4.1) to generate:

$$\ln(P_{i,t}/P_{US,t}) = \alpha_t + \gamma_0 \ln(Y_{i,t}/Y_{US,t}) + \gamma_1 [\text{Year}_t \times \ln(Y_{i,t}/Y_{US,t})] + \eta_{i,t}, \tag{4.3}$$

which identifies the linear time trend in β_t as γ_1 through the interaction of Year_t and $\ln(Y_{i,t}/Y_{US,t})$ and the composite error is given by $\eta_{i,t} \equiv \varepsilon_{i,t} + v_t \ln(Y_{i,t}/Y_{US,t})$, which suffers from heteroskedasticity. α_t represents a year fixed effect. Expressing in this form allows estimation to include both cross-sectional and time-series data dimensions simultaneously.

The results from this alternative specification are shown in Table 4.3, with specifications which also contain country and time fixed effects. Specification (5) includes both fixed effects to prevent bias in the estimates, with comparable results to the simple baseline specification (1). Specifications (2) and (3) may be biased, as these regressions do not contain year fixed effects. However the scale of this bias appears moderate with coefficients similar when the fixed effects are included. Empirical evidence across all specifications points towards a positive time trend in the β_t slope coefficient from canonical HBS regressions over the past 70 years. This is evident in Table 4.3, Panel A. However, as in the previous analysis, when countries of all income levels are considered jointly, this trend appears to shift substantially around the year 2004. This may be seen by comparing the results in Table 4.3, Panel B and Panel C,

with specifications controlling for year fixed effects changing from a positive trend in the HBS slope coefficient, β_t, for the early sample to a negative trend one in the later period. Looking across estimations we observe a substantial increase in the Adjusted R^2 measure whenever country-level fixed effects are included in the specification. This is unsurprising, as it tells us that much of the cross-sectional variation in relative consumer prices is explained by country-specific factors, other than relative real GDP per capita.

Table 4.3 Additional Estimates of Time Trend in HBS Deviations

Method	Baseline	Interaction			
	(1)	(2)	(3)	(4)	(5)
Panel A: 1950-2019					
$\hat{\gamma}_1$	0.56***	0.30***	0.25***	0.37***	0.58***
	(0.03)	(0.01)	(0.01)	(0.02)	(0.02)
N×T	70	10329	10329	10329	10329
Adj-R^2	0.88	0.27	0.67	0.29	0.72
Panel B: 1950-2004					
$\hat{\gamma}_1$	0.66***	0.40***	0.37***	0.58***	0.86***
	(0.04)	(0.02)	(0.01)	(0.04)	(0.02)
N×T	55	7599	7599	7599	7599
Adj-R^2	0.88	0.24	0.69	0.26	0.75
Panel C: 2004-2019					
$\hat{\gamma}_1$	-0.28***	0.17**	0.08**	-0.35***	-0.32***
	(0.06)	(0.07)	(0.04)	(0.12)	(0.05)
N×T	16	2910	2910	2910	2910
Adj-R^2	0.53	0.40	0.91	0.41	0.93
Country FE		✗	✓	✗	✓
Year FE		✗	✗	✓	✓

Source and Notes: Penn World Tables version 10.0 (Feenstra et al., 2015). The table reports coefficients from alternative specifications to uncover a time trend in β_t, with clustered Heteroskedasticity- and Autocorrelation-Consistent (HAC) robust standard errors in brackets. Baseline refers to the OLS regression given in equation (4.2). Interaction refers to the panel OLS regression given in equation (4.3). Coefficients and their standard errors have been multiplied by 100. Coefficients for other variables are suppressed. Regressions also include country and year fixed effects as indicated by Country FE and Year FE. Coefficients which are significantly different from zero at the 1% , 5% and 10% significance level are denoted with asterisks ***, ** and * respectively. The number of observations (countries times time periods) included in the sample is also reported along with the adjusted R^2 regression statistic. The Baseline regression uses a fixed sample of 54 countries for whom data are available in all years from 1950 until 2019 in the first stage of the estimation process, regression (4.1). Interaction regressions use all availavble data (unbalanced panel).

Stylised Fact 2 (LOOP). *The Law Of One Price (LOOP) has weakened over the past 40 years.*

The Law of One Price (LOOP), which predicts that when tradable goods are consumed in multiple physical locations their prices are equal (when expressed in the same currency), is frequently used as a theoretical assumption to generate absolute PPP. A range of studies have empirically tested this hypothesis, with both McCallum (1995) and Engel and Rogers (1996) highlighting LOOP violations across North American cities. Even in a situation with low levels of protectionism and after accounting for trade costs using a proxy distance measure, consumer price differences between US and Canadian cities exhibit a substantial border effect. Between cities, relative price variation is far greater for equidistant cities located in different countries.[18] The size of this effect, in comparison to the contribution from distance, was posited in Obstfeld and Rogoff (2000) as one of the six major puzzles in international macroeconomics. I find that, since then, this puzzle has worsened.

I extend the two-stage method of Engel and Rogers (1996) to investigate time trends in the LOOP, while additionally incorporating multiple African, Australasian, European, and South American cities for a larger cross-section. Countries and cities are selected based upon data availability.[19] To the best of my knowledge there are no other contiguous countries with publicly available data. A map of these city locations is given in Figure 4.2, while a list is given in Table C.2 of the Appendix. Appendix C.1 provides further details on the data used in this section of the paper. Throughout the paper I will refer to city locations, but it should be noted that statistical convention across countries necessitates using a range of geographical statistical region classifications, again based upon availability as outlined in Table C.1. Regions are matched to their most populous conurbation for the purposes of

[18]Further work has found these results to be robust, though perhaps smaller, when actual prices rather than an index is used. Engel et al. (2005) use data taken from the Economist EUI, while Broda and Weinstein (2008) use barcode-level data. Beck et al. (2009, 2016) find similar results for European cities. Using further disaggregated data Engel and Rogers (2001) argue that, other than distance, the difference in price volatility within US cities may be explained by nominal price stickiness.

[19]I identify 208 potential countries for investigation from the UN National Statistical Office Partners website. Whenever hyperlinks are broken I attempt to identify the main statistical office website. From these statistical agencies Consumer Price Index (CPI) data must meet a number of criteria to be useable. Firstly, CPI data (of quarterly frequency or higher) must be jointly distinguishable by expenditure category and physical location (e.g. this test fails in several countries due to geographical separability between the capital city and "rest" of the country). Secondly, data must be freely and publicly accessible (e.g. in Austria this data is not free while in Finland this data is not easily made available to the public). Thirdly, website and statistical table translation into English must be sufficiently possible to identify location and expenditure categories (e.g. Google translation is possible for basic French, German, Italian, Polish, Portuguese, Spanish and Ukrainian, but problematic for data from China and Japan. These latter countries are hence omitted). Fourthly, included countries must be contiguous with at least one other country in the sample (e.g. though data exist for Ghana, India, the Kyrgyz Republic, Lebanon, the Philippines, Turkey and South Sudan, data does not exist for any of their direct neighbours). I identify 21 countries from the full list of 208 which satisfy these conditions. Some German and Swiss data are retrieved from regional statistical offices as directed by their national statistical offices.

geolocation (see footnote in Table C.2). Although only a single border is investigated in Africa and Australasia, multiple national borders are jointly investigated in other continents. I use a similar European sample to Beck et al. (2016), augmenting their dataset to include European countries outside the eurozone (Poland, Switzerland and Ukraine). I am not aware of similar existing exercises in the literature for Africa, Australasia or South America.

After CPI data are collected for a range of city locations, CPI categories and time periods, the first stage of this procedure calculates the natural log of the relative Consumer Price Index of CPI category $i \in [1, I]$ between cities $\{j, k\} \in [1, J]$ for $j < k$ at time t, which is denoted by $\ln P^i_{j,k,t}$, with $P^i_{j,k,t} = P^i_{j,t}/P^i_{k,t}$.[20] Time periods between 1970 Q1 and 2019 Q4.[21] For example, this could represent the natural log of the relative CPI of Housing between Miami and Philadelphia in Q1 1990. All relative price indices are converted into US Dollars when investigating North American borders; Euros while investigating European borders; Australian Dollars when considering the Australia-New Zealand border; Brazilian Reals when considering South America; and South African Rand for the South African-Mozambique border.[22] From these relative price indices, relative measures of quarterly inflation, $\pi^i_{j,k,t}$, are then calculated as:

$$\pi^i_{j,k,t} \equiv \ln P^i_{j,k,t} - \ln P^i_{j,k,t-1}, \tag{4.4}$$

$$\sigma^i_{j,k} \equiv \text{std}(\pi^i_{j,k,t}), \tag{4.5}$$

again for CPI category i, between cities $\{j, k\}$, at time t. The, full sample, time-series standard deviation of these paired city-level, CPI-category relative levels of inflation is denoted as $\sigma^i_{j,k}$. Following the existing literature (Beck et al., 2016; Broda and Weinstein, 2008; Engel and Rogers, 1996), this measure of relative price volatility is the primary object of empirical interest. Engel and Rogers (2001) explain, and justify, using such volatility measures when price indices rather than actual prices are available, to form a "proportional" LOOP test. Low values of $\sigma^i_{j,k}$ may arise due to: (1) "absolute" LOOP; (2) "proportional" LOOP; or (3) constant prices, and hence low inflation in each location. Distinguishing between these three cases is not possible without price level data.

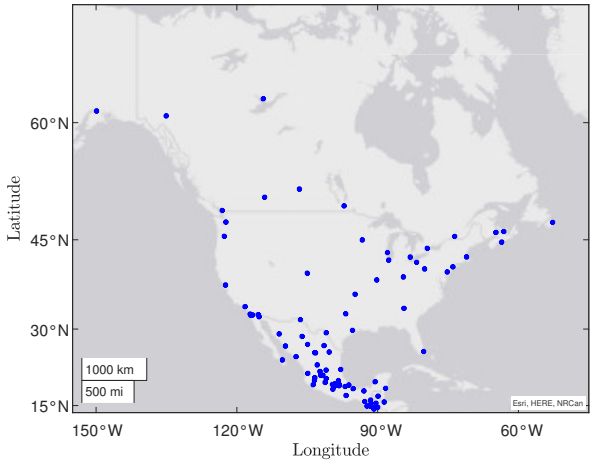
In total, across these five continents, 26,775 paired city-level CPI-category relative price volatility measures then exist. Their average volatilities are displayed in Table 4.4, highlighting how relative price volatility is always larger when cities are located across an international

[20] Cities as in Table C.2; CPI categories as in Table C.3.

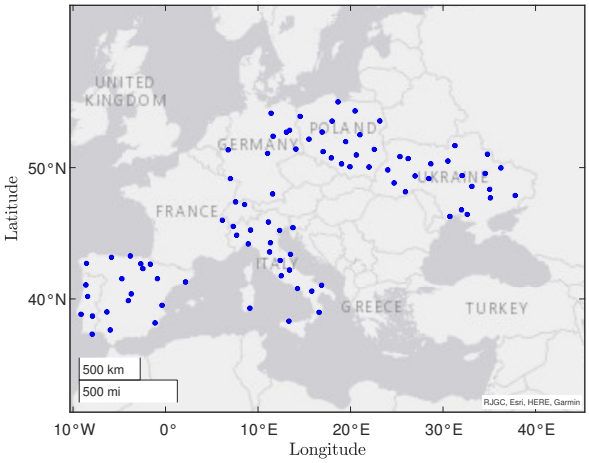
[21] In some cases CPI data are collected at quarterly frequency while in other cases data are collected at monthly frequency. There is no perfect way to convert monthly CPI data to a quarterly frequency. For simplicity I consider the average monthly level of CPI in each quarter.

[22] Exchange rates are taken as monthly or quarterly averages, whichever appropriate, from the Bank of International Settlements (BIS) database.

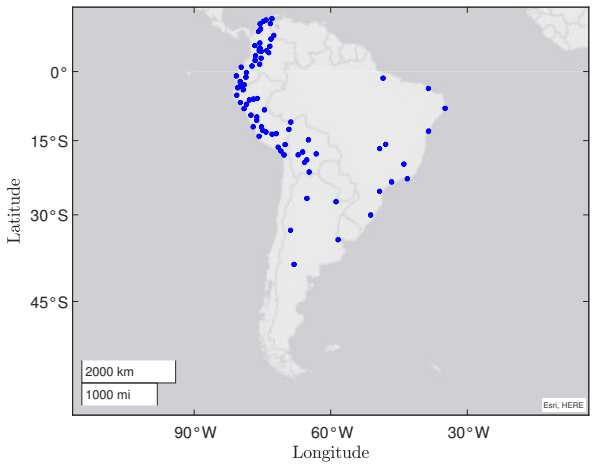
Figure 4.2 City Locations



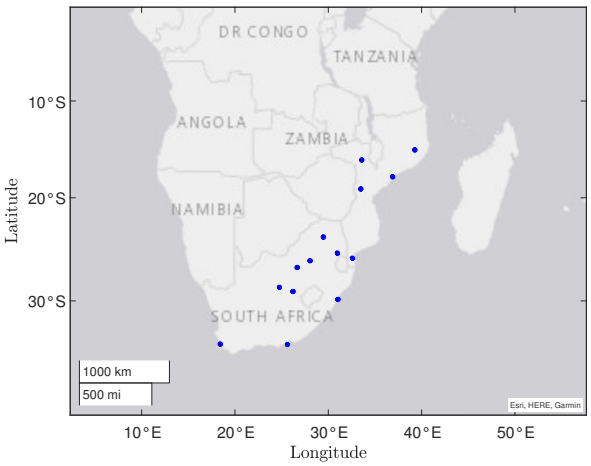
(a) North America.



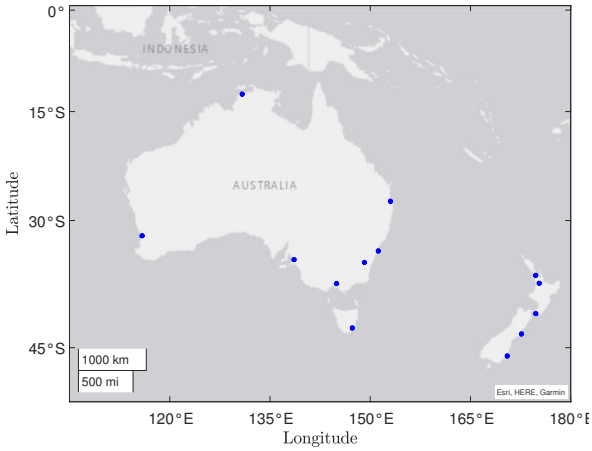
(b) Europe.



(c) South America.



(d) Africa.



(e) Australia & New Zealand.

Sources and Notes: https://download.geonames.org/ and Matlab mapping toolbox. Maps show city-level locations for CPI data used, as **blue dots**.

border. The precise order of magnitude varies from 24% higher for North American CPI Category 4 (Clothing & Footwear) to a maximum of 872% higher in Africa (also for CPI Category 4). There are multiple plausible explanations for these differences. Economic policy changes to tariffs, non-tariff trade barriers, direct and indirect taxation would cause greater volatility in cross-border inflation differences, than domestic. Differences in firms cost structures, transportation costs and productivity cycles, combined with different weights on consumption items, could also cause a structural wedge in cross-border consumer price inflation volatility, over domestic. Nominal consumer price rigidities in domestic currency terms, combined with volatile movements in the nominal exchange rate would also be consistent with lower domestic, than cross-border, consumer price inflation volatility. Finally, cross-border differences in firms markups arising through pricing to market and local distribution costs could also explain this difference. Nevertheless, the empirical regularity of a higher relative price volatility between foreign cities, than domestic, holds both for each of the continents studied and for every consumption expenditure item.

Table 4.4 Relative Inflation Volatility Across City Pairs

	North America		Europe		Australasia		South America		Africa	
CPI Category	Domestic	Cross-Border	Domestic	Cross-Border	Domestic	Cross-Border	Domestic	Cross-Border	Domestic	Cross-Border
Traded										
1	0.013	0.031	0.006	0.039	0.009	0.038	0.011	0.050	0.011	0.089
2	0.019	0.040	0.007	0.047	0.010	0.032			0.011	0.084
3	0.018	0.035	0.010	0.037					0.010	0.079
4	0.037	0.046	0.017	0.097			0.007	0.053	0.007	0.084
5	0.017	0.034	0.019	0.078			0.008	0.052	0.010	0.093
Non-traded										
6	0.016	0.035			0.012	0.037	0.006	0.053	0.011	0.089
7	0.018	0.047	0.009	0.041	0.009	0.030			0.015	0.092
8	0.012	0.031	0.013	0.045					0.017	0.094
9	0.020	0.033	0.010	0.034			0.014	0.056	0.012	0.090
10							0.008	0.056	0.015	0.107
11							0.009	0.053	0.013	0.107

Sources: National authority statistical agencies and BIS. Notes: Table reports simple averages for the standard deviation, $\sigma^{i}_{j,k}$, in relative consumer price inflation measures between cities located within (domestic) and between (cross-border) countries for each CPI category, i. The corresponding CPI category names are given in Table C.3. The sample runs between 1970 Q1 and 2019 Q4.

The latitude and longitude of each city is then appended to the dataset using data provided by https://download.geonames.org/. An established practise across many sub-fields of

economic geography uses latitude and longitudes in the Haversine formula to compute an approximation for the distance between cities, assuming a spherical globe (Ashraf and Galor, 2013; Ellison et al., 2010; Monte et al., 2018). Average distance statistics are displayed in Table 4.5. In every continent, on average domestic cities are located substantially closer together than their foreign counterparts. This arises despite the often large distance between domestic cities, for instance in North America. The North American and Australasian cities included in the dataset are, on average, further apart than those in Europe and Africa.

Table 4.5 Average Distance Measures Between Cities

	North America		Europe		Australasia		South America		Africa	
Measure	Domestic	Cross-Border	Domestic	Cross-Border	Domestic	Cross-Border	Domestic	Cross-Border	Domestic	Cross-Border
Distance (km)	1300	2908	399	1567	1559	3155	668	2716	647	1608
Observations (city-pairs)	1382	2534	655	3173	38	40	627	1929	42	36

Source and Notes: Table reports the average distance between cities located within (domestic) and between (cross-border) countries. It also reports the number of city-pair observations. Distance is calculated using the Haversine formula with latitude and longitude data taken from https://download.geonames.org/.

In the second stage, of the procedure, a cross-sectional regression is performed of the relative price volatility measure on the measure of distance, an international border indicator and an indicator variable for each city, according to:

$$\sigma^i_{j,k} = \beta^i_D D_{j,k} + \beta^i_B B_{j,k} + \sum_{n=1}^{J} \beta^i_n \mathcal{I}_n + \varepsilon^i_{j,k} \tag{4.6}$$

where $D_{j,k}$ is the natural log of the distance between cities j and k; $B_{j,k}$ is an indicator variable for whether cities j and k are located in different countries; and $\mathcal{I}_j$ are a series of city-level indicator variables for whether a city pertains to the volatility measure $\sigma^i_{j,k}$ (effectively city-level fixed effects).

The results of these regressions are shown in Table 4.6, highlighting how both city-distance and the presence of an international border are important in explaining observed relative price volatility. In each continental dataset, relative price volatility tends to increase as cities get farther apart, as demonstrated by the mostly significant and positive estimates of β^i_D in Table 4.6. Although three insignificant negative results are found in non-traded African and South American CPI categories, this effect is otherwise found to be widespread across both traded and non-traded goods for most continents. In addition, over and above the effect of city-distance, crossing an international border is universally found to increase relative price

Table 4.6 Relative Price Volatility Regressions

	North America			Europe			Australasia			South America			Africa		
CPI Category	Distance (β_D^i)	Border (β_B^i)	Adj-R^2	Distance (β_D^i)	Border (β_B^i)	Adj-R^2	Distance (β_D^i)	Border (β_B^i)	Adj-R^2	Distance (β_D^i)	Border (β_B^i)	Adj-R^2	Distance (β_D^i)	Border (β_B^i)	Adj-R^2
Traded															
1	0.06***	1.78***	0.99	1.19***	1.87***	0.73	0.20***	2.69***	1.00	0.47***	3.06***	0.82	0.06	7.02***	1.00
	(0.01)	(0.01)		(0.07)	(0.12)		(0.02)	(0.03)		(0.04)	(0.06)		(0.05)	(0.12)	
2	0.06***	2.04***	0.98	1.12***	3.04***	0.77	0.10***	2.19***	0.99				0.11**	6.34***	1.00
	(0.01)	(0.02)		(0.07)	(0.12)		(0.04)	(0.05)					(0.04)	(0.11)	
3	0.01	1.76***	0.99	1.04***	1.88***	0.76							0.00	6.10***	1.00
	(0.01)	(0.01)		(0.06)	(0.12)								(0.03)	(0.12)	
4	0.04***	1.55***	0.95	2.20***	4.80***	0.78				0.08***	4.00***	0.97	0.06***	6.61***	1.00
	(0.01)	(0.02)		(0.13)	(0.23)					(0.03)	(0.08)		(0.02)	(0.06)	
Non-traded															
5	0.07***	1.95***	0.98	2.35***	4.01***	0.73				-0.03	3.99***	0.97	0.06	7.20***	1.00
	(0.01)	(0.01)		(0.15)	(0.26)					(0.03)	(0.08)		(0.07)	(0.11)	
6	0.01	1.97***	0.97				0.09*	2.53***	0.99	0.13***	4.11***	0.97	0.12	6.55***	0.99
	(0.01)	(0.02)					(0.05)	(0.07)		(0.03)	(0.07)		(0.09)	(0.22)	
7	0.03*	3.06***	0.97	0.79***	2.45***	0.79	0.09**	2.03***	0.99				0.18	5.94***	0.99
	(0.02)	(0.03)		(0.04)	(0.09)		(0.04)	(0.06)					(0.14)	(0.29)	
8	0.04***	2.03***	0.99	1.31***	2.04***	0.70							0.09	6.62***	0.99
	(0.01)	(0.01)		(0.07)	(0.14)								(0.09)	(0.14)	
9	0.07**	1.53***	0.72	0.76***	1.80***	0.78				0.13***	3.74***	0.95	-0.08	7.14***	0.99
	(0.03)	(0.03)		(0.04)	(0.08)					(0.03)	(0.09)		(0.09)	(0.19)	
10										0.21***	3.88***	0.97	-0.03	6.81***	1.00
										(0.03)	(0.09)		(0.06)	(0.23)	
11										0.14***	3.96***	0.96	0.15	6.59***	1.00
										(0.03)	(0.07)		(0.10)	(0.17)	

Notes: Table reports coefficients from the regression, with city-level fixed effects, given in (4.6), and HAC robust standard errors in brackets. Coefficients and their standard errors have been multiplied by 100. Coefficients which are significantly different from zero at the 1% , 5% and 10% significance level are denoted with asterisks ***, ** and * respectively. The adjusted-R^2 regression statistic is also reported.

volatility, as all measures β_B^i in Table 4.6 are positive and highly significant. This effect arises across both traded and non-traded goods categories. These factors, alongside city-level fixed effects, can alone account for the majority of variation in relative prices, as demonstrated by high values for the adjusted-R^2.

To understand how these cross-sectional effects have been changing over time, I next allow for time variation in the border coefficient, β_B^i, by re-estimating the cross-sectional OLS regression given in (4.6) using both overlapping and non-overlapping estimation windows. I then estimate a simple linear time trend in the estimated border effects, $\hat{\beta}_{B,t}^i$. In particular, I run the regression:

$$\ln \hat{\beta}_{B,t}^i = \gamma_0^i + \gamma_1^i \text{Trend}_t + \eta_t^i. \tag{4.7}$$

When using an overlapping estimation window, (4.7) is estimated at a quarterly frequency, with each value of $\hat{\beta}_{B,t}^j$ computed using a 5-year rolling window. Alternatively, when using a non-overlapping estimation window, (4.7) is estimated at an annual frequency, with each value of $\hat{\beta}_{B,t}^j$ computed using the quarterly data for that year (a non-overlapping 1-year window). A trade-off exists here between using more time-series data to accurately measure $\sigma_{j,k}^i$, at a given point in time (and hence the accuracy of $\hat{\beta}_{B,t}^i$ estimation), or using a greater number of time periods in the time-series regression (4.7). Generally, an overlapping sample window is therefore preferred as it restricts the time series of the sample data by less than a non-overlapping sample window. However, using relative price variation data in overlapping sample periods artificially introduces dependence between the time-periods, and hence the non-overlapping sample window provides a useful cross-check.

The main conclusion drawn from this time-series investigation is that there appears to be a positive time trend in the impact arising from the border effect on LOOP violation. This is supported by the results from estimation of (4.7) reported in Table 4.7. With three statistically insignificant exceptions, across each continent studied, and across all traded and non-traded CPI categories, time-trend coefficients, γ_1^i, are positive. The vast majority are also highly statistically significant. The significance level is generally lower in the non-overlapping sample case, partly due to the consequential restriction upon the number of time-series observations. These results are robust to alternative window lengths (5-10 year rolling and 1-5 year fixed).

The increasing importance of international borders in the failure of the LOOP may also be shown graphically. Figure 4.3, panel (a), uses data from North American CPI category 1 (Food and non-alcoholic beverages) to highlight the differences in cross-border vs domestic city pairs, over time. Each dot represents the average relative price volatility for a single city in a single year. The x-axis entry uses only domestic city pairs while the y-axis entry uses

Table 4.7 Estimates of Time Trend in LOOP Border Effect

	North America		Europe		Australasia		South America		Africa	
CPI Category	5-year Rolling (γ_1^i)	1-year Fixed (γ_1^i)	5-year Rolling (γ_1^i)	1-year Fixed (γ_1^i)	5-year Rolling (γ_1^i)	1-year Fixed (γ_1^i)	5-year Rolling (γ_1^i)	1-year Fixed (γ_1^i)	5-year Rolling (γ_1^i)	1-year Fixed (γ_1^i)
Traded										
1	0.38***	3.86**	1.03***	7.00**	-0.01	1.48	0.60***	0.27	1.52***	7.97
	(0.06)	(1.55)	(0.01)	(2.56)	(0.06)	(1.09)	(0.07)	(5.35)	(0.01)	(8.89)
2	0.21***	0.66	0.66***	6.07*	0.22***	3.55**			1.11***	11.65
	(0.03)	(1.31)	(0.01)	(2.99)	(0.08)	(1.53)			(0.10)	(9.44)
3	0.34***	3.77*	1.81***	14.68***					0.97***	7.96
	(0.06)	(2.20)	(0.01)	(2.99)					(0.09)	(6.05)
4	0.26***	1.65*	0.65***	12.32***			0.33***	2.37	1.30***	7.13
	(0.03)	(0.96)	(0.22)	(4.24)			(0.10)	(3.81)	(0.00)	(7.08)
Non-traded										
5	0.34***	3.89***	1.47***	12.17***			0.44***	3.12	1.58***	11.27**
	(0.05)	(1.31)	(0.21)	(3.40)			(0.09)	(3.42)	(0.01)	(3.92)
6	0.40***	3.14**			0.25***	1.04	0.44***	3.22	0.83***	-0.01
	(0.05)	(1.54)			(0.00)	(2.15)	(0.10)	(3.66)	(0.10)	(8.01)
7	0.37***	0.32	0.31**	4.74***	0.14***	-1.09			0.79***	8.72
	(0.11)	(2.07)	(0.15)	(1.41)	(0.01)	(2.33)			(0.08)	(7.26)
8	0.35***	2.93*	1.26***	4.45*					1.69***	7.21
	(0.05)	(1.53)	(0.01)	(2.27)					(0.00)	(11.12)
9	0.23***	1.31	0.67***	6.30***			0.47***	4.02	1.29***	7.05
	(0.05)	(1.14)	(0.10)	(2.03)			(0.12)	(4.16)	(0.00)	(4.80)
10							0.47***	3.09	1.68***	2.77
							(0.10)	(3.40)	(0.01)	(6.80)
11							0.32***	1.42	1.06***	8.46
							(0.12)	(4.09)	(0.20)	(7.98)

Notes: Table reports OLS estimates from the regression given in (4.7), with robust standard errors in brackets. Coefficients and standard errors have been multiplied by 100. Coefficients which are significantly different from zero at the 1% , 5% and 10% significance level are denoted with asterisks ***, ** and * respectively.

only cross-boarder city pairs. Alternatively put, Figure 4.3, plots the points:

$$\left(\frac{\sum_{j=1,j\neq k}^{J} \sigma_{j,k}^{1}(1-B_{j,k})}{\sum_{j=1,j\neq k}^{J}(1-B_{j,k})}, \frac{\sum_{j=1}^{J} \sigma_{j,k}^{1} B_{j,k}}{\sum_{j=1}^{J} B_{j,k}} \right) \tag{4.8}$$

where $B_{j,k}$ is the indicator variable for whether cities j and k are located in different countries. The x-axis component defines average domestic relative price inflation volatility while the y-axis component defines the cross-border counterpart. Each **black dot** represents points from the years between 1980 and 1989 while each **blue dot** represents the average relative price volatility in a single year between 2010 and 2019. In both time periods, the majority of cities are above the 45 degree line, indicating that cross-border average relative inflation volatility exceeds domestic relative inflation volatility. For a given level of domestic relative inflation volatility, cross-border relative inflation volatility has increased between the 1980s and 2010s, as indicated by the steeper slope in the **blue dots**, relative to the **black dots**.

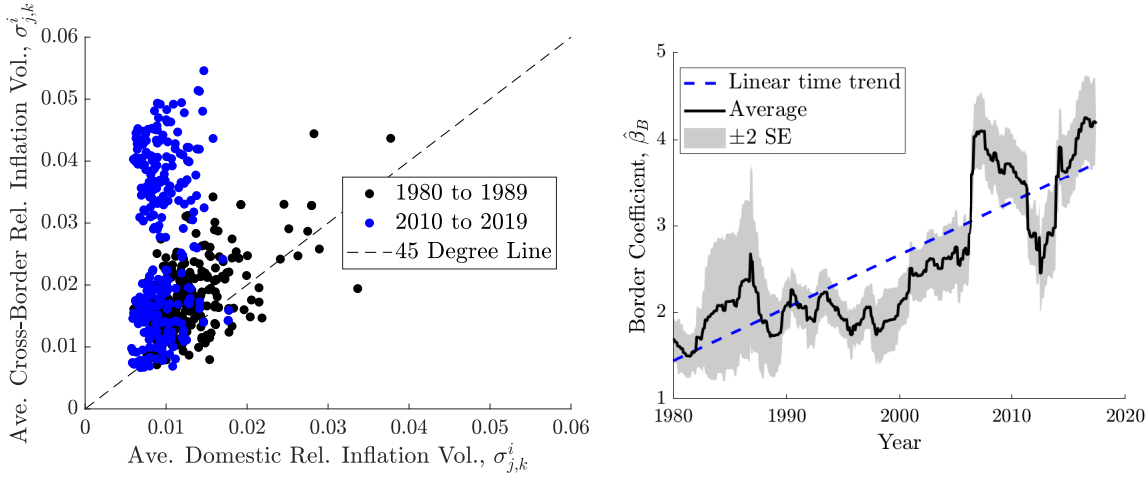
Similarly, the **black solid** line in Figure 4.3, panel (b), displays the average of the coefficient estimates $\ln \hat{\beta}^{i}_{B,t}$ used in the quarterly overlapping 5-year rolling windows. These estimates represent the average across all continents and CPI categories. The **gray region** forms a ±2 standard error band around this central estimate while the linear time-trend is shown in the **blue dashed** line. These estimates of $\hat{\beta}_{B,t}$ do vary over time, as price volatility captures changes in the economy. However, the increasing importance of the border in explaining the failure of the LOOP is clear, as confirmed by the earlier formal statistical tests in Table 4.7.

4.3 Endowment Model

4.3.1 Model

In this section I construct a two-country endowment model to explain the strengthening of the HBS effect alongside the increasing failure of the LOOP, as outlined in section 4.2. The endowment model is particularly useful as an analytical reduced-form model solution exists. This model incorporates a domestic distribution services sector, which results in the failure of the LOOP in equilibrium. An increase in the relative size of this sector can simultaneously explain both stylised facts. This endowment model draws upon, and simplifies, the mechanism at work in Burstein et al. (2003), Burstein et al. (2005) and Corsetti and Dedola (2005), who also use distribution services, in an international context, to enrich the standard new open economy macroeconomics framework. There are two countries in a single time period. Utility

Figure 4.3 Time Trend in LOOP Border Effect



(a) North American City-Level Data.

(b) Rolling Border Effect Estimates.

Notes: Panel (a) shows data from North American CPI Category 1 (Food and non-alcoholic beverages). Each **black dot** represents the average relative inflation volatility for one city, in a single year between 1980-1989. As highlighted described by the cordinates in (4.8). Each **blue dot** represents the average relative price volatility for one city, in a single year between 2010-2019. The **black solid** line in panel (b) shows the average of the 5-year rolling estimates of $\ln \hat{\beta}^i_{B,t}$, from the OLS regression given in equation (4.6). The **gray region** forms a ±2 standard error band around this central estimate. Fitted values and their standard errors are averaged across continents and CPI items. For comparison, a linear time trend is shown in **blue dashed** line.

in the Home country is given by:

$$U = \frac{C_T^{\gamma} C_N^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \tag{4.9}$$

where C_T denotes consumption of a traded good and C_N of a non-traded good, and $0 < \gamma < 1$ is the traded expenditure share. The minimum expenditure problem for one unit of utility, equal to one unit of aggregate hedonic consumption, $C \equiv U$, results in a hedonic consumer price level given by:

$$P = P_T^{\gamma} P_N^{1-\gamma}, \tag{4.10}$$

where P_T represents the final consumption price of the traded good and P_N the price of the non-traded good. A representative household seeks to maximise (4.9) subject to the budget constraint:

$$P_N C_N + P_T C_T = P_N Y_N + \tilde{P}_T \tilde{Y}_T, \tag{4.11}$$

where $\tilde{P}_T$ represents the price faced by domestic distributors of the traded good, Y_N is the endowment of the non-traded good and $\tilde{Y}_T$ is the endowment of thre traded good. Faced with

this constraint, the optimal consumption allocation satisfies:

$$\frac{P_N}{P_T} = \frac{1-\gamma}{\gamma}\frac{C_T}{C_N}. \tag{4.12}$$

As in Burstein et al. (2003), Burstein et al. (2005) and Corsetti and Dedola (2005), although non-tradable goods are sold directly to consumers, bringing one unit of traded goods to market requires $\phi > 0$ units of the non-traded good to be used through domestic distribution services.[23] In other words, the retail production of traded goods, usually left implicit in the literature, arises according to a Leontief (1941) production function in wholesale traded inputs and non-traded distribution services:

$$Y_T = \min\left\{\tilde{C}_T, \frac{\tilde{C}_N}{\phi}\right\}, \tag{4.13}$$

where $\tilde{C}_T$ and $\tilde{C}_N$ are wholesale traded and non-traded goods inputs respectively for the retail distribution of traded goods and Y_T represents the production level of final retail traded goods. This distribution services sector operates under perfect competition such that the Home consumer retail price of traded goods equals:

$$P_T = \tilde{P}_T + \phi P_N, \tag{4.14}$$

which, as $\phi > 0$, exceeds wholesale traded goods prices, through the inclusion of non-traded distribution inputs. In addition, the optimal choice of wholesale inputs from the distribution sector gives:

$$\tilde{C}_N = \phi\tilde{C}_T, \tag{4.15}$$

such that, provided $\phi > 0$ is fixed, wholesale inputs from the traded and non-traded sectors are consumed in constant proportions, as discussed in Tirole (1994).

I assume that the LOOP is obeyed for tradable goods at the wholesale level with:

$$\tilde{P}_T = \tilde{P}_T^*, \tag{4.16}$$

where $\tilde{P}_T^*$ denotes the Foreign price of wholesale traded goods, and throughout this paper a superscript asterisk denote a variable for the Foreign country. Home and Foreign may be assumed to use a common currency such that all nominal prices are expressed in the same

[23] Although $\phi > 0$ is assumed the limiting case of $\phi \to 0$ will also be investigated.

nominal units. In turn, this wholesale traded goods price may be taken as the numéraire:

$$\tilde{P}_T = 1. \tag{4.17}$$

In this static endowment setting the market clearing conditions are given as:

$$Y_N = C_N + \tilde{C}_N, \tag{4.18}$$

$$Y_T = C_T, \tag{4.19}$$

$$\tilde{Y}_T + \tilde{Y}_T^* = \tilde{C}_T + \tilde{C}_T^*, \tag{4.20}$$

for the non-traded, final retail traded and intermediate wholesale traded goods markets respectively.

Finally, for ease of exposition, three auxiliary variables may be defined as:

$$\mathcal{Q} \equiv \frac{P^*}{P}, \tag{4.21}$$

$$\mathcal{Y} \equiv \frac{C_T^{\gamma} C_N^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \tag{4.22}$$

$$\mathcal{Y}^* \equiv \frac{(C_T^*)^{\gamma}(C_N^*)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \tag{4.23}$$

where $\mathcal{Q}$ represents the real exchange rate, with standard definition (4.21).[24] Equations (4.22) and (4.23) define the level of GDP in the Home and Foreign country as $\mathcal{Y}$ and $\mathcal{Y}^*$ respectively. We assume a common preference and production structure between countries, with $\gamma = \gamma^*$ and $\phi = \phi^*$. Consumption is the only source of final demand, and hence equals GDP (as the sum of all final demand) in this economy. Therefore, in nominal terms:

$$P\mathcal{Y} = PC = P_T C_T + P_N C_N, \tag{4.24}$$

where the final equality arises through our definition of the consumption aggregator, $C \equiv U$ and the aggregate hedonic nominal price level, P. These definitions of GDP, in both the Home (4.22) and Foreign (4.23) country, therefore differ from total output, in particular from the non-traded goods sector, which produces to cover a component of intermediate demand from

[24] This endowment model abstracts from the use of a nominal exchange rate as, in this non-monetary setting, multiple possible equilibrium values would be compatible with the equilibrium allocation. An alternative setting with a cash in advance constraint and constant monetary aggregates could be used instead to determine the equilibrium nominal exchange rate without influencing the main results.

traded goods. For example, in nominal terms for the Home country:

$$P_N Y_N = \underbrace{P_N C_N}_{\text{Final Demand}} + \underbrace{P_N \tilde{C}_N}_{\text{Intermediate Demand}} . \tag{4.25}$$

To illustrate the behaviour of this model in a stochastic setting, the exogenous variables Y_N, $\tilde{Y}_T$, Y_N and $\tilde{Y}_T$ could follow a stochastic process, with their deviations from a steady state value and the impact on equilibrium outcomes described below.

4.3.2 Implications

Notation For any variable X, bars denote the non-stochastic steady state levels, $\bar{X}$, while hats denote percentage deviation from their non-stochastic steady state level, $\hat{X}$, (approximately equal to log deviations). Variables are written in upper case with their natural logarithm in lower case, $x \equiv \ln X$.

An equilibrium for the endowment model presented in the previous subsection exists as the solution to the 8 Home country equations (4.10), (4.11), (4.12), (4.13), (4.14), (4.15), (4.19) and (4.22), along with their Foreign country equivalents, and equations (4.16), (4.17), (4.18), (4.20) and (4.21). Appendix C.2 presents the full list of endowment model equations, derives its analytical solution and log-linearises the key results.

To connect the endowment model to the stylised facts presented in section 4.2, consider, as functions of the exogenous variables, the real exchange rate, $\mathcal{Q}$, given in (4.21); the ratio of Foreign and Home national income, $\mathcal{Y}^*/\mathcal{Y}$, i.e. the ratio of (4.23) to (4.22); and the relative Foreign vs Home retail price of tradable consumer goods, P_T^*/P_T, i.e. the ratio of the Foreign and Home equivalents of (4.14). After expressing in terms of the exogenous variables Y_N, $\tilde{Y}_T$, Y_N^* and $\tilde{Y}_T^*$, a first order log-linearisation of $\mathcal{Q}$, $\mathcal{Y}^*/\mathcal{Y}$ and P_T^*/P_T around a symmetric non-stochastic steady state may be found. This is derived in Appendix C.2.3, and results in:

$$\hat{q} \approx \frac{\gamma\theta^2(1-\gamma)}{(\theta-\phi)(\gamma\theta-\phi)}[\hat{\tilde{y}}_T^* - \hat{y}_N^* - \hat{\tilde{y}}_T + \hat{y}_N], \tag{4.26}$$

$$\hat{y}^* - \hat{y} \approx \frac{\gamma\theta-\phi}{\theta-\phi}[\hat{\tilde{y}}_T^* - \hat{\tilde{y}}_T] + \frac{\theta(1-\gamma)}{\theta-\phi}[\hat{y}_N^* - \hat{y}_N], \tag{4.27}$$

$$\hat{p}_T^* - \hat{p}_T \approx \frac{\phi\theta(1-\gamma)}{(\theta-\phi)(\gamma\theta-\phi)}[\hat{\tilde{y}}_T^* - \hat{y}_N^* - \hat{\tilde{y}}_T + \hat{y}_N]. \tag{4.28}$$

where we define θ as the steady state ratio between non-traded and traded endowments, $\theta \equiv \bar{Y}_N/\bar{\tilde{Y}}_T$ and assume this steady state ratio is symmetric across countries, with $\bar{Y}_N/\bar{\tilde{Y}}_T = \bar{Y}_N^*/\bar{\tilde{Y}}_T^*$, which implies $\theta = \theta^*$.

The intuition behind these results may be understood by considering an increase in the endowment of wholesale traded goods in the Foreign country ($\hat{\tilde{y}}_T^* > 0$), with no changes its non-traded endowment or the Home endowments. This greater abundance of Foreign wholesale traded goods causes the Foreign price of non-traded goods, relative to wholesale traded goods to rise, increasing the overall Foreign price level and depreciating the Home real exchange rate in (4.26). As the LOOP does not hold in consumer tradable prices due to non-traded distribution services, this increase in the relative price of non-traded goods, relative to wholesale traded goods, also causes the Foreign consumer retail tradable price to rise in (4.28). The magnitude of both effects, addressed in the next subsection, depends critically upon the relative abundance of the wholesale traded goods endowment, relative to non-traded, θ, the expenditure share of consumption on traded goods, γ, and the size of non-traded distribution requirements to bring these additional wholesale traded products to consumers, ϕ. Finally, an increase in the endowment of wholesale traded goods in the Foreign country will cause Foreign GDP to increase, as in (4.27), where the magnitude of the change to GDP again depends upon θ, γ and ϕ.

In the special nested case without local distribution services, which arises as $\phi \to 0$, the endowment model reduces to the standard HBS model discussed at length in textbooks and considered explicitly, among others, in Bergin et al. (2006). In this case, it is clear from (4.28) that the LOOP holds at the retail level for tradable goods, and from (4.27) that cross-country GDP differences reflect only endowment differences and the consumption expenditure shares, γ and $1-\gamma$, without reference to any production structure.

Theoretical counterparts to $\hat{\beta}$, from regression equation (4.1), and $\hat{\beta}_B$, from regression equation (4.6), discussed in section 4.2, may then be generated from (4.26), (4.27) and (4.28) as:

$$\hat{\beta} \equiv \frac{\mathrm{Cov}[\hat{q}; \hat{y}^* - \hat{y}]}{\mathrm{Var}[\hat{y}^* - \hat{y}]}, \tag{4.29}$$

$$\hat{\beta}_B \equiv \mathrm{std}(\hat{p}_T^* - \hat{p}_T) > 0. \tag{4.30}$$

where both (4.29) and (4.30) represent general theoretical definitions.[25] As previously, $\text{std}(x)$ refers to the standard deviation of x, while $\text{Cov}(x, z)$ represents the covariance between x and z and $\text{Var}(x)$ represents the variance of x.

Moving away from this general case, the remainder of this subsection will frequently make use of two further simplifying assumptions. These allow further intuition to be gained regarding the impact of incorporating a domestic distribution sector into an otherwise standard two country endowment model. The two assumptions are presented below, followed by two analytical results which may be observed as following directly from these assumptions.

Assumption 1 (Finite Prices). *Domestic distribution requirements for traded goods are smaller than the final consumption income share of traded goods multiplied by θ:*

$$\phi < \gamma\theta. \tag{4.31}$$

Assumption 1 is required to ensure prices are positive and finite, and hence the existence of a unique model solution. As domestic distribution requirements, ϕ, increase, an increasing number of non-traded goods are required in order to produce one unit of the domestic traded good. This increases the relative price of non-traded goods given their relative endowment, θ. In the limit, under Cobb-Douglas consumption preferences, as $\phi \to \gamma\theta$, the relative price of non-traded goods becomes infinite. This requirement may be seen directly from the analytical solution to the model presented in Appendix C.2.2. As $\theta > 0$ and $0 < \gamma < 1$, Assumption 1 also implies both $0 \leq \phi < \theta$ and $\phi^2 < \gamma\theta^2$.

Assumption 2 (HBS). *The non-traded endowment in both Home and Foreign is equally far from the non-stochastic steady state:*

$$\hat{y}_N^* = \hat{y}_N. \tag{4.32}$$

Assumption 2 formalises a standard HBS condition which ensures that international relative price variation arises solely due to differences in *tradable* output. Under this assumption

[25] If an economy is in the steady state in period $t-1$, the theoretical definition of β_B, given in (4.30), is justified by appreciating that $\hat{p}_T^* - \hat{p}_T$ may be considered a theoretical counterpart to $\pi^i_{j,k,t}$, from (4.4), whenever i represents a traded goods category, and j and k are in different countries. In that case $\hat{p}_T^* - \hat{p}_T$ and $\pi^i_{j,k,t}$ both represent the change in the natural log of the relative retail consumer price of traded goods between countries. $\hat{p}_T^* - \hat{p}_T$ is the change in relative traded price levels compared to the non-stochastic steady state, while $\pi^i_{j,k,t}$ is the change in relative traded price levels compared to the previous quarter. After accounting for distance and city-level fixed effects, the coefficient $\hat{\beta}_B$ from regression equation (4.6) uncovers the difference in the standard deviation of cross-border and domestic values of $\pi^i_{j,k,t}$. As the model assumes one representative consumer, with no city-level heterogeneity, a model equivalent will set $\text{std}(\pi^i_{j,k,t}) = 0$, in the domestic case with j and k from the same country. Therefore the difference between cross-border and domestic values of $\text{std}(\pi^i_{j,k,t})$ is simply the cross-border value of $\text{std}(\pi^i_{j,k,t})$, and the theoretical equivalent of β_B is as given in (4.30).

(4.29) and (4.30) respectively become:

$$\hat{\beta} = \frac{\gamma\theta^2(1-\gamma)}{(\gamma\theta-\phi)^2} > 0, \tag{4.33}$$

$$\hat{\beta}_B = \left|\frac{\phi\theta(1-\gamma)}{(\theta-\phi)(\gamma\theta-\phi)}\right| \mathrm{std}(\hat{\tilde{y}}_T^* - \hat{\tilde{y}}_T) > 0, \tag{4.34}$$

with the modulus in (4.34) unnecessary once Assumption 1 has been used.

Two analytical results follow directly from these assumptions. Respectively, these show how including a domestic distribution requirement for traded goods increases the theoretical counterparts for both $\hat{\beta}$ and $\hat{\beta}_B$.

Proposition 4.3.1 (Distribution and HBS)**.** *The theoretical size of the HBS effect increases as the size of the non-traded distribution requirement for retailing traded goods increases:*

$$\frac{d\hat{\beta}}{d\phi} = \frac{2\gamma\theta^2(1-\gamma)}{(\gamma\theta-\phi)^3} > 0. \tag{4.35}$$

Proof. Under Assumption 2, differentiation of (4.29) with respect to ϕ is equivalent to differentiation of (4.33), and results in (4.35). Positivity is assured under Assumption 1 and noting that $0 < \gamma < 1$. □

Proposition 4.3.2 (Distribution and LOOP)**.** *The theoretical size of the LOOP failure in the tradable goods sector increases as the size of the non-traded distribution requirement for retailing traded goods increases:*

$$\frac{d\hat{\beta}_B}{d\phi} = \frac{\theta(1-\gamma)(\gamma\theta^2-\phi^2)}{(\theta-\phi)^2(\gamma\theta-\phi)^2} std(\hat{\tilde{y}}_T^* - \hat{\tilde{y}}_T) > 0. \tag{4.36}$$

Proof. Under Assumption 2, differentiation of (4.30) with respect to ϕ is equivalent to differentiation of (4.34), and results in (4.36). Positivity is assured under Assumption 1, noting that this implies $\phi^2 < \gamma\theta^2$ and that $0 < \gamma < 1$. □

Proposition 4.3.1 states that a higher share of distribution services causes relative income differences to be reflected more in relative price level differences. Intuitively this arises as with greater distribution services traded goods prices face a larger wedge between the producer and consumer price of goods, for given income differences. In addition, this larger wedge between consumer and producer prices in each country increases the failure of the LOOP, as highlighted by Proposition 4.3.2. Ultimately, this wedge arises due to international differences in the relative price of non-traded goods. Thus, as local distribution requirements increase,

real income differences are reflected more in overall consumer price level differences, through the non-traded component of the price differential in traded goods.

4.3.3 Graphical Illustration

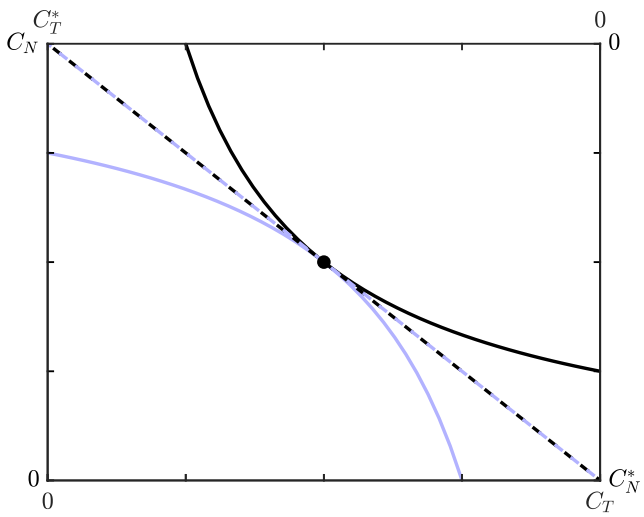
Equilibrium in the two-country endowment model is shown graphically in Figure 4.4, using an Edgeworth box. An initial benchmark, without a domestic distribution sector, assumes symmetric Home and Foreign countries who consequently share a common relative price of traded and non-traded goods. This is shown in Figure 4.4, panel (a), where both Home and Foreign consumption and endowment coincide.

Starting at this point of symmetric endowments, the Foreign country experiences an increase in their traded good endowment, $\tilde{Y}_T^*$, while their non-traded goods endowment, Y_N^*, remains unchanged. This is the classic HBS effect and shown in Figure 4.4, panel (b), where the horizontal dimension of the box is increased to account for a larger traded goods endowment (shown as the hatched section). Assuming the price level of the traded goods to be the numéraire, and with the Home country endowments are unaltered, the position of the Home country's budget line and indifference curve are unchanged in equilibrium.[26] In contrast, the Foreign country's indifference curve shifts outwards to meet the new budget line. Graphically, due to the axis extension, this is shown as the same point in the original space. As the non-traded good has become relatively scarce for the Foreign country, its relative price, P_N^*/P_T^*, increases. This is shown in (C.78) in Appendix C.2.4. In Figure 4.4, panel (b), this reduces the slope of the Foreign budget line relative to the original case which still prevails for Home. At this new equilibrium the Foreign indifference curve is tangent to the new Foreign budget line, with Foreign more willing to forgo a unit of traded consumption to increase non-traded consumption, than Home. An increased endowment of the traded good reduces the relative price of traded goods for the Foreign country, while the price ratio remains unchanged for Home. Although the LOOP holds in this setting for traded goods, non-traded goods prices differ across countries. Alternatively put, differential price ratios across countries are due to the presence of non-traded goods.

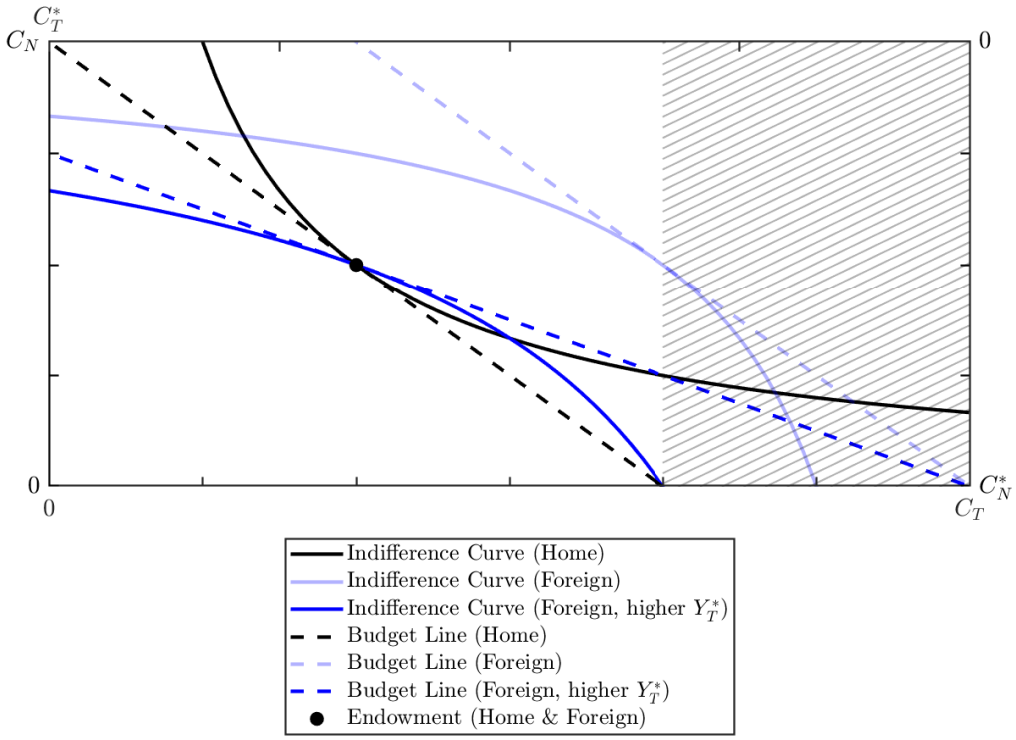
Domestic distribution requirements may also be incorporated into this graphical illustration, as shown in Figure 4.5, panels (a) and (b). Relative to the benchmark case, mandatory use of non-traded goods in the domestic distribution of traded goods reduces the availability of non-traded goods for domestic consumption, resulting in the consumption and endowment points no longer coinciding. This condition reduces utility for both countries, while making non-traded goods relatively scarce and hence increasing their relative price, P_N^*/P_T^*, and

[26] This is a classic result of the chosen preference structure where movements in the real exchange rate perfectly offset changes to overall endowment balances (Cole and Obstfeld, 1991).

Figure 4.4 Impact of Higher Foreign Tradable Endowment, $\tilde{Y}_T^*$, Without Local Distribution Sector



(a) Before change in Foreign tradable endowment, $\tilde{Y}_T^*$.



(b) After increase in Foreign tradable endowment, $\tilde{Y}_T^*$.

Notes: Authors' Edgeworth box representation of the impact of a positive tradable endowment shock to Foreign without a domestic distribution sector. Panel (a) shows the initial case, with symmetric Home and Foreign countries before any change. Panel (b) shows the impact of an increase in the Foreign traded good endowment, $\tilde{Y}_T^*$.

reducing the slope of the Foreign budget line. This is shown in (C.79) in Appendix C.2.4. This impact of including domestic distribution is symmetric for Home, and the slope of the Home budget line also falls.

For an increase in the Foreign endowment of traded goods, as shown in Figure 4.5, panel (b), similar effects arise as in Figure 4.4 panel (b). The relative price of non-traded goods increases. Given the new starting point already has relatively fewer non-traded goods for final consumption, the relative price increase which results is larger than previously. This is shown in (C.80) in Appendix C.2.4. In this setting although the LOOP holds for wholesale traded goods prices it fails for the consumer price of traded goods. The relative difference in budget constraint slopes again reflects differing traded to non-traded relative consumer prices across countries.

4.3.4 Comparative Statics

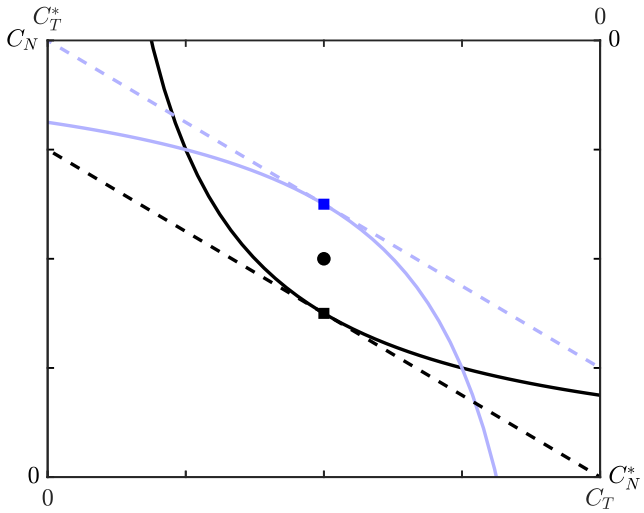
In this subsection I use comparative statics to highlight how this two-country endowment model may be used to closely replicate the changes in both $\hat{\beta}$ and $\hat{\beta}_B$ observed in the data. Assumption 1 holds throughout to ensure a real solution exists. The initial exercise also makes use of Assumption 2.

There are six parameters. Assumption 2 ensures $\hat{\beta}$ and $\hat{\beta}_B$ do not depend upon $\hat{y}_N$ or $\hat{y}^*_N$. For tractability, I normalise $\text{std}(\hat{\tilde{y}}^*_T - \hat{\tilde{y}}_T) = 1$ and assume symmetry across countries. I also assume symmetric endowments in the non-stochastic steady state, implying $\theta = 1$. The remaining parameters, γ and ϕ, are the subjects of our investigation. Figure 4.6 illustrates how a change in the traded goods share, γ, or size of the domestic distribution sector, ϕ, may alter the theoretical values of $\hat{\beta}$ and $\hat{\beta}_B$ from (4.33) and (4.34) respectively. Panel (a) concentrates on $\hat{\beta}$ while panel (b) considers $\hat{\beta}_B$.

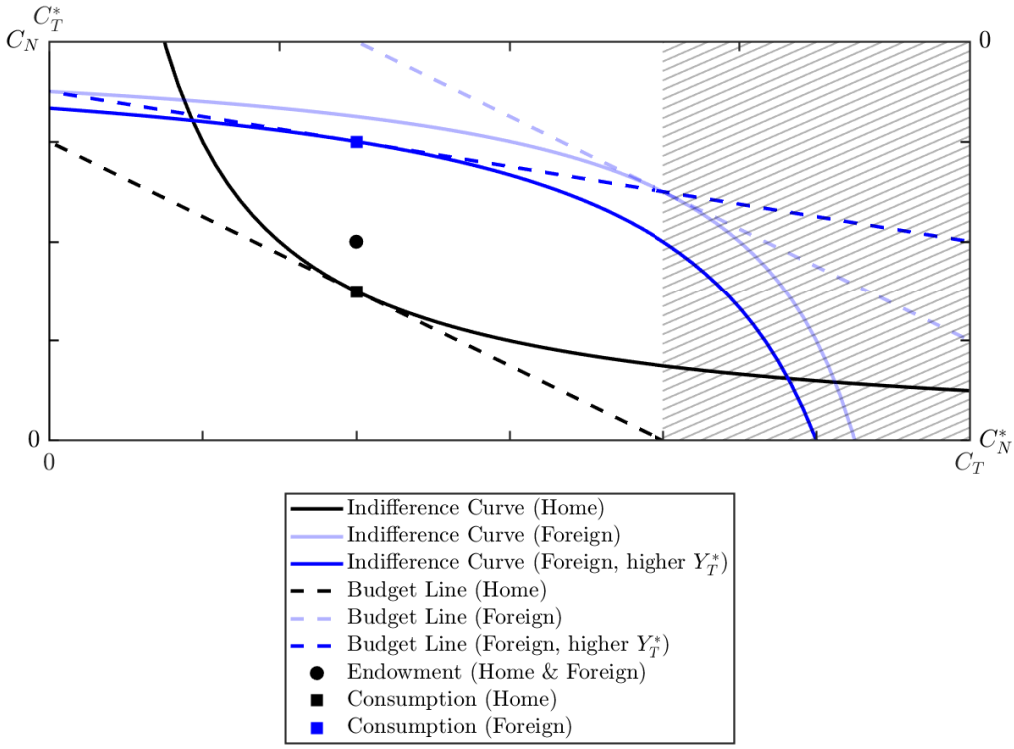
As seen in Figure 4.6, panel (a), the theoretical value of $\hat{\beta}$ displays asymptotic behaviour as γ approaches ϕ from above. Provided Assumption 1 is met, the theoretical estimate of $\hat{\beta}$ is decreasing in γ, the traded goods share of income. An increase in the distributional requirement, ϕ, increases the theoretical estimate of $\hat{\beta}$, shown in panel (a) as the discrete difference between the **black solid** line and the **blue dashed** and **red dotted** lines. In effect, this acts to increase the share of income going to the non-traded goods sector, with the full intuition given in subsection 4.3.3. As observed in Figure 4.6, panel (b), similar effects are true of $\hat{\beta}_B$, which is also decreasing in γ, provided $\phi > 0$. Again, an increase in ϕ raises the estimate of $\hat{\beta}_B$, at every level of γ. Figure 4.6 hence serves as a graphical confirmation of Propositions 4.3.1 and 4.3.2.

In a second exercise, I consider whether the two-country endowment model, presented in subsection 4.3.1, may replicate the two stylised facts reported in section 4.2. The endowment

Figure 4.5 Impact of Higher Foreign Tradable Endowment, $\tilde{Y}_T^*$, With Local Distribution Sector



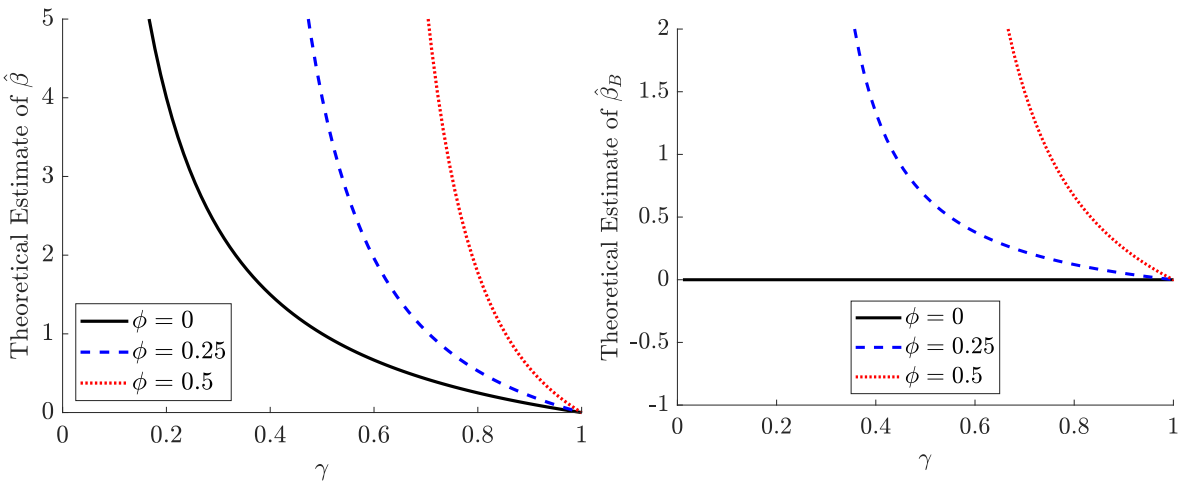
(a) Before change in Foreign tradable endowment, $\tilde{Y}_T^*$.



(b) After increase in Foreign tradable endowment, $\tilde{Y}_T^*$.

Notes: Authors' Edgeworth box representation of the impact of a positive tradable endowment shock to Foreign with a domestic distribution sector. Panel (a) shows the initial case, with symmetric Home and Foreign countries before any change. Panel (b) shows the impact of an increase in the Foreign traded good endowment, $\tilde{Y}_T^*$.

Figure 4.6 Comparative Statics



(a) Sensitivity of $\hat{\beta}$.

(b) Sensitivity of $\hat{\beta}_B$.

Notes: Figure shows the impact of a change in γ and ϕ on the theoretical estimates of $\hat{\beta}$ in (4.33), in panel (a), and $\hat{\beta}_B$ in (4.34), in panel (b), using the two-country endowment model presented in subsection 4.3.1, along with Assumptions 1 and 2, and the normalisation $\text{std}(\hat{\tilde{y}}_T^* - \hat{\tilde{y}}_T) = 1$. The effect of a discrete increase in the distributional requirement, ϕ, from 0 to 0.25 to 0.5 is shown as the difference between the **black solid** line, the **blue dashed** line and the **red dotted** line in each panel.

model is simulated to replicate the changes in the global economy over the past 70 years (from 1950 until 2019), and produce a theoretical counterfactual at an annual frequency.

The US is taken as the Home country and is the primary source of time-series variation, as the model parameters γ and ϕ are calibrated using US data. The income share of traded goods, γ, is computed using aggregate nominal expenditure on traded and non-traded goods from US NIPA Table 2.3.5 (Personal Consumption Expenditures, PCE), which are respectively taken as goods and services. Following Burstein et al. (2003) the distribution *margin* for traded goods:

$$\text{Distribution Margin} = \frac{\text{Retail Price} - \text{Wholesale Price}}{\text{Retail Price}}, \tag{4.37}$$

may be computed using the PCE-bridge of the NIPA Input-Output tables, which contains both nominal retail and wholesale output values. I use a simple cross-industry average. From 1997 onwards data are available at an annual frequency. Prior to this, as the NIPA Input-Output tables are only available at 5-year frequency, data are linearly interpolated for missing dates. An equivalent form of (4.37) in the endowment model is:

$$\text{Distribution Margin} = \phi \frac{P_N}{P_T}. \tag{4.38}$$

Therefore, the parameter value for the distributional requirements, ϕ, is then computed by combining data on the distribution *margin* with the relative aggregate consumer price of traded and non-traded goods, again using the US PCE Tables.[27]

Finally, values for Y_N, Y_T, Y_N^*, and Y_T^* are required. Each year, the US is taken to be at the non-stochastic steady state with $\bar{Y}_N$ and $\bar{Y}_T$ taken from the US PCE NIPA tables. This implies $\hat{y}_N = \hat{y}_T = 0$. The cross-sectional distribution of Foreign countries is taken from the 2017 World Bank's International Comparison Program (ICP) dataset to generate Y_N^* and Y_T^*, for each country in the world, and normalized to target the average value of $\hat{\beta}_B$ in the sample. These ICP data are used as the detailed disaggregated sources for PPP computation in the Penn World Tables (Feenstra et al., 2015), used in section 4.2. From this, $\hat{\beta}$ and $\hat{\beta}_B$ are then computed using (4.29) and (4.30).

The results of this second exercise are shown in Figure 4.7, which qualitatively reproduces both facts observed in the data. Model estimates of $\hat{\beta}$ and $\hat{\beta}_B$ (**blue dashed** lines in Figure 4.7 panels (a) and (b) respectively) are mostly increasing over time and, quantitatively, show a similar secular pattern to their empirical counterparts (highlighted in **black solid** lines). In both case, the theoretical counterfactuals of the model show as a smooth secular trend with less volatility from year-to-year than observed in their empirical counterparts. This arises as the endowment model relies upon six parameters, each of which change gradually throughout these 70 years, while the empirical results are estimates using hundreds (in the case of $\hat{\beta}$) or even many thousands of observations (in the case of $\hat{\beta}_B$).

4.4 Model with Production

In this section I expand upon the endowment model presented in section 4.3. Production is now endogenous, while the failure of the LOOP is an endogenous result arising from optimal product-level markups across countries. Again, there are two countries, Home and Foreign, in a single time period sharing a common currency.

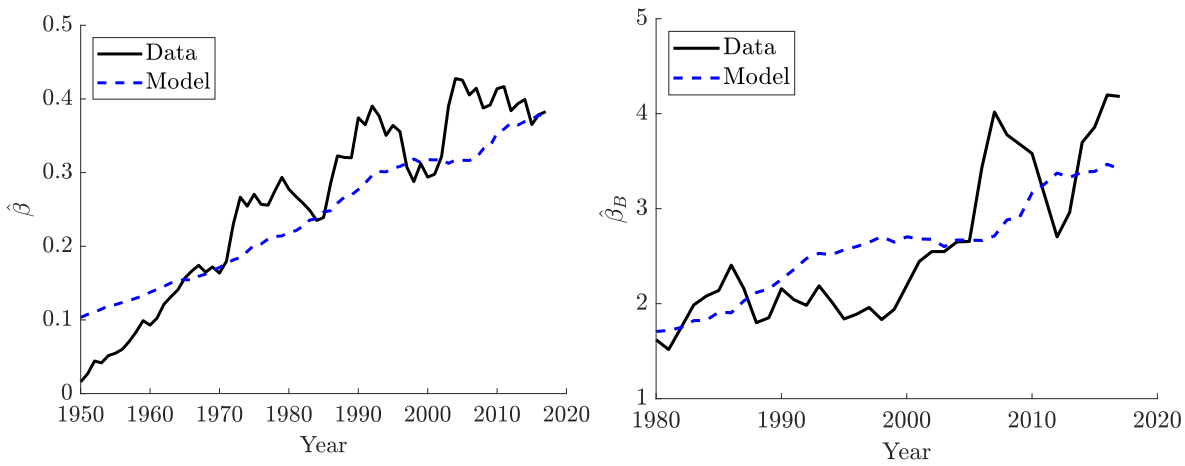
4.4.1 Households

In the Home country, a representative household maximises utility given by:

$$U = \ln C - \frac{\kappa}{1+\psi} L^{1+\psi}, \tag{4.39}$$

[27] NIPA Tables 2.3.3; 2.3.4 and 2.3.5.

Figure 4.7 Theoretical Counterfactuals of the Endowment Model



(a) HBS Effect, $\hat{\beta}$.

(b) LOOP Failure, $\hat{\beta}_B$.

Sources and Notes: Figure shows theoretical counterfactuals from the endowment model against the data counterparts. Panel (a) shows estimates for HBS effect, $\hat{\beta}$, while panel (b) shows estimates for the failure of the LOOP, $\hat{\beta}_B$. In each panel **black solid** lines show data. In panel (a) these data match those shown in Figure 4.1 (c), i.e. the annual estimate of the slope coefficient in the OLS regressions given by equation (4.1), with data taken from Penn World Tables version 10.0 (Feenstra et al., 2015). In panel (b) these data match annual averages of those shown in Figure 4.3 (b), i.e. the average of the 5-year rolling estimates of $\ln \hat{\beta}^i_{B,t}$, from the OLS regression given in equation (4.6). The **blue dashed** lines in this figure show model estimates.

where $\psi \geq 0$ is the inverse of the Frisch elasticity of labour supply and $\kappa > 0$ captures disutility from labour supply, L. Maximisation is subject to a period budget constraint given by:

$$\Pi + WL = PC. \tag{4.40}$$

which states that a household's income from labour, WL, and share of nominal profits, Π, must match their nominal consumption. W represents the nominal wage. As before, P represents the hedonic consumer price level. This results in the standard household optimality condition:

$$W = \kappa L^{\psi} PC. \tag{4.41}$$

Aggregation

As in the endowment model, the index of final *aggregate* consumption, C, is taken to be a Cobb-Douglas measure of tradable and non-tradable goods, with:

$$C = \frac{C_T^{\gamma} C_N^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \tag{4.42}$$

and traded goods expenditure share $0 < \gamma < 1$, which yields consumption demands given by:

$$P_T C_T = \gamma PC, \tag{4.43}$$

$$P_N C_N = (1-\gamma)PC. \tag{4.44}$$

and a standard aggregate consumer price index:

$$P = P_T^{\gamma} P_N^{1-\gamma}. \tag{4.45}$$

The bundle of traded goods is further divided between domestic and foreign products, again according to a Cobb-Douglas measure:

$$C_T = \frac{C_H^{\alpha} C_F^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}, \tag{4.46}$$

where C_H represents consumption of traded goods produced in the Home country, C_F represents consumption of traded goods produced in the Foreign country and the Home country share of traded goods consumption is $0 < \alpha < 1$. This yields consumption demands given by:

$$P_H C_H = \alpha P_T C_T, \tag{4.47}$$

$$P_F C_F = (1-\alpha) P_T C_T. \tag{4.48}$$

and a traded consumer price index:

$$P_T = P_H^{\alpha} P_F^{1-\alpha}. \tag{4.49}$$

The final consumption *sectors* in the model are thus $S \in \{H, F, N\}$. Within each sector households consume brands, $b \in \{h, f, n\}$ according to a CES consumption function:

$$C_S = \left[\int_0^1 C_S(b)^{\frac{\eta-1}{\eta}} db \right]^{\frac{\eta}{\eta-1}}, \tag{4.50}$$

with constant elasticity of substitution $\eta > 1$, where $C_S(b)$ denotes the consumption of brand b within sector S. η is assumed to be constant across sectors. This yields brand demand functions and sectorial consumer price indices respectively given by:

$$C_S(b) = \left(\frac{P_S(b)}{P_S} \right)^{-\eta} C_S, \tag{4.51}$$

$$P_S = \left[\int_0^1 P_S(b)^{1-\eta} db \right]^{\frac{1}{1-\eta}}. \tag{4.52}$$

4.4.2 Firms

In every sector in the Home economy, each producer of a given brand uses a linear production function with labour:

$$\tilde{Y}_S(b) = A_S \tilde{L}_S(b) \tag{4.53}$$

where A_S represents the Home country sector-wide level of labour productivity and $\tilde{L}_S(b)$ is the labour input required to produce a quantity $\tilde{Y}_S(b)$.

Traded Retail Firms

As in the endowment model, and in Burstein et al. (2003), Burstein et al. (2005) and Corsetti and Dedola (2005), although non-tradable goods are sold directly to consumers, the sale of traded goods to households requires use of a perfectly competitive distribution (retail) sector. In this sector, distribution firms produce according to a Leontief (1941) technology:

$$Y_H(h) = \min\left\{\tilde{C}_H(h), \frac{\tilde{C}_N^d(h)}{\phi}\right\}, \qquad \text{and} \qquad Y_F(f) = \min\left\{\tilde{C}_F(f), \frac{\tilde{C}_N^d(f)}{\phi}\right\}, \tag{4.54}$$

where retailers in the Home country use both wholesale traded and domestic non-traded goods as inputs for production. $Y_H(h)$ and $Y_F(f)$ represent the Home country distribution of final traded goods, which use wholesale products from the Home and Foreign country respectively. $\tilde{C}_N^d(h)$ and $\tilde{C}_N^d(f)$ represent the level of the *aggregate* non-traded good used as inputs for the distribution brand h and f respectively. So, the domestic distribution of a quantity, $\tilde{C}_S(b)$, of either Home or Foreign traded products requires $\phi > 0$ units of the domestic non-traded good. Hence, households face retail prices:

$$P_H(h) = \tilde{P}_H(h) + \phi P_N, \tag{4.55}$$

$$P_F(f) = \tilde{P}_F(f) + \phi P_N, \tag{4.56}$$

which strictly exceed the wholesale producer prices of these traded goods, $\tilde{P}_H(h)$ and $\tilde{P}_F(f)$, due to the need for non-traded inputs for distribution. In addition, the optimal choice of wholesale inputs from the distribution sector gives:

$$\tilde{C}_N^d(h) = \phi \tilde{C}_H(h), \tag{4.57}$$

$$\tilde{C}_N^d(f) = \phi \tilde{C}_F(f), \tag{4.58}$$

such that, provided ϕ is fixed, wholesale inputs from the traded and non-traded sectors are again consumed in constant proportions (Tirole, 1994).

Traded Wholesale Firms

Each domestic wholesale producer of a traded goods brand uses the production function (4.53). As sales are made in both the Home and Foreign country, and prices may be set independently across these markets, these firms maximise profits according to:

$$\max_{\tilde{P}_H(h),\tilde{P}^*_H(h)} \tilde{P}_H(h)\tilde{C}_H(h) + \tilde{P}^*_H(h)\tilde{C}^*_H(h) - \frac{W}{A_H}\Big[\tilde{C}_H(h) + \tilde{C}^*_H(h)\Big]. \tag{4.59}$$

Knowing the product distribution process, and competing for consumer demand under monopolistic competition with other brands, each producer of a traded goods brand faces demand from the Home and Foreign country respectively as:

$$\tilde{C}_H(h) = C_H(h) = \left(\frac{\tilde{P}_H(h) + \phi P_N}{P_H}\right)^{-\eta} C_H, \tag{4.60}$$

$$\tilde{C}^*_H(h) = C^*_H(h) = \left(\frac{\tilde{P}^*_H(h) + \phi P^*_N}{P^*_H}\right)^{-\eta} C^*_H, \tag{4.61}$$

which may be shown using (4.51) and (4.55) alongside their Foreign country equivalents. This assumes symmetry across countries, in particular with $\phi = \phi^*$ and $\eta = \eta^*$.

Hence, wholesale prices are optimally set as a markup above marginal costs, W/A_H, as:

$$\tilde{P}_H(h) = \frac{\eta}{\eta - 1}\left(1 + \frac{\phi}{\eta}\frac{A_H P_N}{W}\right)\frac{W}{A_H}, \tag{4.62}$$

$$\tilde{P}^*_H(h) = \frac{\eta}{\eta - 1}\left(1 + \frac{\phi}{\eta}\frac{A_H P^*_N}{W}\right)\frac{W}{A_H}. \tag{4.63}$$

The markup depends not only upon the degree of competitiveness in the traded goods sector, but also on the price of non-traded goods which increases the optimal wholesale price through their role in domestic distribution. Higher domestic distribution costs increase the final consumer price, reducing both retail and wholesale demand. As (4.62) and (4.63) are identical for each brand, h, this indexation in these equations may be suppressed. Furthermore, using this fact, and that (4.52) is homogeneous of degree 1, brand indexation may also be suppressed in (4.55) and (4.56).

Non-traded Firms

From the perspective of non-traded goods producers, the aggregate demand for each non-traded brand is then given as the sum of final and intermediate demand. Using (4.50), (4.51), (4.57) and (4.58) this may be written as:

$$\underbrace{C_N(n)}_{\text{Final demand}} + \underbrace{\tilde{C}_N(n)}_{\text{Intermediate demand}}$$
$$= \underbrace{\left(\frac{P_N(n)}{P_N}\right)^{-\eta} C_N}_{\text{Final demand}} + \underbrace{\phi\left(\frac{P_N(n)}{P_N}\right)^{-\eta}\left(\int_0^1 \tilde{C}_H(h)dh + \int_0^1 \tilde{C}_F(f)df\right)}_{\text{Intermediate demand}}. \tag{4.64}$$

Subject to this condition, non-traded goods firms maximise profits according to:

$$\max_{P_N(n)} P_N(n)C_N(n) + P_N(n)\tilde{C}_N(n) - \frac{W}{A_N}\left[C_N(n) + \tilde{C}_N(n)\right]. \tag{4.65}$$

This results in the standard result that the price is a markup above marginal costs:

$$P_N(n) = \frac{\eta}{\eta - 1}\frac{W}{A_N}. \tag{4.66}$$

Again this expression does not depend upon the brand of non-traded goods, n, which may therefore be suppressed, using the linear homogeneity of (4.52). Using this optimal price for non-traded goods firms, (4.66), alongside the non-traded price index, (4.52), the optimal prices set by traded wholesale firms, (4.62) (4.63), may respectively be rewritten as:

$$\tilde{P}_H = \frac{\eta}{\eta - 1}\left(1 + \frac{\phi}{\eta - 1}\frac{A_H}{A_N}\right)\frac{W}{A_H}, \tag{4.67}$$

$$\tilde{P}_H^* = \frac{\eta}{\eta - 1}\left(1 + \frac{\phi}{\eta - 1}\frac{W^*}{W}\frac{A_H}{A_N^*}\right)\frac{W}{A_H}. \tag{4.68}$$

Finally, noting the symmetry across brands within a sector, profits made by imperfectly competitive wholesale traded and non-traded goods firms are combined and transferred to households as a lump sum payment:

$$\Pi = \left(\tilde{P}_H - \frac{W}{A_H}\right)C_H + \left(\tilde{P}_H^* - \frac{W}{A_H}\right)C_H^* + \left(P_N - \frac{W}{A_N}\right)\left(C_N + \phi(C_H + C_F)\right). \tag{4.69}$$

4.4.3 Equilibrium

Model equilibrium is given whenever households and firms in each country are optimising and all markets clear at a given pricing vector. The wholesale market for each brand, h, of traded goods produced in the Home country clears when production levels are equal to the demand for inputs into retail distribution across both the Home and Foreign country, with:

$$\tilde{Y}_H(h) = \tilde{C}_H(h) + \tilde{C}^*_H(h). \tag{4.70}$$

The corresponding retail market clears whenever distribution matches household demand:

$$Y_H(h) = C_H(h) + C^*_H(h). \tag{4.71}$$

Using (4.51) (4.57) and (4.58), the market for each brand of non-traded goods produced in the Home country will clear whenever:

$$Y_N(n) = C_N(n) + \phi\Big(\frac{P_N(n)}{P_N}\Big)^{-\eta}\Big(\int_0^1 \tilde{C}_H(h)dh + \int_0^1 \tilde{C}_F(f)df\Big). \tag{4.72}$$

Combining labour demand for the production of each brand, the labour market in the Home country will clear whenever:

$$L = \int_0^1 \tilde{L}_H(h)dh + \int_0^1 \tilde{L}_N(n)dn. \tag{4.73}$$

Therefore, an equilibrium for this model with endogenous production exists as the solution to the 14 Home country equations (4.40), (4.41), (4.43), (4.44), (4.45), (4.47), (4.48), (4.49), (4.55), (4.56), (4.66), (4.67), (4.68) and (4.69), the equations (4.53), (4.70), (4.71) and (4.72), which hold for each brand, along with all the Foreign country equivalents. In addition, to close the model we set nominal wages in the Home country as the numéraire:

$$W = 1, \tag{4.74}$$

and assume:

$$\tilde{P}^*_H\tilde{C}^*_H = \tilde{P}_F\tilde{C}_F, \tag{4.75}$$

which ensures the international trade in intermediate goods is balanced across countries.[28] Appendix C.3 presents a full list of endogenous variables and lists the equations which solve the model with endogenous production. In the context of the model with production, whenever the real exchange rate is required, the following definition is used:

$$Q \equiv \frac{P^*}{P}, \tag{4.76}$$

which, as the ratio of final consumer prices in the Foreign and Home countries, is an identical expression to that in the endowment model.

4.5 Results and Sensitivities

This section presents results from model estimation, which is calibrated to match US data at an annual frequency for the 70 years between 1950 and 2019. A full list of the parameters, and their values, is given in Table 4.8. Several parameters are ubiquitous in the international macroeconomics literature (namely ψ, κ, η, and α). For this reason, these parameters are set to standard values and taken as fixed over time.

Table 4.8 Baseline Parameter Calibration

Description	Parameter	Value	Source
Frisch elasticity of labour supply	ψ^{-1}	1	Standard value
Disutility of labour supply	κ	1	Standard value
Brand-level elasticity of substitution	η	6	Standard value
Domestic expenditure share of traded goods	α	0.65	Standard value
Traded goods consumption expenditure share	γ	$0.35^{\dagger}$	US NIPA Table 2.3.5
Distribution margin	ϕ	$0.29^{\dagger}$	US NIPA Table 2.3.5 & IO-bridge
Home traded productivity	A_H	$1.00^{\dagger}$	US NIPA Table 2.3.5
Home non-traded productivity	A_N	$1.77^{\dagger}$	US NIPA Table 2.3.5
Foreign traded productivity	A_F^*	$0.34^{\ddagger}$	IPC
Foreign non-traded productivity	A_N^*	$0.28^{\ddagger}$	IPC

Notes: For parameter values which are adjusted each year in the theoretical counterfactuals † denotes the 1950-2019 average value and ‡ denotes the double average across both the 1950-2019 time period and across countries. NIPA refers to the National Income and Product Accounts. IO-bridge refers to the NIPA Input-Output PCE-bridge. IPC refers to the World Bank's International Comparison Program.

[28] A more complex model would permit persistent current account imbalances to replicate changes in the global economy, particularly in the US since the early 1990s, e.g. see Engel and Rogers (2006). However, as shown empirically in section 4.2 and Table 4.2, capital account restrictions appear to have little empirical impact on the structural trends being examined.

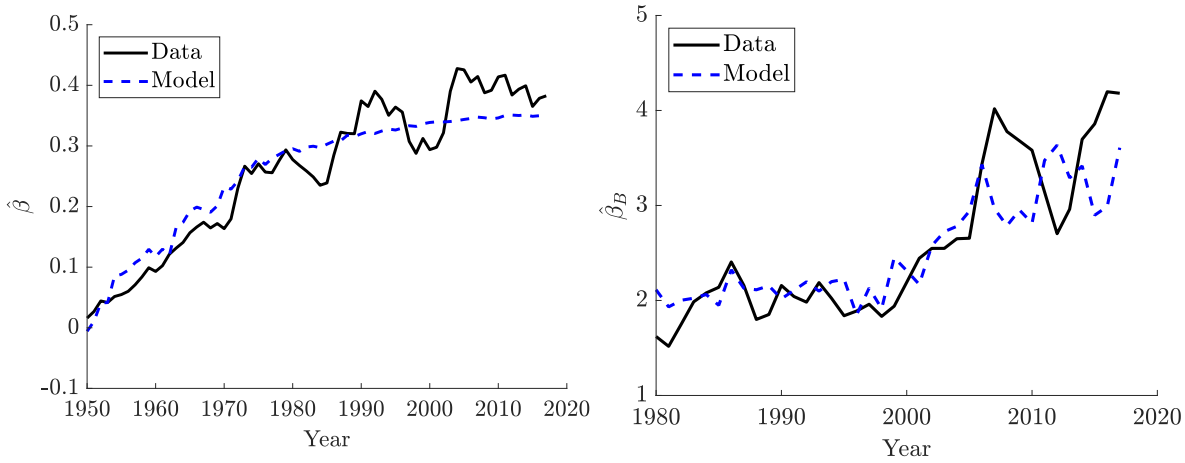
The remaining parameters are replaced each year, based on US data for the Home country and the World Bank's International Comparison Program (ICP) for the Foreign country, as in section 4.3.4. Each year, the income share of traded goods, γ, and domestic distributional requirements, ϕ are computed using the US NIPA tables as described in section 4.3.4. Each year, the Home country productivity variables, A_H and A_N, are computed to ensure Y_H and Y_N match the observed data from the US NIPA tables. The Foreign country productivity variables, A_F^* and A_N^*, are chosen to match observed distribution of Y_F^* and Y_N^* from the 107 countries included in the World Bank's International Comparison Program (ICP). These productivity values are also normalised to ensure $Y_H = 1$ holds on average during the sample years, and to target the average value of $\hat{\beta}_B$ in the sample.

Finally, after simulating the model, theoretical counterparts for $\hat{\beta}$ and $\hat{\beta}_B$ are again constructed using the general definitions (4.29) and (4.30), along with the definition of the real exchange rate, (4.76). These theoretical expressions may then be compared directly to their empirical counterparts, described in section 4.2.

The results of theoretical counterfactuals from the model with production are given in Figure 4.8 and Table 4.9, which both compare the theoretical and empirical counterparts of $\hat{\beta}$ and $\hat{\beta}_B$. Figure 4.8 shows that the baseline specification can simultaneously capture the main trends in the data for both $\hat{\beta}$ and $\hat{\beta}_B$. The cross-country relationship between consumer prices and the ratio of output levels between the Home and Foreign countries has strengthened over time, as shown by the mostly increasing **blue dashed** line in Figure 4.8, panel (a). As shown by the **black solid** line in the same panel, this closely matches the observed empirical counterparts discussed in section 4.2. Similarly, in the model, the Law of One Price in traded goods increasingly fails across the sample period, particularly since 2000. This is shown by the mostly increasing **blue dashed** line in Figure 4.8, panel (b). Again, this closely matches the empirical counterpart as given in the **black solid** line in the same panel. In both cases the dynamics of these relationships are a little more subdued in the baseline counterfactual, than these data show.

A summary of these results is also displayed numerically in Table 4.9. The first two columns highlight the close relationship between data and the theoretical counterfactuals in the model with production, given as column (1). This baseline uses parameters as specified in the main text and Table 4.8. Although the mean of $\hat{\beta}_B$ is a close match to the data, and qualitatively both the data and baseline specification show an increase in $\hat{\beta}_B$, the data highlight a greater difference between the starting and end points than the model is able to replicate. This arises both due to the marginally higher starting value of the theoretical counterpart and due to the model not replicating the amplitude, and indeed phase cycles, of the most recent years.

Figure 4.8 Theoretical Counterfactuals from the Model with Production



(a) HBS Effect, $\hat{\beta}$.

(b) LOOP Failure, $\hat{\beta}_B$.

Sources and Notes: Figure shows theoretical counterfactuals from the model with production against the data counterparts. Panel (a) shows estimates for HBS effect, $\hat{\beta}$, while panel (b) shows estimates for the failure of the LOOP, $\hat{\beta}_B$. In each panel **black solid** lines show data. In panel (a) these data match those shown in Figure 4.1 (c), i.e. the annual estimate of the slope coefficient in the OLS regressions given by equation (4.1), with data taken from Penn World Tables version 10.0 (Feenstra et al., 2015). In panel (b) these data match annual averages of those shown in Figure 4.3 (b), i.e. the average of the 5-year rolling estimates of $\ln \hat{\beta}^i_{B,t}$, from the OLS regression given in equation (4.6). The **blue dashed** lines in this figure show model estimates using the baseline parameter specification.

Table 4.9 also reports several alternative counterfactuals for the model with production. These highlight the role of both distribution services and the expenditure share of traded goods in establishing these patterns. The sensitivity with respect to distribution services is highlighted in theoretical counterfactuals (2) and (3), which respectively set ϕ at the constant values $\phi = 0.29$ and $\phi = 0$. Using $\phi = 0.29$ sets the level of distribution services to the empirical mean observed in the data, turning off any changes throughout the sample period. In contrast, using $\phi = 0$ turns off the impact of distribution services completely. Using the mean of the data, without allowing distribution services to increase through time, marginally increases the model's prediction for the change in the HBS effect, i.e. changes in $\hat{\beta}$. The impact on predictions for the failure of the LOOP, i.e. changes in $\hat{\beta}_B$, are more severe with this variant further under-predicting the increasing failure, compared to the baseline specification. Alternatively, using a model without distribution services, i.e. $\phi = 0$, is problematic for several reasons. As $\phi \longrightarrow 0$ model predictions begin to imply that the level of $\hat{\beta}$ turns negative. As shown in Table 4.9, the case of $\phi = 0$ is associated with $\hat{\beta} = -1$. In addition to the theoretical measures of $\hat{\beta}$ being of opposite sign to their empirical counterparts when distribution services are turned off, neither the HBS effects or LOOP failure change over time, shown in the Table by the fact that the change in both $\hat{\beta}$ and $\hat{\beta}_B$ falls to 0, as these become constant for any value of γ and the productivity parameters. Finally, although it is not possible to turn off traded goods, counterfactual (4) provides a counterpart with a constant value of γ, again at the mean value from the data. In this setting, although PPP deviations, and their trend, are replicated in a pattern similar to the baseline setting, the model's under-prediction of the increasing failure of the LOOP is exacerbated.

Table 4.9 Sensitivity Analysis

	Data	(1) Baseline	(2) Constant ϕ	(3) $\phi = 0$	(4) Constant γ
HBS Effect, $\hat{\beta}$					
average (1950-2019)	0.26	0.26	0.26	-1.00	0.26
change during 1950-2019	0.37	0.36	0.37	0.00	0.33
LOOP Deviations, $\hat{\beta}_B$					
average (1950-2019)	2.49	2.49	2.45	9.19	2.43
change during 1950-2019	2.44	1.43	1.12	0.00	0.93

Notes: This table reports data and various theoretical counterfactuals of the model with production for both $\hat{\beta}$ and $\hat{\beta}_B$. Counterfactual (1) refers to the baseline parameter specification of the model with production, as outlined in Table 4.8 and the main text. Counterfactuals (2) and (4) use constant values for ϕ and γ respectively, taken as the average for the 1950 to 2019 period. Counterfactual (3) uses $\phi = 0$.

Overall, the theoretical counterfactuals in Table 4.9 demonstrate the importance of both an increasing share of distribution services and traded goods expenditure share in replicating the two stylised facts presented in section 4.2.

4.6 Conclusion

In this paper I document how two of the most widely used empirical regularities within international macroeconomics have changed over the past 70 years. PPP is ubiquitous as a fundamental concept and benchmark for most theoretical and empirical work in international macroeconomics. However, the positive cross-country relationship between consumer prices and real output per capita has strengthened (i.e. a rise in the Harrod-Balassa-Samuelson effect). I demonstrate this by showing how empirical estimates of the slope in cross-sectional Ordinary Least Squares (OLS) regressions, which is typically used to highlight violations of PPP, has increased over time, using data from the Penn World Tables. I provide a series of robustness checks for this stylised empirical fact.

I also present a second stylised empirical fact: that border frictions appear to have increased, with international borders effectively becoming wider and an increasing failure of the Law of One Price (LOOP). I construct my own dataset of city-level relative prices using national sources across five continents to document the increasing failure of the LOOP. The magnitude of cross-sectional relative price variation explained by a border effect has increased since the 1980s, when these data series start.

I then demonstrate using a two-country endowment model, with a domestic distribution services sector, how to construct an equilibrium failure of the LOOP. In this setting traded goods require some domestic non-traded services in order for products to reach the consumers. This drives a supply-side wedge between producer and consumer prices, breaking the LOOP. If local distribution requirements increase over time, real income differences are reflected more in overall consumer price level differences, through the non-traded component of the price differential in traded goods. This implies that an increase in the relative size of the distribution services sector can simultaneously explain both stylised facts.

The final sections of the paper expand this idea to a production setting and add monopolistically competitive intermediate goods firms. When local distribution services are required to bring tradable products to market, the optimisation problem of these intermediate goods firms results in endogenous pricing-to-market. After solving and calibrating the model in this richer environment, I demonstrate how it may be used to closely replicate the two stylised facts.

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Appendix A

Product Quality, Measured Inflation and Monetary Policy

A.1 Derivation of Intermediate Firms' First Order Conditions

A.1.1 Sticker Price Adjustment Costs

Under Rotemberg (1982) pricing with sticker price adjustment costs, the real profit flow for an individual intermediate goods producer is given by:

$$D_t(j) = (1+\tau_t)\frac{P_t(j)}{P_t}Y_t(j) - \frac{W_t}{P_t}N_t(j) - \frac{\theta_P}{2}\left(\frac{P_t(j)}{(1+\bar{\Pi}^S)P_{t-1}(j)} - 1\right)^2 Y_t - \frac{\theta_F}{2}\left(\frac{F_t(j)}{(1+\bar{\Pi}^F)F_{t-1}(j)} - 1\right)^2 Y_t. \tag{A.1}$$

Using (2.10) and (2.11) to rewrite labour costs and then using the market clearing condition under monopolistic competition to equate (2.6) and (2.10), this can be rewritten as:

$$D_t(j) = (1+\tau_t)\left[\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t - \frac{\theta_P}{2}\left(\frac{P_t(j)}{(1+\bar{\Pi}^S)P_{t-1}(j)} - 1\right)^2 Y_t - \frac{\theta_F}{2}\left(\frac{F_t(j)}{(1+\bar{\Pi}^F)F_{t-1}(j)} - 1\right)^2 Y_t. \tag{A.2}$$

Under monopolistic competition, the full optimization problem facing intermediate firms is then:

$$\max_{\{P_s(j),F_s(j)\}_{s=t}^{\infty}} \mathbb{E}_t\left[\sum_{s=t}^{\infty}\beta^{s-t}\frac{C_s^{-\sigma}}{C_t^{-\sigma}}\left\{\left[(1+\tau_s)\frac{P_s(j)}{P_s} - MC_s(j)\right]\mathcal{G}_s(j)\left(\frac{P_s(j)}{P_s}\right)^{-\varepsilon} Y_s - \frac{\theta_P}{2}\left(\frac{P_s(j)}{(1+\bar{\Pi}^S)P_{s-1}(j)} - 1\right)^2 Y_s - \frac{\theta_F}{2}\left(\frac{F_s(j)}{(1+\bar{\Pi}^F)F_{s-1}(j)} - 1\right)^2 Y_s\right\}\right], \tag{A.3}$$

where $u_C(C_t) = C_t^{-\sigma}$ is the derivative of the households utility function with respect to consumption and the functional form has been used in the stochastic discount factor.

The first order optimality condition from (A.3) with respect to the sticker price $P_t(j)$ is:

$$
\begin{aligned}
0 = {} & (1+\tau_t)\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}\frac{Y_t}{P_t} - \varepsilon\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon-1}\mathcal{G}_t(j)\frac{Y_t}{P_t} \\
& - \theta_P\left(\frac{P_t(j)}{(1+\bar{\Pi}^S)P_{t-1}(j)} - 1\right)\frac{Y_t}{(1+\bar{\Pi}^S)P_{t-1}(j)} \\
& + \beta\theta_P\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\left(\frac{P_{t+1}(j)}{(1+\bar{\Pi}^S)P_t(j)} - 1\right)Y_{t+1}\frac{P_{t+1}(j)}{(1+\bar{\Pi}^S)[P_t(j)]^2}\right]. \qquad \text{(A.4)}
\end{aligned}
$$

The first order optimality condition from (A.3) with respect to the level of product quality, $F_t(j)$, is:

$$
\begin{aligned}
0 = {} & \left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_{F,t}(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}Y_t + MC_t(j)\frac{\mathcal{H}_{F,t}(j)}{\mathcal{H}_t(j)}\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}Y_t \\
& - \theta_F\left(\frac{F_t(j)}{(1+\bar{\Pi}^F)F_{t-1}(j)} - 1\right)\frac{Y_t}{(1+\bar{\Pi}^F)F_{t-1}(j)} \\
& + \beta\theta_F\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\left(\frac{F_{t+1}(j)}{(1+\bar{\Pi}^F)F_t(j)} - 1\right)Y_{t+1}\frac{F_{t+1}(j)}{(1+\bar{\Pi}^F)[F_t(j)]^2}\right], \qquad \text{(A.5)}
\end{aligned}
$$

using (2.11) and:

$$
MC_{F,t}(j) \equiv \frac{\partial MC_t(j)}{\partial F_t(j)} = -\frac{W_t}{P_t}\frac{\mathcal{H}_{F,t}(j)}{[\mathcal{H}_t(j)]^2} = -MC_t(j)\frac{\mathcal{H}_{F,t}(j)}{\mathcal{H}_t(j)}. \qquad \text{(A.6)}
$$

Next, both first order conditions (A.4) and (A.5) are rearranged. Equation (A.4) is multiplied by $\frac{P_t(j)}{Y_t}$, while (A.5) is multiplied by $\frac{F_t(j)}{Y_t}$. The final equality in (A.6) is also used in (A.5), such

that (A.4) and (A.5) respectively become:

$$
\begin{aligned}
0 &= (1+\tau_t)\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{1-\varepsilon} - \varepsilon\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} \\
&- \theta_P\left(\frac{P_t(j)}{P_{t-1}(j)(1+\bar{\Pi}^S)} - 1\right)\frac{P_t(j)}{(1+\bar{\Pi}^S)P_{t-1}(j)} \\
&+ \beta\theta_P\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{P_{t+1}(j)}{(1+\bar{\Pi}^S)P_t(j)} - 1\right)\frac{P_{t+1}(j)}{(1+\bar{\Pi}^S)P_t(j)}\right], \qquad \text{(A.7)} \\
0 &= \left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]F_t(j)\mathcal{G}_{F,t}(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} \\
&+ MC_t(j)\frac{\mathcal{H}_{F,t}(j)}{\mathcal{H}_t(j)}F_t(j)\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} \\
&- \theta_F\left(\frac{F_t(j)}{(1+\bar{\Pi}^F)F_{t-1}(j)} - 1\right)\frac{F_t(j)}{(1+\bar{\Pi}^F)F_{t-1}(j)} \\
&+ \beta\theta_F\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{F_{t+1}(j)}{(1+\bar{\Pi}^F)F_t(j)} - 1\right)\frac{F_{t+1}(j)}{(1+\bar{\Pi}^F)F_t(j)}\right]. \qquad \text{(A.8)}
\end{aligned}
$$

We assume a symmetric equilibrium where, faced with identical first order conditions, all intermediate goods firms behave identically.[1] All intermediate goods firms charge the same sticker price, $P_t(j)$, and produce the same output at the same level of product quality, $F_t(j)$. Note that $\mathcal{G}_t(j)$, $\mathcal{H}_t(j)$ and $MC_t(j)$ depend upon the index j only through the firm's choice of $F_t(j)$. In this setting we may therefore write the aggregate price level (2.7) as a function of the individual level of prices and quality:

$$P_t = [\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j). \qquad \text{(A.9)}$$

[1]This *assumption* of symmetry is not required whenever (A.7) and (A.8) give a unique solution for $P_t(j)$ and $F_t(j)$. In this case the unique solution ensures a symmetric equilibrium. However, without first specifying the functional forms of $\mathcal{G}_t$ and $\mathcal{H}_t$ this uniqueness is not guaranteed and we therefore assume a symmetric solution to continue with general forms.

Using the assumption of symmetry across intermediate goods firms (A.9) can then be used to rewrite (A.7) and (A.8) respectively, while dropping individual indexation j on quality, as:

$$\begin{aligned}
0 &= (1+\tau_t)(1-\varepsilon) + \varepsilon MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} \\
&- \theta_P \left(\left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}} \right]^{\frac{1}{\varepsilon-1}} \frac{P_t}{(1+\bar{\Pi}^S)P_{t-1}} - 1 \right) \left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}} \right]^{\frac{1}{\varepsilon-1}} \frac{P_t}{(1+\bar{\Pi}^S)P_{t-1}} \\
&+ \beta\theta_P \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \left(\left[\frac{\mathcal{G}_{t+1}}{\mathcal{G}_t} \right]^{\frac{1}{\varepsilon-1}} \frac{P_{t+1}}{(1+\bar{\Pi}^S)P_t} - 1 \right) \left[\frac{\mathcal{G}_{t+1}}{\mathcal{G}_t} \right]^{\frac{1}{\varepsilon-1}} \frac{P_{t+1}}{(1+\bar{\Pi}^S)P_t} \right], \qquad \text{(A.10)} \\
0 &= (1+\tau_t) F_t \mathcal{G}_{F,t} \mathcal{G}_t^{-1} - MC_t F_t \mathcal{G}_{F,t} \mathcal{G}_t^{-\frac{\varepsilon}{\varepsilon-1}} \\
&+ MC_t \frac{\mathcal{H}_{F,t}}{\mathcal{H}_t} F_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} - \theta_F \left(\frac{F_t}{(1+\bar{\Pi}^F)F_{t-1}} - 1 \right) \frac{F_t}{(1+\bar{\Pi}^F)F_{t-1}} \\
&+ \beta\theta_F \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{F_{t+1}}{(1+\bar{\Pi}^F)F_t} - 1 \right) \frac{F_{t+1}}{(1+\bar{\Pi}^F)F_t} \right]. \qquad \text{(A.11)}
\end{aligned}$$

Next we define the gross rate of hedonic aggregate price inflation, product quality improvement and sticker price inflation as:

$$1 + \Pi_t \equiv \frac{P_t}{P_{t-1}}, \qquad \text{(A.12)}$$

$$1 + \Pi_t^F \equiv \frac{F_t}{F_{t-1}}, \qquad \text{(A.13)}$$

$$1 + \Pi_t^S \equiv \frac{P_t(j)}{P_{t-1}(j)} = \left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}} \right]^{\frac{1}{\varepsilon-1}} (1 + \Pi_t), \qquad \text{(A.14)}$$

where (A.14) follows from definitions (A.12) and (A.13) being used alongside the form of pricing under symmetry as given in (A.9). Using definitions (A.12), (A.13) and (A.14) in equations (A.10) and (A.11) we may obtain the intermediate firms' First Order Condition for Prices (FOC P) and quality (FOC F) respectively:

$$\begin{aligned}
(1+\tau_t)(\varepsilon - 1) &= \varepsilon MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} - \theta_P \left(\frac{1+\Pi_t^S}{1+\bar{\Pi}^S} - 1 \right) \frac{1+\Pi_t^S}{1+\bar{\Pi}^S} \\
&+ \beta\theta_P \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{1+\Pi_{t+1}^S}{1+\bar{\Pi}^S} - 1 \right) \frac{1+\Pi_{t+1}^S}{1+\bar{\Pi}^S} \right], \qquad \text{(FOC P)} \\
\frac{(1+\tau_t) F_t \mathcal{G}_{F,t}}{\mathcal{G}_t} &= MC_t F_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} \left(\frac{\mathcal{G}_{F,t}}{\mathcal{G}_t} - \frac{\mathcal{H}_{F,t}}{\mathcal{H}_t} \right) + \theta_F \left(\frac{1+\Pi_t^F}{1+\bar{\Pi}^F} - 1 \right) \frac{1+\Pi_t^F}{1+\bar{\Pi}^F} \\
&- \beta\theta_F \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{1+\Pi_{t+1}^F}{1+\bar{\Pi}^F} - 1 \right) \frac{1+\Pi_{t+1}^F}{1+\bar{\Pi}^F} \right]. \qquad \text{(FOC F)}
\end{aligned}$$

A.1.2 Hedonic Price Adjustment Costs

Under Rotemberg (1982) pricing with hedonic price adjustment costs, the real profit flow for a given intermediate goods producer is given by:

$$D_t(j) = (1+\tau_t)\frac{P_t(j)}{P_t}Y_t(j) - \frac{W_t}{P_t}N_t(j) - \frac{\theta}{2}\left(\frac{1+\Pi_t}{1+\bar{\Pi}} - 1\right)^2 Y_t, \tag{A.15}$$

Using (2.10) and (2.11) to rewrite labour costs and then using the market clearing condition under monopolistic competition to equate (2.6) and (2.10), this can be rewritten as:

$$\begin{aligned} D_t(j) = (1+\tau_t)&\left[\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t \\ &- \frac{\theta}{2}\left(\frac{[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)}{(1+\bar{\Pi})[\mathcal{G}_{t-1}(j)]^{\frac{1}{1-\varepsilon}}P_{t-1}(j)} - 1\right)^2 Y_t, \end{aligned} \tag{A.16}$$

where (A.14) has been used to rewrite the term $1+\Pi_t$, again under the assumption of a symmetric equilibrium.

Under monopolistic competition, the full optimization problem facing intermediate firms is then:

$$\begin{aligned} \max_{\{P_s(j),F_s(j)\}_{s=t}^{\infty}} \mathbb{E}_t\Bigg[\sum_{s=t}^{\infty}\beta^{s-t}\frac{u_C(C_s)}{u_C(C_t)}\Bigg\{&\left[(1+\tau_s)\frac{P_s(j)}{P_s} - MC_s(j)\right]\mathcal{G}_s(j)\left(\frac{P_s(j)}{P_s}\right)^{-\varepsilon} Y_s \\ &- \frac{\theta}{2}\left(\frac{[\mathcal{G}_s(j)]^{\frac{1}{1-\varepsilon}}P_s(j)}{(1+\bar{\Pi})[\mathcal{G}_{s-1}(j)]^{\frac{1}{1-\varepsilon}}P_{s-1}(j)} - 1\right)^2 Y_s\Bigg\}\Bigg]. \end{aligned} \tag{A.17}$$

The first order optimality condition from (A.17) with respect to prices, $P_t(j)$, is:

$$\begin{aligned} &(1+\tau_t)\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}\frac{Y_t}{P_t} - \varepsilon\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon-1}\mathcal{G}_t(j)\frac{Y_t}{P_t} \\ &- \theta\left(\frac{[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)}{(1+\bar{\Pi})[\mathcal{G}_{t-1}(j)]^{\frac{1}{1-\varepsilon}}P_{t-1}(j)} - 1\right)\frac{[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}Y_t}{(1+\bar{\Pi})[\mathcal{G}_{t-1}(j)]^{\frac{1}{1-\varepsilon}}P_{t-1}(j)} \\ &+ \beta\theta\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\left(\frac{[\mathcal{G}_{t+1}(j)]^{\frac{1}{1-\varepsilon}}P_{t+1}(j)}{(1+\bar{\Pi})[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)} - 1\right)Y_{t+1}\frac{[\mathcal{G}_{t+1}(j)]^{\frac{1}{1-\varepsilon}}P_{t+1}(j)}{(1+\bar{\Pi})[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}[P_t(j)]^2}\right] = 0. \end{aligned} \tag{A.18}$$

The first order optimality condition from (A.17) with respect to product quality, $F_t(j)$, is:

$$\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_{F,t}(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t - MC_{F,t}(j)\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} Y_t$$
$$-\frac{\theta}{1-\varepsilon}\left(\frac{[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)}{(1+\bar{\Pi})[\mathcal{G}_{t-1}(j)]^{\frac{1}{1-\varepsilon}}P_{t-1}(j)} - 1\right)\frac{[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)\mathcal{G}_{F,t}(j)Y_t}{(1+\bar{\Pi})[\mathcal{G}_{t-1}(j)]^{\frac{1}{1-\varepsilon}}P_{t-1}(j)\mathcal{G}_t(j)}$$
$$+\frac{\beta\theta}{1-\varepsilon}\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\left(\frac{[\mathcal{G}_{t+1}(j)]^{\frac{1}{1-\varepsilon}}P_{t+1}(j)}{(1+\bar{\Pi})[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)} - 1\right)Y_{t+1}\frac{[\mathcal{G}_{t+1}(j)]^{\frac{1}{1-\varepsilon}}P_{t+1}(j)\mathcal{G}_{F,t}(j)}{(1+\bar{\Pi})[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}+1}P_t(j)}\right] = 0. \tag{A.19}$$

where the definition in (A.6) is used in (A.19), and both (A.18) and (A.19) use the functional form of household utility in the stochastic discount factor with $u_C(C_t) = C_t^{-\sigma}$.

Next both first order conditions (A.18) and (A.19) are rearranged. Equation (A.18) is multiplied by $\frac{P_t(j)}{Y_t}$, while (A.19) is multiplied by $\frac{\mathcal{G}_t(j)}{\mathcal{G}_{F,t}(j)Y_t}$. The final equality in (A.6) is also used in (A.19), such that (A.18) and (A.19) respectively become:

$$(1+\tau_t)\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{1-\varepsilon} - \varepsilon\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}$$
$$-\theta\left(\frac{[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)}{(1+\bar{\Pi})[\mathcal{G}_{t-1}(j)]^{\frac{1}{1-\varepsilon}}P_{t-1}(j)} - 1\right)\frac{[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)}{(1+\bar{\Pi})[\mathcal{G}_{t-1}(j)]^{\frac{1}{1-\varepsilon}}P_{t-1}(j)}$$
$$+\beta\theta\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{[\mathcal{G}_{t+1}(j)]^{\frac{1}{1-\varepsilon}}P_{t+1}(j)}{(1+\bar{\Pi})[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)} - 1\right)\frac{[\mathcal{G}_{t+1}(j)]^{\frac{1}{1-\varepsilon}}P_{t+1}(j)}{(1+\bar{\Pi})[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)}\right] = 0, \tag{A.20}$$

$$\left[(1+\tau_t)\frac{P_t(j)}{P_t} - MC_t(j)\right]\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} + MC_t(j)\frac{\mathcal{H}_{F,t}(j)\mathcal{G}_t(j)}{\mathcal{H}_t(j)\mathcal{G}_{F,t}(j)}\mathcal{G}_t(j)\left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon}$$
$$-\frac{\theta}{1-\varepsilon}\left(\frac{[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)}{(1+\bar{\Pi})[\mathcal{G}_{t-1}(j)]^{\frac{1}{1-\varepsilon}}P_{t-1}(j)} - 1\right)\frac{[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)}{(1+\bar{\Pi})[\mathcal{G}_{t-1}(j)]^{\frac{1}{1-\varepsilon}}P_{t-1}(j)}$$
$$+\frac{\beta\theta}{1-\varepsilon}\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{[\mathcal{G}_{t+1}(j)]^{\frac{1}{1-\varepsilon}}P_{t+1}(j)}{(1+\bar{\Pi})[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)} - 1\right)\frac{[\mathcal{G}_{t+1}(j)]^{\frac{1}{1-\varepsilon}}P_{t+1}(j)}{(1+\bar{\Pi})[\mathcal{G}_t(j)]^{\frac{1}{1-\varepsilon}}P_t(j)}\right] = 0. \tag{A.21}$$

As in section A.1.1, faced with symmetric first order conditions, all intermediate firms behave identically in equilibrium and will charge the same sticker price, $P_t(j)$, and produce the same output at the same level of product quality, $F_t(j)$. The aggregate price level therefore still obeys equation (A.9), which may then be used to rewrite (A.20) and (A.21) respectively, while dropping individual indexation on quality. Using definitions (A.12), (A.13) and (A.14), we obtain the alternative version of the intermediate firms' first order condition for prices

and quality respectively:

$$(1+\tau_t)(\varepsilon-1) = \varepsilon MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} - \theta\left(\frac{1+\Pi_t}{1+\bar{\Pi}} - 1\right)\frac{1+\Pi_t}{1+\bar{\Pi}} + \beta\theta\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{1+\Pi_{t+1}}{1+\bar{\Pi}} - 1\right)\frac{1+\Pi_{t+1}}{1+\bar{\Pi}}\right], \tag{A.22}$$

$$(1+\tau_t)(\varepsilon-1) = (1-\varepsilon)\left[\frac{\mathcal{H}_{F,t}\mathcal{G}_t}{\mathcal{H}_t\mathcal{G}_{F,t}} - 1\right] MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} - \theta\left(\frac{1+\Pi_t}{1+\bar{\Pi}} - 1\right)\frac{1+\Pi_t}{1+\bar{\Pi}} + \beta\theta\mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{1+\Pi_{t+1}}{1+\bar{\Pi}} - 1\right)\frac{1+\Pi_{t+1}}{1+\bar{\Pi}}\right]. \tag{A.23}$$

In a model with hedonic price adjustment costs, these would replace (FOC P) and (FOC F). Taking the difference of (A.22) and (A.23) implies:

$$\frac{1}{\varepsilon-1} = -\frac{\mathcal{H}_{F,t}}{\mathcal{H}_t}\frac{\mathcal{G}_t}{\mathcal{G}_{F,t}}. \tag{A.24}$$

So, with costly hedonic price adjustments, the optimal level of product quality chosen by intermediate goods firms is always consistent with the level of product quality chosen by a benevolent social planner without sticker price or product quality frictions ($\theta_P = \theta_F = 0$). This is characterised by (A.26) in Appendix A.2.

A.2 The Frictionless First-Best Social Planner's Problem

Under flexible prices and quality, with $\theta_P = \theta_F = 0$, the first-best allocation solves the problem of maximising household utility, given production constraints. That is, in this setting, we assume the social planner may overcome frictions caused by the presence of nominal sticker price adjustment and product quality alteration costs facing the intermediate firm.[2] In this setting there are no real resource costs for product quality adjustment. The Lagrangian for this allocation is therefore:

$$\mathcal{L} = \max_{\dot{C},\dot{F},\dot{L}} \left\{ \frac{\dot{C}^{1-\sigma}}{1-\sigma} - \frac{\dot{L}^{1+\eta}}{1+\eta} - \lambda \left(\dot{C} - \dot{\mathcal{G}}^{\frac{1}{\varepsilon-1}} \dot{\mathcal{H}} \dot{L} \right) \right\}, \tag{A.25}$$

where a dot denotes the frictionless first best outcome. The production constraint uses (2.34) and (2.35) alongside the assumption $\theta_F = \theta_P = 0$. The efficient solution is characterised by:

$$\frac{1}{\varepsilon - 1} = -\frac{\dot{\mathcal{H}}_F}{\dot{\mathcal{H}}} \frac{\dot{\mathcal{G}}}{\dot{\mathcal{G}}_F}, \tag{A.26}$$

$$\dot{L} = \left(\dot{\mathcal{G}}^{\frac{1}{\varepsilon-1}} \dot{\mathcal{H}} \right)^{\frac{1-\sigma}{\sigma+\eta}}, \tag{A.27}$$

$$\dot{Y} = \dot{C} = \left(\dot{\mathcal{G}}^{\frac{1}{\varepsilon-1}} \dot{\mathcal{H}} \right)^{\frac{1+\eta}{\sigma+\eta}}. \tag{A.28}$$

These may be solved once the functions are $\mathcal{G}$ and $\mathcal{H}$ are specified. Given that the household optimality condition for consumption and leisure determines the equilibrium real wage, the efficient level of intermediate firms marginal costs, in the frictionless setting, are therefore given as:

$$\dot{MC} = \frac{\dot{W}}{\dot{P}} \frac{1}{\dot{\mathcal{H}}} = \frac{\dot{C}^{\sigma} \dot{L}^{\eta}}{\dot{\mathcal{H}}} = \dot{\mathcal{G}}^{\frac{1}{\varepsilon-1}}. \tag{A.29}$$

using (2.4) and (2.11).

In the standard case where product quality has no impact on demand, i.e. whenever $\mathcal{G} = 1$, then the solution $\dot{MC} = 1$ is efficient. However in this economy, this is replaced by (A.29) as the efficient marginal cost is increasing in the level of product quality, which has additional value to households.

[2] In general the frictionless first-best outcome presented in this appendix does not coincide with the first-best outcome with $\theta_F > 0$. However, as both share the same steady state any log-linearisation around this outcome still provides a good approximate solution for convergence purposes. Moreover this frictionless first best solution is the natural benchmark to use as it aligns directly with the counterpart in the standard NK literature.

Using (A.26), and the functional forms given in section 2.3.1, we are able to characterise the social planner's optimal choice for product quality in a single period as:

$$\dot{F}_t = A_t \left(\frac{\phi}{\kappa(\varepsilon - 1 + \phi)} \right)^{\frac{\nu}{\nu-1}}, \tag{A.30}$$

Hence the solution for the optimal level of the change in product quality between two periods equals:

$$1 + \dot{\Pi}_t^F = \frac{\dot{F}_t}{\dot{F}_{t-1}} = \frac{A_t}{A_{t-1}}. \tag{A.31}$$

We may also use (A.28) and the household Euler equation for bonds, (2.3), to determine the level of the real interest rate, $1 + r_t = \frac{1+i_t}{1+\Pi_{t+1}}$, implicitly set by the social planner in the frictionless first best case, in any given period:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{\dot{C}_{t+1}}{\dot{C}_t} \right)^{-\sigma} (1 + \dot{r}_t) \right] = \beta \mathbb{E}_t \left[\left(\frac{\dot{\mathcal{G}}_{t+1}^{\frac{1}{\varepsilon-1}} \dot{\mathcal{H}}_{t+1}}{\dot{\mathcal{G}}_t^{\frac{1}{\varepsilon-1}} \dot{\mathcal{H}}_t} \right)^{-\frac{\sigma(1+\eta)}{\sigma+\eta}} (1 + \dot{r}_t) \right], \tag{A.32}$$

$$1 = \beta \mathbb{E}_t \left[\left(\left(\frac{\dot{F}_{t+1}}{\dot{F}_t} \right)^{\frac{\phi}{\varepsilon-1}} \left(\frac{A_{t+1}^{\frac{\nu-1}{\nu}} - \kappa \dot{F}_{t+1}^{\frac{\nu-1}{\nu}}}{A_t^{\frac{\nu-1}{\nu}} - \kappa \dot{F}_t^{\frac{\nu-1}{\nu}}} \right)^{\frac{\nu}{\nu-1}} \right)^{-\frac{\sigma(1+\eta)}{\sigma+\eta}} (1 + \dot{r}_t) \right], \tag{A.33}$$

$$1 = \beta \mathbb{E}_t \left[\left(\frac{A_{t+1}}{A_t} \right)^{-\frac{\sigma(1+\eta)(\varepsilon-1+\phi)}{(\sigma+\eta)(\varepsilon-1)}} (1 + \dot{r}_t) \right], \tag{A.34}$$

where the second line imposes the functional forms, given in section 2.3.1, and the final step simplifies using the solution for $\dot{F}_t$ given in (A.30).

Finally, note that in the frictionless first-best allocation without adjustment costs a benevolent social planner may pick any level of sticker prices and inflation. We therefore assume that they choose to implement the central bank's inflation target as $1 + \Pi = 1 + \Pi^*$. Hence, the efficient frictionless level of sticker price inflation, $1 + \dot{\Pi}^S$, will be given by:

$$1 + \dot{\Pi}_t^S = (1 + \Pi^*) \left(\frac{A_t}{A_{t-1}} \right)^{\frac{\phi}{\varepsilon-1}}, \tag{A.35}$$

using (2.20) and (A.30).

In a steady state with $A_t = A_{t-1} = \bar{A}$ the frictionless first-best outcome gives $\dot{\bar{\Pi}}^F = 0$ and $\dot{\bar{\Pi}} = \dot{\bar{\Pi}}^S = \Pi^*$. The case with trend productivity growth is discussed in Appendix A.6.

A.3 Full Set of Model Equations

Table A.1 sets out the complete list of 12 endogenous model variables, which are solved by the set of equations that follow, which have been defined or derived before.

Table A.1 Endogenous Variables

Variable Type	Variable List					Count
Household	C_t	L_t				2
Firms	Y_t	MC_t	Π_t^S	F_t	Π_t^F	5
Equilibrium & Exogenous	$\frac{W_t}{P_t}$	i_t	A_t	Π_t	τ_t	5
Total						12

Household first order conditions:

$$\frac{W_t}{P_t} = C_t^{\sigma} L_t^{\eta}, \tag{A.36}$$

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + i_t}{1 + \Pi_{t+1}} \right]. \tag{A.37}$$

Firm block:

$$Y_t = \mathcal{G}_t^{\frac{1}{\varepsilon-1}} L_t \mathcal{H}_t, \tag{A.38}$$

$$MC_t = \frac{W_t}{P_t} \frac{1}{\mathcal{H}_t}, \tag{A.39}$$

$$\begin{aligned}(1+\tau_t)(\varepsilon - 1) &= \varepsilon MC_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} - \theta_P \left(\frac{1 + \Pi_t^S}{1 + \bar{\Pi}^S} - 1 \right) \frac{1 + \Pi_t^S}{1 + \bar{\Pi}^S} \\ &\quad + \beta \theta_P \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{1 + \Pi_{t+1}^S}{1 + \bar{\Pi}^S} - 1 \right) \frac{1 + \Pi_{t+1}^S}{1 + \bar{\Pi}^S} \right],\end{aligned} \tag{A.40}$$

$$\begin{aligned}\frac{(1+\tau_t) F_t \mathcal{G}_{F,t}}{\mathcal{G}_t} &= MC_t F_t \mathcal{G}_t^{-\frac{1}{\varepsilon-1}} \left(\frac{\mathcal{G}_{F,t}}{\mathcal{G}_t} - \frac{\mathcal{H}_{F,t}}{\mathcal{H}_t} \right) + \theta_F \left(\frac{1 + \Pi_t^F}{1 + \bar{\Pi}^F} - 1 \right) \frac{1 + \Pi_t^F}{1 + \bar{\Pi}^F} \\ &\quad - \beta \theta_F \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}} \right)^{\sigma} \frac{Y_{t+1}}{Y_t} \left(\frac{1 + \Pi_{t+1}^F}{1 + \bar{\Pi}^F} - 1 \right) \frac{1 + \Pi_{t+1}^F}{1 + \bar{\Pi}^F} \right],\end{aligned} \tag{A.41}$$

$$1 + \Pi_t^F = \frac{F_t}{F_{t-1}}. \tag{A.42}$$

Market clearing, definition, and exogenous block:

$$Y_t = C_t + \frac{\theta_P}{2}\left(\frac{1+\Pi_t^S}{1+\bar{\Pi}^S} - 1\right)^2 Y_t + \frac{\theta_F}{2}\left(\frac{1+\Pi_t^F}{1+\bar{\Pi}^F} - 1\right)^2 Y_t, \tag{A.43}$$

$$\ln A_t = \rho_A \ln A_{t-1} + \zeta_t^a, \tag{A.44}$$

$$1+\Pi_t^S = \left[\frac{\mathcal{G}_t}{\mathcal{G}_{t-1}}\right]^{\frac{1}{\varepsilon-1}} (1+\Pi_t), \tag{A.45}$$

$$1+\tau_t = \frac{\varepsilon}{\varepsilon-1}, \tag{A.46}$$

$$\frac{1+i_t}{1+\bar{i}} = \left(\frac{1+\Pi_t}{1+\Pi^*}\right)^{\phi_\pi}\left(\frac{Y_t}{Y_t^n}\right)^{\phi_y} e^{\zeta_t^i}. \tag{A.47}$$

Whenever the model uses hedonic price adjustment costs instead of sticker price and product quality adjustment costs, (A.40), (A.41), and (A.43) are replaced with the alternative intermediate firms' first order conditions, (A.22) and (A.23), in addition to the following market clearing condition under hedonic price adjustment costs:

$$Y_t = C_t + \frac{\theta}{2}\left(\frac{1+\Pi_t}{1+\bar{\Pi}} - 1\right)^2 Y_t. \tag{A.48}$$

A.3.1 Steady State Solution

In this subsection we assume the central bank has non-zero inflation target $\Pi^* \neq 0$ and outline a method to find the deterministic steady state, indicated by an upper bar.[3] This is obtained by dropping the time indices, and may be computed as follows.[4] The Euler condition (A.37) determines the equilibrium real interest rate in the steady state as:

$$1+\bar{r} \equiv \frac{1+\bar{i}}{1+\bar{\Pi}} = \frac{1}{\beta}. \tag{A.49}$$

The policy rule (A.47) shows inflation to be on target in the steady state at $\bar{\Pi} = \Pi^*$, provided $\bar{Y} = \bar{Y}^n$ and $\bar{\zeta}^i = 0$. We will show that the former condition holds while the latter is an assumption. The steady state nominal interest rate is therefore $1+\bar{i} = \frac{1+\Pi^*}{\beta}$. The law of motion for productivity (A.44) shows the steady state level of technology is $\bar{A} = 1$, as $\bar{\zeta}^a = 0$ is assumed, while the government production subsidy choice (A.46) shows the steady state

[3] As seen below, the distinction of whether this inflation target is given in sticker or hedonic terms is irrelevant as in the steady state these perfectly align, due to product quality being stationary.

[4] A similar method is used in Ascari and Rossi (2012). Their analysis aligns with our own model when product quality is constant and when their model is adapted to alter adjustment costs for the introduction of the consumption-production wedge.

equilibrium subsidy sets $1 + \bar{\tau} = \frac{\varepsilon}{\varepsilon-1}$. Assuming a stationary level of product quality, the definition of product quality inflation (A.42) shows that $\bar{\Pi}^F = 0$. Hence, as $\bar{\mathcal{G}}$ only depends upon $\bar{F}$, the relationship between sticker and hedonic inflation measures (A.45) gives $\bar{\Pi}^S = \Pi^*$. This gives 6 of 12 endogenous variables as:

$$\bar{\Pi} = \Pi^*, \tag{A.50}$$

$$1 + \bar{i} = \frac{1 + \Pi^*}{\beta}, \tag{A.51}$$

$$\bar{A} = 1, \tag{A.52}$$

$$\bar{\tau} = \frac{1}{\varepsilon - 1}, \tag{A.53}$$

$$\bar{\Pi}^F = 0, \tag{A.54}$$

$$\bar{\Pi}^S = \Pi^*. \tag{A.55}$$

Using the steady state conditions (A.50) through to (A.55), the remaining 6 model equations in 6 unknown equilibrium values, coming from (A.36), (A.38), (A.39), (A.40), (A.41) and (A.43) become:

$$\frac{\bar{W}}{\bar{P}} = \bar{C}^{\sigma}\bar{L}^{\eta}, \tag{A.56}$$

$$\bar{Y} = \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}}\bar{L}\bar{\mathcal{H}}, \tag{A.57}$$

$$\bar{MC} = \frac{\bar{W}}{\bar{P}}\frac{1}{\bar{\mathcal{H}}}, \tag{A.58}$$

$$1 = \bar{MC}\bar{\mathcal{G}}^{-\frac{1}{\varepsilon-1}}, \tag{A.59}$$

$$\frac{\varepsilon}{\varepsilon - 1} = \bar{MC}\bar{\mathcal{G}}^{-\frac{1}{\varepsilon-1}}\left(1 - \frac{\bar{\mathcal{H}}_F}{\bar{\mathcal{H}}}\frac{\bar{\mathcal{G}}}{\bar{\mathcal{G}}_F}\right), \tag{A.60}$$

$$\bar{Y} = \bar{C}. \tag{A.61}$$

The steady state level of product quality is determined by (A.59) and (A.60) as:

$$\frac{1}{\varepsilon - 1} = -\frac{\bar{\mathcal{H}}_F}{\bar{\mathcal{H}}}\frac{\bar{\mathcal{G}}}{\bar{\mathcal{G}}_F}, \tag{A.62}$$

which matches the frictionless efficient outcome derived in Appendix A.2 equation (A.26). The functional forms as outlined in section 2.3.1 yield:

$$\frac{\bar{\mathcal{H}}_F}{\bar{\mathcal{H}}}\frac{\bar{\mathcal{G}}}{\bar{\mathcal{G}}_F} = \frac{\kappa}{\phi(\kappa - \bar{F}^{\frac{1-\nu}{\nu}})}, \tag{A.63}$$

Combining (A.62) and (A.63), the steady state level of product quality equals:

$$\bar{F} = \kappa^{\frac{\nu}{1-\nu}} \left(\frac{\phi}{\varepsilon - 1 + \phi} \right)^{\frac{\nu}{\nu-1}}, \tag{A.64}$$

Given this solution for $\bar{F}$, it will be useful to consider the analytical solutions for the following:

$$\bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}} = \kappa^{\frac{\nu\phi}{(1-\nu)(\varepsilon-1)}} \left(\frac{\phi}{\varepsilon - 1 + \phi} \right)^{\frac{\nu\phi}{(\nu-1)(\varepsilon-1)}} \tag{A.65}$$

$$\bar{\mathcal{H}} = \left(\frac{\varepsilon - 1}{\varepsilon - 1 + \phi} \right)^{\frac{\nu}{\nu-1}}, \tag{A.66}$$

using the functional forms as outlined in section 2.3.1.

This can be used alongside (A.59) to determine marginal costs as:

$$\bar{MC} = \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}}. \tag{A.67}$$

Next, the real wage is found using (A.58):

$$\frac{\bar{W}}{\bar{P}} = \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}} \bar{\mathcal{H}}. \tag{A.68}$$

Equations (A.56), (A.57) and (A.61) may be used to write output as a function of quality and the real wage. Using (A.68) this becomes:

$$\bar{Y} = \left[\bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}} \bar{\mathcal{H}} \right]^{\frac{\eta+1}{\sigma+\eta}}, \tag{A.69}$$

Using (A.61) consumption is then:

$$\bar{C} = \bar{Y} = \left[\bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}} \bar{\mathcal{H}} \right]^{\frac{\eta+1}{\sigma+\eta}}, \tag{A.70}$$

Finally, using (A.57) the steady state level of employment is:

$$\bar{L} = \left[\bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}} \bar{\mathcal{H}} \right]^{\frac{1-\sigma}{\sigma+\eta}}. \tag{A.71}$$

Taking these relationships together gives the reduced form for the equilibrium as firstly finding the steady state condition for quality using:

$$\frac{1}{\varepsilon - 1} = -\frac{\bar{\mathcal{H}}_F}{\bar{\mathcal{H}}} \frac{\bar{\mathcal{G}}}{\bar{\mathcal{G}}_F}, \tag{A.72}$$

while the remaining equations simplify to become:

$$\bar{\Pi} = \Pi^*, \tag{A.73}$$

$$\bar{\Pi}^S = \Pi^*, \tag{A.74}$$

$$\bar{\Pi}^F = 0, \tag{A.75}$$

$$1 + \bar{i} = \frac{1 + \Pi^*}{\beta}, \tag{A.76}$$

$$\bar{A} = 1, \tag{A.77}$$

$$\bar{\tau} = \frac{1}{\varepsilon - 1}, \tag{A.78}$$

$$\bar{MC} = \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}}, \tag{A.79}$$

$$\frac{\bar{W}}{\bar{P}} = \bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}}\bar{\mathcal{H}}, \tag{A.80}$$

$$\bar{Y} = \bar{C} = \left[\bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}}\bar{\mathcal{H}}\right]^{\frac{\eta+1}{\sigma+\eta}}, \tag{A.81}$$

$$\bar{L} = \left[\bar{\mathcal{G}}^{\frac{1}{\varepsilon-1}}\bar{\mathcal{H}}\right]^{\frac{1-\sigma}{\sigma+\eta}}. \tag{A.82}$$

So, in the steady state all variables are at their frictionless efficient levels, consistent with the choices by a benevolent social planner without sticker price or product quality frictions ($\theta_P = \theta_F = 0$), as shown in Appendix A.2.

Whenever the model uses hedonic price adjustment costs instead of sticker price adjustment costs, the alternative market clearing condition (A.48) and first order conditions (A.22) and (A.23) yield the same steady state. However, hedonic price stickiness will deliver the frictionless efficient level of product quality, not just in the steady state but in every period, as given in (A.62).

A.4 Reduced-Form Model Derivation

This appendix firstly log-linearises the model before combining equations to present a reduced-form model similar to the canonical New Keynesian model. For clarity, wherever possible derivations follow those presented in Galí (2015). Notation follows the main text.

A.4.1 Household Block

Written in logs the first order condition for consumption and labour trade-off, equation (A.36), is:

$$\ln W_t - \ln P_t = \sigma \ln C_t + \eta \ln L_t, \tag{A.83}$$

$$w_t = \sigma c_t + \eta \ell_t. \tag{A.84}$$

Note that, following standard practise, we have defined the log *real* wages as $w_t \equiv \ln W_t - \ln P_t$. As a deviation from steady state level this may be written as:

$$\hat{w}_t = \sigma \hat{c}_t + \eta \hat{\ell}_t. \tag{A.85}$$

A first order log-linear Taylor approximation around the steady state of the household Euler equation (A.37) shows:

$$1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1 + i_t}{1 + \Pi_{t+1}} \right], \tag{A.86}$$

$$\ln 1 = \ln \beta + \ln \mathbb{E}_t \left[e^{-\sigma \ln C_{t+1}} e^{\sigma \ln C_t} e^{\ln(1+i_t)} e^{-\ln(1+\Pi_{t+1})} \right], \tag{A.87}$$

$$0 = \ln \beta + \ln \mathbb{E}_t \left[e^{-\sigma c_{t+1}} e^{\sigma c_t} e^{1+i_t} e^{-(1+\pi_{t+1})} \right], \tag{A.88}$$

$$0 \approx \mathbb{E}_t \left[\sigma \hat{c}_t - \sigma \hat{c}_{t+1} + \hat{i}_t - \hat{\pi}_{t+1} \right], \tag{A.89}$$

$$\hat{c}_t \approx \mathbb{E}_t \left[\hat{c}_{t+1} \right] - \frac{1}{\sigma} \left(\hat{i}_t - \mathbb{E}_t \left[\hat{\pi}_{t+1} \right] \right). \tag{A.90}$$

A.4.2 Firm Block

Given the functional forms specified in section 2.3.1 of the main text, the aggregate production function (equation A.38), marginal costs (A.39) and definition of quality improvement

(A.42) may be specified as:

$$Y_t = F_t^{\frac{\phi}{\varepsilon-1}} L_t \left[A_t^{\frac{\nu-1}{\nu}} - \kappa F_t^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \tag{A.91}$$

$$MC_t = \frac{W_t}{P_t} \left[A_t^{\frac{\nu-1}{\nu}} - \kappa F_t^{\frac{\nu-1}{\nu}} \right]^{-\frac{\nu}{\nu-1}}, \tag{A.92}$$

$$1 + \Pi_t^F = \frac{F_t}{F_{t-1}}. \tag{A.93}$$

Each equation may then be written down directly, in logs, to give:

$$y_t = \ell_t + \frac{\phi}{\varepsilon - 1} f_t + \frac{\nu}{\nu - 1} \ln \left[e^{\frac{\nu-1}{\nu} a_t} - \kappa e^{\frac{\nu-1}{\nu} f_t} \right], \tag{A.94}$$

$$mc_t = w_t - \frac{\nu}{\nu - 1} \ln \left[e^{\frac{\nu-1}{\nu} a_t} - \kappa e^{\frac{\nu-1}{\nu} f_t} \right], \tag{A.95}$$

$$1 + \pi_t^F = f_t - f_{t-1}. \tag{A.96}$$

where the definition for the log of real wages, $w \equiv \ln W_t - \ln P_t$, is used. The first two equations are still non-linear, but around the steady state approximately become:

$$\hat{y}_t \approx \hat{\ell}_t + \frac{\phi}{\varepsilon - 1} \hat{f}_t + \frac{\hat{a}_t - \kappa e^{\frac{\nu-1}{\nu} \bar{f}} \hat{f}_t}{1 - \kappa e^{\frac{\nu-1}{\nu} \bar{f}}}, \tag{A.97}$$

$$\widehat{mc}_t \approx \hat{w}_t - \frac{\hat{a}_t - \kappa e^{\frac{\nu-1}{\nu} \bar{f}} \hat{f}_t}{1 - \kappa e^{\frac{\nu-1}{\nu} \bar{f}}}. \tag{A.98}$$

These may be simplified using equation (2.42) to note that $\bar{f} = \frac{\nu}{\nu-1} \ln \left[\frac{\phi}{\kappa(\varepsilon-1)+\kappa\phi} \right]$, such that $\kappa e^{\frac{\nu-1}{\nu} \bar{f}} = \frac{\phi}{\varepsilon-1+\phi}$ and $1 - \kappa e^{\frac{\nu-1}{\nu} \bar{f}} = \frac{\varepsilon-1}{\varepsilon-1+\phi}$ to show:

$$\hat{y}_t \approx \hat{\ell}_t + \frac{(\varepsilon - 1) + \phi}{(\varepsilon - 1)} \hat{a}_t, \tag{A.99}$$

$$\widehat{mc}_t \approx \hat{w}_t - \frac{(\varepsilon - 1) + \phi}{(\varepsilon - 1)} \hat{a}_t + \frac{\phi}{(\varepsilon - 1)} \hat{f}_t. \tag{A.100}$$

Next attention turns to log-linearisation of the two first order conditions from the intermediate firms, performed around the point $\Pi_t^S = \bar{\Pi}^S$, and $\Pi_t^F = \bar{\Pi}^F$. Starting with (FOC P), and using the functional forms from section 2.3.1 of the main text. Using (A.46) to eliminate

the production subsidy, this approximation follows as:

$$\begin{aligned}\varepsilon &= \varepsilon MC_t F_t^{-\frac{\phi}{\varepsilon-1}} - \theta_P \left(\frac{1+\Pi_t^S}{1+\bar{\Pi}^S} - 1\right)\frac{1+\Pi_t^S}{1+\bar{\Pi}^S} \\ &\quad + \beta\theta_P \mathbb{E}_t \left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \frac{Y_{t+1}}{Y_t}\left(\frac{1+\Pi_{t+1}^S}{1+\bar{\Pi}^S} - 1\right)\frac{1+\Pi_{t+1}^S}{1+\bar{\Pi}^S}\right], \end{aligned} \tag{A.101}$$

$$\begin{aligned}\ln\varepsilon &= \ln\Big[\varepsilon e^{\ln MC_t} e^{-\frac{\phi}{\varepsilon-1}\ln F_t} - \theta_P (e^{\ln(1+\Pi_t^S)-\ln(1+\bar{\Pi}^S)} - 1) e^{\ln(1+\Pi_t^S)-\ln(1+\bar{\Pi}^S)} \\ &\quad + \beta\theta_P \mathbb{E}_t \left[e^{\sigma\ln C_t} e^{-\sigma\ln C_{t+1}} e^{\ln Y_{t+1}} e^{-\ln Y_t} (e^{\ln(1+\Pi_{t+1}^S)-\ln(1+\bar{\Pi}^S)} - 1) e^{\ln(1+\Pi_{t+1}^S)-\ln(1+\bar{\Pi}^S)}\right]\Big], \end{aligned} \tag{A.102}$$

$$\begin{aligned}\ln\varepsilon &= \ln\Big[\varepsilon e^{mc_t} e^{-\frac{\phi}{\varepsilon-1} f_t} - \theta_P (e^{(1+\pi_t^S)-(1+\bar{\pi}^S)} - 1) e^{(1+\pi_t^S)-(1+\bar{\pi}^S)} \\ &\quad + \beta\theta_P \mathbb{E}_t \left[e^{\sigma c_t} e^{-\sigma c_{t+1}} e^{y_{t+1}} e^{-y_t} (e^{(1+\pi_{t+1}^S)-(1+\bar{\pi}^S)} - 1) e^{(1+\pi_{t+1}^S)-(1+\bar{\pi}^S)}\right]\Big], \end{aligned} \tag{A.103}$$

$$0 \approx \varepsilon \bar{MC} \bar{F}^{-\frac{\phi}{\varepsilon-1}} \left(\widehat{mc}_t - \frac{\phi}{\varepsilon-1}\hat{f}_t\right) - \theta_P \hat{\pi}_t^S + \beta\theta_P \mathbb{E}_t \left[\hat{\pi}_{t+1}^S\right]. \tag{A.104}$$

The steady state level of marginal costs, accounting for quality, implies $\bar{MC}\bar{\mathcal{G}}^{-\frac{1}{\varepsilon-1}} = 1$, such that the above expression may be rewritten as:

$$\hat{\pi}_t^S \approx \frac{\varepsilon}{\theta_P}\left(\widehat{mc}_t - \frac{\phi}{\varepsilon-1}\hat{f}_t\right) + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}^S\right]. \tag{Log Linearised FOC P}$$

When quality is constant at the steady state level, $\hat{f}_t = 0$ and therefore $\pi_t^S = \pi_t$, so this Log Linearised FOC P reduces to the Rotemberg (1982) version of the New Keynesian Phillips as a specific case of the more general form. We may also see this by rewriting the Log Linearised FOC P as:

$$\hat{\pi}_t^S = \frac{\varepsilon}{\theta_P}\sum_{s=t}^{\infty}\beta^{s-t}\mathbb{E}_t\left[\widehat{mc}_s - \frac{\phi}{\varepsilon-1}\hat{f}_s\right]. \tag{A.105}$$

Deviations of sticker price inflation are equal to the discounted value of marginal cost deviations, whenever the model is simplified with product quality remaining at the steady state value.

Finally, an approximation of (FOC F) given these functional forms, and the government revenue subsidy, follows as:

$$\frac{\varepsilon\phi}{\varepsilon-1} = MC_t F_t^{-\frac{\phi}{\varepsilon-1}}\left(\phi + \frac{\kappa}{F_t^{\frac{1-\nu}{\nu}} A_t^{\frac{\nu-1}{\nu}} - \kappa}\right) + \theta_F\left(\frac{1+\Pi_t^F}{1+\bar{\Pi}^F} - 1\right)\frac{1+\Pi_t^F}{1+\bar{\Pi}^F}$$
$$- \beta\theta_F \mathbb{E}_t\left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{1+\Pi_{t+1}^F}{1+\bar{\Pi}^F} - 1\right)\frac{1+\Pi_{t+1}^F}{1+\bar{\Pi}^F}\right], \tag{A.106}$$

$$\ln\left(\frac{\varepsilon\phi}{\varepsilon-1}\right) = \ln\Bigg[e^{\ln MC_t} e^{-\frac{\phi}{\varepsilon-1}\ln F_t}\left(\phi + \kappa\left[e^{\frac{1-\nu}{\nu}\ln F_t} e^{\frac{\nu-1}{\nu}\ln A_t} - \kappa\right]^{-1}\right)$$
$$+ \theta_F(e^{\ln(1+\Pi_t^F)-\ln(1+\bar{\Pi}^F)} - 1)e^{\ln(1+\Pi_t^F)-\ln(1+\bar{\Pi}^F)}$$
$$- \beta\theta_F\mathbb{E}_t\left[e^{\sigma\ln C_t} e^{-\sigma\ln C_{t+1}} e^{\ln Y_{t+1}} e^{-\ln Y_t}(e^{\ln(1+\Pi_{t+1}^F)-\ln(1+\bar{\Pi}^F)} - 1)e^{\ln(1+\Pi_{t+1}^F)-\ln(1+\bar{\Pi}^F)}\right]\Bigg], \tag{A.107}$$

$$\ln\left(\frac{\varepsilon\phi}{\varepsilon-1}\right) = \ln\Bigg[e^{mc_t} e^{-\frac{\phi}{\varepsilon-1}f_t}\left(\phi + \kappa\left[e^{\frac{1-\nu}{\nu}f_t} e^{\frac{\nu-1}{\nu}a_t} - \kappa\right]^{-1}\right)$$
$$+ \theta_F(e^{(1+\pi_t^F)-(1+\bar{\pi}^F)} - 1)e^{(1+\pi_t^F)-(1+\bar{\pi}^F)}$$
$$- \beta\theta_F\mathbb{E}_t\left[e^{\sigma c_t} e^{-\sigma c_{t+1}} e^{y_{t+1}} e^{-y_t}(e^{(1+\pi_{t+1}^F)-(1+\bar{\pi}^F)} - 1)e^{(1+\pi_{t+1}^F)-(1+\bar{\pi}^F)}\right]\Bigg], \tag{A.108}$$

$$0 \approx \bar{MC}\bar{F}^{-\frac{\phi}{\varepsilon-1}}\left(\phi + \kappa\left[\bar{F}^{\frac{1-\nu}{\nu}}\bar{A}^{\frac{\nu-1}{\nu}} - \kappa\right]^{-1}\right)\left(\widehat{mc}_t - \frac{\phi}{\varepsilon-1}\hat{f}_t\right)$$
$$- \frac{(\nu-1)\kappa}{\nu}\bar{MC}\bar{F}^{-\frac{\phi}{\varepsilon-1}}\left[\bar{F}^{\frac{1-\nu}{\nu}}\bar{A}^{\frac{\nu-1}{\nu}} - \kappa\right]^{-2}\bar{F}^{\frac{1-\nu}{\nu}}\bar{A}^{\frac{\nu-1}{\nu}}\left(\hat{a}_t - \hat{f}_t\right)$$
$$+ \theta_F\hat{\pi}_t^F - \beta\theta_F\mathbb{E}_t\left[\hat{\pi}_{t+1}^F\right], \tag{A.109}$$

Again, the steady state level of marginal costs, accounting for quality, implies $\bar{MC}\bar{\mathcal{G}}^{-\frac{1}{\varepsilon-1}} = 1$. As discussed in Appendix A.3.1, both productivity and product quality are constant at the steady state, with $\bar{A} = 1$ and $\bar{F} = \kappa^{\frac{\nu}{1-\nu}}\left(\frac{\phi}{\varepsilon-1+\phi}\right)^{\frac{\nu}{\nu-1}}$, such that the above expression may be rewritten as:

$$\hat{\pi}_t^F \approx -\frac{\varepsilon\phi}{\theta_F(\varepsilon-1)}\widehat{mc}_t + \frac{\phi^2\nu\varepsilon - (\nu-1)\phi(\varepsilon-1+\phi)}{\nu\theta_F(\varepsilon-1)^2}\hat{f}_t + \frac{(\nu-1)(\varepsilon-1+\phi)\phi}{\nu\theta_F(\varepsilon-1)^2}\hat{a}_t + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}^F\right], \tag{Log Linearised FOC F}$$

When the hedonic form of price adjustment costs is used, log linearising the two first order conditions from intermediate firms, (Log Linearised FOC P) and (Log Linearised FOC F)

yields:

$$\hat{\pi}_t \approx \frac{\varepsilon}{\theta_P}\left(\widehat{mc}_t - \frac{\phi}{\varepsilon - 1}\hat{f}_t\right) + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}\right], \tag{A.110}$$

$$\hat{a}_t = \hat{f}_t. \tag{A.111}$$

A.4.3 Market Clearing and Exogenous Block

The aggregate goods market condition (2.34) may be rewritten and then approximated as:

$$e^{\ln Y_t} = e^{\ln C_t} + \frac{\theta_P}{2}(e^{\ln(1+\Pi_t^S)-\ln(1+\bar{\Pi}^S)} - 1)^2 e^{\ln Y_t} + \frac{\theta_F}{2}(e^{\ln(1+\Pi_t^F)-\ln(1+\bar{\Pi}^F)} - 1)^2 e^{\ln Y_t}, \tag{A.112}$$

$$\hat{c}_t \approx \hat{y}_t, \tag{A.113}$$

Even though when using the hedonic form of price adjustment costs the market clearing condition (A.43) changes to become (A.48), its log-linearised form is also (A.113).

The technology process (A.44) may be directly written in logs as:

$$a_t = \rho_A a_{t-1} + \zeta_t^a, \tag{A.114}$$

while, in deviations, sticker price inflation (A.45) becomes:

$$\hat{\pi}_t^S \approx \frac{\phi}{\varepsilon - 1}\hat{\pi}_t^F + \hat{\pi}_t. \tag{A.115}$$

Noting that $\bar{\pi} = \pi^*$, the monetary policy rule may be approximated in deviations as:

$$\hat{i}_t \approx \phi_\pi \hat{\pi}_t + \phi_y(\hat{y}_t - \hat{y}_t^n) + \zeta_t^i. \tag{A.116}$$

A.4.4 Full Set of Approximated Equations

The full set of exact and approximated equations is thus given as follows:
Household block:

$$\hat{w}_t = \sigma\hat{c}_t + \eta\hat{\ell}_t, \tag{A.117}$$

$$\hat{c}_t \approx \mathbb{E}_t\left[\hat{c}_{t+1}\right] - \frac{1}{\sigma}\left(\hat{i}_t - \mathbb{E}_t\left[\hat{\pi}_{t+1}\right]\right). \tag{A.118}$$

Firm block:

$$\hat{y}_t \approx \hat{\ell}_t + \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t, \tag{A.119}$$

$$\widehat{mc}_t \approx \hat{w}_t - \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{(\varepsilon-1)}\hat{f}_t, \tag{A.120}$$

$$\hat{\pi}_t^S \approx \frac{\varepsilon}{\theta_P}\left(\widehat{mc}_t - \frac{\phi}{\varepsilon-1}\hat{f}_t\right) + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}^S\right], \tag{A.121}$$

$$\begin{aligned}\hat{\pi}_t^F \approx & -\frac{\varepsilon\phi}{\theta_F(\varepsilon-1)}\widehat{mc}_t + \frac{\phi^2\nu\varepsilon-(\nu-1)\phi(\varepsilon-1+\phi)}{\nu\theta_F(\varepsilon-1)^2}\hat{f}_t \\ & + \frac{(\nu-1)(\varepsilon-1+\phi)\phi}{\nu\theta_F\left(\varepsilon-1\right)^2}\hat{a}_t + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}^F\right],\end{aligned} \tag{A.122}$$

$$\hat{\pi}_t^F = \hat{f}_t - \hat{f}_{t-1}. \tag{A.123}$$

Market clearing and exogenous block:

$$\hat{c}_t \approx \hat{y}_t, \tag{A.124}$$

$$a_t = \rho_A a_{t-1} + \zeta_t^a, \tag{A.125}$$

$$\hat{\pi}_t^S = \frac{\phi}{\varepsilon-1}\hat{\pi}_t^F + \hat{\pi}_t, \tag{A.126}$$

$$\hat{\tau}_t = 0, \tag{A.127}$$

$$\hat{i}_t \approx \phi_\pi\hat{\pi}_t + \phi_y(\hat{y}_t - \hat{y}_t^n) + \zeta_t^i. \tag{A.128}$$

Along with the process for the evolution of y_t^n, these 12 equations fully describe the model. When the hedonic form of price adjustment costs is used (A.121) and (A.122) respectively become:

$$\hat{\pi}_t \approx \frac{\varepsilon}{\theta_P}\left(\widehat{mc}_t - \frac{\phi}{\varepsilon-1}\hat{f}_t\right) + \beta\mathbb{E}_t\left[\hat{\pi}_{t+1}\right], \tag{A.129}$$

$$\hat{a}_t = \hat{f}_t. \tag{A.130}$$

A.4.5 Model Reduction

Combining the market clearing condition and household optimality condition for labour gives:

$$\hat{w}_t = \sigma\hat{y}_t + \eta\hat{\ell}_t, \tag{A.131}$$

which may then be used alongside the marginal cost function to give:

$$\widehat{mc}_t = \sigma\hat{y}_t + \eta\hat{\ell}_t - \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{(\varepsilon-1)}\hat{f}_t, \tag{A.132}$$

The aggregate production function may then be used to eliminate labour to give:

$$\widehat{mc}_t = \sigma\hat{y}_t + \eta\left[\hat{y}_t - \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t\right] - \frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{(\varepsilon-1)}\hat{f}_t, \tag{A.133}$$

$$\widehat{mc}_t = (\sigma+\eta)\hat{y}_t - (\eta+1)\frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{\varepsilon-1}\hat{f}_t. \tag{A.134}$$

This relationship holds in the economy for each given degree of price and product quality adjustment costs. In particular, it will hold whenever there are no costs to adjusting prices or product quality, with $\theta_P = \theta_F = 0$. We can define this theoretical economy as the *frictionless* one, which would occur but for the presence of adjustment costs. Hence, we define the deviation of this frictionless equilibrium solution from the model steady state as $\{\hat{a}_t, \hat{y}_t^n, \hat{f}_t^n, \widetilde{mc}_t^n, \cdots\}$, and show that:

$$f_t^n = a_t + \frac{\nu}{\nu-1}\ln\left[\frac{\phi}{\kappa(\varepsilon-1)+\kappa\phi}\right], \tag{A.135}$$

$$mc_t^n = \frac{\phi}{\varepsilon-1}f_t^n. \tag{A.136}$$

using (A.26) and (A.29), from Appendix A.2 which outlines the first best allocations from the social planner in such a setting. Therefore, given that $\hat{f}_t^n = \hat{a}_t$ and thus $\widehat{mc}_t^n = \frac{\phi}{\varepsilon-1}\hat{a}_t$ implies:

$$\widehat{mc}_t^n = (\sigma+\eta)\hat{y}_t^n - (\eta+1)\frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{\varepsilon-1}\hat{f}_t^n, \tag{A.137}$$

$$\frac{\phi}{\varepsilon-1}\hat{a}_t = (\sigma+\eta)\hat{y}_t^n - (\eta+1)\frac{(\varepsilon-1)+\phi}{(\varepsilon-1)}\hat{a}_t + \frac{\phi}{\varepsilon-1}\hat{a}_t, \tag{A.138}$$

$$\hat{y}_t^n = \frac{(\eta+1)(\varepsilon-1+\phi)}{(\sigma+\eta)(\varepsilon-1)}\hat{a}_t. \tag{A.139}$$

Alternatively:

$$y_t^n = \psi_a a_t + \psi. \tag{A.140}$$

where $\psi_a \equiv \frac{(\eta+1)(\varepsilon-1+\phi)}{(\sigma+\eta)(\varepsilon-1)}$ and $\psi \equiv \bar{y}$. The former is a larger parameter than in the standard New Keynesian case, which has $\phi = 0$ and $\psi_a \equiv \frac{\eta+1}{\sigma+\eta}$. This then gives:

$$mc_t - mc^n \equiv \widetilde{mc}_t = \widehat{mc}_t - \widehat{mc}_t^n = (\sigma+\eta)\tilde{y}_t + \frac{\phi}{\varepsilon-1}\tilde{f}_t. \tag{A.141}$$

When this description for the movement of the difference between current and optimal frictionless marginal costs (A.141) and the goods market clearing condition (A.124), is used in the intermediate firm's FOC for prices (A.121), this gives:

$$\tilde{\pi}_t^S \approx \frac{\varepsilon(\sigma+\eta)}{\theta_P}\tilde{y}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^S\right], \tag{A.142}$$

Then, again using (A.141), and the fact that $\tilde{a}_t = 0$, the intermediate firms' FOC for quality (A.122) becomes:

$$\tilde{\pi}_t^F \approx -\frac{\varepsilon\phi(\sigma+\eta)}{\theta_F(\varepsilon-1)}\tilde{y}_t - \frac{\phi(\nu-1)(\varepsilon-1+\phi)}{\nu\theta_F\left(\varepsilon-1\right)^2}\tilde{f}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^F\right], \tag{A.143}$$

And hence, since $\tilde{\pi}_t = \tilde{\pi}_t^S - \frac{\phi}{\varepsilon-1}\tilde{\pi}_t^F$, the structural form of the Phillips curve becomes:

$$\tilde{\pi}_t = \frac{\varepsilon(\sigma+\eta)}{\theta_P}\left(1+\frac{\phi^2\theta_P}{\theta_F(\varepsilon-1)^2}\right)\tilde{y}_t + \frac{\phi^2(\nu-1)(\varepsilon-1+\phi)}{\nu\theta_F\left(\varepsilon-1\right)^3}\tilde{f}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}\right]. \tag{A.144}$$

The intertemporal IS curve is formed by combining the goods market clearing condition (A.124) and intertemporal Euler equation (A.118) to give:

$$\tilde{y}_t \approx \mathbb{E}_t\left[\tilde{y}_{t+1}\right] - \frac{1}{\sigma}\left(\tilde{i}_t - \mathbb{E}_t\left[\tilde{\pi}_{t+1}\right]\right). \tag{A.145}$$

when expressed as deviations from their natural rates. Then, using the fact that $i_t^n = r_t^n + \pi_t^*$, this may be written as:

$$\tilde{y}_t \approx \mathbb{E}_t\left[\tilde{y}_t\right] - \frac{1}{\sigma}\left(i_t - \mathbb{E}_t\left[\tilde{\pi}_{t+1}\right] - r_t^n - \pi^*\right). \tag{DIS}$$

which is referred to as the Dynamic IS relationship where r_t^n is the natural real interest rate defined in (A.34) and log-linearised as:

$$r_t^n \equiv \rho + \sigma\mathbb{E}_t[\Delta y_{t+1}^n] = \rho + \sigma\psi_a\mathbb{E}_t[\Delta a_{t+1}] \tag{A.146}$$

where $\rho \equiv -\ln\beta$ and the second equality uses the relationship for $\dot{Y}_t$ given in equation (A.28), which is log-linearised and simplified to become (A.139).

The reduced-form model is therefore given as the solution to:

$$\tilde{y}_t = \mathbb{E}_t[\tilde{y}_{t+1}] - \frac{1}{\sigma}(i_t - \mathbb{E}_t[\tilde{\pi}_{t+1}] - r_t^n - \pi^*), \tag{Dynamic IS}$$

$$\tilde{\pi}_t^S = \frac{\varepsilon(\sigma+\eta)}{\theta_P}\tilde{y}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^S\right], \tag{NKPC P}$$

$$\tilde{\pi}_t^F = -\frac{\varepsilon\phi(\sigma+\eta)}{\theta_F(\varepsilon-1)}\tilde{y}_t - \frac{\phi(\nu-1)(\varepsilon-1+\phi)}{\nu\theta_F\left(\varepsilon-1\right)^2}\tilde{f}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^F\right], \tag{NKPC F}$$

$$i_t = \rho + \pi^* + \phi_\pi\tilde{\pi}_t + \phi_y\tilde{y}_t + \zeta_t^i, \tag{Taylor rule}$$

$$r_t^n = \rho + \sigma\psi_a\mathbb{E}_t[\Delta a_{t+1}], \tag{A.147}$$

$$\tilde{\pi}_t^F = \tilde{f}_t - \tilde{f}_{t-1}, \tag{A.148}$$

$$\tilde{\pi}_t^S = \frac{\phi}{\varepsilon-1}\tilde{\pi}_t^F + \tilde{\pi}_t, \tag{A.149}$$

$$a_t = \rho_A a_{t-1} + \zeta_t^a. \tag{A.150}$$

A.5 Determinacy

Given a policy rule without shocks to the nominal interest rate, of the form:

$$i_t = r_t^n + \pi^* + \phi_\pi \tilde{\pi}_t + \phi_y \tilde{y}_t, \tag{A.151}$$

the system of 8 equations in 8 unknowns which forms the reduced model, as outlined at the end of appendix A.4.5, may be reduced further and written in matrix form as:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{\sigma} & 0 & 0 \\ 0 & \beta & 0 & \frac{\phi^2(\nu-1)(\varepsilon-1+\phi)}{\nu\theta_F(\varepsilon-1)^3} \\ 0 & 0 & \beta & -\frac{\phi(\nu-1)(\varepsilon-1+\phi)}{\nu\theta_F(\varepsilon-1)^2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbb{E}_t[\tilde{y}_{t+1}] \\ \mathbb{E}_t[\tilde{\pi}_{t+1}] \\ \mathbb{E}_t[\tilde{\pi}_{t+1}^F] \\ \tilde{f}_t \end{bmatrix}$$
$$+ \begin{bmatrix} -\frac{\sigma+\phi_y}{\sigma} & -\frac{\phi_\pi}{\sigma} & 0 & 0 \\ \frac{\varepsilon(\sigma+\eta)}{\theta_P}\left(1+\frac{\phi^2\theta_P}{\theta_F(\varepsilon-1)^2}\right) & -1 & 0 & 0 \\ -\frac{\varepsilon\phi(\sigma+\eta)}{\theta_F(\varepsilon-1)} & 0 & -1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} \tilde{y}_t \\ \tilde{\pi}_t \\ \tilde{\pi}_t^F \\ \tilde{f}_{t-1} \end{bmatrix}, \tag{A.152}$$

where the Taylor rule (A.151) has been used alongside the Dynamic IS equation (DIS) to eliminate i_t as an endogenous variable. The structural NKPC (A.144) has also been used to simplify algebra. We are interested in the role of monetary policy on the determinacy of this economic system, and hence focus upon the role of ϕ_π and ϕ_y in (A.152), which is written in the form:

$$\mathbf{0} = \mathbf{A}\mathbb{E}_t[\mathbf{x}_{t+1}] + \mathbf{B}\mathbf{x}_t, \tag{A.153}$$

$$\mathbb{E}_t[\mathbf{x}_{t+1}] = -\mathbf{A}^{-1}\mathbf{B}\mathbf{x}_t, \tag{A.154}$$

which is permissible as, by inspection, $\mathbf{A}^{-1}$ exists. Following insights from Blanchard and Kahn (1980), as $\tilde{f}_{t-1}$ is predetermined, uniqueness the rational expectations equilibrium required (exactly) 3 of the eigenvalues from the matrix $-\mathbf{A}^{-1}\mathbf{B}$ to be outside the unit circle.

A.6 Product Quality Growth

Table A.1 in Appendix A.3 gives the complete list of 12 endogenous model variables, which are solved by our model. A variant of this framework, with trend growth in product quality, may be obtained when three small alterations are made to the model.

Firstly, given the additive separability in household utility between consumption and hours of labour supply, a restriction to logarithmic consumption ($\sigma = 1$) is required for consistency with a balanced growth path, as noted in King et al. (1988). This restriction is necessary as, along the balanced growth path, hours worked are unable to grow, or fall, continuously. So income and substitution effects must have precisely offsetting effects on labour supply to accommodate an increasing trend in the marginal productivity of labour.

Secondly, the model is given trend productivity growth by modifying the equation for technology process (A.44) to a random walk with positive drift:

$$\ln A_t = \ln(1 + g_A) + \ln A_{t-1} + \varepsilon_t^a, \tag{A.155}$$

with exogenous growth parameter $g_A > 0$ and ε_t^a as an *iid* normally distributed random variable with mean 0 and standard deviation σ_A^2. In this way, along a balanced growth path, productivity grows over time at the exogenous rate g_A, as:

$$A_t = (1 + g_A)A_{t-1}e^{\varepsilon_t^a}. \tag{A.156}$$

Thirdly, the model is recast using stochastically de-trended variables. The unit root in (A.155) will influence the solution for the remainder of the system. This renders the system unsolvable in the form outlined in Appendix A.3. Instead, assuming the functional forms for $\mathcal{G}$ and $\mathcal{H}$ as outlined in section 2.3.1, an alternative model may be solved using stochastically de-trended variables which are given by $\check{C}_t \equiv C_t A_t^{-\frac{\varepsilon-1+\phi}{\varepsilon-1}}$, $\check{Y}_t \equiv Y_t A_t^{-\frac{\varepsilon-1+\phi}{\varepsilon-1}}$, $\check{W}_t/P_t \equiv W_t A_t^{-\frac{\varepsilon-1+\phi}{\varepsilon-1}}/P_t$, $\check{MC}_t \equiv MC_t A_t^{-\frac{\phi}{\varepsilon-1}}$, $\check{F}_t \equiv F_t A_t^{-1}$, and $\check{A}_t \equiv A_t/A_{t-1}$. The equation for technology progress may then be written in stationary form as:

$$\ln \check{A}_t = \ln(1 + g_A) + \varepsilon_t^a. \tag{A.157}$$

The functional forms assume $\mathcal{H}(F_t, A_t)$ is homogenenous of degree 1. Hence $\check{\mathcal{H}}_t \equiv \mathcal{H}(F_t, A_t)/A_t = \mathcal{H}(\check{F}_t, 1)$ and $\check{\mathcal{G}}_t \equiv \mathcal{G}(F_t)/A_t^{\phi} = \mathcal{G}(\check{F}_t)$ are the chosen stationary equivalents. It is useful to define $1 + \check{\Pi}_t^F \equiv (1 + \Pi_t^F)\check{A}_t^{-1}$ and assume intermediate firms face an analogous but stationary real costs of adjustment. The remaining variables, L_t, τ_t, i_t, $1+\Pi_t$ and $1+\Pi_t^S$, remain stationary and do not require further manipulation. Using hedonic price adjustment costs does not alter

the proposed definitions for de-trended variables or any of our main conclusions about that form of price adjustment costs. In this setting both total factor productivity, as captured by A_t, and product quality, F_t, grow at the same exogenous rate, $g_A > 0$, while output and consumption grow more quickly at the rate $g_C = g_Y = \frac{\varepsilon-1+\phi}{\varepsilon-1}$.

The full model, discussed in section A.3, may be recast using these three changes (the parameter restriction $\sigma = 1$; alternation to the productivity process; and stochastically de-trended variable notation). Once expressed in terms of the stochastically de-trended variables, 5 of the 12 model equations are no longer isomorphic to their original form. Equations (A.37), (A.41), (A.43), (A.44) and (A.45) respectively become:

$$1 = \beta \mathbb{E}_t \left[\frac{\check{A}_{t+1}\check{C}_t(1+i_t)}{\check{C}_{t+1}(1+\Pi_{t+1})} \right], \tag{A.158}$$

$$\begin{aligned} \frac{\varepsilon\phi}{\varepsilon-1} &= \check{MC}_t \check{F}_t^{-\frac{\phi}{\varepsilon-1}} \left(\phi - \frac{\kappa}{\check{F}_t^{\frac{1-\nu}{\nu}} - \kappa} \right) + \theta_F \left(\check{A}_t \frac{1+\check{\Pi}_t^F}{1+\bar{\check{\Pi}}^F} - 1 \right) \check{A}_t \frac{1+\check{\Pi}_t^F}{1+\bar{\check{\Pi}}^F} \\ &\quad - \beta\theta_F \mathbb{E}_t \left[\frac{\check{C}_t \check{Y}_{t+1}}{\check{C}_{t+1}\check{Y}_t} \left(\check{A}_{t+1} \frac{1+\check{\Pi}_{t+1}^F}{1+\bar{\check{\Pi}}^F} - 1 \right) \check{A}_{t+1} \frac{1+\check{\Pi}_{t+1}^F}{1+\bar{\check{\Pi}}^F} \right], \end{aligned} \tag{A.159}$$

$$\check{Y}_t = \check{C}_t + \frac{\theta_P}{2} \left(\frac{1+\Pi_t^S}{1+\bar{\Pi}^S} - 1 \right)^2 \check{Y}_t + \frac{\theta_F}{2} \left(\check{A}_t \frac{1+\check{\Pi}_t^F}{1+\bar{\check{\Pi}}^F} - 1 \right)^2 \check{Y}_t, \tag{A.160}$$

$$\ln \check{A}_t = \ln(1+g_A) + \varepsilon_t, \tag{A.161}$$

$$1 + \Pi_t^S = \check{A}_t^{\frac{\phi}{\varepsilon-1}} (1+\check{\Pi}_t^F)^{\frac{\phi}{\varepsilon-1}} (1+\Pi_t). \tag{A.162}$$

The log-lineraised form of these equations are given as:

$$\hat{\check{c}}_t \approx \mathbb{E}_t \left[\hat{\check{c}}_{t+1} \right] - \left(\hat{i}_t - \mathbb{E}_t \left[(\hat{\pi}_{t+1} - \hat{\check{a}}_{t+1}) \right] \right), \tag{A.163}$$

$$\hat{\check{a}}_t + \hat{\check{\pi}}_t^F \approx -\frac{\varepsilon\phi}{\theta_F(\varepsilon-1)} \widehat{\check{mc}}_t + \frac{\phi^2\nu\varepsilon - (\nu-1)\phi(\varepsilon-1+\phi)}{\nu\theta_F(\varepsilon-1)^2} \hat{\check{f}}_t + \beta \mathbb{E}_t \left[\hat{\check{a}}_{t+1} + \hat{\check{\pi}}_{t+1}^F \right], \tag{A.164}$$

$$\hat{\check{y}}_t = \hat{\check{c}}_t, \tag{A.165}$$

$$\hat{\check{a}}_t = \varepsilon_t, \tag{A.166}$$

$$\hat{\pi}_t^S = \frac{\phi}{\varepsilon-1} (\hat{\check{a}}_t + \hat{\check{\pi}}_t^F) + \hat{\pi}_t. \tag{A.167}$$

As a result equations the reduced model becomes:

$$\tilde{\check{y}}_t = \mathbb{E}_t[\tilde{\check{y}}_{t+1}] - (i_t - \mathbb{E}_t[\tilde{\pi}_{t+1}] - \check{r}_t^n - \pi^*), \tag{A.168}$$

$$\tilde{\pi}_t^S = \frac{\varepsilon(\sigma+\eta)}{\theta_P}\tilde{\check{y}}_t + \beta\mathbb{E}_t\left[\tilde{\pi}_{t+1}^S\right], \tag{A.169}$$

$$\tilde{\check{\pi}}_t^F = -\frac{\varepsilon\phi(\sigma+\eta)}{\theta_F(\varepsilon-1)}\tilde{\check{y}}_t - \frac{\phi(\nu-1)(\varepsilon-1+\phi)}{\nu\theta_F\left(\varepsilon-1\right)^2}\tilde{\check{f}}_t + \beta\mathbb{E}_t\left[\tilde{\check{\pi}}_{t+1}^F\right], \tag{A.170}$$

$$i_t = r_t^n + \pi^* + \phi_\pi\tilde{\pi}_t + \phi_y\tilde{y}_t + \varepsilon_t, \tag{A.171}$$

$$\check{r}_t^n = \rho + \mathbb{E}_t[\check{a}_{t+1}], \tag{A.172}$$

$$\tilde{\check{\pi}}_t^F = \tilde{\check{f}}_t - \tilde{\check{f}}_{t-1}, \tag{A.173}$$

$$\tilde{\pi}_t^S = \frac{\phi}{\varepsilon-1}\tilde{\check{\pi}}_t^F + \tilde{\pi}_t, \tag{A.174}$$

$$\check{a}_t = \ln(1+g_A) + \varepsilon_t. \tag{A.175}$$

This version of the model shares many similarities with that discussed in the main text. Once difference is the absence of an amplification channel from productivity shocks to the natural rate of interest, $\check{r}_t^n$. This difference arises as the model accounts for this change during the process of being made stationary. For instance, in each period output is divided by $A_t^{-\frac{\varepsilon-1+\phi}{\varepsilon-1}}$, rather than the standard A_t. In this model with a unit root, a shock to productivity will now *permanently* alter the level of output.

A.7 Product Quality and the Skill Premium

This appendix details an alternative scenario for the production process of intermediate firms, which is isomorphic to the form discussed in the main text. A typical intermediate firm, j, faces a production function given by:

$$Y_t(j) = N_t(j)\mathcal{H}(A_t, F_t(j)) = N_t(j)\left[A_t^{\frac{\nu-1}{\nu}} - \kappa[F_t(j)]^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}, \tag{A.176}$$

where $N_t(j)$ is the level of labour input and the function $\mathcal{H}(\cdot)$ takes the place of the standard aggregate productivity level. The functional form of $\mathcal{H}_t(j)$, as a CES aggregate of economy-wide productivity and the decision of which level of quality to produce, is given in the second equality, with $\nu < 0$ and $\kappa > 0$, and is the object of discussion here. This function shifts the production process depending on both the economy-wide level of productivity, A_t and the level of quality firm j chooses to produce, $F_t(j)$.

A.7.1 Differentiated Labour

Consider an alternative model where household labour hours supplied to each firm is comprised of both unskilled, $N_{U,t}(j)$, and skilled, $N_{S,t}(j)$, workers, according to a CES aggregate:

$$N_t(j) = \left[N_{U,t}(j)^{\frac{\nu-1}{\nu}} + \kappa N_{S,t}(j)^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}. \tag{A.177}$$

where $N_t(j)$ is the total amount of labour supplied by the representative household to firm j. The parameter $\nu < 0$ is the elasticity of substitution between labour types, and captures the difficulty (or ease) through which workers are able to reallocate their labour supply between skilled and unskilled job roles.

Intermediate firms take these two labour inputs to produce two outputs, the quantity and quality of their products. Unskilled labour may be used to produce physical units, according to linear production technology:

$$Y_t(j) = A_t N_{U,t}(j). \tag{A.178}$$

Simultaneously, the quality of each unit produced is linear in the share of skilled workers in the firm's labour force:

$$F_t(j) = A_t \frac{N_{S,t}(j)}{N_t(j)}, \tag{A.179}$$

such that whenever a greater proportion of the labour force is highly skilled, the quality of all units produced increases. In equilibrium, the need to follow skill differentials, may be

eliminated as the production constraints (A.177), (A.178), and (A.179) may be combined to produce (A.176) .

These microeconomic foundations of this choice for the functional form of $\mathcal{H}(\cdot)$ clarify how workers may be seen to produce a form of TFP which is also valued by the household. Production of the quantity of goods in restricted by the level of quality as workers are moved away from the production process towards increasing the quality of production. Effective TFP, $\mathcal{H} \equiv \left[A_t^{\frac{\nu-1}{\nu}} - \kappa[F_t(j)]^{\frac{\nu-1}{\nu}}\right]^{\frac{\nu}{\nu-1}}$, therefore falls as the effort and labour inputs used to produce high quality products reduce the number of material items available at each productivity level.

Once the model in the main text has been solved, there are no further requirements to clear the skilled and unskilled labour markets. This is because any equilibrium levels of F_t, Y_t and N_t found in a solution may be combined with the market supply for skilled and unskilled labour to find the equilibrium wage in each labour market, $W_{S,t}$ and $W_{U,t}$, respectively. Given these equilibrium prices, either labour demand or supply for each sector may subsequently be used to determine the equilibrium quantity of skilled and unskilled labour. In other words, given any solution for N_t, the balance between skilled and unskilled workers is determined by the balance between quantity and quality of the products, Y_t and F_t.

A.7.2 Product Quality and the Skill Premium

One implication of the above formulation is a close link between the level of quality and the labour skill premium – the ratio between the wages of skilled and unskilled workers. This may be seen using the supply function for each labour skill type:

$$\frac{N_{U,t}(j)}{N_t(j)} = \left(\frac{W_{U,t}}{W_t}\right)^{-\nu}, \qquad \text{and} \qquad \frac{N_{S,t}(j)}{N_t(j)} = \left(\frac{W_{S,t}}{W_t}\right)^{-\nu}, \tag{A.180}$$

where $W_t = \left[W_{U,t}^{1-\nu} + W_{S,t}^{1-\nu}\right]^{\frac{1}{1-\nu}}$. Taking the ratio between sectors highlights:

$$\frac{N_{U,t}(j)}{N_{S,t}(j)} = S_t^{\nu}, \tag{A.181}$$

where $S_t \equiv \frac{W_{S,t}}{W_{U,t}}$ is the definition of the skill premium. To demonstrate the relationship between quality and the skill premium, the production function for quality may be used to show:

$$F_t(j) = A_t \frac{N_{S,t}(j)}{N_t(j)} = A_t\left[\left(\frac{N_{U,t}(j)}{N_{S,t}(j)}\right)^{\frac{\nu-1}{\nu}} + \kappa\right]^{-\frac{\nu}{\nu-1}} = A_t\left[S_t^{\nu-1} + \kappa\right]^{-\frac{\nu}{\nu-1}}. \tag{A.182}$$

Using this relationship, and the efficient steady state level of quality derived in Appendix A.2, $\bar{F} = \left(\frac{\phi}{\kappa(\varepsilon-1)+\kappa\phi}\right)^{\frac{\nu}{\nu-1}}$, the efficient steady state level of the skill premium is given as:

$$\bar{S} = \left(\frac{\phi}{\kappa(\varepsilon-1)}\right)^{\frac{1}{1-\nu}}, \tag{A.183}$$

which, assuming $\nu < 0$, shows:

$$\frac{d\bar{S}}{d\phi} = \frac{1}{(1-\nu)\kappa(\varepsilon-1)}\left(\frac{\phi}{\kappa(\varepsilon-1)}\right)^{\frac{\nu}{1-\nu}} > 0. \tag{A.184}$$

This shows a higher preference for product quality, with ϕ increasing, will increase the equilibrium skill premium. Log-linearising the relationship between quality and the skill premium around this steady state gives:

$$\hat{f}_t = \hat{a}_t - \frac{\nu(\varepsilon-1)}{\varepsilon-1+\phi}\hat{s}_t. \tag{A.185}$$

Finally, using the fact that $f_t^n = a_t + \bar{f}$, which implies $s_t^n = \bar{s}$, yields:

$$\tilde{f}_t = -\frac{\nu(\varepsilon-1)}{\varepsilon-1+\phi}\tilde{s}_t, \tag{A.186}$$

such that any implications of the model for product quality may be directly matched to implications for the skill premium. For example, if the product quality increases by more than it's corresponding natural level following a productivity increase, this means the worker skill premium is also anticipated to increase by more than it's natural level, given that $\nu < 0$.

Calibration

This formulation of a link between product quality and the skill premium is useful in the context of scarce reliable macroeconomic data on product quality. Instead, empirical estimates of the skill premium may be used to calibrate the model. In particular, ϕ may be used to match the equilibrium size of the skill premium while ν may be used to match it's cyclical properties.

Initially note that we use κ to normalise $\bar{F} = 1$, with:

$$\bar{F} = 1 = \left(\frac{\phi}{\kappa(\varepsilon-1)+\kappa\phi}\right)^{\frac{\nu}{\nu-1}} \qquad \rightarrow \qquad \kappa = \frac{\phi}{\varepsilon-1+\phi} \tag{A.187}$$

Then, as the size of the skill premium is approximately $\exp^{0.65}$, this gives:[5]

$$\bar{S} = \exp^{0.6} = \left(\frac{\varepsilon - 1 + \phi}{\varepsilon - 1}\right)^{\frac{1}{1-\nu}} \qquad \rightarrow \qquad \phi = (\varepsilon - 1)(\exp^{0.6(1-\nu)} - 1). \tag{A.188}$$

As noted in Acemoglu (2003), conventional wisdom is that the skill premium increases when skilled workers become relatively more-not relatively less-productive. In the setting outlined above this is consistent with $\nu < -1$ and therefore and elasticity of substitution between skilled and unskilled workers greater than 1. This represents the second of Tinbergen's earnings inequality forces. Given a fixed technology skill bias, an increase in relative supply of labour skilled labour reduces the skill premium with elasticity ν.

[5] As identified using college vs. non-college graduated for instance in Figure 1 of Acemoglu (2003) and Acemoglu and Autor (2011).

Appendix B

The Impact of Large Scale Asset Purchases on Wealth Inequality

B.1 Data Description

B.1.1 Survey of Consumer Finances

Household wealth data are primarily taken from the US Survey of Consumer Finances (SCF), published by the Federal Reserve. This survey is ideally suited to the study of wealth inequality using household balance sheets, as it includes a stratified sample of the wealthiest households whose data would be difficult to match using standard sampling techniques. The SCF is conducted triennially and currently comprises around 6,000 US households, starting in the most comparable form in 1989. Households are usually interviewed between May and December of each survey year. In the current form households are only included for one wave, with the re-interview of households for the 2009 panel an exceptional case.

The calculation of net worth uses the methodology of Kuhn and Ríos-Rull (2016), matching their results with few differences.[1] Where small differences exist these are likely accounted for by data truncation in the public record, not affecting Kuhn and Ríos-Rull (2016). Following this methodology, wealth is calculated at a household level, with a definition that includes the value of financial and real assets of all kinds, net of debts. This includes property; businesses (including farms); checking accounts, certificates of deposit, and other banking accounts; IRA/Keogh accounts, money market accounts, mutual funds, bonds and stocks, cash and call money at the stock brokerage, and all annuities, trusts, and managed investment accounts; vehicles; the cash value of term life insurance policies and other policies; money owed to others; pension plans accumulated in accounts; and other assets. Debts include the value of property debts (including mortgages, home equity, and HELOCs); credit card debts; instalment loans; loans taken against pensions; loans taken against life insurance; margin loans and other miscellaneous debts.

Whenever percentiles are used a 100 household average around the percentile of interest is taken to reduce cross-sectional noise, bracketing the percentile of interest by approximately 1pp. Where small differences between the 2007 panel and main survey exist, these are likely accounted for by the exclusion of households who were not re-interviewed in 2009. Where real values are reported, nominal wealth is deflated using the PCE deflator.

[1] For example their Table 2 (Quantiles of the 2013 earnings, income, and wealth distributions) and Table 4 (Concentration and skewness decomposition).

Additional Tables

The evolution of US household wealth (net worth) is shown in Table B.1. Over the past quarter of a century, total nominal US household wealth has increased more than fivefold. However the evolution of aggregate household wealth masks a substantial degree of heterogeneity. For instance, households in the 90th percentile of today's wealth distribution are close to 10 times wealthier than the median household, with a net worth of $1.1mn, compared to $114,600 at the 50th percentile. Heterogeneity among households is therefore important in describing the evolution of wealth. When examined by wealth percentile, the high proportion of wealth accruing to the top of the wealth distribution can be seen. Indeed, in nominal terms, those at the bottom of the wealth distribution were less wealthy in 2019, than in 1989, with negative net worth.

Table B.1 US Household Net Wealth ($ thousand)

Percentile / Year	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
10	0.0	0.0	0.0	0.0	0.0	0.1	0.0	-2.1	-3.0	-2.2	-1.3
20	2.0	3.0	4.5	4.7	6.4	6.0	6.3	3.6	3.7	4.0	5.6
30	10.0	12.3	15.0	16.4	22.3	22.7	23.3	13.5	13.4	17.4	21.4
40	25.2	26.8	31.5	38.7	47.9	51.0	60.7	37.4	35.7	46.4	63.6
50	46.1	48.0	56.0	71.4	84.2	90.8	115.7	73.1	77.2	93.4	114.6
60	76.1	77.6	84.2	108.7	136.7	152.7	195.3	132.4	140.8	164.7	191.9
70	112.6	116.2	123.9	167.8	224.1	250.8	294.5	229.9	233.2	271.2	303.5
80	184.9	182.9	189.9	268.2	370.6	428.3	486.2	400.0	410.7	478.2	537.4
90	351.2	341.7	356.2	494.8	731.2	827.4	869.2	918.0	919.4	1150.9	1142.7
Total ($ trillion)	16.3	20.0	21.0	29.7	42.9	50.2	66.0	59.3	64.5	86.7	94.8

Sources: US SCF and author's calculations.

These data also highlight how unusual changes in the wealth distribution have been since the financial crisis. Until 2007 nominal household wealth had broadly increased for every decile in the distribution. Yet, between 2004 and 2013, wealth has fallen for every decile other than the wealthiest. Households at the lower end of the wealth distribution saw proportionally larger declines in net wealth. The least wealthy households substantially increased the nominal value of debt. For example, in 2010 households at the 10th percentile had a negative net worth of $2,100, decreasing to a negative net worth of $3000 by 2013. Overall, total nominal household wealth had not recovered to its previous peak by 2013, but far exceeded this in the 2016 and 2019 surveys, driven predominantly by increases for the wealthiest households.

The composition of US household net worth across SCF samples is shown in Table B.2. The various sub-components are then constructed to form a measure of household liquid and illiquid assets. Between 2007 and 2013 the share of household liquid assets in total wealth rapidly increased, from 11.0% to 16.3%, while that of illiquid assets has fell, from 89.0% of net worth to 83.7%. Since then the share of liquid wealth has fallen slightly towards a more historically normal level. During the same period, between 2007 and 2013, the percentage share of the population with negative illiquid asset holdings rapidly increased from 6.0% to 10.6% in 2013. There has since been some reduction in this debt overhang, but at 9.1% in 2019 the level remains substantially above historical values.

Table B.2 Percentage Composition of Aggregate Household Net Wealth

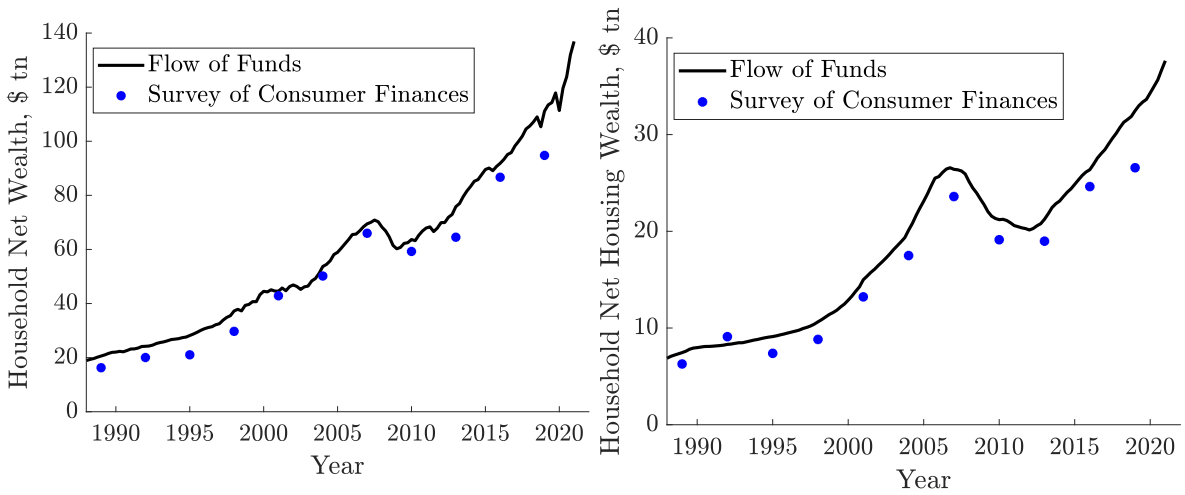
Year	1989	1992	1995	1998	2001	2004	2007	2010	2013	2016	2019
Liquid Wealth											
Consumer Debt	-1.8	-1.1	-1.3	-1.2	-0.8	-0.9	-0.9	-0.9	-0.7	-0.7	-0.7
Deposits	14.1	12.5	13.1	13.8	13.9	13.4	11.9	15.3	17.1	14.9	15.0
Total Liquid	12.3	11.4	11.8	12.5	13.1	12.5	11.0	14.4	16.3	14.2	14.3
Negative†	25.7	25.1	27.1	21.8	21.4	23.6	22.9	23.4	19.8	20.8	21.5
Illiquid Wealth											
Housing	38.6	45.4	35.1	29.7	30.8	34.9	35.8	32.3	29.4	28.4	28.0
Vehicle	3.2	3.1	4.0	3.2	2.9	2.8	2.5	3.0	2.8	2.3	2.2
Pension	3.6	3.8	5.0	10.0	10.6	7.4	7.2	8.4	7.8	7.9	7.6
Illiquid Financial	19.2	15.6	22.9	24.0	23.5	21.7	19.4	19.2	21.4	24.1	23.6
Business	23.1	20.7	21.2	20.6	19.1	20.7	24.2	22.7	22.2	23.2	24.2
Total Illiquid	87.7	88.6	88.2	87.5	86.9	87.5	89.0	85.6	83.7	85.8	85.7
Negative†	2.2	4.6	4.3	5.9	4.3	4.6	6.0	9.3	10.6	10.0	9.1

Sources and Notes: US SCF and author's calculations. Reported as percentage of total net worth unless otherwise indicated by † to denote percentage of population.

Comparison to US Flow of Funds

As also demonstrated in Kuhn and Ríos-Rull (2016), the aggregate measures in the SCF closely match those available from an alternative source, the US Flow of Funds (FoF) (Table B100). This is true both for the aggregate net household wealth, shown in Figure B.1, panel (a), and the housing component of net wealth, shown in Figure B.1, panel (b).

Figure B.1 Alternative Measures of Household Net Wealth



(a) Total Wealth.

(b) Housing Wealth.

Sources: US Flow of Funds and Survey of Consumer Finances.

B.1.2 International Comparison - UK Wealth and Assets Survey

The UK Wealth and Assets Survey (WAS) collects data on assets, liabilities and the distribution of net wealth from a sample of over 30,000 households. The survey is conducted in biannual waves, the first of which started in 2006. The WAS is published by the UK Office for National Statistics (ONS).

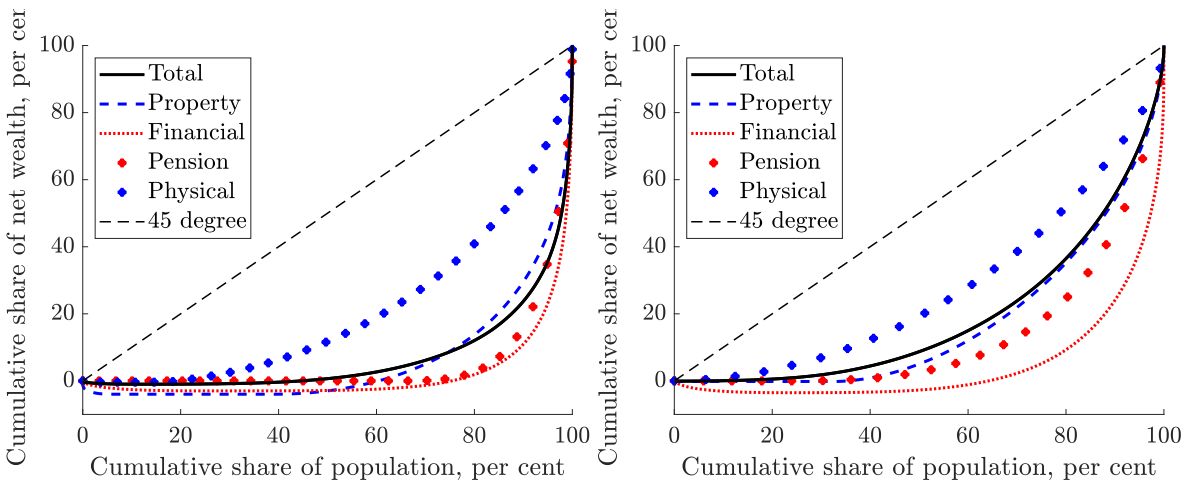
Although the main text focusses on the US context, a similar pattern of inequality measures holds for UK data, with physical and housing assets the most evenly distributed throughout the population, while pension and financial assets are less evenly distributed. Table B.3 shows composite measures of wealth inequality, by subcomponent of net wealth for the 2013 US SCF and Wave 4 of the UK WAS, which was conducted between 2012 and 2014. These data show the US to have a higher absolute level of wealth inequality, which holds in most components. The substantially higher inequality of US households' pension wealth, compared to the UK is indicative of different institutional arrangements. Figure B.2 shows the same data plotted as Lorenz curves.

Table B.3 Wealth Inequality, by Component

	Physical	Housing	Pension	Financial	Total
US					
Gini	0.58	0.88	0.90	0.97	0.86
Coef. of Var.	4.54	4.97	4.52	9.03	7.07
UK					
Gini	0.45	0.66	0.73	0.91	0.63
Coef. of Var.	1.11	1.88	2.00	18.84	3.17

Sources: US SCF (2013), UK WAS (Wave 4, 2012-2014).

Figure B.2 Household Net Wealth Inequality, by Component

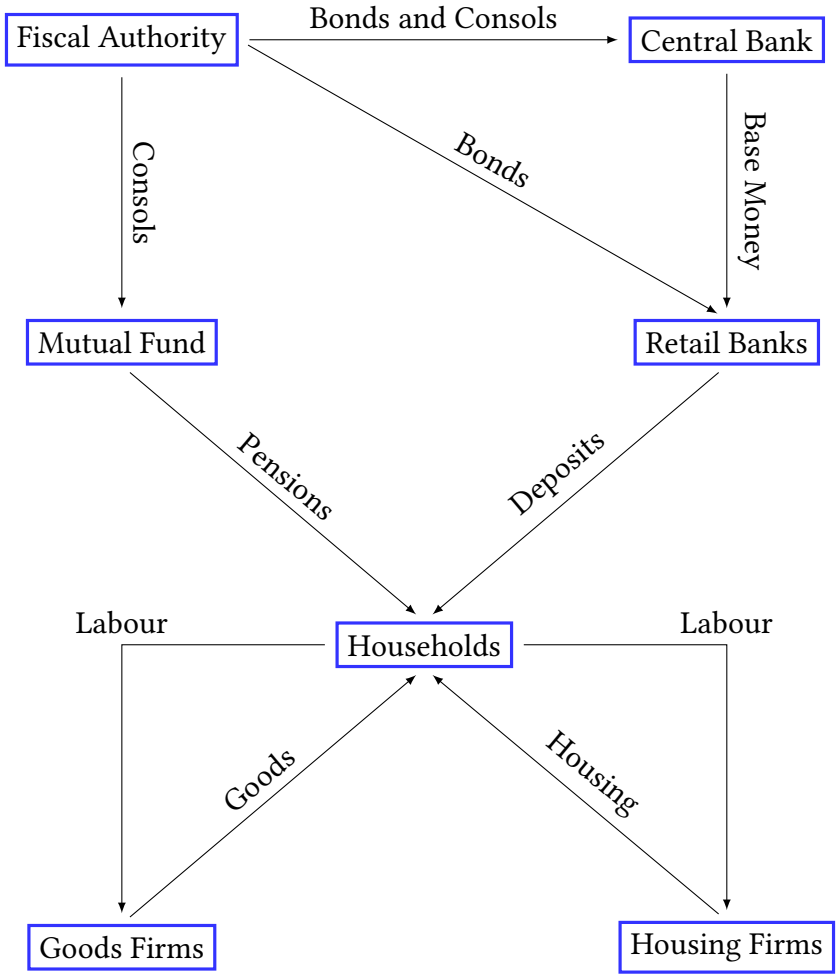


(a) US. (b) UK.

Sources: US SCF (2013), UK WAS (Wave 4, 2012-2014).

B.2 Model Schematic

Figure B.3 Model Schematic



Notes: Figure shows all economic agents along with goods and asset flows between them. Arrows begin at the economic agent which sells the good or asset.

B.3 Model in Aggregate Steady State

This appendix outlines the key equations presented in section 3.4 of the main text under the condition of an aggregate steady state. An aggregate steady state arises whenever aggregate variables are constant. Although aggregate variables are stationary, households may move between income states and asset positions. Indexation is therefore maintained for household variables, while time subscripts are dropped for aggregate variables.

B.3.1 Model Equations in Aggregate Steady State

Table B.4 sets out the complete list of 25 endogenous model variables, which are solved by the set of equations that follow.

Table B.4 Endogenous Variables

Sector	Variables						Count
Households	$c_{i,t}$	$d_{i,t+1}$	$h_{i,t+1}$	$x_{i,t}$	$\ell_{i,t}$	$\zeta_{i,t}$	6
Firms	n^c	n^h	w	p^h			4
Government and Central Bank	τ	m_s	s				3
Retail Banks and Mutual Fund	a_b	m_b	d_b	π_b	π_m		
	r^a	r^m	r^d	r^b			9
Market Clearing	n^c	n^h	$\pi_{i,t}$				3
Total							25

Each household, i, assumes r^d, p^h, w and τ are given and solves:

$$v(x,h,\zeta) = \max_{c,\ell,a',h'} \left\{ u(c,h,1-\ell) + \beta\mathbb{E}[v(x',h',\zeta')] \right\}, \tag{B.1}$$

subject to:

$$c + d' + p^h h' + \phi(h',h) = x + \tau + \pi + w\zeta\ell. \tag{B.2}$$

$$x' = (1+r^d)d' + p^h(1-\delta)h', \tag{B.3}$$

$$x' \geq (1-\Phi^d)p^h(1-\delta)h', \tag{B.4}$$

$$h' \geq 0, \tag{B.5}$$

$$\zeta \qquad \text{As specified in Table 3.8.} \tag{B.6}$$

This yields solutions for all 6 household variables and their distribution.

The optimality conditions for firm's may be summarised as:

$$w = z^c, \tag{B.7}$$

$$p^h = \frac{z^c}{z^h}. \tag{B.8}$$

Government and Central Bank:

$$\tau = s - r^a a_g - r^b b_g, \tag{B.9}$$

$$m_s = \lambda^a a_g + \lambda^b b_g, \tag{B.10}$$

$$s = (r^a - r^m)\lambda^a a_g + (r^b - r^m)\lambda^b b_g. \tag{B.11}$$

Retail Banks and Mutual Fund:

$$d_b = a_b + m_b, \tag{B.12}$$

$$\pi_b = (r^a - r^d)a_b + (r^m - r^d)m_b, \tag{B.13}$$

$$r^a = r^d, \tag{B.14}$$

$$r^m = r^d, \tag{B.15}$$

$$\pi_m = r^b b_m, \tag{B.16}$$

$$1 = \beta(1 + r^b). \tag{B.17}$$

Market clearing:

$$\pi_{i,t} = \begin{cases} \pi_b + \pi_m \left(\int_{\zeta_{i,t}=0} dF_t\right)^{-1} & \text{if} \quad \zeta_{i,t} = 0, \\ \pi_b & \text{otherwise,} \end{cases} \tag{B.18}$$

$$z^c n^c = \int_i c_{i,t} + \phi(h_{i,t+1}, h_{i,t}) dF, \tag{B.19}$$

$$z^h n^h = \int_i h_{i,t+1} - (1 - \delta) h_{i,t} dF, \tag{B.20}$$

$$n^c + n^h = \int_i \zeta_{i,t} \ell_{i,t} dF, \tag{B.21}$$

$$d_b = \int_i d_{i,t} dF_t, \tag{B.22}$$

$$a_g = a_b + \lambda^a a_g, \tag{B.23}$$

$$b_g = b_m + \lambda^b b_g, \tag{B.24}$$

$$m_s = m_b. \tag{B.25}$$

B.3.2 Model Solution in Aggregate Steady State

First note that the following variables may be written directly as functions of exogenous variables:

$$m_s = m_b = \lambda^a a_g + \lambda^b b_g, \tag{B.26}$$

$$a_b = (1 - \lambda^a) a_g, \tag{B.27}$$

$$b_m = (1 - \lambda^b) b_g, \tag{B.28}$$

$$w = z^c, \tag{B.29}$$

$$p^h = \frac{z^c}{z^h}, \tag{B.30}$$

$$1 + r^b = 1/\beta, \tag{B.31}$$

$$\zeta \qquad \text{As specified in Table 3.8.} \tag{B.32}$$

Combining the central bank and retail bank balance sheets, (B.10) and (B.12) respectively, with the liquid asset market and money market clearing conditions, (B.23) and (B.25) respectively, gives:

$$d_b = a_g + \lambda^b b_g, \tag{B.33}$$

such that an expansion of the central bank's balance sheet will increase the supply of deposits, as λ^b increases. Next, the profit flow of retail banks, (B.13), may be simplified using the optimality conditions for asset holdings, (B.14) and (B.15), as:

$$\pi_b = 0, \tag{B.34}$$

a result of the perfect competition in retail banking.

Combining government seigniorage revenue, (B.11) with the retail banks optimality conditions for asset holdings, (B.14) and (B.15), and the mutual fund's optimality condition, (B.17), the level of seignorage and government transfers, (B.9), respectively may be written as:

$$s = \left(\frac{1-\beta}{\beta} - r^d\right) \lambda^b b_g, \tag{B.35}$$

$$\tau = -\left(\frac{1-\beta}{\beta} - r^d\right) \left(1 - \lambda^b\right) b_g - r^d \left(a_g + b_g\right). \tag{B.36}$$

Then, using the mutual fund's budget constraint, (B.16), and first order condition, (B.17), along with the market clearing condition for illiquid assets, (B.24), the mutual fund lump-sum

dividends are:

$$\pi_m = \left(\frac{1-\beta}{\beta}\right)(1-\lambda^b)b_g. \tag{B.37}$$

Combining our results thus far, the individual level of household transfers (B.18) may be rewritten using the equilibrium level of banking profits, (B.34), and lump-sum dividends from the mutual fund, (B.37), along with the productivity process for $\zeta_{i,t}$:

$$\pi_{i,t} = \begin{cases} \left(\frac{1-\beta}{\beta}\right)(1-\lambda^b)b_g\left(\int_{\zeta_{i,t}=0} dF_t\right)^{-1} & \text{if} \quad \zeta_{i,t}=0, \\ 0 & \text{otherwise,} \end{cases} \tag{B.38}$$

Integrating this transfer scheme across all individuals gives:

$$\int_i \pi_{i,t} dF = \left(\frac{1-\beta}{\beta}\right)(1-\lambda^b)b_g. \tag{B.39}$$

Finally, given r^d, (B.14) and (B.15) give:

$$r^a = r^d, \tag{B.40}$$

$$r^m = r^d. \tag{B.41}$$

We may then find $c_{i,t}$, $d_{i,t+1}$, $h_{i,t+1}$, $x_{i,t}$, $\ell_{i,t}$ using the above solutions for w, p^h, m_b, m_s, a_b, d_b, π_b, τ, s, $\zeta_{i,t}$, $\pi_{i,t}$, r^a and r^m, along with the household problem. Alternatively put, given r^d, every aggregate price and quantity relevant for the household problem may be found. In a second step, once the household problem is solved, the remaining aggregate variables may be found. These are n^c and n^h, which are found using the market clearing conditions (B.19) and (B.20):

$$n^c = (1/z^c)\int_i c_{i,t} + \phi(h_{i,t+1}, h_{i,t})dF, \tag{B.42}$$

$$n^h = (1/z^h)\int_i h_{i,t+1} - (1-\delta)h_{i,t}dF. \tag{B.43}$$

B.3.3 Walras' Law

To show Walras' law holds for the labour market, begin with the household budget constraint, (3.2), and integrate this across all households:

$$\begin{aligned}&\int_i c_{i,t} + d_{i,t+1} + p_t^h h_{i,t+1} + \phi(h_{i,t+1}, h_{i,t}) dF \\ &= \int_i (1 + r_t^d) d_{i,t} + p_t^h (1 - \delta) h_{i,t} + \tau_t + \pi_{i,t} + w_t \zeta_{i,t} \ell_{i,t} dF. \end{aligned} \tag{B.44}$$

Using the market clearing condition for goods, (B.19), housing, (B.20), deposits (B.22), this simplifies to:

$$z_t^c n_t^c + p_t^h z_t^h n_t^h + d_{b,t+1} = (1 + r_t^d) d_{b,t} + \tau_t + \int_i \pi_{i,t} + w_t \zeta_{i,t} \ell_{i,t} dF. \tag{B.45}$$

In a steady state equilibrium, this may again be simplified using the equilibrium condition for bank deposits, (B.33), equilibrium fiscal transfers (B.36), and the integrated form of the idiosyncratic transfer mechanism, (B.39), to show:

$$z^c n^c + p^h z^h n^h = w \int_i \zeta_{i,t} \ell_{i,t} dF. \tag{B.46}$$

Finally using the firm's first order conditions for c and h respectively, (B.7) and (B.8), this becomes:

$$n^c + n^h = \int_i \zeta_{i,t} \ell_{i,t} dF. \tag{B.47}$$

which is exactly the labour market clearing condition, (3.52), above.

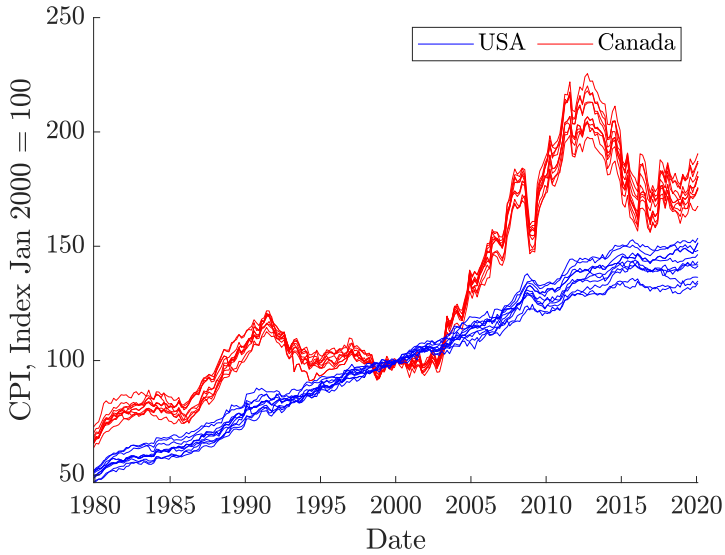
Appendix C

The Rise of Harrod-Balassa-Samuelson

C.1 Data Appendix for Stylised Fact 2 (LOOP)

This appendix provides greater detail on the data used in the paper for Stylised Fact 2 (LOOP). Figure C.1 presents an example of the price data being used. This exists in similar form for all CPI categories and continents as discussed in the main text. Table C.1 provides more information on data availability. Table C.2 provides a complete list of the cities used in the empirical section of this paper. Table C.3 outlines the CPI categories used, while Figure C.2 shows how these categories contribute to an average of 90.1% of consumer spending in the US.

Figure C.1 City-Level CPIs for Food and Non-alcoholic Beverages in the USA and Canada



Source and Notes: BLS and CANSIM. Figure shows the Consumer Price Indices (CPIs) for the Food and non-alcoholic beverages category across North American cities.

Table C.1 Data Availability

Country	Start Date	Frequency	Region
Argentina	2016 Q4	Monthly	Statistical Regions
Australia	1972 Q4	Quarterly	Cities
Brazil	2004 Q1	Monthly	Cities
Canada	1978 Q3	Monthly	Statistical Regions
Colombia	2007 Q1	Monthly	Cities
Ecuador	2005 Q1	Monthly	Cities
Germany	1995 Q1	Monthly	NUTS Level 1 Regions
Guatemala	2001 Q1	Monthly	Statistical Regions
Italy	1999 Q1	Monthly	NUTS Level 2 Regions
Mexico	1969 Q1	Monthly	Cities
Mozambique	2007 Q1	Monthly	Cities
New Zealand	1974 Q4	Quarterly	Statistical Regions
Peru	1997 Q1	Monthly	Statistical Regions
Poland	2005 Q1	Quarterly	Cities
Portugal	1992 Q1	Monthly	NUTS Level 2 Regions
South Africa	2008 Q1	Monthly	Statistical Regions
Spain	1999 Q1	Monthly	NUTS Level 2 Regions
Switzerland	2000 Q1	Monthly	Cities
USA	1970 Q1	Various†	Cities
Ukraine	2006 Q4	Monthly	Cities

Notes: Monthly data are converted to quarterly frequency. NUTS refers to Nomenclature of Territorial Units for Statistics. †USA has a unique reporting schedule. Some cities are sampled every month, while others are sampled in only odd or even months. Further, some cities are sampled at bi-annual frequency. For this latter group data are interpolated to generate quarterly measures. Many US cities change their reporting schedule during the sample.

Table C.2 List of Cities

City	Canada	Guatemala	Mexico	USA	Germany	Italy	Poland	Portugal	Spain	Switzerland	Ukraine
1	Calgary	Coban	Acapulco	Anchorage	Berlin	Ancona	Bialystok	Coimbra	Barcelona	Basel	Cherkasy
2	Charlottetown	Escuintla	Aguascalientes	Atlanta	Dusseldorf	Aosta	Bydgoszcz	Evora	Gasteiz	Geneva	Chernihiv
3	Halifax	Guatemala City	Campeche	Boston	Erfurt	Bari	Gdansk	Faro	Logrono	Zurich	Chernivtsi
4	Moncton	Huehuetenango	Chetumal	Chicago	Kamenz	Bologna	Katowice	Lisbon	Madrid		Dnipro
5	Montreal	Jalapa	Chihuahua	Cincinnati	Magdeburg	Cagliari	Kielce	Porto	Merida		Donetsk
6	Saskatoon	Puerto Barrios	Ciudad Acuna	Cleveland	Munich	Catanzaro	Krakow		Murcia		Ivo
7	St. John's	Quetzaltenango	Ciudad Juarez	Dallas	Potsdam	Florence	Lodz		Oviedo		Kharkiv
8	Toronto	San Benito	Colima	Denver	Saarbrucken	Genoa	Lublin		Pamplona		Kherson
9	Vancouver		Cordoba	Detroit	Schwerin	L'Aquila	Olsztyn		S. de Compostela		Khmelnytskyi
10	Whitehorse		Cortazar	Houston		Milan	Opole		Santander		Kyiv
11	Winnipeg		Cuernavaca	Kansas		Naples	Poznan		Seville		Lutsk
12	Yellowknife		Culiacan	Los Angeles		Palermo	Rzeszow		Toledo		Lviv
13			Durango	Miami		Perugia	Szczecin		Valencia		Mykolayiv
14			Fresnillo	Milwaukee		Potenza	Warsaw		Valladolid		Odessa
15			Guadalajara	Minneapolis		Rome	Wroclaw		Zaragoza		Oleksandriya
16			Hermosillo	New York		Trento	Zielona Gora				Poltava
17			Huatabampo	Philadelphia		Trieste					Rivne
18			Iguala	Pittsburgh		Turin					Sumy
19			Jacona	Portland		Venice					Vinnytsya
20			Jimenez	San Diego							Zaporizhzhya
21			La Paz	San Francisco							Zhytomyr
22			Leon	Seattle							
23			Matamoros	St. Louis							
24			Merida								
25			Mexicali								
26			Mexico City								
27			Monclova								
28			Monterrey								
29			Morelia								
30			Oaxaca								
31			Puebla								
32			Queretaro								
33			S.A. Tuxtla								
34			San Luis Potosi								
35			Tampico								
36			Tapachula								
37			Tehuantepec								
38			Tepatitlan								
39			Tepic								
40			Tijuana								
41			Tlaxcala								
42			Toluca								
43			Torreon								
44			Tulancingo								
45			Veracruz								
46			Villahermosa								

	Australia	New Zealand	Argentina	Brazil	Colombia	Ecuador	Peru	Mozambique	South Africa
1	Adelaide	Auckland	Buenos Aires	Belem	Armenia	Ambato	Abancay	Chimoio	Bloemfontein
2	Brisbane	Christchurch	Cordoba	Belo Horizonte	Barranquilla	Cuenca	Arequipa	Nampula	Cape Town
3	Canberra	Dunedin	Corrientes	Brasilia	Bogota	Esmeraldas	Ayacucho	Quelimane	Durban
4	Darwin	Hamilton	Mendoza	Curitiba	Bucaramanga	Guayaquil	Cajamarca	Tete	Johannesburg
5	Hobart	Wellington	Neuquen	Fortaleza	Cali	Loja	Cerro de Pasco		Kimberley
6	Melbourne		San Miguel	Goiania	Cartagena	Machala	Chachapoyas		Klerksdorp
7	Perth			Porto Alegre	Cucuta	Manta	Chiclayo		Mbombela
8	Sydney			Recife	Florencia	Quito	Cusco		Polokwane
9				Rio de Janeiro	Ibague		Huancavelica		Port Elizabeth
10				Salvador	Manizales		Huancayo		
11				Sao Paulo	Medellin		Huanuco		
12					Monteria		Huaraz		
13					Neiva		Ica		
14					Pasto		Lima		
15					Pereira		Moquegua		
16					Popayan		Moyobamba		
17					Quibdo		Piura		
18					Riohacha		Pucallpa		
19					Santa Marta		Puerto Maldonado		
20					Sincelejo		Puno		
21					Tunja		Tacna		
22					Valledupar		Trujillo		
23					Villavicencio		Tumbes		
24							Yurimaguas		

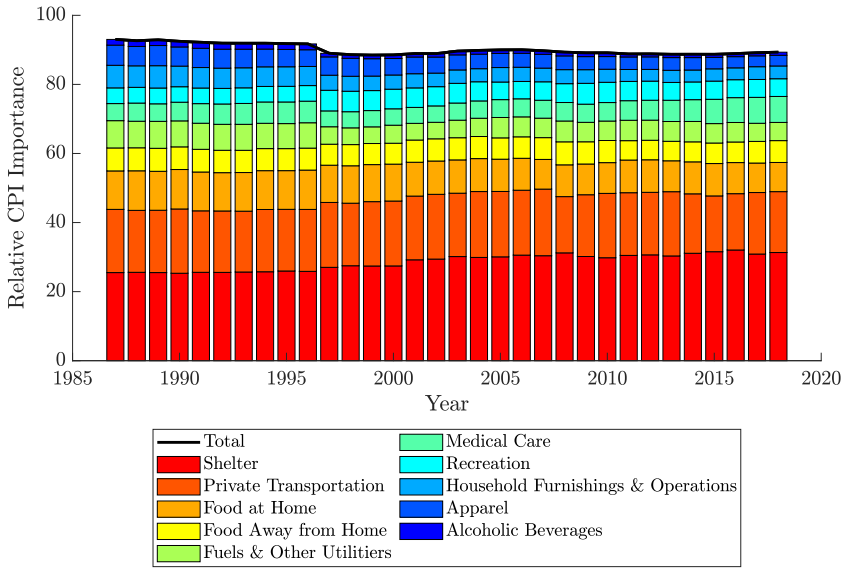
Notes: Statistical convention across countries necessitates using a range of geographical statistical region classifications as outlined in Table C.1. Regions are matched to most their populous conurbation for the purposes of geolocation. USA data is matched between the (older) 27 Metropolitan Statistical Areas and (newer) 23 Core Based Statistical Areas (CBSAs) according to the Bureau of Labour Statistics 2018 Geographic Revisions, as outlined at https://www.bls.gov/cpi/additional-resources/geographic-revision-2018.htm.

Table C.3 CPI Categories

Number	CPI Category
Traded	
1	Food and non-alcoholic beverages
2	Alcohol and tobacco
3	Household equipment
4	Clothing and footwear
Non-traded	
5	Housing
6	Health
7	Recreation
8	Restaurants and hotels
9	Transportation
10	Education
11	Communications

Notes: Whenever possible CPI data uses the United Nation's Classification of Individual Consumption by Purpose (COICOP) at the 2 digit (division) level. This classification system is widely used. Whenever this is not possible CPI data are instead sorted into their closest comparable COICOP category. In the USA CPI COICOP data are only available for at national level, consequently 'Alcohol and tobacco' refers only to alcoholic beverages, while 'Housing' refers to shelter without utility bills. All CPI data are non-seasonally adjusted. The first four categories are generally considered tradable goods, while the remaining categories are considered non-tradable services. A more detailed discussion of this split, with reference to underlying trade data, may be found in Andrade and Zachariadis (2016); Crucini and Telmer (2020).

Figure C.2 USA CPI Weights



Source and Notes: BLS CPI Historical Relative Importance Tables. Figure shows the relative importance (contribution) of categories included in main analysis to the overall headline US consumer spending.

C.2 Endowment Model

C.2.1 Endowment Model Equations

Table C.4 sets out the complete list of the 21 endogenous model variables, which are solved by the 8 Home country equations (4.10), (4.11), (4.12), (4.13), (4.14), (4.15), (4.19) and (4.22) from the main text, along with their Foreign country counterparts, and equations (4.16), (4.17), (4.18), (4.20) and (4.21). For ease of reference, the full set of model equations, including the Foreign country counterparts, are presented below.

Table C.4 Endogenous Variables

Country	Variables					Count
Home	C_T	C_N	$\tilde{C}_T$	$\tilde{C}_N$	Y_T	
	P	P_T	P_N	$\tilde{P}_T$	$\mathcal{Y}$	10
Foreign	C_T^*	C_N^*	$\tilde{C}_T^*$	$\tilde{C}_N^*$	Y_T^*	
	P^*	P_T^*	P_N^*	$\tilde{P}_T^*$	$\mathcal{Y}^*$	10
International	$\mathcal{Q}$					1
Total						21

Model equations, household's block:

$$P_N C_N + P_T C_T = P_N Y_N + \tilde{P}_T \tilde{Y}_T, \tag{C.1}$$

$$P_N^* C_N^* + P_T C_T^* = P_N^* Y_N^* + \tilde{P}_T^* \tilde{Y}_T^*, \tag{C.2}$$

$$\gamma P_N C_N = (1-\gamma) P_T C_T, \tag{C.3}$$

$$\gamma P_N^* C_N^* = (1-\gamma) P_T^* C_T^*, \tag{C.4}$$

Distribution and pricing block:

$$P = P_T^{\gamma} P_N^{1-\gamma}, \tag{C.5}$$

$$P^* = (P_T^*)^{\gamma} (P_N^*)^{1-\gamma}, \tag{C.6}$$

$$Y_T = \min\left\{\tilde{C}_T, \frac{\tilde{C}_N}{\phi}\right\}, \tag{C.7}$$

$$Y_T^* = \min\left\{\tilde{C}_T^*, \frac{\tilde{C}_N^*}{\phi}\right\}, \tag{C.8}$$

$$P_T = \tilde{P}_T + \phi P_N, \tag{C.9}$$

$$P_T^* = \tilde{P}_T^* + \phi P_N^*, \tag{C.10}$$

$$\tilde{C}_N = \phi \tilde{C}_T, \tag{C.11}$$

$$\tilde{C}_N^* = \phi \tilde{C}_T^*, \tag{C.12}$$

$$\tilde{P}_T = 1, \tag{C.13}$$

$$\tilde{P}_T^* = \tilde{P}_T. \tag{C.14}$$

Market clearing:

$$Y_N = C_N + \tilde{C}_N, \tag{C.15}$$

$$Y_T = C_T, \tag{C.16}$$

$$Y_T^* = C_T^*, \tag{C.17}$$

$$\tilde{Y}_T + \tilde{Y}_T^* = \tilde{C}_T + \tilde{C}_T^*. \tag{C.18}$$

Auxiliary variable definitions:

$$\mathcal{Q} = \frac{P^*}{P}, \tag{C.19}$$

$$\mathcal{Y} = \frac{C_T^{\gamma} C_N^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \tag{C.20}$$

$$\mathcal{Y}^* = \frac{(C_T^*)^{\gamma} (C_N^*)^{1-\gamma}}{\gamma^{\gamma}(1-\gamma)^{1-\gamma}}, \tag{C.21}$$

C.2.2 Endowment Model Solution

To solve the Home country household's problem, the budget constraint (C.1) is used alongside the first order condition (C.3) to show:

$$C_T = \gamma \left(\frac{P_N}{P_T} Y_N + \frac{\tilde{P}_T}{P_T} \tilde{Y}_T \right), \tag{C.22}$$

$$C_N = (1-\gamma) \left(Y_N + \frac{\tilde{P}_T}{P_N} \tilde{Y}_T \right), \tag{C.23}$$

which, given endowments, Y_N and $\tilde{Y}_T$, and prices P_N, P_T and $\tilde{P}_T$, will determine the household's choice of consumption of the traded and non-traded goods respectively. Through an analogous derivation, using (C.2) and (C.4) symmetric conditions hold for the Foreign country:

$$C_T^* = \gamma \left(\frac{P_N^*}{P_T^*} Y_N^* + \frac{\tilde{P}_T^*}{P_T^*} \tilde{Y}_T^* \right), \tag{C.24}$$

$$C_N^* = (1-\gamma) \left(Y_N^* + \frac{\tilde{P}_T^*}{P_N^*} \tilde{Y}_T^* \right), \tag{C.25}$$

Equations (C.22), (C.23), (C.24) and (C.25) may be rewritten using the Home and Foreign retail consumer pricing conditions (C.9) and (C.10), the choice of numéraire (C.13) and LOOP in wholesale traded goods pricing (C.14) as:

$$C_T = \gamma \left(\frac{P_N}{1+\phi P_N} Y_N + \frac{\tilde{Y}_T}{1+\phi P_N} \right), \tag{C.26}$$

$$C_N = (1-\gamma) \left(Y_N + \frac{\tilde{Y}_T}{P_N} \right), \tag{C.27}$$

$$C_T^* = \gamma \left(\frac{P_N^*}{1+\phi P_N^*} Y_N^* + \frac{\tilde{Y}_T^*}{1+\phi P_N^*} \right), \tag{C.28}$$

$$C_N^* = (1-\gamma) \left(Y_N^* + \frac{\tilde{Y}_T^*}{P_N^*} \right), \tag{C.29}$$

which gives four equations in the six unknowns C_T, C_N, C_T^*, C_N^*, P_N and P_T^*.

Using the production functions for the domestic distribution of traded goods, (C.7) and (C.8), alongside the optimal input choices, (C.11) and (C.12), the market clearing conditions for wholesale traded goods in the Home and Foreign country, (C.16) and (C.17) respectively,

become:

$$Y_T = \tilde{C}_T = \frac{\tilde{C}_N}{\phi} = C_T, \tag{C.30}$$

$$Y_T^* = \tilde{C}_T^* = \frac{\tilde{C}_N^*}{\phi} = C_T^* \tag{C.31}$$

In this form, these two market clearing conditions for domestic distribution may be used to rewrite the market clearing condition Home non-traded goods (C.15) and traded goods (C.18) and as:

$$Y_N = C_N + \phi C_T, \tag{C.32}$$

$$\tilde{Y}_T + \tilde{Y}_T^* = C_T + C_T^*. \tag{C.33}$$

which may then be used alongside (C.26) to (C.29) to solve the system. Note that the market clearing condition for Foreign non-traded goods, the Foreign equivalent of equation (C.15), will hold by Walras' law.

Using (C.26), (C.27) and (C.32), the equilibrium price of non-traded goods in the Home country, P_N, may be derived as:

$$P_N = \frac{(1-\gamma)\tilde{Y}_T}{\gamma Y_N - \phi \tilde{Y}_T}, \tag{C.34}$$

Then, using (C.26), (C.28) and (C.33) along with (C.34), the equilibrium price of non-traded goods in the Foreign country, P_N^*, may be derived as:

$$P_N^* = \frac{(1-\gamma)\tilde{Y}_T^*}{\gamma Y_N^* - \phi \tilde{Y}_T^*}. \tag{C.35}$$

Finally, given these price levels, (C.26) to (C.29) may be used to find household consumption of each good, while the aggregate hedonic consumer price levels for each country may be found using (C.5) and (C.6) and the consumer price of traded goods for each country may be found using (C.9) and (C.10). Definitions (C.19), (C.20) and (C.21) may be used to find the

auxiliary variables. The full analytical reduced-form model solution is then given as:

$$\tilde{P}_T = 1, \tag{C.36}$$

$$\tilde{P}_T^* = 1, \tag{C.37}$$

$$P_N = \frac{(1-\gamma)\tilde{Y}_T}{\gamma Y_N - \phi \tilde{Y}_T}, \tag{C.38}$$

$$P_N^* = \frac{(1-\gamma)\tilde{Y}_T^*}{\gamma Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.39}$$

$$P_T = \gamma \frac{Y_N - \phi \tilde{Y}_T}{\gamma Y_N - \phi \tilde{Y}_T}, \tag{C.40}$$

$$P_T^* = \gamma \frac{Y_N^* - \phi \tilde{Y}_T^*}{\gamma Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.41}$$

$$P = \frac{(Y_N - \phi \tilde{Y}_T)^\gamma \tilde{Y}_T^{1-\gamma} \gamma^\gamma (1-\gamma)^{1-\gamma}}{\gamma Y_N - \phi \tilde{Y}_T}, \tag{C.42}$$

$$P^* = \frac{(Y_N^* - \phi \tilde{Y}_T^*)^\gamma (\tilde{Y}_T^*)^{1-\gamma} \gamma^\gamma (1-\gamma)^{1-\gamma}}{\gamma Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.43}$$

$$C_T = Y_T = \tilde{C}_T = \frac{\tilde{C}_N}{\phi} = \tilde{Y}_T, \tag{C.44}$$

$$C_T^* = Y_T^* = \tilde{C}_T^* = \frac{\tilde{C}_N^*}{\phi} = \tilde{Y}_T^*, \tag{C.45}$$

$$C_N = Y_N - \phi \tilde{Y}_T, \tag{C.46}$$

$$C_N^* = Y_N^* - \phi \tilde{Y}_T^*, \tag{C.47}$$

$$\mathcal{Q} = \left(\frac{Y_N^* - \phi \tilde{Y}_T^*}{Y_N - \phi \tilde{Y}_T}\right)^\gamma \left(\frac{\tilde{Y}_T^*}{\tilde{Y}_T}\right)^{1-\gamma} \frac{\gamma Y_N - \phi \tilde{Y}_T}{\gamma Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.48}$$

$$\mathcal{Y} = \frac{\tilde{Y}_T^\gamma (Y_N - \phi \tilde{Y}_T)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}, \tag{C.49}$$

$$\mathcal{Y}^* = \frac{(\tilde{Y}_T^*)^\gamma (Y_N^* - \phi \tilde{Y}_T^*)^{1-\gamma}}{\gamma^\gamma (1-\gamma)^{1-\gamma}}. \tag{C.50}$$

C.2.3 Log-linearisation

To connect the endowment model to the stylised facts presented in section 4.2, consider, as functions of the exogenous variables, the real exchange rate, $\mathcal{Q}$, given in (C.48); the ratio of

national income, $\mathcal{Y}^*/\mathcal{Y}$, i.e. the ratio of (C.50) to (C.49):

$$\frac{\mathcal{Y}^*}{\mathcal{Y}} = \left(\frac{\tilde{Y}_T^*}{\tilde{Y}_T}\right)^{\gamma} \left(\frac{Y_N^* - \phi\tilde{Y}_T^*}{Y_N - \phi\tilde{Y}_T}\right)^{1-\gamma}; \tag{C.51}$$

and the relative price of tradable consumer goods, P_T^*/P_T, i.e. the ratio of (C.41) to (C.40):

$$\frac{P_T^*}{P_T} = \left(\frac{Y_N^* - \phi\tilde{Y}_T^*}{Y_N - \phi\tilde{Y}_T}\right)\left(\frac{\gamma Y_N - \phi\tilde{Y}_T}{\gamma Y_N^* - \phi\tilde{Y}_T^*}\right). \tag{C.52}$$

Log-linearisation of (C.48), (C.51) and (C.52) then proceeds as follows:

Notation Bars denote the non-stochastic steady state levels, $\bar{x}$, hats denote percentage deviation from their non-stochastic steady state level, $\hat{x}$, ($\approx$ to log deviations). Variables are written in upper case with their logarithm in lower case.

Step 1 Take natural logs of (C.48), (C.51) and (C.52) respectively and, where needed, replace variables with $X = \exp^{\ln X}$:

$$\begin{aligned}
\ln Q &= \gamma\left[\ln(\exp^{\ln Y_N^*} - \phi\exp^{\ln \tilde{Y}_T^*}) - \ln(\exp^{\ln Y_N} - \phi\exp^{\ln \tilde{Y}_T})\right] \\
&\quad + (1-\gamma)(\ln \tilde{Y}_T^* - \ln \tilde{Y}_T) \\
&\quad + \ln(\gamma\exp^{\ln Y_N} - \phi\exp^{\ln \tilde{Y}_T}) - \ln(\gamma\exp^{\ln Y_N^*} - \phi\exp^{\ln \tilde{Y}_T^*}), && \text{(C.53)} \\
\ln(\mathcal{Y}^*/\mathcal{Y}) &= \gamma(\ln \tilde{Y}_T^* - \ln \tilde{Y}_T) \\
&\quad + (1-\gamma)\left[\ln(\exp^{\ln Y_N^*} - \phi\exp^{\ln \tilde{Y}_T^*}) - \ln(\exp^{\ln Y_N} - \phi\exp^{\ln \tilde{Y}_T})\right], && \text{(C.54)} \\
\ln(P_T^*/P_T) &= \ln(\exp^{\ln Y_N^*} - \phi\exp^{\ln \tilde{Y}_T^*}) + \ln(\gamma\exp^{\ln Y_N} - \phi\exp^{\ln \tilde{Y}_T}) \\
&\quad - \ln(\exp^{\ln Y_N} - \phi\exp^{\ln \tilde{Y}_T}) - \ln(\gamma\exp^{\ln Y_N^*} - \phi\exp^{\ln \tilde{Y}_T^*}). && \text{(C.55)}
\end{aligned}$$

Step 2 Take derivatives of (C.53), (C.54) and (C.55) with respect to $\ln Y_N$, $\ln \tilde{Y}_T$, $\ln Y_N^*$ and $\ln \tilde{Y}_T^*$. This gives:

$$\frac{\partial \ln \mathcal{Q}}{\partial \ln Y_N^*} = \gamma \frac{Y_N^*}{Y_N^* - \phi \tilde{Y}_T^*} - \frac{\gamma Y_N^*}{\gamma Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.56}$$

$$\frac{\partial \ln \mathcal{Q}}{\partial \ln \tilde{Y}_T^*} = -\gamma\phi \frac{\tilde{Y}_T^*}{Y_N^* - \phi \tilde{Y}_T^*} + (1-\gamma) + \frac{\phi \tilde{Y}_T^*}{\gamma Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.57}$$

$$\frac{\partial \ln \mathcal{Q}}{\partial \ln Y_N} = -\gamma \frac{Y_N}{Y_N - \phi \tilde{Y}_T} + \frac{\gamma Y_N}{\gamma Y_N - \phi \tilde{Y}_T}, \tag{C.58}$$

$$\frac{\partial \ln \mathcal{Q}}{\partial \ln \tilde{Y}_T} = \gamma\phi \frac{\tilde{Y}_T}{Y_N - \phi \tilde{Y}_T} - (1-\gamma) - \frac{\phi \tilde{Y}_T}{\gamma Y_N - \phi \tilde{Y}_T}. \tag{C.59}$$

$$\frac{\partial \ln(\mathcal{Y}^*/\mathcal{Y})}{\partial \ln Y_N^*} = \frac{(1-\gamma) Y_N^*}{Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.60}$$

$$\frac{\partial \ln(\mathcal{Y}^*/\mathcal{Y})}{\partial \ln \tilde{Y}_T^*} = \frac{\gamma Y_N^* - \phi \tilde{Y}_T^*}{Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.61}$$

$$\frac{\partial \ln(\mathcal{Y}^*/\mathcal{Y})}{\partial \ln Y_N} = -\frac{(1-\gamma) Y_N}{Y_N - \phi \tilde{Y}_T}, \tag{C.62}$$

$$\frac{\partial \ln(\mathcal{Y}^*/\mathcal{Y})}{\partial \ln \tilde{Y}_T} = -\frac{\gamma Y_N - \phi \tilde{Y}_T}{Y_N - \phi \tilde{Y}_T}. \tag{C.63}$$

$$\frac{\partial \ln(P_T^*/P_T)}{\partial \ln Y_N^*} = \frac{Y_N^*}{Y_N^* - \phi \tilde{Y}_T^*} - \frac{\gamma Y_N^*}{\gamma Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.64}$$

$$\frac{\partial \ln(P_T^*/P_T)}{\partial \ln \tilde{Y}_T^*} = -\frac{\phi \tilde{Y}_T^*}{Y_N^* - \phi \tilde{Y}_T^*} + \frac{\phi \tilde{Y}_T^*}{\gamma Y_N^* - \phi \tilde{Y}_T^*}, \tag{C.65}$$

$$\frac{\partial \ln(P_T^*/P_T)}{\partial \ln Y_N} = -\frac{Y_N}{Y_N - \phi \tilde{Y}_T} + \frac{\gamma Y_N}{\gamma Y_N - \phi \tilde{Y}_T}, \tag{C.66}$$

$$\frac{\partial \ln(P_T^*/P_T)}{\partial \ln \tilde{Y}_T} = \frac{\phi \tilde{Y}_T}{Y_N - \phi \tilde{Y}_T} - \frac{\phi \tilde{Y}_T}{\gamma Y_N - \phi \tilde{Y}_T}. \tag{C.67}$$

Step 3 Simplify and evaluate derivatives (C.56) to (C.67) at the non-stochastic steady state $\bar{\tilde{Y}}_T$, $\bar{Y}_N$, $\bar{\tilde{Y}}_T^*$, $\bar{Y}_N^*$. We will assume symmetry between countries in this steady state, and define:

$$\theta \equiv \frac{\bar{Y}_N}{\bar{\tilde{Y}}_T} = \frac{\bar{Y}_N^*}{\bar{\tilde{Y}}_T^*}. \tag{C.68}$$

to simplify algebra. This gives:

$$\frac{\gamma\theta^2(1-\gamma)}{(\theta-\phi)(\gamma\theta-\phi)} = \frac{\partial \ln \mathcal{Q}}{\partial \ln \tilde{Y}_T^*} = -\frac{\partial \ln \mathcal{Q}}{\partial \ln Y_N^*} = \frac{\partial \ln \mathcal{Q}}{\partial \ln Y_N} = -\frac{\partial \ln \mathcal{Q}}{\partial \ln \tilde{Y}_T}, \tag{C.69}$$

$$\frac{\theta(1-\gamma)}{\theta-\phi} = \frac{\partial \ln(\mathcal{Y}^*/\mathcal{Y})}{\partial \ln Y_N^*} = -\frac{\partial \ln(\mathcal{Y}^*/\mathcal{Y})}{\partial \ln Y_N} \tag{C.70}$$

$$\frac{\gamma\theta-\phi}{\theta-\phi} = \frac{\partial \ln(\mathcal{Y}^*/\mathcal{Y})}{\partial \ln \tilde{Y}_T^*} = -\frac{\partial \ln(\mathcal{Y}^*/\mathcal{Y})}{\partial \ln \tilde{Y}_T}, \tag{C.71}$$

$$\begin{aligned}\frac{\phi\theta(1-\gamma)}{(\theta-\phi)(\gamma\theta-\phi)} &= \frac{\partial \ln(P_T^*/P_T)}{\partial \ln \tilde{Y}_T^*} = -\frac{\partial \ln(P_T^*/P_T)}{\partial \ln Y_N^*} \\ &= \frac{\partial \ln(P_T^*/P_T)}{\partial \ln Y_N} = -\frac{\partial \ln(P_T^*/P_T)}{\partial \ln \tilde{Y}_T}.\end{aligned} \tag{C.72}$$

Step 4 Note, and apply, the formula for a multivariate first order Taylor expansion:

$$f(X,Y) \approx f(\bar{X},\bar{Y}) + \frac{\partial f}{\partial X}(\bar{X},\bar{Y})(X-\bar{X}) + \frac{\partial f}{\partial Y}(\bar{X},\bar{Y})(Y-\bar{Y}). \tag{C.73}$$

Step 5 Apply the multivariate Taylor expansion, (C.73), to the reduced-form model equations (C.53), (C.54) and (C.55). Evaluate these approximations around the steady state using (C.69) to (C.72), and rewrite in deviation form, using the notation $\hat{x} \equiv \ln X - \ln \bar{X}$. Together this gives:

$$\hat{q} \approx \frac{\gamma\theta^2(1-\gamma)}{(\theta-\phi)(\gamma\theta-\phi)}[\hat{\tilde{y}}_T^* - \hat{y}_N^* - \hat{\tilde{y}}_T + \hat{y}_N], \tag{C.74}$$

$$\hat{y}^* - \hat{y} \approx \frac{\gamma\theta-\phi}{\theta-\phi}[\hat{\tilde{y}}_T^* - \hat{\tilde{y}}_T] + \frac{\theta(1-\gamma)}{\theta-\phi}[\hat{y}_N^* - \hat{y}_N], \tag{C.75}$$

$$\hat{p}_T^* - \hat{p}_T \approx \frac{\phi\theta(1-\gamma)}{(\theta-\phi)(\gamma\theta-\phi)}[\hat{\tilde{y}}_T^* - \hat{y}_N^* - \hat{\tilde{y}}_T + \hat{y}_N], \tag{C.76}$$

which are (4.26), (4.27), (4.28) from the main text.

C.2.4 Graphical Illustration Equations

Using the solution to the endowment model, given in section C.2.2, a change in the Foreign level of tradable output, with and without a domestic distribution channel, results in the following changes. Equations (C.39) and (C.41) may be combined to show:

$$\ln(P_N^*/P_T^*) = \ln\left(\frac{(1-\gamma)\tilde{Y}_T^*}{\gamma(Y_N^* - \phi\tilde{Y}_T^*)}\right), \tag{C.77}$$

while equations (C.38) and (C.40) show a symmetric condition for the Home country. The differentials of (C.77) are then given as:

$$\frac{d\ln(P_N^*/P_T^*)}{d\ln\tilde{Y}_T^*} = \frac{\theta}{\theta-\phi} > 0, \tag{C.78}$$

$$\frac{d\ln(P_N^*/P_T^*)}{d\phi} = \frac{1}{\theta-\phi} > 0, \tag{C.79}$$

$$\frac{d^2\ln(P_N^*/P_T^*)}{d\ln\tilde{Y}_T^* d\phi} = \left(\frac{\theta}{\theta-\phi}\right)^2 > 0, \tag{C.80}$$

where the definition of θ has again been used and positivity of these results is assured provided $\phi < \theta$, which holds under Assumption 1. Whenever $\phi = 0$ the wholesale tradable goods endowment $\tilde{Y}_T^*$ is replaced by the equivalent retail tradable goods endowment, Y_T, and (C.78), (C.79) and (C.80) remain positive with all three derivatives equal to 1.

C.3 Model with Production

C.3.1 Model Equations and Solution

Table C.5 sets out the complete list of the 30 endogenous model variables, which are solved by the set of equations that follow.

Table C.5 Endogenous Variables

Country	Variables					Count
Home	C	C_T	C_N	C_H	C_F	
	P	P_T	P_N	P_H	P_H^*	
	$\tilde{P}_H$	$\tilde{P}_H^*$	L	W	Π	15
Foreign	C^*	C_T^*	C_N^*	C_H^*	C_F^*	
	P^*	P_T^*	P_N^*	P_F^*	P_F	
	$\tilde{P}_F^*$	$\tilde{P}_F$	L^*	W^*	Π^*	15
Total						30

Model equations, household block:

$$\Pi + WL = PC, \tag{C.81}$$

$$W = \kappa L^{\psi} PC, \tag{C.82}$$

$$P_T C_T = \gamma PC, \tag{C.83}$$

$$P_N C_N = (1-\gamma) PC, \tag{C.84}$$

$$P_H C_H = \alpha P_T C_T, \tag{C.85}$$

$$P_F C_F = (1-\alpha) P_T C_T, \tag{C.86}$$

$$P = P_T^{\gamma} P_N^{1-\gamma}, \tag{C.87}$$

$$P_T = P_H^{\alpha} P_F^{1-\alpha}. \tag{C.88}$$

Firm block:

$$P_N = \frac{\eta}{\eta - 1}\frac{W}{A_N}, \tag{C.89}$$

$$P_H = \tilde{P}_H + \phi P_N, \tag{C.90}$$

$$P_H^* = \tilde{P}_H^* + \phi P_N^*, \tag{C.91}$$

$$\tilde{P}_H = \frac{\eta}{\eta - 1}\left(1 + \frac{\phi}{\eta - 1}\frac{A_H}{A_N}\right)\frac{W}{A_H}, \tag{C.92}$$

$$\tilde{P}_H^* = \frac{\eta}{\eta - 1}\left(1 + \frac{\phi}{\eta - 1}\frac{W^*}{W}\frac{A_H}{A_N^*}\right)\frac{W}{A_H}, \tag{C.93}$$

$$\Pi = \left(\tilde{P}_H - \frac{W}{A_H}\right)C_H + \left(\tilde{P}_H^* - \frac{W}{A_H}\right)C_H^* + \left(P_N - \frac{W}{A_N}\right)\left(C_N + \phi(C_H + C_F)\right). \tag{C.94}$$

Market clearing:

$$W = 1, \tag{C.95}$$

$$\tilde{P}_H^* C_H^* = \tilde{P}_F C_F. \tag{C.96}$$

Similar conditions arise for the Foreign country. If output levels are desired, we may also add the block:

$$Y_N = A_N L_N, \tag{C.97}$$

$$Y_H = A_H L_H, \tag{C.98}$$

$$Y_H = C_H + C_H^*, \tag{C.99}$$

$$L = L_N + L_H. \tag{C.100}$$