



# Sensitivity analysis methods for building energy models: Comparing computational costs and extractable information



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## ABSTRACT

Though sensitivity analysis has been widely applied in the context of building energy models (BEMs), there are few studies that investigate the performance of different sensitivity analysis methods in relation to dynamic, high-order, non-linear behaviour and the level of uncertainty in building energy models. We scrutinise three distinctive sensitivity analysis methods: (a) the computationally efficient Morris method for parameter screening, (b) linear regression analysis (medium computational costs) and (c) Sobol method (high computational costs). It is revealed that the results from Morris method taking the commonly used measure for parameter influence can be unstable, while using the median value yields robust results for evaluations with small sample sizes. For the dominant parameters the results from all three sensitivity analysis methods are in very good agreement. Regarding the evaluation of parameter ranking or the differentiation of influential and negligible parameters, the computationally costly quantitative methods provide the same information for the model in this study as the computationally efficient Morris method using the median value. Exploring different methods to investigate higher-order effects and parameter interactions, reveals that correlation of elementary effects and parameter values in Morris method can also provide basic information about parameter interactions.

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## 1. Introduction

With the energy demand of buildings accounting for a major part of total energy consumption and consequently for a significant amount of CO<sub>2</sub>-emissions worldwide there is an increasing interest in building energy performance [1–3]. With various types of simulation software packages available building energy models have become useful and easy-to-employ tools for a broad spectrum of applications, such as estimating potential reductions in the heating and cooling demand of buildings for retrofit options or system performance optimisation, and exploring options for energy supply systems at the design stage [4–7].

As many input parameters in these models are associated with a certain degree of uncertainty, due to variable conditions or lack of knowledge about the exact parameter value, sensitivity analysis of model parameters represents an important step in the modelling process in order to obtain credible results and valuable informa-

tion, as well as to increase the confidence in the model results [8–10]. The most common purpose for applying sensitivity analysis is to identify those input parameters that have a large impact on model outcomes in combination with an uncertainty analysis of these parameters [11–13]. Other possible applications include the assessment of uncertain model parameters that have a negligible impact on the model outcome, so that they might be neglected in further model evaluations or could be used to simplify a specific model [14,15]. Furthermore, sensitivity analysis can be a useful tool for the diagnosis of modelling errors, identification of weak model components and to improve the understanding of the relationship between model inputs and outputs [9].

Sensitivity analysis methods can be broadly separated into local methods that are computationally fast to apply, but explore only a reduced space of uncertain inputs, and global methods, which examine the influence of uncertain parameters over the whole parameter range [10,16]. Commonly applied global methods in building energy simulations cover parameters screening methods [5,11,17], regression-based methods [4,9,13,18] and variance-based methods [9,14,15]. A comprehensive overview on different sensitivity analysis methods used in building energy analysis is presented by Tian [16] including a detailed description of the theoretical framework of each method and of existing studies applying

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## Nomenclature

### Variables, parameters and indices

$A$	area of window openings ( $\text{m}^2$ )
$\mathbf{A}$	Sample matrix for model parameters
$A_{\text{eff}}$	Effective area of window openings ( $\text{m}^2$ )
$\mathbf{B}$	Sample matrix for model parameters
$c_p$	Specific heat capacity of water ( $\text{kJ}/(\text{kg K})$ )
$C_D$	Discharge coefficient (–)
$C_v$	Effectiveness of window openings (–)
$dH_{\text{NPL}}$	Difference in height of neutral pressure level (m)
$E(\cdot)$	Conditional expectation
$f$	Mass flow rate in radiant ceiling ( $\text{kg/s}$ )
$f_{\text{win}}$	Fraction of window area for air inlet (–)
$F$	Regression model
$g$	Gravitational acceleration ( $\text{m/s}^2$ )
$i, j$	Parameter/input variable index
$k$	Number of model input variables
$N$	Number of lines in sample matrix, equals number of required model evaluations
$p$	Number of grid levels for factorial sampling
$Q_{\text{heat}}$	Heat input to radiant ceiling (W)
$Q_{\text{th}}$	Air flow rate due to wind pressure ( $\text{m}^3/\text{s}$ )
$Q_w$	Air flow rate due to thermal buoyancy effect ( $\text{m}^3/\text{s}$ )
$r_w$	Wind reduction factor (–)
$R^2$	Coefficient of determination
$S_H$	Higher-order variance-based sensitivity index
$S_i$	First-order variance-based sensitivity index
$S_{ij}$	Second-order variance-based sensitivity index
$S_{Ti}$	Total variance-based sensitivity index
$t$	Morris method
$T_i$	Indoor air temperature ( $^{\circ}\text{C}$ )
$T_{\text{in}}$	Inlet temperature of radiant ceiling ( $^{\circ}\text{C}$ )
$T_o$	Outdoor air temperature ( $^{\circ}\text{C}$ )
$T_{\text{out}}$	Outlet temperature of radiant ceiling ( $^{\circ}\text{C}$ )
$U$	Measured wind speed (m/s)
$U_{\text{eff}}$	Effective wind speed at window height (m/s)
$V$	Variance
$X$	Vector of model input variables
$\mathbf{X}$	Sample matrix of input variables values
$X_{\text{min}}$	Vector with lower bounds of input variables
$X_{\text{max}}$	Vector with upper bounds of input variables
$Y$	Vector of model output variable

### Greek letters

$\mu^*$	Absolute mean value of elementary effects
$\sigma$	Standard deviation of elementary effects
$\chi^*$	Absolute median of elementary effects

### Abbreviations

BEM	Building energy model
EE	Elementary effect
LHS	Latin hypercube sampling
SA	Sensitivity analysis
SRC	Standardised regression coefficient

specific methods. A widely used method for parameter screening is Morris method, which provides a qualitative measure to rank parameter according to their effect on the model outcome [8,19]. In combination with the factorial sampling plan proposed by Morris [19], which is derived from one-factor-at-a-time sampling methods and defines several sequences, called trajectories, of step-wise parameter changes, it offers a good compromise between accuracy and efficiency and represents a suitable screening method for

larger parameter sets in computationally intensive models [5,16]. Regression or variance-based sensitivity methods use larger Monte Carlo or quasi-random Latin hypercube samples (LHS) of uncertain parameters and consequently require more computational time, but the results from these methods generally provide more quantitative information. They potentially offer more detailed insights into non-linear parameter behaviour and higher-order parameter interactions, which characterise the effect on model outcome due to simultaneous changes in two (second-order) or more (higher-order) parameters [20].

Though some form of parameter screening or sensitivity analysis is usually applied to building energy models, there are few studies that investigate or compare the performance of different sensitivity analysis methods in relation to dynamic, high-order, non-linear behaviour and the level of uncertainty in building energy models [9]. Indeed, model inputs associated with the building and its operation often exhibit large ranges of uncertainty, which are likely to impact the sample size needed to obtain robust sensitivity results. In a recent study Kristensen and Petersen compare the parameter rankings obtained by local sensitivity analysis, Morris method and Sobol method [21]. Their results indicate that for predominantly additive, reduced-order BEM the ranking of important parameters obtained by Morris method is in most cases equal to the ranking by Sobol method, while the local method identifies the correct cluster of important parameters, but fails to provide a reliable ranking.

In this study we scrutinise three distinctive sensitivity analysis methods: (a) the computationally efficient Morris method for parameter screening, (b) linear regression analysis (medium computational costs) and (c) a variance-based method (high computational costs) through a case study using the TRNSYS simulation model. The performance of Morris Method is investigated for transient dynamic thermal building models with different numbers of trajectories to identify the number of trajectories needed for a robust ranking of the parameters. In addition, we explore extensions to the standard Morris method to increase the robustness of the parameter ranking for a low number of trajectories. Results obtained from the Morris method are compared to quantitative sensitivity measures, such as the standardised regression coefficient (SRC) and Sobol's sensitivity indices, with regard to the parameter ranking and differentiation of important and non-influential parameters. The sensitivity results from regressions analysis is also investigated over a range of sample sizes, as the latter is a critical factor for the applicability of these methods. In addition, we examine different ways for the characterisation of higher-order effects between uncertain model parameters by looking at correlations in the results from Morris method and higher-order effects obtained with Sobol method. Finally, we discuss the specific advantages and potential short-comings of each method, and the resulting implications for the applicability of each method to specific questions with regard to building energy modelling.

## 2. Building energy model

To test the performance and applicability of different sensitivity analysis methods we apply them to an energy model of the Architecture Studio building at the University of Cambridge. The studio building is built of two storeys: the lower one is used as a workshop with an area of approx.  $85 \text{ m}^2$ , and the upper one contains the undergraduate design studio with an area of approx.  $325 \text{ m}^2$ , which is formed of one large open space room. The energy model of the building is set up using the commercial software package TRNSYS (Version 17.02) and evaluated to estimate the annual heating demand of the first floor, which is used as the quantity of interest in the sensitivity analysis. The heating demand for a given set point

**Table 1**

List of parameters and assigned uncertainty ranges used for the sensitivity analysis. Lower and upper outputs for the annual heating demand were calculated with the corresponding lower and upper bounds of each parameter and assigning average values to the remaining inputs. Output ranges were calculated as difference between each lower and upper output, respectively.

Parameter	Lower bound	Upper bound	Unit	Lower output (kJ/year)	Upper output (kJ/year)	Output range (kJ/year)
All parameters				$1.29 \times 10^8$	$2.32 \times 10^9$	$2.19 \times 10^9$
Set point temperature	18	22	°C	$2.32 \times 10^8$	$1.43 \times 10^9$	$1.20 \times 10^9$
Radiative proportion of heating system	0.2	0.6	–	$6.25 \times 10^8$	$1.27 \times 10^9$	$6.48 \times 10^8$
discharge coefficient	0.5	0.8	–	$5.98 \times 10^8$	$1.17 \times 10^9$	$5.69 \times 10^8$
Infiltration rate	0.10	1.25	$\text{h}^{-1}$	$1.18 \times 10^9$	$7.29 \times 10^8$	$4.49 \times 10^8$
Thermal capacitance	500	50,000	kJ/K	$1.24 \times 10^9$	$7.93 \times 10^8$	$4.44 \times 10^8$
$dH_{NPL}$	0.5	0.7	m	$6.99 \times 10^8$	$8.80 \times 10^8$	$1.81 \times 10^8$
Wind reduction factor	0.2	0.6	–	$7.40 \times 10^8$	$9.08 \times 10^8$	$1.67 \times 10^8$
$C_v$ factor variation	–0.1	0.1	–	$7.80 \times 10^8$	$8.04 \times 10^8$	$2.39 \times 10^7$
Occupant heat gain	0.5	52.9	kJ/h	$7.93 \times 10^8$	$7.92 \times 10^8$	$3.10 \times 10^5$
Appliances heat gain	0.7	40.6	kJ/h	$7.93 \times 10^8$	$7.92 \times 10^8$	$2.37 \times 10^5$
Lighting heat gain	0.01	5	kJ/h	$7.93 \times 10^8$	$7.93 \times 10^8$	$2.95 \times 10^4$

temperature is calculated in hourly time steps by the building component (Type 56) in TRNSYS using actual weather data from 2013, which was recorded by a weather station on top of the Computer Laboratory building in West Cambridge, approx. 2.5 km from the studio building. Internal heat gains are also defined in the building component by daily schedules for occupancy, lighting and the use of appliances, such as computer and laptops.

The studio room has several windows, which can be manually operated by the students. The natural ventilation rate for these windows is calculated in a separate MATLAB component (Type 155) that takes into account air exchange through windows due to indoor-outdoor pressure difference induced by wind pressure  $Q_w$  [22,23] and due to thermal buoyancy effects  $Q_{th}$  [24,25], and applying superposition of  $Q_w$  and  $Q_{th}$  according to the ASHRAE Handbook [22].

The air exchange rate due to wind pressure  $Q_w$  is calculated according to [22,23]:

$$Q_w = C_v A_{eff} U_{eff} \quad (1)$$

With effectiveness of window openings  $C_v$  (–) as a function of the incident angle of the wind on the window surface [24], and area of window openings  $A_{eff}$  ( $\text{m}^2$ ). The effective wind speed  $U_{eff}$  (m/s) at window height differs from the wind speed measured at roof height and is estimated as [22]:

$$U_{eff} = U \cdot r_w \quad (2)$$

With the wind speed at the weather station  $U$  (m/s) and a wind reduction factor  $r_w$  (–) that accounts for the reduction in wind speed due to the built environment surrounding the studio building. The fraction of window area that is effectively available for inflowing air is estimated by:

$$A_{eff} = f_{win} \cdot A \quad (3)$$

With  $f_{win}$  (–) assumed to be 0.5, so that  $A_{eff}$  represents half of the total window area  $A$  ( $\text{m}^2$ ) [11]. The air exchange rate due to thermal buoyancy effects  $Q_{th}$  is calculating differently depending on the relation of indoor and outdoor air temperatures [24,25]:

$$Q_{th} = C_D A_{eff} \sqrt{2gdH_{NPL}(T_i - T_o)/T_i}, \text{ for } T_i > T_o \quad (4a)$$

$$Q_{th} = C_D A_{eff} \sqrt{2gdH_{NPL}(T_o - T_i)/T_o}, \text{ for } T_o > T_i \quad (4b)$$

With discharge coefficient  $C_D$  (–), gravitational acceleration  $g$  ( $\text{m}/\text{s}^2$ ) and height between midpoint of opening and neutral pressure level  $dH_{NPL}$  (m). We assume that windows are opened mainly to enable space cooling, so that windows are opened when the room temperature exceeds  $23^\circ\text{C}$ , and are closed again when the indoor

temperature drops below  $22^\circ\text{C}$ . These temperature values are chosen in such way to avoid the opening of windows while heating is supplied to the room, which would result in an unreasonably high heating demand over the specified time interval of one hour.

For the sensitivity analysis we select 11 uncertain parameters in our TRNSYS model to enable detailed analysis with manageable computational effort. The set of selected parameters is specific to the examined model and its components, such as calculation of natural ventilation rate, but at the same time it reflects a typical set of uncertain parameters applied in sensitivity analysis of building energy models (Table 1). The uncertainty ranges for these parameters were intentionally defined rather broad in order to examine the performance of different SA methods under unfavourable conditions that correspond to applications with limited information about the actual parameter values. All uncertain parameter are assumed to have a uniform distribution, as suggested by Eisenhower et al. [26] for not specifically known uncertainty distributions or large parameter ranges.

The definition of the ranges for set point temperature and infiltration rate is based on typical literature values [5,11,12]. For the radiative part, which defines whether heat supply from the radiator to the room is dominated by radiative or convective portion, we set a range of  $\pm 50\%$  of the default value of 0.4 in the TRNSYS building component. As lower bound for the internal thermal capacitance of the design studio we select the default value in TRNSYS of 500 kJ/K. For the upper bound of the range we estimate the potential maximum thermal capacitance of the room by assuming specific volumes for materials that can be found in the design studio in form of furniture and building materials, such as wood, paper, cardboard, metal etc., and assigning typical thermal properties to each material [27]. The range of values for occupant heat gain is estimated by assigning different metabolic rates for certain types of work, which are specified within TRNSYS based on ISO 7730, to a variable number of occupants that is estimated based on the number of working desks available in the room. The potential range of the heat gain caused by the use of appliances is estimated by assuming that an uncertain number of laptops is used in the studio and by varying the TRNSYS base value for the heat gain from laptops by  $\pm 50\%$ . Likewise, the range in the heat gain from different sources of lighting in the studio is estimated as the TRNSYS default value  $\pm 50\%$ . The ranges for uncertain parameters related to the calculation of the natural ventilation rate, such as discharge coefficient  $C_D$ ,  $dH_{NPL}$ , wind reduction factor  $r_w$  and variation in  $C_v$ , are based on the literature used for setting up the ventilation model [22–24]. To test the capability of the SA method to clearly identify parameters with no influence on the model output, we select  $dH_{NPL}$  as a ‘test parameter’, which is varied in the sample matrix used for SA, but

is at the same time fixed in the natural ventilation model. Thermal properties of building materials, such as specific heat, conductivity and thermal absorptance, are not included in the set of uncertain parameters in this study. For the specific case study building the values for the physical properties of windows, walls, etc. could be obtained within comparatively small uncertainty ranges from the available reports and technical specifications by constructors and manufacturers. Yet, disregarding uncertain parameters in the sensitivity analysis can cause a fraction of the total output uncertainty to remain unexplained in the results, which has to be considered for the interpretation of the results. In this building, supply and return temperatures of the heating system are continuously monitored, and we use this information to gain confidence in the fidelity of modelled outputs. The amount of heat supplied to the Studio room in 2013,  $Q_{heat}$ , was calculated based on the measured temperature difference between supply and return flow to the radiant ceiling ( $T_{in} - T_{out}$ ), the overall flow rate of the radiant ceiling  $f$  and the heat capacity of water  $c_p$ .

$$Q_{heat} = (T_{in} - T_{out}) \cdot f \cdot c_p \quad (5)$$

The evaluation of the TRNSYS model using the mean values of the parameter ranges in Table 1 reveals a coefficient of variation of the root mean square error (CVRMSE) of 0.2 for the modelled monthly heat consumption compared to the measured heating consumption data from 2013. Based on this value it is reasonable to assume that the model is a sufficiently accurate mathematical representation of the building under investigation in order to perform sensitivity analysis.

Table 1 also includes a preliminary assessment of the influence of each parameter on the model output by calculating the modelled annual heating demand using the lower and upper bound for each parameter, respectively, while taking the mean values of the remaining parameters. This represents a basic form of sensitivity analysis with one change in each parameter value covering the whole parameter range. The overall range between the heating demand for the lower and upper bound of each parameter gives a rough estimate for the parameter sensitivities and suggests a large influence from heating related parameters (set point temperature and radiative proportion), less influence from ventilation related parameters (discharge coefficient,  $dH_{NPL}$ , wind reduction factor, etc.) and only minor impact of parameters related to internal heat gains (occupants, appliances, lighting).

### 3. Methods for sensitivity analysis

#### 3.1. Parameter screening with Morris method

For the description of the different approaches and measures for sensitivity analysis we consider the outcome of a building energy model as mathematical function  $Y(\mathbf{X})$  with  $Y$  as a vector of one model output, as we focus on the annual heating demand, and  $\mathbf{X}$  as a  $N \times k$  matrix of model inputs  $X$  with  $N$  samples of  $k$  input parameters defined within the parameter space by the lower and upper bounds for each parameter,  $X_{min}$  and  $X_{max}$ , respectively.

Morris [19] proposed an efficient parameter screening method in combination with a factorial sampling strategy in order to identify parameters that can be fixed at any value within their range without affecting the variance of the model outcome. For sampling the parameter space is discretized by transforming the input parameters into dimensionless variables in the interval (0;1) and dividing each parameter interval into a number of  $p$  levels, which form a regular grid in the unit-length hypercube  $H^k$ . The starting point for sampling on this grid is randomly chosen and each sample differs only in one coordinate from the preceding one [19]. A sequence of  $k + 1$  points, in which each parameter changes only

once by a pre-defined value  $\Delta_i$ , is called a trajectory. One point in this trajectory represents one evaluation run of the model. The magnitude of variation in the model output due to the pre-defined variation of one parameter  $X$  is called elementary effect (EE) [19]:

$$EE_i = \frac{Y(X + e_i \Delta_i) - Y(X)}{\Delta_i} \quad (6)$$

where  $e_i$  is a vector of zeros, except for the  $i$ -th component that equals  $\pm 1$  and represents an incremental change in parameter  $i$  [5].

While one trajectory allows the evaluation of one elementary effect for each parameter  $i$ , a set of  $t$  trajectories enables statistical evaluation of the finite distribution of the elementary effects. Commonly used statistical measures for the evaluation of the EEs include the absolute mean  $\mu^*$  [8] and the standard deviation  $\sigma$  [19]:

$$\mu^*_i = 0.5 \sum_{t=1}^r |EE_{it}| \quad (7)$$

$$\sigma = \sqrt{\frac{1}{(r-1)} \sum_{t=1}^r (EE_{it} - \mu_i)^2} \quad (8)$$

where the index  $t$  indicates a set of multiple trajectories. The criterion  $\mu^*$  indicates the magnitude of influence of a parameter on the model outcome and is often used to rank the parameters according to their importance. The standard deviation  $\sigma$  is a measure for the spread in the model outcome due to changes in a specific parameter. It indicates that the magnitude of influence of a parameter is dependent on the values of the remaining parameters, and can be interpreted as a measure for non-linearity and parameter interactions [19]. Although the measure  $\mu^*$  is often used in literature to identify influential parameters, it is technically a measure to identify 'unimportant' or negligible parameters [8]. Other measures suggested for the ranking of important parameters include for instance the ratio  $\mu^*/\sigma$  [12,28]. With an adapted sampling strategy Morris method can also be used to investigate higher-order effects, such as second-order parameter interaction effects [5,29]. The computational costs for Morris method (given as the number of required simulation runs) depend on number of parameters  $k$  and number of  $t$  trajectories and are given by  $t^*(k+1)$ .

#### 3.2. Standardised regression coefficients

Regression and variance-based sensitivity analysis methods are based on the decomposition of a square integrable mathematical function  $Y(\mathbf{X})$  into [9]:

$$Y(\mathbf{X}) = f_0 + \sum_{i=1}^k f_i(X_i) + \sum_{j>i}^k f_{ij}(X_i, X_j) + f_{12\dots k}(X_1, \dots, X_k) \quad (9)$$

Assuming independence of the individual input factors the decomposition is unique [30,31]. To estimate sensitivity indices based on regression analysis, the model response  $Y(\mathbf{X})$  is approximated by a linear multidimensional model  $F(\mathbf{X})$  with a regression coefficient  $f_i$  for each input parameter  $X_i$  [9]:

$$F(\mathbf{X}) = f_0 + \sum_{i=1}^k f_i(X_i) \quad (10)$$

In order to make the estimated regression coefficients comparable they are standardised using the variance of the model response

$V(Y(\mathbf{X}))$  and the variance of the corresponding input parameter  $V(X_i)$  [9]:

$$SRC_i = \frac{f_i V(X_i)}{V(Y(\mathbf{X}))} \tag{11}$$

The absolute value of the standardised regression coefficient SRC represents a measure for parameter importance with higher SRC values indicating more influence on the model outcome, while the sign of the SRC value indicates whether the model outcome increases or decreases as the value of the input factor changes [13]. However, the applicability of the SRC is limited to a model response  $Y(\mathbf{X})$  that can be sufficiently represented by the fitted regression model, so that the SRCs are not reliable when the building model is highly non-linear [9]. In order to measure how well the approximated linear model fits the building model we use the coefficient of determination  $R^2$ , which indicates how much of the building model variance  $V(Y(\mathbf{X}))$  can be explained by the variance of the linear model  $V(F(\mathbf{X}))$  [32]:

$$R^2 = \frac{V(F(\mathbf{X}))}{V(Y(\mathbf{X}))} \tag{12}$$

Low  $R^2$  values indicate an insufficient fit of the regression model with the outcome of the building model, possibly due to missing relevant parameters in the SA, or due to non-linear effects or parameter interactions that are not captured by the regression analysis. Saltelli et al. [32] define a threshold of  $R^2 = 0.7$  for the acceptance of a regression model and the resulting SRCs.

The SRCs provide an easy to interpret quantitative measure for the influence of input parameters on model outcomes with moderate computational costs, because in contrast to variance-based methods only one set of parameter samples and model outcomes corresponding to  $N$  simulation runs is required to calculate the SRCs. To build a sample that is representative for the parameter space, we use Latin hypercube sampling to create the  $N \times k$  matrix of model inputs, while a factorial sampling plan is used in combination with Morris method. Latin hypercube sampling provides a better coverage of the parameter space than random Monte Carlo samples, because it initially partitions the parameter space into equally probable areas and then takes random samples within these multidimensional areas. Thus, clumping of samples in certain regions is avoided so that less samples are needed to sufficiently cover the parameter space [33,34].

### 3.3. Variance-based sensitivity indices

The analysis of variance-based sensitivity measures is based on the functional decomposition scheme in Eq. (9) that leads to the ANOVA decomposition of total model variance [10,30,35]:

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \dots + V_{12\dots k} \tag{13}$$

where  $V_i = V(E(Y|X_i))$  is the contribution of each parameter to the total variance of the output with the inner expectation operator denoting that the mean of  $Y$  is taken over all possible values of  $X_i$ , and  $V_{ij} = V(E(Y|X_i, X_j)) - V_i - V_j$  is the joint effect of the parameter pair  $(X_i, X_j)$  on the outcome, and accordingly for all other higher-order effects.

A normalized variance-based sensitivity measure for the first order effect of a model input parameter  $X_i$  can be written as [35]:

$$S_i = \frac{V_{X_i}(E_{X_{\sim i}}(Y|X_i))}{V(Y)} \tag{14}$$

where  $\mathbf{X}_{\sim i}$  denotes a matrix of all parameters but  $X_i$ . In practice this means that  $S_i$  is calculated using a sample matrix, where the values

of  $X_i$  remain unchanged and all other parameters are varied according to their ranges. Thus,  $S_i$  is an estimate for the expected fraction of variance in the model output that could be removed if the true value of  $X_i$  was known and can be used for identifying parameters that have a significant impact on the model outcome [10,36]. For linear and additive models the  $S_i$  indices for all parameters add up to 1, while values for the sum of all  $S_i$  that are lower than 1 indicate higher-order interaction effects between the parameters [10].

Another frequently used variance-based sensitivity measure is the total effect index [36,37]:

$$S_{Ti} = \frac{E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i}))}{V(Y)} = S_i + S_{ij} \dots + S_{ij\dots k} \tag{15}$$

The total effects index  $S_{Ti}$  measures all effects involving the factor  $X_i$ , i.e. first, second and all other higher order effects due to parameter interactions. As indicated by the inner conditional variance operator in Eq. (15) the calculation of  $S_{Ti}$  is performed by varying the values for  $X_i$  over the parameter space and keeping all other parameters unchanged. Accordingly, the total effects is a measure to identify unimportant parameters, i.e. parameters that can be fixed at any value within their uncertainty ranges without affecting the variance of the output significantly [36]. For additive models the parameter indices  $S_{Ti}$  equal the first order measures  $S_i$ , and if the values for  $S_{Ti}$  are significantly larger than the main effects  $S_i$  the investigated model is non-additive.

In addition to the first-order and total effect sensitivity we define another measure derived from these indices, which quantifies the overall involvement of a parameter in higher-order effects as:

$$S_H = S_{Ti} - S_i = S_{ij} + \dots + S_{ij\dots k} \tag{17}$$

Based on Eq. (13) variance-based sensitivity measures can also be defined for other higher-order or interaction terms, for instance for the second-order effects  $S_{ij}$  for parameter pairs [10]:

$$S_{ij} = \frac{V_{X_{ij}}(E_{\mathbf{X}_{\sim ij}}(Y|X_i, X_j))}{V(Y)} - S_i - S_j \tag{16}$$

The  $S_{ij}$  measure represents the fraction of variance in the model outcome caused by the interaction of parameter pair  $(X_i, X_j)$ . Without the subtraction the first variance term denotes the potential reduction in model variance if the true values of  $X_i$  and  $X_j$  were known. Accordingly, in the calculation procedure for this sensitivity index the parameter values for  $X_i$  and  $X_j$  are kept fixed while the other input parameters are varied.

The described sensitivity indices  $S_i$ ,  $S_{Ti}$ ,  $S_H$  and  $S_{ij}$  can be estimated by different numerical approaches, such as Fourier Amplitude Sensitivity Test (FAST) [16,38,39] and Sobol's method [31], which is implemented in this study by using Monte Carlo integration. There also exist different numerical estimators for  $S_i$  and  $S_{Ti}$ , which are described in detail by Saltelli et al. [35]. In this study we apply the formula for  $S_i$  introduced by Saltelli et al. [35] (Eq. (18)) and an estimator for  $S_{Ti}$  developed by Jansen [40] (Eq. (19)), and for the second-order effects  $S_{ij}$  we define Eq. (20) analogous to the calculation of first-order effects with Eq. (18):

$$V_{X_i}(E_{\mathbf{X}_{\sim i}}(Y|X_i)) = \frac{1}{N} \sum_{j=1}^N f(\mathbf{B})_j \left( f(\mathbf{A}_B^{(i)})_j - f(\mathbf{A})_j \right) \text{ for } S_i \tag{18}$$

$$E_{\mathbf{X}_{\sim i}}(V_{X_i}(Y|\mathbf{X}_{\sim i})) = \frac{1}{2N} \sum_{j=1}^N \left( f(\mathbf{A})_j - f(\mathbf{A}_B^{(i)})_j \right)^2 \text{ for } S_{Ti} \tag{19}$$

$$V_{X_{ij}}(E_{\mathbf{X}_{\sim ij}}(Y|X_i, X_j)) = \frac{1}{N} \sum_{w=1}^N f(\mathbf{B})_j \left( f(\mathbf{A}_B^{(ij)})_w - f(\mathbf{A})_w \right) \text{ for } S_{ij} \tag{20}$$

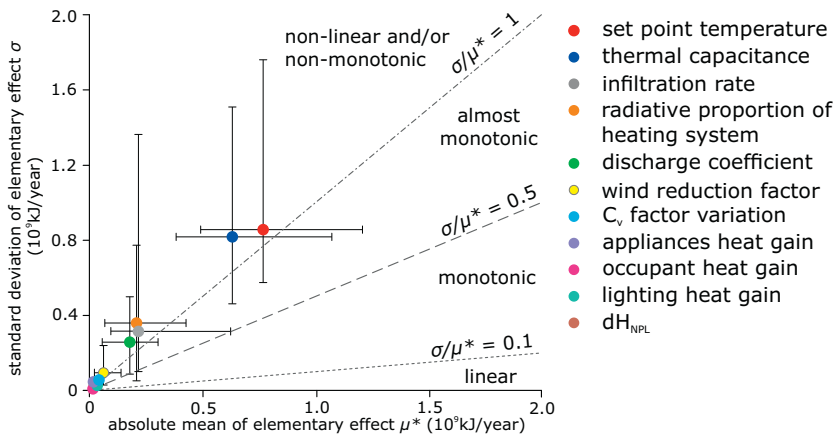


Fig. 1. Results for parameter screening with Morris method from 10 screening runs with 10 trajectories each assigning the parameter ranges from Table 1.

The sample matrixes **A** and **B** are set up in the following way: both matrixes are generic sample matrixes with  $i$  columns, corresponding to the number of parameters  $k$ , and a number of lines  $N$ , which indicates the number of sampled simulation runs.  $\mathbf{A}_B^{(i)}$  is a matrix, where all columns are taken from **A** except for the  $i$ -th column, which is taken from matrix **B** [35], and accordingly  $\mathbf{A}_B^{(ij)}$  is a matrix with columns  $i$  and  $j$  taken from **B** and the remaining columns are taken from **A**. By using these matrixes the schemes of changing certain parameters are implemented as described above.

The sample matrixes are again generated by applying Latin hypercube sampling to ensure a good coverage of the parameter space. The computational costs for all Sobol sensitivity indices would add up to  $2^{k-1} \cdot N$  simulation runs [9]. Because of the computational time of dynamic building energy models we focus on the analysis of first-order, second-order and total effects. With  $2 \cdot (k+1) \cdot N$  model evaluations, a sample size of  $N=500$  and  $k=11$  parameter result in 12,000 simulation runs required for the calculation of  $S_i$ ,  $S_{Ti}$  and  $S_H$  [35]. The additional estimation of the second-order indices  $S_{ij}$  for all 55 pairs of 11 parameters requires  $k \cdot (k-1) / 2 \cdot N$  model evaluations, which add up to additional 27,500 simulation runs.

## 4. Results and discussion

### 4.1. Parameter screening with Morris method

As aforementioned, the performance of Morris Method is investigated with different numbers of trajectories to identify the number of trajectories needed for a robust ranking of the parameters. The results from 10 independent evaluations of Morris method, each with 10 trajectories, are summarised in Fig. 1, where each point represents the mean of the 10 absolute mean values  $\mu^*$  (x-axis) and standard deviations  $\sigma$  (y-axis) of the elementary effects for each parameter. The error bars indicate the overall range of  $\mu^*$  and  $\sigma$  values. Note that according to Eq. (6) the elementary effect has the same physical unit as the model outcome or quantity of interest, which is the annual heating demand (kJ/year) in our case. The points plotted in Fig. 1 thus represent the magnitude of variation in annual heating demand due to changing the value of one input parameter at a time.

For our model the highest  $\mu^*$  values are found for the set point temperature and thermal capacitance. Another group of parameters with slightly lower values includes radiative proportion of the heating system, discharge coefficient and infiltration rate, while the remaining parameters are identified as negligible parameters. The test parameter  $dH_{NPL}$ , which has no influence on the model

outcome is correctly identified as unimportant with zero values for  $\mu^*$  and  $\sigma$ . Overall, the ranking in Fig. 1 is in good agreement with the rough sensitivity estimates from Table 1 and findings from previous studies, which often identify set point temperature, infiltration, thermal properties, such as thermal conductivity of building components or internal thermal mass, and ventilation related parameters as influential for building energy models [5,11–14].

According to a classification scheme proposed by Garcia Sanchez et al. [5] the ratio  $\sigma/\mu^*$  allows the characterisation of the model parameters in terms of (non-)linearity, (non-) monotony or possible parameter interactions (Fig. 1). The average values for  $\mu^*$ ,  $\sigma$  and  $\sigma/\mu^*$  for the 10 Morris method evaluations from Fig. 1 are given in Table 2. For our test model all parameters exhibit a  $\sigma/\mu^*$  ratio  $>1$ , except for the heat gain by appliances, which suggests that most parameters exhibit either non-linear behaviour, interaction effects with other parameters or both.

Another observation from Fig. 1 is that the results from Morris method regarding both  $\mu^*$  and  $\sigma$  vary significantly across the 10 independent evaluations as indicated by the large error bars. Small variations in the results, especially for a low number of trajectories, are expected because the resulting  $\mu^*$  and  $\sigma$  values depend on the absolute values of the elementary effects, which vary with the randomly picked parameter combinations in each trajectory. The overlapping error bars in Fig. 1 however indicate that parameter rankings may not be consistent across the 10 independent evaluations.

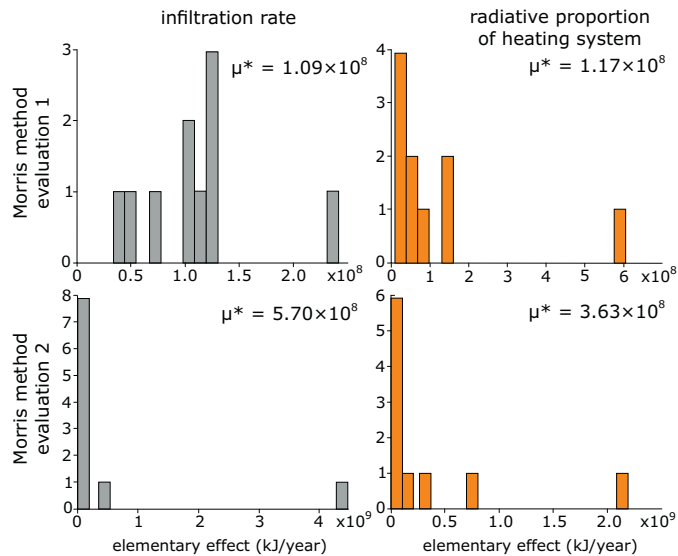
To investigate the cause for this inconsistency in parameter rankings we analyse the elementary effects (EE) defined by the Morris Method in more detail (Eq. (6)). The evaluation of the EE with  $\mu^*$  and  $\sigma$  (given in Eqs. (7) and (8), respectively) is based on the assumption that the set of EEs across the trajectories has a continuous and non-skewed distribution that can be summarised accurately by the measures of absolute mean and standard deviation. Fig. 2 shows two sets of histograms of the EEs obtained from two runs of the Morris method. Each set shows the distribution of EEs for two parameters: infiltration rate and radiative proportion of the heating system. The corresponding values of  $\mu^*$  per parameter (Eq. (7)) are also listed in Fig. 2.

If one were to use the  $\mu^*$  value as the measure for overall parameter importance, the radiative proportion of the heating system would be identified as the more influential parameter from the results of one run of the Morris Method (shown in the top two histograms). However, results obtained from a second run would rank the infiltration rate as more influential (the two lower histograms in Fig. 2). All histograms show distributions that are discontinuous, with frequently occurring outliers.

**Table 2**

Results for parameter screening with Morris Method. Values are given as average values and ranges for the absolute mean  $\mu^*$  and standard deviation  $\sigma$  of the elementary effects from 10 independent screenings runs.

Parameter	average $\mu^*$ (kJ/year)	average $\sigma$ (kJ/year)	average $\sigma/\mu^*$	range $\mu^*$ (kJ/year)	range $\sigma$ (kJ/year)
Set point temperature	$7.69 \times 10^8$	$8.54 \times 10^8$	1.11	$7.12 \times 10^8$	$1.18 \times 10^9$
Thermal capacitance	$6.34 \times 10^8$	$8.16 \times 10^8$	1.29	$6.87 \times 10^8$	$1.05 \times 10^9$
Infiltration rate	$2.19 \times 10^8$	$3.20 \times 10^8$	1.46	$5.30 \times 10^8$	$1.26 \times 10^9$
Radiative proportion of heating system	$2.11 \times 10^8$	$3.60 \times 10^8$	1.71	$3.56 \times 10^8$	$7.21 \times 10^8$
Discharge coefficient $C_D$	$1.80 \times 10^8$	$2.58 \times 10^8$	1.43	$2.44 \times 10^8$	$4.09 \times 10^8$
Wind reduction factor	$6.60 \times 10^7$	$9.37 \times 10^7$	1.42	$1.14 \times 10^8$	$2.08 \times 10^8$
$C_v$ factor variation	$1.40 \times 10^7$	$2.24 \times 10^7$	1.60	$2.21 \times 10^7$	$4.37 \times 10^7$
Appliances heat gain	$3.90 \times 10^6$	$6.54 \times 10^5$	0.17	$5.27 \times 10^7$	$2.62 \times 10^6$
Occupant heat gain	$3.87 \times 10^6$	$1.14 \times 10^7$	2.95	$4.40 \times 10^7$	$1.39 \times 10^8$
Lighting heat gain	$2.56 \times 10^6$	$8.02 \times 10^6$	3.13	$3.75 \times 10^7$	$1.18 \times 10^8$
$dH_{NPL}$	0	0	–	0	0



**Fig. 2.** Two sets of histogram plots showing the distribution of 10 absolute elementary effect values for the infiltration rate and the radiative proportion of the heating system from two different evaluations of Morris method, and the resulting values for the absolute mean value  $\mu^*$  in kJ/year.

**Fig. 2** indeed demonstrates that parameter rankings obtained by the Morris method can be biased by the occurrence (or absence) of outliers and the resulting skewness in the EE distributions. This varying occurrence of outliers in individual Morris method runs is a consequence of the low number of trajectories in combination with a comparably large parameter space (**Table 1**), which results in a low number of sampled EEs per parameter. Thus, picking certain random parameter combinations, such as low internal gains, a high infiltration rate, and a high set point temperature, can cause extreme values for model outputs, as in the bottom left histogram in **Fig. 2**.

There are several potential strategies to overcome this issue. The most obvious one is to increase the number of trajectories so that the resulting EE distributions become more continuous and presumably more robust. **Fig. 3(a)–(b)** show results obtained from Morris method runs by incrementally increasing the number of trajectories from 10 to 150. **Fig. 3(a)** plots the absolute mean values  $\mu^*$  and **Fig. 3(b)** plots the relative ranking per parameter for each Morris Method run. Crossing lines in these two plots indicate changes in parameter ranking between individual evaluations with the number on the right hand side of **Fig. 3(b)** stating the total number of occurring changes in rank for each parameter.

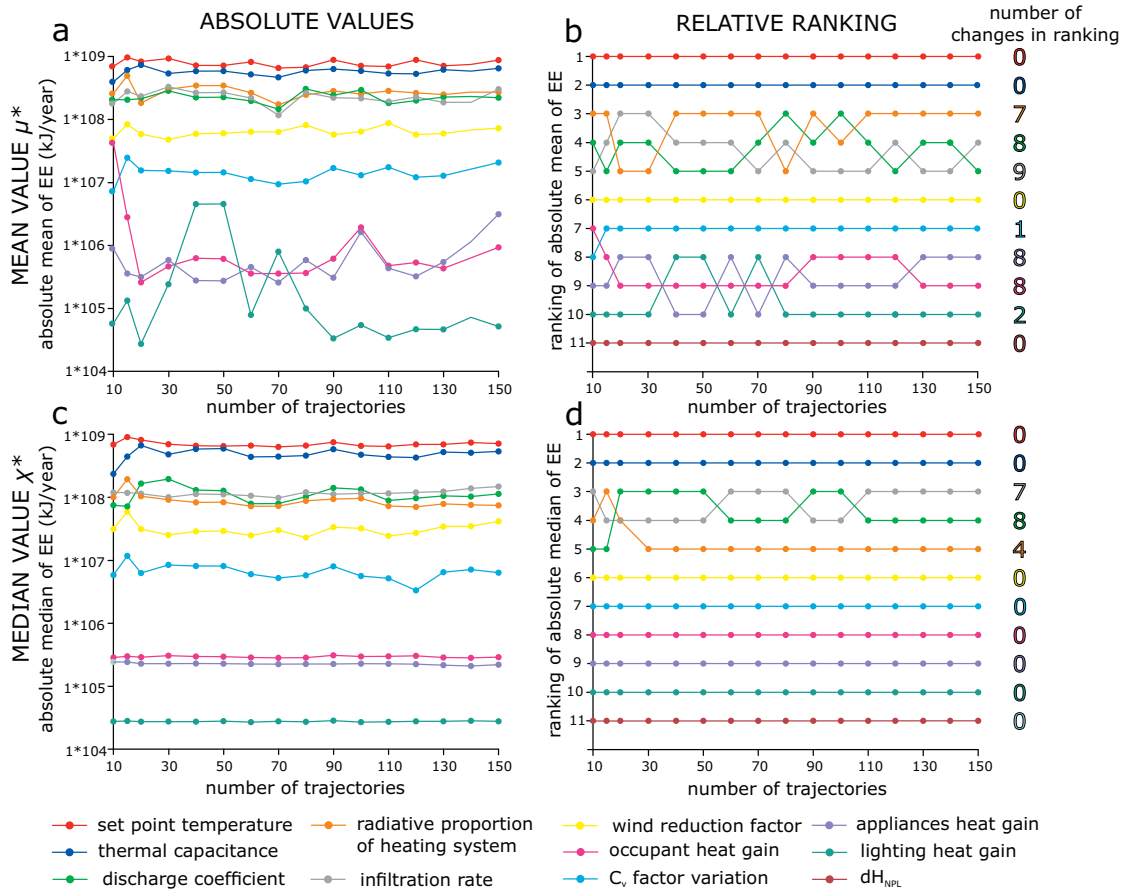
Note in **Fig. 3(a)** and **(b)** that variations in  $\mu^*$  result in different parameter rankings also for evaluations with large numbers of trajectories, though it seems that the changes become less frequent for runs with more than 120 trajectories. Changes in parameter rank-

ing are particularly frequent for the following three parameters: radiative proportion of the heating system, discharge coefficient, and infiltration rate. This might be due to the fact that increasing the number of trajectories results in less discontinuous EE distributions, but does not tackle the problem of their skewness. In addition, increasing the number of trajectories by large amounts naturally increases the number of required simulation runs, which would cancel out the original advantage of low computational costs for Morris method.

Another possible solution to this problem is to explore a more robust importance measure for the evaluation of the elementary effects: one that is less influenced by the type of distribution of the EE and by outliers. Instead of using the absolute arithmetic mean we suggest the use of the absolute median  $\chi^*$  to characterise the EE distribution in order to identify negligible parameters. The median of a certain number of data samples is defined as the number dividing the total number of ordered observations in half, which makes it a robust measure of location for skewed distributions with reduced importance being attached to outliers [41]. The median values for the four EE distributions in **Fig. 2** are  $1.12 \times 10^8$  and  $1.09 \times 10^8$  kJ/year for the infiltration rate and  $5.11 \times 10^7$  and  $5.46 \times 10^7$  kJ/year for the radiative proportion of the heating system for the two runs of Morris method, respectively. Indeed, using the absolute median  $\chi^*$  as a measure of elementary effect results in the same ranking of the two parameters in both Morris method runs, with the infiltration rate being rated more influential than the radiative proportion of the heating system.

In order to assess the performance of the absolute median  $\chi^*$  in comparison to  $\mu^*$ , **Fig. 3(c)** and **(d)** show the  $\chi^*$  values and the resulting parameter rankings for the same trajectories that were previously characterised by the mean value (**Fig. 3a** and **b**). The rather straight lines in **Fig. 3c** for each parameter indicate that the  $\chi^*$  values from the individual Morris method evaluations are quite similar in contrast to the varying  $\mu^*$  values in **Fig. 3a**. This is particularly obvious for the less important parameters, such as heat gains from appliances, occupants and artificial lighting. Regarding the resulting parameter rankings based on the median value (**Fig. 3d**) there are also less crossing lines than for the  $\mu^*$  values (**Fig. 3b**), which is also reflected by lower numbers shown on the far right side of the plot. The high number of zero values here indicates a robust ranking of most parameters with the median value as measure of (un)importance. Changes in ranking only occur for discharge coefficient, radiative proportion and infiltration rate, because their absolute values of  $\chi^*$  are so similar that even minor variations in the  $\chi^*$  values can cause a change in the ranking (**Fig. 3c**). This indicates that these three parameters have a very similar influence on the model outcome.

In addition to a distinct and robust ranking of the parameters, another important aspect of the outcome from sensitivity analysis is the differentiation between influential and negligible parameters. Few quantitative criteria for the separation of influential and

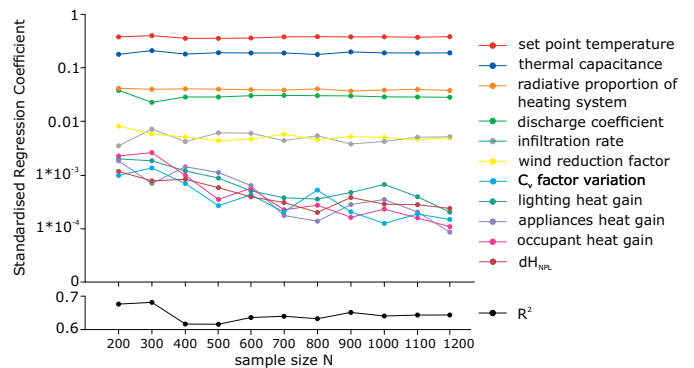


**Fig. 3.** Comparison of Morris method results for an increasing number of trajectories, showing absolute values (a,c) and the resulting ranking of parameters (b,d) using the mean value of the elementary effects (a,b) and the median value (c,d). The numbers on the right-hand side indicate the number of changes in the ranking, taking the most often occurring rank as base rank.

negligible parameters based on results from Morris method exist so far and their applicability seems rather case-specific. For instance, Yang and Becerik-Gerber [12] apply Morris method with 5 trajectories and define parameters with an absolute mean to standard deviation ratio greater than 0.1 as influential. Using the average values for  $\mu^*$  and  $\sigma$  for the parameters in Table 1 this would result in  $\mu^*/\sigma$  ratios ranging from 0.32 to 5.96. Thus, this measure would suggest that all parameters (except  $dH_{NPL}$ ) have a significant influence on the model outcome, which seems rather unrealistic given the preliminary sensitivity estimates in Table 1 and the spread of  $\mu^*$  and  $\chi^*$  values in Fig. 3. Furthermore, there is no correlation between  $\mu^*$  or  $\chi^*$  and the ratio  $\mu^*/\sigma$  in our study. In order to validate the parameter ranking obtained with Morris method and the median of the elementary effects they are compared to other sensitivity measures in the next sections.

#### 4.2. Standardised regression coefficients SRC

Standardised regression coefficients (SRC) were calculated for the parameters and ranges shown in Table 1 by applying linear regression analysis (Eqs. (10) and (11)) for different sizes of LHS samples ranging from  $N=200$  to  $N=1200$  simulations runs. Fig. 4 shows the obtained SRC values per parameter as a function of sample size (x-axis) and the coefficient of determination  $R^2$  of each evaluation (bottom of plot), which indicates the amount of model variance that can be explained by the corresponding regression model. While SRC values are sometimes given in % in literature, they are given in absolute values throughout this study in order



**Fig. 4.** Results for standardised regression coefficients (SRC) for uncertain parameters obtained by linear regression analysis with different LHS sample sizes. Please note that the SRC values are plotted on a logarithmic scale. The bottom plot shows the coefficient of determination  $R^2$  for each model evaluation.

to facilitate comparison with the results from Morris method and Sobol's sensitivity indices.

The most dominant parameters exhibiting the highest SRC values in all evaluations are the set point temperature and thermal capacitance with almost constant values of  $\sim 0.37$  and  $\sim 0.19$ , respectively. The radiative proportion of the heating system ( $\sim 0.04$ ) and discharge coefficient ( $\sim 0.03$ ) show smaller regression coefficients indicating less influence on the model outcome, while the wind reduction factor and infiltration rate have only minor influence given SRC values of  $\sim 0.005$ , which varies slightly with the evaluation of different sample sizes. The remaining parameters



**Table 3**

Overview on the parameter rankings obtained by the investigated sensitivity analysis methods. The ranking given for Morris method refers to the most often occurring rank obtained using the median value for different numbers of trajectories. The ranking given for the SRC method refers to the ranking obtain with  $N = 500$ .

Morris method	SRCs	Sobol first-order effects	Sobol total effects
set point temperature	set point temperature	set point temperature	set point temperature
thermal capacitance	thermal capacitance	thermal capacitance	thermal capacitance
infiltration rate	radiative proportion	radiative proportion	radiative proportion
discharge coefficient	discharge coefficient	discharge coefficient	infiltration rate
radiative proportion	infiltration rate	infiltration rate	discharge coefficient
wind reduction factor	wind reduction factor	lighting heat gain	wind reduction factor
$C_v$ factor variation	appliances heat gain	occupant heat gain	occupant heat gain
occupant heat gain	lighting heat gain	appliances heat gain	$C_v$ factor variation
appliances heat gain	$dH_{NPL}$	$C_v$ factor variation	appliances heat gain
lighting heat gain	occupant heat gain	$dH_{NPL}$	lighting heat gain
$dH_{NPL}$	$C_v$ factor variation	wind reduction factor	$dH_{NPL}$

show a negligible effect ( $<0.005$ ) on the model outcome, and a significant decrease in SRC values with increasing sample size. The reason for this is that with a low number of data samples used for the calculation of the SRCs there is a chance of obtaining minor spurious correlations for technically non-influential parameter due to extreme model output values or outliers. As shown in Fig. 4, with increasing sample size the separation of non-influential and influential parameters becomes more distinct. Based on the SRC results from evaluations with  $N > 500$  it seems reasonable to state that 0.01 (or 1%) is a rough threshold for obtaining robust and meaningful SRC values. Parameters with  $SRC > 0.01$  show quite consistent values throughout the evaluations with different LHS sample sizes, while parameters with  $SRC < 0.01$  show very inconsistent values and have presumably no measurable impact on the model outcome. Mara and Tarantola [9] also apply regression analysis to a building energy model and define parameters with a regression coefficient lower than 0.05 (5%) as non-significant for their model.

The parameter  $dH_{NPL}$  exhibits a very low SRC value in all evaluations and is part of the group of parameters with low and inconsistent values, indicating negligible impact on model output. Contrary to the results from Morris method, the SRC results for the test parameter exhibit small positive values and not zero values. This is due to the simultaneous variation of all parameters in the sample matrix, so that the regression analysis detects small spurious correlation between the  $dH_{NPL}$  value and the model output. Because of this small SRC value it is not possible to differentiate between a negligible parameter and a parameter without any effect on the model outcome caused by a model error.

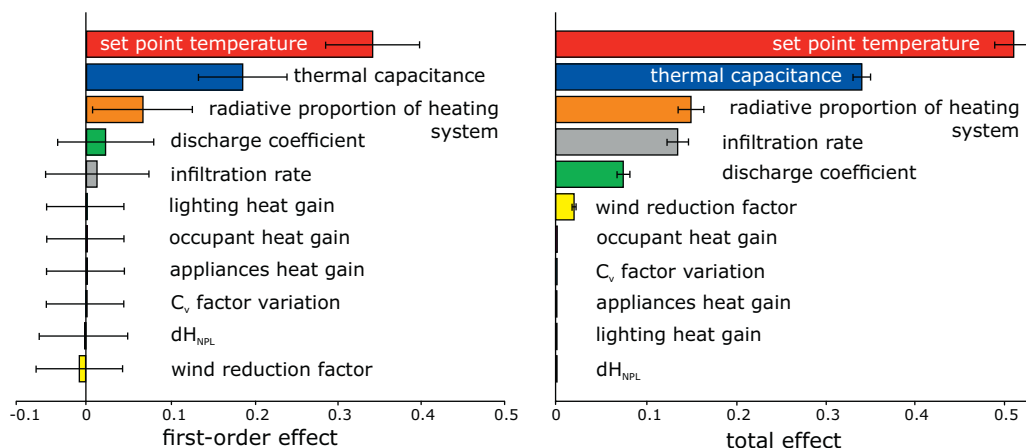
The coefficients of determination  $R^2$  are significantly lower than 1 for all evaluated runs suggesting that higher-order effects due to parameter interactions or non-linear behaviour play an impor-

tant role in the building model, which is in accordance with the high  $\sigma$  values obtained from the Morris method. Though the sum of SRC shows some variation between the runs, there seems to be no dependency on the size of the sample matrix, indicating that the low  $R^2$  are not related to insufficient number of sample size. Hygh et al. [4] reported  $R^2$  values of 0.498 and 0.816 for an office building model and proposed the use of a forward stepwise multivariate regression analysis, which can potentially increase the model fit and results in  $R^2$  values up to 0.96.

Despite the low  $R^2$  values below 0.7 and the apparently insufficient fit of the linear regression model, the SRC results are in good agreement with the ranking from Morris method that show the same general trend of set point temperature and thermal capacitance as dominating factors, radiative proportion of heating system and discharge coefficient with slightly lower values, significantly lower values for wind reduction factor and very low sensitivity indices for the rest of the parameters (Table 3). One exception in this pattern is the SRC value of the infiltration rate, which is ranked considerably lower than by Morris method (Table 3). One explanation for the lower ranking in regression analysis could be non-linear or non-monotonic behaviour possibly associated with the wide range of possible values assigned to the infiltration rate and possible interaction effects with other parameters.

4.3. Variance-based sensitivity indices

The results for variance-based sensitivity analysis with Sobol's method are shown in Fig. 5 as first-order indices  $S_i$  and total effects  $S_{Ti}$ . Due to the high computational costs, (6500 simulation runs for  $S_i$  and  $S_{Ti}$  with  $N = 500$ ), no analysis of the variation in  $S_i$  and  $S_{Ti}$  with different sample sizes was performed. Based on the results from



**Fig. 5.** Results for first-order and total effects with Sobol method showing the mean values for  $S_i$  and  $S_{Ti}$  and the corresponding standard deviations from the resampling analysis as error bars.

regression analysis we chose a sample size of  $N = 500$  for the LHS samples for calculation of the variance-based sensitivity indices. To provide an estimate for the variability of the indices a resampling analysis is applied to the matrixes with the original  $N = 500$  simulation runs and the standard deviations of 400 evaluations with subsamples of the original sample matrixes used for  $S_i$  and  $S_{Ti}$  are displayed in form of error bars in Fig. 5.

The first-order sensitivity index  $S_i$  represents a measure for parameter importance, and thus reveals that three of the parameters have a direct impact on the model outcome: set point temperature, thermal capacitance, and radiative proportion of the heating system. The remaining parameters show very low  $S_i$  values indicating that the possible reduction in the variance of the model outcome by knowing the true values of these parameters is very low. The infiltration rate, which was identified as a not-insignificant parameter by the results of Morris Method, shows a very low  $S_i$  value, which is consistent with the results from regression analysis (Fig. 4, Table 3), and the similar way of sampling in these two methods by varying several parameter values at the same time. The test parameter  $dH_{NPL}$  is identified as a non-important parameter, but due to the simultaneous variation of several parameters in the sample matrix there is a residual reduction in variance associated with the parameter resulting in a very small  $S_i$  value. The rather large error bars of the  $S_i$  values of most parameters suggest a certain variability in the  $S_i$  values and an indistinct ranking for those parameters with overlapping error bars (Fig. 5). For the less influential parameters the error bars also cover ranges of negative values, and the wind reduction factor even exhibits a slightly negative average  $S_i$  value.

Negative values for sensitivity indices can occur in this type of variance decomposition analysis, when the difference between the modelled output from the reference matrix  $\mathbf{A}$  is on average larger than the model outcomes in the column of matrix  $\mathbf{A}_B$  referring to the specific parameter under investigation (Eq. (18)). With the generally large error bars for the  $S_i$  values parameters with a low  $S_i$  seem to have equal possibility of positive and negative values depending on the set of model outputs used for calculation of the  $S_i$ . Closer inspection of the model output matrices reveals significantly skewed distributions of the annual heating demand, which will affect the robustness of the resulting  $S_i$  values as their calculation is based on using the mean value to estimate the variance of the model outcome distributions (Eq. (18)).

A  $S_i$  index value close to zero indicates that the parameter has no effect in the model outcome on the first-order. Yet, it might have an effect on a higher level, when changes in other parameters occur at the same time, so that the parameter might not be generally negligible. Taking the infiltration rate and discharge coefficient in our study as an example this could be caused by the dependency of the effect of infiltration and ventilation on the thermal capacitance of the space, which quantifies the ability of the room to store thermal energy. Depending on the amount of thermal energy stored for instance in furniture or other office components, the magnitude of the effect of ventilation and infiltration on the heating demand will be different. Knowledge of the true values for infiltration rate or ventilation-related parameters and fixing the value in the model according to the procedure above, while varying the value for thermal capacitance, would thus not necessarily reduce the variation in the model outcome. However, varying the value for infiltration rate or discharge coefficient with fixed values for all other model parameters will cause variations in the annual heating demand, as shown in the results from Morris method.

Other studies applying Sobol's first-order effects in sensitivity analysis for building energy models are for instance the work by Spitz et al. [14], who identify six parameters as influential with  $S_i > 0.05$ , such as heating capacity, internal gains and thermal properties. Peleš et al. [15] calculate the first-order index  $S_i$  for 217

parameters of different energy end uses in an office building model, of which only 19 parameters show importance for one or more outputs.

The total effects index  $S_{Ti}$ , as a measure of negligible model inputs, identifies 5 parameters as non-negligible: set point temperature, thermal capacitance, radiative proportion of the heating system, infiltration rate and discharge coefficient. The wind reduction factor is almost negligible and the other parameters are clearly negligible (Fig. 5). The ranking of the parameters as per  $S_{Ti}$  is in general similar to the ranking by  $S_i$  values, although the  $S_{Ti}$  values for most non-negligible parameters (including infiltration rate) are higher than the  $S_i$  indices, suggesting higher-order effects (Table 3). The most remarkable increase from  $S_i$  to  $S_{Ti}$  value is found for the infiltration rate, which indicates that though the parameter has no direct impact on the model outcome, it is not a negligible parameter and has a significant impact in form of interactions with other parameters. The test parameter  $dH_{NPL}$  is assigned a very low  $S_{Ti}$  value, and is correctly identified as an unimportant model input parameter, but as before in the regression analysis and first-order index  $S_i$ , there is no possibility to use the sensitivity results to detect faults in parameter or model settings.

The parameter rankings from Morris method and Sobol's total effects are almost identical regarding the non-negligible parameters, which can be explained by the similar way of parameter variation in the sample matrixes used for calculation and that both methods aim to identify unimportant parameters (Table 3). In addition, the relative difference between the individual  $S_{Ti}$  values is very similar to the gaps between the absolute median values from Morris method. For instance, in both cases there is a clear difference between the corresponding importance measure for set point temperature and thermal capacitance, while the parameters radiative proportion of heating system and infiltration rate show very similar influence. The presence of higher-order effects observed in the  $S_i$  and  $S_{Ti}$  indices for almost all parameters is also in accordance with the high values for standard deviations found by the Morris method. Campolongo et al. [8] compared results from Morris method using  $\mu^*$  against variance-based total effects on analytical test functions from Morris [19] and also found a good proportionality between their results.

#### 4.4. Higher-order parameter interaction effects

In order to investigate the parameter interactions, which cause the difference between first-order effects  $S_i$  and total effects  $S_{Ti}$ , in more detail we now examine the higher-order effects obtained from the Sobol method as general interaction sensitivity index  $S_{Hj}$  and second-order index  $S_{ij}$ . The calculation of Sobol effects requires large computational efforts, therefore we also explore a more efficient way to investigate the interaction effect between two parameters by utilising the results generated from the Morris method. For a given pair of two parameters, we suggest evaluating the correlation coefficient of the first parameter (its value) with the elementary effect (EE) of the other parameter as an indication of second-order effects.

Although the calculation of the correlation coefficients for a pair of two parameters is computationally trivial, one needs a rather large number of trajectories to interpret the correlations meaningfully. Therefore, we use the input parameter matrixes and their corresponding EEs obtained from sensitivity analysis runs ranging from 50 to 150 trajectories to compute the corresponding Pearson's correlation coefficients (Fig. 6). For each set of trajectories correlation coefficients are calculated between the elementary effects for each parameter  $i$  and the corresponding parameter values resulting in a matrix of correlation coefficient for each sensitivity run. We then select from these results the coefficients with a  $p$ -value  $< 0.005$  to calculate the sum of absolute correlation coefficients for each

matrix with elementary effects      matrix with parameter values

$$EE_t = \begin{bmatrix} EE_1^i & EE_1^{i+1} & \dots & EE_1^k \\ EE_2^i & EE_2^{i+1} & \dots & EE_2^k \\ \vdots & \vdots & \ddots & \vdots \\ EE_t^i & EE_t^{i+1} & \dots & EE_t^k \end{bmatrix} \quad X_t = \begin{bmatrix} X_1^i & X_1^{i+1} & \dots & X_1^k \\ X_2^i & X_2^{i+1} & \dots & X_2^k \\ \vdots & \vdots & \ddots & \vdots \\ X_t^i & X_t^{i+1} & \dots & X_t^k \end{bmatrix}$$

calculation of correlation coefficients  $\gamma$  per column  
for  $t = 50, 60, \dots, 150$

$$Y_t = \begin{bmatrix} Y_{i,i}(EE^i, X^i) & Y_{i+1,i}(EE^{i+1}, X^i) & \dots & Y_{k,i}(EE^k, X^i) \\ Y_{i,i+1}(EE^i, X^{i+1}) & Y_{i+1,i+1}(EE^{i+1}, X^{i+1}) & \dots & Y_{k,i+1}(EE^k, X^{i+1}) \\ \vdots & \vdots & \ddots & \vdots \\ Y_{i,k}(EE^i, X^k) & Y_{i+1,k}(EE^{i+1}, X^k) & \dots & Y_{k,k}(EE^k, X^k) \end{bmatrix}$$

combination of all correlation matrices  $Y_{50}, Y_{60}, \dots, Y_{150}$

$$Y_{\text{mean}} = \text{mean}(Y_{50}, Y_{60}, \dots, Y_{150})$$

Fig. 6. Calculation process of correlation coefficients for interaction effects from Morris method.

parameter. Table 4 lists the sum of correlation coefficients of the top six parameters obtained from this process and the corresponding Sobol higher-order indices.

High  $S_H$  values in Table 4 indicate that a parameter is involved in parameter interactions between two or more parameters, while a large sum of correlation coefficients indicates interactions in a slightly different form: for instance, that by increasing the thermal capacitance values, the change in the heating demand due to an incremental change in the set point temperature also increases. The ranking of parameters based on their higher-order effects is identical to the ranking from the correlation coefficients and indicates that 5 of the 11 model parameters show significant higher-order effects with  $S_H > 0.05$  corresponding to a sum of correlation coefficients larger than approx. 2.5. In general, all important parameters in our model are involved in higher-order parameter interaction effects. In particular, the infiltration rate is ranked higher based on the  $S_H$  index than for its individual effects  $S_i$  or  $S_{T_i}$  indicating significant involvement in interaction effects. This observation is also in agreement with the large range of the standard deviation of infiltration rate from Morris method (Fig. 1).

Calculating the second-order effects index  $S_{ij}$  allows a more detailed assessment of higher-order effects with regard to two-way parameter interactions. Fig. 7 shows seven second-order sensitivity indices formed by six parameters in our building energy model. Set point temperature, which ranked highest for the higher-order effect measure  $S_H$ , shows very significant interaction effects with two parameters: the thermal capacitance and the radiative proportion of the heating system. Indeed, in our building model simultaneous changes in set point temperature and the ability of the room to store thermal energy, which is quantifies by the thermal capacity, or the way heat is delivered to the room, described

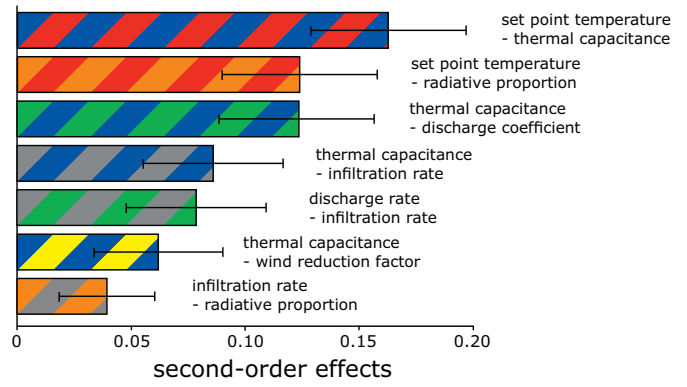


Fig. 7. Second-order effects from Sobol method formed by six model parameters.

through the radiative proportion of the heating system, will have a larger impact on the space heating demand than varying one parameter individually. Generally, the thermal capacitance shows interaction effects with heating-related as well as infiltration and ventilation related parameters, which reveals that the thermal mass in the model has a very diverse and complex impact on the model outcome. The infiltration rate exhibits interaction effects not only with the thermal mass, but also with ventilation related parameters, indicating an intensified effect on the heating demand when both ventilation and infiltration rates increase (or decrease) at the same time.

Parameters involved in parameter interactions on different levels in this study are mostly parameters that also show significant individual effects. This was also found by Garcia Sanchez et al. [5], who analysed second-order elementary effects from Morris method with an adapted factorial sampling plan, which changes two parameter values at a time. In their model of an apartment building the second-order effects also confirm first-order effects, i.e. the individually influential parameters also show the most significant interaction effects, with the interaction effect sometimes exceeding the first-order effect.

### 5. Summary and conclusion

The comparison of three methods frequently used for sensitivity analysis of building energy models based on a test model provides descriptive examples of specific advantages and disadvantages of the individual methods, which are mostly related to consistent and robust identification of (non) negligible parameters and identification of important parameter interactions. Building energy models (BEM) often contain non-linear parameter effects and large uncertainties in parameter values, and therefore the results from sensitivity analysis can be misleading. The comparisons presented in this paper demonstrate potential pitfalls for sensitivity analysis of BEMs with a limited number of uncertain parameters and wide uncertainty ranges, and presents techniques that can support evaluation and interpretation of the results.

Table 4

Higher-order sensitivity indices  $S_H$  from Sobol method and sum of significant, absolute correlation coefficients (with a p-value < 0.005) between elementary effects and parameter input values from Morris method. The standard deviation for the  $S_H$  originates from the resampling analysis for  $S_i$  and  $S_{T_i}$  used for the calculation of  $S_H$ .

Parameter	higher-order effects sensitivity index $S_H$	sum of average correlation coefficients
Set point temperature	0.17 ± 0.07	4.28
Infiltration rate	0.12 ± 0.07	3.85
Thermal capacitance	0.09 ± 0.06	3.76
Radiative proportion of heating system	0.08 ± 0.07	3.58
Discharge coefficient	0.05 ± 0.04	2.95
Wind reduction factor	0.01 ± 0.04	2.33
$C_v$ factor variation	0.001 ± 0.04	1.90

The parameter ranking obtained by Morris method using the absolute mean value  $\mu^*$  of the elementary effects as a criterion is shown to be potentially unstable, when parameter ranges are rather large and non-linear parameter behaviour occurs. We show that, for a low number of trajectories, the use of the median value  $\chi^*$  improves the robustness of the parameter ranking. We also demonstrate that any potential variability (or inconsistency) in parameter rankings obtained from Morris method can be assessed at early stages in the analysis with a low number of trajectories by repeated Morris method runs at computational costs that are still lower than variance based sensitivity analysis (e.g. six evaluations of Morris method with 10 trajectories and 11 parameters need 720 model runs, compared to approx. 6000 runs for Sobol's  $S_i$ ). Yet, interpretation of the results should take in account a potential variability in the results, which might lead to ambiguous rankings, especially if two or more parameters exhibit very similar  $\chi^*$  values. Yet, for our test model 10 trajectories are sufficient to get a robust ranking, except for those parameters with very similar median values, which was confirmed by the rankings obtained by regression analysis and variance-based sensitivity analysis. Based on the comparison with results from quantitative methods it seems valid to state that parameters with absolute median values that are more than one order of magnitude lower than those of the dominating parameter can be regarded as negligible for the model outcome. The distribution of the relative difference of individual  $\chi^*$  values is very similar to the difference in the quantitative sensitivity indices from Sobol total effects, which represent a quantitative measure for negligible parameters. Although the results from Morris method are not standardized with regard to parameter ranges or variation in model outcome, so that the absolute  $\chi^*$  values depend on the magnitude of the elementary effects used in the calculation, the  $\chi^*$  values can be used to draw conclusions about the magnitude of parameter influence. Another advantage of Morris method is that due to the factorial sampling with one parameter value changing at a time, parameters with absolutely no influence on the model outcome are assigned zero values for mean, median and standard deviation, and also parameters with a minor influence on the model outcome show a clear and robust ranking.

The parameter ranking based on the standardised regression coefficients (SRC) from linear regression analysis is overall in good agreement with the ranking from Morris method, despite being only defined for linear components of the model. The parameter infiltration rate is ranked lower by the SRC than in Morris method, which is presumably due to non-monotonic or non-linear effects that are not captured by linear regression analysis. Furthermore, the SRC is a measure for parameter importance calculated from a sample matrix, where all parameter values are varied simultaneously, while the Morris method aims to identify negligible parameters. For our test model the regression analysis shows a consistent ranking with a clear differentiation of negligible and non-negligible for a sample size of  $N=400$  or more, except for parameters that show very similar SRC values. With 400 or more simulation runs needed for a robust evaluation the computational costs of regression analysis are generally higher than for Morris method, while, besides representing normalized, quantitative values, the SRCs provide basically the same information as the results from Morris method. Regarding the test parameter dHNPL in our model, the regression analysis provides no possibility to identify potential parameter or model faults.

The evaluation of the direct impact of parameters on the model outcome using Sobol first-order effects reveals a parameter ranking identical to the SRC, which is also a measure for parameter importance. Although the  $S_i$  values and the SRCs are both based on the analysis of model variance, there are discrepancies in the values for our model, which are potentially caused by the assumption of a linear regression model for the SRC, while the  $S_i$  index is

model independent. A resampling analysis shows that also Sobol's quantitative sensitivity indices exhibit a certain variability in the quantitative values and parameter ranking with the chosen sample size of  $N=500$ . This variability is similar to the variations in results from Morris method and is most likely due to the significant skewness of the model outcome vectors used for assessing the variance components. The large positive skewness observed in the model outcome is presumably caused by the dynamic and non-linear nature of the building energy model. For the interpretation of the first-order sensitivity indices the specific definition of 'direct impact' on model outcome has to be considered. As demonstrated for the parameter infiltration rate in this study, a low  $S_i$  value does not necessarily imply that a parameter is negligible, which would be indicated by a low total effect index  $S_{Ti}$  value. In addition to the  $S_i$  and  $S_{Ti}$  index we introduce a sensitivity index for higher-order effects  $S_H$ , which quantifies the involvement of a parameter into interaction effects at all levels and can be calculated at no additional costs.

As an alternative for the assessment of higher-order effects with Sobol method we explore the use of correlation analysis between parameter values and elementary effects based on Morris method. The ranking of the sum of absolute correlation coefficients is identical to the ranking from Sobol higher-order effects  $S_H$ . Thus, the correlation analysis with the results from Morris method represents a useful tool with no additional computational costs, which can give detailed insights into parameters behaviour and provide additional information about parameter behaviour and model complexity that might also help to select the most meaningful parameter combination for further analysis.

Regarding the investigation of the ranking of important parameters or the differentiation of influential and negligible parameters the computationally more intensive quantitative methods, such as linear regression analysis or Sobol method, provide the same information for the model in this study as the computationally efficient Morris method using the median value. However, the computationally costly detailed evaluation of first-order and higher-order, such as  $S_{Ti}$  and  $S_{ij}$ , effects can reveal interesting insights into complex parameter behavior, and provides quantitative sensitivity measures that are useful to compare parameter sensitivity on different models or other quantities of interest. In addition, this study highlights the importance of verifying that underlying statistical assumptions or conditions that apply to most sensitivity analysis methods are fulfilled by the outcome of the dynamic, non-linear building energy models.

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