Industrial cyberespionage in research and development races

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Abstract
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1 Introduction

This paper is devoted to the issues of modern research and development (R&D) races.
Specifically, we focus on the incentives behind industrial cyberespionage and how it in-
fluences R&D investments, research companies’ profits, and the end-product quality. We
define industrial cyberespionage as the practice of obtaining competitors’ secrets and
information stored in digital formats without permission in order to gain an economic advantage. It can be performed using various hacking techniques, malicious software or social engineering methods. Cyberespionage may be wholly conducted online or involve infiltration by computer-trained spies and moles. Seemingly any modern espionage is cyberespionage since it is improbable that it would not involve at least some information communication technology (ICT).

It is straightforward to find examples of industrial cyberespionage in the news feed. For example, in October 2018, the supply chain of technological giant Super Micro was infiltrated by Chinese adversaries. They created a backdoor in some of Super Micro’s server equipment that would allow them to steal corporate secrets in the future (Lynch, 2018). In November 2018, the US justice department filed criminal charges against Fujian Jinhua, a Chinese state-owned company, for the alleged theft of trade secrets worth up to $8.75 billion from Micron, an American semiconductor manufacturer (Shubber, 2018).

Industrial espionage has caused an enormous loss for the global economy. For instance, The Commission on the Theft of American Intellectual Property (2017) estimated that espionage via hacking costs between $180 billion and $540 billion annually to the US economy.

The situation aggravates every year with an increasing number of organisations and individuals being affected. The current COVID-19 crisis makes this even more apparent. During the crisis, national security agencies reported a significant increase in cyberespionage activity over the globe. People stay at home during lockdowns, stressed, and often violate good cybersecurity practices. It creates an excellent attack surface for a series of social-engineering attacks performed by state-backed hacking groups (Warrell & Manson, 2020).
Cyberespionage has become an inalienable part of contemporary competition. Nonetheless, the phenomenon has not been rigorously studied under the competitive framework. We fill this gap by developing a multi-stage R&D race model with endogenous espionage decisions. The model is built on a rank-order tournament framework by Lazear and Rosen (1981), which is consistent with innovative activity as it embeds a degree of uncertainty in companies’ decisions and has a relativistic award mechanism that resembles the patenting process.

We extend Lazear and Rosen’s framework to consider two ex-ante identical companies that compete in an R&D race over two stages for a better innovation to acquire the market. During the initial research stage, rivals invest in research to obtain superior technology—a technological edge. In order to obtain a technological edge, a company must outperform its opponent by a substantial margin, which we call a technological step. During the following development stage, companies invest again to perfect their technologies and release them to the market.

Espionage incentives may arise from the difference in relative positions of competitors who race to innovate. If the gap between rivals is sufficiently large, it will create a strong incentive for the underdog company to steal the leader’s intellectual property. Thus, cyberespionage might increase an underdog’s chances of success and its expected profits. For example, an attacker can obtain classified information to manufacture copycat products and gain market share.

If one of the companies gains a technological edge in the research stage, it utilises this advantage in the development stage to achieve better innovation and increase its chances of winning the market. A company that has not gained a technological edge (an underdog) can choose to perform a cyberattack at a fixed cost to steal a fraction of the
competitor’s technological edge in an attempt to equalise the chances in the development stage. If neither company gains a technological edge in the research stage, the race continues with the symmetric development stage, where both companies have the same available technologies, and nobody performs industrial espionage. By the end of the game, the company with a better innovative product wins the race and obtains all the market to itself. The framework is stated formally and in greater detail at the beginning of Section 2.

The study makes two main contributions. First, we demonstrate that industrial espionage has an ambiguous influence on the overall investments exerted in the race. Espionage negatively influences the investments in the research stage and positively influences the investments in the development stage. Intuitively, none of the companies would be willing to invest in the technological edge during the initial research stage, knowing that it is most likely to be stolen. However, if one firm gets far ahead in the research stage, its rival might “give up” in the development stage. On the other hand, if it catches up with the opponent (through espionage or otherwise), it will want to compete because it has a realistic chance of winning. Therefore, by regulating the power of industrial espionage through legal means, a policymaker can control competitive incentives in the R&D race and maximise investments. Cyberespionage can maximise firms’ payoffs from participating in the R&D race and make the race more attractive for them.

Second, we investigate the welfare implications of cyberespionage. Social welfare is measured by the quality of the innovative product delivered to the market by the winner of the race. We demonstrate that, under specific conditions, dosed espionage might be beneficial for society. It generally happens when the technological edge awarded at the initial stage is too large, which decreases competitive incentives at the second development
stage. In turn, diminished efforts at the development stage decrease the overall quality of an end-user product. Thus, industrial cyberespionage might counter the inefficient (from a market perspective) technological edge, motivate competition, and foster higher quality products. It also means that by choosing an optimal value of technological edge (e.g. by setting an optimal duration of patent protection), policymakers can make industrial cyberespionage obsolete while encouraging competition.

The paper is built as follows. This section provides an introduction and a review of related literature. Section 2 formalises the model and provides the results. The social welfare analysis is presented in 3. Section 4 concludes. For those interested in computing equilibria and related proofs, see Appendix A.1. The derivation of the Social Welfare Function and related proofs can be found in Appendix A.2.

1.1 Related literature

This paper contributes to a rich strand of literature about industrial espionage in R&D races. The early works by Matsui (1989) and Solan and Yariv (2004) could be attributed to information economics as the espionage mechanism was modelled as a tool which ex-ante uninformed players could use to obtain information about the actions of their opponents. Matsui (1989) studied a two-person repeated game where espionage happened with some exogenously set probability and provided players with an opportunity to learn about their opponents’ actions. It was demonstrated that dosed espionage could lead to a Pareto-efficient subgame perfect equilibrium in which both players increase their payoffs. Solan and Yariv (2004) modelled strategic espionage in a sequential non-zero-sum game and demonstrated that a firm is incentivised to conduct espionage only if its rival has a first-mover advantage. Even though we use a very different espionage mechanism, the
findings of those papers are echoed in the presented framework. Similarly to Matsui (1989), we find that espionage can work as a payoff-increasing collusion device. The assumptions behind the espionage mechanism in our framework absorbed the findings of Solan and Yariv (2004)—in our model, espionage incentives arise when one of the companies is sufficiently behind in the innovative process.

More recent work presented by Barrachina et al. (2014) studied the effects of espionage on market entry deterrence. They studied an asymmetric two-player game in which a monopoly incumbent may choose to extend its capacity in order to deter the entrance of a potential entrant. The entrant is unaware of the incumbent’s choice. However, he can choose to use “The Intelligence System”—a device that provides a noisy signal about the incumbent’s actions. Similarly to Matsui (1989), they found that espionage makes the market more competitive and leads to higher payoffs, which could also be observed in our framework under certain circumstances.

Another popular way of modelling espionage was initially presented by Whitney and Gaisford (1996, 1999). The authors utilised a modified Cournot game in which espionage could decrease the marginal cost of a firm by providing access to a better production technology retrieved from the competitor. They primarily focused on the influence of espionage on social welfare. It was argued that espionage could be considered as a technology transfer, which inevitably leads to increased competitive incentives, higher payoffs, and higher social welfare. Our framework similarly depicts espionage, allowing companies to steal technologies from each other.

Unlike the papers mentioned above, we study espionage in a dynamic setting. In our model, the espionage decision is made in the middle of the race and only if one of the ex-ante identical companies has achieved superior technology. This allows for a more flexible
model that distinguishes the effects of espionage on innovative incentives in a race for a technological edge and competitive incentives in a race for a market share. Additionally, our approach reveals that, depending on its power, espionage can influence social welfare both positively and negatively.

Billand et al. (2012) and Chen (2016) have also studied espionage in modified Cournot settings. Billand et al. (2012) studied an oligopolistic competition with espionage in a multi-market setting. Espionage was modelled as an exogenous boost that companies attain if they choose to spy on their competitors. The magnitude of a boost did not depend on the effort levels of the opponents and simply increased in the number of compromised rivals. This framework did not allow for the investigation of the relationship between espionage and competitive incentives but, similarly to Whitney and Gaisford (1996, 1999), demonstrated that spying is beneficial for social welfare. Similarly to Solan and Yariv (2004) and to our framework, Chen (2016) assumed that espionage incentives arise from relative differences in opponents’ technological levels. In Chen’s framework, one of the companies was assumed to possess a more sophisticated technology and be capable of producing products at a lower marginal cost. They demonstrated that espionage reduces rivals’ competitive incentives, which contradicts the findings of our study. This difference arises from a static setting of the model. By allowing for the endogenous technological edge award mechanism, we demonstrate that the influence of espionage is ambiguous: it decreases innovative incentives in the initial race for better technology and increases the incentives in the following stage where the technologies are implemented in a final product.

Send (2022) also considered a one-shot R&D race with espionage in which one of the companies is assumed to be more technologically advanced. The author, however, drew
ideas from a contest theory and employed a modified version of a Tullock contest (Tullock, 2001). Similarly to Chen (2016), a more advanced company is assumed to have a smaller marginal cost of innovative efforts. Espionage was modelled as a device which copies the opponent’s research effort and adds it to the company’s own. The authors found that the disadvantaged player is more likely to win the race if espionage is possible. It was also demonstrated that espionage strictly decreases overall R&D investment and the leading company’s expected payoff. On the contrary, our model demonstrates that espionage’s influence on overall efforts and payoffs is ambiguous. The difference can be once more attributed to the static nature of Send’s model. This model can be considered a variation of the second stage of the R&D race presented in the paper and does not capture our main results.

As mentioned before, this study builds on the ideas of tournament theory originally introduced in seminal articles by Lazear and Rosen (1981) and Nalebuff and Stiglitz (1983). The original papers studied wage differentiation based on relative performance differences between employees with a framework widely recognised as a rank-order tournament. They analysed optimal prize (wage) structure in a static setting to find methods to increase employees’ competitive incentives and productivity. Since then, their model has found numerous applications in various fields unrelated to personnel economics. The random element of the original rank-order tournament makes it a good fit to model an innovation process. Additionally, we modify an originally static setting into a dynamic one. The dynamic structure of the framework is essential for the investigation as incentives behind cyberespionage attacks arise when competitors find themselves in uneven positions in the middle of the R&D race.

Several other papers studied dynamics in tournaments. Denter and Sisak (2015, 2016)
and Siegel (2014) have analysed head starts in multi-stage contests under symmetric and asymmetric assumptions. In their papers, the authors discovered that awarding a small head start to one of the players whenever they interact over multiple stages is optimal, even in the case of symmetric players. This research contributes to the literature by introducing an endogenous award mechanism, which allows players to obtain a head start by performing a cyberespionage attack.

There were a small number of papers in tournament theory which focused on cheating in contests—a phenomenon which resembles cyberespionage. Berentsen and Lengwiler (2005) in their article on fraudulent accounting and doping in sports utilised a modified rank-order tournament model in which they implemented a cheating mechanism—the doping game. They investigated the evolutionary heterogeneous players’ framework where contestants can improve their performance by using performance-enhancing drugs. They described cases where high-ability contestants are more likely to cheat than low-ability ones. The players could boost their performance by adding an exogenously defined variable to their effort levels. However, the model assumes ex-ante heterogeneous players and does not account for uncertainty in a race: the “doped” player wins over any clean player with certainty. Moreover, the cheating mechanism in the framework does not depend on the opponent’s performance—any player can boost her effort levels regardless of how successful the opponent is. This makes the model unsuitable for studying cyberespionage in R&D races.

The doping game was also studied by Kräkel (2007). The author analysed the dynamics of two asymmetric players in the tournament with cheating. The paper discovered three determinants of a player’s cheating decision: the likelihood effect (cheating increases the winning probability), the cost effect (cheating influences the effort costs) and
the windfall-profit or base wages effect (the decrease in base salary due to the possibility of getting caught). However, the cheating mechanism broadly remained similar to the predecessors.

Rather than study cheating, Curry and Mongrain (2000) focused on ways to prevent it. They demonstrated that prize structure plays a crucial role in determining incentives for cheating. It is possible to decrease cheating solely by setting up a correct prize structure instead of investing in sophisticated monitoring systems. Unlike the model presented in the paper, they assumed that both effort and cheating decisions are simultaneous, which might not be the case in many real-life situations (e.g. R&D).

The series of papers on cheating, enforcement and audit in contests by Stowe and Gilpatric (2010) and Gilpatric (2011) is closely related to our research as well. In the 2010 paper, the authors analysed the incentives behind prohibited behaviour in rank-order tournaments while the base effort was considered as given. Their framework should be considered the final stage of the tournament, where effort levels are already known, and the players only make a dichotomous choice on whether to cheat or not. The latter paper described how the prize structure, uncertainty, monitoring efficiency, number of contestants, and associated penalty impact cheating. They found that greater enforcement might harm players’ productivity, which partially echoes our results. However, due to the limitations of a static framework, the authors did not capture the cases in which cheating positively influences contestants’ effort levels.

Therefore, this study contributes to the literature by studying the dynamic interaction between contestants, their relative effort levels and payoffs, and how they are affected by the presence of espionage (or cheating). It investigates the influence of espionage on social welfare by converting the relative outcomes of rank-order tournament into absolute ones,
which, to our best knowledge, has not been done before.

2 The model of economic cyberespionage

Two players, firm 1 and firm 2, engage in an R&D race over two stages: the initial research stage and the following development stage. Participating in the research stage and the development stage, the companies invest $x_i$ and $\hat{x}_i$ (where $i = 1, 2$), respectively, which increases their chances of gaining a superior innovation by the end of the race. If one of the companies pulls ahead after the research stage and achieves an advantage (a technological edge), its opponent might choose to carry out a cyberespionage attack to steal a fraction of the advantage and increase the chances of winning the race. During the race, the competitors experience random shocks, which account for the uncertain nature of R&D (Baker et al., 1986). In line with the classical framework of price competition by Bertrand (1989), the company which delivers superior technology by the end of the development stage remains the only one active on the market and collects all of the profits to itself. Society benefits from the quality of the new technology delivered to the market by the winner of the race. The social welfare function is formally defined in Section 3.

Figure 1 outlines the timings of the game.

![Figure 1: Timings of the R&D race. Circles represent the timings for firms’ investment decisions.](image)

Similar to Lazear and Rosen (1981), shocks ($\epsilon_i$ and $\hat{\epsilon}_i$) are drawn from a known distri-
distribution with zero mean, infinite support, and variance \( \frac{1}{2} \sigma^2 \). It is assumed that shocks are independent and identically distributed (i.i.d.) across firms and stages. They represent the luck of competitors and can influence the research either positively or negatively. The shocks for the research and development stages are denoted \( \epsilon_i \) and \( \hat{\epsilon}_i \), respectively. Throughout the paper, we use the notations \( f(\cdot) \) and \( F(\cdot) \) to denote the probability density function (PDF) and the cumulative distribution function (CDF) of the difference between two shocks.

A firm’s output of the **research stage** is expressed as the sum of investment and the random shock:

\[
q_i = x_i + \epsilon_i. \tag{1}
\]

The structure of the research stage closely resembles the original model of Lazear and Rosen (1981). However, it uses an alternative award mechanism to account for two possibilities in an innovation race: concurrent failure and simultaneous invention, both of which can be treated as a similar event from a relative standpoint (Lemley, 2011). A company is said to be the winner of the research stage if it outperforms its opponents by a substantial margin \( k \), e.g. firm \( i \) wins in the research stage if \( q_i - q_{-i} > k \). We call variable \( k \) a technological step. Firm \( i \) is then awarded a technological edge \( h \), which adds on to the output of the development stage and provides an advantage over its opponent. Conversely, if the output difference is not significant, \( q_i - q_{-i} < k \), participants achieve parity, and nobody gains a technological edge \( h \). Note that technological step \( k \) and technological edge \( h \) are assumed to be discrete. While one could argue that innovation is a continuous phenomenon, innovation often happens in discrete steps. For instance, microprocessor manufacturers compete to achieve a better fabrication process, which enables smaller silicon transistors on a printed circuit board (PCB). The advancement of
the process is measured in nanometers and usually happens in discrete steps (e.g. 14 nm, 10 nm, 7 nm). In turn, the advanced fabrication process allows for microprocessors with more cores and threads—which are also discrete characteristics (Shrout, 2018).

The race then proceeds to the final stage of the R&D process—the development stage. During this stage, the companies implement their innovations into end-user commercial technologies. A company with a better end-user product wins the market, the value of which is normalised to 1, \( w = 1 \).

The development stage depends on the outcome of the research phase. If neither of the companies has gained a technological edge, the game follows a standard rank-order tournament pattern. Each firm then produces a product of quality:

\[
\hat{q}_i = \hat{x}_i + \hat{\epsilon}_i. \tag{2}
\]

If one of the firms obtains a technological edge \( h \), the underdog firm might choose to carry out a cyberespionage attack to retrieve a fraction \( \alpha \in [0, 1] \) of the opponent’s head start at a fixed cost \( A \) and boost her chances of winning the race. This relatively simple cyberattack mechanism captures the fact that the cyberattacks are mainly performed by hired third-party hacking groups as companies rarely participate in the offensive operations themselves.

Without loss of generality, assume that firm 1 is the leader while firm 2 is the underdog. Then their output levels are:

\[
\hat{q}_1^a = \hat{x}_1 + \hat{\epsilon}_1 + h, \tag{3}
\]
\[
\hat{q}_2^a = \hat{x}_2 + \hat{\epsilon}_2 + (\alpha h), \tag{4}
\]

where \( \alpha h = 0 \) if the underdog decides not to attack. The company that achieves a higher output in the development stage wins the main prize.
The R&D effort \( x \) (either \( x_i \) or \( \hat{x}_i \)), is costly and is produced at cost \( c(x) \). The cost function is assumed to be convex in \( x \): \( c'(x) > 0 \) and \( c''(x) > 0 \) for all \( x > 0 \). As the study focuses on the process dynamics, sunk costs are omitted from the research: \( c(0) = 0 \). The cost function’s convexity indicates that the marginal cost of the R&D effort increases, and every additional investment unit becomes less efficient. For instance, the effectiveness of the investment in developing a new matrix for a computer screen tends to zero as soon as the pixels become indistinguishable to a human eye. Despite the convexity of the cost function, competitors’ payoff functions do not need to be quasiconcave in investments.

We also impose the following assumption:

**Assumption 1** The second derivative of the cost function is greater than the marginal benefits for any \( x \in \mathbb{R} \):

\[
\max f'(x) < \min c''(x).
\]

The assumption guarantees that \( c'(\cdot) \) crosses \( f(\cdot) \) only once from below (see Figure 2), which ensures the existence of interior pure strategy equilibrium in every stage.

The overall payoffs of the game are equal to:

\[
E[\Pi_i] = P^S \hat{P}_i^s + P^A_1 \hat{P}_1^a + P^A_2 \left( \hat{P}_2^a - (A) \right) - c(\hat{x}_i) - c(x_i),
\]

where \( P^S \) is the probability that neither of the companies has gained a technological edge in the research stage and \( \hat{P}_i^s \) is the probability of achieving a superior innovation in the symmetric development stage from an even position; \( P^A_1 \) is the probability of gaining a technological edge in the research stage and \( \hat{P}_1^a \) is the probability of achieving a superior innovation in the development stage from a leading position; \( P^A_2 \) is the probability of losing a technological edge to the opponent in the research stage and \( \hat{P}_2^a \) is the probability of achieving a superior innovation in the development stage from an underdog position;
A is a cost of cheating, which equals to 0 if the underdog decides not to engage in cyberespionage. Note that cheating might only occur when the development stage of the game is asymmetric and, therefore, incorporated in terms \( \hat{P}_1 \) and \( \hat{P}_2 \).

The study uses the Subgame Perfect Nash Equilibrium (SPNE) as a solution concept and assumes that each company rationalises against its opponent’s optimal investment level, representing a firm’s inability to influence the market. The game is solved by backward induction beginning at the development stage.

2.1 Development stage

Depending on the outcome of the research stage, there are three possible cases: (i) symmetric case if nobody has a head start (technological edge) in the development stage, \( HS = 0 \), (ii) asymmetric case with cyberespionage and the leader’s head start, \( HS = h(1 - \alpha) \), and (iii) asymmetric case without cyberespionage and the leader’s head start, \( HS = h \).

2.1.1 Symmetric case, \( HS=0 \)

In the symmetric case both companies have the same marginal incentives. Thus, their behaviour should also be identical if Assumption 1 is satisfied. The firms’ expected payoff functions are then:

\[
\hat{\Pi}_i = \hat{P}_i - c(\hat{x}_i),
\] (6)
where $\hat{P}_s^i$ is the probability of winning the game. It can be expressed as:

$$
\hat{P}_s^i = P(\hat{q}_i > \hat{q}_{-i})
= P(\hat{x}_i - \hat{x}_{-i} > \hat{\epsilon}_i - \hat{\epsilon}_{-i})
= P(\hat{x}_i - \hat{x}_{-i} > \hat{\zeta})
= F(\hat{x}_i - \hat{x}_{-i}),
$$

where $\hat{\zeta} = \hat{\epsilon}_{-i} - \hat{\epsilon}_i$ is the convolution of two random variables. As shocks are i.i.d., $E(\hat{\zeta}) = 0$ and $E(\hat{\zeta}^2) = \sigma^2$. Substituting (7) into (6) yields:

$$
\hat{\Pi}_s^i = F(\hat{x}_i - \hat{x}_{-i}) - c(\hat{x}_i).
$$

Each player then chooses $\hat{x}_i$ to maximise (8), taking its opponent’s strategy as given.

Given that Assumption 1 holds and that the objective function is twice differentiable, maximisation requires:

$$
\frac{\partial \hat{\Pi}_s^i}{\partial \hat{x}_i} = f(\hat{x}_i - \hat{x}_{-i}) - c'(\hat{x}_i) = 0,
$$

$$
\frac{\partial^2 \hat{\Pi}_s^i}{\partial \hat{x}_i^2} = f'(\hat{x}_i - \hat{x}_{-i}) - c''(\hat{x}_i) < 0,
$$

where (9) is the set of first-order conditions (FOCs) and (10) is the set of second-order conditions (SOCs).

We find that if the solution exists, both firms choose identical investment strategies. The result should be intuitive, considering the symmetry of marginal incentives of firms. Note that the marginal increase in the probability of winning the market by a firm is precisely the density of the uncertainty shock evaluated at the relative investment level.

Since participants also have the same well-behaved cost functions, their marginal costs are also identical. Consequently, given any prior distribution $F(\cdot)$, firms have identical marginal incentives and choose the same strategies. Thus, the equilibrium must be symmetric. This finding is summarised in the proposition below.
Lemma 1. The equilibrium of the symmetric case of the development stage exists and is unique if Assumption 1 is satisfied. In the equilibrium of the subgame, both companies choose identical investments:

\[ \hat{x}_i^* = c^{-1} [f(0)]. \]  \hspace{1cm} (11)

Proof of Lemma 1. See Appendix A.1.

The intersection of marginal benefits \( f(\cdot) \) and marginal costs \( c'(\cdot) \) might not be unique. In Figure 2, graph (a) demonstrates a case of the unique intersection, while graph (b) shows a case where the intersection is not unique.

![Figure 2: The uniqueness of Nash equilibrium depending on the slopes of PDF and cost functions.](image)

Assumption 1 ensures that marginal costs intersect with marginal benefits only once “from below”, as illustrated in Figure 2 (a). It rules out the possibility of multiple intersections, as shown in Figure 2 (b), and guarantees that the solution for the system
of the FOCs is indeed an equilibrium.

Substituting optimal investment levels (11) into (8) returns the equilibrium payoffs for the symmetric case of the stage:

$$\hat{\Pi}_i^S = \frac{1}{2} - c \left[ e^{-1} [f(0)] \right].$$

(12)

It is clear that if neither of the companies gains a technological advantage in the research stage, the winner is decided by a coin toss. Thus, in equilibrium, the development stage has zero influence on the expected winner of the race: both choose the same strategies, have the same marginal incentives, and zero net effect.

### 2.1.2 Asymmetric case involving cyberespionage, HS=h(1-\(\alpha\))

If one of the competitors gains a technological edge in the research stage, the contenders continue the race from uneven positions. Without loss of generality, we assume that firm 1 is the leader who has gained the technological edge \(h\) in the previous stage. Then, firm 2 is the underdog and may choose to carry out a cyberespionage attack and retrieve a portion \(\alpha\) of the competitor’s head start if the attack costs \(A\) are sufficiently small. Assuming sufficiently small attack costs, the firms produce innovative products of qualities \(\hat{q}_1^a\) and \(\hat{q}_2^a\) respectively:

\[
\hat{q}_1^a = \hat{x}_1 + \hat{\epsilon}_1 + h,
\]

\[
\hat{q}_2^a = \hat{x}_2 + \hat{\epsilon}_2 + \alpha h.
\]

The firms’ payoffs for the asymmetric case are then:

$$\hat{\Pi}_1^a = \hat{P}_1^a - c(\hat{x}_1),$$

$$\hat{\Pi}_2^a = \hat{P}_2^a - c(\hat{x}_2) - A,$$

(13)
where $A$ is the cyberattack costs and $\hat{P}_a^i$ are the probabilities of winning the stage. The leader then wins with the probability:

$$\hat{P}_1^a = P(\hat{q}_1 > \hat{q}_2)$$

$$= P(\hat{x}_1 - \hat{x}_2 + h - \alpha h > \hat{\epsilon}_1 - \hat{\epsilon}_2)$$

$$= P \left( \Delta \hat{x} + h(1 - \alpha) > \hat{\epsilon} \right)$$

$$= F(\Delta \hat{x} + h(1 - \alpha)),$$

while the underdog wins with the probability:

$$\hat{P}_2^a = 1 - \hat{P}_1^a$$

$$= 1 - F(\Delta \hat{x} + h(1 - \alpha)).$$

Substituting equations (14) and (15) into payoffs (13) yields:

$$\hat{\Pi}_1^a = F(\Delta \hat{x} + h(1 - \alpha)) - c(\hat{x}_1),$$

$$\hat{\Pi}_2^a = 1 - F(\Delta \hat{x} + h(1 - \alpha)) - c(\hat{x}_2) - A.$$ (16)

The firms then maximise their expected payoffs by choosing optimal investment levels $\hat{x}_a^i$, which must satisfy the following FOCs:

$$\frac{\partial \hat{\Pi}_1^a}{\partial \hat{x}_1} = f'(\Delta \hat{x} + h(1 - \alpha)) - c''(\hat{x}_1) = 0,$$

$$\frac{\partial \hat{\Pi}_2^a}{\partial \hat{x}_2} = f'(\Delta \hat{x} + h(1 - \alpha)) - c''(\hat{x}_2) = 0,$$ (17)

and SOCs:

$$\frac{\partial^2 \hat{\Pi}_1^a}{\partial \hat{x}_1^2} = f''(\Delta \hat{x} + h(1 - \alpha)) - c'''(\hat{x}_1) < 0,$$

$$\frac{\partial^2 \hat{\Pi}_2^a}{\partial \hat{x}_2^2} = -f''(\Delta \hat{x} + h(1 - \alpha)) - c'''(\hat{x}_2) < 0.$$ (18)

Even though the firms’ profit functions are now asymmetric, marginal incentives are still identical. The marginal increase in the probability of winning the game is the density
function evaluated at the relative investment level, which is influenced by technological edge $h$ and espionage attack power $\alpha$. Given that the shock is drawn out of a distribution symmetric around zero, marginal benefits are precisely the same. The cost functions are symmetric by assumption. Therefore, it must be the case that both firms again choose identical investments. The finding is summarised below.

**Lemma 2** The equilibrium of the asymmetric case of the development stage involving cyberespionage exists and is unique if Assumption 1 is satisfied. In the equilibrium of the subgame, both companies choose identical investments:

$$ \hat{x}_i^a = c^{-1} [f (h(1 - \alpha))] .$$

(19)

**Proof of Lemma 2.** See Appendix A.1.

Substituting optimal investment levels (19) into (16) yields equilibrium payoff levels:

$$ \hat{\Pi}_1^A = F (h(1 - \alpha)) - c [c^{-1} [f (h(1 - \alpha))]], $$

$$ \hat{\Pi}_2^A = 1 - F (h(1 - \alpha)) - c [c^{-1} [f (h(1 - \alpha))] - A. $$

(20)

In the asymmetric case of the race, the leader has more chances to win the market due to the technological edge, while the underdog might balance the game by cyberattack means. The underdog’s binary decision about espionage is investigated in the subsection below.

It also follows from (11) and (19) that companies are exerting less effort in the asymmetric case than in the symmetric one. Thus, uncertainty encourages R&D companies to invest more.
2.1.3 Asymmetric case not involving cyberespionage, HS=h

It is straightforward to obtain the equilibrium payoffs for the fair instance of the asymmetric case (without cyberespionage). Substituting $\alpha = 0$ and $A = 0$ to (17) yields a solution to the game with no espionage:

$$\hat{x}_i^f = c^{-1} [f(h)].$$  \hspace{1cm} (21)

**Proposition 1** The equilibrium investments in the asymmetric case with cyberespionage are higher than those in the asymmetric case without cyberespionage.

Proposition 1 immediately follows from (19), (21) and the fact that $\hat{\zeta}$ has a single-picked distribution with zero mean and infinite support.

The intuition behind the proposition should be straightforward. When the gap between the leader and the underdog is too large after the research stage, it decreases investment incentives for both of them: the leader does not need to invest a lot as she is already far ahead, and the underdog is not motivated to invest in a surely lost game. By diminishing the gap, cyberespionage encourages players to invest more and stimulates competition.

Optimal payoffs for the asymmetric case without cyberespionage are:

$$\hat{\Pi}_1^F = F(h) - c \left[ c^{-1} [f(h)] \right],$$

$$\hat{\Pi}_2^F = 1 - F(h) - c \left[ c^{-1} [f(h)] \right].$$  \hspace{1cm} (22)

The existence and uniqueness of the equilibrium are guaranteed if:

$$f' (\hat{x}_i - \hat{x}_i^f) < c'' (\hat{x}_i^f),$$

which is always true by Assumption 1.
2.1.4 Cyberespionage decision

It is evident that the underdog will choose to perform a cyberespionage attack only if:

$$\Pi_2^A > \Pi_2^F,$$

or, alternatively, if:

$$A < F(h) - F(h(1 - \alpha)) + c \left[ c^{-1} [f(h)] - c \left[ c^{-1} [f (h(1 - \alpha))] \right] \right] = AT. \quad (23)$$

The sign of the right-hand side (RHS) of the inequality above is always positive if Assumption 1 is satisfied. It implies that the underdog always chooses to perform an attack when it is sufficiently cheap. This finding is summarised in the proposition below.

**Proposition 2** Cyberespionage decision threshold, \(AT\), (23) is always positive if Assumption 1 is satisfied.

**Proof of Proposition 2.** See Appendix A.1.

Note that Proposition 2 may not hold if Assumption 1 is relaxed. This case is worth investigating in future research as it might reveal scenarios in which the underdog chooses not to perform an attack even if it is free. Following the result of Proposition 1, cyberespionage may substantially increase the investments of the leader, which may decrease the expected profits of the underdog even further. In this case, cyberespionage may not be performed even if the underdog is paid for it.

From now on, we assume zero cyberattack costs. The assumption allows us to focus on the particular case of the R&D race with industrial espionage and neglects the less interesting case where cyberespionage never occurs in equilibrium (which is also formally equivalent to the standard rank-order tournament). The assumption is summarised below and imposed until the end of the paper.
Assumption 2 The cheating is costless, $A = 0$.

2.2 Research stage

The study now has all the necessary ingredients to solve the research stage of the game. Since both companies are assumed to start their research simultaneously without prior knowledge, the stage is symmetric. It is said that the firm gains a technological edge $h$ if its output is at least $k$ higher than the opponent’s:

$$q_i - q_{-i} > k,$$

or:

$$
\Delta x + \zeta > k,
$$

where $\Delta x = x_i - x_{-i}$ and $\zeta = \epsilon_i - \epsilon_{-i}$. There are three possible outcomes of the stage for each player:

1. neither firm gains a technological edge and the game continues symmetrically with probability:

$$P^S = P(-k < \Delta x + \zeta < k)$$

$$= P(-\Delta x - k < \zeta < -\Delta x + k)$$

$$= F(-\Delta x + k) - F(-\Delta x - k);$$

2. firm $i$ gains a technological edge and becomes the leader of the race, while firm $-i$ becomes the underdog with probability:

$$P^A_i = P(k < \Delta x + \zeta)$$

$$= P(-\zeta < \Delta x - k)$$

$$= F(\Delta x - k);$$
(3) firm $i$ loses the technological edge to firm $-i$ and continues the race as the underdog with probability:

$$P_2^A = P(\Delta x + \zeta < -k)$$

$$= P(\zeta < -\Delta x - k)$$

$$= F(-\Delta x - k).$$

The distribution of the new random variable $\zeta$ is symmetric around zero $E(\zeta) = 0$ and has a variance $E(\zeta^2) = \sigma^2$ as shocks are assumed to be i.i.d.

Combining outcome possibilities with equilibrium payoffs (12), (20) yields the following expected payoffs at the beginning of the initial stage:

$$\Pi_i = P^S\Pi^S + P_1^A\Pi_1^A + P_2^A\Pi_2^A - c(x_i)$$

$$= (F(-\Delta x + k) - F(-\Delta x - k))\Pi^s +$$

$$+ F(\Delta x - k)\Pi_1^A + F(-\Delta x - k)\Pi_2^A - c(x_i).$$

(24)

The firms then choose the optimal investment levels $x_i^*$, which, assuming differentiability of the objective function, must satisfy two sets of conditions:

(1) FOCs:

$$\frac{\partial \Pi_i}{\partial x_i} = f(\Delta x + k) \left( \Pi^S - \Pi_2^A \right) + f(\Delta x - k) \left( \Pi_1^A - \Pi^S \right) - c'(x_i) = 0$$

(25)

(2) and SOCs:

$$\frac{\partial^2 \Pi_i}{\partial x_i^2} = f'(\Delta x + k) \left( \Pi^S - \Pi_2^A \right) +$$

$$+ f'(\Delta x - k) \left( \Pi_1^A - \Pi^S \right) - c''(x_i) < 0.$$  

(26)

Thus, we demonstrate that the research stage of the game also has a unique symmetric equilibrium. The intuition remains the same: marginal incentives are equal for both firms.

Proposition 3 summarises this claim.
**Proposition 3** The equilibrium of the research stage exists and is unique if Assumption 1 is satisfied. In the equilibrium the firms choose identical investments:

\[ x_i^* = c^{r-1} \left[ f(k) \left( 2F(h(1 - \alpha)) - 1 \right) \right]. \tag{27} \]

Moreover, investments decrease in cyberespionage power \( \frac{\partial x_i^*}{\partial \alpha} < 0 \).

**Proof of Proposition 3.** See Appendix A.1. ■

The intuition behind the fact that the research stage investments decrease in cyberespionage power is that the firms are unwilling to invest in the research, a large share of which will be stolen. Therefore, cyberespionage diminishes competitive incentives in the initial stage. The combination of the results of Propositions 1 and 3 implies that cyberespionage has opposite effects on the investments in the research and development stages.

It is now possible to write down the optimal expected payoff at the beginning of the race. Substituting optimal effort levels (27) and payoffs of the latter stage into (26) and simplifying yields:

\[
\Pi_i = \frac{1}{2} + 2F(-k) \left( c \left[ c^{r-1} [f(0)] \right] - c \left[ c^{r-1} [f(h(1 - \alpha))] \right] \right) - \\
- c \left[ c^{r-1} [f(0)] \right] - c \left[ c^{r-1} [f(k) (2F(h(1 - \alpha)) - 1)] \right].
\tag{28}
\]

Thus, both players initially have the same chance to win the race. However, it is unclear how cyberespionage attacks influence the firms’ expected payoffs and investment levels. The following subsection investigates this matter.

### 2.2.1 Espionage externalities

The results suggest that cyberespionage power has opposite effects on the investment levels in the research and the development stages. A policymaker can optimise the overall
R&D race investments by influencing cyberespionage power $\alpha$. It can be regulated by legal instruments that impose obligations to businesses to implement cybersecurity measures to protect their sensitive information. For instance, the EU has two primary regulative documents that cover cybersecurity: General Data Protection Regulation (GDPR) and Network and Information Security Regulations (DCMS, 2018; Van Alsenoy, 2019). Both documents provide guidelines that aid companies in coping with common cyberattacks and ensuring the security of private and sensitive information. It is relatively hard to quantify the effects of the new cybersecurity law. Still, there is evidence that those laws are beneficial. Cisco (2019) reports that GDPR compliant companies were 15% less likely to have experienced a breach in the last 12 months, had 3 hours shorter average service downtimes, and lost significantly less money to breaches. Moreover, $63$ million in fines were issued in the first year after the release of GDPR, which was insignificant compared to overall losses due to cyberattacks. This supports Assumption 2 about negligible cyberespionage costs (Sobers, 2018).

Therefore, by changing the severity of the cybersecurity landscape, policymakers can regulate espionage power $\alpha$ and influence the competitive incentives in the R&D market. Well-designed cybersecurity law may then be used as a balancing device that erodes asymmetries and intensifies competition.

The research demonstrates that there exists a non-zero cyberespionage power $\alpha^I \in (0, 1)$ that maximises the overall expected investments if the technological edge $h$ is too large. The result implies that “dosed” industrial espionage can promote competition by balancing out the effects of espionage on investments in the research and development stages. Proposition 4 summarises the results and Figure 3 illustrates them.

To allow for in-depth inference and simplify the notation, we assume the normality of
shocks and impose a quadratic cost function.

**Assumption 3** Shocks are Normally Distributed: $\epsilon_i, \hat{\epsilon}_i \sim N(0, \frac{1}{2} \sigma^2)$ and cost function is quadratic $c(x) = \frac{x^2}{2}$.

![Figure 3: Influence of attack power on the overall investment.](image)

**Proposition 4** There exists a technological edge $\bar{h}$ such that overall effort is maximised by

$$\alpha^I = 1 - \frac{1}{\bar{h}} \sqrt{\frac{1}{2\pi} \frac{\sigma}{\Phi(-k)}} \exp \left( - \frac{k^2}{2\sigma^2} \right),$$

if $h > \bar{h}$ and $\alpha^I = 0$ otherwise. $\Phi(\cdot)$ is a CDF of normal distribution with zero mean and standard deviation $\sigma$.

**Proof of Proposition 4.** See Appendix A.1.

We now demonstrate that effort-maximising espionage power, $\alpha^I$, (i) weakly increases in technological edge $h$ and (ii) decreases in $k$. The first relationship should be straightforward as the effort-maximising espionage power $\alpha^I$ should increase in the distance between

---

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the actual \( h \) and \( \bar{h} \). It happens because if the awarded technological edge is too large, it severely diminishes investment incentives in the development stage, as the underdog has little chance of winning. In this case, cyberespionage allows the underdog to level with the leader and continue the race. It balances out the depressing effect of the large technological edge in the development stage. The second relationship is a consequence of the first. A larger technological step \( k \) means a higher “price” that companies need to pay to be awarded a technological edge \( h \). It makes the companies more reluctant to invest in the research stage and reduces the weight of the technological edge, driving \( \alpha^I \) down. Those results are summarised below in Proposition 5.

**Proposition 5** The effort-maximising espionage power \( \alpha^I \) increases in technological edge, \( h \), and decreases in technological step, \( k \).

**Proof of Proposition 5.** See Appendix A.1. ■

The influence of cyberespionage power on a company’s expected payoff in the race is not straightforward either. The research investigates the issue under three different espionage regimes: (i) ultimate espionage power \( \alpha^* = 1 \), (ii) interim power with \( \alpha^* \in (0, 1) \), and (iii) absolute fairness with \( \alpha = 0 \).

The ultimate espionage regime completely diminishes the importance of the initial research stage. No firm would invest in the research stage, knowing that every single bit of the developed innovation will be stolen. The whole competition is then decided in the development stage of the game. We now demonstrate that such a high \( \alpha \) can never maximise a firm’s payoffs. The result is formalised below.

**Lemma 3** Choosing the ultimate espionage power, \( \alpha^* = 1 \), is never optimal for payoff maximisation.

In order to simplify further derivations, let $H = (1 - \alpha)h$. Proposition 6 demonstrates that there always exists a unique optimal value of head start, $H^*$, that maximises the firms’ expected payoffs. If the technological edge is too small, $h \leq H^*$, the payoffs are maximised under the regime of absolute fairness as there is no point in diminishing it even further from its optimal value by choosing a positive cyberespionage power. This finding echoes the known result from the competition theory that it is always optimal to reward one of the contesters with a small head start to intensify competition (Denter & Sisak, 2016). On the contrary, if the technological edge is sufficiently large, $h > H^*$, it negatively influences the firms’ payoffs by diminishing the competition in the development stage. Then choosing an interim espionage power regime (with $\alpha^* \in (0, 1)$) can reduce the technological edge to the payoff-maximising value $H^*$. The results are illustrated by Figure 4.

It follows that policymakers can influence firms’ decisions to enter the R&D race by imposing stimulating cybersecurity laws. This would be especially interesting to consider in an N-players setting, which will be considered in future research.

Observe also that cyberespionage might serve as a collusion device. It diminishes the investment levels in the initial research stage, but does not influence the prize the winner of the race receives. Therefore, under certain conditions, espionage can reduce the competitive incentives of firms while increasing their expected payoffs at the beginning of the game—imposing a self-policing cartel.

Proposition 6 Joint profit maximising espionage power is equal to 0, $\alpha^* = 0$, if technological edge is less or equal than optimal head start value, $h \in (0, H^*]$, and obtains its intermediate value $\alpha^* \in (0, 1)$ if $h \in (H^*, \infty)$. Moreover, $\alpha^*$ always exists regardless of $h$
Figure 4: Influence of technological edge on firm’s payoff.

and $k$ and is a global maximum.

**Proof of Proposition 6.** See Appendix A.1. ■

Depending on the size of payoff maximising $H^*$, $k$ can influence $\alpha^*$ either positively or negatively. For instance, if $H^*$ is sufficiently small, then optimal espionage power increases in the technological step, $\frac{\partial \alpha^*}{\partial k} > 0$. The phenomenon occurs because the extra investments needed to gain a technological edge surpass the added benefits, making the symmetric case more appealing to the competitors.

If the optimal technological edge is already sufficiently large, the relationship becomes complicated. However, it still follows a logical pattern: the $\alpha^*$ increases in distance between $h$ and its optimal value $H^*$ and decreases otherwise. Proposition 7 formalises the findings and provides more details.

**Proposition 7** The technological step $k$ has an ambiguous influence on payoff maximising cyberespionage power $\alpha^*$. It does not influence the optimal attack level if $h < H^*$. If $h > H^*$ then the sign depends on the values of $k$ and $H^*$:

1. if $H^*$ is sufficiently small and $q(H^*) \in \left(0, \frac{\sqrt{2}}{\sqrt{e\pi \sigma^2}}\right)$ then $\frac{\partial \alpha^*}{\partial k} > 0$;
2. If $H^*$ is sufficiently large and $q(H^*) \in \left( 0, \frac{\sqrt{7}}{\sqrt{e \pi \sigma^2}} \right)$ then (1) $\frac{\partial \alpha^*}{\partial k} < 0$ if $k \in (k_1^*, k_1^*)$,

(2) $\frac{\partial \alpha^*}{\partial k} > 0$ for any $k \in (0, k_1^*) \cup (k_2^*, \infty)$,

where $q(H) = \frac{f'(H)}{1-2f'(H)}$.

Proof of Proposition 7. See Appendix A.1.

There may also exist an optimal value of the technological step $k^*$, which maximises the firm’s expected payoff. The presented framework can be adopted to study cyberespionage in relation to the patent regimes, with $h$ being interpreted as the patent length and $k$ as the patent breadth (or patentability threshold)—the dimensions of a patent identified by Scotchmer (2004). We will pursue this line of logic in future research.

### 3 Social welfare

The research adopts the expected quality (output) function to measure the social benefits of the race. We assume that society benefits from the quality of innovation delivered to the market by the winner of the R&D race and loses the cumulative costs that both companies incurred in the R&D race (which could be spent on something else otherwise).

While the model measures the performance of the companies in relative terms, absolute investments and quality of the end-product may be crucial from a social perspective. For instance, it is arguably of minor importance for a car manufacturer to reduce the gasoline its vehicles consume, as far as the car of its competitor uses more. However, any additional economised litre of gasoline matters for the environment and influences social welfare.

The social welfare function (SWF) is then expressed as expected output of the winner of the race accounting for all exogenous shocks that might happen during the R&D race.
subtracting expected overall costs of both competitors:

\[ W = X_i + E^S + E^L + E^U - C, \]  

(29)

where \( X_i \) is the overall effort exerted by the winner of the race, \( E^S \) is a contribution of symmetric case shocks to the output of the game, \( E^L \) is a contribution of asymmetric case shocks when a leader wins the race, \( E^U \) is a contribution of asymmetric case shocks when an underdog wins the race, and \( C \) is the overall expected costs of the R&D race. See Appendix A.2 for a complete SWF derivation.

Note that technological edge \( h \) influences the SWF (29) only indirectly through the overall expected effort exerted by the winner of the race. It is not, however, present in the function as a separate term. The technological edge is a measure of the relative advantage of one firm over another. Given that society is interested in the absolute quality of innovation, the inclusion of the technological edge into the function is unnecessary.

### 3.1 Espionage and welfare

Now we focus on the relationship between the end-product quality and cyberespionage power. The function (29) then can be simplified by excluding terms that are not functions of \( \alpha \).

Let,

\[
g(k) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \epsilon_i f(\epsilon_i) f(\epsilon_{-i}) d\epsilon_i d\epsilon_{-i}
\]

\[
= - \int_{-\infty}^{\infty} \int_{-\infty}^{\epsilon_i-k} \epsilon_{-i} f(\epsilon_{-i}) f(\epsilon_i) d\epsilon_{-i} d\epsilon_i > 0,
\]

and

\[
i(H) = \int_{-\infty}^{\infty} \int_{\hat{\epsilon}_2 - H}^{\hat{\epsilon}_1 + H} \hat{\epsilon}_1 f(\hat{\epsilon}_1) f(\hat{\epsilon}_2) d\hat{\epsilon}_1 d\hat{\epsilon}_2
\]

\[
= \int_{-\infty}^{\infty} \int_{\hat{\epsilon}_1 + H}^{\hat{\epsilon}_2 - H} \hat{\epsilon}_2 f(\hat{\epsilon}_2) f(\hat{\epsilon}_1) d\hat{\epsilon}_2 d\hat{\epsilon}_1 > 0.
\]
It is then sufficient to study the properties of the following function:

\[ \psi = F(H)(2g(k) + f(k) + F(-H)f(k)^2) + F(-k)(2i(H) + f(H) - f(H)^2). \] (30)

The study now demonstrates that the welfare-maximising value of cyberespionage power, \( \alpha^W \), depends on the size of the technological edge: \( \alpha^W \in (0, 1) \) if the technological edge is sufficiently large and \( \alpha^W = 0 \) if it is sufficiently small. The finding is summarised in a subsequent proposition.

**Proposition 8** Social welfare maximising espionage power is equal to \( \alpha^W = 0 \), if technological edge is less or equal to the socially optimal head start value, \( h \in (0, H^W] \), and attains its intermediate value \( \alpha^W \in (0, 1) \) if \( h \in (H^W, \infty) \).

**Proof of Proposition 8.** See Appendix A.2.2.

The result of Proposition 8 echoes the results about effort and payoff maximisation derived in the previous section. The regime of absolute fairness (\( \alpha = 0 \)) can benefit social welfare only if the technological edge is sufficiently low \( h \leq H^W \). However, it achieves the opposite result if the technological edge is sufficiently large. In this case, choosing an interminable espionage power regime (\( \alpha^W \in (0, 1) \)) leads to higher social welfare. As with payoff maximisation, choosing the regime of absolute espionage power (\( \alpha = 1 \)) is never optimal as it completely deteriorates the investments in the initial research stage leading to low-quality end-user products. The findings align with the empirical results described by Busso and Galiani (2014).

Of course, increasing the power of cyberespionage in R&D competition if the technological edge is too big might not be an option for policymakers. Still, if we interpret a technological edge (\( h \)) as a patent length—the dimension that policymakers can control—the model yields some insights into patent law regulations. By choosing an optimal patent
regime and IP laws (e.g. trade secret laws) for a specific R&D industry, policymakers might maximise investment levels, firms’ payoffs, and, as a result, the quality of the end-products while making cyberespionage worthless. Those ideas will be investigated in more detail in future research.

4 Conclusion and discussion

In this paper, we explored the influence of industrial cyberespionage on the R&D incentives, the R&D race outcome, and social welfare. The study finds that cyberespionage power has an ambiguous effect on the R&D investments in the dynamic race. Espionage diminishes the competitive incentives in the initial research stage while increasing them in the development stage. Companies will be reluctant to invest in the research if their technology will most likely be stolen. On the contrary, espionage can serve as a balancing device in the development stage: by decreasing the gap between the leader and the underdog, providing the latter with a fighting chance. Therefore, by regulating the cyberespionage power via legal means, policymakers can maximise the overall R&D investments in the race.

Additionally, cyberespionage can positively influence competitors’ payoffs. By choosing an appropriate espionage regime, policymakers can make the R&D race more attractive to the participants and motivate competition. Moreover, cyberespionage can lead to higher quality innovation and produce social benefits. The social welfare function is defined as the absolute measure of the quality of the innovation delivered to the market by a winner of the race. Thus, imposing an appropriate espionage regime can sufficiently increase overall competitive incentives, positively influencing the expected quality of the innovation.
It is rather challenging to retrieve empirical evidence favouring the proposed theory due to the secrecy of the industrial espionage phenomenon. It would require a detailed data set that includes variables on individual companies’ R&D efforts (e.g. R&D expenses, research human capital, base stock of knowledge), R&D profit breakdowns, and the information on industrial espionage decisions, which is probably the most difficult to obtain.

However, it might be possible to study the dynamics of social welfare function expressed in economic growth terms (e.g. TFP or GDP in per capita terms) and research it against indices of industrial espionage. This would also require a data set containing information on the flows of the stolen sensitive data, which is not currently available.

The study also identified three possible vectors for future research. Firstly, it is worth exploring the link between the R&D process, innovation economics, and optimal patent design. The technological step $k$ might be alternatively interpreted as a patentability threshold, and the technological edge $h$ can be regarded as a function of patent length. The framework then can be easily modified to study the optimal patent design in the aggressive digital environments (Scotchmer, 2004).

Secondly, the framework can be extended by allowing firms to diversify investment into various technologies during the research stage. The product design is rarely built on a single technology and often incorporates multiple innovations that synergise together in a unique design. Thus, the expansion can lead to insights about optimal research budget allocation and strategic data organisation for enhanced cybersecurity.

Thirdly, the model can be extended with a more sophisticated cyberattack mechanism. Instead of using exogenously defined attack power, one could use a more sophisticated endogenously defined function that might capture the offensive and defensive efforts of
participants or affiliation mechanisms, and social planners’ investments into industrial espionage enforcement, in the fashion of Stowe and Gilpatric (2010).

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A Mathematical Appendix

A.1 Equilibrium derivation and analysis

Proof of Lemma 1. Assumption 1 and the fact that $c'(\cdot)$ is monotone guarantee the existence of a unique pure strategy equilibrium. It is easy to observe that in pure strategy equilibrium, it must be the case that $\hat{x}_i = \hat{x}_{-i}$ as FOCs (9) are symmetric for both players.

Given that $f(\hat{x}_i - \hat{x}_{-i})$ is a symmetric function with zero mean, both companies then choose:

$$\hat{x}_{is}^* = c^{-1}[f(0)].$$

SOCs (10) are always satisfied around $\hat{x}_{is}^*$ as $f'(0) = 0$ and $c'(\hat{x}_i) > 0$ for all $\hat{x}_i > 0$ by Assumption 1.

Notice that corner solutions $\hat{x}_i = 0$ cannot be an equilibrium since the density functions are assumed to have infinite support (are unbounded) and $c'(0) = 0$.

Proof of Lemma 2. The proof follows the same pattern as the proof for the symmetric case of the stage. Assumption 1 and the monotonicity of a cost function guarantee a single intersect from below of marginal benefits and marginal costs, so the equilibrium is unique. It follows from the symmetry of the FOCs that both companies will choose the same levels of investment:

$$\hat{x}_{ia}^* = c^{-1}[f(h(1 - \alpha))].$$
Corner solutions $\hat{x}_i = 0$ cannot be an equilibrium since density functions are assumed to have an infinite support (are unbounded) and $c'(0) = 0$.

Assumption 1 also guarantees that the set of SOCs is always satisfied:

$$f'(h(1 - \alpha)) < c''(\hat{x}_i^\alpha).$$  \hspace{1cm} (31)

**Proof of Proposition 2.** The sufficient condition for the proposition to be true is:

$$\frac{\partial}{\partial x} \left( F(x) + c \left[ c'^{-1} [f(x)] \right] \right) > 0,$$  \hspace{1cm} (32)

or:

$$-f(x) > \frac{f(x)f'(x)}{c''[c'^{-1} [f(x)]]}.$$  

Simplifying yields:

$$c''[c'^{-1} [f(x)]] > f'(x),$$  \hspace{1cm} (33)

which is always true via Assumption 1.

**Proof of Proposition 3.** Observe that companies are identical at the beginning of the race. Moreover, they have symmetric marginal benefits and identical monotone cost functions. Thus, it must be the case that both companies choose identical equilibrium strategies.

Substituting $x_i = x_{-i}$, (12), and (20) into (25) and simplifying yields:

$$x_i^* = c'^{-1} [f(k) (2F(h(1 - \alpha)) - 1)] .$$

In order to ensure the uniqueness of the equilibrium, it is sufficient to impose the following set of second-order conditions, which guarantees the single intersect of marginal benefits
and marginal costs:

\[ f'(x_i - x_i^* + k) (\Pi^S - \Pi^A_2) + f'(x_i - x_i^* - k) (\Pi^A_1 - \Pi^S) < c''(x_i). \]  \tag{34}  

Inequality (34) is satisfied if:

\[
\max_{x_i} f'(x_i - x_i^* + k) (\Pi^S - \Pi^A_2) + f'(x_i - x_i^* - k) (\Pi^A_1 - \Pi^S) < c''(x_i). \tag{35}
\]

Term \( t_1 \) of inequality (35) is always positive given that condition (32) of Proposition 2 always holds under Assumption 1:

\[ t_1 = F(h(1 - \alpha)) + c \left[ c^{-1} \left[ f(h(1 - \alpha)) \right] \right] - \frac{1}{2} - c \left[ c^{-1} [f(0)] \right] > 0. \]

Term \( t_2 \) of inequality (35) is always positive given the monotonicity of the cost function and the fact that the random variable \( \zeta \) has a single-peaked distribution with zero mean:

\[ t_2 = F(h(1 - \alpha)) - c \left[ c^{-1} \left[ f(h(1 - \alpha)) \right] \right] - \frac{1}{2} + c \left[ c^{-1} [f(0)] \right] > 0. \]

Observe now that inequality (35) always holds if:

\[
\max f'(\cdot) \underbrace{\left( \Pi^A_1 - \Pi^A_2 \right)}_{\rho} < \min c''(x_i), \tag{36}
\]

where \( \rho = 2F(h(1 - \alpha)) - 1. \)

It is easy to spot that the maximum value that \( \rho \) can attain is 1. It follows that (36) always holds whenever:

\[
\max f'(\cdot) < \min c''(x_i), \tag{37}
\]

which is precisely the form of Assumption 1.

It immediately follows that SOCs are automatically satisfied under Assumption 1.

\footnote{At this step, we assumed that both \( f'(x_i - x_i^* + k) \) and \( f'(x_i - x_i^* - k) \) attain their maximum possible values \(- \max f'(x_i - x_i^* + k) = \max f'(x_i - x_i^* - k) = f'(\cdot)\).}
A corner solution \((x_i^* = 0)\) cannot be an equilibrium since the density functions are assumed to have infinite support (are unbounded) and \(c'(0) = 0\). It is easy to verify by substituting \(x_i = x_{-i} = 0\) into the FOCs.

It is also trivial to demonstrate that a firm’s investment in the research stage decrease in an espionage power \(\alpha\):

\[
\frac{\partial x_i^*}{\partial \alpha} = -2hf(k)f(h(1 - \alpha)) (c'^{-1})' [f(k)(2F(h(1 - \alpha)) - 1)] < 0.
\]

Proof of Proposition 4. The overall investments are defined as the sum of expected investments in the research and the development stages:

\[
X \equiv 2x_i^* + 2P^S\dot{x}_i^s + 2P^A\dot{x}_i^a + 2P^A\dot{x}_i^a.
\]

Expanding the function and substituting \(\Delta x = 0\) yields:

\[
X = \frac{2}{c} (f(k)(2F(h(1 - \alpha)) + A - 1) + (F(k) - F(-k)) f(0) + 2F(-k)f(h(1 - \alpha)).
\]

Taking the first derivative of (39) w.r.t. to \(\alpha\) and setting it to zero yields:

\[
\frac{\partial X}{\partial \alpha} = \frac{2h}{c} \left( -f(k)f(h(1 - \alpha)) \left( \frac{2F(-k)f'(h(1 - \alpha))}{0} \right) \right) = 0.
\]

The derivative has an ambiguous sign. It is caused by the fact that espionage power has an opposite effect on investment levels in different stages: an increase in \(\alpha\) causes a decrease in optimal investment level during the research stage and an increase during the development stage.

Simplifying and rearranging (40) yields the following candidate solution:

\[
\alpha' = 1 - \frac{1}{h} \sqrt{\frac{2}{\pi \eta}} \exp \left( - \frac{k^2}{2\sigma^2} \right),
\]
where $\eta = \text{erfc} \left( \frac{k}{\sqrt{2} \sigma} \right)$. The candidate solution is bounded from above; however, it does not have a defined lower bond. Therefore, $\alpha^i$ might take negative values, which contradicts the initial assumption (that $\alpha \in [0, 1]$). Thus, the interior solution indeed exists only for sufficiently large $h$:

$$h > \sqrt{\frac{2 \sigma}{\pi \eta \pi}} \exp \left( -\frac{k^2}{2 \sigma^2} \right) \equiv \bar{h}.$$  

Otherwise, it is optimal to choose $a^i = 0$ to maximise the overall investment. Verifying the sign of the second-order condition around $a^i$ confirms that it is indeed a maximum point:

$$\frac{\partial^2 X}{\partial \alpha^2} = -\sqrt{\frac{2 \pi \eta \pi}{\pi \sigma^3}} \exp \left( -\frac{\exp \left( \frac{k^2}{\sigma^2} \right)}{\pi \eta^2} \right) < 0. \tag{42}$$

The solution assuming normally distributed shocks can then be summarised as follows:

$$\begin{cases} 
\alpha^I = \alpha^i, & \text{if } h > \sqrt{\frac{1}{2\pi \Phi(-k)}} \exp \left( -\frac{k^2}{2\sigma^2} \right), \\
\alpha^I = 0, & \text{otherwise,}
\end{cases}$$

where $\Phi$ is the CDF of the normal distribution with standard deviation $\sigma$ and zero mean.

\[\blacksquare\]

**Proof of Proposition 5.** Candidate solution (41) increases in $h$, however the influence of innovation threshold $k$ is not clear. Consider the following equation that was derived from (40):

$$\frac{f(k)}{F(-k)} = \frac{f'(h(1 - \alpha))}{f(h(1 - \alpha))}.$$  

Taking the derivative of the left-hand side of the equation w.r.t. $k$ yields:

$$\frac{\partial}{\partial k} \left( \frac{f(k)}{F(-k)} \right) = \frac{f'(k)F(-k) + f(k)^2}{F(-k)^2}. \tag{43}$$

I now make use of two facts:

$$f'(k) = -kf(k),$$

43
and
\[ \lambda(-k) = \frac{f(-k)}{1 - F(k)} \forall k. \]
which could be immediately recognised as the inverse Mill’s ratio. Simplifying (43) yields:
\[ \lambda(-k) (\lambda(-k) - k) > 0, \tag{44} \]
which is positive for \( \forall k > 0 \) given that normal distribution is symmetric around zero and the fact the inverse Mill’s ratio is the expectation of the truncated normal:
\[ E[\zeta|\zeta > k] = \frac{f(k)}{1 - F(k)} = \frac{f(-k)}{F(-k)} = \lambda(-k) > k. \]

Consider now the derivative of the right-hand side of equation (43) w.r.t. \( \alpha \):
\[ \frac{\partial}{\partial a} \left( f'(h(1-\alpha)) \right) = (\alpha - 1)h < 0. \tag{45} \]
Combining results (44) and (45) it is evident that there is an inverse relationship between innovation threshold \( k \) and optimal attack power \( \frac{\partial \alpha^*}{\partial k} < 0. \)

**Proof of Lemma 3.** Consider \( \Upsilon \) that is a simplified version of \( \Pi_i (28) \), which includes only members that are the functions of \( \alpha \):
\[ \Upsilon \equiv -f(h(1-\alpha))^2 + \kappa F(h(1-\alpha)) F(-h(1-\alpha)), \tag{46} \]
where \( \kappa = \frac{2f(k)^2}{F(-k)}. \) Taking the first derivative and setting it to zero yields:
\[ \frac{\partial \Pi_i}{\partial \alpha} = \frac{\partial \Upsilon}{\partial \alpha} = hf(h(1-\alpha)) (2f'(h(1-\alpha)) + \kappa(1 - 2F(-h(1-\alpha))) = 0. \]
It is easy to spot that \( \alpha = 1 \) is a candidate maximum point. It is now necessary to verify a second-order condition:
\[ \frac{\partial^2 \Upsilon}{\partial \alpha^2} = -h^2 f'(h(1-\alpha)) (2f'(h(1-\alpha)) + \kappa(1 - 2F(-h(1-\alpha))) + \\
+ hf(h(1-\alpha)) (-2f''(h(1-\alpha)) - 2\kappa f(h(1-\alpha))) < 0, \]
which simplifies around $\alpha = 1$ to:

$$\frac{\partial^2 \Upsilon}{\partial \alpha^2} \bigg|_{\alpha=1} = 2hf(0) (-f''(0) - \kappa f(0)) < 0. \quad (47)$$

It must then be true that:

$$2f(k)^2 > -\frac{f''(0)}{f(0)} F(-k). \quad (48)$$

Rearranging (48) yields:

$$2f(k)^2 - \frac{f''(0)}{f(0)} F(k) > -\frac{f''(0)}{f(0)}. \quad (49)$$

Given that $-\frac{f''(0)}{f(0)} = \frac{1}{\sigma^2}$ inequality (49) becomes:

$$K \equiv 2\sigma^2 f(k)^2 - F(k) > 1. \quad (50)$$

Consider now the first derivative of $K$ w.r.t. $k$:

$$\frac{\partial K}{\partial k} = f(k) \left(4\sigma^2 f'(k) - 1\right), \quad (51)$$

which is always negative for any $k$ in $\mathbb{R}^+$. As $k \geq 0$, it must be the case that $K$ attains its maximum at $k = 0$. Evaluating (50) at $k = 0$ yields:

$$f(0)^2 > \frac{1}{4\sigma^2},$$

or

$$\frac{1}{2\pi\sigma^2} > \frac{1}{4\sigma^2},$$

which never holds. Therefore, $\alpha^* = 1$ is always a local minimum point. $\blacksquare$

**Proof of Proposition 6.** Consider $\Upsilon$ that is a simplified version of $\Pi_i$ (28), which includes only terms that are the functions of $\alpha$:

$$\Upsilon \equiv -f \left(h(1-\alpha))^2 + \kappa F \left(h(1-\alpha)) F \left(-h(1-a)\right), \quad (52)$$
where $\kappa = \frac{2f(k)^2}{F(-k)}$. Substituting $H = h(1 - \alpha)$ yields:

$$\Upsilon = -f(H)^2 + \kappa F(H)F(-H).$$

(53)

The first-order condition is then:

$$\frac{\partial \Upsilon}{\partial H} = f(H)\left(\kappa F(-H) - \kappa F(H) - 2f'(H)\right) = 0,$$

(54)

It is evident that $H = 0$ is a candidate solution for the FOC. However, it follows from Lemma 3 that $H = 0$ is a point of local minimum. Observe now that:

$$\left.\frac{\partial \Upsilon}{\partial H}\right|_{H \to \infty} = -\kappa f(H)F(H) < 0.$$

(55)

Combining equation (55) and results from Lemma 3 it should be evident that the function $\Upsilon'(H)$ should intersect axis of abscissas at some point $H^* \in (0, \infty)$. Moreover, the dynamics of the function suggest that $H^*$ should be indeed the local maximum point.

Given that $H$ is a linear combination of $h$ and $\alpha$ it follows that:

$$\begin{cases} 
\alpha^* = \frac{h - H^*}{H^*}, & \text{if } h > H^*, \\
\alpha^* = 0, & \text{if } h \leq H^*. 
\end{cases}$$

It is now sufficient to show that $H^*$ always exists. By rearranging (54), observe that the FOC is satisfied whenever:

$$\frac{f'(H)}{1 - 2F(H)} = \frac{\kappa}{2q(H)}.$$

(56)

It is easy to spot that function $q(H)$ obtains its maximum at $H = 0$ as $\max f'(H) = f'(0) = 0$ and $1 - 2F(H)$ obtains its minimum positive value when $H = 0$. I now demonstrate that this condition is always satisfied as

$$\max_H q(H) = q(0) = \frac{1}{\sigma^2} > \kappa, \forall H \& \forall k,$$

46
which follows from Lemma 3. Consider also the sign of:

\[
q'(H) = -\frac{\exp\left(-\frac{H^2}{\sigma^2}\right) \left(-2H\sigma - \exp\left(-\frac{H^2}{2\sigma^2}\right) (H^2 - \sigma^2) \text{erf}\left(\frac{H}{\sqrt{2}\sigma}\right)\right)}{2\pi\sigma^5 \text{erf}\left(\frac{H}{\sqrt{2}}\right)}.
\] (57)

Simplifying (57) yields:

\[
q'(H) \equiv -2H\sigma - \exp\left(-\frac{H^2}{2\sigma^2}\right) (H^2 - \sigma^2) \text{erf}\left(\frac{H}{\sqrt{2}\sigma}\right) < 0,
\] (58)

which is always negative given that:

\[
\max_H q(H) = \lim_{H \to 0} \left(-2H\sigma - \exp\left(-\frac{H^2}{2\sigma^2}\right) (H^2 - \sigma^2) \text{erf}\left(-\frac{H}{\sqrt{2}\sigma}\right)\right) = 0.
\] (59)

Therefore, \(q'(H) < 0\) and \(q(H)\) is a monotone strictly decreasing function of \(H\). Given that the right-hand side of (56) is a constant, there must be a unique value \(H^*\) that satisfies the equation, and \(H^*\) must be a global maximum. 

\[\Box\]

**Proof of Proposition 7.** Rearranging (56) yields:

\[
Q \equiv F(-k)q(H) - f(k)^2 = 0.
\]

I now implicitly differentiate \(Q\) to obtain:

\[
\frac{\partial H^*}{\partial k} = -\frac{\frac{\partial Q}{\partial k}}{\frac{\partial Q}{\partial H} q'(H) F(-k)}.
\] (60)

It follows from Proposition 6 that the denominator of the equation is negative. Therefore, the sign of (60) depends solely on the sign of the numerator, which, however, is not obvious. We consider it separately:

\[
\frac{\partial Q}{\partial k} = f(k) (-q(H) - 2f'(k)).
\]

The sign of the equation above depends on the value of \(q(H)\). Consider two possibilities:

1. if \(H^*\) is sufficiently small and \(q(H^*) \in \left(\frac{\sqrt{2}}{\sqrt{\pi\sigma^2}}, \frac{1}{2\sigma^2}\right)\) then \(Q'(k) < 0\);
2. if $H^*$ is sufficiently large and $q(H^*) \in \left(0, \frac{\sqrt{2}}{\sqrt{\pi a^2}}\right)$ then given the shape of $-2f'(k)$ it intersects $q(H^*)$ twice at $k_1^*$ and $k_2^*$ with $k_1^* < k_2^*$. It then gives three intervals: (1) $Q'(k) < 0$ for any $k \in (0, k_1^*)$, (2) $Q'(k) > 0$ for any $k \in (k_1^*, k_2^*)$, and (3) $Q'(k) < 0$ for any $k \in (k_2^*, \infty)$.

As mentioned before, the sign of $\frac{\partial H^*}{\partial k}$ follows the same pattern. It is now straightforward to translate the result into espionage power terms. Observe that $\frac{\partial H^*}{\partial k} = 0$ if $h < H^*$.

Therefore, consider only the cases with $h > H^*$:

1. if $H^*$ is sufficiently small then $\frac{\partial \alpha^*}{\partial k} > 0$;

2. if $H^*$ is sufficiently large then (1) $\frac{\partial \alpha^*}{\partial k} < 0$ if $k \in (k_1^*, k_1^*)$, (2) $\frac{\partial \alpha^*}{\partial k} > 0$ for any $k \in (0, k_1^*) \cup (k_2^*, \infty)$.

A.2 Social welfare function derivation and analysis

A.2.1 Derivation of social welfare function

SWF is derived as an overall output exerted by the winner of the game minus the overall expected costs exerted by both competitors during the game. Due to the complexity of the function, the derivation is divided into five parts and then combined: (1) overall effort exerted by the winner of the game, (2) contribution of the symmetric case shocks to the overall expected shock, (3) contribution of asymmetric case shocks when a leader wins, (4) contribution of asymmetric case shocks when an underdog wins, and (5) overall expected costs.

1. Overall effort Both players have similar expected investment levels at the beginning of the race. Moreover, the winner will choose the same level of investment as the loser.
Expected investments of the winner are then expressed as:

\[ X_i \equiv x_i^* + P^S \hat{x}_i^* + P^A_1 \hat{x}_i^* + P^A_2 \hat{x}_i^* . \]

or expanding:

\[ X_i = f(-k)(2F(H) - 1) + (F(k) - F(-k))f(0) + 2F(-k)f(H). \] (61)

2. Contribution of symmetric case shocks

The contribution of winning firm shocks to the overall expected shock in case the winner is decided in the symmetric case of the development stage is:

\[ E^S = P^S (E(\epsilon_i | - k < \epsilon_i - \epsilon_{-i} < k) + E(\hat{\epsilon}_i | \hat{\epsilon}_i > \hat{\epsilon}_{-i})) = \]

\[ = P^S \left( \frac{\int_{-\infty}^{\epsilon_{-i} + k} \int_{\epsilon_{-i} + k}^{\epsilon_i} f(\epsilon_i) f(\epsilon_{-i}) d\epsilon_i d\epsilon_{-i}}{F(k) - F(-k)} + 2 \int_{-\infty}^{\infty} \int_{\epsilon_{-i}}^{\infty} \hat{\epsilon}_i f(\hat{\epsilon}_i) f(\hat{\epsilon}_{-i}) d\hat{\epsilon}_i d\hat{\epsilon}_{-i} \right) . \]

Substituting \( P^S = F(k) - F(-k) \) into the equation yields:

\[ E^S = 2(F(k) - F(-k)) \int_{-\infty}^{\infty} \int_{\epsilon_{-i}}^{\infty} \hat{\epsilon}_i f(\hat{\epsilon}_i) f(\hat{\epsilon}_{-i}) d\hat{\epsilon}_i d\hat{\epsilon}_{-i} . \] (62)

Asymmetric case shocks contributions

The asymmetric development stage occurs with the probability \( P^A = 2P^A_1 = 2P^A_2 \). There are two sub-cases to consider in the asymmetric development stage: (3) the leader wins, or (4) the underdog wins.

3. Leader wins

Contribution of the asymmetric case shocks to the overall shock if an underdog wins:

\[ E^L = P^A \hat{P}^a_1 (E(\epsilon_i | \epsilon_i > \epsilon_{-i} + k) + E(\hat{\epsilon}_1 | \hat{\epsilon}_1 > \hat{\epsilon}_2 - H)) = \]

\[ = P^A \hat{P}^a_1 \left( \frac{\int_{-\infty}^{\epsilon_{-i} + k} \int_{\epsilon_{-i} + k}^{\epsilon_i} f(\epsilon_i) f(\epsilon_{-i}) d\epsilon_i d\epsilon_{-i}}{F(-k)} + \frac{\int_{-\infty}^{\infty} \int_{\epsilon_2 - H}^{\epsilon_{-i} + k} \hat{\epsilon}_1 f(\hat{\epsilon}_1) f(\hat{\epsilon}_2) d\hat{\epsilon}_1 d\hat{\epsilon}_2}{F(H)} \right) . \] (63)
Substituting $P^A = 2F(-k)$ and $\hat{P}^a_1 = F(H)$ in (63) and simplifying yields:

$$E^L = 2F(H) \int_{-\infty}^{\infty} \int_{\epsilon_i+k}^{\infty} \epsilon_i f(\epsilon_i) f(\epsilon_{-i}) d\epsilon_i d\epsilon_{-i} +$$

$$+ 2F(-k) \int_{-\infty}^{\infty} \int_{\epsilon_2-H}^{\infty} \hat{\epsilon}_1 f(\hat{\epsilon}_1) f(\hat{\epsilon}_2) d\hat{\epsilon}_1 d\hat{\epsilon}_2.$$  (64)

4. **Underdog wins**  Similarly, the contribution of shocks of underdog-winner of the race to the overall expected shock is as follows:

$$E^U = P^A \hat{P}^a_2 \left( E(\epsilon_{-i} | \epsilon_{-i} < \epsilon_i - k) + E(\hat{\epsilon}_2 | \hat{\epsilon}_2 > \hat{\epsilon}_1 + H) \right) =$$

$$= P^A \hat{P}^a_2 \left( \frac{\int_{-\infty}^{\epsilon_i-k} \int_{-\infty}^{\epsilon_{-i}} \epsilon_{-i} f(\epsilon_{-i}) f(\epsilon_i) d\epsilon_{-i} d\epsilon_i}{F(-k)} + 
\frac{\int_{-\infty}^{\infty} \int_{\hat{\epsilon}_2 + H}^{\infty} \hat{\epsilon}_2 f(\hat{\epsilon}_2) f(\hat{\epsilon}_1) d\hat{\epsilon}_2 d\hat{\epsilon}_1}{F(-H)} \right),$$  (65)

where $\hat{P}^a_2 = 1 - \hat{P}^a_1 = F(-H)$ is the probability an underdog wins in the asymmetric development stage: Expanding (65) and simplifying yields:

$$E^U = 2F(-H) \int_{-\infty}^{\infty} \int_{-\infty}^{\epsilon_i-k} \epsilon_{-i} f(\epsilon_{-i}) f(\epsilon_i) d\epsilon_{-i} d\epsilon_i +$$

$$+ 2F(-k) \int_{-\infty}^{\infty} \int_{\hat{\epsilon}_1 + H}^{\infty} \hat{\epsilon}_2 f(\hat{\epsilon}_2) f(\hat{\epsilon}_1) d\hat{\epsilon}_2 d\hat{\epsilon}_1.$$  (66)

5. **Overall expected costs**  Overall expected costs of the R&D race can be expressed as:

$$C \equiv c(x^*_i) + P^S c(\hat{x}^*_i) + P^A_1 c(\hat{x}^*_i) + P^A_2 c(\hat{x}^*_i).$$  (67)
A.2.2 Social Welfare Function

The expected quality social welfare function can be derived by combining (61), (62), (64), (66), and (67):

\[
W = f(-k)(2F(H) - 1) - f(-k)^2(2F(H) - 1) + \\
+ (F(k) - F(-k))(f(0) - f(0)^2) \\
+ 2F(-k)(f(H) - f(H)^2) \\
+ 2(F(k) - F(-k)) \int_{-\infty}^{\infty} \int_{\hat{\epsilon}_{-i}}^{\hat{\epsilon}_{i}} \hat{\epsilon}_i f(\hat{\epsilon}_i)f(\hat{\epsilon}_{-i})d\hat{\epsilon}_id\hat{\epsilon}_{-i} + \\
+ 2F(H) \int_{-\infty}^{\infty} \int_{\epsilon_{-i}+k}^{\epsilon_{-i}} \epsilon_i f(\epsilon_i)f(\epsilon_{-i})d\epsilon_i d\epsilon_{-i} + \\
+ 2F(-k) \int_{-\infty}^{\infty} \int_{\hat{\epsilon}_{2-H}}^{\hat{\epsilon}_{2}} \hat{\epsilon}_2 f(\hat{\epsilon}_1)f(\hat{\epsilon}_2)d\hat{\epsilon}_2 d\hat{\epsilon}_1 + \\
+ 2F(-H) \int_{-\infty}^{\infty} \int_{\epsilon_{-i}+H}^{\epsilon_{-i}} \epsilon_{-i} f(\epsilon_{-i})f(\epsilon_i)de_{-i}de_i + \\
+ 2F(-k) \int_{-\infty}^{\infty} \int_{\hat{\epsilon}_{1}+H}^{\hat{\epsilon}_{1}} \hat{\epsilon}_1 f(\hat{\epsilon}_2)f(\hat{\epsilon}_1)d\hat{\epsilon}_1 d\hat{\epsilon}_2.
\]

(68)

Proof of Proposition 8. It is sufficient to study the properties of the simplified function, which consist only of the members of SWF that are functions of attack power \(\alpha\):

\[
\psi = F(H)(2g(k) + f(k) + F(-H)f(k)^2) + \\
+ F(-k)(2i(H) + f(H) - f(H)^2)
\]

(69)

Consider the FOC of the function (69) w.r.t. \(H\):

\[
\frac{\partial \psi}{\partial H} = f(H)(2(g(k) - F(-k))f'(H)) + f(-k)^2(1 - 2F(H)) + f(-k) + \\
+ F(-k)(2i'(H) + f'(H)) = 0,
\]

(70)

Studying the FOC around \(H = 0\) yields:

\[
\left. \frac{\partial \psi}{\partial H} \right|_{H=0} = (f(-k) + 2g(k))f(0) > 0.
\]
Therefore, the function is always positive around $H = 0$. Therefore, $H = 0$ is neither a maximum nor a minimum point. It is also evident that:

$$\frac{\partial \psi}{\partial H} \bigg|_{H \to \infty} = 0.$$  

Now I consider the sign of the $\frac{\partial \psi}{\partial H}$ as it approaches 0. Expanding FOC’s terms and simplifying yields:

$$\frac{\partial \psi}{\partial H} = \exp \left( -\frac{H^2 + k^2}{\sigma^2} \right) \exp \left( \frac{H^2}{2\sigma^2} \right) \left( \kappa^1 - 2\sqrt{2}\sigma \text{erf} \left( \frac{H}{\sqrt{2}\sigma} \right) \right) +$$

$$+ H\sqrt{\pi} \text{erfc} \left( \frac{k}{\sqrt{2}\sigma} \right) \exp \left( \frac{k^2}{\sigma^2} \right) \left( 4\exp \left( -\frac{H^2}{2\sigma^2} \right) - 2\sqrt{2}\pi\sigma - 2\exp \left( \frac{H^2}{4\sigma^2} \right) \sqrt{\pi}\sigma^3 \right).$$  

where:

$$\kappa^1 = 4\exp \left( \frac{k^2}{2\sigma^2} \right) \sqrt{\pi}\sigma^2 \left( 1 + \sqrt{2} \exp \left( \frac{k^2}{4\sigma^2} \right) \right).$$

The overall sign of the FOC is negative as $Q^1|_{H \to \infty} = 0^+$ and $Q^2|_{H \to \infty} = -\infty$:

$$\frac{\partial \psi}{\partial H} \bigg|_{H \to \infty} < 0.$$  

Therefore, given the function is smooth and continuous, there will always be a value of $H^W$ between 0 and $\infty$ that maximises the social welfare function. Therefore, there exists a value of $a^W = \frac{h - H^W}{h}$ that maximises the social welfare function for any $h > H^W$. ■