Lateral Thermal Buckling of Pipelines

Dissertation submitted to the University of Cambridge for the degree of Doctor of Philosophy

by

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To my parents,
with thanks for all their support.
Acknowledgements

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Declaration

The author declares that, except for commonly understood ideas and concepts, or where specific reference is made to the work of other authors, the contents of this dissertation are original and include nothing that is the outcome of work done in collaboration. The research described in this dissertation has been carried out in the Department of Engineering at the University of Cambridge. This dissertation has not previously been submitted, in part or in whole, to any other university or institution for any degree, diploma, or other qualification. This dissertation is presented in 136 pages and contains approximately 40000 words including references and appendices.

David Miles
Abstract

Subsea oil and gas exploration is increasingly moving into deeper water, where trenching of a pipeline for protection and to mitigate against upheaval buckling becomes increasingly impractical. In addition, the exploration of new reservoirs at higher temperatures and pressures than before leaves a submarine pipeline on the seabed more susceptible to lateral thermal buckling.

A novel small-scale compressible base model, with an expanded polystyrene base compressed beneath a silicone rubber strip, has been developed to represent the constrained thermal loading of a pipeline lying on the seabed. This physical model is used, in addition to a nonlinear finite-element analysis, for a case study of a real buckled pipeline. Dimensional analysis is used to provide a means of comparing the post-buckled behaviour of the model strip with that of the full-size pipeline. There is good agreement between the results of the post-buckled behaviour for the physical and finite-element models, and these results compare well with the survey data for the buckled real pipeline. General results from the physical model are also presented for strips with differing geometric and material properties, laid both straight and on a scaled lay-away curve.

A useful measure of the evolution of a buckle, the free end displacement, is introduced. This is the axial displacement of the free end of a cut pipe, constrained to remain straight while undergoing thermal loading. This measure used in a study of the parameters which affect the far-post-buckling behaviour of a beam on a frictional foundation. The phenomenon of buckle lobe extinction, when a buckle lobe stops growing, is discovered for certain combinations of beam bending stiffness, axial friction coefficient and lateral friction coefficient. When the buckle length, buckle amplitude and free end displacement are formed into non-dimensional groups with these three parameters, curves for many parameter combinations are found to fall onto a single curve. The conditions for buckle lobe extinction, in terms of these dimensionless groups, may be determined directly from this universal curve.

Finally, the closely-related problem of the stability of a pipeline being built up a slope is investigated. A case study is made of a real pipeline, incorporating numerical and physical models and also a simplified analytical model. These models correlate well with each other, and enable the conditions for collapse of the real pipeline to be predicted.

Keywords: buckle lobe extinction, dimensional analysis, far-post-buckling behaviour, finite-element model, free end displacement, lateral thermal buckling, lay-away curve, small-scale compressible base model, submarine pipeline, stability on slope, thermal loading.
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Nomenclature

The final column denotes the dimension of each physical entry: 
$F' =$ force, $L =$ length, $\Theta =$ temperature, and $[-] =$ dimensionless.

All quantities refer to a general beam except if they are specific to the silicone rubber strip from the experiments, or to a full-size pipeline.

<table>
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<tr>
<th>Alphabetical</th>
<th>Dimension</th>
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<td>$a$</td>
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</tr>
<tr>
<td>$b$</td>
<td>width of silicone rubber strip</td>
</tr>
<tr>
<td>$C$</td>
<td>a constant</td>
</tr>
<tr>
<td>$d$</td>
<td>depth of silicone rubber strip</td>
</tr>
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<td>$D$</td>
<td>nominal diameter of pipeline</td>
</tr>
<tr>
<td>$D'$</td>
<td>dimensionless group</td>
</tr>
<tr>
<td>$E$</td>
<td>Elastic modulus</td>
</tr>
<tr>
<td>$(EA)$</td>
<td>axial stiffness of beam</td>
</tr>
<tr>
<td>$(EI)$</td>
<td>bending stiffness of beam</td>
</tr>
<tr>
<td>$f$</td>
<td>frictional force per unit length of beam</td>
</tr>
<tr>
<td>$F$</td>
<td>concentrated lateral force</td>
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<tr>
<td>$k$</td>
<td>foundation stiffness for the elastic foundation approach</td>
</tr>
<tr>
<td>$L$</td>
<td>length of finite beam</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure excess between inside and outside of pipeline</td>
</tr>
<tr>
<td>$P$</td>
<td>axial force in buckled region of beam</td>
</tr>
<tr>
<td>$r$</td>
<td>radius of steel reinforcing wire for silicone rubber strip</td>
</tr>
<tr>
<td>$R$</td>
<td>radius of curved layout of beam</td>
</tr>
<tr>
<td>$s$</td>
<td>coordinate along the centreline of beam</td>
</tr>
<tr>
<td>$t$</td>
<td>thickness of pipeline</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature for thermal loading model</td>
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<tr>
<td>$u$</td>
<td>displacement of a point, relative to the base, in the $x$-direction</td>
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\( v \) displacement of a point, relative to the base, in the \( y \)-direction [\( L \)]

\( w \) weight per unit length of beam (taking account of additional distributed loading and upthrust when submerged) [\( FL^{-1} \)]

\( x, y \) Cartesian coordinates of a point on a beam; or

\( \text{transformed} \) Cartesian coordinates of a point on the silicone rubber strip [\( L \)]

\( X, Y \) \textit{global} coordinate of a point on the silicone rubber strip, measured by a digitiser [\( L \)]

**Greek**

\( \alpha \) coefficient of linear thermal expansion [\( \Theta^{-1} \)]

\( \delta \) end shortening of compressible base model [\( L \)]

\( \Delta \) prefix indicating a change in value of variable

\( \varepsilon \) strain [-]

\( \theta \) slope angle [-]

\( \lambda \) length of buckled lobe [\( L \)]

\( \mu \) general coefficient of friction [-]

\( \nu \) Poisson’s ratio of beam or pipe material [-]

\( \xi \) “elastic slip” of beam on frictional foundation [\( L \)]

**subscripts**

\( a \) in the local axial direction

\( b \) buckling

\( c \) fully constrained

\( e \) extinct

\( el \) elastic foundation

\( eff \) effective

\( E \) Euler

\( fm \) fully mobilised

\( i \) initial value

\( l \) in the local lateral direction

\( r \) silicone rubber

\( T \) thermal
in the vertical plane
steel wire
general numerical subscripts
Chapter 1

Introduction

This dissertation describes an investigation into a technique of modelling a pipeline in the laboratory, specifically the lateral, or horizontal, thermal buckling of a submarine pipeline lying on the seabed, due to increasing temperature or pressure in the pipeline.

1.1 The Use of Untrenched Pipelines

In the oil and gas exploration and production industry, the most common transportation method of products from source to market-place, sometimes hundreds of kilometres distant, has been the steel pipeline. The fact that much offshore production has traditionally come from relatively shallow water (for example the North Sea) means that these pipelines have been susceptible to damage from shipping in the form of anchor and lost cargo impact, or trawling. This problem may be avoided by burying the pipeline in a trench in the seabed. However, this is an expensive operation, increasingly so as the depth and length of the pipeline increase.

With the limited lifespan of many production fields coming to an end, exploration of high pressure / high temperature (HPHT) fields is becoming increasingly common. These HPHT fields, and also the deep-water fields such as West of Shetland, require thicker (or sometimes insulated, double-walled) pipes to withstand the more extreme operating conditions that would be encountered; hence the pipeline would not be so prone to damage. In some cases it is therefore possible to forego trenching and lay the pipeline directly on the seabed. However, this can lead to a problem which does not occur with trenched and covered pipelines, namely lateral buckling.
1.2 Buckling of Untrenched Pipelines

A section of a pipeline is effectively restrained by the long lengths of pipe laid on either end of it. As hot oil is pumped at pressure through the pipe, this restraint leads to compressive stresses being set up in the pipe. The problem is especially bad at the upstream end of the pipeline, where the oil is at its hottest. These compressive stresses can eventually lead to a buckling instability of the pipeline.

The most important difference between the untrenched and trenched cases is the very much smaller lateral restraint of an untrenched pipeline. The trenched pipeline can rely on the mass of earth at the side of the trench to provide this support, while for the untrenched pipe restraint is provided mostly by lateral friction between the pipe and the seabed. Some additional restraint can be provided if the pipe has to move up a sloping seabed, and also by the fact that over a period of time the pipe embeds itself slightly into the material on the sea floor. Generally, for friction between the pipe and seabed, the lateral coefficient of friction $\mu < 1$. This means that the restraint for upheaval buckling (due to submerged weight) is greater than the restraint for lateral buckling (due to $\mu$ times the submerged weight), and the pipe will tend to buckle laterally on the seabed. The tendency to buckle laterally is exacerbated if the pipeline is initially laid with a significant out-of-straightness. In this dissertation, such initial out-of-straightness is described by the term “initial imperfection.”

In order to ease the restraint and so allow the pipe to expand longitudinally, expansion spools have traditionally been used in shallower water. However, these spools require manual intervention for their construction; this becomes increasingly unattractive in deeper water. An alternative to the expansion spool is the lay-away expansion curve. This is a large-radius bend in the pipe (occasionally an S-bend) located at the end of the pipeline, which allows the pipe to move radially outwards in order to moderate compressive stress levels.

It is worth noting that an untrenched pipeline will not be just a steel cylinder lying on the seabed; it will have additional layers around the steel. Some of these layers, such as a concrete weighting/armour layer will, as well as increasing the submerged weight of the pipeline, increase somewhat the bending and axial stiffnesses of the pipeline. However, only the weight effect of such armour has been taken into account in the analyses to be presented here.

1.3 Objectives of the Dissertation

The work presented in this dissertation concerns the phenomenon of lateral buckling; but it does not attempt to deal in detail with buckle initiation, which
is sensitive to initial conditions. The buckling of pipelines in service is dictated by complex geometrical, loading and foundation conditions which cannot readily be ascertained. This means that although the various expressions put forward so far by other workers for critical buckling loads give useful reference values, the actual behaviour of the pipeline can be significantly different from what has been assumed in the various analyses. The pipeline operators usually only know that buckling has occurred by surveying the line after the event. The initial conditions will probably not be accurately known, and the conditions for which an analysis is performed may be significantly different from those at which buckling first occurred.

The main objectives of the work described here are:

i) to demonstrate that the key characteristics of a buckled pipeline, such as buckle amplitude and wavelength, can be predicted from a physical model;

ii) to study the post- and far-post-buckled behaviour of model beams in an attempt to provide a general, simplified characterization of buckle evolution; and

iii) to use this new characterization to predict a worst-case ultimate state for a buckled pipeline.

By taking this less theoretical view, and assuming that the ill-defined nature of the seabed and initial layout will never allow the possibility of lateral buckling being “designed out” altogether, then it should be possible to provide confidence that a given buckled pipeline is still fit for its primary purpose of fluid transport, and that it will never reach a point at which the possibility of failure becomes an issue.

1.4 Outline of the Dissertation

This dissertation is divided into seven chapters. Chapter 1 provides a brief overview of the problem of lateral buckling of submarine pipelines, and the motivation behind the work to be presented.

The published literature is reviewed in Chapter 2. Although the review is not exhaustive, it gathers together key ideas in the field of beam instability. This includes upheaval, or vertical, buckling in addition to lateral buckling of both pipelines and railway tracks.

Chapter 3 describes a novel physical model used to perform pseudo-thermal buckling experiments in the laboratory. The measurement of the material properties of the model and methods of data analysis are discussed, as well as the
concept of dimensional analysis as a method of comparing the behaviour of the model strip with that of a full-size beam, or pipeline.

Methods of analysing the problem of thermally-induced buckling, by means of finite-element analysis, are presented in Chapter 4. These methods are compared, and used to study the effects of various parameter changes. A useful measure of the evolution of a buckle, the free end displacement, is introduced.

Chapter 5 presents the results of a case study of a buckled pipeline, including a comparison of the physical and numerical modelling of the post-buckled behaviour. This is then extended to far-post-buckled behaviour, where the phenomenon of *buckle lobe extinction* is discovered. Finally, general results from the physical model are presented for straight model strips with differing geometries and material properties, and also for strips laid on a curve.

Chapter 6 is a self-contained case study of a different pipeline instability problem, namely the collapse of a pipeline being built up an uneven slope. This study, based on a real, practical problem, required urgent analysis in an attempt to understand the underlying causes of the collapse. A simple physical model is presented, and the results obtained from this are compared with a simple analytical model. From this comparison, a possible cause of the pipeline collapse is surmised.

Finally, Chapter 7 provides a short summary with conclusions on the work presented, along with a recommendation for future work in this area.
Chapter 2

Literature Review

2.1 Introduction

The literature on the buckling of submarine pipelines covers three distinct types of buckling behaviour. Perhaps the most widely covered of the three is a shell-type buckling of the pipe cross-section — see, for example, Kamalarasa & Calladine (1988). It is possible for this type of buckle, once it has been initiated, to propagate along the pipe under the action of external pressure; this can lead to rupture of the pipe. The propagating failure has high safety and economic costs, reflected in the depth of study by researchers. Mitigating measures such as local stiffening in the form of buckle arresters (Park & Kyriakides 1997) are now fairly commonplace.

Looking now to beam-type buckling, rather than the buckling of a cylindrical shell, the main area of focus has been upheaval buckling of a pipeline trenched into the seabed, on account of constrained thermal expansion. The upheaval can result in a portion of the pipe pushing up through the seabed surface, where it is no longer protected, and is thus at risk of serious damage from trawler gear and dropped objects. These impacts may also lead to eventual rupture of the pipe.

The study of lateral buckling, at least for pipelines, has remained a little more limited. This is possibly because lateral buckling is only a problem when the pipe is lying on top of the seabed, with the decision not to trench it suggesting that the pipe has been designed to allow for possible impact, or that the impact dangers are not present. Although oil companies remain secretive on the subject of lateral buckling (which is, after all, a failure of the original pipe design,) it is seen as a nuisance rather than an imminent danger. Even if the pipeline material approaches its yield stress somewhere along its length during buckling, the line may still be fit for its purpose of safely transporting oil or gas. This is in contrast with the associated problem of lateral buckling of railway lines, where even a
small buckle may cause the derailment of a train. Consequently, railway lines have received more attention from researchers.

The analysis of lateral buckling falls broadly into two distinct models; the elastic-base model and the rigid-base model. These are studied in more depth below. There is also a significant pool of experimental and survey data on large-scale pipe-seabed interaction, which provides information on material and frictional characteristics of pipelines lying on the seabed.

In the following literature review, some of the notation has been changed from the original in order to provide consistency between analyses.

### 2.2 Elastic Foundation Model

#### 2.2.1 Beam on a Linear Elastic Foundation

The analysis of a beam on an elastic foundation is a preliminary to either lateral or upheaval buckling. Hetényi (1946), like others before*, considered the general solution of an initially straight elastic beam with lateral bending stiffness $EI$ under an axial load $P$. The beam rests on an elastic foundation of lateral stiffness $k$ per unit length.

![Beam on elastic foundation](image)

**Figure 2.1:** Beam on elastic foundation.

The governing equation for an infinitely long beam under these conditions (shown in Figure 2.1 with a lateral point load $P$) is, for small deflections,

$$EI \frac{d^4 y}{dx^4} + P \frac{d^2 y}{dx^2} + ky = 0,$$

(2.1)

*See Kerr (1975) for a more extensive review of earlier work.*
the solution to which is a product of exponential and sinusoidal functions of $x$. In the absence of the lateral point load $F$, which is not present in Equation (2.1) but comes in via boundary conditions, the critical (eigenvalue) buckling load is

$$P_{cl} = 2\sqrt{kEI},$$

(2.2)

and the half-wavelength (or lobe length) of the non-decaying sinusoidal eigenmode is

$$\lambda = \pi \sqrt{\frac{EI}{k}}.$$  

(2.3)

The main limitation of this kind of model for real pipeline buckling is the implicit linear-elastic foundation. A real foundation only exhibits such behaviour over very small displacements before changing quite markedly, typically reaching a frictional force plateau. Furthermore, this model, and Equation (2.1), do not take account of initial imperfections in the line of the beam, which lower the critical buckling load. The model also predicts that the beam will take up a periodic post-buckled geometry, which is not observed in buckled pipelines or railway tracks. Kerr (1974) reviewed and compared different elastic models for upheaval buckling, and concluded that the assumption of continuous elastic support during buckling is not admissible.

## 2.3 Rigid Base Model

### 2.3.1 Martinet’s Railway Track Model

Martinet (1936) was among the first researchers to examine both lateral and upheaval buckling of railway tracks on a rigid base. Instead of using elastic lateral and axial foundation forces, which depend on displacements, Martinet considered distributed friction forces in the lateral and axial directions. This is basic Coulomb friction where the friction forces are fully mobilized for all deflections, shown in Figure 2.2 as the full line.

For this analysis, it is assumed that a buckle has already formed; and the aim is to find the conditions which minimise the compressive axial force in the track. The force away from the buckle can then be found, and will be equal to the thermally-induced compression force ($EA \alpha \Delta T$). For a single buckle (the mode 1 buckle shown in Figure 2.3,) which is identical to the shape analysed for upheaval buckling, the minimum compressive force within the buckle is found to be

$$P = C_1 \left( \frac{EI}{\lambda^2} \right);$$

(2.4)
and the compressive force in the beam at a point where there is no axial slippage is given by

$$P_c = P + C_3 \mu_a w \lambda \left[ 1 + C_2 \frac{EA(\mu_tw)^2\lambda^5}{\mu_a w (EI)^2} - 1 \right]; \quad (2.5)$$

and the lateral displacement at the centre of the buckle is

$$a = C_4 \left( \frac{\mu_tw \lambda^4}{EI} \right). \quad (2.6)$$

The constants in these equations are $C_1 = 80.73$, $C_2 = 1/15641$, $C_3 = 0.5$ and $C_4 = 1/415$. It is clear that Equation (2.5) is rather cumbersome to use.

The single buckle mode-shape is not appropriate for lateral buckling; it requires the application of large point loads at the ends of the buckle to maintain lateral equilibrium. Martinet further examined a second form of lateral buckling (the mode 3 buckle shown in Figure 2.3) which produced slightly lower critical temperature rises than the first form, but it was pointed out that for the buckle to go straight to the second form without passing through the first form it would be necessary to apply three lateral deflections of appropriate size and sense to the track, and this was deemed unlikely to occur in practice. However, mode 3, and also higher odd mode numbers (not shown in Figure 2.3) are similar to the shapes that are seen in the small-scale experiments of the present work.
2.3.2 Refinements to the Rigid Base Model

Hobbs (1984) used the same general approach of a beam on a rigid base with Coulomb friction to analyse the problem of in-service buckling of pipelines, both upheaval and lateral. Unlike Martinet, Hobbs considered asymmetric as well as symmetric buckling modes; these modes had been analysed earlier for the case of railway tracks by Kerr (1978a). The modes are shown in Figure 2.3. For lateral equilibrium, a concentrated lateral force is required at the end of each odd mode-shape, but this force reduces as the number of lobes increase, becoming zero for mode $\infty$. The values obtained for compressive loads and buckle amplitude are of the same form as Equations (2.4–2.6), but with different constant $C_1 \ldots C_4$. The appropriate constants can be found from Hobbs (1984), Table 1.

![Diagram of lobes for various modes](image)

Figure 2.3: Definition of lobe length $\lambda$ for various modes (after Hobbs).

Since the critical axial force is not significantly different between symmetric and asymmetric modes, the actual mode of buckling is generally dependent on a series of unknown factors such as horizontal out-of-straightness of the pipeline or non-uniform seabed features. Kaye, LeMarchand, Blondin & Carr (1995) suggest that for design purposes the critical mode shape is the one which induces the greatest curvature; this turns out to be the symmetric mode, for which mode 3 is put forward as a suitable idealisation.

A significant difference between pipeline buckling and railway track buckling is the additional contribution of internal pressure to the axial compressive force in the pipeline. This component of the compressive force may be taken into account as an additional temperature by dividing the free axial strain, which
when constrained produces the force, by the coefficient of thermal expansion of the pipeline material, and using this to find an effective temperature rise

\[ \Delta T_{\text{eff}} = \Delta T + \frac{pD}{4Et\alpha} (1 - 2\nu). \]  

(2.7)

This effective temperature can then be used in the various railway-track analyses. It should be noted that the main component of the axial compressive force in typical submarine pipeline applications is due to the constrained thermal expansion rather than the pressurization.

As mentioned earlier, this type of analysis assumes that a buckle has already formed, and finds the conditions under which it can exist. It does not say anything about the initial conditions which led to the final buckled configuration; indeed, it is inherently impossible to determine pre-buckled conditions from such post-buckled analysis.

Hobbs considered the qualitative effects of imperfections on the buckling behaviour of a pipeline, supporting the behaviour observed by Tvergaard & Needleman (1981); that is, unstable behaviour with a high critical temperature for smaller imperfections, and a stable equilibrium path for larger imperfections. For any imperfection, the behaviour was found to be asymptotic to the equilibrium path for a perfect system at larger buckled amplitudes. This is shown schematically in Figure 2.4.

![Diagram](image)

**Figure 2.4:** Imperfection sensitivity (after Hobbs).
2.3.3 Deflection-Dependent Friction

The rigid-base analyses mentioned in the previous section employed idealised Coulomb friction forces; but real pipeline-seabed interaction can be much more complicated than this. Measured pipeline and railway responses on various soils have shown a high degree of deflection-dependence.

Taylor & Ben Gan (1986) suggested a lateral friction force/deflection characteristic with an exponential form

\[
\frac{f_l}{\mu lw} = 1 - exp\left(-\frac{m w}{v_{fm}}\right)
\]

(2.8)

where the fully mobilized lateral friction force \( \mu lw \) and the lateral displacement at full friction mobilization \( v_{fm} \) are taken from a particular set of experimental data. The value of \( m \) is calculated by a best-fit to the data set. An equivalent expression can be used for axial friction, with a different value of \( m \). Full mobilization of friction was seen to occur at a much lower displacement in the axial direction than in the lateral direction, and is very close to full mobilization in the axial direction at a much lower percentage of that displacement*.

With the rigid-base analysis repeated, taking account of the full frictional behaviour in both the axial and lateral directions, the following points were concluded by Taylor & Ben Gan.

i) The loci of temperature rise against buckle amplitude for the modes studied are hardly changed by using the modified axial friction behaviour. This is probably because the modified behaviour is very close to the standard fully mobilized friction behaviour.

ii) Using the modified lateral friction behaviour, a critical temperature is found at which a perfectly straight pipeline would snap through to a stable post-buckled state. Without the modified lateral behaviour, the temperature versus buckle amplitude loci for all buckling modes are asymptotic to the temperature axis, rather like the solid line in Figure 2.4(b).

iii) The snap-through state of a perfect pipeline is the state to which all loci of imperfect pipelines should converge; at this point all the previous imperfection history is lost.

iv) The critical temperature for the pipeline is sensitive to the lateral friction curve used. As \( v_{fm} \) increases, the critical temperature decreases, and can eventually reach stable post-buckling behaviour where the critical temperature is below the temperature rise considered by Kerr (1978a) to be safe from buckling.

*The value of \( m \) in Equation (2.8) was taken as 25 for axial friction and 6 for lateral friction for the particular experimental data studied.
A much simpler approach to computations of this sort is to incorporate an “elastic slip” into the Coulomb friction model. This is shown by the chain-dotted line in Figure 2.2. The friction force is assumed to rise elastically up to the fully mobilized limit over a displacement which is called the elastic slip, $\xi$. This displacement is equivalent to $v_{fm}$ from Equation 2.8. Unloading is also elastic, shown by the dashed line, and it is this which gives rise to hysteretic behaviour with cyclic loading. In the associated field of upheaval buckling, Maltby & Calladine (1995b, 1995a) demonstrated that the exact shape of the soil response (linear or exponential) up to full mobilization has little effect on the buckling analysis; it is dominated by the fully-mobilized plateau value of friction.

If a beam were perfectly straight, then it would be expected to remain straight and not buckle at all. However, Kerr (1974) suggests that this is not the case; the buckling behaviour is dominated by the linear part of the soil response, and buckling would occur at a load

$$P = 2\sqrt{\frac{EI\mu w}{\xi}}. \tag{2.9}$$

Equation (2.9) is identical to Equation (2.2), with $k = \mu w/\xi$, and this provides an upper limit to the buckling load for a pipe.

### 2.3.4 Simple Behaviour of a Beam on a Rough Surface

Nikitin (1992) assessed the simpler problem of a beam on a frictional foundation subject to a monotonically-increasing concentrated lateral load $F$, but no applied axial load*. An infinite beam was split into an infinite number of regions of lateral friction loading with alternating directions. The boundaries of these regions are characterised by discrete points where the velocity-like term $(\partial y/\partial F) \to 0$, and the lateral friction forces are assumed to be fully mobilized immediately to each side of the force reversal point. An approximate solution for the system was obtained for a finite number of force reversals subject to appropriate boundary conditions at the loading point and at the end of the final force reversal, such that the final slope was deemed to be “sufficiently small”. A key quantity from the analysis is the lateral deflection at the load point

$$v = 0.068 \left( \frac{F^4}{16(\mu w)^3 EI} \right). \tag{2.10}$$

Note that no account was taken of the effect of axial friction — for the small lateral deflections which this analysis assumes it is not too important, but as the

---

*Also see Nikitin, Fischer, Oberaigner, Rammerstorfer, Seitzberger & Mogilevsky (1996) for an analysis on bending due to non-uniform thermal loading
deflection increases there will be some amount of axial feed-in from the straight part of the beam beyond the laterally-displaced region.

Despite the fact that this analysis assumes small displacements and an infinite beam, this equation will be useful for comparison with the results from the physical model and the finite-element analysis. The comparison will be made in Section 5.1.1. The lateral-loading analysis was extended by Stupkiewicz & Mróz (1994) for cyclic loading of the beam.

The complementary analysis for cyclic axial loading of a beam on a frictional foundation was covered by Jarzębowski & Mróz (1994). The loading history (loading, unloading, reloading) must be carefully considered in order to predict the extent of slip zones along the length of the beam. For a beam given an end load $P_1$, unloaded to $P_2$ ($P_2 < P_1$) and reloaded to $P_3$ ($P_2 < P_3 < P_1$), then the displacement of the loaded end of the beam is

$$u = \frac{P_1^2}{2EA\mu_aw} - \frac{(P_1 - P_2)^2}{4EA\mu_aw} + \frac{(P_2 - P_3)^2}{4EA\mu_aw}$$

where the three terms represent the three loading stages. Again, this is useful for comparison purposes with simple experiments, and will be discussed in Section 5.1.2.

### 2.4 Buckle Localization

Tvergaard & Needleman (1981), in a study of lateral buckling of railway tracks, considered the effect of localization on buckling behaviour. They noted that the critical bifurcation mode predicted by the elastic foundation analysis is periodic, while observed track buckling modes are localized; and that the bifurcation temperature from the elastic analysis is significantly larger than the observed track buckling temperature. The linear-elastic springs of the elastic foundation analysis were replaced by “elastic-plastic” springs, with a nonlinear first-loading curve and elastic unloading/reloading curve. The form of the foundation response was chosen empirically to give good agreement with the measured behaviour of a particular railway track.

Initially a range of periodic imperfections, with varying wavelengths and initial amplitudes, were examined. The following general trends were found by Tvergaard & Needleman.

1. The smaller the initial imperfection amplitude, the larger the critical temperature rise for buckling. As the imperfection amplitude becomes larger, eventually there is no longer a peak in the plot of temperature rise against lateral deflection amplitude, representing a stable solution for increasing temperature.
ii) The lack of a maximum point in the plot of temperature rise against lateral deflection amplitude does not necessarily mean that there will not be a maximum on a plot of axial compressive force against lateral deflection amplitude. In fact, a maximum compressive force was observed for several numerical tests where the equilibrium solution was stable with increasing temperature.

iii) For conditions where both temperature and compressive force maxima were observed, the force maximum occurred slightly before the temperature maximum.

iv) If the half wavelength of the initial imperfection is reduced, keeping the amplitude constant, then the critical temperature decreases. Also, the imperfection amplitude necessary for a stable equilibrium solution with steadily increasing temperature is reduced.

v) For any particular imperfection amplitude there is a certain critical half-wavelength where a local temperature maximum just occurs; and this wavelength is larger than the bifurcation wavelength for a perfectly straight track.

The phenomenon of buckle localization (Tvergaard & Needleman 1980, Tvergaard & Needleman 1981) was also investigated in relation to periodic imperfections. It was shown that if a maximum axial compressive force is reached, then the periodic pattern can bifurcate into a localized pattern shortly after that force maximum. Localization means that one particular lobe of the periodic pattern will begin to grow at the expense of the lobes around it. This is analogous to the tensile phenomenon of plastic necking. This pure form of localization, associated with an unstable equilibrium path, can develop with equal likelihood in one of a number of locations. Hunt, Bolt & Thompson (1989) note that apparent localization can occur simply on account of local stress variations throughout the structure, and that this apparent localization may occur under a continuously increasing load (a stable equilibrium path) with a unique post-buckling solution. For real pipelines laid imperfectly on the seabed, this is perhaps the kind of behaviour to be expected.

Tvergaard & Needleman studied buckle localization numerically, considering the post-bifurcation behaviour of a track with a very small deviation from a periodic imperfection. The length of track modelled was found not to have an effect on the location of the bifurcation point, but on a plot of temperature rise against maximum lateral deflection, the curve for a longer track falls below that for a shorter track at large lateral deflections. The authors state that “the length necessary to represent a very long track corresponds to the distance from the localized buckle at which no noticeable restoring force in the axial direction
develops in the foundation," although tests for this condition were not presented for any of the numerical experiments.

In a refinement of the work by Tvergaard & Needleman, the effect of the size of beam imperfection has been examined by Maltby (1992). Experimental work on the effect of the imperfection amplitude on the buckling load has been performed by Allan (1968).

2.5 Effect of Beam Imperfections

Allan (1968) performed a series of experiments in which thin strips, under a uniform vertical load \( w \), were compressed. The strips rested on a rigid foundation. Initial prop imperfections of height \( v_o \), up to roughly the thickness of the strip, could be inserted beneath the strip. The results of these experiments fitted well with the solution of the governing equation, subject to the appropriate boundary conditions,

\[
P = 3.95 \sqrt{\frac{wEI}{v_o}}. \tag{2.12}
\]

A similar relationship for a half-wavelength of an initially-imperfect buckled pipe on a frictional foundation has been discussed (Maltby 1992, Maltby & Calladine 1995b) in which the lowest axial force at which a buckle can localize is given by

\[
P = \sqrt{\frac{EI\mu_aw}{v_o}}, \tag{2.13}
\]

where \( \mu_a \) is the axial coefficient of friction between the pipe and the soil. Again the axial load is inversely proportional to the square root of the initial imperfection height, and here depends only on the plateau value of the lateral friction response. Note that the term \( v_o \) is not quite the same in each equation. In Equation (2.12) it is the amplitude of a full-wave imperfection, while in Equation (2.13) it is the amplitude of a half-wavelength. Equation (2.13) is based on a specific imperfection half-wavelength for which a maximum in the curve of compressive axial force in the buckle just disappears, and is therefore the lowest unstable buckling load.

The actual imperfection can be of several different forms, with different initial states of stress. A stress-free imperfect beam can be laid onto a perfect base, and remain stress-free; or a stress-free perfect beam can be laid onto an imperfect base, and so become stressed as it adopts the shape of the imperfection. The base imperfection could be a simple prop which applies a point load to the beam, or a fully-contacting "hump," as discussed by Yun & Kyriakides (1985) and Ju & Kyriakides (1988). The main finding of that work was that although the nature of the imperfection (initially stressed or unstressed) and the size of the imperfection
(length and amplitude) both affect the axial force at which buckling occurs, the post-buckling behaviour is not very sensitive to changes in these parameters.

Of course, these investigations all relate to upheaval buckling. The same level of parameter insensitivity does not seem to apply to lateral buckling, where more than one lobe will form. It appears that, unlike upheaval buckling, where just one uplifted lobe is formed which eventually settles into a preferred buckled geometry, the formation of outer lobes in lateral buckling cases prevents the central lobe from reaching an unique, imperfection-independent geometry. The results from imperfection studies by Tvergaard & Needleman (1980), mentioned further in Section 2.4, bear this out. Where more than one half wavelength is studied, different imperfection heights slightly alter the post-buckling behaviour of the central lobe (which was the only lobe considered in that study).

2.6 Industrial Pipeline Studies

As well as previous, mostly analytical, studies into lateral buckling, there have also been several studies by members of the oil industry consisting of field and laboratory tests of the interaction between pipelines and soil.

2.6.1 The PIPESTAB Project

The Pipeline Stability Design (PIPESTAB) Project (Brennodden, Sveggen, Wagner & Murff 1986, Lambrakos, Remseth & Verley 1987, Wagner, Murff, Brennodden & Sveggen 1987, Wolfram, Getz & Verley 1987) included both analytical and experimental investigations into the on-bottom stability design of submarine pipelines subject to storm loading. Wolfram et al. (1987) give a general outline of the aims of the project, and the key findings, the most useful of which concerns the pipe-soil interaction model (Brennodden et al. 1986, Wagner et al. 1987).

Experiments were conducted on short pipeline sections resting on five typical types of offshore soil (loose fine sand, loose and dense coarse sands, and soft and stiff clays.) Some typical results for soil resistance coefficient against lateral pipeline displacement are shown in Figure 2.5.

In both cases it is clear that ideal Coulomb friction cannot properly describe the pipe-soil interaction. For the tests on sand, the soil resistance was higher for breakout after cyclic loading than for breakout under monotonic loading. This is attributed to penetration of the pipe into the soil and hence passive loading from a layer of soil to the side of the pipe. This is probably the reason behind the higher soil resistance for the loose sand, since the pipe penetration was found to be higher in that case. For the tests on clay, a very different type of behaviour
Figure 2.5: Soil resistance coefficient against lateral displacement for a) coarse sand b) soft clay, taken from Brennodden et al. (1986).

was encountered. Consolidation leads to increased shear strength beneath the pipe, and also clay-pipe adhesion, and so the peak breakout resistance was relatively high. However, after breakout, or with initial cyclic loading, the effect of consolidation was found to be lost, and a common soil resistance was approached at increasing lateral displacements. For soft clay, again cyclic loading increased pipe penetration and therefore the peak breakout soil resistance; but this effect was not present for stiff clay. It is interesting to note that the initial slopes of the plots in Figure 2.5 are very similar, despite the very different subsequent behaviours, possibly suggesting a fairly uniform initial foundation stiffness \( k \); but this cannot be checked reliably due to the limited amount of experimental data published for foundation stiffness.

The occurrence of penetration is a drawback for the simple Coulomb friction model in that finite lateral soil resistances were found for near-zero vertical contact forces between the pipe and soil (Wagner et al. 1987). A suggested improvement, which does not take account of the initial linear behaviour, is

\[
f_l = \mu_l w + f_r
\]

where \( f_r \) is a lateral passive soil resistance term, a function of the force required to move a volume of the appropriate soil, and of the loading history. It is not obvious that this approach is useful for general pipeline design, depending as it does on a specific history; but it might be useful for analysis of a pipeline which
has already experienced some lateral deflection, and for which survey information may be available.

The PIPESTAB project also involved the development of a generalised pipeline response under hydrodynamic loading (Lambrakos et al. 1987), which considered scaling parameters by means of dimensionless groups. This is examined in more detail in Section 3.1.2.

2.6.2 Other Studies

Lyons (1973) discussed the results from small- and large-scale tests on lateral sliding of a pipe on sand and soft clay, and concluded that while Coulomb friction analysis is valid for sliding on sand, it is not appropriate for soft clay. Whereas the effect of pipe diameter, surface roughness and sand coarseness hardly affected the lateral coefficient of friction on sand, for clay, increasing the pipe diameter or decreasing the submerged weight decreased the lateral soil resistance. These findings correspond well with the PIPESTAB conclusions about the effect of pipe penetration into various soils.

Anand & Agarwal (1981) presented data from lateral- and axial-pull tests on both model and short prototype pipes. The results from the model tests typically show the horizontal load rising to a maximum and then remaining approximately constant in both the lateral and axial assays. Coefficients of friction were found for both sand ($\mu_a \approx 0.60, \mu_t \approx 0.15$) and for silt ($\mu_a \approx 0.80, \mu_t \approx 0.20$). The same behaviour was noted for the prototype pipe pulled axially, with a similar coefficient of friction on sand, but much lower on silt. For the prototype pipe pulled laterally, again the coefficient of friction was much the same as for the model tests, but this time the result for the silty soil was higher. This was attributed to the different composition of silty soils in the model and prototype tests.

Interestingly, on the basis of these experiments, a short concrete-coated pipeline was installed in a sand-filled trench using the bottom-pull method. The load exerted by the winch during installation was used to find an axial coefficient of friction for the concrete-coated pipe on the sand. It was found that $\mu_a \approx 1.0$, which is significantly higher than the results obtained for both the model and prototype pipe tests. Although this difference was attributed to deposition of silty clay in the trench, therefore increasing the adhesion between the coating and the soil, there are many possible external factors in the field, such as the pipe-end getting caught on uneven ground, which are not easily taken into account for smaller-scale tests. Also, it is not clear whether the winch load presented is the horizontal load required to move the pipe, or the total winch force applied at an angle from the pipe to the winch barge. If the latter is correct, the axial coefficient of friction would have been overestimated.
2.7 Conclusions

The two main types of buckling model presented here stem from distinctly different assumptions. The frictional foundation models take a buckle which has already formed as a starting point, and then attempt to find conditions for which, theoretically, the buckle cannot occur. The elastic foundation models focus on the conditions when buckling occurs from the initial, possibly imperfect, shape. An important phenomenon in the case of a beam with multiple imperfections is localization, where one imperfection grows at the expense of others; this is associated with a peak in the axial compressive force in the beam.

One area of research not encountered during the literature survey was “far-post-buckling” behaviour. For railway tracks any buckle, however small, requires the track to be replaced; but a pipeline is often still serviceable after buckling. It would be useful in a pipeline to know how the buckle would develop; for instance, would the initial buckle begin to stabilise in a safe state and another buckle initiate elsewhere, or would the initial buckle continue to grow until failure?

The industrial studies demonstrate that there are significant problems associated with trying to obtain a single number for a coefficient of friction to be used either in design or in analysis. The behaviour of pipelines on different soils is very complicated, and is often different in the lateral and axial directions. Combined with the fact that a submarine pipeline is laid imperfectly (the imperfection sizes being unknown), an analysis of submarine pipelines which is simple, yet correlates well with real pipeline data, has proven to be a real challenge.
Chapter 3

A Physical Model

The main idea behind any form of physical modelling in the laboratory is the attempt to reproduce, under controlled and repeatable conditions, phenomena which are observed in real life. Physical models can also be used for validation of theoretical schemes.

The choice of model type and geometric scale is driven by two main factors. The first is the requirements of the modelling process. If, hypothetically, the phenomena which are to be observed are at a small scale, then the modelling process should attempt to reproduce these phenomena at a scale which is more easily seen or measured. It would probably be impractical to model the entire full-scale situation; one would concentrate on a particular part of the problem. However, this leads to the need for careful application of boundary conditions to account for those parts which have not been included in the model. Dimensional analysis can be used to suggest scaling factors for various parameters to produce "similar" results.

The second factor is practicality. In this case, to model the lateral buckling of pipelines on the seabed on a laboratory bench, it would obviously be impractical to carry out full-scale tests with steel pipes in water. The length of pipe required to contain the buckling phenomenon would be too great (tens or hundreds of metres) and the equipment needed to provide the buckling force too large. A typical laboratory bench is only around 3 m long with a reasonably small load limit. Kerr (1978b) details problems encountered with full-size test rigs for studying the related problem of railway-track buckling. Associated with practicality comes simplicity. When building the apparatus for physical modelling, it is important to make everything as simple as possible, while allowing the phenomena to be observed and measured to the required degree of accuracy. Hence, if only qualitative observations are required, the apparatus may not need

*The values of certain dimensionless groups are the same for the prototype (full scale) and for the model.
be as complicated as if detailed measurements are necessary. There is also a “law of diminishing returns” as far as instrumentation is concerned. It is often better to begin with very basic instrumentation, which can later be upgraded, than to set out initially looking for state-of-the-art technology which will often be costly and time-consuming to set up and run, and may not in the event prove to be as useful as was first thought.

3.1 Scaling from Real Pipelines

The most immediate problem with physically modelling the subsea behaviour of pipelines is the sheer scale of the problem. Observed deviations of pipelines from their as-laid positions can extend to more than 100 m in length, and require very large forces to produce them. If the pipeline is laid in a curve, this could typically have a diameter of 4000 m. In the laboratory, where space is usually at a premium, the more compact a model can be made, the better. However, scaling from the real pipeline is not as straightforward as simply reducing all dimensions by a certain factor.

Although many pipelines, and especially in-field flowlines, use standard pipe sizes (nominal outside diameters), the pipe thickness can vary within these sizes (D/t typically in the region 18–25). For export pipelines, larger diameters are encountered which, for some current applications, may be custom-built. Hence, if a specific length scale is required, each individual pipe needs to be modelled separately; and if it passes over different seabed formations, even different parts of the same pipeline may need to be modelled. Therefore, for convenience, a model should be made specifically for a particular set of pipeline parameters.

3.1.1 Pipeline Data

Some data exist for lateral buckling of submarine pipelines, but they are not readily accessible. Although it is not generally seen as a major failure, a lateral buckle is treated by the oil companies as something which they do not wish to publicise. As mentioned in Chapter 1, lateral buckling is chiefly associated with pipelines in more inaccessible locations, such as deep water, where the risk of damage to the pipeline from external causes is low enough not to warrant the use of trenching. Most pipeline coordinate data are obtained using ROVs (Remotely-Operated Vehicles) which can be made to follow the alignment of the pipeline using video camera images, while its global position is monitored. The high cost of mobilizing an ROV and ancillary equipment means that, after an initial survey of the as-installed alignment, a pipeline will be checked and data obtained only if a problem is perceived.
Even if a set of data is available, errors may be present, and the accuracy may be somewhat limited. As a particular example (Andrew Palmer & Associates Ltd. 1989), a pipeline lateral movement of concern might be as little as 0.6 m in amplitude and 80 m long. The data from the ROV gives lateral coordinates to about the nearest 0.05 m and longitudinal coordinates to the nearest 1 m. So while the error in longitudinal readings could be only 1 or 2 percent, the error in the lateral direction could be nearly 10 percent. This would have a large impact on subsequent curvature and stress calculations.

The interface conditions between the pipeline and the seabed play a large part in the lateral behaviour of the pipeline. Table 3.1 gives typical maximum and minimum values for the coefficients of friction between pipelines and soil, as given in BS 8010 Part 3 (British Standards Institution 1993). The table shows the large range of the friction coefficient in just one moderately small geographical region. Other typical values, from small- and large-scale testing, were reported in Section 2.6.

<table>
<thead>
<tr>
<th></th>
<th>axial friction coeff.</th>
<th>lateral friction coeff.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>minimum</td>
<td>maximum</td>
</tr>
<tr>
<td>non-cohesive soil (sands)</td>
<td>0.55</td>
<td>1.2</td>
</tr>
<tr>
<td>cohesive soil (clays)</td>
<td>0.3</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Table 3.1: Typical effective coefficients of friction for North Sea applications, taken from BS 8010:3 (1993).

### 3.1.2 Dimensional Analysis

The usual way for authors in this field to present their data is in terms of a particular size of pipe or railway track; and this is only to be expected where the analysis is intended to present a clear assessment of a real situation. However, for the sake of generality, and to enable comparisons to be made between the work of different authors, it is often beneficial to make use of dimensionless variables. This is also the most appropriate way to approach the problem of scaling in physical modelling.

The Pi Theorem of Buckingham can be used to determine the number of dimensionless groups required completely to express a problem (Taylor 1974). If, for a particular set of physical quantities $q$, the equation

$$\phi(q_1, q_2, \ldots, q_n) = 0$$

is the only relationship between the quantities, then the solution has the form

$$\phi(\Pi_1, \Pi_2, \ldots, \Pi_m) = 0$$
where the $\Pi$s are the independent dimensionless products of the $q$s. If the $q$s can be expressed in $k$ dimensions such as mass, length and time, then

$$m = n - k.$$ 

Since the lateral buckling of pipelines is being considered as a quasi-static problem, and acceleration due to gravity will remain constant, force can be taken as a dimension in its own right.

There is, however, a problem with this approach. As the physical system gets more complicated, the number of dimensionless groups $m$ will increase. The more dimensionless groups that exist, the harder it is to create a model where the values of all the dimensionless groups are equal to those of the full-scale system. This is especially true when the materials used for the model are quite different from the materials in the actual pipeline/seabed system. The more sophisticated the model is to be, the smaller this problem becomes.

Looking now at a beam on a frictional foundation, dimensionless groups $\mathcal{D}$ involving the key variables may be assembled:

$$\mathcal{D}_1 = \lambda \left( \frac{\mu_a w}{EI} \right)^{\frac{1}{3}}, \quad \mathcal{D}_2 = a \left( \frac{\mu_a w}{EI} \right)^{\frac{1}{3}}, \quad \mathcal{D}_3 = \frac{\mu_t}{\mu_a},$$

$$\mathcal{D}_4 = \left( \frac{(EI)(\mu_a w)^2}{(EA)^3} \right)^{\frac{1}{3}}, \quad \mathcal{D}_5 = \frac{P}{EA}.$$ 

where $\lambda$ is a buckle lobe length, $a$ is a buckle lobe amplitude, $\mu_a$ is the axial coefficient of friction, $\mu_t$ is the lateral coefficient of friction, $EI$ is the beam bending stiffness, $EA$ is the beam axial stiffness, and $P$ is the axial force at the centre of a buckle lobe.

Note that pipeline geometric and material properties are described purely in terms of bending and axial stiffnesses $EI$ and $EA$, so the diameter and thickness do not appear here explicitly. The use of dimensionless groups involving stiffnesses rather than the real geometrical data preserves the generality of the problem so that, for example, the analysis is equally applicable to railway track studies. Other quantities which have a minimal effect on the data to be presented may be laid aside.

For a beam laid in a curve, the lay-radius $R$ would then need to be included; this would introduce one more dimensionless group $\mathcal{D}_6$, possibly similar to group $\mathcal{D}_1$, with $R$ replacing $\lambda$. A point to notice here is that while groups $\mathcal{D}_1$ and $\mathcal{D}_2$ involve quantities which are initially unknown (the length and amplitude of a buckle), group $\mathcal{D}_6$ uses a radius which may be chosen. Consequently it would be possible to use this group as a basis for deciding on the properties of a model for a particular prototype to a particular length scale.
The choice of dimensionless groups is not absolutely fixed, although any particular dimensionless group can be formed by a combination of other dimensionless groups from an independent set. For example, Lambrakos et al. (1987) make use of the dimensionless groups

\[ I_l = \frac{EI D}{l^4 w}, \quad P_l = \frac{Pl^2}{EI} \]

for bending and tension effects, where \( l \) is a length “representative of the distance from the end restraint beyond which the pipeline stiffness does not significantly affect the displacement,” but a length such as a buckle half-wavelength \( \lambda \) would have been equally valid. In that case \( P_l \) could be formed by the product \( D_5 D_1^2 D_4^{-1} \).

### 3.2 Modelling the Pipeline and the Seabed

In the present investigation, the final model for the pipeline on the seabed was arrived at in two stages. First a physical pipeline model was created, but there was then a problem with how to load it for the experiments. A solution was found by making use of a novel seabed model.

#### 3.2.1 The Pipeline Material

The pipeline model was designed specifically to be able to investigate lateral buckling phenomena on a 3 m laboratory bench, so it needed to be lightly loaded to avoid the use of large or heavy loading equipment. The first stage of the design process was to find data on real pipelines, and to put the data into selected dimensionless groups. These were initially based on the curved layout of a pipeline, and so the curve radius could be included in the groups.* Table 3.2 shows the properties of two particular pipelines under typical working conditions, along with the values of two dimensionless groups. The values for the dimensionless groups are broadly similar for the two real pipelines.

The preliminary assumptions for the model were that the coefficients of friction would be close to 1; that the linear scale for the curve should be around 1/1000, which is reasonable for the laboratory setting; and that the model pipe should have a diameter (or width if not circular) in the region 1-10 mm for ease of handling. For instance, trying to model the Palmer pipeline mentioned above with a width of 10 mm on a curve of 4 m, the first dimensionless group requires the model material to have a value of specific modulus \((E/\rho)\) of the order of

*This is in contrast with other length variables such as the buckle length, which are unknown before the buckle occurs, as discussed in Section 3.1.2
Table 3.2: Examples of pipeline properties under typical working conditions.

$10^{-3} \text{ MPa/km}^3$. Ashby (1992) has composed material selection charts, one of which shows $(E/\rho)$. Most materials fall far above the required value (for example, steel has $(E/\rho) \approx 30 \text{ MPa/km}^3$). A suitable material would need be a soft elastomer. Silicone rubber, being easily moulded to any shape and easily handled, seems particularly appropriate. In fact, it is easier to mould a rectangular strip than a circular one, and so this was the approach taken.

There are considerable benefits in the choice of a silicone rubber strip as a model for a pipeline:

- Very flexible in bending. Low axial forces in the strip will produce lateral deflections which can be easily measured.

- Easy to manufacture. The rubber is formed by adding a hardener to a viscous rubber solution. In this way it is possible to manufacture a strip of any reasonable required length, size and shape.

- Lightweight and easily handled. Long lengths of the strip can be moved and manipulated with ease, and the loading on any apparatus would not be too large.

However, as with the choice of material for almost any modelling process, there are also some problems associated with silicone rubber:

- Low axial stiffness. Axial strains of the order of 10% are readily achievable, and this causes problems when trying to model dimensionless groups involving the axial stiffness of a pipeline $(EA)$. To match both dimensionless groups in Table 3.2 for the Palmer pipe would require a $1 \mu m$ wide strip lying on a curve of radius $1 \text{ cm}$, to an order of magnitude.

\[
\begin{array}{|c|c|c|}
\hline
& \text{Svanø (1992)} & \text{Palmer (1989)} \\
\hline
EI & 1.87 \text{ GN.m}^2 & 0.30 \text{ GN.m}^2 \\
EA & 19.05 \text{ GN} & 6.828 \text{ GN} \\
\mu_t, \mu_a & 1, 1^{\frac{1}{3}} & 0.7, 0.7 \\
R & 3000 \text{ m} & 4000 \text{ m} \\
\hline
D_6 = R \left( \frac{\mu_a w}{EI} \right)^{\frac{1}{3}} & 30.6 & 61.2 \\
D_4 = \left( \frac{(EI)(\mu_a w)^2}{(EA)^3} \right)^{\frac{1}{3}} & 1.02 \times 10^{-5} & 1.03 \times 10^{-5} \\
\hline
\end{array}
\]

\text{Assumed values}
Easily damaged. Although silicone rubber is very flexible, once it has become damaged by a small tear, failure can be rapid. These tears are all too easily produced while removing excess material ("flash") from the moulding process.

- Creeps noticeably at room temperature. The strip must be stored straight and flat, since any imperfections caused by poor storage can have a detectable effect on results.

- Relatively costly. Ashby (1992) suggests that silicone rubber is between 20 and 50 times more expensive than mild steel, per unit volume; but this is not really a problem for the small amounts required for the present experiments.

There is a solution to the first (and most important) problem if a composite, rather than silicone rubber alone, is considered. It is necessary to increase significantly the axial stiffness of the strip while changing the bending stiffness as little as possible. A simple solution is to mount an axial stiffening fibre (steel wire) centrally down the strip. Equations (3.1) and (3.2) give the bending and axial stiffnesses for the strip.

\[
EI = E_r \left( \frac{bd^3}{12} \right) + E_w \left( \frac{\pi r^4}{4} \right) \approx E_r \left( \frac{bd^3}{12} \right), \tag{3.1}
\]

and

\[
EA = E_r (bd) + E_w (\pi r^2). \tag{3.2}
\]

where \( r \) is the radius of the stiffening wire, \( b \) and \( d \) are the width and depth of the silicone rubber strip, and \( E_r \) and \( E_w \) are the elastic moduli of the rubber and stiffening wire (\( r \approx 10^{-2} b, d \) and \( E_w \approx 10^5 E_r \)).

**Figure 3.1:** Cross-section of mould for forming silicone rubber strips.
The strips were manufactured in a 2 m long mould, shown in cross-section in Figure 3.1. Three sides of the mould were formed from long mild steel bars, and the fourth from a flexible perspex strip. The central wire was passed through holes in the end stops, and it rested on intermediate silicone rubber pads cut to the correct height. The wire was then put under a low tension to remove any sag between pads. Small blobs of solder were placed at approximately 200 mm intervals along the wire in order to help increase the mechanical bond between the rubber and the wire. The wire was attached to the pads with adhesive and coated with Ambersil Silcoset Primer, a bonding agent for use with silicone rubber.

The silicone rubber solution was prepared from Ambersil Silcoset 105 with 1% dibutyltin dilaurate curing agent, and placed in a bell jar vacuum to remove most of the air trapped during mixing. It was poured into the mould and any remaining air bubbles removed before the perspex strip was placed on top. The solution was left overnight to set, and then removed from the mould to cure in air for a further 24 hours.

Target points were then marked on the surface of the silicone rubber strip, to allow its position on the base representing the seabed to be determined. Most of the strips that were made, including test pieces, were cast in the standard white colour. The target points were formed by making pin pricks centrally along the axis of the strip, and filling the small cavities with black ink. In order to provide contrast against a light-coloured base, some pieces of rubber strip were coated with a further thin layer of silicone rubber solution, into which some black oil-based paint had been mixed. Once this layer had dried, target points were marked on the surface with a fine-tipped permanent white ink marker pen. The second method is more time-consuming, but the better contrast with the white polystyrene base makes it easier to take photographs of the buckled strip if required.

The spacing of the target points was determined by trial experiments. If the spacing is too large, the buckled shape may not be sufficiently well defined; but if the spacing is too small, data analysis becomes difficult as small errors in recording data points lead to large fluctuations upon numerical differentiation. An acceptable spacing was found to be 50 mm, generally giving 3–5 data points per buckle lobe.

Appendix A lists the properties of the three silicone rubber strips used in these experiments; reference will be made to the strip names listed there throughout the remainder of this dissertation.

3.2.2 The Seabed Material

So far, discussion has been confined to a model for the pipeline; but it is also necessary to find a material to model the seabed upon which the pipeline lies.
There are several different approaches to this search for a suitable material. The first is to use some kind of soil. In the field of hydraulics, for example, there are dimensionless groups such as dimensionless shear stress, as defined by Raudkivi (1976), which may be used to determine the required particle size for the model soil. This type of scaling generally produces a reduction in bed material size rather less than the linear scale factor, but considering that seabeds typically consist of sands or silty clays, the required material would most likely need to be a very fine clay. This would be inconvenient in use, and time-consuming to reconfigure between experiments, and so this approach was not followed.

A different approach is to search for a material which, in conjunction with the silicone rubber strip chosen as the pipeline model, produces interaction phenomena similar to those encountered with real pipelines. The main requirements of the seabed material are that it should not be too abrasive, so as not to cause deterioration of the silicone rubber strip over prolonged use; and that it should be easy to handle and not require significant reworking between experiments.

A suitable starting point for this search is a material which would produce a similar coefficient of friction when used with the silicone rubber as for a pipeline on the seabed. Recalling Table 3.1, a value which falls within the range of practical coefficients is around $\mu = 0.6$. Preliminary material tests were carried out by dragging the silicone rubber strip on the test surface, to see the qualitative behaviour, and also by taking crude readings of axial friction forces by pulling the strip by a spring balance. These readings were too crude to distinguish between static and dynamic coefficients of friction.

Flat aluminium sheet gave a value of coefficient of friction which was too low ($< 0.2$), while a polymer foam covering over the aluminium gave a value which was too high ($> 1.0$). Both of these materials also give rise to an undesirable stick-slip behaviour. Expanded polystyrene sheet, of the kind used in packaging and house insulation, appeared to give $\mu_a \approx 0.55$, although this was later revised downwards (see Section 3.4.3). In fact, it is not vital to have a particular value of coefficient of friction, as long as the value can be reliably determined. The overriding advantage of the expanded polystyrene sheet only became evident when trying to develop a method for “thermally loading” the silicone rubber strip, leading to the compressible base model.

### 3.3 The Compressible Base Model

In order to model the lateral buckling of a pipeline under thermal and internal pressure loading, it is necessary to find a way of applying a thermal-like compressive load to the silicone rubber strip. At its most basic, this would equate to a uniform compressive strain along the length of the strip. This is not quite what
is encountered in a real pipeline. The temperature would decay non-uniformly along the length of the pipe due to heat loss to the cold surrounding water, but over a length of the order of a small number of buckle wavelengths the assumption of uniform thermal strain should be acceptable. The precise rate of temperature decay along the pipeline depends on many factors such as pipeline inlet temperature, ambient seawater temperature, pipeline size, coating thickness etc. For example, from Andrew Palmer & Associates Ltd. (1989), a 10% reduction in temperature at the well end (where the rate of change of temperature is a maximum) occurs over a distance of 600 m, compared with a a typical buckle length of 100 m.

With silicone rubber being a thermal insulator (its thermal conductivity is around 1/400 that of mild steel), it is not practical to heat the strip up, and since the strip is centrally reinforced, it is also impractical to keep its centre hollow and apply the loading through pressurization.

One of the advantages of expanded polystyrene is that it has a low modulus of elasticity (approximately $3 \times 10^6 \, Nm^{-2}$), so it can be compressed to a strain of several percent by a relatively low force. A small specimen ($49 \, mm \times 49 \, mm \times 80 \, mm$) was tested in an Instron machine to a strain of 1.25%. Although the first loading curve was nonlinear, subsequent cycles were linear on loading and unloading almost down to zero load.

Therefore, to model thermal loading, the strip may be loaded by compressing the polystyrene base beneath it. This is the principle of the compressible base model. A schematic of the model is shown in Figure 3.2.

![Figure 3.2: Schematic plan view of the compressible base model.](image-url)
3.3.1 Model Construction

The base is formed from four sheets of expanded polystyrene (each 1200 mm × 450 mm × 25 mm), which are sandwiched together in two layers of two, using wood adhesive; the final base size was 2400 mm × 450 mm × 50 mm. Before assembly, two recesses were cut along the length of each sheet to form 10 mm square voids through the middle of the sandwich. Threaded steel rods (9 mm diameter) were passed along these voids. One end of each rod was firmly screwed into a heavy steel end block. This was the “fixed end” of the apparatus; the end block was clamped securely to the bench. The other end of each rod was passed through another steel block, the “moving end,” to which the chain drive was fitted. As well as providing the means of compressing the base, the rods, in tension under loading, also provide the polystyrene with some buckling restraint.

![Figure 3.3: Chain drive for compressible base model.](image)

The chain drive is shown in Figure 3.3. A tapped 12-tooth sprocket is threaded onto each rod, and these are driven by standard chain from an 8-tooth sprocket attached to the handle. The tapped sprockets advance equally along the two threaded rods, providing equal base shortening, $\delta$, across the width of the base. The end block movement can easily be set to an accuracy of 0.05 mm; this corresponds to a base compression of 8 micro-strain. The reverse movement of the block was provided by the compressive load on the polystyrene base pushing the block against the sprockets.

Measurements of the strip’s layout were made by using an x-y digitiser. It rested on an aluminium I-beam laid on its side, which formed a track for the digitiser as well as the guide for the moving steel block of the compressible base to butt against. To cover the entire length of the base, the digitiser had to be able to move between set positions on the track. It rested on three PTFE sliding feet: one was a simple slider at the front, and two were v-notch sliders at the
rear to prevent yaw of the digitiser while it was moved. The digitiser is described further in Section 3.6.1.

The bench was liberally dusted with French chalk where the base rested on it, in order to eliminate any points of high friction, and therefore to allow almost uniform compression of the polystyrene. The performance of the compressible base over a loading/unloading cycle is shown in Figure 3.4.

![Figure 3.4: Loading and unloading performance of the compressible base.](image)

End shortening $\delta$ for a) 0 mm : 2 mm : 20 mm, 20 mm : 5 mm : 30 mm.

End shortening $\delta$ for b) 30 mm : -5 mm : 0 mm.

The light lines show the ideal uniform compression of the base, and the dark lines show the actual compression, measured to the nearest 0.25 mm at 13 points uniformly spaced along the base. It can be seen that the assumption of uniform strain is fair, the error reaching a maximum of approximately 15% near the centre of the base at the higher end shortenings. Loading the base up to an end compression of 30 mm (corresponding to a strain of 0.0125) and then unloading resulted in a net shortening of 3.5 mm (a strain of 0.0015). This was recoverable over a period of time.

In fact, the performance of the compressible base was good provided that there was a pre-compression of around 5 mm and it was not left in a compressed state for longer than necessary for an experiment.
3.3.2 Model Limitations

The search for a simple physical model inevitably means that the model will have some limitations. Probably the most serious problem is the difference between pure thermal loading and unidirectional compression. The problem could be eliminated by applying, somehow, equal bidirectional compression to the polystyrene base, followed by bidirectional stretching of the data; but the complexity of producing such a system, able to cope with long specimens, meant that this was not incorporated into the model.

The constrained thermal strain loading of a real pipeline is directed along the pipe, in other words in the tangential direction. For the unidirectional compression of the model, this would be factored by the cosine of the rotation angle at the particular point. If small rotations are assumed (conventionally less than 0.1 radians), then the difference is very small (less than 1/2%), but for larger rotations (0.5 radians is not exceptional for these experiments) the difference is around 10%.

Another problem with the model is associated with uniform strain along the base, which means that a point on the base moves by an amount proportional to its distance from the fixed end. Suppose that a thermal expansion of 1% in the pipe is simulated by a 1% shortening of the compressible base. Strictly, to obtain the correct geometry for the thermal-loading case, the shape of the buckled strip should be stretched by 1% in the direction of the main axis of the base. This is the basis of the coordinate transformation, which will be discussed in Section 3.6.3. However, if a certain buckle lobe has amplitude \( a \) and length \( \lambda \) after this stretching, then as it lies on the base it has length 0.99\( \lambda \). The amplitude remains unchanged. Therefore the actual curvature of the strip is roughly 2% higher for the strip physically lying on the base than for the strip after coordinate transformation. Although this is not a large discrepancy in itself, it may affect the evolution of the buckle lobe under increased end shortening. The actual differences between these two loading methods will be considered further in Section 4.4.3.

3.4 Measurement of Key Parameters

3.4.1 Measurement of Weight and Stiffness

As mentioned in Section 3.2.1, the physical properties of silicone rubber cast in the laboratory cannot be controlled very easily, and so they need to be measured individually for each strip which is produced. The weight is measured simply by placing the strip on a set of accurate scales, measuring to the nearest 0.1 gramme. This is then divided by the length of the strip to give the weight per unit length.
Since the experiments take place in air, this is taken to model the submerged weight per unit length of a pipeline, \( w \).

The cross-section dimensions of the strip as manufactured were determined at several sections along its length by use of a micrometer. This enabled the cross-section profile to be compared against the original design. In practice the cross-section dimensions were uniform along the strip to within 4%.

The bending stiffness of the strip was measured using the apparatus shown in Figure 3.5. The deflection, \( h \), of a cantilever section of the strip of known length, \( L \), was determined by slowly raising the strip until the end of the cantilever section was just touching the flat block. The strip was then raised clear of the block, and the slowly lowered until the end of the cantilever section was again just touching. The deflections in both cases were noted, and the procedure was repeated several times to find an average deflection. This was repeated for a range of values of \( L \). The strip was tested at both ends, and for all four orientations of the cross-section. This procedure could not be repeated for the centre of the strip, which will be of most interest during the experiments, so the average value from the ends was taken. The strips were examined for external imperfections, but internal imperfections, such as small voids, could not be easily detected.

![Figure 3.5: Apparatus to measure bending stiffness.](image)

For small deflections, the self-weight deflection of a cantilever beam is given by

\[
h = \frac{wL^4}{8EI}
\]  

This can be written as

\[
\log(h) = 4\log(L) + \log\left(\frac{w}{8EI}\right),
\]

which allows \( EI \) to be found from the intercept of a best-fit line of slope 4 through the data points on a log-log plot. The value obtained in this way can be compared
with the theoretical value given by Equation (3.1). In all cases there was a
good agreement between the experimental and theoretical bending stiffnesses,
and because of this (and the difficulty in performing an axial stiffness test on the
intact strip) the theoretical value of axial stiffness $EA$ from Equation (3.2) was
used.

### 3.4.2 Measurement of Lateral Friction Behaviour

The apparatus used to measure the lateral frictional response of a silicone
rubber strip on an expanded polystyrene base is shown in Figure 3.6. The 300 $mm$
strip under test is placed in front of a perspex bar, which is then pulled by the
cantilever force transducer. The outputs from both the force transducer and the
displacement transducer were fed to a datalogger.

![Diagram of apparatus to measure lateral friction behaviour]

**Figure 3.6:** Plan view of apparatus to measure lateral friction behaviour.

Before performing these experiments it was necessary to calibrate both trans­
ducers. A Novotechnik Linopot potentiometer-type displacement transducer was
used to measure known displacements over its full travel, and the resulting volt­
age output noted. The performance of the transducer was linear apart from at
the extremes of its travel, and so it was set to read in the centre of its travel
for the actual experiments. The load transducer was made from a thin strip of
steel, 160 $mm$ long, with strain gauges mounted top and bottom close to the can­
tilever root. The output from the strain gauges went into a half-bridge resistor
circuit, from which the bending strain could be determined. The cantilever load
transducer performed linearly (bending strain proportional to end load) over the range of loads expected during the experiments.

Once the two transducers had been calibrated independently, the interaction between them and the rest of the apparatus (in the absence of the test strip) was established. The first measurement was the load-deflection characteristic of the cantilever. This was performed by firmly fixing the perspex strip to the base, and the slowly moving the base of the cantilever in the direction of the arrow in Figure 3.6. The results of this allowed the displacement measured by the displacement transducer to be split into components due to lateral movement of the strip and to deflection of the cantilever. The displacement transducer was not set up to read the displacement of the perspex bar directly, which would have made the data analysis more straightforward, because of the load it would impose due to the spring-loaded plunger.

The perspex bar was initially placed several centimetres behind the rubber strip, and pulled slowly forward until it contacted the strip. This allowed the frictional force due to the perspex bar alone to be determined. Two silicone rubber "brakes" were attached to the back of the bar, projecting below the lower surface of the bar, so that they dragged on the polystyrene. This was done in order mostly to remove the stick-slip behaviour which was present when the perspex bar alone was pulled along the lubricated PTFE pads. The point at which the perspex bar made contact with the silicone rubber strip was very well defined in the data. Once the force/displacement characteristics of the force transducer and the perspex bar had been taken into account, the force/displacement characteristic of the silicone rubber strip alone could be determined. Figure 3.7 shows this characteristic for two different pulling velocities. The force has been divided by the weight of the strip to give a coefficient of friction.

The strip was pulled by moving the base of the force transducer by means of a horizontal screw system. This was turned by hand at as uniform a rate as possible. Figure 3.7(a) shows the results of a pull velocity of approximately $0.4 \text{ mms}^{-1}$. There is a slight trend of increasing average lateral friction coefficient with lateral displacement, with $\mu_l \approx 0.5$ after 10 mm of movement. The experiment was then repeated at a lower pull velocity of approximately $0.2 \text{ mms}^{-1}$ (Figure 3.7(b)). This time the average lateral friction coefficient was fairly uniform with $\mu_l \approx 0.33$. This appears to be a reasonable approximation of quasi-static conditions. There is some oscillation in the measured friction coefficient, mostly due to the grainy nature of the expanded polystyrene surface. Thus the leading edge of the rubber strip bumps into the slightly raised cell boundaries of the polystyrene. Another possible contribution to the oscillating reading is the small amount of remaining stick-slip behaviour of the perspex bar on the PTFE pads. The amount of elastic slip before full mobilization of friction may also be determined from the initial slope of Figure 3.7(b). Here, $\xi \approx 0.3 \text{ mm}$. 

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**3.4.3 Measurement of Axial Friction Behaviour**

The apparatus used to measure the axial frictional response of a silicone rubber strip on an expanded polystyrene base is shown in Figure 3.8. Here, the 300 mm strip under test is pulled directly by the cantilever force transducer. The outputs from both the force transducer and the displacement transducer were fed to a datalogger.

The same force and displacement transducers were used as for the experiments described in Section 3.4.2. The experiments were started with slack in the cotton thread between the force transducer and the strip. The force transducer was then moved forward slowly, by means of a horizontal screw system, to take up the slack, and then the strip was pulled across the polystyrene base. The screw system was turned by hand at as uniform a rate as possible. Figure 3.9(a) shows the results for a pull velocity of approximately 0.4 mms\(^{-1}\).

The friction appears to settle to a value of \(\mu_l \approx 0.55\). The experiments were repeated turning the screw more slowly, giving a pull velocity of approximately 0.1 mms\(^{-1}\) (Figure 3.9(b)). In this case the friction coefficient settles down to a steady value over a much shorter displacement, with \(\mu_l \approx 0.36\). Again, this is taken as the value under quasi-static conditions. The initial curve on the plot is
Figure 3.8: Apparatus to measure axial friction behaviour.

Figure 3.9: Axial frictional behaviour of silicone rubber strip on expanded polystyrene base.
a result of the slack being taken in the cotton. Hence an “elastic slope” line has been added to the figure. It is the displacement between the point where this line intersects the horizontal axis and where the friction coefficient curve becomes non-linear at the top end which is taken as the elastic slip. Here, $\xi \approx 0.5 \text{ mm}$. A comparison with Figure 3.7 shows that there is much less oscillation in the force reading. This is because only the end of the strip, not a long edge, was being pulled against the raised cell boundaries of the polystyrene. Also, the perspex bar was not present in this experiment.

It can be seen that Figure 3.7(b) and Figure 3.9(b) are quite similar. Therefore, for the sake of simplicity common values were taken for both the lateral and axial directions. Reasonable values are $\mu_l = \mu_a = 0.35$ and $\xi = 0.4 \text{ mm}$. The different strips used during the physical modelling were all tested individually. They exhibited similar behaviour, with similar coefficients of friction.

### 3.5 Model Preparation

The physical model was designed to be very easy to set up for performing experiments. The base was given an initial pre-compression of approximately 5 mm to ensure that it was already into its uniform-straining region. The strip was laid centrally on the base against a long, straight steel bar. The ends of the strip were pinned to the base, either with a nail through a piece of copper tube glued to the larger strips, or a sharpened bolt through a nut fixed to the smaller strip.

These end fixings were meant to allow the ends of the strip to rotate if necessary; but it was found in preliminary tests that the strip would often rotate at one of the ends very early in the experiment if that end was slightly disturbed during this setting-up process. It was decided therefore to prevent this possibility in one of two ways. One is to place rigid guides either side of the strip over some chosen distance; the guides were dusted with French chalk to reduce axial friction with the strip. This method was used to investigate the feeding-in of straight material into a buckle. The other method was to apply lead weights to the last 30 mm at each end of the strip, thereby effectively making the ends encastre with respect to the base.

An initial imperfection was applied at a point by laterally displacing the strip by a known amount and then allowing it to relax back to an imperfect equilibrium position. This was normally done in order to control the position of eventual buckling; the imperfection was usually placed at the centre of the strip in order to keep the buckle as far away from the ends as possible.

The strip could also be laid in a curve, by using one of a set of specially made wooden templates ranging from 2.5 m to 4.0 m in radius. Imperfections
could be added either by pushing at a point as with the straight strip, or by
pushing drawing pins into the template before laying the strip against it. With
the second method, it was difficult to try and lay the strip with a symmetrical
imperfection as far as imperfection length was concerned. These experiments
were only performed as supplementary to the main experiments, in order to test
the assertion that a curved layout has little effect in eliminating unstable buckling
behaviour (Martinet 1936, Hobbs 1984).

3.6 Data Acquisition

The data collected during the compressible base experiments consists of the
end shortening of the base (δ), which is measured by a displacement transducer,
and global coordinates of points on the silicone rubber strip (X, Y) from the
digitiser; these measuring devices are both shown in Figure 3.2. It was found
not to be practical to put strain gauges on the silicone rubber strip in order to
measure the axial force in it.

3.6.1 The Digitiser

The digitiser used was an acoustic-type SAC GP-9. A cursor produces a
high-frequency burst of pulsed sound which is picked up by two microphones,
one at each end of the digitiser case. The position of the cursor is then found
automatically by triangulation. The claimed accuracy of the device was ±0.2 mm
over the entire coverage area (1200 mm × 900 mm). The data were passed from
the digitiser to a personal computer via the DataTalk software package.

The cursor was raised on blocks so that the crosshairs marked on the base of
the clear plastic alignment section were 1 mm above the surface of the rubber
strip. A second set of crosshairs was added to the cursor, 30 mm above the
original crosshairs and rotated by 45°, to reduce the effect of parallax while taking
readings. Simple tests were performed, repeatedly returning to set points, on a
large sheet of graph paper, over different parts of the coverage area. From this,
the error from physically lining the cursor up on a point, in addition to the error
due to the digitiser itself, could be found. Over much of the coverage area, all
the digitiser data fell randomly within a circle of approximately 0.3 mm radius.
The error was worse very close to one or other of the microphones; therefore
these areas were avoided during the experiments. Care was taken to perform
experiments during periods of relative quiet in the laboratory, since the readings
were subject to error due to extraneous sound.

For many tests, it was only necessary to measure the buckled region in the
central part of the strip (approximately 1.1 m in length); and for such measure-
ments the digitiser could be fixed in a single position. However, in order to cover
the entire polystyrene base, the digitiser had to be moved longitudinally in the
middle of recording the position of a strip. End stops were attached to the track
so that the digitiser could be moved reliably between two known positions. In
order to match the two sets of readings together, points common to both data
sets were matched. These included points on the strip, points pinned into the
polystyrene, and a global reference point fixed to the bench. The matching was
done to give an overall least-squares error in both the X and Y directions. Unfortu­
nately, on account of the small amount of overlap between digitiser positions to
get the required overall coverage, it was not practical to match rotations between
the two data sets by this method. If necessary, this was done by matching the
rotation of a least-squares line fitted to the second half of the data with the line
fitted to the first half, for the initial straight layout.

3.6.2 Data Processing

For the experimental data obtained from the physical model, data processing
has two main objectives. The first is to process the data in an attempt to recover
the true coordinates of a point from the digitised coordinates, which include
a random error as discussed in Section 3.6.1. This is especially important in
the initial stages of buckling, when small displacements occur, and the error
inherent in the position of a digitised point becomes important. The second
objective is to interpolate a smooth curve between data points. Various filtering
and interpolation schemes were employed, some more successfully than others.

For interpolation, the most basic class of function used for curve fitting is the
interpolating polynomial, which is easily manipulated mathematically. Unfortu­
nately, if fitting a large number of data points, the behaviour of the polynomial
may become too oscillatory: an n-degree polynomial may cross the x-axis up
to n times. These oscillations are artefacts of the computational scheme, and
bear no relation to a best-fit curve through the data. Piecewise polynomial (pp­)
functions, where polynomial segments are joined together smoothly, preserve the
ease of using standard polynomial functions while avoiding the main problems.
The simplest pp-function is the linear interpolating spline (straight line fit), but
the most popular pp-function is the cubic interpolating spline.

The cubic interpolating spline is used in the MATLAB numeric computation
software package (The MathWorks Inc. 1992), so this would be a convenient tool
to use. However, since the beam-bending equation involves a fourth derivative
do displacement, this term would become zero if a cubic spline were used; so a
higher-order pp-polynomial is strictly necessary. A quintic spline would allow a

*In this case smoothness is a measure of the continuity of first and higher derivatives of the
fitted function.
variation of shear force (which varies as $d^3y/dx^3$) without jumps. A jump would imply a concentrated load at a point – something which is not possible in the buckled region of the strip.

A different method for joint error-correction and interpolation is curve-fitting using a smoothing spline. The theory behind smoothing splines is explained in de Boor (1978), and makes use of the smoothing function

$$F = p \sum_{i=0}^{N} \left( \frac{Y_i - f(X_i)}{\delta Y_i} \right)^2 + (1 - p) \int_{X_0}^{X_N} \left( f''(X) \right)^2 dX$$

where $f(X)$ is the smoothing spline function,

$\delta Y_i$ is an estimate on the variance in the value of $Y_i$,

and $p$ is a smoothing parameter ($p \in [0, 1]$).

The best smoothing spline $f(X)$ is the one which minimises the function $F$. The first term is an “error squared” term, and would be minimised by the use of an interpolating spline passing through all the data points. The second term involves the squared curvature, and is proportional to the energy stored due to bending of an initially-straight rod made to conform to the shape of $f(X)$. Obviously this would be minimised by a straight line. The value of the smoothing parameter $p$ can be varied to achieve a balance between these two extreme splines. As $p \to 1$, the interpolating spline is dominant; and the straight line dominates as $p \to 0$. Unfortunately, in using this scheme on the present empirical data, the smoothing spline function was very sensitive to the smoothing parameter, and quite difficult to control. Also, the $X$ coordinates are used as the basis of this method, and only the values of the $Y$ coordinates are changed.

Perhaps a better way of processing the raw data is to use a travelling-window filter. A suitable candidate is a Savitzky-Golay filter, which is discussed by Davies (1997). This approach is based on a least-squares polynomial fit to the data points which lie within the travelling window. The window covers $(2r+1)$ data points, and a polynomial with $p$ coefficients is fitted. This is illustrated for $r = 3$ in Figure 3.10.

This scheme involves replacing data point $x_n$ with the value at that point of a polynomial fitted to the data $Z$ lying within the window.

$$Ma = z,$$

where $a = (a_0, a_1, \ldots, a_{p-1})^T$,

$z = (x_{n-r}, \ldots, x_n, \ldots, x_{n+r})^T$ (equally spaced),

and $M_{ij} = (i - r - 1)(j - 1)/((j - 1)!$
This expression can be taken to represent a Taylor expansion. The least-squares solution for \( \mathbf{a} \) comes from

\[
\mathbf{a} = (\mathbf{M}^T \mathbf{M})^{-1} \mathbf{M}^T \mathbf{z}.
\]

Since we are looking at a window centred on the \( n \)\(^{th} \) data point, \( a_0 \) will give a filtered value of \( x_n \). Estimates of the first, second \( \text{etc.} \) derivatives come from \( a_1, a_2/2 \) and so on. The amount of filtering depends on the choice of \( r \) and \( p \). Obviously if \( p \geq (2r+1) \) an interpolating polynomial is obtained, and so the data remain unfiltered. Also, as \( p \) is reduced, the implicit assumption is that the data are sufficiently oversampled within the window becomes more important. For the sample rate in these experiments (1 reading every 50 mm), this assumption is not valid. A further condition is useful for deciding the values of \( r \) and \( p \). Each of the \( X \) and \( Y \) coordinates of the raw data can be filtered independently since the error in one direction does not appear to be related to the error in the other direction; the mean error in each direction is approximately zero. As previously noted, the digitised data point generally lies somewhere within a circle some 0.3 mm in radius from the true point. Consequently, the filtered data point (which ideally would lie on the true point) should lie within a circle 0.3 mm in radius from the digitised data point. Various values for \( r \) and \( p \) can be tried with actual raw data to see when this condition is mostly met. Reasonable filtering was obtained for \( (2r+1) = 7 \) and \( p = 5 \). The standard Savitzky-Golay filter does not work for the \( r \) data points at each end of the data set. In general this is not a concern since the area of interest lies in the centre of the data range.
3.6.3 Coordinate Transformation

An important point to consider is the transformation between coordinate systems for the silicone rubber strip on a compressible base and for the corresponding thermally-loaded beam on a fixed base. As a point on the rubber strip moves when the base is shortened by an amount \( \delta \), its global coordinates change from \((X_1, Y_1) = (x_1, y_1)\) to \((X_2, Y_2)\), relative to a global origin \(O\) placed at the fixed end of the base. The subscripts 1 and 2 denote the coordinates before and after the base shortening. In the meantime, a point on the base beneath the strip has moved by an amount \((-X_1\delta/L, 0)\). This amount therefore needs to be added back onto the global coordinate for the point on the rubber. Applied along the length of the strip, this is equivalent to applying a stretch to the \(X\)-axis equal in magnitude to the compressive strain in the base. No change is made to the \(Y\)-axis, so \(Y = y\). Hence

\[
(X_2, Y_2) + (X_1\delta/L, 0) = (x_2, y_2) = (x_1, y_1) + (u, v) \tag{3.5}
\]

The filtered and transformed data were fitted with an interpolating quintic spline. The matrix definition of the spline process is rather too large to include here; it is defined in Ahlberg, Nilson & Walsh (1967), pages 160–165. The \(x\)-axis grid for spline-fitting was chosen to be uniform to allow for comparison between successive data sets.

Other schemes were tried, such as attempting to represent the buckle shapes as a series of orthogonal Hermite polynomials, but the Savitzky-Golay filter was the most straightforward and successful. In summary, the final data processing scheme was chosen to be:

1) Digitiser position matching of the two half-sets of data, if necessary.

2) Savitzky-Golay filter on \(X\) \((r = 3, p = 5)\).

3) Savitzky-Golay filter on \(Y\) \((r = 3, p = 5)\).

4) Coordinate transformation \((X, Y) \implies (x, y)\).

5) Quintic interpolating spline fit to \((x, y)\).
4.1 Introduction

Finite-element (FE) modelling plays many different roles in modern engineering. Although it was originally developed for stress analysis of problems too complicated for traditional analytical procedures, the scope of the method has been widened to encompass many other fields such as heat transfer and fluid flow. Indeed, FE packages have become standard tools in research and design. In the field of structures and stress analysis, a large proportion of FE analysis assumes linear-elastic, small-deformation behaviour, and there are great benefits associated with this: the solution can be arrived at quickly without load incrementation and iteration schemes, and superposition of load cases is valid. However, even though a linear analysis can produce a simple approximation of the behaviour of a structure, there are some cases where it is necessary to model not only more complicated material properties but also the effects of large rotations and deformations.

Geometrically nonlinear FE analysis takes into account the effect of changes in geometry on the load-deflection characteristics of a structure (Hinton 1992). For lateral buckling of a submarine pipeline the effect of large displacements and rotations, coupled with the hysteretic effects of friction, are clearly very important. For this type of analysis, the nodal displacements are taken into account when forming the equilibrium equations which are to be solved at any particular stage.

For the FE modelling presented here, the commercially-available software package ABAQUS/Standard (Hibbitt, Karlsson & Sorensen Inc. 1995) was used. It supports both linear and nonlinear analysis. Appendix B lists the various finite-element beam models which have been used; reference will be made to the model names listed there throughout the remainder of this dissertation.
4.2 FE Models of a Beam on a Frictional Foundation

Within the scope of this dissertation, any effects involving changes in the cross-section of a pipeline, such as ovalization, have been ignored. Therefore it is appropriate to model a pipeline on the seabed as a long, slender beam lying on a frictional foundation. The assumption for a beam element that the dimensions of the cross-section are small compared to typical dimensions along the axis of the beam reduces the problem to one dimension. The deformation of the beam is estimated from solution variables which are functions only of position along the length of the beam.

A basic approach for a beam is the Euler-Bernoulli assumption, namely that sections originally normal to the axis of the unstressed beam remain normal to that axis, and undeformed when the beam distorts. This is generally accurate for slender beams. The beam element chosen here was type ‘B31H’, a 2-node linear beam element, with hybrid formulation treating the axial force as a degree of freedom additional to the nodal displacements and rotations. The use of a hybrid beam element is recommended in the ABAQUS User’s Manual for extremely slender beams such as pipelines. The element is formulated using the Euler-Bernoulli assumption for arbitrarily large rotations and axial strains. Exact details of the beam element formulation are presented in the ABAQUS Theory Manual (Hibbitt, Karlsson & Sorensen Inc. 1995).

For a general beam analysis, the beam cross-section may be entered as one of many standard sections, for instance a rectangular or, rather more usefully for pipeline analysis, a pipe section. The x and y nodal coordinates follow the centreline of the beam. The composite strip used in the physical experiments was modelled by a rectangular beam (for the silicone rubber) and a circular beam (for the stiffening wire), sharing common nodal points.

4.2.1 Thermal Loading Model (TLM)

In the thermal loading model (referred to subsequently as TLM), the compressive axial force necessary for buckling was generated by applying a temperature rise of $\Delta T_{\text{eff}}$ (Equation (2.7)) to a constrained length of beam*. In order to apply a thermal load it is necessary to fix the coefficient of linear thermal expansion ($\alpha$) for the beam. In the case of a real pipeline, the value of $\alpha$ for the pipeline material can be used. For the composite beam, which is not tested under real

---

*The FE models were intended to provide a link to the physical model. Hence no attempt was made to simulate the effect of a length of additional straight beam beyond the end of the model by means of, say, a spring.
thermal loading, an arbitrary value of $\alpha$ is used. For the model of the composite beam presented here, a value of $10^{-5} \, ^\circ C^{-1}$ was used for both the rectangular and circular cross-section beams.

The base was created as a large rigid plane, with the friction modelled as a finite-sliding interaction between the deformable beam and the rigid base. The contact between the beam and the base was controlled by defining them as a slave/master contact pair. The friction can be defined by coefficients of friction in the two principal directions, tangential and normal to the local beam axis, and by an elastic slip, as defined in Figure 2.2. The value of elastic slip, $\xi$, applies to displacements in both principal directions; it cannot be specified for each direction independently without additional programming to redefine the frictional behaviour within ABAQUS. This was not thought to be necessary for the elastic slips of similar magnitude found in the physical model, in contrast with real pipeline design where a factor of 10 between axial and lateral friction mobilization displacements is commonly used. The B31H element is used for a beam in space (i.e. three dimensions), while lateral buckling is essentially a two-dimensional problem on a plane. The coordinate of the beam in the third dimension, normal to the plane of the rigid base, was fixed at a distance from the base such that the contact force between the beam and the base is equivalent to the weight per unit length of the beam.

To keep the size of the computation as small as possible, only half of the beam was modelled. Boundary conditions were imposed at the centre point to model either symmetry or antisymmetry. The conditions for symmetry are shown in Figure 4.1; end 1 would be the centre point of a full beam, and end 2 is the encastré end.

![Figure 4.1: Schematic diagram of the thermal loading model (TLM).](image-url)
The beam-element length was chosen so that there was a sufficient number of elements defining a characteristic buckled lobe; but it was found that if the element length is reduced too much, the small increase in accuracy does not warrant the larger increase in computational time. On some occasions it is helpful to vary the element length along the length of the beam, especially when long beams are modelled. Thus nodes can be concentrated towards end 1 where the buckle forms and greater resolution is required, and the node spacing gradually increased, towards the end 2 which is expected to remain more-or-less straight. Model TLM3 employs this nodal biasing, with the end 2 element being (arbitrarily) 100 times longer than the end 1 element. The element length increases in a geometric progression, with adjacent elements differing in length by the bias factor. In the case of model TLM3, this factor was 0.99426.

4.2.2 Compressible Base Model (CBM)

The physical model described in Chapter 3 employs a compressible polystyrene base to model the effect of thermal expansion. Therefore, a compressible base model (referred to subsequently as CBM) was also developed in order to model the unidirectional compression of the base with a beam resting on top. The strip was modelled by collinear beam elements, as before, but the base was modelled by ‘C3D8R’ brick elements. Since no output information was required from the base, the choice of base element was not too important. The 8-node reduced integration brick element was appropriately shaped and is not too computationally expensive. In contrast to the TLM described in the previous section, the entire length of rubber strip was modelled in the CBM, rather than just one half. The main reason behind this practice was to find out if the compressible base method of applying a compressive load to the beam would preserve the exact symmetry of the symmetrical buckling case, with lobes either side of the main central lobe attaining the same lengths and amplitudes. The ends of the collinear beams were tied to nodes on the base, making all active degrees of freedom (displacements and rotations) at the end of the beam the same as in the base immediately beneath it; thus the beam is effectively encastré at both ends. The end conditions for the physical model are not perfect (the strip is pinned to the base and then weighted down, as described in Section 3.5, but this is not perfectly encastré), so the “ideal” end condition is the most suitable for comparing model runs with changes in key parameters.

Since the elastic modulus of the expanded polystyrene is of similar order to that of the silicone rubber strip, significant distortion of the base elements around the tie nodes occurs, which affects the displacement linearity of the base. The elastic modulus of the base material was therefore artificially raised by a factor of $10^3$ to reduce such distortions to insignificant levels. The Poisson’s ratio for the expanded polystyrene was set to zero; it would be very small for this type
of cellular material, and the restraint applied by the loading rods in the physical model would make any small lateral strain in the base difficult to quantify. This condition means that it is not necessary to model the full width of the base, but just a width sufficient to contain the buckled strip over a full analysis. Figure 4.2 shows the dimensions used for the model. Compression of the base was applied in the X-direction only.

Figure 4.2: Schematic diagram of the compressible base model.

With the base no longer rigid, it was necessary to model the friction as finite sliding between two deformable bodies, and the more straightforward “contact pair” approach used above is not currently available within ABAQUS for this particular interaction problem. The contact was achieved by applying single-node ‘ISPl’ contact elements to the beam nodes, which interacted with a slide plane formed by the top surface of the base elements. This method of contact application is much more computationally expensive than the contact pair approach used for the TLM.

The main aim of this model was to provide a suitable link between the TLM and the physical model. If the CBM correctly modelled the physical model, and the results from TLM and CBM were found to correlate well, then the much simpler thermal loading model could be used for the parameter studies.
4.3 Linear Eigenvalue Analysis

As an initial calculation, a linear eigenvalue analysis of the beam on an elastic foundation was carried out for the thermal loading model with a perfectly straight beam. The foundation stiffness $k = \mu w/\xi$, which is the stiffness of the frictional model while in the elastic slip range. This type of analysis is commonly used to obtain an estimate of the critical buckling load of axially-loaded structures. In the case of a long, slender structure like a pipeline, it is likely that there are many closely-spaced eigenmodes, and associated eigenvalues. It is helpful to apply a preload in this case, to just below the critical buckling load, and then perform the eigenvalue analysis on a small perturbation load. For the thermal loading, an estimate of the critical temperature is given by

$$T_b = T_{preload} + (\text{eigenvalue} \times T_{perturbation}).$$

(4.1)

Table 4.1 presents values of critical temperature, and the associated buckled lobe length, for various values of preload and perturbation temperatures. Only the first symmetric mode was used for the calculation of these values. The length of beam under consideration was also changed. The last row of the table shows the central lobe length and buckling temperature predicted for an infinite beam on an elastic foundation.

<table>
<thead>
<tr>
<th>beam half-length [m]</th>
<th>No. of elements</th>
<th>$T_{pre.}$ [°C]</th>
<th>$T_{pert.}$ [°C]</th>
<th>lobe length [m]</th>
<th>buckling temp. [°C]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>5</td>
<td>0.1</td>
<td>0.0780</td>
<td>16.223</td>
</tr>
<tr>
<td>2.5</td>
<td>500</td>
<td>5</td>
<td>0.1</td>
<td>0.0777</td>
<td>16.196</td>
</tr>
<tr>
<td>2.5</td>
<td>500</td>
<td>5</td>
<td>0.5</td>
<td>0.0777</td>
<td>16.196</td>
</tr>
<tr>
<td>2.5</td>
<td>2000</td>
<td>5</td>
<td>0.1</td>
<td>0.0785</td>
<td>16.181</td>
</tr>
<tr>
<td>2.5</td>
<td>500</td>
<td>15</td>
<td>0.1</td>
<td>0.0781</td>
<td>16.188</td>
</tr>
<tr>
<td>10</td>
<td>2000</td>
<td>5</td>
<td>0.1</td>
<td>0.0780</td>
<td>16.189</td>
</tr>
<tr>
<td>10</td>
<td>2000</td>
<td>15</td>
<td>0.1</td>
<td>0.0780</td>
<td>16.182</td>
</tr>
<tr>
<td>$\infty$</td>
<td></td>
<td></td>
<td></td>
<td>0.0782</td>
<td>16.197 from eqn.(2.2)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>from eqn.(2.3)</td>
</tr>
</tbody>
</table>

Table 4.1: Linear eigenvalue analysis.

The differences between the critical temperatures and lobe lengths in all the above finite-element cases are very small. There does not seem to be a clear pattern for the difference in the buckling temperature and lobe length with changing beam half-length, number of elements, and preload and perturbation temperatures. The differences may well be accounted for in part by small numerical rounding errors in the analyses. It could be expected that a higher node density,
or a higher preload and smaller perturbation temperature, would bring the results of the numerical analysis closer to the values from an elastic analysis; but that does not seem to be the case. However, these differences were small enough to not be considered significant: the computation scheme is evidently not sensitive, within reason, to the element size or the details of the perturbation scheme.

4.4 Nonlinear Analysis

In order to investigate inelastic pre- and post-buckling behaviour of the beam on a frictional base, it is necessary to run a nonlinear analysis. Only geometric and frictional non-linearities are considered — the silicone rubber and steel composite strip may be taken as behaving linear-elastically in these circumstances. This analysis involves incrementing any loading and solving the equilibrium equations subject to prescribed tolerances on the convergence of force residuals and incremental displacements.

In ABAQUS, the Riks method may be selected for solving problems where there is a possibility of unstable behaviour. This is particularly applicable to buckling when small initial imperfections are involved; for larger imperfections, the force/displacement behaviour is normally fully stable for the applied loading.

Figure 4.3: Schematic example of unstable static response.
Figure 4.3 shows a schematic example of an unstable static force/displacement curve. The unstable snap-through for a force-controlled system is shown by the dashed line; the part of the curve which cannot be followed is shown dotted. The Riks method allows the entire response curve to be followed, whether stable or not, as long as the response does not contain any sudden bifurcations. The algorithm increments the arc length along the equilibrium solution path; both the load and the displacement are treated as unknowns in the scheme.

A general strategy that is widely used (e.g. Tvergaard & Needleman (1981)) is to take an initial imperfection in the form of one of more of the eigenmodes obtained from the linear analysis. However, in this particular study, the initial imperfection was put into the perfectly straight beam by imposing a lateral displacement to the central node, and then releasing to allow the strip to relax back to a new, imperfect equilibrium position. This method is equivalent to the way in which an imperfection is imposed on the strip in the physical model, and is similar to the imperfection which would be found in a real pipeline if it were forced sideways by an obstacle on the seabed. In this respect it is perhaps a more realistic type of imperfection than the slowly-decaying sinusoidal curves of the first few eigenmodes.

4.4.1 Example Analysis of the TLM

A typical analysis of the thermal loading model consists of 4 consecutive steps, as follows.

1) Apply a lateral displacement to the end of the half-beam, at the point where the symmetry boundary conditions are imposed.

2) Release the half-beam and allow it to reach equilibrium — this is the “initial imperfection” for the beam.

3) (a) If the analysis is to use an initially-stressed beam, proceed straight to step 4).

(b) If the analysis is to use an initially-unstressed beam, save the displaced nodal coordinates and use them in as the nodal input to a new analysis consisting of step 4) only.

4) Apply a linearly-changing temperature rise over the course of the step; this temperature rise is uniform along the length of the half-beam.

Depending on the size of the initial imperfection, the temperature application step could consist of either a monotonically increasing temperature (for large imperfections) or a Riks step with the temperature as an unknown (for small imperfections giving rise to an unstable equilibrium solution).
An example analysis using model TLM2 is now presented. A lateral displacement of 2 mm was applied, and the beam was then relaxed to give an imperfection amplitude of 1.20 mm. This is the same size as the smallest deliberate initial imperfection imposed of the physical model. For an amplitude of this size, there was no unstable jump in the response; this is similar to the response shown schematically by the right-hand dashed line of Figure 2.4(b). The temperature was applied gradually to a final value of 1250 °C. This value is meaningless in itself because of the arbitrary value of the coefficient of linear thermal expansion given to the beam. The constrained thermal load at this temperature in a perfectly straight beam would be $EA \times 1.25\%$; which is the same as the compressive load in a beam on the compressible base for an end shortening of 30 mm. This end shortening is around the maximum which can be applied to the physical compressible base before the handle of the chain drive (Figure 3.3) becomes too hard to turn any further. The lightly-shaded line in Figure 4.4 shows the beam at the end of step 2, and the darker lines show the buckle growth in 15 equal stages over step 3. The half-beam which was analysed here has been mirrored about $x = 0$ to aid comparison with the full-length beam of the compressible base model.

![Buckle growth for TLM2 example analysis](image)

**Figure 4.4:** Buckle growth for TLM2 example analysis. The $y$–axis scaling is 25 times the $x$–axis scaling.

On initial buckling, the beam adopts what is essentially a “mode 3” geometry (Figure 2.3), with only a very small overshoot beyond the first pair of side lobes. These lobes, on either side of the central lobe 1, are each named lobe 2. As the
temperature increases, further pairs of lobes progressively start to grow. Various other features may be seen in Figure 4.4: they will be described in detail below.

4.4.2 Example Analysis of the CBM

A typical analysis of the compressible base model consists of 4 consecutive steps, as follows.

1) Apply a lateral displacement to the central node of the beam.

2) Release the beam and allow it to reach equilibrium — this is the “initial imperfection” for the beam.

3) (a) If the analysis is to use an initially-stressed beam, proceed straight to step 4).
   (b) If the analysis is to use an initially-unstressed beam, save the displaced nodal coordinates and use them in as the nodal input to a new analysis consisting of step 4) only.

4) Apply a steadily increasing displacement to the nodes at one end of the compressible base.

![Graph](image)

**Figure 4.5:** Buckle growth for CBM2 example analysis. The $y$–axis scaling is 25 times the $x$–axis scaling.
An example analysis using model CBM2 is now presented. A lateral displacement of 2 mm was applied, and the beam relaxed to give an imperfection amplitude of 1.20 mm. The end of the compressible base was moved gradually to a final value of 30 mm. The lightly-shaded line in Figure 4.5 shows the beam at the end of step 2, and the darker lines show the buckle growth in 15 equal stages over step 3. These curves have had the coordinate transformation $(X, Y) \rightarrow (x, y)$ applied to them in exactly the same way as for the physical model (Section 3.6.3)

Figure 4.5 appears to be very similar to Figure 4.4; but there are some small differences, which will be explored in the following section.

A typical CBM analysis takes up to 7 times as long to run as the equivalent TLM analysis.

### 4.4.3 Comparisons between the Example Analyses

In order to compare the two types of finite-element model, there must first be a common reference quantity to define the application of load. For the TLM, the temperature in the beam is increased over the load step, while for the CBM it is the base shortening which is changed. One such reference quantity, which can readily be determined in both cases, is the compressive load in a hypothetical short beam which is fixed to the base at both ends and constrained to remain straight. For thermal loading, this load would be equal to $(EA\alpha \Delta T_{eff})$, while for the CBM it would be equal to $(EA\delta / L)$. This “constrained compressive load” would be equivalent to the load in a beam, remote from the buckled region, if a sufficiently long beam were being modelled.

Figure 4.6 shows the development of buckle lobe amplitudes $(a)$ and half-wavelengths $(\lambda)$ with increasing constrained compressive load. Only lobes 1–3, as indicated in Figure 4.4, have been included for the sake of clarity.

The main point to notice is that the behaviour of the CBM is very close to the behaviour of the TLM. The difference between the lobe amplitudes varies between 0.5% and 2.5% by the end of the calculation; the amplitude is higher for the CBM in lobe 1, and lower in the other 2 lobes. The CBM consistently has a longer lobe length for all lobes, on average 1% longer by the end of the step. This difference is slightly less than the 1.25% stretch (the coordinate transformation) applied to the CBM results at the end of the step. Also the CBM does not give a perfectly symmetrical buckled geometry; but the differences in equivalent lobe lengths and amplitudes for lobes either side of the central lobe are insignificant ($\ll 1\%$).

These differences are not easily attributable to one specific feature of either model; they are a combination of the different way the friction is applied in each
model and the differences between thermal and compression modelling already mentioned in Section 3.3.2.

A further comparison can be made between the two models by considering the axial force at the centre of each beam. For the way the models are constructed, the total axial force is the sum of the forces calculated for the two co-nodal beams which make up each beam model. Figure 4.7 shows the development of this force
over the course of the analysis.

The axial load at the centre of the buckle is practically identical for both models. Note that the two points which show positive (tensile) axial force are for the application of a lateral displacement of the centre of the beam, and the subsequent relaxation, in order to form an initial imperfection.

### 4.4.4 Conclusion

The unidirectional compressible base model has been shown by these computations to be in very good agreement with the thermal loading model over the practical range of base end displacements of the physical model. Therefore, if a full-scale thermal loading problem can be reduced to a suitable model scale by means of dimensional analysis, with the relevant parameters such as frictional forces known reasonably accurately, then the physical compressible base model should give a good representation of the behaviour of the full-scale pipeline.

Since the thermal loading model uses a rigid base and the easily implemented “contact pair” approach to define the beam/base interaction, there are great savings in the computational time required for TLM analyses over CBM analyses; this is therefore the preferred finite-element model for any further numerical analysis.
4.5 A Measure for Buckle Evolution

At this point it is convenient to define another term which is useful for comparing the evolution of buckle growth in different situations: the free end displacement (f.e.d.). The f.e.d. is the amount that the cut end of half of a perfectly straight, encastré beam would displace axially under loading. For the thermal loading model in Figure 4.1, this would be the axial displacement of end 1 with the axial constraint removed, and is calculated from Equation (4.2):

$$\text{f.e.d.} = \begin{cases} 
\frac{EA\alpha^2}{2\mu_a w} \frac{\Delta T^2}{T_c^2} & : 0 < \Delta T \leq \Delta T_c \\
\frac{EA\alpha^2}{2\mu_a w} \frac{\Delta T^2}{T_c^2} + \alpha(\Delta T - \Delta T_c) \frac{L}{2} & : \Delta T > \Delta T_c
\end{cases}$$

(4.2)

where

$$\Delta T_c = \frac{\mu_a w L}{2EA\alpha}$$

is the temperature at which the axial friction force is fully mobilized along the entire length $\frac{L}{2}$ of the half-beam. The calculations leading to Equation (4.2) are straightforward. As the temperature in the beam starts to increase, there is a region of slip near the free end of the beam with no slip beyond. In the slip region, the axial compressive force $P$ is equal to $\mu_a w x$, with $P = 0$ at the free end ($x = 0$) and $P = EA\alpha \Delta T$ in the non-slip region. This force distribution is clearly shown later in this section as the total shaded area in Figure 4.10. The free end displacement is proportional to the shaded area.

These equations assume ideal Coulomb friction. In order to take account of elastic slip, an adjustment should be made. For small temperature rises, while the foundation forces are totally in the elastic range (f.e.d. $\leq \xi$), the axial displacement along the strip is governed by

$$\frac{\partial^2 u}{\partial x^2} + \frac{k u}{EA} = 0$$

(4.3)

where $k = \mu_a w / \xi$, and, for the appropriate boundary conditions, the free end displacement is given by

$$\text{f.e.d.} = \sqrt{\frac{EA}{k \alpha}} \Delta T \tanh \left( \sqrt{\frac{k L}{EA}} \right)$$

(4.4)

The f.e.d. therefore depends the elastic slip and the length of the beam, as well as the axial friction force and the axial stiffness.

Once the free end displacement has exceeded the elastic slip, then its value should be determined jointly from equation (4.3) to the point where $u(x) = \xi$, and using the ideal friction analysis behind equation (4.2) beyond that point.
Figure 4.8: Variation in the free end displacement with temperature rise for “long” and “short” beams. The point at which axially friction is fully mobilized along the entire length of the short beam is marked ‘*’.

Figure 4.8 shows how the f.e.d. varies with temperature for particular cases of a “long” and a “short” beam, as described below. These examples come from the analysis into the effect of model length to be presented in Section 4.6.4. For ideal Coulomb friction, and within the temperature range shown, the long beam (dotted line) has yet to reach $\Delta T_c$, and the region of axial slip does not extend to the encastré end. The short beam (solid line) reaches $\Delta T_c$ at approximately 13°C, which is where the solid line deviates from the dotted line. This is when the region of axial slip reaches the encastré end. The solid curve is linear beyond this point.

Although these two methods of calculating the f.e.d. are significantly different for temperature rises corresponding to the early stages of buckling, for higher temperature rises the f.e.d. is sufficiently well approximated by Equation 4.2. This is also a much simpler calculation, of course, especially in cases when it is not easy to measure, or even guess, the value of the elastic slip of the foundation response.

The f.e.d. is a better measure than, say, temperature or fully constrained axial load for the evolution of buckles in beams of finite length, since it takes into account the change in behaviour once the full friction has been mobilized over
the entire length of the beam; in effect it provides a way of comparing long and short beams on an equal footing.

There is a close empirical relationship between the free end displacement and the change in contour length of a buckled beam during a completed buckling event. This is demonstrated in Figure 4.9, where the f.e.d. is shown plotted against the change in contour length for an example analysis of a beam of half-length 12 m and elastic slip $\xi = 0.4 \text{ mm}$ (model TLM3, which is used for the parameter studies to be introduced in Section 4.6) loaded to an equivalent compressible base strain of 0.17%.

![Figure 4.9: Relationship between the change in contour length along the beam and the free end displacement for model TLM3 loaded to an equivalent compressible base strain of 0.17%.

The contour length along the beam was computed using Pythagoras' Theorem to calculate the distances between nodes, and these were summed over the length of the beam."

It is clear from Figure 4.9 that the relationship is not one of precise equality; indeed, exact equality is not expected. If the change in contour length is to be equal to the f.e.d., this is equivalent to taking the displaced free end of a straight strip and moving it back to $x = 0$, allowing lateral deflections to occur, but

*For model TLM3, the node spacing is very small in the buckled region where the beam is curved, and gets larger in the straight region near the encastré end. This summation will give a very good approximation to the contour length.
keeping the axial force distribution along the length of the beam the same. For a buckled beam, even for ideal Coulomb friction and infinite length, the force at the centre of the buckle never falls to zero. The idealised situation is illustrated in Figure 4.10.

![Diagram of axial force distribution](image)

**Figure 4.10:** Schematic of an idealized axial force distribution in a half-beam. The lightly-shaded area is proportional to the change in contour length of the beam, while the total shaded area is proportional to the free end displacement (Equation (4.2)).

The entire shaded area is equal to \((\text{f.e.d.} \times EA)\). The more darkly-shaded area is the difference due to the buckled region where the load at the centre of the buckle is non-zero and does not remain constant throughout the region (a), while the small elastic slip produces a slight curve in the force distribution close to fully-restrained end (b). If the buckled beam were straightened out, while maintaining the same force distribution, the end displacement (equal to the change in contour length) would be slightly smaller than the f.e.d. due to the small difference mentioned above. Nevertheless, Figure 4.9 shows that f.e.d. is a useful measure of the current state of a buckle. F.e.d. will be used extensively in the following sections.

### 4.6 Parameter Studies

A great advantage of finite-element modelling is the ease with which the model parameters can be altered. With the physical model, the only parameter which could be easily changed is the unit weight of the strip, by addition of a flexible weight layer to the top of the strip. Changes to the bending and axial stiffnesses
would require the manufacture of a new strip, and changes to the axial and lateral friction properties would require either the strip or the compressible base to have its surface roughened or smoothed.

Within the finite-element model, in contrast, many parameters can be changed easily and independently, including:

i) initial imperfection length, \( \lambda_i \)

ii) initial imperfection amplitude, \( a_i \)

iii) elastic slip, \( \xi \)

iv) length of beam model, \( L \)

v) initial stress in beam

vi) bending stiffness, \( EI \)

vii) axial stiffness, \( EA \)

viii) axial coefficient of friction, \( \mu_a \)

ix) lateral coefficient of friction, \( \mu_l \)

x) weight per unit length, \( w \).

Items i), ii), iii) and v) have been shown in previous studies to have differing degrees of effect on the onset of initial buckling, but to have a limited effect on the far-post-buckling behaviour, at least as far as upheaval buckling is concerned (Ju & Kyriakides 1988, Maltby 1992). Item iv) does have an effect (Kerr 1979), but use of the free end displacement (f.e.d.) takes account of it. It was therefore decided to examine briefly the effects of these five parameters on lateral buckling behaviour in the remainder of this chapter, and then to examine the effects of the remaining five parameters on the far-post-buckling behaviour in Chapter 5.

Since the thermal loading model (TLM) and the compressible base model (CBM) have been found to be broadly comparable, and the parameter studies only really need to be consistent within themselves, it was decided to use the TLM for this study, because it is much simpler and quicker to run. Model TLM3 (half-length 12 m) was used for these analyses, although only 0.25 m adjacent to the centre of the beam, which is the main region of interest, will be examined in the following sections. The beam was laterally displaced by 3 mm at the centre point, and then relaxed to 2.06 mm. This imperfection is larger than the one used previously in Sections 4.4.1 and 4.4.2 in order to ensure that, for ease of comparison, the same number of equal load increments are present in each step over the range of imperfection sizes and model lengths to be analysed, with no
convergence problems. This imperfect geometry was recorded and, after scaling, used directly as a nodal input to the finite-element model. Note that in this case the beam is un\textit{stressed} at the start of the thermal loading step; this is equivalent to laying the beam against an imperfect form, as described for the curved strips in Section 3.5.

In Sections 4.6.1–4.6.4, each of the first four parameters listed above will be varied one at a time. Comparisons will be made by plotting variables such as the lobe lengths, lobe amplitudes, and the central compressive load against the free end displacement. A comparison between initially stressed and unstressed beams with identically-shaped initial imperfections will be made in Section 4.6.5.

In all five cases it will turn out that soon after buckling (f.e.d. > 1.5 \textit{mm}) the beam/foundation interaction has effectively lost its “memory” of the initial imperfections. This is true for the side lobes (lobe 2), although there are cases when the central lobe continues to be affected; these cases will be discussed as they arise.

\section*{4.6.1 Initial Imperfection Length}

The imperfect geometry was scaled in the $x$-direction by length factors of 0.25, 0.50, 0.75, 1.00 and 1.25. A length factor of 1.00 gives the “natural” imperfection length obtained by the initial displacement and relaxation process as described in Section 3.5; in this case $\lambda = 0.232$ m. A length factor of 0.5 corresponds to a contraction of the “natural” imperfection length to a half while keeping the size of the amplitude the same. Figure 4.11 shows some results from these analyses.

The smaller subplots of Figure 4.11 show the progressive growth of the imperfection into a buckled state. The initial shapes, with different lengths, are clearly visible. The post-buckled shapes are broadly similar. For the length factor of 0.25 (and to some extent 0.50), the central lobe continues to be distorted by the initial imperfection. The larger subplot shows the axial force at the centre of the beam against f.e.d. The light dashed line shows how the axial force would grow for the same beam constrained to remain straight.

The larger the length factor, the lower the curvature in the imperfection; hence the axial compressive load should reach a higher value before deviating from the constrained line. Interestingly, the lowest maximum compressive force does not correspond to the lowest length factor. The two shortest imperfections are able to sustain a higher load as the imperfection grows both axially and laterally towards a preferred size. The beams with imperfection length factors of 1.00 and 1.25 exhibit a more rapid reduction in compressive load. The axial loads in all the beams appear to be tending towards a common curve, although the still-distorted shape of the central lobe in the two shorter cases means that this convergence has not occurred during the loading “period” presented here.
Figure 4.11: Effect of initial imperfection length on buckle shape and central axial load. For each length factor, the buckle shape evolution is shown by curves for the initial layout and successively after five equal temperature rises.

At this stage, it is worth pointing out that since the temperature (and therefore f.e.d.) are increasing over the thermal loading step of the finite-element analysis, it is often easier to make simple comparisons in terms of time-like concepts such as “loading period” rather than an absolute change in temperature or f.e.d.

Note that the f.e.d. only reaches 1.7 mm at the right of the plot — this value was chosen so that the f.e.d. was still varying parabolically with temperature rise for the 12 m long half-beam (i.e. the axial slip region has not reached the encastré end — see Figure 4.8) so that a later comparison with the parabolic → linear f.e.d./temperature relation of shorter beams can be made. Also, this is approximately the level of f.e.d. at which the physical model simulates the operating conditions of the pipe in a case study to be described in Chapter 5.

The change in amplitude and length for both the central lobe (lobe 1) and first side lobe (lobe 2) with increasing f.e.d. is shown in Figure 4.12. Only lobes 1
and 2 have grown significantly enough for the temperature rise analysed here to permit worthwhile analysis. In Figure 4.12(b) the lobe 1 curves are demarcated by the two circles. Because the initial imperfection initiates the growth of lobe 1, and this in turn affects the growth of lobe 2, it is perhaps not surprising that the variation in lobe length and amplitude is much greater for the central lobe than it is for the side lobe, at least this early in the evolution of the buckled shape. For lobe 2, the situation is more clear. By the time the f.e.d. = 0.5 mm, the lobe length is practically identical is each case, and there is little variation in the amplitude. This suggests that lobe 2 and any lobes more remote from the centre of the beam would not be influenced by the size of the initial imperfection. Lobe 1 retains the influence of the initial imperfection for a longer period; for instance, when the f.e.d. = 1.5 mm the extent of the variation in lobe length against the length for the “natural” imperfection is +6%, −8%. As mentioned in Chapter 2, it is almost
certainly the case that, unlike upheaval buckling on a rigid base where generally just one uplifted lobe is formed which eventually settles into a preferred buckled geometry, the formation of outer lobes in lateral buckling cases prevents the central lobe from reaching an unique, imperfection-independent geometry. The behaviour of lobe 2 is very similar to the imperfection-independent post-buckled behaviour discussed by Ju & Kyriakides (1988) for upheaval buckling.

4.6.2 Initial Imperfection Amplitude

A similar analysis was performed for a variation of the amplitude of the initial imperfection. Amplitude factors of 0.50, 1.00 and 1.50 were applied to the imperfection with a common length factor of 1.00. Plots of the central axial load, lobe amplitudes and lobe lengths against f.e.d. are presented in Figure 4.13.

![Figure 4.13: Effect of initial imperfection amplitude on buckle lobe amplitudes, lengths and the central axial load. The curve for a straight, fully-constrained beam is shown for the central axial load.](image-url)
The axial load follows the same trend as for the changing wavelengths; the smaller the amplitude factor (and hence the smaller the initial curvature in the imperfection), the larger the maximum compressive force. The maximum force occurs just beyond the departure from the "fully constrained" line.

The change in amplitude and length for lobe 2 again falls within a narrow band over the range of amplitudes shown. In this parameter study, the worst agreement is for the amplitude evolution of lobe 1, where a case with larger initial amplitude retains a larger amplitude throughout the analysis. When the f.e.d. reaches 1.5 mm, the variation in lobe amplitude from the amplitude for the "natural" imperfection is ±5%; although this difference reduces as the f.e.d. increases further.

4.6.3 Elastic Slip

Model TLM3, with the initially-unstressed "natural" imperfection, was used again to study the effect of elastic slip in the foundation. As discussed in Chapter 2, it has been reported that, for upheaval buckling in the presence of an initial imperfection, the buckling load depends only on the plateau value of the lateral frictional force, and not on the shape of the initial foundation force-deflection response.

Figure 4.14 presents plots of the central axial load, lobe amplitudes and lobe lengths against f.e.d. for different values of elastic slip. Actually, there is a slight variation in the peak force reached; the smaller elastic slip gives rise to a larger compressive load. There does not appear to be a simple power-law relation between the peak load and the elastic slip (cf. Equation 2.9 with a -0.5 power for a perfectly straight beam); this may be a result of using the single imperfection shape, formed on a foundation with $\xi = 0.4$ mm, rather than a "natural" imperfection shape for the specific elastic slip value concerned.

However, in the far-post-buckling region, any small effect due to the elastic slip very quickly disappears. This range has not been shown on the figure, in order to allow clearer presentation of the differences in peak central compressive load. Since the far-post-buckled region will be the main focus of interest in the following chapter, the effect of the elastic slip will not be considered further.
4.6.4 Length of Beam

The effect on buckling behaviour of the overall length of test pieces of railway track has been discussed in some detail by Kerr (1979). It was found that the shorter the length of track, the smaller the post-buckled lobe amplitude for a certain temperature rise, and the higher the central axial compressive force.

Model TLM3 was used as above in order to investigate the effect of overall length on the behaviour of a pipeline. Length and amplitude factors of 1.00 for the initial out-of-straightness imperfection were used. The length of the half-beam was successively taken as 12 m to 5 m, 2.5 m and 1 m. The relationship between the temperature rise and the free end displacement was shown earlier for two of these lengths in Figure 4.8. This behaviour can be seen clearly in Figure 4.15, where the central axial load, lobe amplitudes and lobe lengths have been plotted against temperature rise, curtailed to 40°C for clarity.

As explained in Section 4.5, the usefulness of the free end displacement as a
measure of the buckle evolution is that it provides a common reference for models of beams with different finite lengths. The central axial load, lobe amplitudes and lobe lengths from Figure 4.15 are replotted against f.e.d. in Figure 4.16.

For the beams of half-length 12 m, 5 m and 2.5 m the analyses were run for the same temperature rise as used in the previous sections, and hence the free end displacements are different in each case. For the shortest beam (half-length 1 m) the temperature rise was increased so that the maximum f.e.d. was approximately the same as for the longest half-beam, to allow a better comparison between the two extreme lengths chosen. Note that at initial buckling, the length \((\frac{1}{2} \lambda_1 + \lambda_2)\) from the centre of the beam to the end of lobe 2 (lobe 3 is not significant at this stage) is approximately 0.2 m, which is one-fifth of the length of the shortest half-beam but only one-sixtieth of the length of the longest half-beam considered here.

Several points emerge from this study. An important one is that the f.e.d. approach does prove to be very good at allowing direct comparisons to be made
Figure 4.16: Effect of beam length on buckle lobe amplitudes, lengths and the central axial load, plotted against free end displacement.

between analyses when equal temperature rises lead to dissimilar lobe growths. However, there is still a slight discrepancy between the different examples just after buckling on all the plots.

It should be pointed out at this stage that the “short” beams analysed here are significantly longer in relation to the buckled length than many of the short railway tracks considered by Kerr (1979). In the most extreme cases that he reports, the buckled region could occupy practically the entire track length. The f.e.d. approach would not be valid in such a situation; it presupposes that the buckled region is short compared with the length of the beam as a whole (cf. Figure 4.10); however, it does not require the beam to be so long that the load reaches the fully restrained value of $EA\alpha\Delta T$ within its length.
4.6.5 Initial Imperfection Stress

Figure 4.17 shows the difference in behaviour between two beams with identically sized initial imperfections. One is initially unstressed, while the other is left with a tensile axial load in the imperfection from where the beam was pulled laterally to form the imperfection. A larger temperature rise was applied to these beams than to the beams in the previous sections in order to show clearly the differences in the growth in lobe amplitudes.

![Central axial load and Lobe amplitudes](image)

**Figure 4.17:** Effect of initial imperfection stress on buckle lobe amplitudes, lengths and the central axial load.

The initially-stressed beam reaches a greater maximum compressive load, and approaches, but never quite reaches, the central axial load of the initially-unstressed beam. The first two lobes settle down to similar lengths very quickly; similar behaviour can be seen in Figure 7 of the paper by Ju & Kyriakides (1988) on upheaval buckling.

However, the growth of the lobe amplitudes is very different between the two beams. The main central lobe of the initially-stressed beam tends towards a constant amplitude by the end of the analysis, while the equivalent lobe of the
Initially-unstressed beam is around 20% higher at the same stage in the analysis, and still growing. This behaviour is not evident from Ju & Kyriakides (1988), but may be due to the differences between lateral and upheaval buckling; the axial tension in the imperfection region appears to increase the size of lobe 2, and to some extent the smaller, currently insignificant lobes 3 onwards, which in turn continue to grow at a slightly faster rate and act as a "buffer zone" to the growth of lobe 1. The initial stress state does, therefore, have a significant effect on the ultimate buckle lobe shape. However, the initially-unstressed state chosen for most of the parameter studies to allow a fair comparison between calculations with different initial imperfections is actually the most onerous state - it produces a higher central lobe amplitude and lower buckling load. In fact, in pipeline design, the beneficial effect of tensile residual loads from pipelaying are often ignored so as to give the most conservative design conditions.

4.7 Conclusions

The comparison between the two types of finite-element model, namely the thermal loading model (TLM) and the compressible base model (CBM), has shown that their behaviour is very similar in all respects. This led to the decision to use the TLM model, which is much simpler and quicker to run, for the parameter studies.

A partial parameter study has been carried out on the finite-element model. This has identified several model parameters which play little part in the far-post-buckled behaviour of a buckled beam; and therefore they will not be considered any further. The length of the beam under test did indeed affect the post-buckled behaviour of the beam when the buckle evolution is measured against temperature rise, but the use of the free end displacement as a measure of loading has effectively enabled the beam length to be eliminated from consideration.

The stage is set, therefore, for a study in Chapter 5 of the effect of varying parameters that do have a significant effect on the overall post-buckled behaviour of a beam.
Chapter 5

Results and Comparisons

This chapter summarises results obtained from both the physical and finite-element models of a nominally straight pipeline lying on the seabed. Some simple preliminary experiments are presented first, to validate the behaviour of the finite-element model against the physical model. These are followed by comparisons between lateral buckling experiments performed with the physical model and the finite-element model, particularly in relation to a case study of a particular buckled submarine pipeline. Then the results of the remaining parameter studies performed with the finite-element model are presented, where the concept of lobe extinction is introduced. Finally, some results from the physical model are presented for a silicone rubber strip lying on a horizontal curve.

5.1 Preliminary Experiments

Some simple preliminary experiments, separately pulling a strip laterally at a point and axially at the end, were performed in order to judge the behaviour of the physical model against both theoretical and finite-element model results. The unstiffened strip U1 (1.8 m long), as detailed in Appendix A, was used for these experiments. This was because, for the axial-pull experiment at least, it is the only strip which would displace significantly at the end without the whole strip sliding, due to its low axial stiffness.

5.1.1 Lateral-Pull Experiment

The first of the preliminary experiments is the lateral pull, where a lateral point load is applied to the strip at its midpoint, as shown schematically in Figure 5.1. A small metal clip was glued to the side of the strip, over which a hook was placed. The hook was attached to a cotton thread passing over a
low-friction pulley to a very light loading pan. The clip was attached near the bottom of the strip to avoid the tendency of the strip to overturn. The cotton thread was kept horizontal so as not to lift the bottom edge of the strip.

The strip was laid straight, and one half of its length digitised, as explained in Chapter 3. Measurement of only half of the strip eliminates the need to move the digitiser between readings; and since the size of the lateral displacement at the point of load application was of main concern here, any slight asymmetry in the deflected shape was not considered too important for this particular experiment. Masses were then added to the loading pan, and the strip redigitised. A typical loading sequence is presented in Figure 5.2 for both the physical experiment and finite-element analysis, with lateral loads of 0, 10, 15 and 20 grammes. The half-strip has been reflected about the loading point to give the appearance of a full strip. The finite-element model TLM1, described in Appendix B, was used to predict the behaviour of the strip using the values measured in Section 3.4 for the geometric and material properties of the strip.

The agreement between the sets of curves is excellent as far as the displaced length is concerned. It is good for the lateral deflection of first two load increments; but it becomes slightly worse for the third increment. One possible reason for this is that the parameters measured in Section 3.4 were not entirely accurate. This is especially true of the interface relationship between the silicone rubber and the expanded polystyrene which, as well as the velocity-dependence noted in Section 3.4.2, also exhibits a degree of time-dependence, or creep. Table 5.1 shows the lateral displacement of the loaded point at various loads for two different experiments. The second column shows the lateral displacement within two minutes of the load being applied (that is approximately the time taken to digitise half of the strip). The third column shows the lateral displacement taken when
Figure 5.2: Experimental and FE lateral-pull tests.

Table 5.1: Time dependence of the lateral-pull experiment.

<table>
<thead>
<tr>
<th>Mass on pan [g]</th>
<th>Central lateral displacement ( v ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>reading taken instantly ( \frac{dv}{dt} \leq 1\text{mm}/\frac{1}{2}\text{hr} )</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------------------------------------</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>6.5</td>
</tr>
<tr>
<td>15</td>
<td>19.3</td>
</tr>
<tr>
<td>20</td>
<td>41.7</td>
</tr>
</tbody>
</table>

The point is moving at less than approximately \( 1\text{mm}/\frac{1}{2}\text{hr} \) (an arbitrary limit for the “stopped” condition). The final column is the lateral displacement predicted by from Equation (2.10), based on small deflections and an infinite beam.

The instantaneous displacements are roughly two-thirds of the “stopped” displacements, but this time-dependent “creep” continues over time. Since the best agreement with the FE static analysis was obtained by taking the readings immediately after the application of a load, this was the policy adopted for the main series of experiments.

The central deflection obtained from Equation (2.10) is not relevant for the
higher lateral loads; the effect of large displacements and the finite length of the silicone rubber strip mean that the underlying assumptions are not valid. But for the 10 gramme load, the displacements for both the physical and the finite-element models are close to the value obtained from the equation.

5.1.2 Axial-Pull Experiment

The second preliminary experiment was an axial-pull test, which is shown schematically in Figure 5.3. The short piece of copper tube was glued to the end of the silicone rubber strip. The cotton thread ran over a pulley to the light loading pan.

![Figure 5.3: Preliminary axial-pull experiment.](image)

The strip was laid straight and digitised along its length. Masses were then added to the loading pan and the strip redigitised. A point to note with this experiment is the fact that the maximum load is limited by axial equilibrium of the strip when friction is fully mobilized along its length. For \( \mu_a = 0.35 \), the maximum mass with which the strip can be loaded without sliding along its entire length is just in excess of 60 g. In the experiments presented here the loading steps were 0 \( \rightarrow \) 20 \( \rightarrow \) 30 \( \rightarrow \) 40 \( \rightarrow \) 50 \( \rightarrow \) 60 \( \rightarrow \) 0 g. Figure 5.4 shows the axial displacement of the loaded end against the applied axial load. The data points shown are the mean of 5 data sets, and the bars show the maxima and minima within those 5 data sets. The solid line is the curve obtained from finite-element analysis, using model TLM1. The dotted line is the result of applying Equation (2.11).

Figure 5.4(a) shows the results using an axial coefficient of friction of 0.35, which was the value obtained from experiments described in Section 3.4.3; Figure 5.4b shows results using a slightly higher value of \( \mu_a = 0.45 \). It is clear that \( \mu_a = 0.35 \) does not adequately describe the behaviour of the strip on loading, although the residual end displacement on unloading falls very close to the value predicted by both FE and Equation (2.11). Using \( \mu_a = 0.45 \) gives a better agree-
Figure 5.4: Experimental and FE axial-pull tests.

ment between the experimental strip and FE beam behaviours; but the agreement is not so good for unloading. The actual "coefficient of friction" for this experiment probably lies somewhere between these two limits; this is a static experiment, as opposed to the dynamic, albeit very low velocity (quasi-static), experiments previously performed to measure friction. The value of $\mu_a = 0.35$ will still be used throughout this chapter, subject to the caveat that the coefficient of friction between silicone rubber and expanded polystyrene is extremely difficult to determine accurately.

The difference between the solid and dotted lines is due to the inclusion of elastic-slip behaviour in the finite-element model. This explains the higher axial displacements for the finite-element model on loading, and also the more rapid reduction in displacement on initial unloading.
5.2 A Case Study

5.2.1 Introduction

As well as studying the general far-post-buckling behaviour of beams on a frictional foundation, to be described in Section 5.3, the compressible base model was used to study a particular case of a North Sea flowline*. Data on the pipe are presented in Appendix C. Three significant lateral movements had been reported along the line, spaced at approximately 0.5 km intervals.

The main problem with data on submarine pipeline buckling is that one can never be entirely certain what the loading conditions were when buckling was initiated. This would require constant monitoring, which is, of course, totally impractical. The line may be surveyed after installation and, if a problem is detected, periodically thereafter in order to check on its condition. It is generally assumed that buckling was initiated at the most extreme operating conditions measured prior to the survey which revealed the buckling. However, it is possible that less extreme operating conditions actually initiated the buckling, and the subsequent higher temperature conditions just caused the buckle to grow. It is this uncertainty which makes it very difficult to compare the behaviour of a model with an operating pipeline.

The value given in Table C.1 of Appendix C for $\Delta T_{\text{eff}}$ is based on quoted operational conditions at the wellhead of the particular flowline, although it was not clear if these were extreme conditions, or the conditions measured contemporaneously with the ROV survey. The temperature at the buckle locations would be slightly less than this due to heat loss along the pipe length.

It is clear from the three buckles (amplitudes 2 m, 4m, 2 m and lengths 29 m, 20 m, 70 m respectively, in order away from the hotter end of the pipe) that there is not a clear pattern to the buckle shapes observed. There are many possible reasons for this, among them obstacles on the seabed or very different length imperfections induced during pipelaying1.

5.2.2 Model Test

The values of the five dimensionless groups formed in Section 3.1.2 are presented in Table 5.2. For groups $D_1$ and $D_2$ three values are given, one for each buckle, in the same order as listed above.

The model strip S2, 2 m long, was designed to give a linear scale factor of (1/250), so that the entire strip would model 0.5 km of pipeline, which is the

*A flowline is typically a small diameter pipe used for intra-field applications.

1The conclusion of Section 4.6.1 that the initial imperfection length is not important is only applicable for far-post-buckled behaviour.
approximate distance between independent buckles measured on the flowline. It is reasonable to assume — certainly for the central lobe — that the end of this half-length of flowline is effectively encased, since the flowline material would be equally likely to feed into the buckled region either side, and hence would feed into neither. The value of \( D_4 \), which contains the main geometrical and material properties of the beam/pipe, was kept the same in both cases. This required the strip to be rectangular so that a standard steel bar size could be used to form the mould (see Section 3.2.1). The strip was to be tested with the shorter cross-sectional edge in contact with the base, and hence with bending about the minor axis. For the strip \textit{as cast}, the following values were obtained:

\[
D_4 = \left( \frac{EI}{E} \right)^{1/3} \left( \frac{\mu w}{\mu_k w} \right)^{1/3} = 5.16 \times 10^{-6}
\]

and

\[
\text{linear scale factor} = \left( \frac{\left( \frac{\mu w}{EI} \right)_{\text{pipe}}}{\left( \frac{\mu w}{EI} \right)_{\text{strip}}} \right)^{1/3} = 1/247.
\]

A typical set of curves from strip S2 on the compressible base is shown in Figure 5.5(a). The initial geometry for the central 1.2 m of the strip is represented by the dotted line, and the solid lines show the deformed shape at base overall end shortenings of \( \delta = 0, 1, 2, 3, 4, \) and 5 mm. The strip was given an initial lateral deflection of 3 mm at the centre; this relaxed to 2.4 mm. The imperfection was made as large as this in order to ensure that the lobes would grow centrally on the strip, and would not be initiated near the ends due to smaller imperfections formed while laying out the strip. The ends were pinned and weighted as in previous experiments.

The most striking feature of this set of measurements is the asymmetry in the growth of the lobes either side of the main lobe. In this case the faster growth

### Table 5.2: Dimensionless groups for case study.

<table>
<thead>
<tr>
<th>Dimensionless Groups</th>
<th>( \frac{\lambda \left( \frac{\mu w}{EI} \right)^{1/3}}{\mu_k} )</th>
<th>( \frac{\lambda}{\mu} \left( \frac{\mu w}{EI} \right)^{1/3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_1 )</td>
<td>1.19, 0.82, 2.87</td>
<td>0.082, 0.164, 0.082</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>1</td>
<td>5.0 \times 10^{-6}</td>
</tr>
<tr>
<td>( D_3 )</td>
<td>\frac{\mu_k}{\mu}</td>
<td>970 \times 10^{-6}</td>
</tr>
</tbody>
</table>
The shape of the lobe on the left may have been due to a slight lateral imperfection during the initial setting out of the strip (visible on Figure 5.5(a) near the crossover between the central lobe and the left-hand side lobe) which interacted with the larger central imperfection applied later. Another possibility is that the strip is flawed in some way just to the left of the centre. Finite-element analysis of a strip with the wire axis locally displaced a small distance from the axis of the rubber gives similar asymmetric buckled shapes; but the asymmetry does not develop as quickly as it did with the physical model, so this would have only a small effect, if any, for this particular experimental assay.

Plots of lobe growth against the free end displacement are presented in Fig-
Figure 5.6: Buckle lobe amplitudes and lengths of the physical and FE models for the case study, as shown in Figure 5.5. The lighter-shaded circles represent the average of the lobe 2 data.

Because of the asymmetry in the growth of the two side lobes in the physical model, a point for each lobe has been plotted, although they are both denoted “lobe 2.” These points lie either side of the curve obtained from the finite-element analysis. The lighter-shaded circles represent the average of the lobe 2 data from the physical model; it is clear that this average falls very close to the finite-element prediction. The amplitude and length of lobe 1 follow the general trend of the finite-element result, but fall consistently low. In the case of the amplitude, the difference averages 1 mm. There is no clear, single factor which could be causing this; it is a combination of small errors in the measurement of the properties of the silicone rubber strip on the expanded polystyrene base, and unquantifiable imperfections in the rubber strip.

When lateral imperfections of the order of 1 mm were imparted to the strip at the centre, in those cases when the strip did actually buckle at the location
of the imposed imperfection rather than at another, unintentional imperfection*, it moved suddenly (an unstable jump) to a larger lateral displacement at an end shortening of 1.4–1.6 mm; from the finite-element model, the amplitude rises rapidly at an end shortening of around 0.4 mm for this size of imperfection. It is reasonable to conclude that, although the distribution of compressive strain throughout the base is comparatively uniform even at 2 mm of shortening, some of the shortening would have been taken up by small movement in the imperfectly encastré ends. It is perhaps unreasonable to assume that a base strain of 0.01% could be fully and evenly imparted to the rubber strip.

In a number of other experimental assays, still with a single lateral imperfection at the centre, the strip buckled into an antisymmetric shape, much like mode 4 in Figure 2.3. This happened at almost exactly the same end shortening as for the symmetric buckle. Although the buckling loads given by Equation (2.5) are very similar for the various modes listed — which would explain the similar end shortening — the buckled shape is very dependent on the distribution of initial imperfections (e.g. Kerr 1978a, Tvergaard & Needleman 1981). It appears that some small, unintentional initial imperfection must have been present in these tests.

5.2.3 Finite-Element Modelling of the Flowline

Several finite-element analyses were performed, using the material and geometrical properties of the actual flowline. These included several Riks analyses to highlight the difference in behaviour between an unstable and a stable buckle. The half-length of the model was chosen to be 250 m, which gives an overall pipe length of 0.5 km, as for the physical model. Figure 5.7 shows the central axial load, lobe amplitudes and lobe lengths for three different imperfection sizes. Note that the amplitudes given are those after the relaxation stage. The two smaller imperfection amplitudes give rise to unstable behaviour characterized by a temporary reversal in the direction of f.e.d. The light-shaded line represents the f.e.d. which is equivalent to $\Delta T_{eff} = 83^\circ C$.

Putting aside for a moment the assumptions of no stiffening from the pipe’s armour coating, and the unknown friction conditions, there are several main points to notice. The first is that, at the f.e.d. under consideration, the amplitude of the main lobe as computed is indeed approximately 2 m, which was measured in two of the three buckles. However lobe 2 has grown to 1 m. This should be easily measurable by ROV, but there are no records of any side lobes in the flowline survey. The computed lobe lengths are 30–40 m, which is quite central within the

*This imperfection could be present from manufacturing the strip, or may have been imposed accidentally while setting up for an experiment.
range of lobe lengths reported*. An important point here is that the operating condition quoted in Appendix D, and marked on Figure 5.7, appears to be considerably beyond the condition required for buckling; the effective temperature rise at buckling, when the axial load in the central lobe begins to fall, is of the order of $10-20^\circ C$, depending on the exact nature of the initial imperfections.

The size of the variation of the measured lobe amplitudes and lengths is shown more clearly in Figure 5.8 where they have been plotted in a non-dimensional form, as discussed in the following section.

*Late in the writing of this dissertation, data for buckles on an identical flowline within the same field, under similar operating conditions, were obtained. Four buckles were recorded, with central lobe lengths ranging from 40–45 m and amplitudes from 1.4–2.8 m.
5.2.4 Dimensional Analysis

In order to compare results from the physical model with the data from the actual flowline, it is useful to consider the results in terms of dimensionless groups. For the lobe amplitudes and lengths, dimensionless groups $D_1$ and $D_2$ are useful, and $D_3$ could be used for cases where the axial and lateral coefficients of friction are different. The f.e.d., which already takes into account the axial stiffness $EA$, could be formed into a dimensionless group as $(f.e.d.)(\mu w/EI)^{1/3}$ following the same reasoning behind the formation of $D_1$ and $D_2$ from $a$ and $\lambda$, two other quantities with dimensions of length.

Figure 5.8 shows the finite-element runs from the previous two sections compared in this way. The solid line is from the flowline model with the large (0.5 m) initial imperfection amplitude, while the dashed line is from the TLM model of the physical experiment, where the initial imperfection amplitude is 2 mm. These particular analysis runs were chosen so that the initial imperfection amplitudes were identical when non-dimensionalized using group $D_1$. The circles represent the recorded data from the ROV survey of the flowline.

Again, the different values of the parameters such as elastic slip and initial imperfection heights and lengths, mentioned in the parameter studies of Chapter 4, mean that there is some variation in behaviour up to the point when the peak central axial load is reached. However, beyond the peak central axial load, the parameter studies showed that the non-dimensional central axial loads in the post-buckled region tended towards a common curve for a range of different imperfection sizes. In Figure 5.8, the two curves do indeed tend towards a common curve.

In order to ascertain whether the buckled pipeline is still fit for service, it is necessary to determine the stress state in the main buckled lobe. From Figure 5.7, at the stated operation conditions, the central axial compressive load is approximately 80 kN. The central curvature is approximately 0.024 m$^{-1}$ for the case with an initial imperfection height of 0.50 m. With the pressure difference $p = 8.4 \times 10^6 \, Nm^{-2}$, the hoop, axial and bending stresses may be calculated:

\[
\begin{align*}
\sigma_h & \approx 44.5 \times 10^6 \, Nm^{-2} \\
\sigma_a & \approx -10.7 \times 10^6 \, Nm^{-2} \\
\sigma_b & \approx \pm 424 \times 10^6 \, Nm^{-2}
\end{align*}
\]

The worst von Mises stress

\[\sigma_{vM} = \sqrt{0.5((\sigma_a + \sigma_b)^2 + \sigma_h^2 + (\sigma_a + \sigma_b - \sigma_h)^2)}\]

occurs centrally on the inside of the curved buckle lobe where the compressive bending stress coincides with the compressive axial stress; in this case,
Figure 5.8: Non-dimensional plots of buckle lobe amplitudes, lengths, and central axial load for the case study. The FE analyses from Figure 5.6 and Figure 5.7 (0.50 m initial imperfection) are shown, along with the flowline survey data.

\[ \sigma_{\text{SMYS}} \approx 458 \times 10^6 \text{ Nm}^{-2} \]. The specified minimum yield stress (SMYS) of the pipeline material (API 5L X65 steel) is \( 448 \times 10^6 \text{ Nm}^{-2} \). The von Mises stress just exceeds the material yield stress and some plastic deformation would be expected — this is not to say that failure is inevitable, but certainly mitigating measures should be put in place to prevent further amplitude growth, and hence increased curvature, which could lead to local shell-buckling of the pipeline.

### 5.2.5 Discussion

With a good agreement between the physical model and its representative finite-element model, and a good agreement between the non-dimensional results of finite-element computations for both the small-scale physical model and the flowline itself, there is evidently some merit in using the physical model as a way
of visualising the buckling and post-buckled behaviour of a real pipeline. The
main drawback of this kind of modelling is the lack of detailed data from the
flowline itself. The recorded values of lobe amplitude and length do indeed lie in
the general area of the physical finite-element model predictions, but up to 70%
away. It is not possible to say whether this is due to major inadequacies in the
modelling, such as the unknown friction and initial imperfection conditions of the
flowline, or to the difficulty in trying to determine accurately the buckle length
and amplitude of the flowline from the survey data, all of which would have a
significant effect on the dimensional analysis.

In general, for a long pipeline, when the axial load in the buckle falls and
the thermal load in the rest of the pipeline continues to rise, there could be
other imperfections which would initiate buckling elsewhere. This would inhibit
the amount of pipe available to slip into the original buckle, feeding its growth.
Indeed, this is the type of behaviour which the flowline in the case study ex­
hibits. The buckles with smaller amplitudes have longer lengths; as discussed in
Section 4.6.1, for a given imperfection amplitude, a longer imperfection initiates
buckling at a higher axial load. Hence the outer two buckles, which are smaller in
amplitude and longer in length than the centre buckle, may have formed sometime
after the first buckle; but this can only be a matter for speculation. No analysis
was carried out into the formation of independent buckle groups, because the
small-scale physical model is not long enough for this to be practical.

As a note of caution, no account has been taken of cyclic loading; monotonic
loading has been assumed throughout this analysis. It is likely that the "ratchet­
ing" effect of cyclic loading on a frictional foundation plays an important rôle in
lobe development, but a computational study of the phenomenon would require
a detailed loading history.

5.2.6 Far-Post-Buckled Behaviour of the Physical Model

It is interesting to examine further the far-post-buckled behaviour of the sili­
cone rubber model, although this goes far beyond the current or possible future
loading conditions of the flowline itself. Figure 5.9 shows the evolution of the
buckled geometry of the physical strip up to an end shortening $\delta = 30 \text{ mm}$,
compared to the results of a finite-element analysis using model TLM2 (cf. Fig­
ures 5.5 and 5.6 showing early behaviour). Note that only the central region
of the strip was digitised; after the loss of end points due to filtering, only the
central 0.8 m is presented here. The growth in the amplitude and length of lobes
1, 2 and 3, as defined in Figure 4.4, are also plotted against $\text{f.e.d}$. The amplitude
and length values from the physical model, lobes 2 and 3, are averaged with the
values from the corresponding lobes on the other side of the central lobe 1. The
exception is to this is the length of lobe 3, where only the left-hand lobe could
Figure 5.9: Far-post-buckled behaviour of the case study pipeline model.
be measured from the filtered data; this accounts for the larger scatter of these data points.

There is a reasonable agreement between the lobe amplitudes and lengths from the physical model and the FE model. Certainly the general trends of both sets of data are similar. The rate of growth of the amplitude of lobe 1 begins to decrease, and the length of lobe 1 has become constant at $\lambda = 0.165$ m. For the flowline, this would correspond to an ultimate main lobe length of 41 m. The amplitudes of lobes 1 and 3 are very close to the FE predictions, but the amplitude of lobe 2 of the strip is generally lower than the FE prediction. The length of lobe 2 is also levelling out to around 10% less than the predicted length. It is not clear why this lobe should deviate from the prediction to a greater extent than the others.

Far-post-buckled behaviour, including the ultimate geometry of buckled lobes, will be discussed further in Section 5.3.

5.3 Further Parameter Studies — Far-Post-Buckling Behaviour

Having considered the behaviour of different strips separately, it would be useful to have some direct comparison between them. The parameter studies, which were begun in Section 4.6, were therefore extended to include material and geometric parameters which have a significant effect on the far-post-buckling behaviour of a beam or pipe. Rather than treating each of these parameters individually, their combined effect was examined.

The focus of this section moves more away from the behaviour of beams during and just after buckling, and examines the far-post-buckling behaviour of a beam on a frictional foundation. Indeed, it leads to the characterisation of an interesting phenomenon, lobe extinction, which will be discussed in Section 5.3.2.

The parameters studied here are items vi) to ix) from the list in Section 4.6, namely $EI$, $EA$, $\mu_a$ and $\mu_f$. Since $w$, the weight per unit length of the beam, is only important when combined with a coefficient of friction to give a friction force per unit length, it is not varied here. The analyses were performed on the TLM4 model, as described in Section 4.2.1 and Appendix B. The parameters were varied according to the nominal factors set out in Appendix D. In fact, in order to simplify the input, the bending stiffness $EI$ was changed by applying the nominal factor to the height of the beam elements representing the rubber strip. The axial stiffness $EA$ was changed by applying the square root of the nominal factor to the radius of the beam elements representing the steel wire. Obviously, these two operations are not independent; changing the wire diameter
alters the bending stiffness by a small amount. This was taken into account when calculating the actual $EI$ and $EA$ for each analysis.

The analyses listed in Appendix D represent a selection which, given a particular initial imperfection, did not run into numerical difficulties. For instance, increasing the nominal factor on the bending stiffness to 1.5 led to convergence problems, even with increasingly small load increments and the Riks analysis option active. The Riks analysis can run into problems if there is a sudden bifurcation in the static response, or even if the response exhibits high curvature requiring the load increment to become very small.

5.3.1 Qualitative Discussion

Figure 5.10 presents six examples, chosen from many, to show how the evolution of buckled lobes is affected by different parameter values.

The buckle shapes are shown at 10 equal increments of thermal loading. The temperature to which the model was raised was the same as for the TLM2 model for the far-post-buckled analysis of Section 5.2.6; but because of the greater length of beam, the free end displacement is greater and the buckle pattern has evolved further. Only $1.5\,m$ of the $2.5\,m$ half-length is shown here for the sake of clarity, since all lateral deflections in the last $1\,m$ are negligible in size.

Analysis 1 is the “standard case,” with all the parameter factors equal to 1. The way in which the buckles move outwards from the centre as they grow can be clearly seen, as can the way in which the growth in both amplitude and length eventually dies out in the earlier lobes. Although the temperature rise for this analysis is the same as that for the FE analysis shown in Figure 5.9, the longer length on the model used here means that the final f.e.d. is greater, and the buckle pattern has evolved more. The amplitudes of the “live” lobes i.e. those which have not yet become extinct, decay to negligible levels over 5 lobe lengths (approximately $0.7\,m$).

Analyses 2 and 6 both show very different behaviour, with a common factor being that the lateral coefficient of friction is larger than the axial coefficient of friction. Material feeds easily into the central lobes which were present initially, and outer lobes cannot grow easily against the higher lateral friction. In this case, it is easier for an existing lobe to grow further than for a new lobe to grow. The central lobe continues to grow in amplitude throughout the thermal loading, and a very high rate of decay in lobe amplitude over the length of the beam means that only 2 or 3 significant side lobes are evident for the particular temperature rise imposed.

Analysis 11, which has modest increases in both $EI$ and $\mu_a$, is only shown for half the temperature rise of the others, i.e. for five increments. Even at this
Figure 5.10: Buckle evolution for various parameter changes. The analysis number refers to the table of analyses in Appendix D. The numbers enclosed in square brackets above the plots are the nominal factors applied to $EI$, $EA$, $\mu_a$ and $\mu_f$ respectively, using Analysis 1 as the reference. Thus, for example, in Analysis 2 the lateral coefficient of friction was double the value used for Analysis 1. Analysis 11 is only shown for half the temperature rise of the others — see the text for an explanation.
stage in the thermal loading, noticeable lobes have appeared as far as 1 m from the central lobe. As the temperature rises, the buckled region extends further to the right and eventually there is a further localization at one of these outer lobes. This behaviour is associated with an increase in the axial coefficient of friction with respect to the lateral coefficient of friction; there is less material feeding into the buckled region, and, in comparison to Analyses 2 and 6, the outer lobes form easily.

Analysis 17, which has half the axial stiffness of the standard case, appears to be practically identical to Analysis 1 in respect of its geometrical evolution.

Analysis 18 has a factor of 1.2 applied to the bending stiffness of the standard case, and no other changes. At first glance it too looks very similar to the standard case, but there are small differences in the amplitudes of corresponding lobes, and the lobes are longer; this can be seen clearly by looking at the position of lobe \( L_1 \) in relation to the x-axis tick mark at 0.5 m. However, the rate of decay of the live lobes appears to be rather similar to that of the standard case.

### 5.3.2 Lobe Extinction

"Lobe extinction" is characterized by a stage in the loading history where a particular lobe becomes practically unaffected by what is happening in the rest of the beam. The lobe length becomes constant, and no more material feeds tangentially into the lobe; in effect the ends of the lobe have become pinned to the foundation. If the temperature rises further, then there will still be a small change in the lobe amplitude to accommodate the change in contour length of the lobe due to thermal expansion; but this is a small effect in comparison with those lobes into which material is being fed from neighbouring lobes.

Figure 5.11 shows how the rate of change of the tangential displacement of node points on the beam varies with the f.e.d. for the standard case (Analysis 1). \( \frac{du}{d(f.e.d.)} \) represents a tangential "velocity" — a change in tangential displacement with respect to the loading parameter instead of time — which shows how material feeds into the various lobes as the thermal loading proceeds.

When \( \frac{du}{d(f.e.d.)} = 0 \) at a point on the beam, then there is no further feed-in of beam material through that point. The three vertical dotted lines mark where the right hand ends of lobes 1, 2 and 3 are located at the end of the analysis, i.e. at the stage where these lobes are "extinct".

It is clear that at extinction both the ends of the lobe and the centre of the lobe have stopped moving tangentially, and there is a slight movement in between as the lobe grows to accommodate additional thermal expansion. The long straight part of each curve passes through \( \frac{du}{d(f.e.d.)} = 0 \) at \( x = 2.5 \) m (the encastré end), and when extrapolated, passes through \( \frac{du}{d(f.e.d.)} = -1 \) at \( x = 0 \). This arises from the linear increase in additional tangential displacement with length.
Figure 5.11: The variation of tangential "velocity" over the length of the half-beam of Analysis 1. The "velocity" is defined as the rate of change of tangential displacement with respect to the loading parameter, in this case the free end displacement. The early variation of tangential "velocity" has been omitted here for clarity, but is explained in the text.

Once the axial friction is fully mobilized along the length of the beam, i.e. when the f.e.d. becomes linear with temperature in Equation (4.2). To the right of lobe 3, in the curve furthest to the right, there is a net displacement (negative, so to the left), meaning that beam material is passing through these lobes; they are still "live." In the earlier stages, prior to the first curve of Figure 5.11, and before the axial friction is fully mobilized along the entire length of the strip, the maximum magnitude of $\frac{du}{d(f.e.d.)}$, initially in the region of lobes 1 and 2, increases in magnitude up to approximately 0.9. The rate of change of $u$ in the rest of the beam similarly increases, i.e. the slope of the right-hand side of the graph gets steeper until it reaches the slope shown in Figure 5.11. At the same time, the rate of change of $u$ in the buckled region slowly starts to fall — in the first curve shown in Figure 5.11 $\frac{du}{d(f.e.d.)}$ at the centre of lobe 2 has fallen in magnitude from around 0.9 to 0.67. These earlier curves, which would cross those curves shown in Figure 5.11, have been omitted in the interest of clarity.
Figure 5.12 shows how the amplitudes and lengths of the first 3 lobes vary as the temperature, and hence free end displacement, rises. For clarity, the curves shown are only for the six cases presented in Figure 5.10. Obviously, the two upper lobe 1 amplitude curves are for Analyses 2 and 6, as are the two lower lobe 3 amplitude curves. There is a significant scatter in the other curves, although this is less pronounced for the lobe lengths, and in particular lobe 1. It was mentioned in Section 4.6.1 that the length of lobe 1 is affected by the length of the initial imperfection, and the growth of the side lobes seems to prevent the central lobe from attaining its preferred length.

With the exception of Analyses 2 and 6, by the end of each computation the amplitudes of lobes 1 and 2 have reached a “plateau,” and the growth of lobe 3 is slowing. This happens after the respective lobe lengths have become constant. In fact, the amplitude does not quite reach a constant, but continues to rise very slowly, as mentioned earlier in this section.

Consider for a moment the case of half of a sine wave, of fixed length $\lambda$, which is indeed a reasonable approximation to the shape of an extinct lobe. Then

$$y = a \sin \left( \frac{\pi x}{\lambda} \right).$$  \hspace{1cm} (5.2)

The “excess length” of the curve, or the contour length of the curve less $\lambda$, is given, to a good approximation when $a \ll \lambda$, by

$$\int_0^\lambda \frac{1}{2} \left( \frac{\pi a}{\lambda} \cos \left( \frac{\pi x}{\lambda} \right) \right)^2 \, dx = \frac{\pi^2 a^2}{4\lambda}.$$  \hspace{1cm} (5.3)

If there is now a temperature rise $\Delta T$ which causes a change in amplitude of $\Delta a$, then

$$\frac{\pi^2}{4\lambda} (a + \Delta a)^2 - \frac{\pi^2 a^2}{4\lambda} = \alpha \Delta T \left( \lambda + \frac{\pi^2 a^2}{4\lambda} \right).$$  \hspace{1cm} (5.4)

Hence

$$\frac{\Delta a}{\alpha \Delta T} = \frac{2\lambda^2}{\pi^2 a} \left( 1 + \frac{\pi^2 a^2}{4\lambda^2} \right);$$  \hspace{1cm} (5.5)

and since $a^2 \ll \lambda^2$, certainly in the majority of the 19 cases examined here,

$$\frac{\Delta a}{\alpha \Delta T} \approx 0.2 \frac{\lambda^2}{a}. $$  \hspace{1cm} (5.6)

For this particular model, the free end displacement varies as $\Delta T$ when lobe extinction is encountered, and so $da/d(f.e.d.)$ should vary from Equation (5.6) by a factor of $L$; and indeed this turns out to be a very good approximation to what is found numerically in extinct lobes. The growth of the amplitude of extinct lobes with increasing f.e.d. can be seen most clearly in the non-dimensional plot of Figure 5.14 in the following section.
Figure 5.12: Buckle lobe amplitudes and lengths for the numerical analyses presented in Figure 5.10. Only the first 3 lobes are considered in each case.
5.3.3 Dimensional Analysis

As with the three-way comparison between the physical model, the finite-element model and the flowline of the case study in Section 5.2, it is appropriate to make use of dimensional analysis in developing the present parameter study.

Appropriate groups have already been put forward for making the lobe lengths and amplitudes non-dimensional, namely dimensionless groups $D_1$ and $D_2$ from Section 3.1.2; and in Section 5.2 $(f.e.d.)(E_1/w/EI)^{1/3}$ was found to be suitable for making the free end displacement non-dimensional. It was also found that doubling or halving the axial stiffness $EA$ had a negligible effect on the shape of the buckles, affecting only the temperature at which they form; and indeed the use of the free end displacement already takes that into account for the post-buckled behaviour.

It is desirable to do a systematic dimensional analysis of the situation. Consider a relationship of the form

$$\lambda = G_1(f.e.d., EI, \mu_a w, \mu_l w),$$

where $\lambda$ is the length of a given lobe, and where $G_1$ signifies a function. There are 5 physical quantities here, and 2 dimensions (viz. force and length), and so by Buckingham’s Pi rule (Section 3.1.2) there are 3 independent dimensionless groups. Hence the relationship may be expressed in the following non-dimensional form:

$$\lambda \left( \frac{\mu_a w}{EI} \right)^{1/3} = G_2 \left( f.e.d. \left( \frac{\mu_a w}{EI} \right)^{1/3}, \left( \frac{\mu_a w}{\mu_l w} \right) \right)$$

Here $(EI/\mu_a w)^{1/3}$, which has the dimension of length, has been used to make the lengths $\lambda$ and f.e.d. dimensionless.

When $\lambda(\mu_a w/EI)^{1/3}$ is plotted against $(f.e.d.)(\mu_a w/EI)^{1/3}$ for a particular lobe, the various curves shown in the lower plot of Figure 5.12 are found all to have a similar shape in the post-buckled region. If logarithmic scales are used, as in Figure 5.13, it is not difficult to find the form of the relationship between these curves with respect to the ratio of friction forces.

In Figure 5.13 the curves group together, for post-buckling behaviour, according to the ratio of axial and lateral friction forces; the values of $(\mu_a w/\mu_l w)$ decrease from left to right. All the curves for which the second lobe becomes extinct (i.e. for which the lobe length stops growing) become horizontal at approximately the same value of $\log_{10}(\lambda(\mu_a w/EI)^{1/3})$. This implies that the ratio of axial and lateral friction has a negligible effect on the dimensionless extinct lobe length. A horizontal line, plotted at 0.1 on the vertical axis so as to pass through all the curves in the post-buckled region, enables the scaling factor for the dimensionless f.e.d. (the loading parameter in this case) to be determined in
Figure 5.13: Log-log plot of dimensionless lobe length against dimensionless f.e.d. for lobe 2. The 19 analyses listed in Table D.1 in Appendix D are included. The curves can be seen to fall into seven groups according to the ratio of axial and lateral friction forces, indicated in order.

It turns out that the difference in the values of $\log_{10}(f.e.d.(\mu_l w/EI))$ between the different groups of curves is described well by $2\log_{10}(\mu_a w/\mu_l w)$. The same conclusion is reached by a similar study of lobes 1 and 3. Hence, Equation (5.7) may be rewritten as

$$\lambda \left( \frac{\mu_a w}{EI} \right)^{\frac{1}{3}} = G_3 \left( \frac{\text{f.e.d.}}{EI} \right) \left( \frac{\mu_a w}{\mu_l w} \right)^2. \quad (5.8)$$

This relationship, shown in the lower plot of Figure 5.14, includes all of the data in the lower plot of Figure 5.12, and many more besides.

The large variation between curves in the early stages of the numerical calculations (before and during buckling — see Figure 5.13) stem from the initial
conditions of the curve. When the initial length $\lambda_1$ is non-dimensionalized by the factor $(\mu_4 w/E) \frac{1}{2}$, the initial conditions of $\lambda_1(\mu_4 w/E)^{\frac{1}{2}}$ are not the same for each calculation. In the early stages of the calculation, the stiffness and friction parameters further change the evolution of the lobe length. While the data in the early stages of the analyses are rather sparse, and therefore preclude detailed examination, very general trends can be related to particular parameters by comparison to the standard case, Analysis 1, for which the initial slope on Figure 5.13 is approximately 0.06.

- Consider Analyses 11, 9 and 13 for which the factors on $EI$, $\mu_4$ and $\mu_1$ respectively are increased by a factor of 1.2, with no other changes. The initial slopes on Figure 5.13 are 0.06, 0.06 and 0.02. This shows there is a significant effect from the lateral friction in the early stages of the analysis.

- Consider Analyses 2 and 3 where the values of $(\mu_4 w/\mu_1 w)$ are $(1/2)$ and $(0.5/1)$. The initial slopes are -0.19 and 0.09. Again, the lateral friction is significant, and the lobe length is actually decreasing in Analysis 2.

- Consider Analyses 16 and 17 where the factor on $EI$ is changed to 2 and 0.5. The initial slopes are 0.15 and -0.07. Therefore, the axial stiffness is significant in the early stages of the analysis — the lobe length grows more rapidly with a higher axial stiffness.

These differences will not be considered further here; the analysis will concentrate on the far-post-buckled region where the effects of these differences have died out.

So far only the lobe length $\lambda$, and how it is related to the other parameters, has been discussed. However, the lobe amplitude $a$ can be considered in the same way. This time, a good approximation for the relationship between the dimensionless lobe amplitude and dimensionless f.e.d. is

$$a \left(\frac{\mu_4 w}{EI}\right)^{\frac{1}{3}} \left(\frac{\mu_4 w}{\mu_1 w}\right) = G_4 \left(\text{f.e.d.} \left(\frac{\mu_4 w}{EI}\right)^{\frac{1}{3}} \left(\frac{\mu_4 w}{\mu_1 w}\right)^2\right) \tag{5.9}$$

This is shown in the upper plot of Figure 5.14. The dimensionless variable on the right is exactly the same as before; but the dimensionless group containing $a$ involves the ratio of friction forces $(\mu_4 w/\mu_1 w)$.

Instead of having to use the above expressions, which are rather cumbersome, it it convenient to define a shorthand notation for Equations (5.8) and (5.9), such that

$$\mathcal{L} = G_3(\mathcal{F}) \tag{5.10}$$

$$\mathcal{A} = G_4(\mathcal{F}) \tag{5.11}$$
Figure 5.14: Non-dimensional plots of buckle lobe amplitudes and lengths for the parameter study. The 19 analyses listed in Table D.1 in Appendix D are included. Only the first 3 lobes are considered in each case.
where $\mathcal{L}$, $A$ and $F$ are the modified dimensionless groups involving the lobe amplitude, lobe length and free end displacement respectively, such that:

$$\mathcal{L} = \lambda \left( \frac{\mu_a w}{EI} \right)^{\frac{1}{3}}, \quad (5.12)$$

$$A = a \left( \frac{\mu_a w}{EI} \right)^{\frac{1}{3}} \left( \frac{\mu_a w}{\mu_l w} \right), \quad (5.13)$$

$$F = (\text{f.e.d.}) \left( \frac{\mu_a w}{EI} \right)^{\frac{1}{3}} \left( \frac{\mu_a w}{\mu_l w} \right)^2. \quad (5.14)$$

The curves of $\mathcal{L}$ and $A$ against $F$ are plotted in Figure 5.14 for the 19 different analyses (see Table D.1 in Appendix D). They do not exactly coincide. For the non-dimensional lobe length $\mathcal{L}$ there is evidence that the third dimension cannot be dealt with so easily as for the amplitude. But it is also clear that for any given lobe, the length $\lambda$ stops changing at a smaller free end displacement than the amplitude. The correlation between curves is not especially good for lobe 1. This goes back to the problem of the influence of the initial imperfection length and the central lobe not reaching its preferred length because of the formation of side lobes (Section 4.6.1). In fact, these analyses were repeated for beams where the initial imperfection was imposed by laterally deflecting the beam and allowing it to relax as part of the analysis. This resulted in the initial imperfection length for each analysis being the natural length*, and reduced the variation in the non-dimensional lobe length of lobe 1. However, this method of applying the imperfection also resulted in a different initial stress state for each beam and, as shown in Section 4.6.5, this had a large effect on the growth of the lobe amplitude, resulting in a much larger scatter in the non-dimensional amplitudes of all the three lobes under consideration.

Another effect, this time in relation to the non-dimensional amplitude $A$, may be seen in the upper plot of Figure 5.14, with the curves which terminate at $F \approx 0.08$ (including Analysis 6) and look to break away from the main set of curves, high in lobe 1 and low in lobe 3. These are for the cases where $\left( \frac{\mu_a w}{\mu_l w} \right) = 0.25$. At such a ratio of axial and lateral friction forces, this analysis would seem to be inapplicable for finding the extinction behaviour; and the fortunate reduction from 3 dimensionless groups to 2 seems to become invalid. But recalling Table 3.1 of typical friction coefficients for North Sea applications, and taking the minimum axial friction coefficient and the maximum lateral friction coefficient (conditions which would probably not be encountered together) then the smallest value of $\left( \frac{\mu_a w}{\mu_l w} \right)$ is 0.4 for clay and 0.6 for sand. To first order, then, the correlation between the curves for the lobe amplitude ($A$ against $F$) is excellent.

*Note that the natural length is the imperfection length that a lateral pull and release produces, which is not the same as the preferred wavelength which the lobe would ideally grow to (like a single upheaval buckle in the absence of cover.)
Despite these differences, it is possible to obtain general values for the extinct lobe amplitude and length \( a_e \) and \( \lambda_e \). For example, for the first lobe, the values at extinction are

\[
A_e \approx 0.24 \implies a_e \approx 0.24 \left( \frac{EI}{\mu_0 w} \right)^{\frac{1}{3}} \left( \frac{\mu_1 w}{\mu_0 w} \right)
\]

and

\[
L_e \approx 1.65 \implies \lambda_e \approx 1.65 \left( \frac{EI}{\mu_0 w} \right)^{\frac{1}{3}}
\]

at

\[
F_e \approx 0.15 \implies \text{(f.e.d.)}_e \approx 0.15 \left( \frac{EI}{\mu_0 w} \right)^{\frac{1}{3}} \left( \frac{\mu_1 w}{\mu_0 w} \right)^2.
\]

By comparison, the lobe length has stopped changing much earlier, at \( F \approx 0.05 \). Similarly for lobe 2,

\[
A_e \approx 0.24, \text{ and } L_e \approx 1.68 \text{ at } F_e \approx 0.25.
\]

These values should be compared to the values for the stated operating conditions of the case study flowline, namely \( F \approx 0.01 \), with \( A \approx 0.10 \) and \( L \approx 1.62 \). Although the flowline conditions are far from the conditions which would result in the extinction of lobe 1, that lobe has attained almost half the amplitude that it might ultimately reach, and almost exactly the ultimate length. However, since other independent buckles had formed elsewhere on the flowline, the ultimate state of a single set of buckle lobes is not really indicative of a possible ultimate state of the flowline.

The actual forms of the functions \( L = G_3(\mathcal{F}) \) and \( A = G_4(\mathcal{F}) \) are not simple, and Figure 5.14 shows that they are different for each lobe. Therefore, at present, values of \( A \) and \( L \) for a particular value of \( \mathcal{F} \) should be read directly from Figure 5.14.

No mention has been made yet of the change in axial compressive load at the centre of lobe 1 as it approaches extinction. Previously, in Section 3.1.2, it was suggested that the dimensionless group \( D_0 = P/EA \) would be suitable for comparing the change of axial load with respect to a non-dimensional free end displacement. Indeed, that measure was suitable when comparing the loads from the finite-element model analyses in the case study (Figure 5.8). However, this was in the early stages of buckling when the axial stiffness is important to the load at buckling. As the lobe moves towards extinction, the axial load in the buckle depends on the lobe geometry and the bending stiffness of the beam. This suggests that an alternative way of non-dimensionalizing the axial load is required. A good candidate for this is the well-known Euler buckling load, \( P_E = (\pi^2 EI/\lambda^2) \).
Figure 5.15 shows the compressive axial load at the centre of lobe 1 non-dimensionalized in this way; again this is a 2-D representation of a 3-D surface. $\mathcal{P} = (P/P_E)$ is plotted against $\mathcal{F}$. There is scatter between the curves at lower values of $\mathcal{F}$ — this is in direct relation to the scatter observed in the value of $\mathcal{L}$ for lobe 1. In the two-thirds of the analysed cases for which the central lobe reaches extinction, $\mathcal{P}$ can be seen to be settling towards a constant value of $\frac{4}{3}$, or $P_e = \frac{4}{3}P_E$.

Although this particular non-dimensional grouping yields a very simple rule for the ultimate load in the centre of a buckle in terms of the Euler buckling load, it seems to defy any obvious explanation at present.

At this juncture it is worth clarifying which length has been used to calculate the Euler buckling load. At present, the lobe length $\lambda$ has been used. The most appropriate length for an equivalent Euler strut is between successive points of inflexion. As the central lobe tends towards extinction, the points of inflexion tend towards the zero-crossing points by which $\lambda$ is defined, so it is quite reasonable to use the lobe length when considering extinction; thus the expression $P_e = \frac{4}{3}P_E$ still holds. However, in the earlier stages of the central lobe’s evolution, the point
of inflexion lies within the lobe. Hence the Euler length would be smaller, the Euler load larger, and therefore $\mathcal{P}$ would be smaller. Although it has not been examined, differing relationships between the Euler length and lobe length at a particular value of the non-dimensional loading parameter $\mathcal{F}$ could possibly reduce some of the scatter in Figure 5.15.

### 5.4 Further Experiments with Straight Strips

Further experiments were conducted on the other two silicone rubber strips listed in Appendix A, initially laid straight on the compressible base. This series was not intended to model the behaviour of a particular pipeline; it was to be used for comparison with the results from strip S2 used for the case study in Section 5.2. A typical buckled shape for each of strips U1 and S1 is presented in Figure 5.16.

![Buckle shapes for stiffened (S1) and unstiffened (U1) strips.](image)

**Figure 5.16:** Buckle shapes for stiffened (S1) and unstiffened (U1) strips. For the stiffened strip the curves shown are at $\delta = 0, 2, 3, 4, 5, 6, 7, 8, 9, 10 \text{ mm}$, and for the unstiffened strip at $\delta = 0, 5, 10, 15, 20, 25, 30 \text{ mm}$.

Since the axial stiffnesses are very different, the base shortenings at which the buckle shapes are shown are not the same in each case. Note that the peak of the central lobe moves along the strip slightly in both cases when the unstable jump occurs. It is not clear why this is so, although it may be due to irregularities within the strips. Unfortunately, after data filtering was applied, the right-hand
side lobe is not quite complete for higher base end shortenings. The general buckle shapes should be compared with those of Figure 5.9 for strip S2.

Using the same size of initial imperfection as for the parameter studies with the smaller strip S2, both of these strips exhibit a jump from an unstable to a stable equilibrium position. The change in amplitude and lobe length during this jump is much larger for the unstiffened strip. For the unstiffened strip, there was more scatter in the base end shortening at which the jump occurred, varying from $\delta = 25.0 \, \text{mm}$ to $29.1 \, \text{mm}$. This is perhaps not too surprising, because it is much harder to set up a strip with low axial stiffness so that it is as near to being unstressed as possible. For the axially-stiffened strip, the base end shortening fell between 2.6 $\, \text{mm}$ and 2.9 $\, \text{mm}$ at the jump.

In order to compare these results more than just qualitatively, it is necessary to work again in terms of a free end displacement, and to non-dimensionalize the f.e.d. and the lobe lengths and amplitudes in terms of $\mathcal{F}$, $\mathcal{L}$ and $\mathcal{A}$. These curves are presented in Figure 5.17 for lobes 1 and 2 of the buckled strip. The lobe amplitudes and lengths obtained for strip S2, as shown in Figure 5.9, are also included. Note that none of the strips have reached $\mathcal{F} = 0.15$, the condition for extinction of the central lobe from Equation (5.17).

The lightly-shaded lines on the figure represent a change between unbuckled and buckled states for each strip. In all cases, readings could not be taken immediately before and after buckling occurred, so these lines are in no way supposed to represent the exact path of the unstable jump in $(\mathcal{A}, \mathcal{F})$ or $(\mathcal{L}, \mathcal{F})$ space; they are merely included in the plots to aid clarity.

Strips S1 and S2 have quite similar axial stiffnesses (around 4% difference), and the values of $(\mu a w/EI)^{1/3}$ are 6.9 and 10.1, while the axial stiffness of strip U1 is around $1/18$th that of strip S1, and $(\mu a w/EI)^{1/3}$ is 7.1

The most striking feature of the plot of $\mathcal{A}$ against $\mathcal{F}$ is how well all the post-buckled curves fit together, especially for lobe 1. For lobe 2, the curve for strip S2 lies below the other two curves, but certainly they all lie broadly within the same band*. Because of the very low axial stiffness of strip U1, its pre-buckled phase is much longer; a much higher temperature, or in this case base end shortening, is required to achieve the buckling load. Even though the initial imperfection amplitudes were similar in all three cases, it is not at all effective in reducing the size of the large unstable jump for the compliant strip. Thinking back to Figure 2.4(b), strip U1 follows a curve similar to the left-hand dashed line denoted “initially crooked”, and strip S2 similar to the right-hand dashed line denoted “initially very crooked”.

*In fact, it was seen in Figure 5.9 that the curve for lobe 2 falls significantly below that for the representative FE model, so perhaps in this particular case lobe 2 has not grown quite how it would normally be expected to.
Figure 5.17: Non-dimensional plots of lobe lengths and amplitudes for various silicone rubber strips. These curves represent the experimental results presented in Figures 5.9 and 5.16. The dimensionless groups used are the same for the parameter study of Section 5.3.

If, for strip U1, the imperfection is not effective in preventing a large unstable jump, this means that a reasonable approximation to the critical base shortening could perhaps be obtained from Equation (2.9), Kerr’s upper limit for the buckling load of a perfectly straight beam. Inserting the appropriate values gives $P = 1.88 \, N$, which, to get the required compressive strain in the compressible base, requires an end shortening of $30.7 \, mm$. The observed base shortenings which produce buckling in the unstiffened strip are very close to this upper limit.
Looking now to the plot of $L$ against $F$, again there is very good agreement between strips S1 and S2 with respect to the lengths of the first two lobes, all levelling out to $L \approx 1.6$. The non-dimensional lobe lengths for strip U1 lie above, at $L \approx 1.8$. As with most of the other differences between these curves, it is difficult to attribute them to a single factor. Since the axial stiffness is a factor of 18 different to the other strips, perhaps there is, after all, a very small dependence of the lobe length on $EA$.

5.5 Experiments with Curved Strips

As well as experiments with the silicone rubber strips laid straight, a small amount of physical modelling was performed with the strips laid on a curve of constant radius. This represents a lay-away curve used to attempt to control a lateral displacement of an axially compressed pipeline (Svanø, Bjærum & Stokkeland 1992). The purpose of these tests was simply to report on the observed differences between buckle evolution on straight and curved rubber strips, and to observe whether the buckle length is rather insensitive to the radius of the curve, as stated by Hobbs (1984).

Long plywood templates were cut to nominal radii between 2.5 m and 4 m, and smoothed to take out any major imperfections. The rubber strips were laid up against these templates. Obviously, the uniaxial compression of the base no longer applies a uniform axial compression to the strip, but in the central region of interest, the angle between the axis of the strip and the axis of the base is small enough for this non-uniformity to be neglected, especially at smaller base end deflections where the effect on the centre of the strip by the outer parts of the strip is likely to be small.

Lateral buckling of curved railway tracks was analysed by Martinet (1936), using the same approach as for straight tracks (Section 2.3.1), but only for a mode 1-type buckle, since the analysis becomes much more complicated for higher-order modes. The following general trends were noted by Martinet as the radius of curvature of the lay-away curve decreases.

- The critical buckling temperature rise decreases.
- The buckle length increases slightly.
- The buckle amplitude decreases.
- The curvature at the centre of the buckle decreases.
- The stress at the centre of the buckle decreases.
Figure 5.18: Buckle evolution for strip S2 on a curve of radius a) $R = \infty$ (i.e. straight), b) $R = 4 \text{ m}$ and c) $R = 2.5 \text{ m}$. An initial imperfection is present. The base shortening $\delta$ is $0, 1, 2, 3, 4, 5 \text{ mm}$ in a) and $0, 1.5, 2, 3, 4, 5 \text{ mm}$ in b) and c).
Figure 5.18 show the experimentally observed evolution of a buckle at curve radii of $R = \infty$, $R = 4\,\text{m}$ and $R = 2.5\,\text{m}$, using strip S2. Only the central region of interest is presented. The initial imperfection was somewhat larger for the straight strip (this is the same data set used for the case study of Section 5.2) and the lobes grew steadily, without a sudden jump. The buckles on the curved strip did form suddenly, at a base end shortening of $\delta = 1.5\,\text{mm}$ in both cases. The buckle shapes are shown relative to the original curved layout, with the lateral displacement taken radially from that initial curve. The first point to notice is that at a base end shortening of $5\,\text{mm}$ (the outer curve), the amplitude of the central lobe is very similar in each case, but the lobe length, measured between points of zero net radial displacement, does indeed get longer as the curve radius decreases. The central lobe lengths are $0.163\,\text{m}$, $0.195\,\text{m}$ and $0.211\,\text{m}$ for curve radii $\infty$, $4\,\text{m}$ and $2.5\,\text{m}$ respectively.

An important difference between the buckle shape at different radii is the behaviour of the side lobes. For the straight strip, the first pair of side lobes for $\delta = 5\,\text{mm}$ have reached about half the height of the main lobe; but as the curve radius decreases the height of the first side lobe decreases significantly. In fact, when $R = 2.5\,\text{m}$, the side lobes only encroach on the area within the original curve by around $1\,\text{mm}$. At the same time, there is an overall movement of the entire strip in the outwards radial direction; this can be observed at the ends of the plots. At a base end shortening of $5\,\text{mm}$, the outward movement is almost $1\,\text{mm}$ for the strip with the largest initial curvature. This suggests that, ideally, the analysis of buckled lobes should be performed with respect to a curved axis which moves with the overall radial movement of the strip.

The change in curvature due to bending at the centre of the central lobe for $\delta = 5\,\text{mm}$ does fall as the curve radius decreases, from $4.9\,\text{m}^{-1}$ for the straight strip to $3.9\,\text{m}^{-1}$ for the $2.5\,\text{m}$ radius.

Further experiments were performed with strip U1, as detailed in Appendix A, which has a larger cross-section than strip S2 but is unstiffened. One advantage of this strip is that the base end displacement is greater before buckling occurs, so that the difference in critical end shortening for different radii is magnified.

In a series of 10 experiments, 5 with $R = 4\,\text{m}$ and 5 with $R = 2.5\,\text{m}$, and all with the same nominal initial imperfection amplitude and length, there was very good repeatability in determining the lengths of the central lobes when $\delta = 20\,\text{mm}$. Table 5.3 shows these values, along with the predicted values from Martinet (Equations (2.4)–(2.5), modified for a curved beam). It is clear that the agreement between the experimental and theoretical lengths and end shortenings are not particularly good; indeed, even with an initial imperfection, the silicone rubber strips buckle at a higher-than-predicted load, and adopt a shape in which the lobe length is significantly shorter. However, as mentioned previously, Martinet’s equations are based on a mode 1 shape at initial buckling,
and clearly the buckles seen in the physical model are at least mode 3 or even mode 5; so like is not being compared with like. Figure 6 of Hobbs (1984) shows that the critical length of a mode 1 buckle can be much larger than a mode 3 buckle, by around 50% for his particular example of a straight pipeline.

Finally, two examples are presented for the buckling on a curve of strip $S_1$, the axially-stiffened version of strip $U_1$. The buckle evolution for curves of radius $R = 4 \, m$ and $R = 2.5 \, m$ are compared with finite-element predictions using model TLM5. Again, the same pattern of general behaviour can be seen. For the experimental data, at $\delta = 10 \, mm$, the main lobe lengths, measured between points of zero radial displacement, are both of a similar order of length as the equivalent unstiffened strip, around $25 \, mm$ longer in each case ($0.327 \, m$ for $R = 4 \, m$, $0.377 \, m$ for $R = 2.5 \, m$). The main amplitude is larger for the higher-radius curve ($0.021 \, m$) than for the lower-radius curve ($0.023 \, m$). Again, the net outward radial displacement can be seen clearly for both cases, as can the much smaller side lobes for the smaller radius curve.

The finite-element modelling produces quite similar buckle shapes. The difference in the lengths of the main lobes is almost the same ($0.353 \, m$ for $R = 4 \, m$ compared to $0.391 \, m$ for $R = 2.5 \, m$), and the lobe amplitudes are higher than those from the physical model, by $2 \, mm$ in each case. These differences of up to 10% are not too surprising in light of the many factors previously discussed, such as the reliability of friction measurements, which can each contribute in a small way to differences between physical and numerical analyses.

In summary, there are differences in the evolution of the buckle shapes as the radius of the curve is changed, and there is probably a case for including the

<table>
<thead>
<tr>
<th>$R , [m]$</th>
<th>Experimental</th>
<th>Martinet</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$\delta , [mm]$</td>
<td>$\lambda , [m]$</td>
</tr>
<tr>
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<td>0.292</td>
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<tr>
<td>4</td>
<td>0.292</td>
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<tr>
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<tr>
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<td>0.361</td>
<td>9.9</td>
</tr>
</tbody>
</table>

Table 5.3: Experimental and theoretical lobe lengths for strip $U_1$ laid on a curve.
curve radius $R$ into an additional parameter study to find out how strong the effect of radius is compared to other geometric and material parameters.

Figure 5.19: Comparison of experimental and FE buckle evolution for strip S1 on a curve of radius $R = 4 \, m$ and $R = 2.5 \, m$. A small initial imperfection is present. The base shortening $\delta$ is $0 : 1 : 10 \, mm$ in each case.
5.6 Conclusions

The computations using results from the physical model under simple preliminary tests were related to simple theory for a beam on a frictional foundation, and were found to correlate quite well with the theoretical results, despite their implicit simplifications of ideal Coulomb friction and small deflections. This provided confidence that the silicone rubber strip on expanded polystyrene was behaving as expected.

A combination of physical and numerical models has been used in a case study to predict the post-buckling behaviour of a submarine pipeline, and these predictions have been compared to survey data on the actual geometry and operating conditions of the pipeline. Although the non-dimensional results from the finite-element computation for the pipeline offer a good agreement with the non-dimensional results from the survey data, there is much merit in using the small-scale physical model as a means of visualising the buckling and post-buckling behaviour of a pipeline. It is certainly much faster to carry out repeated experimental assays to determine the general effect of different initial conditions, and the lengths and amplitudes of the various buckle lobes can be measured directly from the silicone rubber strip. The finite-element model was validated using data from the physical model, thus providing a clear link between the small-scale model and the pipeline survey data.

The finite-element parameter study of Chapter 4 was extended to involve geometric and material properties which have a significant effect on the far-post-buckling behaviour of a beam on a frictional foundation. One finding of this work was the phenomenon of lobe extinction, characterized by a stage in the loading history where a particular lobe becomes practically unaffected by what is happening in the rest of the beam. The point of extinction in the loading process was found to depend on the key parameters $\mu_0w$, $\mu w$ and $EI$. The extinct lobe amplitudes and lengths were also found to depend on the same three key parameters. Hence, the dimensionless groups $\mathcal{F}$, $\mathcal{A}$ and $\mathcal{L}$ were formed from these parameters and the free end displacement, lobe amplitude and lobe length respectively to allow different cases to be directly compared. Common curves, spanning a range of cases with different geometric and material properties, were found for the post-buckled evolution of both the dimensionless lobe amplitude and length, and also for the dimensionless central axial load in lobe 1, $\mathcal{P}$.

Further experiments were performed with the compressible base model, using straight silicone rubber strips with different cross-sections and stiffnesses. The evolution of buckles on these strips was plotted in terms of the dimensionless groups $\mathcal{F}$, $\mathcal{A}$ and $\mathcal{L}$, and, at least for post-buckling behaviour, were found to approximately lie on the same common curves.
Finally, the effect of an initial large-radius curve on the buckling behaviour of
the silicone rubber strip was examined. As the radius of curvature of the lay-away
curve is decreased, significant differences from the behaviour of the straight strip
become apparent:

- The length of the main buckle lobe increases; the lobe amplitude is almost
  the same for the three radii examined, and hence the additional curvature
  in the strip due to bending while buckling is reduced.
- There is a tendency for the main curve to move radially outwards as a whole
  prior to buckling.
- The side lobes are much less pronounced; the initial curvature means that
  it easier for the buckle lobes to grow on the outside of the curve.

Overall, the good correspondence, at least to a first order, between the phys­
ical model, the numerical model and the survey data has been encouraging. The
universal dimensionless plots of Figure 5.14 should be useful as a guide for ex­
amining a buckled pipeline configuration in relation to its extinct ultimate state.
Chapter 6

Pipelines on a Slope - A Case Study

Although this chapter does not relate to the modelling of lateral buckling of pipelines on the seabed, it does examine the closely-related problem of the stability of a pipeline built up a slope.

6.1 Introduction to the Problem

It is increasingly common for land reserves of oil and gas to be found in remote locations, with the land between these reserves and the coastal facilities which serve them very difficult to cross. Perhaps the most difficult terrain to cross is mountainous. Although pipeline routes can be planned to mitigate the worst effects of mountain ranges, steep slopes will almost invariably be encountered at some point. Such slopes can be in excess of $30^\circ$ from horizontal and are generally traversed without difficulty during the construction stage; but occasionally a number of factors can contribute to stability problems of a pipeline on a mountainside.

When a pipeline is constructed up a slope, it is common practice to anchor a length of pipe securely near the base of the slope, and then add further lengths of pipe up the slope, supporting the line off the ground on temporary supports while the lengths of pipe are being joined together, usually by welding. It is clear that if the component of weight of the pipe is greater than the limiting value of the friction force between the pipe coating and the supports acting up the slope, then the pipeline will go into compression (more highly loaded at the base), and that this can eventually lead to mechanical instability of the line. If on the other hand the pipeline is anchored at the top of the slope and constructed down, then it will go into tension and the stability problem is no longer present. Instability
is only a problem during the construction phase, since the pipelines are generally trenched and buried along almost their entire length during operation.

The analysis of just such a mechanical instability, during the construction of a pipeline up a hill*, formed part of the industrial component of the CASE award which funded the work for this dissertation. The study involved the design and construction of a physical model to provide both qualitative and quantitative information on the onset of unstable behaviour of a slender beam on a slope. The results from simple numerical and analytical models were compared with the results from the physical model. The pipeline data could then be used in the numerical model, and hence possible causes of instability could be identified.

6.2 Key Factors for Instability

6.2.1 Pipeline Route

Because of the rugged terrain in mountainous areas, pipelines do not take a direct route up and down slopes, but weave about to avoid natural and man-made obstacles and to take an easier path through a particularly difficult region. This gives rise to both horizontal and vertical curvatures at various points along the pipeline, possibly coinciding in some places. The profile of the particular failed pipeline is shown in Figure 6.1.

At the point $x \approx 220 \text{ m}$ an overbend (i.e. the slope becoming more steep downhill) coincides with a small horizontal curve. If there is a compressive force in the pipe at a point where there is a curve, there will be a resulting force tending to try to move the pipe in the direction outwards from the centre of curvature. For horizontal curves this results in a lateral load on the supports; for an overbend the compressive thrust produces a force normal to the support, directed outwards. Therefore the net normal support reaction is reduced. If simple Coulomb friction is assumed, this could result in a reduced limiting frictional force between the pipe and the support. Conversely, at a sagbend (i.e. the slope becoming less steep downhill) the net normal support reaction is increased.

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*The exact details of the pipeline are not for general release; consequently no reference is included here.
6.2.2 Friction Between the Pipeline and the Support

Although the presence of friction is an integral part of the construction (since a higher friction force reduces the compressive load, other things being equal), it is probably the least understood factor. One of the main difficulties here is that a steel pipeline will invariably have some kind of polymeric protective coating, for example medium density polyethylene (MDPE), which has complicated frictional properties characterised by

\[ \mu = \frac{\tau_0}{p} + \alpha, \]  

(6.1)

where \( \mu \) is the coefficient of friction, \( p \) is the contact pressure, and \( \tau_0 \) and \( \alpha \) are material constants (Briscoe 1981). The situation is further complicated by temperature and sliding velocity effects, whether a thick or thin film of the polymer is being considered, and the presence of contaminants.

Equation (6.1) suggests one possible reason why the coefficient of friction might be badly overestimated. Field tests, carried out as simply as possible, would normally involve finding the angle of sliding for a small sample of the support material on the polymer. But during the construction process, the situation
is reversed as the heavy pipe pushes the polymer against the support. Obviously
the contact loads in these two cases are several orders of magnitude apart, and
although the contact areas will also be quite different, the contact pressure for
the test situation is less than under the construction loading. Since $p$ is in the
denominator in Equation (6.1), an underestimate of the true contact pressure
leads to an overestimate in $\mu$.

Returning to the pipeline in this study, when a length of pipe was tilted until
one of the wooden support planks began to slide down it, $\mu$ was measured from
the angle of sliding as 0.4. When a test was later carried out for two lengths of
pipe resting on wooden supports, with one support built up a little at a time until
the pipe began to slide, $\mu$ was found to be around 0.2. If the contact surfaces
were wetted, this value was reduced by roughly 40% to 0.13.

### 6.2.3 Method of Construction

The way in which the pipeline is constructed may influence its overall sta-
bility. As mentioned in Section 6.1, this may be simply to the direction of
construction, over which there may not be any control. In the present case, the
construction programme dictated that the particular section of pipeline under
consideration must be constructed uphill, thereby giving rise to a build-up of
compressive loads in the pipeline. The risk of an unstable failure can be cut by
reducing the distance between fully restrained anchor points; but this increases
both construction time and cost.

As well as the general construction method, specific parts of the construction
may also prove to be factors in a collapse in their own right. A good example of
this is the temporary supports on which the pipeline is built. A typical support
might be a tower of crossed thick wooden planks. The resistance of the support
to movement of the pipe depends on its rigidity. If the planks are simply piled
one on top of the other, rather than being securely fastened together, then the
support will be lacking in rigidity. From full-scale experiments, the slope at which
the short length of pipe is just on the point of sliding is reduced, in this specific
case from 11.5° to 10°. Hence the friction between the pipe coating and the
support may never be fully mobilized before there is significant movement of the
pipeline.

The welding strategy is also important in determining the loading on the
pipeline. The lengths of pipe are initially connected with only one or two passes
of the welding machine. Welding is completed in staggered stages behind the main
construction front. This leads to a series of incomplete welds extending many pipe
lengths from the free end, and an associated magnification of the stresses in the
joints from the build-up of the compressive load. This can heighten the risk of
an unstable failure of the pipeline in the under-strength region.
6.2.4 Temperature Effects

In common with many other large structures, small changes in ambient temperature can have a significant effect on the behaviour of an exposed pipeline. Because of the long length of the pipeline, a small thermal strain could produce a large unconstrained thermal expansion, but this expansion is opposed by friction between the pipe and the supports. The friction force is therefore now acting down the slope; it is this reversal in the direction of the friction force at the supports which can lead to problems, since it increases the compression force acting downhill of a support where this friction force reversal has occurred.

6.3 A Physical Model

A physical model was proposed to study the stability of a long beam on a slope. It was designed to be as simple as possible, and not to model any real situation in particular, but rather just to show the general behaviour of a beam on a slope. The experimental setup is shown in Figure 6.2.

6.3.1 The Slope

The Dexion frame was constructed to allow a Dexion flat bed to be raised to an arbitrary slope between 5° and 45°. The bed (overall length 2.4 m) could be raised continuously through this range and locked in place at intervals of approximately 0.5°. The slope angle was measured from a simple inclinometer hanging beneath the bed. The beam was the axially-stiffened silicone rubber strip S1, as described in Appendix A, which was available from the previous lateral buckling experiments. The slope profile was formed from expanded polystyrene sheet, which is easily cut and shaped on a bandsaw, and can be smoothed with emery paper. Although discrete supports are shown in the figure, experiments could also be carried out with the beam resting directly on the polystyrene, or on another material pinned to the polystyrene. A heavy steel end block prevented the lower end of the strip from moving down the slope. The strip was laid as straight as possible down the slope, although there were inevitably small lateral deviations. The size and position of any such deviations would dictate whether they had a significant effect on the outcome of the experiment.

A problem with using the silicone rubber directly in contact with the polystyrene has been mentioned previously in Section 3.4.2, namely a velocity dependence of the friction coefficient. Because of this, and also because of a wish to replicate in some way the discrete nature of the pipeline supports, the silicone rubber strip was supported above the polystyrene surface at 100 mm intervals, as described below. The bend region was formed by two equal circular arcs,
Figure 6.2: Elevation of experimental setup for slope stability experiments. The height of the triple layer of polystyrene has been exaggerated.

Giving a reverse-S shape. Experiments were performed for nominal arc radii of $R_v = 0.40 \, \text{m}$ and $R_v = 0.66 \, \text{m}$, although there were local variations in curvature due to the “saw and smooth” method of manufacture; and, of course, the use of discrete supports meant that the profile of the strip did not follow precisely the profile of the polystyrene base.

6.3.2 The Supports

Three different support systems were used in order to give a range of friction coefficients. They are referred to as “high-“, “medium-“ and “low-friction“ for ease of reference, even though the friction coefficients may all be considered to be low values. The three types of support are shown in Figure 6.3.

The high-friction support is a PTFE pad on top of a square wooden block (side 10 mm); the medium-friction support is a stack of two PTFE pads; and the low-friction support is a PTFE pad on top of a 20 mm length of drinking straw,
Figure 6.3: Support systems for beam on slope a) "high" friction, b) "medium" friction, c) "low" friction.

acting as a roller. The rollers were made as light as possible so that movement of the rollers due to their own weight acting down the slope did not contribute significantly to the overall movement of strip-on-support system. It should be noted that slip always occurred first between the PTFE pad and the support, and never between the PTFE pad and the silicone rubber strip. The coefficients of friction for these support systems were measured using the method depicted in Figure 6.4.

A short length of polystyrene sheet was restrained from sliding at one end by a heavy block, and the other end was rested on a pantographic trestle so that the slope angle, measured by a simple inclinometer, could be steadily increased. A 300 mm length of silicone rubber strip was rested on four supports set at 100 mm intervals. In the case of the high- and medium-friction supports, drawing pins were stuck into the polystyrene on the downhill side to prevent the wooden blocks or lower PTFE pads from sliding or rolling independently of the rubber strip. Obviously, for the low-friction support case, where the light roller should be rolling against the PTFE pad, this condition was not applied. Each support system was tested a number of times, and the angle at which the strip first started to move was noted, with the average angle being converted to a coefficient of friction. The values obtained are summarised in Table 6.1. In the high- and medium-friction cases, the axial and lateral coefficients of friction were equal. For the low-friction case, the values are different; the coefficient is low in the

*This is done in full appreciation of the difficulty in obtaining a single "coefficient of friction" for frictional systems involving polymers.
rolling direction, but laterally, when the PTFE pad slides across the roller, the coefficient of friction is higher.

![Diagram of experiment setup]

**Figure 6.4:** Method for measuring friction between strip and supports.

<table>
<thead>
<tr>
<th></th>
<th>high friction PTFE/wood</th>
<th>medium friction PTFE/PTFE</th>
<th>low friction PTFE/roller</th>
</tr>
</thead>
<tbody>
<tr>
<td>axial coefficient</td>
<td>0.28</td>
<td>0.19</td>
<td>0.05</td>
</tr>
<tr>
<td>lateral coefficient</td>
<td>0.28</td>
<td>0.19</td>
<td>0.19</td>
</tr>
</tbody>
</table>

**Table 6.1:** Coefficients of friction for different support systems.

### 6.3.3 Isotropic Low Friction

For the low-friction support case, it would be preferable to have isotropic friction in order to provide a more direct comparison with the isotropic high- and medium-friction support systems.

Two methods were attempted to obtain an isotropic low-friction contact between the strip and the slope. The first was to produce an ice slope. The polystyrene was placed in a coldroom (-30°C) and sprayed with water to build up layers of ice over a short period of time. However, because the polystyrene is a poor conductor of heat, even leaving it in the coldroom overnight only cooled the surface, and consequently the rate of melting was quite high once the slope was moved into the ambient-temperature laboratory for experiments. The silicone rubber strip soon melted a rectangular track into the ice (thus providing lateral
restraint and removing the isotropy of the ice resistance), and very quickly melted through to the polystyrene at the overbend. This produced a large local increase in the friction force. There also appeared to be a suction effect between the melt water and the rubber; indeed, the overall performance of the ice experiment was comparable with the medium friction supports, but much less predictable.

In another attempt to produce an isotropic low-friction surface, the polystyrene was covered in a layer of smooth polyester film, coated with a film of undiluted detergent. The tilt test described above showed the coefficient of friction to be approximately 0.03 (a 2° slope), and so the initial setup of the strip was critical. Slight horizontal curves grow rapidly with little increase in slope.

Figure 6.5: Collapse angles and locations for rubber strip on detergent ($R_v = 0.40 \text{ m}$) - plan view.

Figure 6.5 shows a plan view of the higher-curvature profile, with arrows pointing to the approximate position where the strip first appeared to move, and annotated with the slope angle at which the movement was seen. There does not appear to be any pattern to these collapses (except for the fact that they mostly occur either in the region of high vertical curvature, or on the lower slope where the compressive force is largest). Most of these collapses are almost certainly due to imperfections in the initial layout. The predicted angle of collapse for this level of friction is 27°. This approach was not pursued any further.

6.4  A Numerical Model

In addition to the physical experiments, a two-dimensional (2-D) numerical analysis was performed for the problem of pipeline stability on a slope. The effect of temperature was neglected, although it could easily have been incorporated if necessary.
The analysis was performed on an *Excel* spreadsheet with the facility to perform iterative calculations. The main inputs to the model consisted of the lengths of individual pipes in the pipeline as constructed, the change in vertical angle over pipe length and the coefficient of friction between pipe and support. The first two items were obtained from the bending schedule from the pipeline contractor. For the analysis of the physical model, these inputs were obtained from the 2-D coordinates of the support points when the bed was laid horizontal. An overall angle of rotation, as measured by the inclinometer in the physical model, could then be applied to the numerical model. An input/output diagram showing the iterative solution process is presented in Figure 6.6.

![Input/output diagram of physical quantities for spreadsheet iterative solution.](image)

The length of the pipe and its associated vertical bend allow the normal force due to weight and the local vertical curvature to be calculated. Summing from the free end, a first-pass cumulative compressive load can also be obtained. From this, the normal force due to compression in a curve, and hence the net normal force can be found at each pipe support; and using the coefficient of friction, the friction force is found. The friction force at a particular support will affect the value of the compressive load further down the pipeline, and so the forces must be calculated iteratively from the free end. One iteration per support is necessary in order to arrive at a consistent solution. Care was taken to check for the condition where the compressive load at a point was lower than the friction force. In this case, no compressive load is transferred below this point.
6.5 Results and Analysis

The slope angle for collapse of the physical model was measured by slowly and steadily raising the flat bed until the strip became unstable and fell off its supports. The collapse angle was recorded from the inclinometer. Note that this is the angle of inclination of the flat bed. The slope angle $\theta$, defined in Figure 6.7 as the angle from the top of the slope to the point of interest on the overbend, is $\tan^{-1}(75/1000) \approx 4^\circ$ lower than this, due to the $75 \, mm$ height difference between the two halves of the slope when the flat bed is horizontal. The experiments were repeated six times and an average slope calculated. Some of these experiments were recorded on a video camera so that the actual mechanism of failure could be seen in slow motion. The video showed that in almost all cases the strip quivered above a support in the overbend region immediately prior to collapse. The collapse angle is compared against the result of the 2-D numerical analysis in Table 6.2; these values include the $4^\circ$ correction.

<p>| overbend | support scheme | coefficient of friction | slope at failure, $\theta$ | difference |</p>
<table>
<thead>
<tr>
<th>radius $R_v$ [m]</th>
<th></th>
<th></th>
<th>experimental</th>
<th>numerical</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>a</td>
<td>0.28</td>
<td>35°</td>
<td>35°</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.19</td>
<td>31°</td>
<td>31°</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>0.05 (0.19)</td>
<td>27°</td>
<td>25°</td>
</tr>
<tr>
<td>0.66</td>
<td>a</td>
<td>0.28</td>
<td>39°</td>
<td>44°</td>
</tr>
<tr>
<td></td>
<td>b</td>
<td>0.19</td>
<td>33°</td>
<td>40°</td>
</tr>
<tr>
<td></td>
<td>c</td>
<td>0.05 (0.19)</td>
<td>32°</td>
<td>34°</td>
</tr>
</tbody>
</table>

Table 6.2: Slope at failure for various slope profiles and friction conditions.

The support scheme labels refer to Figure 6.3.

For the higher-curvature profile ($R_v = 0.40 \, m$), the agreement between the physical and numerical models is excellent. The low-friction result is not quite as close as the other two, but this may be due to the anisotropy of the support system. The results for the lower-curvature profile ($R_v = 0.66 \, m$) are significantly different from the numerical predictions. This suggests that there are different factors which are important in the two cases. With the higher-curvature profile, the effect of the vertical curvature dominates any other factors such as horizontal imperfections, and so the 2-D numerical model is an excellent representation of the physical model in this case. For the lower-curvature profile, the compressive force needs to be larger to reach the same net normal load at the support on the overbend. For a comparable horizontal imperfection, the horizontal thrust in the latter case would be larger, possibly exceeding the limiting lateral friction force. Had time permitted, an intermediate radius would have been tested to help determine where the change-over between these two phenomena lies.
For the real pipeline, the 2-D numerical model predicts that the net normal force exerted by the pipe on the support is as low as 640 N per metre of pipe. Making the simplifying assumption that this would be reduced to zero by applying an additional compressive force due to constrained temperature rise, such that $(EA_0\Delta T/k) = 640 N$, the temperature rise required would only be around $1^\circ C$. Obviously, the temperature rise would not be fully constrained (the pipe can expand towards the free end, reversing the friction direction as it happens), but this gives an indication of how close the pipeline is to becoming unstable.

6.5.1 Simplified Analysis

Figure 6.7 defines some quantities for a simplified analysis of a beam on a slope. The angle $\psi(s)$ is the local angle to the horizontal at a pipe contour length $s$, measured from the top of the pipe. The angle $\theta(B)$ is the angle from the horizontal of a line joining the top of the pipe to the point of interest, B, on the slope. Notice that, in order to keep the following calculations as simple as possible, this line has been drawn tangentially to the overbend, assuming that this will be the point of first uplift of the pipe from its supports. The actual point of uplift might occur slightly away from this point, with a local slope $\psi \neq \theta$, but it is an acceptable approximation here. $R_v$ is the radius of curvature in the overbend region and length $l$ is the straight line distance between A and B.

![Figure 6.7: Simple 2-D analysis of beam on a slope.](image-url)
At point B on the overbend, the compressive force is given by

$$P = \int_A^B w(\sin \psi - \mu \cos \psi) ds = wl(\sin \theta - \mu \cos \theta), \quad (6.2)$$

and the condition for uplift (zero normal load) at B is given, approximately, by

$$\frac{P}{R_v} - w \cos \theta = 0. \quad (6.3)$$

Hence the critical radius of curvature at a length \( l \) down the slope is

$$R_v = l(\tan \theta - \mu). \quad (6.4)$$

From Figure 6.7, it is easy to show that

$$h_1 = l(\sin \theta - \mu \cos \theta) = l \cos \theta(\tan \theta - \mu) \quad (6.5)$$

and so the critical overbend radius can be found by considering the vertical height of the point below the free end (\( R_v = h_1 / \cos \theta \)). If \( \theta \) is not too large, then only \( h_1 \) need be measured. If the beam was resting on frictionless supports, the critical radius would be the whole of this height; the presence of friction reduces the value of \( h_1 \) and hence \( R_v \).

If the beam were to experience a temperature rise sufficient to fully mobilize the friction force between the beam and supports due to the extension of the beam towards the free end, at least above the point under consideration, then the situation becomes very different. The friction forces act down the slope, and so the minus sign in Equations (6.2) and (6.4) must be replaced by a plus sign; and so Equation (6.5) is replaced by

$$h_2 = l \cos \theta(\tan \theta + \mu). \quad (6.6)$$

The critical overbend radius has therefore increased by \( 2l \mu \cos \theta \) because of the thermal expansion.

For the model slope with, for example, \( R_v = 0.40 \) m and \( \mu = 0.19 \), the value of \( h_1 / \cos \theta \) turns out to be 0.41 m.

For the case-study pipeline, it is found from the profile that \( h_1 / \cos \theta = 44 \) m; but \( h_2 / \cos \theta = 135 \) m, which corresponds very well with the actual maximum overbend radius of approximately 140 m. This suggests that heating of the pipeline is a feasible initiating factor for unstable collapse of a pipeline built to the particular profile of Figure 6.1. It is unfortunate that the small horizontal curve and the limit of completed welds coincided at this point, and probably also contributed in some way to the collapse.
6.6 Conclusions

The 2-D numerical model gave very good agreement with the 3-D physical model for high vertical curvature of the overbend, but it was not so good for lower vertical curvature. This is probably due to the heightened effect of horizontal imperfections for the lower-curvature tests, which the numerical model does not take into account. The numerical model indicated that the real pipeline was still stable as constructed, but again, no account was taken of horizontal curvature or below-strength welds.

The simplified model, by which the criticality of a vertical curve may be checked simply from the difference in elevation between the curve and the top of the pipe, appears to perform well despite the assumptions made, and could be a useful tool for planning the route of a pipeline in mountainous regions.

For actually designing and constructing a pipeline up a slope, the following key points emerge:

- It is important to obtain a reliable value for the coefficient of friction for a pipe sliding on a temporary support of any kind, and especially for the built-up wooden-trestle type used here, rather than for a small piece of support material sliding on a pipe.

- The dangerous coincidence of a vertical overbend and a horizontal curve should be avoided if at all possible.

- A small temperature rise can have a very large effect on how close to becoming unstable an exposed pipeline is.
Chapter 7

Summary and Conclusions

The work presented in this dissertation has fallen broadly into two related areas. The first, the use of a small-scale physical model to predict the behaviour of a submarine pipeline lying directly on the seabed, has been shown to provide a useful way of visualizing the lateral buckling response. The second, the study of the far-post-buckling phenomenon of lobe extinction, has revealed an unexpected and simple relationship between the thermal load and the evolution of the buckle shape over a range of geometric and material parameters.

7.1 Physical Modelling

7.1.1 Present Study

A novel method for modelling the behaviour of a pipeline lying on the seabed, subjected to thermal loading and internal pressurization, has been developed. The pipeline has been modelled by a long rectangular silicone rubber strip, axially stiffened by a fine steel wire. The seabed has been modelled by a base of expanded polystyrene, which was uniaxially compressed beneath the rubber strip to provide an equivalent pseudo-thermal loading on the strip.

Coordinate data, obtained from an x-y digitiser, were compared with the results of finite-element analysis of both a compressible base model and a pure thermal loading model. The good agreement between all three sets of results acts not only as a validation of the compressible base finite-element model against the physical model, but also demonstrates that the pseudo-thermal loading of the uniaxially compressible base is an excellent representation of true thermal loading.

A case study, performed for a North Sea pipeline which had undergone lateral buckling, brought together actual survey data, the physical model and the finite-
element model. The main emphasis of this work was on the evolution of buckles once they had initiated, rather than the initiation process itself — which has tended to be the focus of railway-track buckling analyses. Dimensional analysis allowed dimensionless groups to be formed for the important buckle parameters such as amplitude, length and axial force, so that the full-scale pipeline and small-scale rubber strip could be directly compared. The results of this comparison are encouraging, despite the several large assumptions made about the conditions under which the actual pipeline buckled.

Physical modelling of the related problem of pipeline collapse on a slope has shown that performing simple experiments can provide a clue to the possible cause of a collapse, and that a very good estimate of the collapse conditions may be obtained from very simple theory.

7.1.2 Possible Model Modifications

In its current configuration, the compressible base model is not ideally suited to studying the onset of buckling in detail; in this dissertation it has been used mostly to look at post-buckling behaviour. This is because, for stiffened silicone rubber strips which have similar ratios of axial to bending stiffness as steel pipelines, the base shortening is only about 1.5 mm over a 2.4 m length (0.06% strain) when buckling occurs. In order to investigate buckle initiation in detail, a sufficient number of data sets are necessary around the critical base shortening; these would necessarily be taken at small changes of base shortening, say 0.2 mm, and the effect of the error in the digitiser readings between successive data sets would become much more significant, even with data filtering.

A model test-bed with a longer base would increase the end shortening for an equivalent buckling strain, and should allow greater resolution of the exact shortening which produces buckling. However, the problem of ensuring that the strain is evenly distributed along the length of the base still remains. A modification to the base arrangement, with certain points along the base forced to move in proportion to the moving end, is proposed. This could perhaps be achieved using a screw drive with special proportional-pitch threads at the various points to control the axial movement of the expanded polystyrene base.

Another advantage with a longer base is that a longer strip could be used. Notwithstanding the increased difficulty of manufacturing such a strip, this would bring the benefit of reducing the effect of the artificially-anchored ends on the buckle at the centre of the strip, and would also allow the study of independent buckle groups.

It would also be reasonably straightforward to replace the handle of the chain drive by an electric motor, perhaps with computer control, to enable a series of
cyclic loading tests to be performed, and to see how the "extinct" buckle lobes are affected by different load histories.

7.2 Parameter Studies

7.2.1 Present Study

The parameter study, by means of finite-element computations, has demonstrated the sensitivity of the behaviour of the buckling strip to changes in the different geometric and material parameters of the model system. Many parameters, such as the initial imperfection size and stress and the axial stiffness of the strip, were found to have an effect on the load at buckling, and also the initial buckled configuration of the strip. However, these same parameters were found mostly to have only a very weak effect on the far-post-buckled evolution of the buckle shape. The overall length of the beam did have a significant effect on the buckle evolution; but a transformation in the loading parameter from temperature rise to free end displacement was able to account for this difference.

The three parameters found to have a significant effect on the post-buckled shape of the buckle were the bending stiffness of the strip and the axial and lateral friction forces per unit length between the strip and the base. Using the concept of dimensional analysis, it was possible to use these parameters to form dimensionless groups for the buckle lobe amplitudes and lengths, the central axial compressive load, and the free end displacement. These dimensionless groups allowed the results from 19 numerical analyses, varying the chosen parameters, to be plotted together on a single set of axes. It was found that, despite a small amount of scatter, the plots for all the different cases tend to fall on the same general set of curves, suggesting that the remaining parameters do indeed have only a small effect on the buckling behaviour. The parameter study showed that as the loading progresses, buckle lobes reach a state of extinction (i.e. there is no further axial feed-in of beam material into such a lobe), and the conditions at extinction for each lobe may be determined from the dimensionless plots of lobe amplitude, length and axial force.

7.2.2 Possible Further Analysis

The present parameter study has neglected the effect of those parameters which have only a small effect on the far-post-buckled behaviour. The inclusion of these parameters would undoubtedly remove some of the scatter observed in reducing the plots from the different numerical analysis cases to a single set of dimensionless curves, but at the cost of increasingly complicated empirical
relationships. Also, there is already some evidence that a few, the cases examined (e.g. those with a low axial friction to lateral friction ratio) seem to deviate from the general trend at a certain point; and so the range of parameter values could be widened to examine this.

Certain aspects of the parameter study would benefit from additional or more detailed analysis. The effect of a lay radius on the strip was shown to have a significant effect on the buckling load, and a larger than expected effect on the buckle lobe length. It would be useful to see if the radius $R$, in non-dimensional form, would still allow a simple empirical relationship between loading evolution and buckle growth.

The emphasis of this dissertation has been on far-post-buckling behaviour, at least as far as the parameter study has been concerned. A similar study could be performed for buckling and near-post-buckling behaviour to determine how the buckle shape and axial load vary within and between these different regimes — something which is much more applicable to the safe operation of buckled pipelines.

However, the idea of extinction is very valuable as a practical concept. It provides simple limits to the dimensions of a buckled lobe. At present these limits are empirical in nature. It would be useful to examine in more detail the relationships between $A$, $L$ and $F$, and also to investigate a theoretical basis to the expression for the extinct lobe load in terms of the Euler load, i.e. $P_e = \frac{a}{3} P_E$. It seems possible that there is a simple explanation — that has not yet been found — for these simple relationships between $A$, $L$ and $F$ for extinct buckle lobes.

The ratio of axial to lateral friction coefficients has a significant effect on the rate of growth of buckle lobes with thermal loading ($F$ is a function of $(\mu_a w/\mu_l w)^2$) and on the extinct amplitude of the buckle lobes ($A$ is a function of $(\mu_a w/\mu_l w)$). It would be worthwhile to engage in a larger study of actual pipelines, preferably using the results of drag tests on lengths of real pipe rather than assumed values for the coefficients of friction.
References


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Raudkivi, A. (1976), Loose boundary hydraulics, Pergamon Press.


Appendix A

Properties of Silicone Rubber Strips

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<th>Strip properties</th>
<th>Strip name</th>
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<th>S1</th>
<th>S2</th>
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<tr>
<td>Strip length $L [m]$</td>
<td></td>
<td>1.8</td>
<td>1.8</td>
<td>2</td>
</tr>
<tr>
<td>Strip width $b [mm]$</td>
<td></td>
<td>9.3</td>
<td>9.3</td>
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</tr>
<tr>
<td>Strip depth $d [mm]$</td>
<td></td>
<td>9.5</td>
<td>9.6</td>
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<tr>
<td>Wire radius $r [mm]$</td>
<td></td>
<td>—</td>
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</tr>
<tr>
<td>Axial stiffness $EA [N]$</td>
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<tr>
<td>Bending stiffness $EI [Nm^2]$</td>
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<td>Weight per unit length $w [Nm^1]$</td>
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<td>1.01</td>
<td>1.04</td>
<td>0.393</td>
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</table>

Table A.1: Properties of silicone rubber strips

This table lists the measured properties for the silicone rubber strips used in the compressible base model.
Appendix B

Finite Element Beam Models

<table>
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<th>Model properties</th>
<th>TLM1</th>
<th>TLM2*</th>
<th>TLM3*</th>
<th>TLM4*</th>
<th>TLM5*</th>
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<tr>
<td>$L$ [m]</td>
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<td>24</td>
<td>5</td>
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<tr>
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<td>800</td>
<td>1000</td>
<td>180</td>
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<tr>
<td>$b$ [mm]</td>
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<td>5.35</td>
<td>5.35</td>
<td>9.6</td>
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<tr>
<td>$d$ [mm]</td>
<td>9.6</td>
<td>6.35</td>
<td>6.35</td>
<td>6.35</td>
<td>9.6</td>
</tr>
<tr>
<td>$r$ [mm]</td>
<td>—</td>
<td>0.0625</td>
<td>0.0625</td>
<td>0.0625</td>
<td>0.0625</td>
</tr>
<tr>
<td>$EA$ [N]</td>
<td>147</td>
<td>2630</td>
<td>2630</td>
<td>2630</td>
<td>2720</td>
</tr>
<tr>
<td>$EI$ [Nm$^2$]</td>
<td>$1.13 \times 10^{-3}$</td>
<td>$1.32 \times 10^{-4}$</td>
<td>$1.32 \times 10^{-4}$</td>
<td>$1.32 \times 10^{-4}$</td>
<td>$1.13 \times 10^{-3}$</td>
</tr>
<tr>
<td>$w$ [Nm$^{-1}$]</td>
<td>1.01</td>
<td>0.393</td>
<td>0.393</td>
<td>0.393</td>
<td>1.04</td>
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</table>

Table B.1: Finite element beam models

The finite element modelling was performed using the ABAQUS/Standard package. Table B.1 details the finite-element models referred to in the main text. The models were based on the nominal (design) properties of the silicone rubber strips in Appendix A. All beams consist of B31H elements modelling the silicone rubber strip. Those beams marked thus (*) also have co-nodal B31H elements modelling the steel stiffening wire. TLM refers to the thermal loading model with a rigid base (Section 4.2.1) and CBM refers to the compressible base model (Section 4.2.2). In those cases where a half-beam is modelled, the length and number of elements (or collinear pairs) listed in the table have been doubled to aid comparison. Only one compressible base model has been used; it is numbered according to its corresponding thermal loading model.
Appendix C

Lateral Buckling Case Study

Data

<table>
<thead>
<tr>
<th>Pipe Properties</th>
<th></th>
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<tbody>
<tr>
<td>Outer diameter</td>
<td>$D$</td>
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<tr>
<td>Wall thickness</td>
<td>$t$</td>
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<tr>
<td>Axial stiffness</td>
<td>$EA$</td>
</tr>
<tr>
<td>Bending stiffness</td>
<td>$EI$</td>
</tr>
<tr>
<td>Submerged weight</td>
<td>$w$</td>
</tr>
<tr>
<td>Axial coefficient of friction</td>
<td>$\mu_a$</td>
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<tr>
<td>Lateral coefficient of friction</td>
<td>$\mu_l$</td>
</tr>
<tr>
<td>Elastic slip</td>
<td>$\xi$</td>
</tr>
<tr>
<td>Axial stiffness</td>
<td>$1.57 \times 10^9 , N$</td>
</tr>
<tr>
<td>Bending stiffness</td>
<td>$4.62 \times 10^6 , Nm^2$</td>
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<tr>
<td>Submerged weight</td>
<td>$457 , Nm^{-1}$</td>
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<tr>
<td>Axial coefficient of friction</td>
<td>$0.7^*$</td>
</tr>
<tr>
<td>Lateral coefficient of friction</td>
<td>$0.7^*$</td>
</tr>
<tr>
<td>Elastic slip</td>
<td>$0.01 , m^*$</td>
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</table>

<table>
<thead>
<tr>
<th>Buckle conditions</th>
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<tbody>
<tr>
<td>Pressure difference</td>
<td>$p$</td>
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<tr>
<td>(at wellhead)</td>
<td>$8.4 \times 10^6 , Nm^{-2}$</td>
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<tr>
<td>Temperature rise</td>
<td>$\Delta T$</td>
</tr>
<tr>
<td>(at wellhead)</td>
<td>$77^\circ C$</td>
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<tr>
<td>Effective temperature rise</td>
<td>$\Delta T_{eff}$</td>
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<td></td>
<td>$83^\circ C$</td>
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<tr>
<td>Buckle amplitude</td>
<td>$a$</td>
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<td></td>
<td>$2, 4, 2 , m$</td>
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<tr>
<td>Buckle length</td>
<td>$\lambda$</td>
</tr>
<tr>
<td></td>
<td>$29, 20, 70 , m$</td>
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</tbody>
</table>


†Assumed value: Taylor & Ben Gan (1986)

Table C.1: Flowline data for case study

This table contains the data for the pipeline used as the case study for lateral buckling, as discussed in Section 5.2.
Appendix D

Data for Far-Post-Buckling FE Analysis

<table>
<thead>
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<th>Analysis No.</th>
<th>$EI$</th>
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</table>

Table D.1: Data for far-post-buckling analysis

This table lists the nominal factors applied to the standard parameters for the finite element model TLM4, listed in Table B.1. These analyses were run to study the effect of changing the listed parameters on the far-post-buckled behaviour of a beam.
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